A Brute Force, Computational Proof to the Big Nineteen Postulate

J. R. Porzio, E. S. Sikes
Department of Tomfoolery and Useless Statistics
Royal Cribbage Society
Worcester, MA 01609
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A brute force method simulation was conducted to determine the existence of the infamous "Big 19" in the card game Cribbage. Every possible hand is counted via computer and the score is recorded. The results and statistics from these measurements are discussed here. All code written for simulation and analysis can be obtained by contacting the authors.

I. INTRODUCTION

Created in the late 17^{th} century by Sir John Suckling as a spin off from the card game "noddy", cribbage is one of the most popular card games in the English speaking world. The game is distinct from many other card games because it features two distinct phases in which players can gain points in different ways. In the first phase, "Pegging", players follow a rotation terminated by the dealer to lay cards in their hand on the table in an attempt to score points. This is the only phase of the game where players interact with each other. In the second, "Counting", phase players count their hands for points. The dealer counts a second hand called the "crib", the game's namesake. Score is kept on a cribbage board using pegs. Throughout the game, the goal of each player or team of players is to reach 121 points first. The aspect of the game that this paper seeks to analyze is the counting phase of the game.

The experiment and arguments discussed in this paper seek to tie together many of the proposed but unverified suppositions that have accumulated over the last several decades of the cribbage tradition observed by the Porzio extended family and friends. Below is described a set of instructions to count every possible cribbage hand and record the multiplicity of each value using the Python Programming Language. An object oriented approach is used. A discussion of the results in the context of what has colloquially become to be known as the *Big Nineteen Postulate*. This paper seeks to definitively confirm or deny the existence of "The Big 19" in cribbage.

II. THEORY

A. State of the Art

It has been postulated that the rules of counting the points in a player's hand only allow for a certain discrete set of attainable scores that each player can earn. That is to say that the player's hand's score is bounded by zero and some upper limit. This idea was proposed by the authors of this paper in the form of the Porzio-Sikes Lemma, which states that the set possible scores, P, a player can have are at lowest 0 points, and at highest 29 points:

$$P = [0, 1, 2, ..., 29] \tag{1}$$

where all members of the set are positive integers with no exclusions. The Porzio-Sikes Lemma is at odds with the generally accepted DSP Theory proposed by J. Doyle, G. Sikes, and J. A. Porzio, also known as the *Big Nineteen Postulate*; which states that no hand in cribbage yields a value of 19 points:

$$P = [0, 1, 2, ..., 18, 20, ..., 29] \tag{2}$$

Neither the Porzio-Sikes Lemma nor DSP Theory attempt to address the issues of multiplicity of the possible point values contained in set P.

B. The Combinatorics of Cribbage

In the absence of a mathematical, closed form proof of the Big 19, which this paper does not hope to achieve, another way to confirm or deny the existence of the Big 19 and bounds of the set P is to actually count all of the possible hands in cribbage. The simplifying assumption that players' hands are non-interacting is made to allow for smaller calculations. Although there is no single player form of cribbage, calculations are performed for every possible hand that can be dealt in cribbage. That is, every possible combination of 5 cards.

It can easily be shown that there are a finite number of hands in cribbage. Each player is dealt a 4 card hand and allowed access to a common card to including in the scoring of their hand. This for each combination of 5 cards from a 52 card deck there are 5 possible hands. One hand for each of the 5 cards being chosen as the common card. Thus, the total number of hands we will need to count, N, is:

$$N = 5\binom{52}{5} = 5\left(\frac{52!}{(52-5)!(5)!}\right) \tag{3}$$

read "5 times 52 choose 5". N has a value of $N = 12,994,800$. While it is doubtful that any one player can be dealt each of these possible hands in their lifetime, it is not outside the realm of possibility that a computer can count each of the just shy of 13 million hands possible in cribbage. This is a trivial amount of computations even on a simple machine.

C. Counting Hands

In order to count all of the possible hands explicitly, the challenge we need to overcome is teaching the computer how to count each hand. For the purpose of creating a concise and clear way to code these instructions it is convenient to think of a player as being able to earn points from combinations of two, three, four, or five cards at one time. Players can gain points from groups of two cards through pairs, 15's and the right jack. Players can gain points from groups of three cards through 15's and runs of three. Players can gain points from groups of four cards with 15's, runs of four, and in hand flushes. Players can gain points from groups of five cards with 15's, runs of five, and flushes.

