Final Project
Heat Gradients
Phys 430

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Abstract

The goal of this experiment was solving for the heat diffusivity constant for aluminum of type 6061. A bar that best represented a one-dimensional system was used. Temperature sensors Dallas DS18B20s were used and data was collected using the Arduino Uno micro-controller. Data analysis was performed using computational methods in python to find a $\kappa_{exp} = (2.5 \pm .53) \times 10^{-5} \frac{m^2}{s}$. The published value is $\kappa_{published} = 6.4 \times 10^{-5} \frac{m^2}{s}$. The values did not agree within error and this was most likely due to the experimental setup.

1 Background

The one-dimensional heat equation is

$$\frac{c_p \rho}{k} \frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} = 0, \tag{1}$$

where c_p is the heat capacity, ρ is the mass density, k is the thermal conductivity, and T is the temperature. In this experiment, the heat equation was numerically solved to find the diffusivity constant κ which is

$$\kappa = \frac{c_p \rho}{k} \tag{2}$$

2 Experiment

2.1 Experimental Setup

In this experiment, we are exploring temperature gradients of a one dimensional Aluminum type 6061 bar. The temperature sensor used in this experiment was the Dallas DS18B20. The experiment was performed on a thin and long bar to closely simulate a one-dimensional bar. The goal of this experiment is to show the steady state temperature gradient on a one dimensional bar. A simplified version of the general heat equation was used,

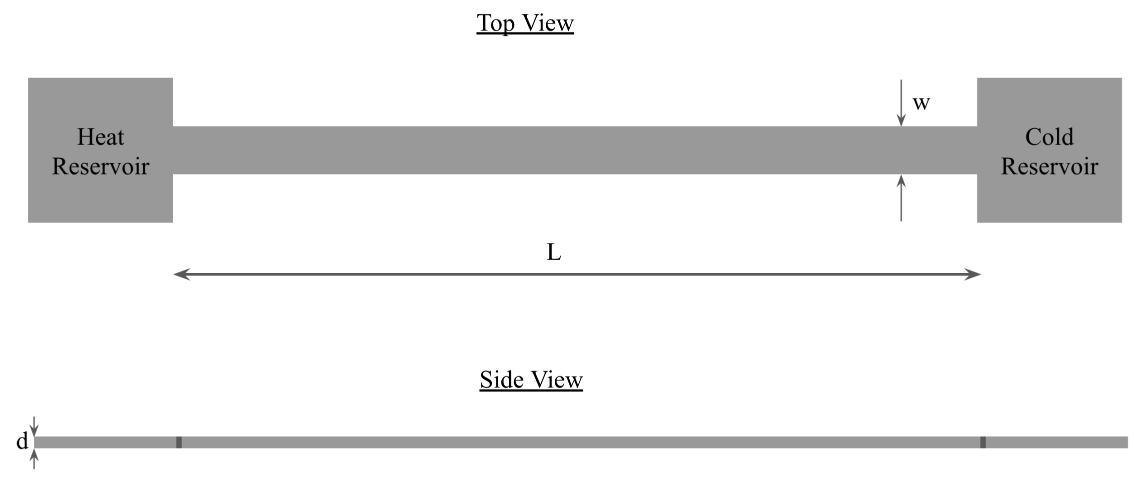

Figure 1: Side view and top view of bar used in the experiment

In order to approximate a one-dimensional bar, some conditions were met:

$$L >> d$$
$$L >> w.$$

The following values were found for L, w, & d (which were averaged out from three measurements for each):

$$L = 159mm$$
$$w = 5.06mm$$
$$d = 1.55mm$$

To further support the bar is one-dimensional, we looked at the time constant for heat diffusion in each direction. Solving the heat equation with the intention of finding the time constant for heat diffusion, τ, we get:

$$\tau = \frac{c_p \rho L^2}{k\pi^2}. \tag{3}$$

The method of theoretically finding τ can be found in the Appendix. Using published values for c_p, ρ, and k of Aluminum 6061, the time constants for each axis were

$$\tau_d = 3.609x10^{-2}$$
$$\tau_w = 4.107x10^{-1}$$
$$\tau_L = 4.055x10^{2}$$

Using $c_p = 897$ J/(kgK), $\rho = 2.70$ g/cm, and $k = 151$ W/(mK)

Due to the bar being thinner than the DS18B20 sensors, only ten sensors were used. If too many sensors were placed, there would be more plastic than there is aluminum and the temperature data would deviate from matching a one-dimensional temperature distribution for aluminum. The sensors were kept on the bar using varnish. Figure 2 shows the sensor setup used.

