Exploiting Ridge Structure in Chance-Constrained Design Under Uncertainty

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Outline of our Design Uncertainty Approach

1. Construct ridge approximations
   - Determine a 'good' set of samples $\{\mathbf{x}_i\}_{i=1}^{M}$ using **stretched sampling**
   - Using input-output pairs $\{\mathbf{x}_i, f(\mathbf{x}_i)\}_{i=1}^{M}$ build a **polynomial ridge approximation**
   - Build **bounding response surfaces**
2. Solve reliability based design optimization (RBDO)
   - Approximate expectation objective with linear function
   - Chance-constraints become linear inequality constraints with an **empirical safety factor**
   - Estimate solution to RBDO by solving a **linear program**
3. Verify the solution
   - Sample the model at the design point
   - Estimate failure probabilities using linear approximation

Building Ridge Approximations

Building Polynomial Ridge Approximations

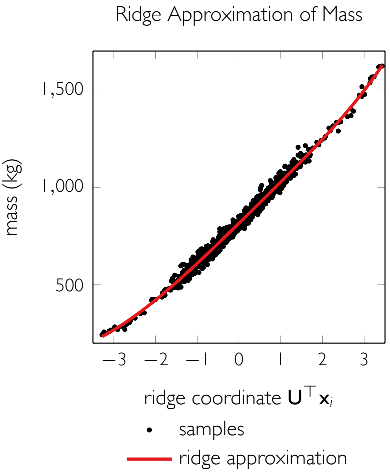
Given samples of f evaluated at $\{\mathbf{x}_i\}_{i=1}^M$, solve

$$\underset{\substack{g\in\mathcal{P}_p(\mathbb{R}^n) \\ \operatorname{Range}\mathbf{U}\in\mathbb{G}(n,\mathbb{R}^m)}}{\text{minimize}} \sum_{i=1}^{M} \left| f(\mathbf{x}_i) - g(\mathbf{U}^\top \mathbf{x}_i)\right|^2$$

where

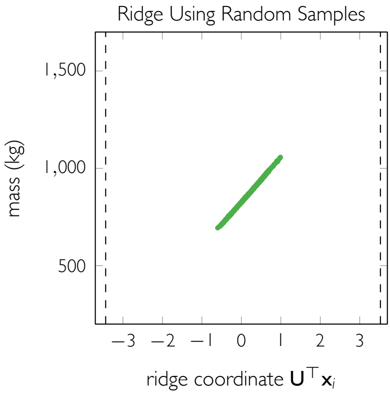
$\mathcal{P}_p(\mathbb{R}^n)$	polynomials of total degree p on $\mathbb{R}^n$
$\mathbb{G}(n,\mathbb{R}^m)$	Grassmann manifold of n-dimensional subspaces of $\mathbb{R}^m$

Efficient computation using Variable Projection and Grassmann optimization [HC18]



[HC18] Hokanson & Constantine, *Data-Driven Polynomial Ridge Approximation Using Variable Projection*, SISC 2018

Random Sampling Yields Poor Approximations



Concentration of measure ensures most samples fall in the interior

- - - boundary of domain • random samples

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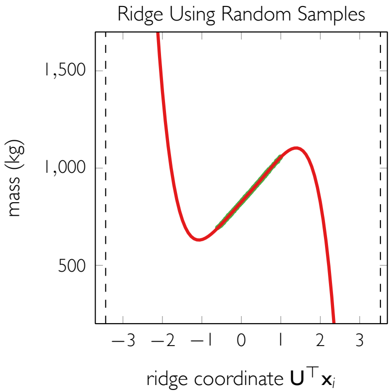

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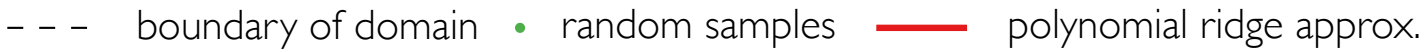

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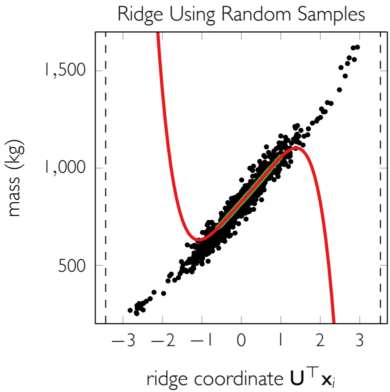

Concentration of measure ensures most samples fall in the interior resulting in poor extrapolation to boundaries

– – – boundary of domain • stretched samples • random samples —— polynomial ridge approx.

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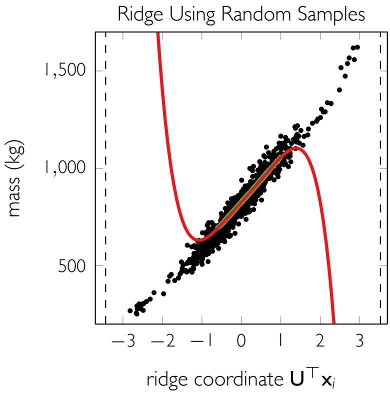

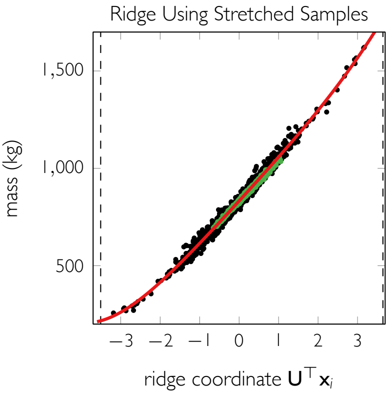

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Stretched sampling ensures a more even distribution of samples along ridge direction and a better global response surface

- - - boundary of domain • stretched samples • random samples — polynomial ridge approx.

Basic Stretched Sampling

Stretched sampling is **sequential maximin sampling** with a **variable metric** given by ridge direction $\mathbf{U}$

Stretched Sampling

Input : Domain $\mathcal{D} \subset \mathbb{R}^m$, function f, initial samples $\{\mathbf{x}_i\}_{i=1}^{M_0}$

Output : Stretched samples $\{\mathbf{x}_i\}_{i=1}^{M} \subset \mathcal{D}$

for $k = M_0, M_0 + 1, \ldots, M - 1$ **do**

Fit ridge approximation $f(\mathbf{x}_i) \approx g_k(\mathbf{U}_k^\top \mathbf{x}_i)$;

Sample $\mathbf{x}_{k+1} = \operatorname*{argmax}_{\mathbf{x} \in \mathcal{D}} \min_{i=1,\ldots,k} \|\mathbf{U}_k^\top (\mathbf{x} - \mathbf{x}_i)\|_2$;

- Sampling requires $\mathcal{D}$ to be **bounded**
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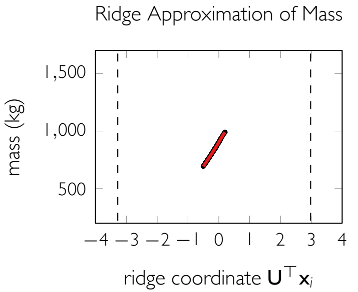

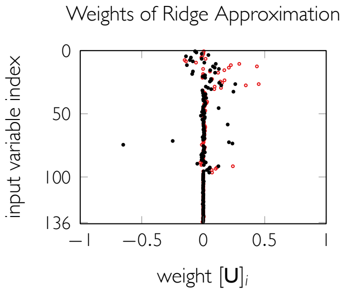

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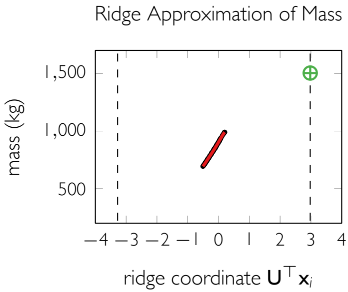

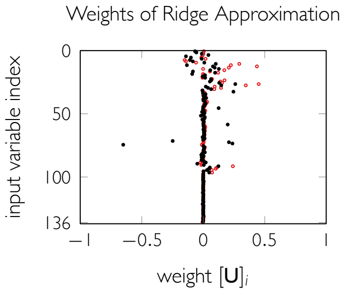

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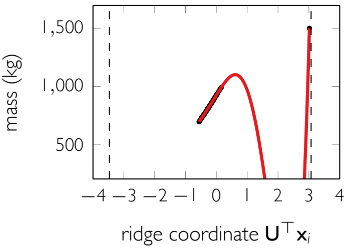

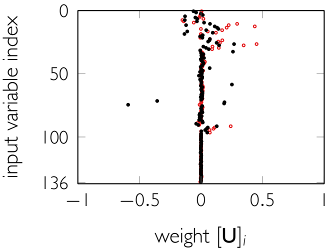

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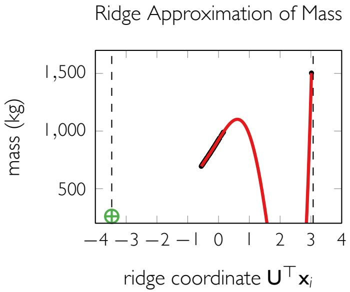

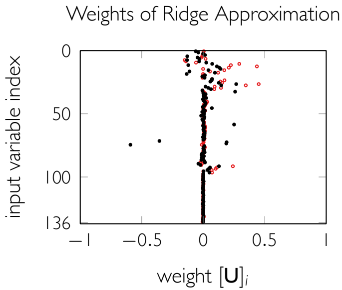

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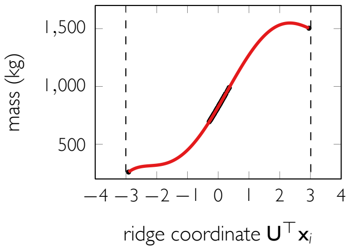

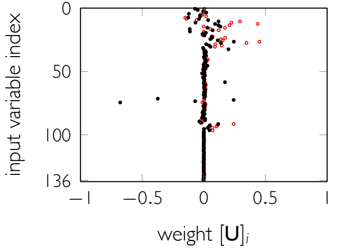

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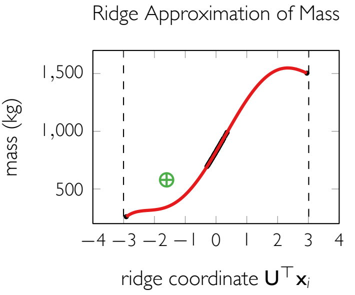

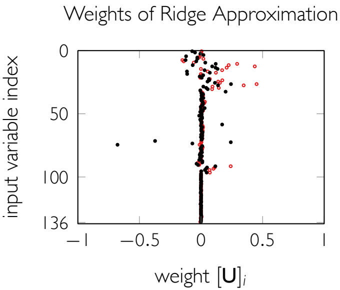

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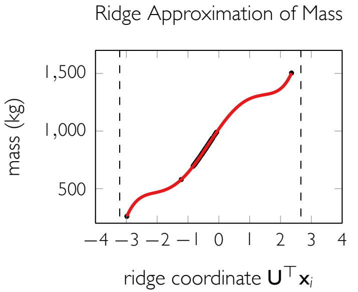

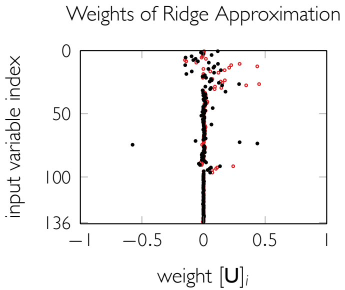

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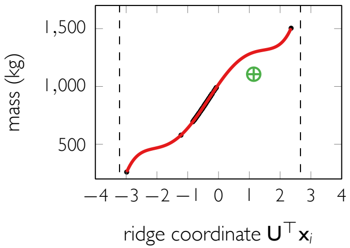
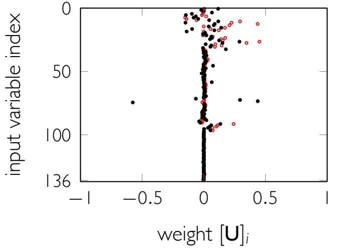
Ridge Approximation of Mass



Weights of Ridge Approximation



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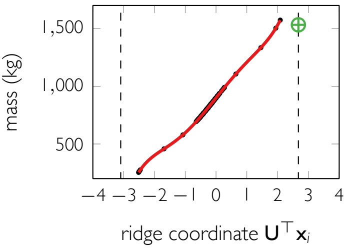

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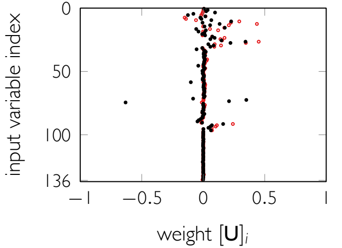

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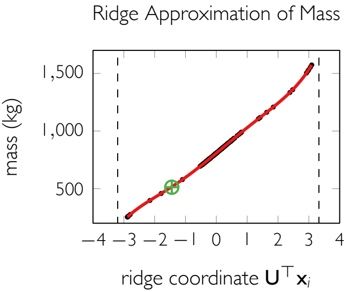

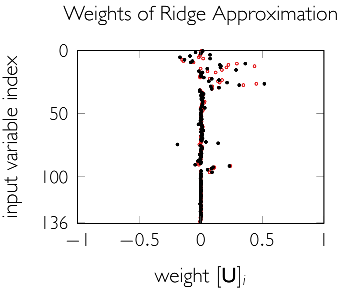

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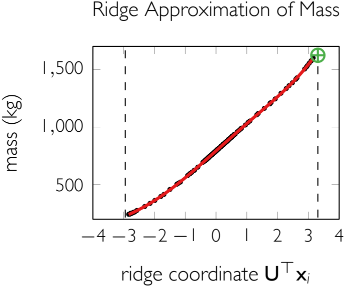

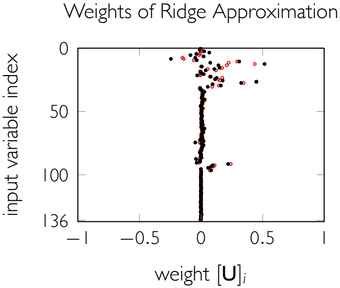

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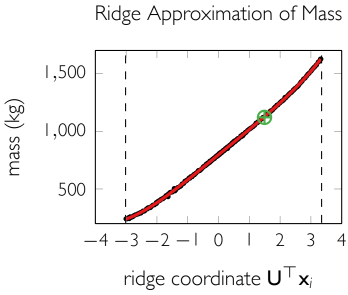
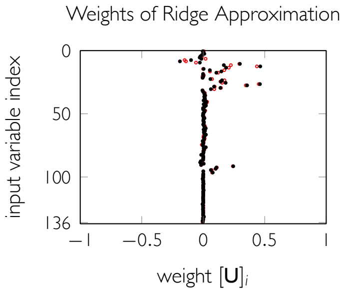
$k = 300$