There are some interesting points that make scripting a hand counting method easier. Three and four of a kind function simply as a set of all possible combinations of pairs. Three of a kind has 3 possible pairs and four of a kind has 6 possible pairs. In light of this, it is not necessary to consider three and four of a kind as separate modes of gaining points. It is sufficient only to count all possible pairs in a hand. 15's are possible with any number of cards, so the same algorithm can be used for each combination of either 2, 3, 4, or 5 cards at at time.

Runs prove to be the most challenging scoring mode to quantify. Runs of four will *always* contain two runs of three. Runs of five *always* contain 2 runs of four and 3 runs of three. A combinatoric approach (which is utilized in the code for this experiment) needs to be able to screen our the lower order runs and *NOT* count them towards the player's score. Because of this some preprocessing is required before counting the points due to runs. This makes up the bulk of the computation time. Computation time is reduced with careful placements of break statements in order to avoid checking for consecutive cards in a run that do not need to be checked for.

III. EXPERIMENT

To accomplish the counting of every possible hand in cribbage the authors implement the object oriented capabilities of the Python Programming Language. A Card and Hand class are constructed. The card class has members number, suit, and value. The card's number is between 1 and 13 to signify Ace through King. This quantity is used in constructing runs and pairs. The card's value is the value used when summing cards together for 15's (i.e. the value of all face cards is 10). The Hand class is simply a list of 5 cards, with the last card assumed to be the common card. The Hand class has a method countHand() which returns a value as per the counting rules discussed above.

A traditional deck of 52 cards is created as a list of card objects. A list of hand objects is created using the Python itertools module. Each hand is this list is then counted five times (once for each possible arrangement of the common card). The score for each hand is recorded in a dictionary whose keys are the score of each hand and whose values are the multiplicity of each score. When all the hands are finished counting, the dictionary contains all of the possible scores and the number of different hands that can produce each score. These are the data displayed in this paper.

IV. RESULTS AND ANALYSIS

The results of the simulation can be found in Table I. The simulation (without aggressive optimization) runs in just under 15 minutes on a standard MacBook Pro. After implementing optimization computing and storing a hash map of the results for every hand takes around 3.5 minutes. The hash map can be used to do look ups for more efficient counting (at about 4x the speed as computing the results originally).

The data show that the average score for a randomly dealt hand is 4.769 with a standard of deviation of 3.125 points. Note that the standard of deviation (while still carrying useful information) is not the 68% confidence bound because the data are not a normal distribution. The most common score (the mode) is 4 points, with a 21.9% chance of being dealt. It is worth noting that these are the statistics of the hand being *dealt*. Since players are allowed to throw away at least one card before play begins the actual distribution of scores in a real game of cribbage will be preferentially skewed to higher values. As such, these results serve as an upper limit for the likelihood of encountering higher scoring hands and a lower limit for the likelihood of encountering a lower scoring hand in a real game.

These numbers are known exactly and reported to 3 significant digits because of the finite and integer calculations necessary for this experiment. From the data we can see that the Big Nineteen Postulate holds true, but is not sufficient at characterizing the entire behavior of all cribbage hands. In addition to "The Big 19", scores of 25, 26 and 27 also have zero multiplicity, meaning a hand with those scores will never be dealt.

From these data we can correct DSP Theorem. The authors posit with these data as verification that the set of possible scores in cribbage, P, is defined as:

$$P = [0, 1, 2, ..., 18, 20, ..., 24, 28, 29] \tag{4}$$

which shall be known as the Porzio-Sikes Law. The Porzio-Sikes Lemma is also verified because the definitive upper bound on the score for a hand is indeed 29.

Interesting behaviors can be seen in the distribution of multiplicities. All the multiplicities can be see in figure 1. Because of the huge discrepancies in the multiplicities of the lower scoring and the higher scoring hands, a more convenient plot can be seen in figure 2, which displays the data with a logarithmic y-axis. The data indicate hands with odd scores are less common that those with even scores. This is likely due to the right jack, runs of

Score	Multiplicity
0	1009008
1	99792
2	2813796
3	505008
4	2855676
5	697508
6	1800268
7	751324
8	1137236
9	361224
10	388740
11	51680
12	317340
13	19656
14	90100
15	9168
16	58248
17	11196
18	2708
19	0
20	8068
21	2496
22	444
23	356
24	3680
25	0
26	0
27	0
28	76
29	4

TABLE I: List of the possible scores a player can have in a single hand with the number of different hands that can give the player that score.