Figure 2: Sensor setup on one-dimensional aluminum bar

The purpose of the dumbbell shape for the bar was to have the heat flow as evenly as possible to ensure temperature does not vary on the w & d axes. To accomplish this, the entirety of the reservoirs had to be at the same temperature. The cold reservoir easily had temperature distributed along it since an ice bath of water was used. As for the hot reservoir, a heat clamp was made with a welding clamp. To raise the temperature of the clamp, high power resistors were connected to the ends of the clamp. Varying the current across the resistors allowed for different temperature equilibriums to be reached. See below for the the hot and cold reservoirs.

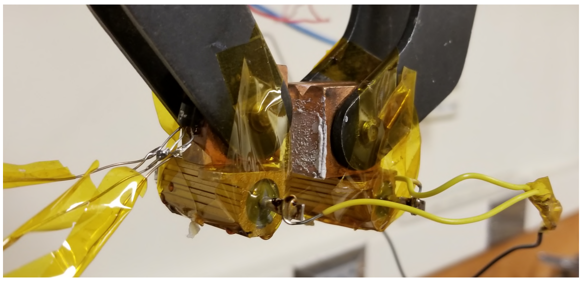

Figure 3: Heating clamp for hot reservoir

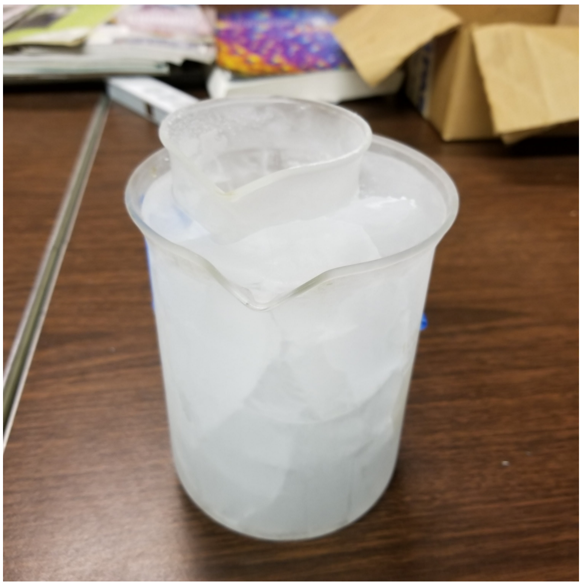

Figure 4: Cold reservoir

This setup, again, helped get temperature to diffuse evenly from the hot reservoir to the cold. Since aluminum will heat up quickly, air can act as a heat sink which then changes the temperature of the bar and lowers the equilibrium temperature gradient overall. To avoid losing this energy, the bar itself was isolated from the air using polystyrene. An image of the entire setup is shown below.

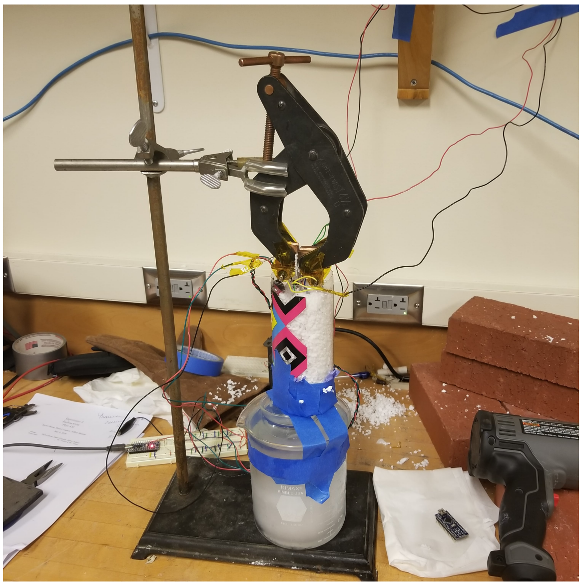

Figure 5: Experimental Setup

2.2 Data Acquisition

The data was acquired using an Arduino Uno micro-controller. The communication between the sensors and the Arduino was feasible using only one input since each DS18B20 sensor has an assigned serial address. The sensors function properly between $-10°C$ and $85°C$. Therefore, the cold reservoir was simply an ice bath and the hot reservoir was always set to temperatures below $85°$.