Basic Stretched Sampling

Stretched sampling is **sequential maximin sampling** with a **variable metric** given by ridge direction $\mathbf{U}$

Stretched Sampling

Input : Domain $\mathcal{D} \subset \mathbb{R}^m$, function f, initial samples $\{\mathbf{x}_i\}_{i=1}^{M_0}$

Output : Stretched samples $\{\mathbf{x}_i\}_{i=1}^{M} \subset \mathcal{D}$

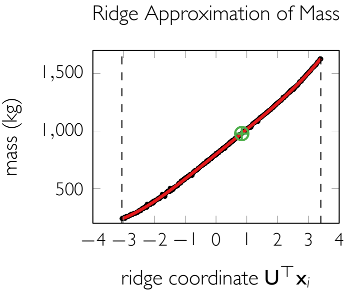
for $k = M_0, M_0 + 1, \ldots, M - 1$ **do**

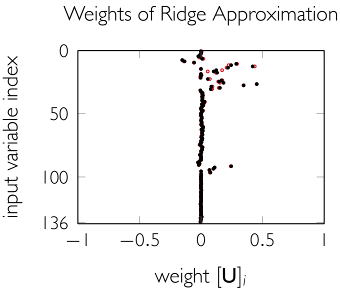
Fit ridge approximation $f(\mathbf{x}_i) \approx g_k(\mathbf{U}_k^\top \mathbf{x}_i)$;

Sample $\mathbf{x}_{k+1} = \underset{\mathbf{x}\in\mathcal{D}}{\operatorname{argmax}} \min_{i=1,\ldots,k} \|\mathbf{U}_k^\top(\mathbf{x} - \mathbf{x}_i)\|_2$;

- Sampling requires $\mathcal{D}$ to be **bounded**
- Uniformly randomly select $\mathbf{x}_{k+1}$ from $\mathcal{D} \bigcap \{\mathbf{x} : \mathbf{U}_k^\top \mathbf{x} = \mathbf{b}\}$
- Since $\mathbf{U}_k$ depends on data already collected, we cannot precompute samples $\{\mathbf{x}_i\}_{i=1}^{M}$

$k = 400$





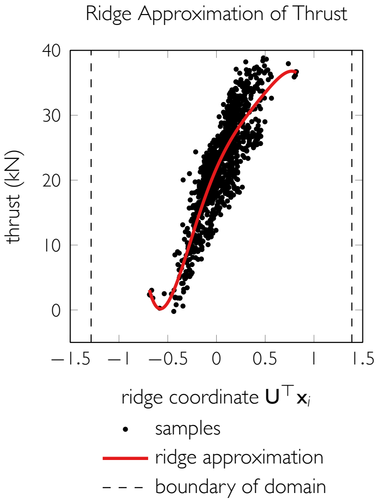
Parallel and Multi-objective Stretched Sampling

- MULTI-F has 16 quantities of interest that are **evaluated simultaneously**
- To make efficient use of resources we sample in parallel for each objective

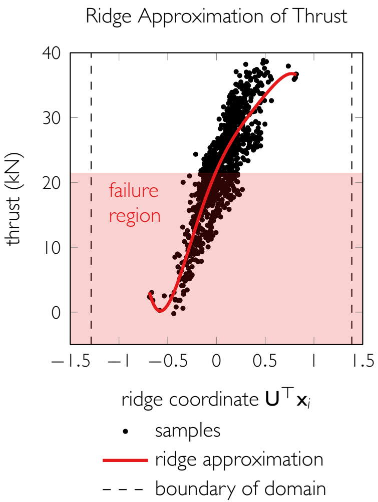
Parallel & Multi-objective Stretched Sampling

Input : Domain $\mathcal{D} \subset \mathbb{R}^m$, functions $\{f_j\}_{j=1}^{N_f}$, initial samples $\{\mathbf{x}_i\}_{i=1}^{M_0}$
Output : Stretched samples from $\mathcal{D}$
Start evaluating $f_j(\mathbf{x}_i)$ for $i = 1, \ldots, M_0$ and $j = 1, \ldots, N_f$;
while $k < M$ **do**
    if *a worker is avalible* **then**
        for $j = 1, \ldots, N_f$ **do**
            Fit ridge approximation $f_j(\mathbf{x}_i) \approx g_{j,k}(\mathbf{U}_{j,k}^\top \mathbf{x}_i)$ where $f_j(\mathbf{x}_i)$ exists;
            Construct candidate sample $\mathbf{x}_{k+1}^{(j)} = \operatorname*{argmax}_{\mathbf{x} \in \mathcal{D}} \min_{i=1,\ldots,k} \|\mathbf{U}_{j,k}^\top (\mathbf{x} - \mathbf{x}_i)\|_2$;
        Choose furthest point in any ridge: $\mathbf{x}_{k+1} = \operatorname*{argmax}_{j=1,\ldots,N_f} \min_{i=1,\ldots,k} \|\mathbf{U}_{j,k}^\top (\mathbf{x}_{k+1}^{(j)} - \mathbf{x}_i)\|_2$;
        Start evaluating $f_j(\mathbf{x}_k)$ for $j = 1, \ldots, N_f$;

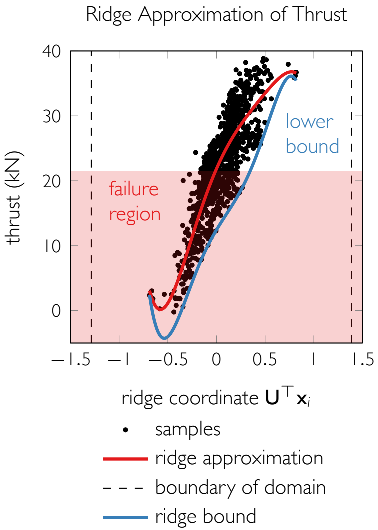
Bounding Ridge Approximations



Bounding Ridge Approximations



Bounding Ridge Approximations



To satisfy constraints conservatively we construct a **bounding ridge approximation**:

Bounding Ridge Approximation

$$\underset{g \in \mathcal{P}_p(\mathbb{R}^n)}{\text{minimize}} \sum_{i=1}^{M} |f(\mathbf{x}_i) - g(\mathbf{U}^\top \mathbf{x}_i)|^2$$

$$\text{such that } f(\mathbf{x}_i) \leq g(\mathbf{U}^\top \mathbf{x}_i) \quad i = 1, \ldots, M$$

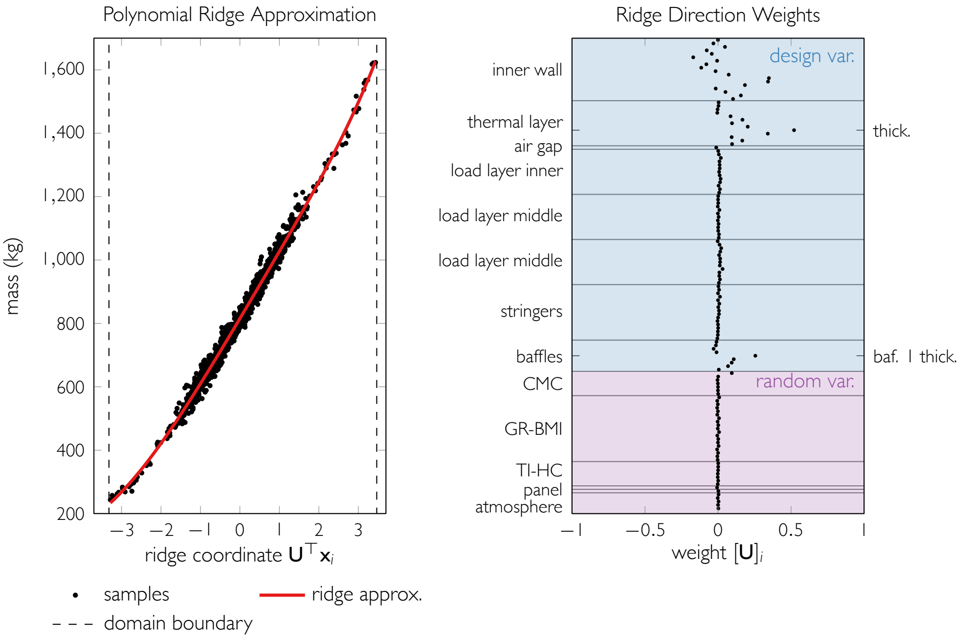
Solved using **constrained least squares**

Ridge Approximations and Ridge Bounds for MULTI-F

$f_k(\mathbf{x}) = f_k(\mathbf{x}_d, \mathbf{x}_r)$	**quantity of interest**
$\mathbf{x} = [\mathbf{x}_d, \mathbf{x}_r]$	**input variables**
$\mathbf{x}_d \in \mathcal{D}_d := \{\mathbf{x} : \boldsymbol{\ell} \leq \mathbf{x} \leq \mathbf{u}, \mathbf{A}\mathbf{x} \leq \mathbf{b}\} \subset \mathbb{R}^{96}$	**design variables**
$\mathbf{x}_r \in \mathcal{D}_r \subset \mathbb{R}^{40}$ with density function $p(\mathbf{x}_r)$	**random variables**

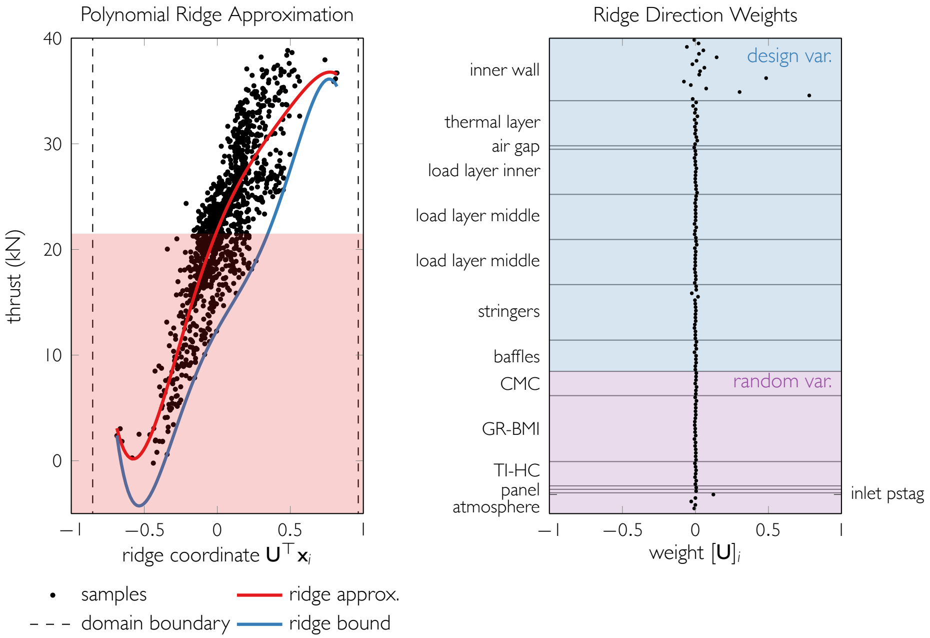
Built using **1150 3D RANS Coarse** samples

Mass



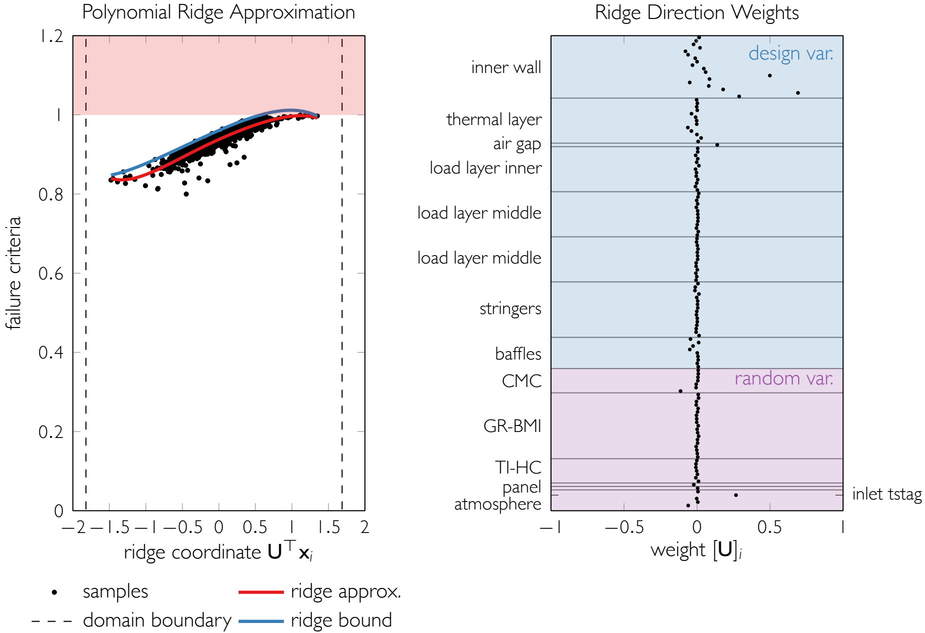
Polynomial Ridge Approximation
mass (kg)
1,600
1,400
1,200
1,000
800
600
400
200
−3
−2
−1
0
1
2
3
ridge coordinate $\mathbf{U}^\top \mathbf{x}_i$
Ridge Direction Weights
design var.
random var.
inner wall
thermal layer
air gap
load layer inner
load layer middle
load layer middle
stringers
baffles
CMC
GR-BMI
TI-HC
panel
atmosphere
thick.
baf. 1 thick.
−1
−0.5
0
0.5
1
weight $[\mathbf{U}]_i$
samples
ridge approx.
domain boundary

