

# CRYPTOGRAPHY 099

DIFFIE-HELLMAN-MERKLE KEY EXCHANGE

WHO WHAT WHERE WHY HOW

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# WHY?

- WE AGREE ON A SECRET KEY.
- THEN USE THIS SECRET KEY TO ENCRYPT AND DECRYPT OUR MESSAGES.
- BUT HOW DO I TELL YOU WHAT THE SECRET KEY IS?
- THIS IS KNOWN AS THE KEY DISTRIBUTION PROBLEM.

# WHO?

- WHITFIELD DIFFIE
- INDEPENDENT SECURITY EXPERT & CRYPTOGRAPHER
- STUDIED MATHEMATICS AND GRADUATED FROM MIT IN 1965

# WHO?

- MARTIN HELLMAN
- PROFESSOR AT STANFORD UNIVERSITY
- PEERS CALLED HIM CRAZY FOR RESEARCHING CRYPTOGRAPHY SINCE NSA WAS DOING THE SAME (WOULD BE IN COMPETITION).

# WHO?

- RALPH MERKLE
- ALSO AT STANFORD UNIVERSITY AND WORKED WITH HELLMAN
- HELLMAN WAS HIS DOCTORAL ADVISOR

# WHERE?

- STANFORD UNIVERSITY
- WHEN DIFFIE AND HELLMAN MET, THEY INSTANTLY WORKED WELL TOGETHER.
- HELLMAN COULDN'T AFFORD TO HIRE DIFFIE, SO DIFFIE ENROLLED AS A GRADUATE STUDENT
- DIFFIE'S WIFE WAS ALSO IN ACADEMIA AND HER INCOME HELPED DIFFIE STAY INDEPENDENT TO GET TO THIS POINT AND CONTINUE ON.

# WHAT?

- REMEMBER THE KEY DISTRIBUTION PROBLEM (OR KEY-EXCHANGE PROBLEM)?
- CAN WE SOLVE THIS IN THE REAL WORLD WITH PHYSICAL BOXES & LOCKS?
- YES!
- I PUT MY MESSAGE IN A BOX, ADD MY PAD LOCK AND SEND IT TO YOU (USPS).
- YOU RECEIVE IT, ADD YOUR PAD LOCK AND SEND IT BACK TO ME.
- I RECEIVE IT, REMOVE MY PAD LOCK AND SEND IT TO YOU.
- YOU RECEIVE IT, REMOVE YOUR PAD LOCK AND READ MY MESSAGE.

# WHAT?

- DIFFIE USED THIS IDEA AS INSPIRATION AND NEEDED A ONE-WAY MATHEMATICAL FUNCTION.
- EASY TO DO ONE-WAY, BUT DIFFICULT TO DO THE OTHER WAY (I.E. UNDO).
- MULTIPLICATION IS A TWO-WAY FUNCTION – JUST DIVIDE TO UNDO.
- MULTIPLYING TWO LARGE PRIMES IS EASY – BUT FINDING THEM IS HARD (ONE-WAY FUNCTION).
- ANOTHER CRYPTIC SYSTEM RELIES ON THIS FACT.

# WHAT?

- THE FOCUS CAME TO MODULAR ARITHMETIC AS A WAY TO CREATE THEIR ONE-WAY FUNCTION.
- IN COMPUTER SCIENCE, WE KNOW THIS AS MODULUS OR REMAINDER FUNCTION.
- IS THE VARIABLE  $i$  EVEN? SIMPLY TEST IF:  $i \% 2 == 0$
- WE ALSO LEARNED MODULAR ARITHMETIC AS KIDS. WHAT TIME IS 14 O'CLOCK?
- ALSO KNOWN AS CLOCK ARITHMETIC AND A GOOD VISUALIZATION

# WHAT?

- AFTER YEARS OF RESEARCH, HELLMAN DEVISED A STRATEGY TO SOLVE THE KEY-EXCHANGE PROBLEM IN 1976.
- IT IS QUITE ELEGANT, FAIRLY SIMPLE AND REGARDED AS ONE OF THE BIGGEST BREAKTHROUGHS IN CRYPTOGRAPHY IN THE PAST FEW CENTURIES.
- COMPUTERS AND MODERN ENCRYPTION HAS MADE CRYPTANALYSTS OBSOLETE (I.E. BREAKING THE CIPHER - ALAN TURING AND HIS WORK DURING WWII AGAINST ENIGMA).
- TODAY, WE SAFE-GUARD KEYS AND STRENGTHEN ENCRYPTION.
- SOCIAL HACKING HAS BECOME MORE POPULAR – IT IS THE LEAST RESISTANT.