The Arduino continuously obtained data when the code was uploaded and the sensors were connected. This data stream was then tapped into using python by open the serial port to store data. Due to data coming in based on the serial address rather than physical location, the data arrays had to be reorganized using a python script. The data was then plotted and analyzed using python.

3 Data and Results

In order to ensure the data was accurate, two tests were done to attempt to calibrate the sensors. The first test was run at room temperature for 10 minutes with new data coming being processed every second. This was done to detect modest fluctuations between sensors. Below is a 30 second slice from the room temperature data where every column is a different sensor starting from sensor one on the left.

24.5	25	25	24.5	24.5	25	25	25	25	25
25	25	25	24.5	24.5	25	25	25	25	25
25	25	25	24.5	24.5	25	25	24.5	25	25
24.5	25	25	24.5	24.5	25	25	24.5	25	25
24.5	25	25	24.5	24.5	24.5	25	25	25	25
24.5	25	25	24.5	24.5	24.5	25	25	25	25
24.5	25	25	24.5	24.5	25	24.5	24.5	25	25
24.5	25	25	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	25	24.5	24.5	25	25
24.5	25	25	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	25	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	25	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25
24.5	25	24.5	24.5	24.5	24.5	24.5	24.5	25	25

Figure 6: Room Temperature Calibration Results

The sensors have an resolution through the program of measuring 0.5 degree changes. Within the calibration tests, these fluctuations of 0.5 degrees happened at random and can be attributed to noise from the environment.

The next test done was accomplished by placing the sensors inside a plastic bag, then submerging it into an ice bath. Below is another 30 second slice while in the bath.

1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1
1	1	1	0.5	0.5	0.5	0.5	0.5	1	1

Figure 7: Ice Bath Calibration Results

After running the calibration tests, the setup for hot and cold reservoirs was used to obtain a gradient along the bar. First, the "hot reservoir" clamp was heated to approximately 86°C and the cold reservoir was cooled to approximately 0°C. When ready, the program was run, and the bar was clamped into the hot reservoir to begin data collection for 10 minutes. Thermal paste was applied to the heat clamps in order to create the best thermal contact. The following chart is a 30 second increase that starts from room temperature. The final column is the time step per read.

22	21	20.5	19	18	16	14.5	13	11	8	23.10700011
22	21	20.5	19	18	16	14.5	13	11	8	24.27900004
22.5	21	20.5	19	18	16	14.5	13	11	8	25.45200014
22.5	21	20.5	19	18	16	14.5	13	11	8	26.62400007
22.5	21	20.5	19	18	16	14.5	13	11	8	27.796
23	21	20.5	19	18	16	14.5	13	11	8	28.96700001
23	21	20.5	19	18	16	14.5	13	11	8	30.1400001
23.5	21.5	21	19	18	16	14.5	13	11	8	31.31400013
23.5	21.5	21	19	18	16	14.5	13	11	8	32.48799992
24.5	21.5	21	19	18	16	14.5	13	11	8	33.65799999
25	22	21	19	18	16	14.5	13	11	8	34.829
26	22.5	21	19	18	16	14.5	13	11	8	36.00300002
27	22.5	21	19	18	16	14.5	13	11	8	37.17499995
27.5	23	21	19	18	16	14.5	13	11	8	38.34700012
28.5	23.5	21.5	19.5	18	16	14.5	13	10.5	7.5	39.51900005
29	24	21.5	19.5	18	16	14.5	13	11	7.5	40.69199991
30	24.5	21.5	19.5	18	16	14.5	13	10.5	7.5	41.86400008
31	25	21.5	19.5	18	16	14.5	13	10.5	7.5	43.03699994
32.5	26	22	19.5	18	16	14.5	13	10.5	7.5	44.20799994
34	26.5	22	20	18	16	14.5	13	10.5	7.5	45.38000011
36	27.5	22.5	20	18	16.5	14.5	13	10.5	7.5	46.55299997
38	29	22.5	20	18	16.5	14.5	13	10.5	7.5	47.72600007
40	30	23	20	18	16.5	14.5	13	10.5	7.5	48.89700007
41.5	31.5	23.5	20.5	18.5	16.5	14.5	13	10.5	7.5	50.06800008
43.5	33	24	20.5	18.5	16.5	14.5	13	10.5	7.5	51.24099994
45.5	34.5	24.5	21	18.5	16.5	14.5	13	10.5	7.5	52.41400003
47	36	25	21.5	18.5	16.5	15	13	10.5	7.5	53.58500004
48.5	37	25.5	21.5	19	16.5	15	13	10.5	7.5	54.75800014

Figure 8: Thermal Paste Run

After the data was collected, a 3D graph of the temperature vs. time vs. sensor position was created.