Thrust

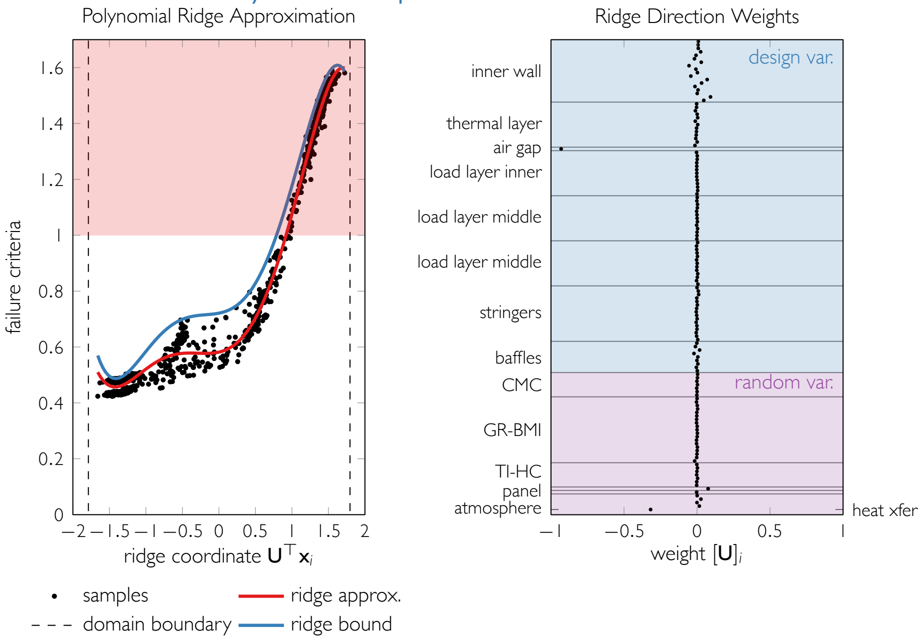


Polynomial Ridge Approximation
40
30
20
10
0
thrust (kN)
−1
−0.5
0
0.5
1
ridge coordinate $\mathbf{U}^\top \mathbf{x}_i$
samples
domain boundary
ridge approx.
ridge bound
Ridge Direction Weights
design var.
random var.
inner wall
thermal layer
air gap
load layer inner
load layer middle
load layer middle
stringers
baffles
CMC
GR-BMI
TI-HC
panel
atmosphere
inlet pstag
−1
−0.5
0
0.5
1
weight $[\mathbf{U}]_i$