# HOW?

- PRIMER - DRE'S HACKWEEK WINNER:
- E.G. POOL = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
- E.G. TICKET\_HASH = 2134 (I.E. REALLY BIG NUMBER)
- LET VOLUNTEER = POOL[TICKET\_HASH % POOL.LENGTH]
- PICK ANY LARGE NUMBER AND VOLUNTEER WILL ALWAYS BE IN RANGE [0 : 11]

# HOW?

- BACK TO DIFFIE-HELLMAN - HELLMAN CAME UP WITH THE FOLLOWING:
- TWO PARTIES AGREE ON VALUES  $a$  (GENERATOR) AND  $p$  (LARGE PRIME) AND CAN BE PUBLIC.
- EACH PARTY PICKS A SECRET NUMBER (BOB PICKS  $x$ , ALICE PICKS  $y$ ) BETWEEN 1 AND  $p-2$
- BOB COMPUTES  $PK_B = a^x \text{ MOD } p$  AND ALICE COMPUTES  $PK_A = a^y \text{ MOD } p$
- EACH PARTY PUBLICLY SENDS THEIR  $PK_B$  AND  $PK_A$  (INSIDE THE CLOCK,  $< p$ )
- BOB COMPUTES  $PK_A^x \text{ MOD } p$  AND ALICE  $PK_B^y \text{ MOD } p$  (KEY EXCHANGE COMPLETE!)
- PROOF:  $PK_A^x \text{ MOD } p = PK_B^y \text{ MOD } p = a^{x^y} \text{ MOD } p = a^{y^x} \text{ MOD } p = a^{(x*y)} \text{ MOD } p = a^{(y*x)} \text{ MOD } p$  (EQUAL BY COMMUTATIVE PROPERTY)

# HOW?

- THE PRIME NUMBER  $P$  HAS TO BE VERY LARGE – 2048 BITS (\$100M CAN BREAK 1024 BITS – NSA, AMAZON, PUTIN, ETC.)
- BREAKING THE ENCRYPTION IS CALLED “SOLVING THE DISCRETE LOGARITHM PROBLEM”).
- REMEMBER THE NUMBER ‘ $a$ ’ CHOSEN AS A GENERATOR FOR  $a^x \bmod p$ ?  
 $a$  IS USUALLY A SMALLER PRIME NUMBER AND IT MUST PRODUCE A CYCLIC “TABLE” WHEN COMPUTING VALUES  $a^x \bmod p$ .
- BASICALLY, THIS “TABLE” IS A MATHEMATICAL SET THAT IS BETWEEN 0 AND  $p-1$
- TODAY ELLIPTIC-CURVE DIFFIE–HELLMAN (ECDH) IS USED (FASTER AND MORE SECURE AGAINST ATTACKS).

# NEXT?

- DIFFIE CAME UP WITH THE IDEA OF PUBLIC-KEY CRYPTOGRAPHY(ASYMMETRIC ENCRYPTION)
- ENCRYPT A MESSAGE WITH A PUBLIC KEY AND DECRYPT IT WITH THE PRIVATE KEY
- EVENTUALLY A SOLUTION WAS INVENTED IN 1977 BY A TEAM OF THREE AT MIT: ADLEMAN, RIVEST AND SHAMIR.
- ADLEMAN WAS THE MATHEMATICIAN ON THE PAPER AND BEING HUMBLE, DIDN'T WANT HIS NAME ON IT.
- BUT RIVEST INSISTED SO THERE WAS A COMPROMISE – ADLEMAN'S NAME WOULD BE LISTED LAST. SO TODAY, WE DON'T KNOW THIS SYSTEM AS ASR, BUT RSA.

# QUESTIONS?

- THE CODE BOOK BY SIMON SINGH (HIGHLY RECOMMEND)
- ELEMENTARY NUMBER THEORY AND ITS APPLICATIONS BY KENNETH H. ROSEN