Distribution.png

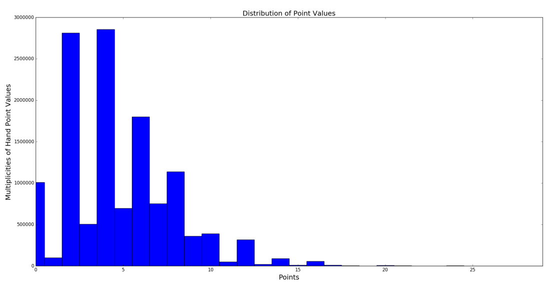

FIG. 1: This is a plot of the multiplicities of each score. The relative heights of each score's bar is directly proportional to the probability that a hand of that score will be dealt. It should be noted that the scores greater than 15 are difficult to see on this scale.

Distribution semilogy.png

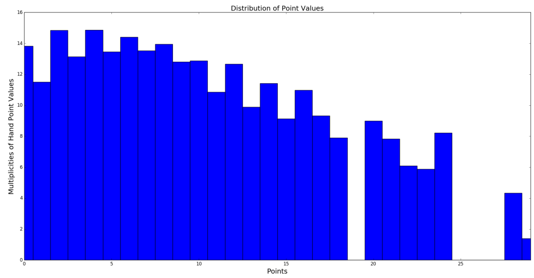

FIG. 2: This is a plot of the logarithm multiplicities of each score. The relative heights of each score's bar is proportional to the order of magnitude of the probability that a hand of that score will be dealt.

Distribution semilogy evens.png

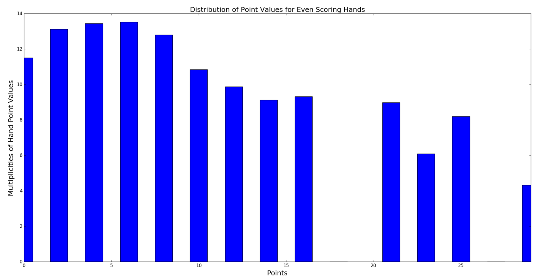

FIG. 3: This is a plot of the logarithm multiplicities of the Even scoring hands. The relative heights of each score's bar is proportional to the order of magnitude of the probability that a hand of that score will be dealt. Three distinct regions (from 0 to 18, 20 to 24, and 28) can be seen.

three, and full flushes, the only modes of scoring that award an odd number of points.

The data show a maximum multiplicity around a score of 4 points, and tapering off from there. It can be seen that the probability dies off as the score approaches 18. However, after 19 the multiplicities abruptly jump up again for the hands that score in the 20 to 24 range. It happens again after 25,26, and 27. This effect can be seen more clearly when only looking at the even scoring hands, as seen in figure 3. The higher density of the 20-24 point hands is most likely attributed to the "super run" hands, in which double runs of 4 or triple runs of three happen with cards between 5 and 9, which allows for multiple 15's in addition to the high scoring runs. It is also well known that the hands who score 28 and 29 are the 5-5-5-5-Face Card hands. The four of a kind and enormous 15 potential make these types of hands the only hand capable of reaching such high scores. The 29 points hands include a right jack as the face card.

V. CONCLUSIONS

We have provided data which show the existence of the "Big 19", but it seems that the exclusiveness of certain scores was not totally understood. We can now say with certainty that there exists a "Big 19", "Big 25,", "Big 26", and "Big 27" and that the highest possible score in cribbage is 29. Defined here, the Porzio-Sikes Law totally and accurately encompasses the behavior of all cribbage hands. This definition corrects and summaries the work done by J. Doyle, G. Sikes, and J. A. Porzio in addition to J. R. Porzio and E. S. Sikes.

The three distinct regions in the data plots show an interesting relationship between the impossible scores and the multiplicity of the possible scores. The first and second region are separated by "The Big 19", after which the multiplicity jumps up. Then, after the impossible scores of 25, 26, and 27 the multiplicity them jumps again! It appears that the impossible scores punctuate and separate three separate regimes of hands. The hands scoring 0 to 18 are of no notable type. However the hands scoring between 20 and 24 are almost certainly of the "super run" category. Lastly, the hands scoring 28 and 29 are almost certainly of the 5-5-5-5-Face Card variety. These data suggest the different hand categories (15 heavy, run heavy, super run, 5-5-5-5-Face Card) are fundamentally higher scoring than others.

The behavior of cribbage hands is now well understood. Future work involves taking a deeper look at the structure of the different regimes of cribbage hands to determine the true distribution of the different modes of scoring. It is now supposed that the impossible scores of 19, 25, 26, and 27 are somehow fundamental in breaking up the different regimes of the possible hands.