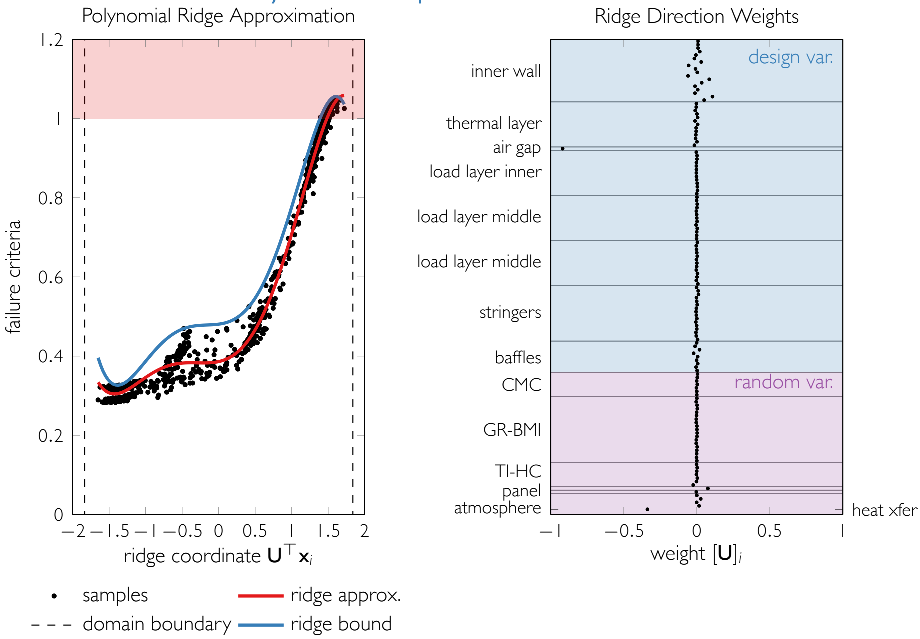
Thermal Layer Temperature Failure



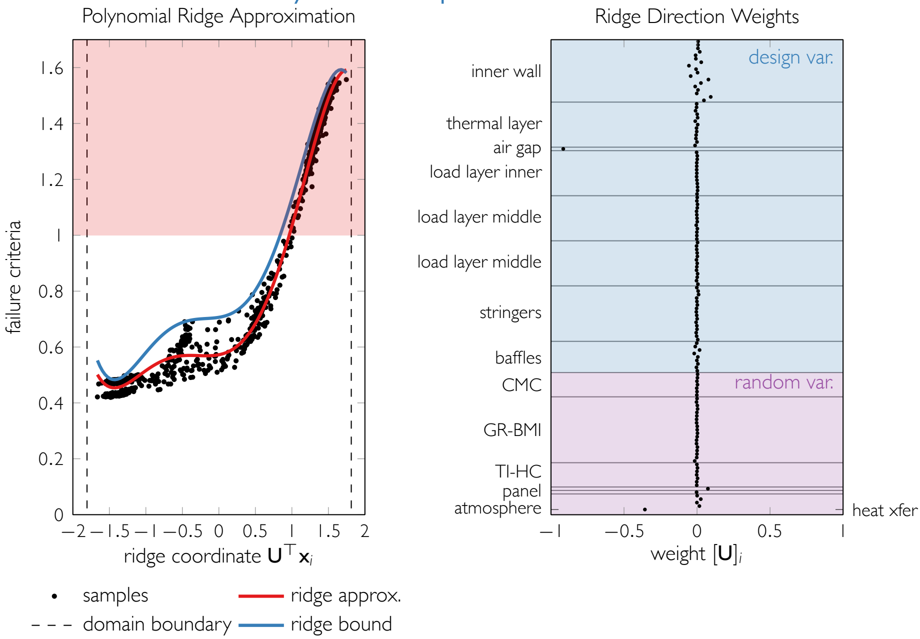
Inside Load Layer Temperature Failure



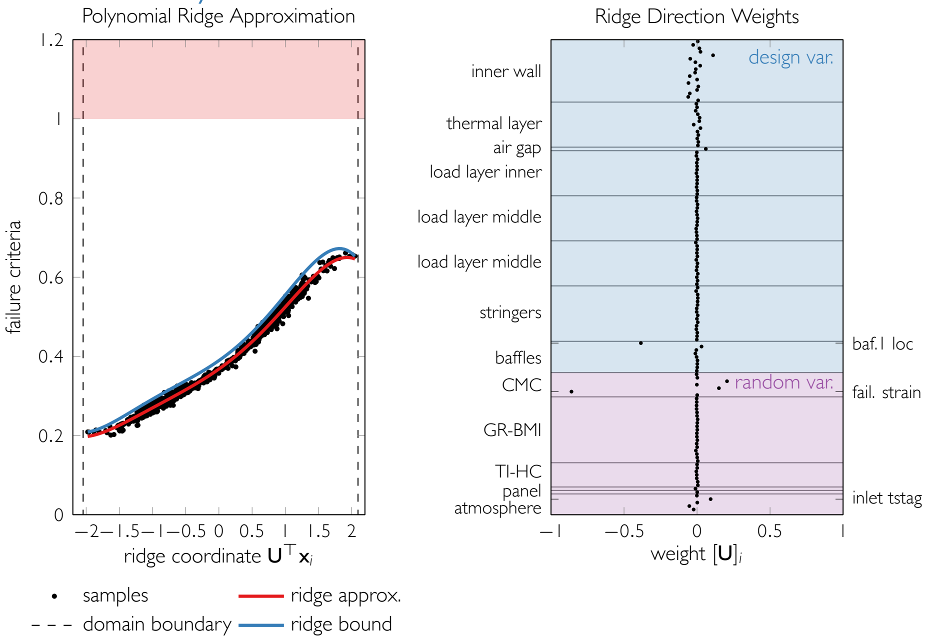
Middle Load Layer Temperature Failure



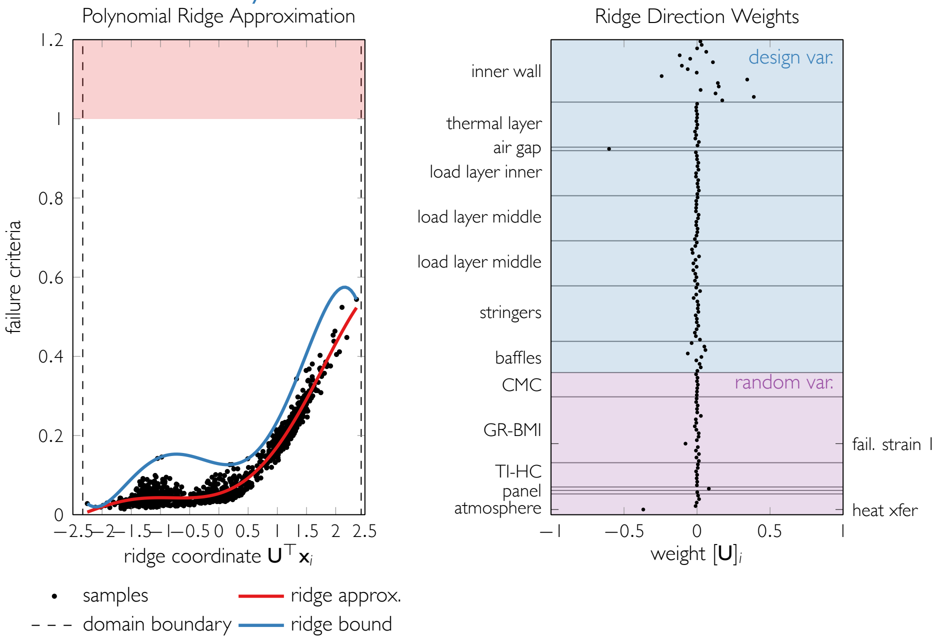
Outside Load Layer Temperature Failure



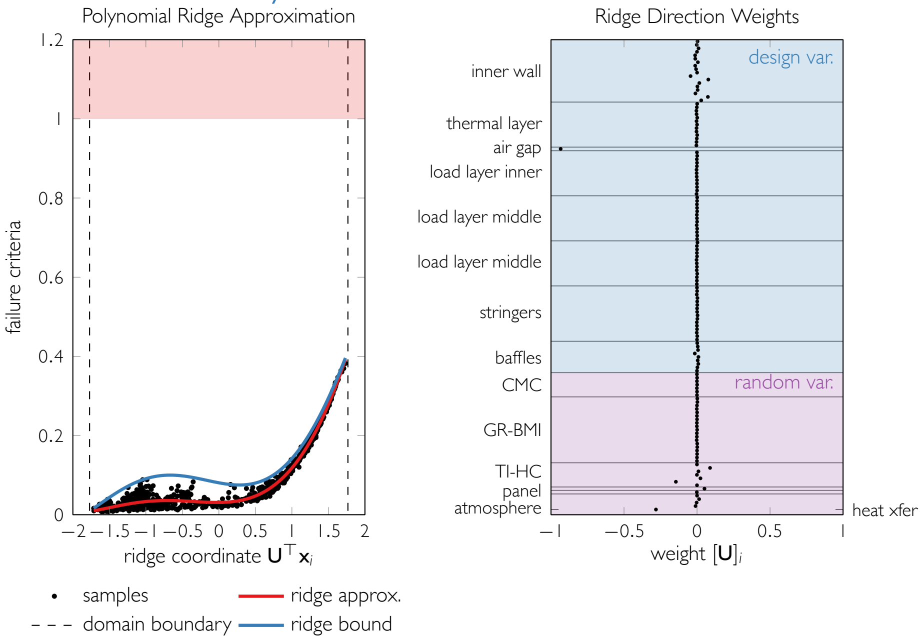
Thermal Layer Structural Failure



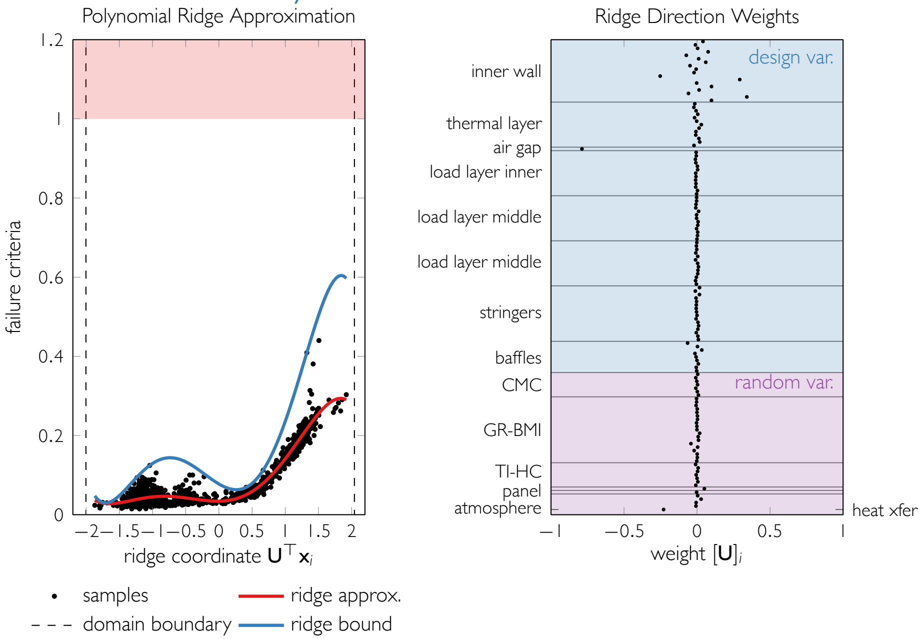
Inside Load Layer Structural Failure



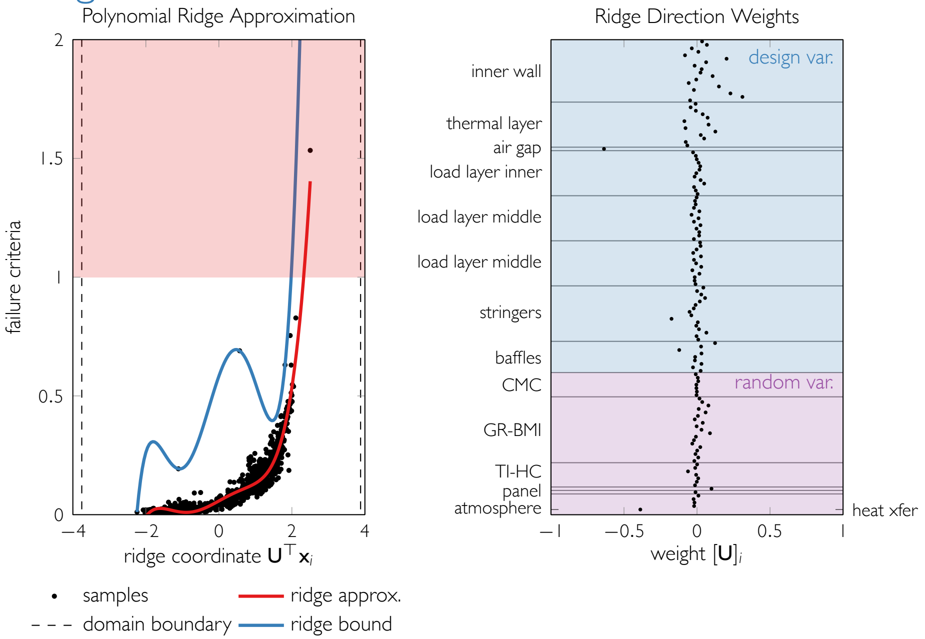
Middle Load Layer Structural Failure



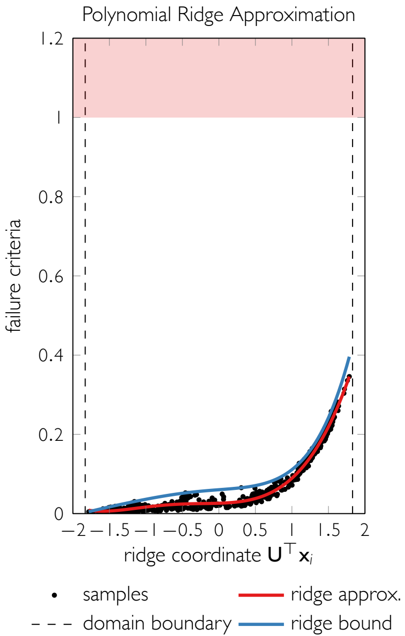
Outside Load Layer Structural Failure

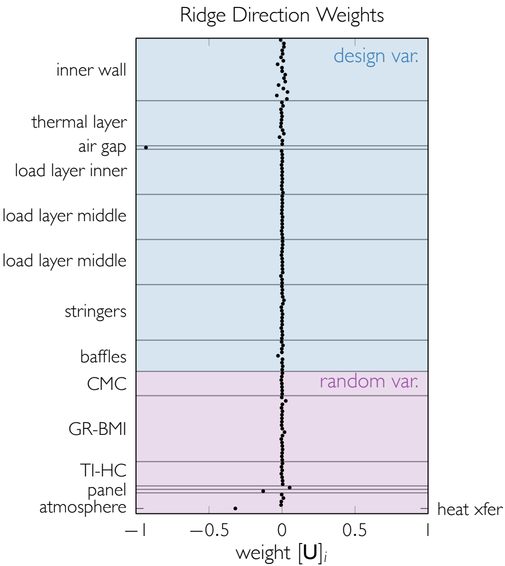


Stringers Structural Failure

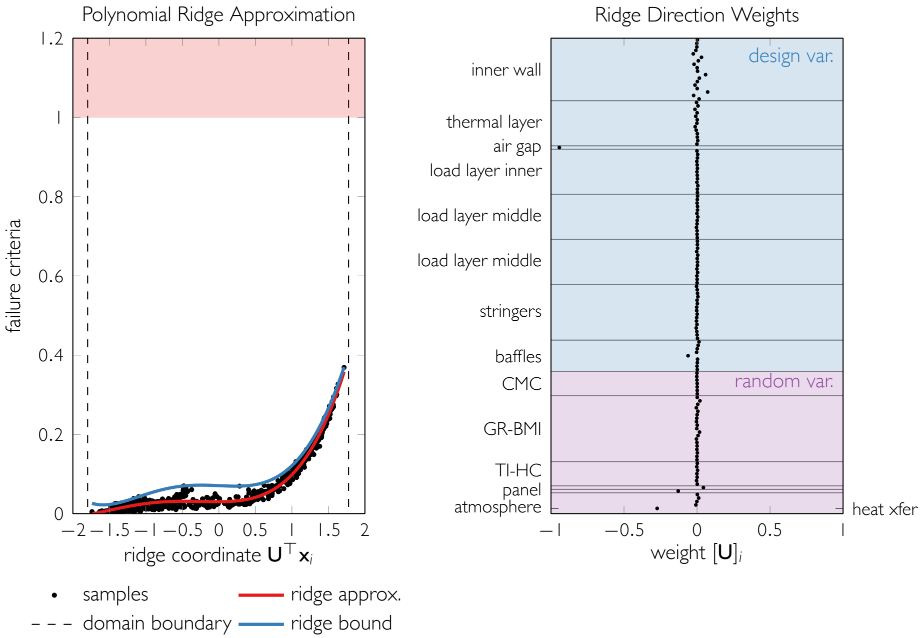


Baffle | Structural Failure

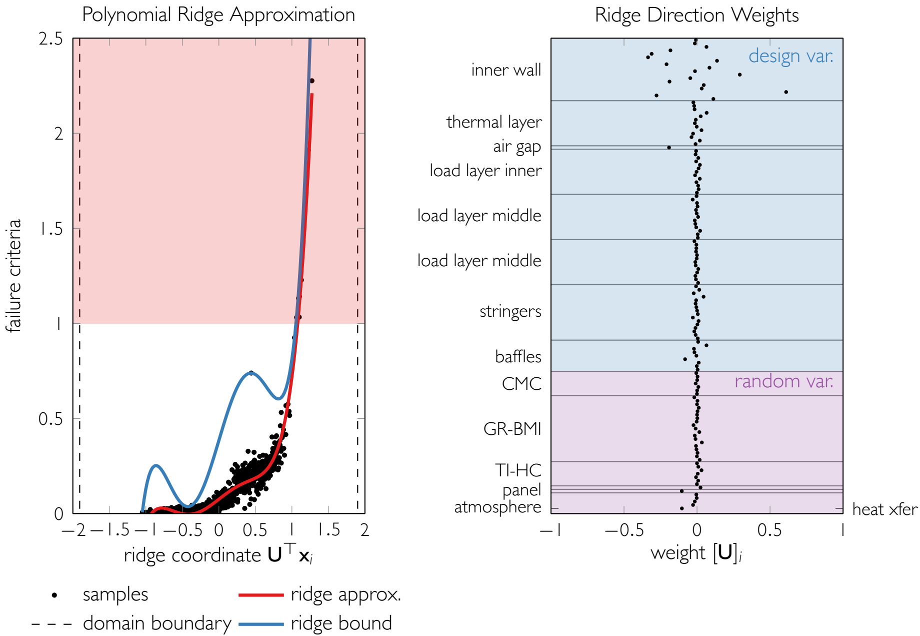




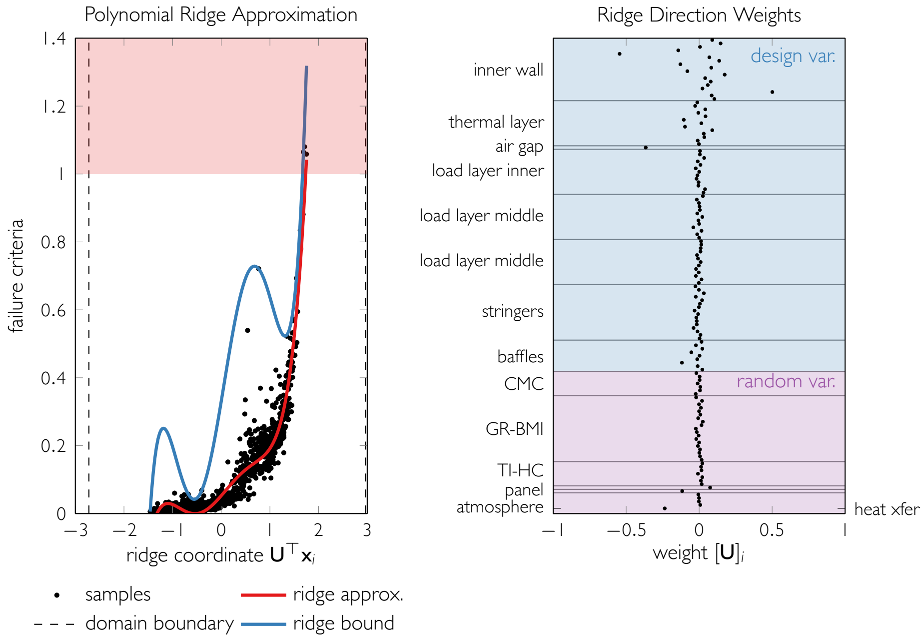
Baffle 2 Structural Failure



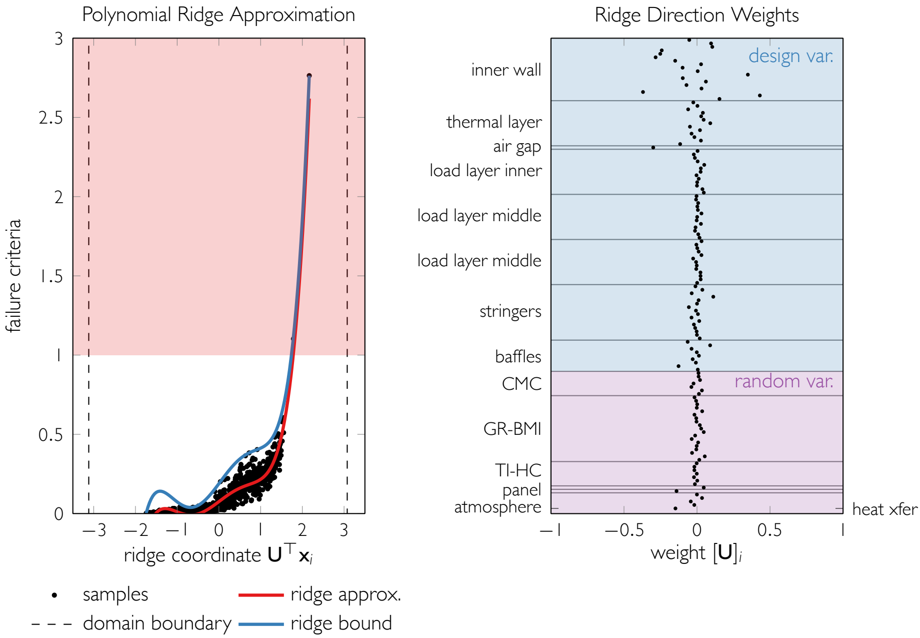
Baffle 3 Structural Failure



Baffle 4 Structural Failure



Baffle 5 Structural Failure



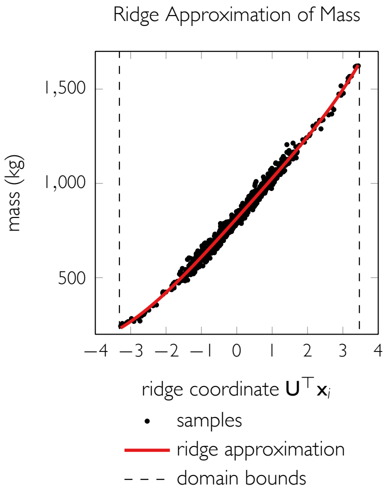
Solving the MULTI-F
Design Under Uncertainty Problem

Reliability Based Design Optimization

A Chance-Constrained Design Under Uncertainty Problem

$\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}} \; \mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)]$	mass
such that $\mathbb{P}_{\mathbf{x}_r}[f_1(\mathbf{x}_d, \mathbf{x}_r) < 21500] < \tau$	thrust
$\mathbb{P}_{\mathbf{x}_r}[f_2(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	thermal layer temperature failure
$\mathbb{P}_{\mathbf{x}_r}[f_3(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	inside load layer temperature failure
$\mathbb{P}_{\mathbf{x}_r}[f_4(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	middle load layer temperature failure
$\mathbb{P}_{\mathbf{x}_r}[f_5(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	outside load layer temperature failure
$\mathbb{P}_{\mathbf{x}_r}[f_6(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	thermal layer structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_7(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	inside load layer structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_8(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	middle load layer structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_9(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	outside load layer structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_{10}(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	stringers structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_{11}(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	baffle 1 structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_{12}(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	baffle 2 structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_{13}(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	baffle 3 structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_{14}(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	baffle 4 structural failure
$\mathbb{P}_{\mathbf{x}_r}[f_{15}(\mathbf{x}_d, \mathbf{x}_r) > 1] < \tau$	baffle 5 structural failure

Approximating Objective $\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}}\, \mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)]$



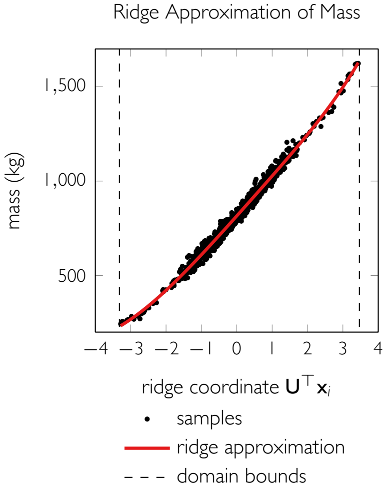
Approximating Objective $\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}}\, \mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)]$

Replace f_0 with **ridge approximation**:

$$f_0(\mathbf{x}) \approx g_0(\mathbf{U}_0^\top \mathbf{x})$$

$$f_0(\mathbf{x}_d, \mathbf{x}_r) \approx g_0\left(\begin{bmatrix} \mathbf{U}_{0,d} \\ \mathbf{U}_{0,r} \end{bmatrix}^\top \begin{bmatrix} \mathbf{x}_d \\ \mathbf{x}_r \end{bmatrix} \right)$$

$$= g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbf{U}_{0,r}^\top \mathbf{x}_r)$$



Approximating Objective $\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}}\ \mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)]$

Replace f_0 with **ridge approximation**:

$$f_0(\mathbf{x}) \approx g_0(\mathbf{U}_0^\top \mathbf{x})$$

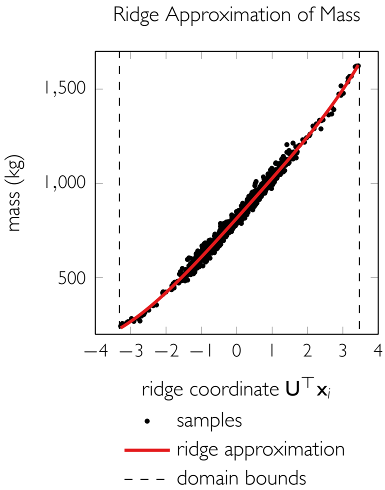
$$f_0(\mathbf{x}_d, \mathbf{x}_r) \approx g_0\left(\begin{bmatrix} \mathbf{U}_{0,d} \\ \mathbf{U}_{0,r} \end{bmatrix}^\top \begin{bmatrix} \mathbf{x}_d \\ \mathbf{x}_r \end{bmatrix} \right)$$

$$= g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbf{U}_{0,r}^\top \mathbf{x}_r)$$