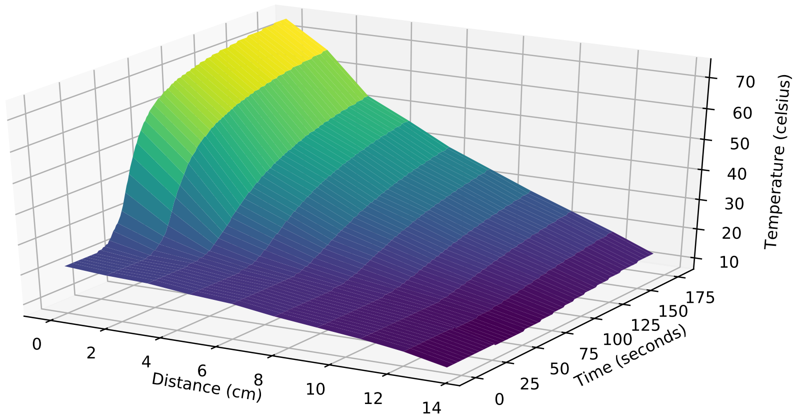

Figure 9: 3D plot of temperature change based on position on bar and time

This data was then used to find the heat diffusivity constant, κ. κ is the specific heat times the mass density divided by the thermal conductivity.

$$\kappa = \frac{c_p \rho}{k} \tag{4}$$

That means the heat equation can be written in terms of the thermal diffusivity constant:

$$\kappa \frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} = 0 \tag{5}$$

Therefore, we can solve for the diffusivity constant in terms of the spacial and tensile derivatives. This can be found with the data from figure 9. The central difference method was used to find the derivatives in python and obtained a value of:

$$\kappa_{exp} = (2.5 \pm .53) \times 10^{-5} \frac{m^2}{s} \tag{6}$$

The published value for κ †:

$$\kappa_{published} = 6.4 \times 10^{-5} \frac{m^2}{s} \tag{7}$$

4 Discussion

The method in which the hardware part of the experiment was setup introduced the largest amount of error. Various aspects such as how well insulated the bar was, how the sensors effected the heat distribution, and how well the sensors made thermal contact.

†Wikipedia contributors. (2018). Thermal diffusivity. *Wikipedia, The Free Encyclopedia*

Appendices

A Time Constant

In this experiment we are solving the one-dimensional heat equation.

$$\frac{c_p\rho}{k}\frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} = 0. \tag{8}$$

Where c_p is the heat capacity, ρ is the mass density, k is the thermal conductivity, and T is the temperature. We then assume a temperature function of the form:

$$T(x,t) = X(x)G(t) \tag{9}$$

Plugging this into the (8) and separating variables yields,

$$\frac{c_p\rho}{k}\frac{G'}{G} = \frac{X''}{X} \tag{10}$$

The equation above implies that the LHS and the RHS must be constant. With knowledge of what will happen to the ODE on the RHS of equation (10), we set them equal to $-\gamma^2$. For reasons explained in the next section, we are interested in finding the time constant for heat diffusivity. The LHS of equation (10) gives the following ODE:

$$G' + \gamma^2\frac{c_p\rho}{k}G = 0 \tag{11}$$

Which has a solution of the form:

$$G = C_1 e^{-\gamma^2(\frac{k}{c_p\rho})t} \tag{12}$$

That implies that the time constant we are interested in is

$$\tau = \frac{c_p\rho}{k\gamma^2} \tag{13}$$

Finding γ with known terms is easily done. We use the ODE that comes from the RHS of equation (10).

$$X'' + \gamma^2 X = 0 \tag{14}$$

This has a solution of the form:

$$X = C_2\cos\gamma x + C_3\sin\gamma x \tag{15}$$

We are only interested in finding gamma and we can find that using the usual boundary conditions for a 1 dimensional bar, $C_2 = 0$ and the argument inside the sine function at the boundary L (the edge of the bar) is set equal to π (we are not interested in every possible solution, only one that allows us to get γ in terms of known constants). Therefore,

$$\gamma = \frac{\pi}{L} \tag{16}$$

Which then gives us a time constant of

$$\tau = \frac{c_p\rho L^2}{k\pi^2} \tag{17}$$