Approximate **mean**

$$\mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)] \approx \mathbb{E}_{\mathbf{x}_r}[g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbf{U}_{0,r}^\top \mathbf{x}_r)]$$

$$\approx g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbb{E}_{\mathbf{x}_r}[\mathbf{U}_{0,r}^\top \mathbf{x}_r])$$



Approximating Objective $\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}} \, \mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)]$

Replace f_0 with **ridge approximation**:

$$f_0(\mathbf{x}) \approx g_0(\mathbf{U}_0^\top \mathbf{x})$$

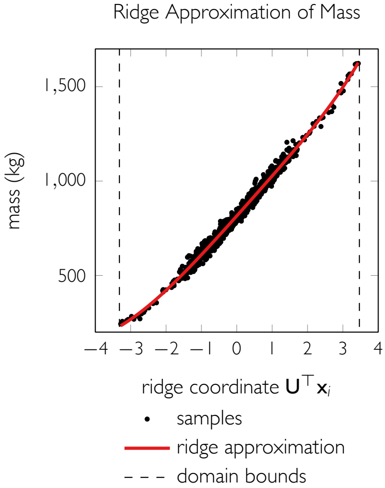
$$f_0(\mathbf{x}_d, \mathbf{x}_r) \approx g_0\left(\begin{bmatrix} \mathbf{U}_{0,d} \\ \mathbf{U}_{0,r} \end{bmatrix}^\top \begin{bmatrix} \mathbf{x}_d \\ \mathbf{x}_r \end{bmatrix} \right) = g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbf{U}_{0,r}^\top \mathbf{x}_r)$$

Approximate **mean**

$$\mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)] \approx \mathbb{E}_{\mathbf{x}_r}[g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbf{U}_{0,r}^\top \mathbf{x}_r)] \approx g_0(\mathbf{U}_{0,d}^\top \mathbf{x}_d + \mathbb{E}_{\mathbf{x}_r}[\mathbf{U}_{0,r}^\top \mathbf{x}_r])$$

As g_0 is **monotonic**, this yields a linear objective

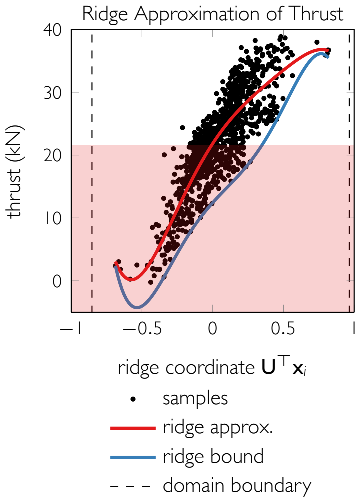
$$\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}} \, \mathbb{E}_{\mathbf{x}_r}[f_0(\mathbf{x}_d, \mathbf{x}_r)] \Rightarrow \underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}} \; g'(0)\mathbf{U}_{0,d}^\top \mathbf{x}_d$$



Approximating Constraint $\mathbb{P}_{\mathbf{x}_r}[f_k(\mathbf{x}_d, \mathbf{x}_r) > \alpha_k] < \tau$

Replace f_1 with **ridge bound**:

$$f_1(\mathbf{x}_d, \mathbf{x}_r) \gtrsim g_1(\mathbf{U}_{1,d}^\top \mathbf{x}_d + \mathbf{U}_{1,r}^\top \mathbf{x}_r)$$



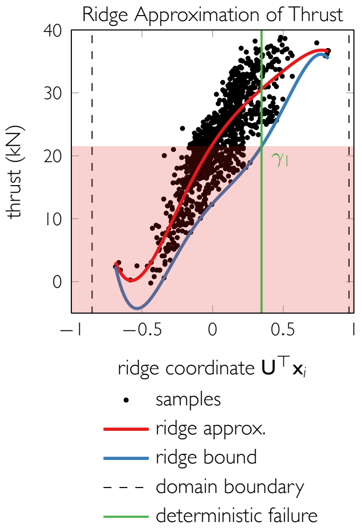
Approximating Constraint $\mathbb{P}_{\mathbf{x}_r}[f_k(\mathbf{x}_d, \mathbf{x}_r) > \alpha_k] < \tau$

Replace f_1 with **ridge bound**:

$$f_1(\mathbf{x}_d, \mathbf{x}_r) \gtrsim g_1(\mathbf{U}_{1,d}^\top \mathbf{x}_d + \mathbf{U}_{1,r}^\top \mathbf{x}_r)$$

Compute deterministic failure point γ_1

$$g_1(\gamma_1) = \alpha_1 = 21500$$



Approximating Constraint $\mathbb{P}_{\mathbf{x}_r}[f_k(\mathbf{x}_d, \mathbf{x}_r) > \alpha_k] < \tau$

Replace f_1 with **ridge bound**:

$$f_1(\mathbf{x}_d, \mathbf{x}_r) \gtrsim g_1(\mathbf{U}_{1,d}^\top \mathbf{x}_d + \mathbf{U}_{1,r}^\top \mathbf{x}_r)$$

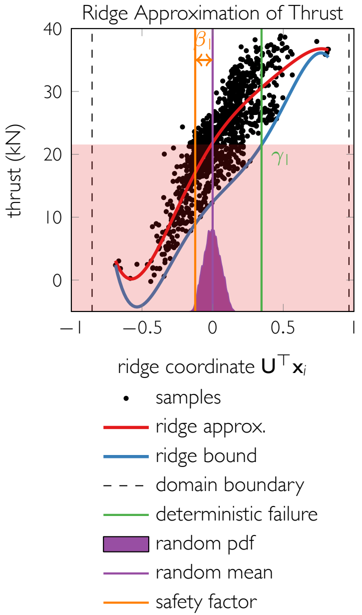
Compute deterministic failure point γ_1

$$g_1(\gamma_1) = \alpha_1 = 21500$$

Compute **empirical safety factor**

$$\beta_1 = \operatorname*{argmax}_{\beta \in \mathbb{R}} \beta \text{ such that } \mathbb{P}_{\mathbf{x}_r}[\mathbf{U}_{1,r}^\top \mathbf{x}_r > \beta] < \tau$$

(e.g., compute using estimated CDF of $\mathbf{U}_{1,r}^\top \mathbf{x}_r$)



Approximating Constraint $\mathbb{P}_{\mathbf{x}_r}[f_k(\mathbf{x}_d, \mathbf{x}_r) > \alpha_k] < \tau$

Replace f_1 with **ridge bound**:

$$f_1(\mathbf{x}_d, \mathbf{x}_r) \gtrsim g_1(\mathbf{U}_{1,d}^\top \mathbf{x}_d + \mathbf{U}_{1,r}^\top \mathbf{x}_r)$$

Compute deterministic failure point γ_1

$$g_1(\gamma_1) = \alpha_1 = 21500$$

Compute **empirical safety factor**

$$\beta_1 = \operatorname*{argmax}_{\beta \in \mathbb{R}} \beta \text{ such that } \mathbb{P}_{\mathbf{x}_r}[\mathbf{U}_{1,r}^\top \mathbf{x}_r > \beta] < \tau$$

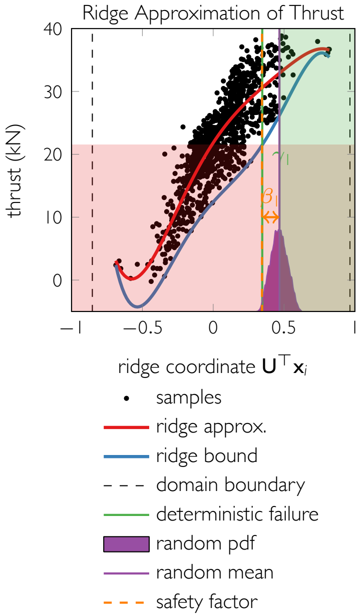
(e.g., compute using estimated CDF of $\mathbf{U}_{1,r}^\top \mathbf{x}_r$)

Add **linear constraint** buffered by safety factor:

$$\mathbf{U}_{1,d}^\top \mathbf{x}_d + \beta_1 \geq \gamma_1$$

$$g_1(\mathbf{U}_{1,d}^\top \mathbf{x}_d + \beta_1) \geq \alpha_1 = 21500$$

$$\mathbb{P}_{\mathbf{x}_r}[g_1(\mathbf{U}_{1,d}^\top \mathbf{x}_d + \mathbf{U}_{1,r}^\top \mathbf{x}_r) \leq 21500] < \tau$$



Approximating Constraint $\mathbb{P}_{\mathbf{x}_r}[f_k(\mathbf{x}_d, \mathbf{x}_r) < \alpha_k] < \tau$

Replace f_2 with **ridge bound**:

$$f_2(\mathbf{x}_d, \mathbf{x}_r) \lesssim g_2(\mathbf{U}_{2,d}^\top \mathbf{x}_d + \mathbf{U}_{2,r}^\top \mathbf{x}_r)$$

Compute deterministic failure point γ_2

$$g_2(\gamma_2) = \alpha_2 = 1$$

Compute empirical safety factor

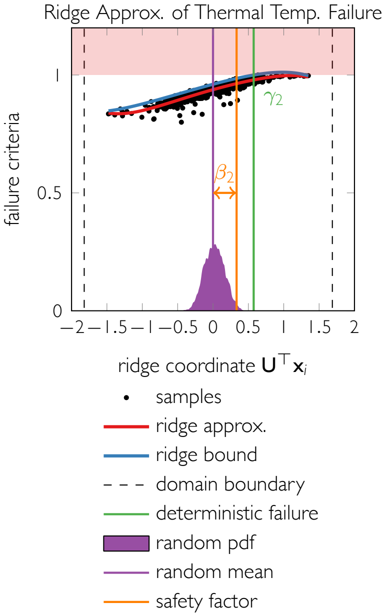
$$\beta_2 = \operatorname*{argmin}_{\beta \in \mathbb{R}} \beta \text{ such that } \mathbb{P}_{\mathbf{x}_r}[\mathbf{U}_{2,r}^\top \mathbf{x}_r > \beta] < \tau$$

(e.g., compute using estimated CDF of $\mathbf{U}_{2,r}^\top \mathbf{x}_r$)
Add linear constraint buffered by safety factor:

$$\mathbf{U}_{2,d}^\top \mathbf{x}_d + \beta_2 \leq \gamma_2$$

$$g_2(\mathbf{U}_{2,d}^\top \mathbf{x}_d + \beta_2) \leq \alpha_2 = 1$$

$$\mathbb{P}_{\mathbf{x}_r}[g_2(\mathbf{U}_{2,d}^\top \mathbf{x}_d + \mathbf{U}_{2,r}^\top \mathbf{x}_r) \leq 1] < \tau$$



Approximating Constraint $\mathbb{P}_{\mathbf{x}_r}[f_k(\mathbf{x}_d, \mathbf{x}_r) < \alpha_k] < \tau$

Replace f_2 with **ridge bound**:

$$f_2(\mathbf{x}_d, \mathbf{x}_r) \lesssim g_2(\mathbf{U}_{2,d}^\top \mathbf{x}_d + \mathbf{U}_{2,r}^\top \mathbf{x}_r)$$

Compute deterministic failure point γ_2

$$g_2(\gamma_2) = \alpha_2 = 1$$

Compute empirical safety factor

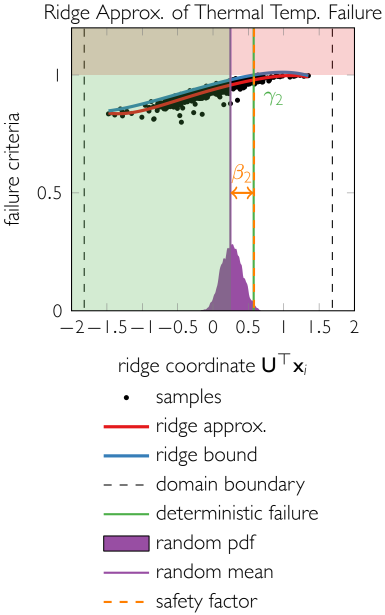
$$\beta_2 = \operatorname*{argmin}_{\beta \in \mathbb{R}} \beta \text{ such that } \mathbb{P}_{\mathbf{x}_r}[\mathbf{U}_{2,r}^\top \mathbf{x}_r > \beta] < \tau$$

(e.g., compute using estimated CDF of $\mathbf{U}_{2,r}^\top \mathbf{x}_r$)
Add linear constraint buffered by safety factor:

$$\mathbf{U}_{2,d}^\top \mathbf{x}_d + \beta_2 \leq \gamma_2$$

$$g_2(\mathbf{U}_{2,d}^\top \mathbf{x}_d + \beta_2) \leq \alpha_2 = 1$$

$$\mathbb{P}_{\mathbf{x}_r}[g_2(\mathbf{U}_{2,d}^\top \mathbf{x}_d + \mathbf{U}_{2,r}^\top \mathbf{x}_r) \leq 1] < \tau$$



Linear Program for RBDO

$$
\begin{array}{rlr}
\underset{\mathbf{x}_d \in \mathcal{D}_d}{\text{minimize}} & g_0'(0)\mathbf{U}_{d,0}^\top \mathbf{x}_d & \text{mass} \\
\text{such that} & \mathbf{U}_{1,d}^\top \mathbf{x}_d + \beta_1 \geq \gamma_1 & \text{thrust} \\
& \mathbf{U}_{2,d}^\top \mathbf{x}_d + \beta_2 \leq \gamma_2 & \text{thermal layer temperature failure} \\
& \mathbf{U}_{3,d}^\top \mathbf{x}_d + \beta_3 \leq \gamma_3 & \text{inside load layer temperature failure} \\
& \mathbf{U}_{4,d}^\top \mathbf{x}_d + \beta_4 \leq \gamma_4 & \text{middle load layer temperature failure} \\
& \mathbf{U}_{5,d}^\top \mathbf{x}_d + \beta_5 \leq \gamma_5 & \text{outside load layer temperature failure} \\
& \mathbf{U}_{6,d}^\top \mathbf{x}_d + \beta_6 \leq \gamma_6 & \text{thermal layer structural failure} \\
& \mathbf{U}_{7,d}^\top \mathbf{x}_d + \beta_7 \leq \gamma_7 & \text{inside load layer structural failure} \\
& \mathbf{U}_{8,d}^\top \mathbf{x}_d + \beta_8 \leq \gamma_8 & \text{middle load layer structural failure} \\
& \mathbf{U}_{9,d}^\top \mathbf{x}_d + \beta_9 \leq \gamma_9 & \text{outside load layer structural failure} \\
& \mathbf{U}_{10,d}^\top \mathbf{x}_d + \beta_{10} \leq \gamma_{10} & \text{stringers structural failure} \\
& \mathbf{U}_{11,d}^\top \mathbf{x}_d + \beta_{11} \leq \gamma_{11} & \text{baffle 1 structural failure} \\
& \mathbf{U}_{12,d}^\top \mathbf{x}_d + \beta_{12} \leq \gamma_{12} & \text{baffle 2 structural failure} \\
& \mathbf{U}_{13,d}^\top \mathbf{x}_d + \beta_{13} \leq \gamma_{13} & \text{baffle 3 structural failure} \\
& \mathbf{U}_{14,d}^\top \mathbf{x}_d + \beta_{14} \leq \gamma_{14} & \text{baffle 4 structural failure} \\
& \mathbf{U}_{15,d}^\top \mathbf{x}_d + \beta_{15} \leq \gamma_{15} & \text{baffle 5 structural failure}
\end{array}
$$

Designs for MULTI-F

Estimating Failure Probabilities

How do we evaluate the chance constraint $\mathbb{P}_{\mathbf{x}_r}[f_i(\mathbf{x}_d, \mathbf{x}_r) > 1]$?

- Sample model at design point $\mathbf{x}_d$ and randomly in $\mathbf{x}_r$ 100 times
- Build a linear model of $f_i(\mathbf{x}_d, \mathbf{x}_r)$:

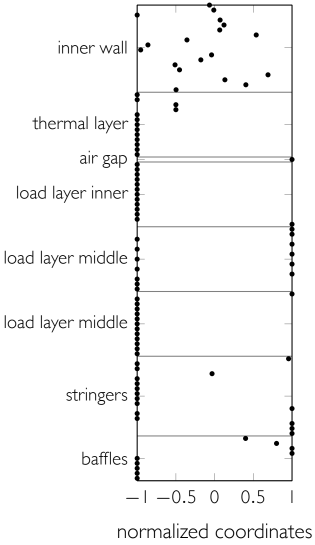
$$f_i(\mathbf{x}_d, \mathbf{x}_r) \approx g(\mathbf{x}_r) := \alpha + \mathbf{a}^\top \mathbf{x}_r$$

- Estimate failure criteria with surrogate using Monte-Carlo

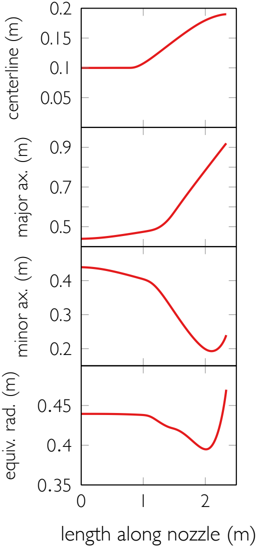
$$\mathbb{P}_{\mathbf{x}_r}[f_i(\mathbf{x}_d, \mathbf{x}_r) > 1] \approx \mathbb{P}_{\mathbf{x}_r}[g(\mathbf{x}_r) > 1] \approx \sum_{i=1}^{N} [\alpha + \mathbf{a}^\top \mathbf{x}_r^{(i)} > 1]$$

Design with Failure Probability $\tau = 10^{-1}$

Design Variables



Inner Wall Shape

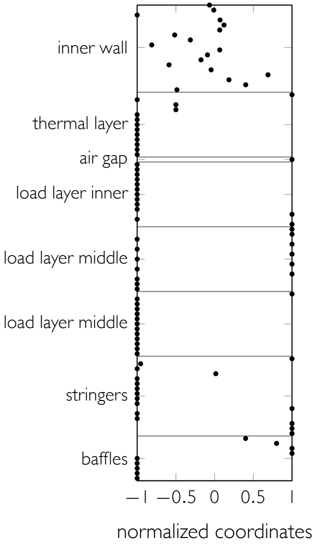


Estimated Failure Probabilities

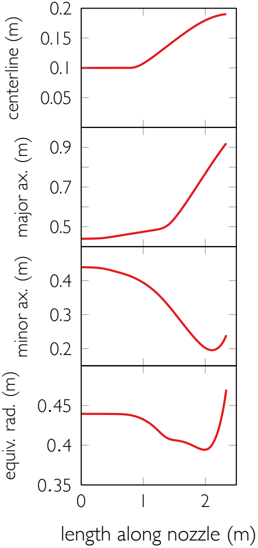
QoI	$\mathbb{E}$ or $\mathbb{P}$
mass	$2.66 \cdot 10^{2}$
thrust	$3.55 \cdot 10^{-5}$
thermal temp. fail.	$1.44 \cdot 10^{-5}$
inside load temp. fail.	$0.00 \cdot 10^{0}$
middle load temp. fail.	$0.00 \cdot 10^{0}$
outside load temp. fail.	$0.00 \cdot 10^{0}$
thermal layer struct. fail.	$0.00 \cdot 10^{0}$
inside load struct. fail.	$0.00 \cdot 10^{0}$
middle load struct. fail.	$0.00 \cdot 10^{0}$
outside load struct. fail.	$0.00 \cdot 10^{0}$
stringers struct. fail.	$0.00 \cdot 10^{0}$
baffle 1 fail.	$0.00 \cdot 10^{0}$
baffle 2 fail.	$0.00 \cdot 10^{0}$
baffle 3 fail.	$0.00 \cdot 10^{0}$
baffle 4 fail.	$0.00 \cdot 10^{0}$
baffle 5 fail.	$0.00 \cdot 10^{0}$

Design with Failure Probability $\tau = 10^{-2}$

Design Variables



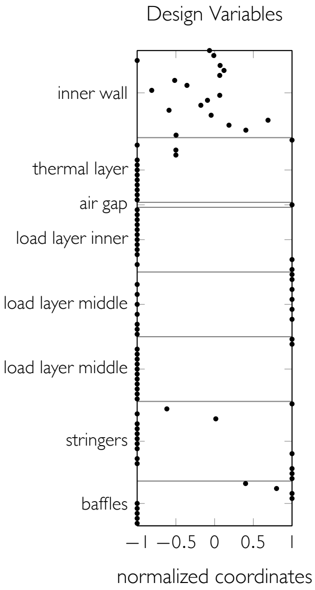
Inner Wall Shape

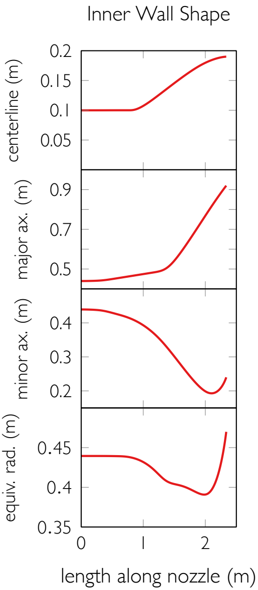


Estimated Failure Probabilities

QoI	$\mathbb{E}$ or $\mathbb{P}$
mass	$2.67 \cdot 10^{2}$
thrust	$4.41 \cdot 10^{-5}$
thermal temp. fail.	$1.33 \cdot 10^{-5}$
inside load temp. fail.	$0.00 \cdot 10^{0}$
middle load temp. fail.	$0.00 \cdot 10^{0}$
outside load temp. fail.	$0.00 \cdot 10^{0}$
thermal layer struct. fail.	$0.00 \cdot 10^{0}$
inside load struct. fail.	$0.00 \cdot 10^{0}$
middle load struct. fail.	$0.00 \cdot 10^{0}$
outside load struct. fail.	$0.00 \cdot 10^{0}$
stringers struct. fail.	$0.00 \cdot 10^{0}$
baffle 1 fail.	$0.00 \cdot 10^{0}$
baffle 2 fail.	$0.00 \cdot 10^{0}$
baffle 3 fail.	$0.00 \cdot 10^{0}$
baffle 4 fail.	$0.00 \cdot 10^{0}$
baffle 5 fail.	$0.00 \cdot 10^{0}$

Design with Failure Probability $\tau = 10^{-3}$



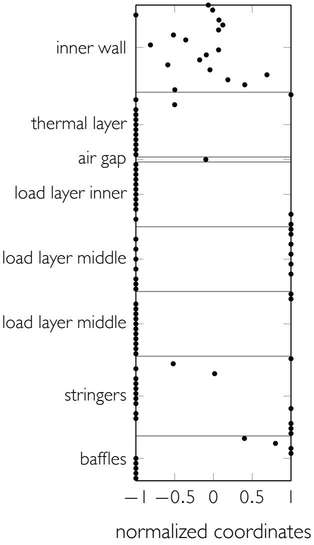


Estimated Failure Probabilities

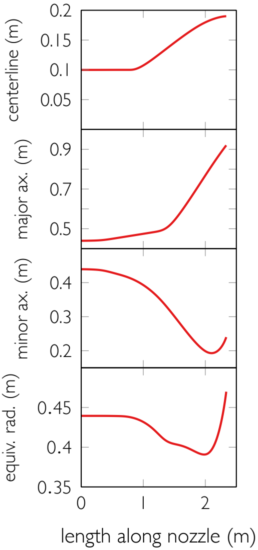
QoI	$\mathbb{E}$ or $\mathbb{P}$
mass	$2.71 \cdot 10^{2}$
thrust	$7.55 \cdot 10^{-5}$
thermal temp. fail.	$1.19 \cdot 10^{-5}$
inside load temp. fail.	$0.00 \cdot 10^{0}$
middle load temp. fail.	$0.00 \cdot 10^{0}$
outside load temp. fail.	$0.00 \cdot 10^{0}$
thermal layer struct. fail.	$0.00 \cdot 10^{0}$
inside load struct. fail.	$0.00 \cdot 10^{0}$
middle load struct. fail.	$0.00 \cdot 10^{0}$
outside load struct. fail.	$0.00 \cdot 10^{0}$
stringers struct. fail.	$0.00 \cdot 10^{0}$
baffle 1 fail.	$0.00 \cdot 10^{0}$
baffle 2 fail.	$0.00 \cdot 10^{0}$
baffle 3 fail.	$0.00 \cdot 10^{0}$
baffle 4 fail.	$0.00 \cdot 10^{0}$
baffle 5 fail.	$0.00 \cdot 10^{0}$

Design with Failure Probability $\tau = 10^{-4}$

Design Variables



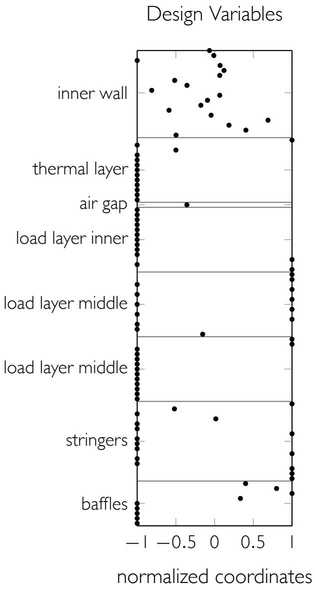
Inner Wall Shape

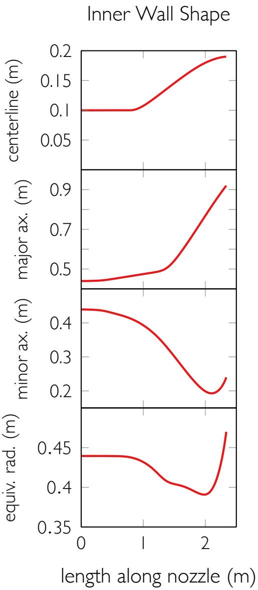


Estimated Failure Probabilities

QoI	$\mathbb{E}$ or $\mathbb{P}$
mass	$2.71 \cdot 10^{2}$
thrust	$7.59 \cdot 10^{-5}$
thermal temp. fail.	$5.70 \cdot 10^{-6}$
inside load temp. fail.	$0.00 \cdot 10^{0}$
middle load temp. fail.	$0.00 \cdot 10^{0}$
outside load temp. fail.	$0.00 \cdot 10^{0}$
thermal layer struct. fail.	$0.00 \cdot 10^{0}$
inside load struct. fail.	$0.00 \cdot 10^{0}$
middle load struct. fail.	$0.00 \cdot 10^{0}$
outside load struct. fail.	$0.00 \cdot 10^{0}$
stringers struct. fail.	$0.00 \cdot 10^{0}$
baffle 1 fail.	$0.00 \cdot 10^{0}$
baffle 2 fail.	$0.00 \cdot 10^{0}$
baffle 3 fail.	$0.00 \cdot 10^{0}$
baffle 4 fail.	$0.00 \cdot 10^{0}$
baffle 5 fail.	$0.00 \cdot 10^{0}$

Design with Failure Probability $\tau = 10^{-5}$

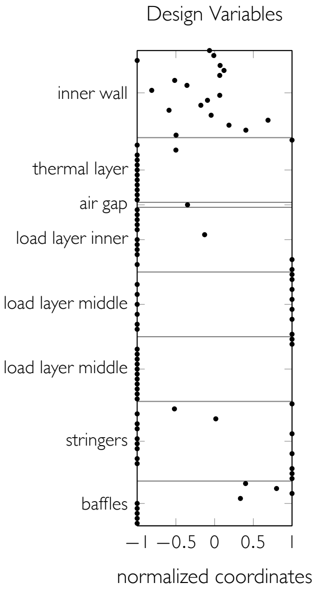


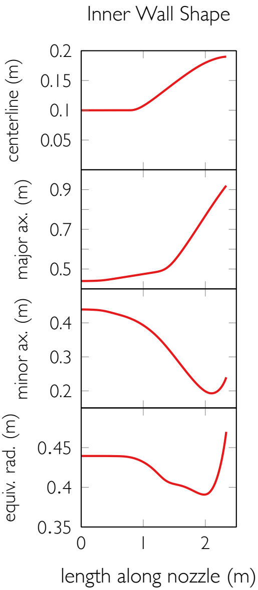


Estimated Failure Probabilities

QoI	$\mathbb{E}$ or $\mathbb{P}$
mass	$2.75 \cdot 10^{2}$
thrust	$7.70 \cdot 10^{-5}$
thermal temp. fail.	$1.53 \cdot 10^{-4}$
inside load temp. fail.	$0.00 \cdot 10^{0}$
middle load temp. fail.	$0.00 \cdot 10^{0}$
outside load temp. fail.	$0.00 \cdot 10^{0}$
thermal layer struct. fail.	$0.00 \cdot 10^{0}$
inside load struct. fail.	$0.00 \cdot 10^{0}$
middle load struct. fail.	$0.00 \cdot 10^{0}$
outside load struct. fail.	$0.00 \cdot 10^{0}$
stringers struct. fail.	$0.00 \cdot 10^{0}$
baffle 1 fail.	$0.00 \cdot 10^{0}$
baffle 2 fail.	$0.00 \cdot 10^{0}$
baffle 3 fail.	$0.00 \cdot 10^{0}$
baffle 4 fail.	$0.00 \cdot 10^{0}$
baffle 5 fail.	$0.00 \cdot 10^{0}$

Design with Failure Probability $\tau = 10^{-6}$

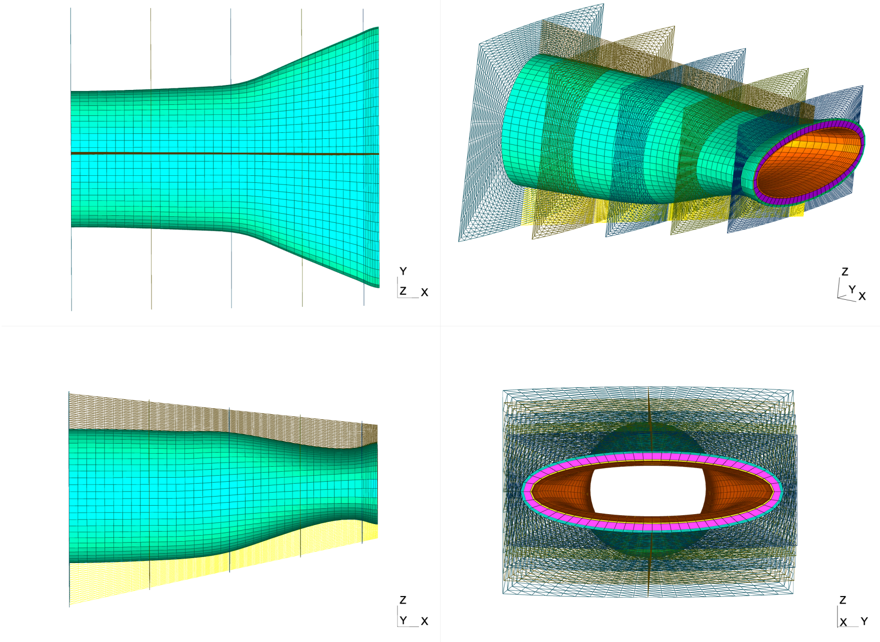




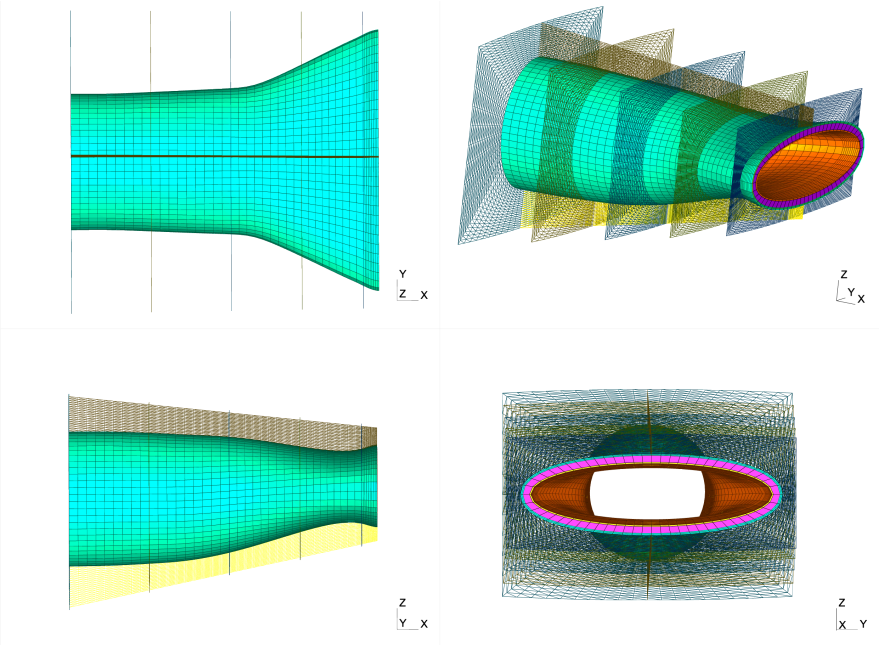
Estimated Failure Probabilities

QoI	$\mathbb{E}$ or $\mathbb{P}$
mass	$2.78 \cdot 10^{2}$
thrust	$7.45 \cdot 10^{-5}$
thermal temp. fail.	$1.57 \cdot 10^{-4}$
inside load temp. fail.	$0.00 \cdot 10^{0}$
middle load temp. fail.	$0.00 \cdot 10^{0}$
outside load temp. fail.	$0.00 \cdot 10^{0}$
thermal layer struct. fail.	$0.00 \cdot 10^{0}$
inside load struct. fail.	$0.00 \cdot 10^{0}$
middle load struct. fail.	$0.00 \cdot 10^{0}$
outside load struct. fail.	$0.00 \cdot 10^{0}$
stringers struct. fail.	$0.00 \cdot 10^{0}$
baffle 1 fail.	$0.00 \cdot 10^{0}$
baffle 2 fail.	$0.00 \cdot 10^{0}$
baffle 3 fail.	$0.00 \cdot 10^{0}$
baffle 4 fail.	$0.00 \cdot 10^{0}$
baffle 5 fail.	$0.00 \cdot 10^{0}$

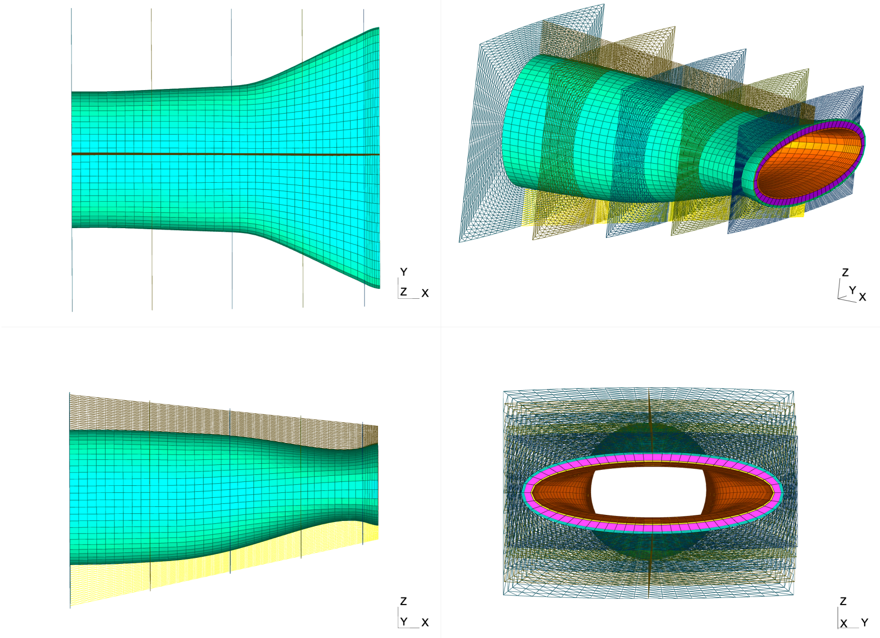
Design with Failure Probability $\tau = 10^{-1}$



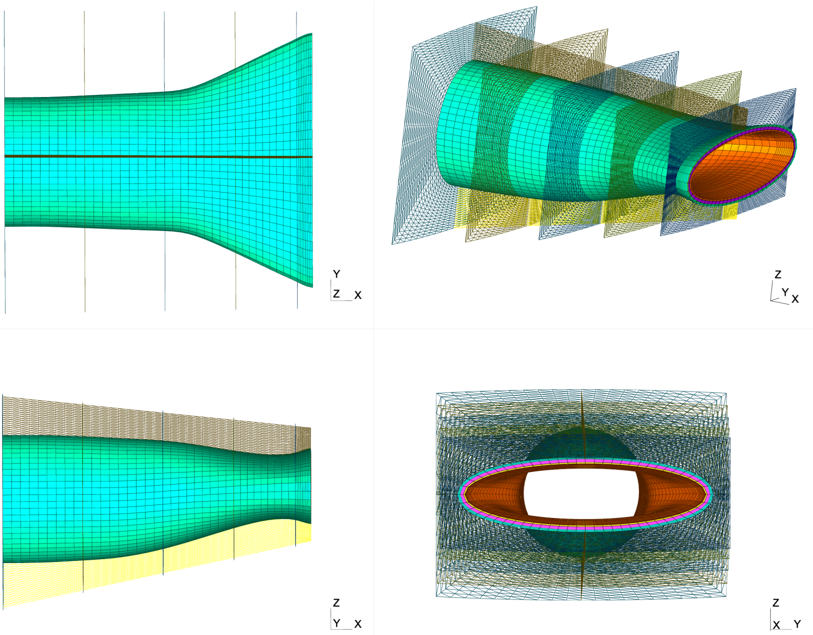
Design with Failure Probability $\tau = 10^{-2}$



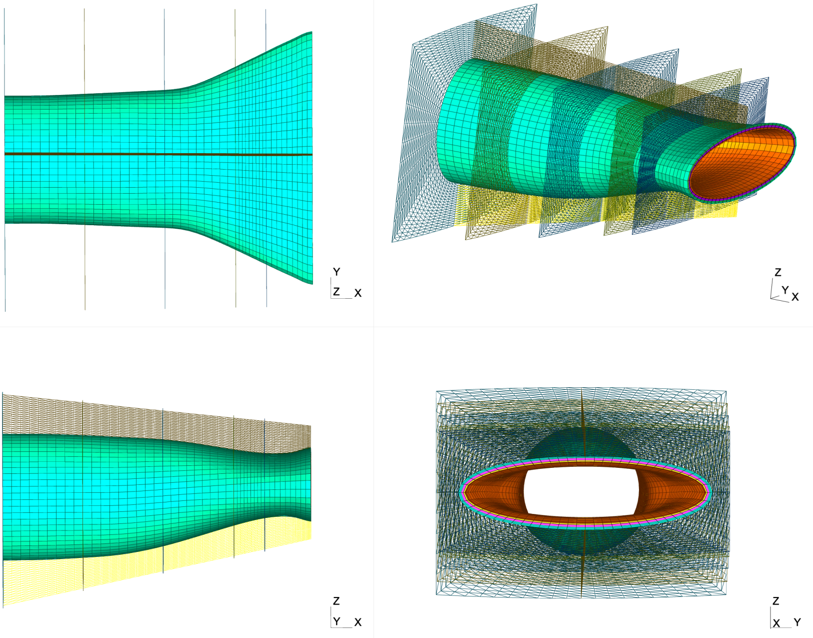
Design with Failure Probability $\tau = 10^{-3}$



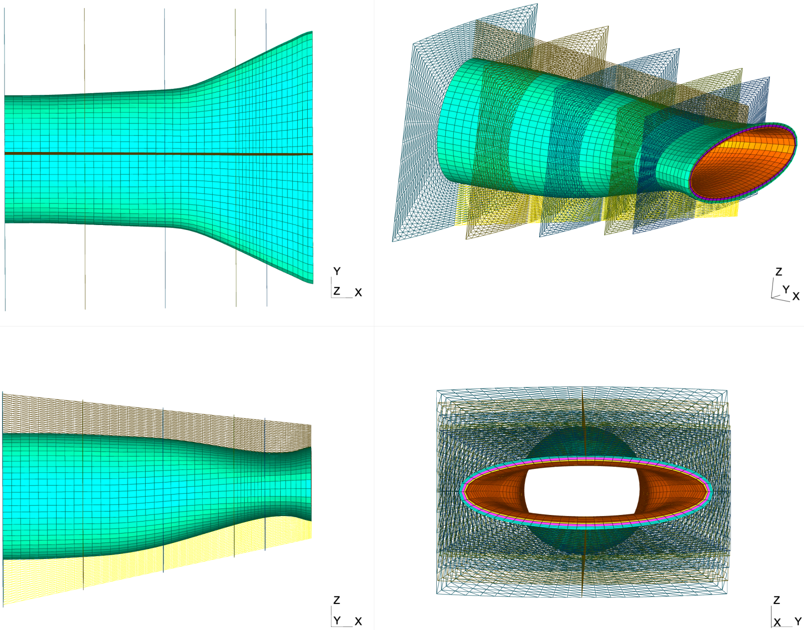
Design with Failure Probability $\tau = 10^{-4}$



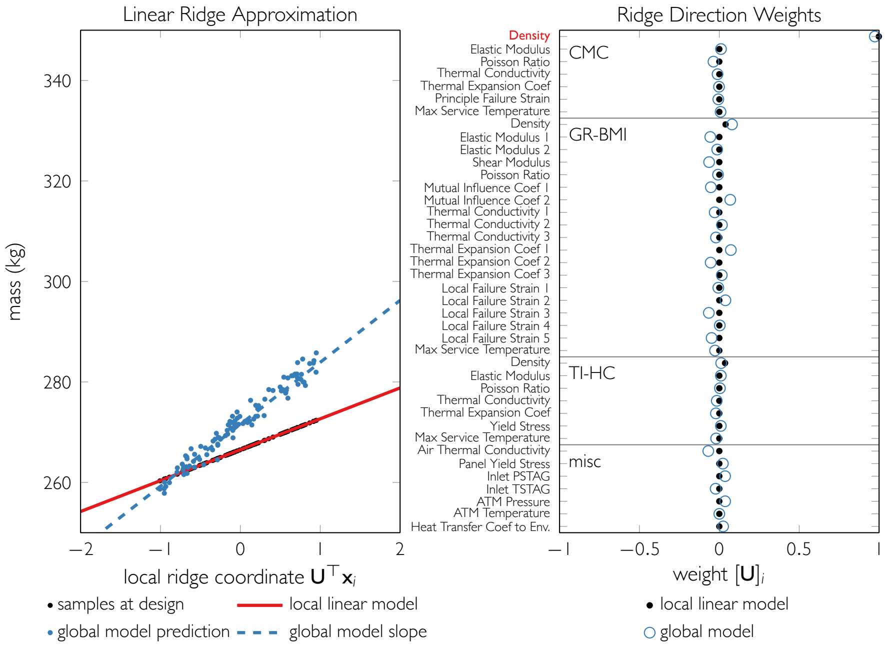
Design with Failure Probability $\tau = 10^{-5}$



Design with Failure Probability $\tau = 10^{-6}$

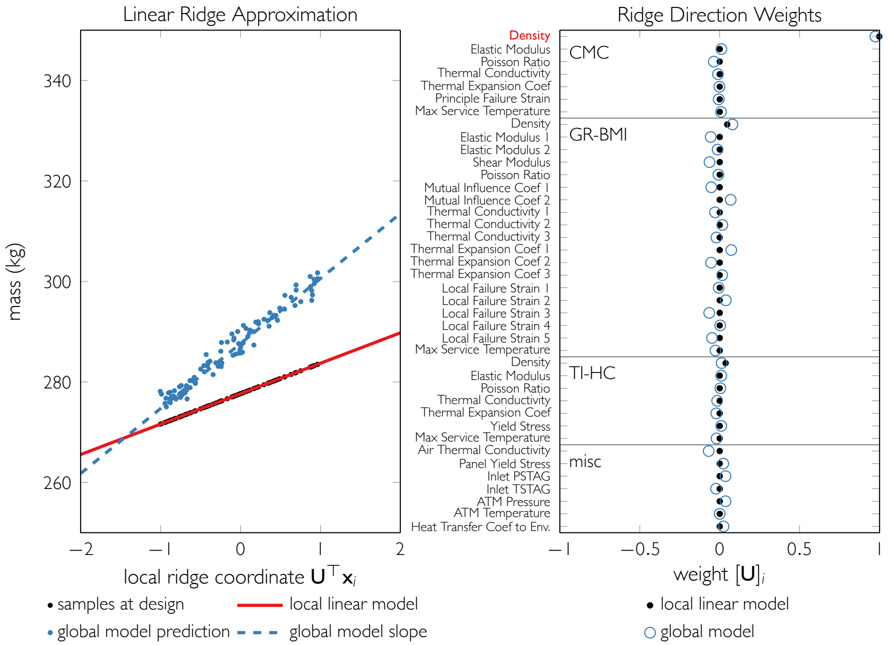


Mass $\tau = 10^{-1}$



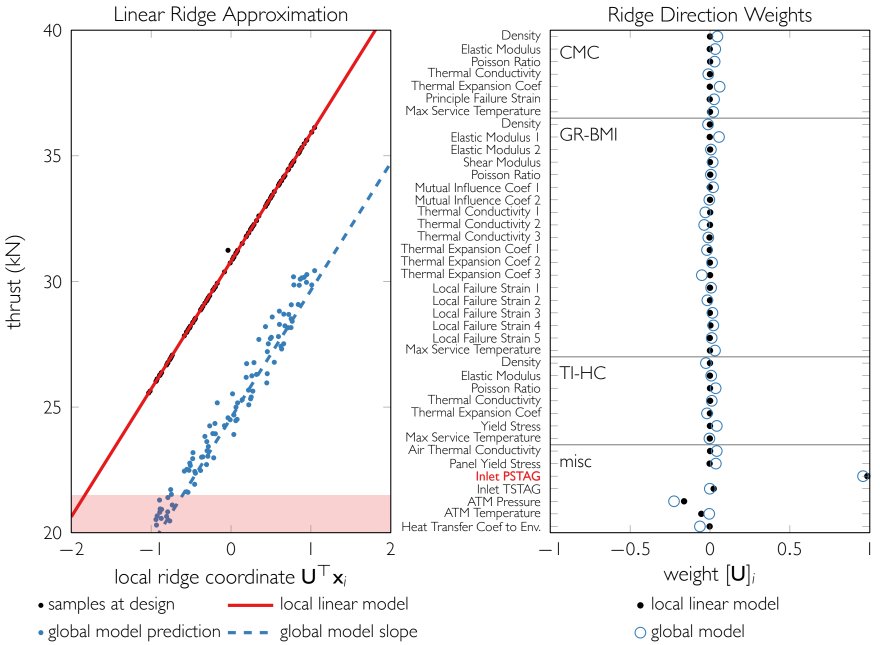
Linear Ridge Approximation
Ridge Direction Weights
mass (kg)
340
320
300
280
260
−2
−1
0
1
2
local ridge coordinate $\mathbf{U}^\top \mathbf{x}_i$
Density
Elastic Modulus
Poisson Ratio
Thermal Conductivity
Thermal Expansion Coef
Principle Failure Strain
Max Service Temperature
Density
Elastic Modulus 1
Elastic Modulus 2
Shear Modulus
Poisson Ratio
Mutual Influence Coef 1
Mutual Influence Coef 2
Thermal Conductivity 1
Thermal Conductivity 2
Thermal Conductivity 3
Thermal Expansion Coef 1
Thermal Expansion Coef 2
Thermal Expansion Coef 3
Local Failure Strain 1
Local Failure Strain 2
Local Failure Strain 3
Local Failure Strain 4
Local Failure Strain 5
Max Service Temperature
Density
Elastic Modulus
Poisson Ratio
Thermal Conductivity
Thermal Expansion Coef
Yield Stress
Max Service Temperature
Air Thermal Conductivity
Panel Yield Stress
Inlet PSTAG
Inlet TSTAG
ATM Pressure
ATM Temperature
Heat Transfer Coef to Env.
CMC
GR-BMI
TI-HC
misc
−1
−0.5
0
0.5
1
weight $[\mathbf{U}]_i$
samples at design
global model prediction
local linear model
global model slope
local linear model
global model

Mass $\tau = 10^{-6}$

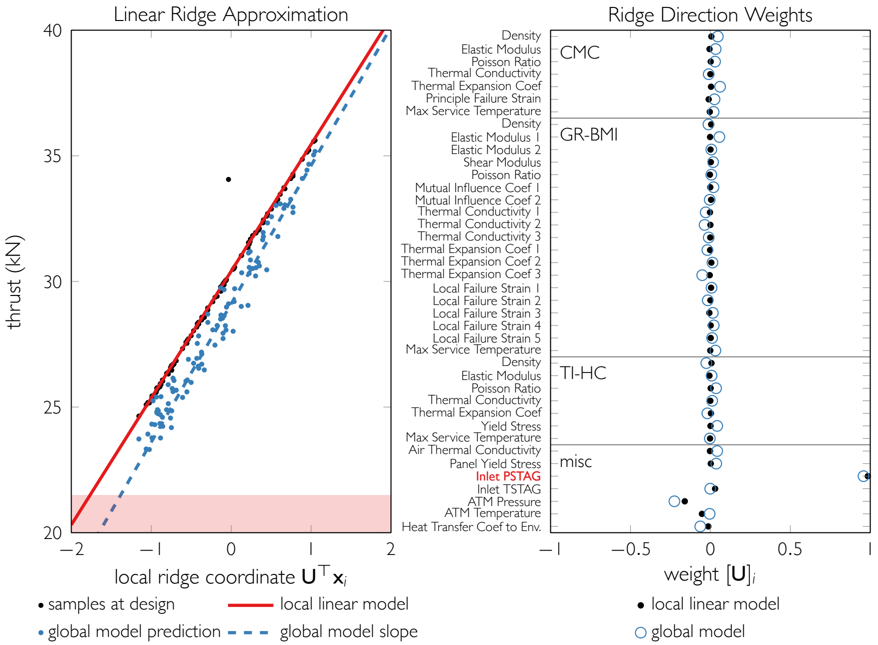


Linear Ridge Approximation
mass (kg)
340
320
300
280
260
−2
−1
0
1
2
local ridge coordinate $\mathbf{U}^\top \mathbf{x}_i$
Ridge Direction Weights
Density
Elastic Modulus
Poisson Ratio
Thermal Conductivity
Thermal Expansion Coef
Principle Failure Strain
Max Service Temperature
Density
Elastic Modulus 1
Elastic Modulus 2
Shear Modulus
Poisson Ratio
Mutual Influence Coef 1
Mutual Influence Coef 2
Thermal Conductivity 1
Thermal Conductivity 2
Thermal Conductivity 3
Thermal Expansion Coef 1
Thermal Expansion Coef 2
Thermal Expansion Coef 3
Local Failure Strain 1
Local Failure Strain 2
Local Failure Strain 3
Local Failure Strain 4
Local Failure Strain 5
Max Service Temperature
Density
Elastic Modulus
Poisson Ratio
Thermal Conductivity
Thermal Expansion Coef
Yield Stress
Max Service Temperature
Air Thermal Conductivity
Panel Yield Stress
Inlet PSTAG
Inlet TSTAG
ATM Pressure
ATM Temperature
Heat Transfer Coef to Env.
CMC
GR-BMI
TI-HC
misc
−1
−0.5
0
0.5
1
weight $[\mathbf{U}]_i$
samples at design
global model prediction
local linear model
global model slope
local linear model
global model

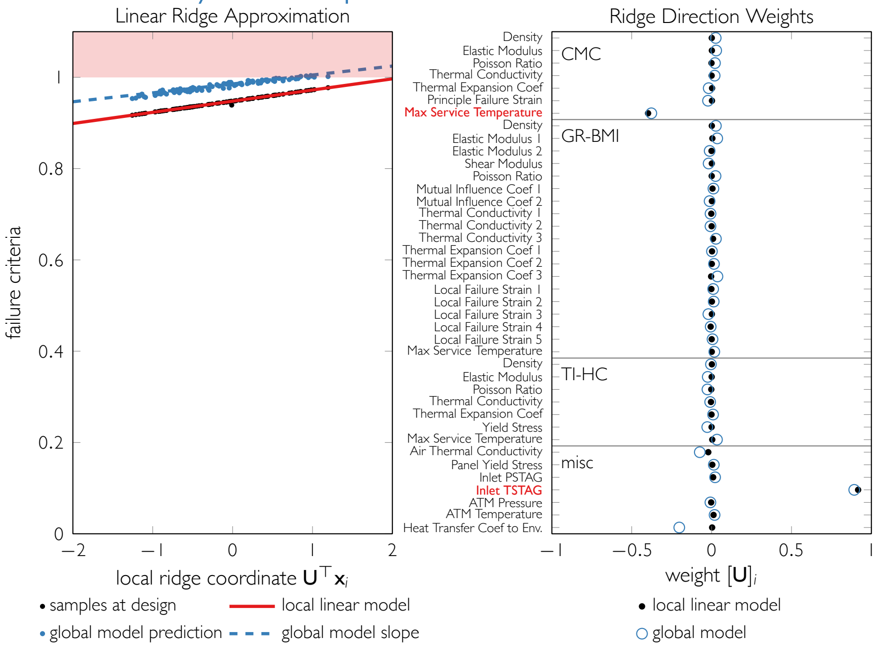
Thrust $\tau = 10^{-1}$



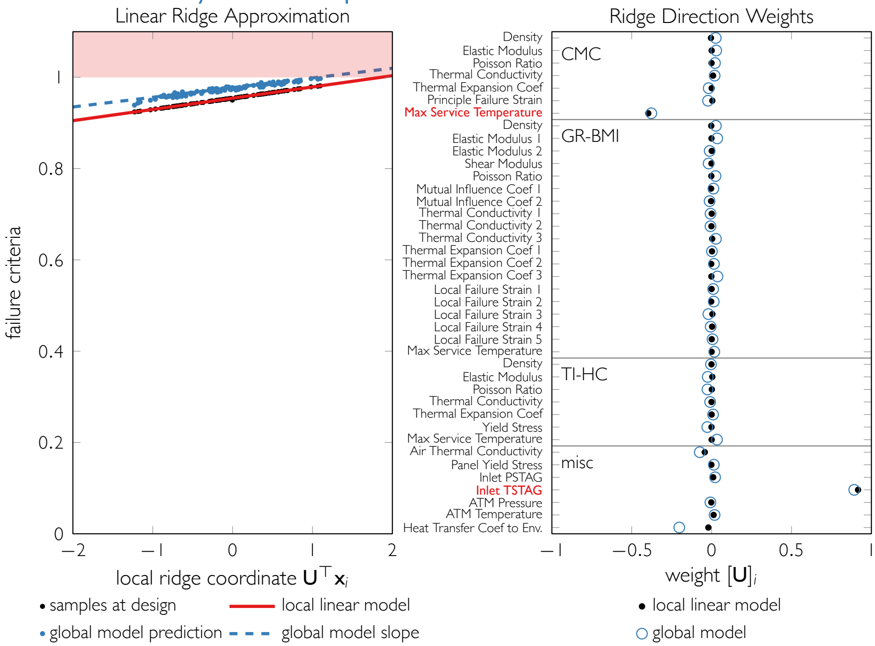
Thrust $\tau = 10^{-6}$



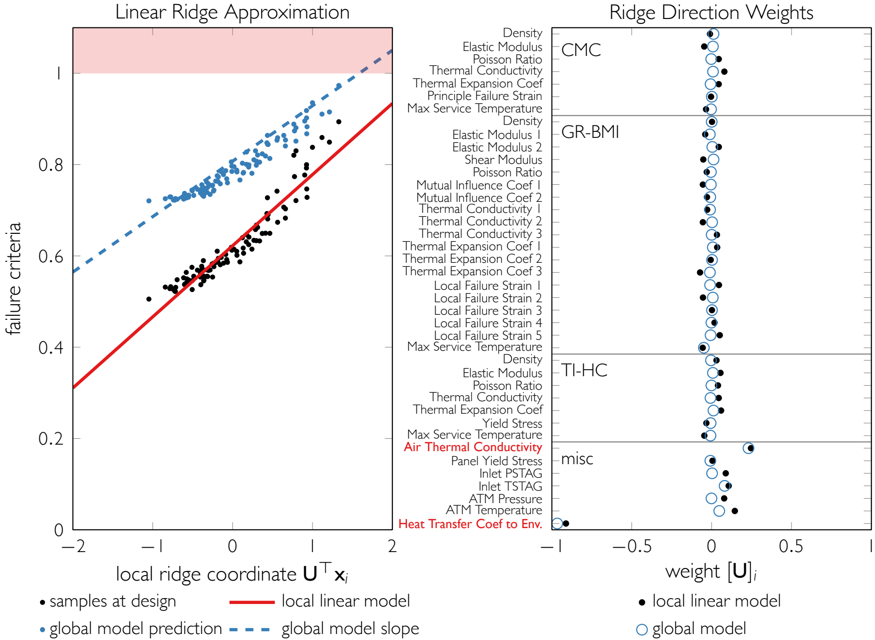
Thermal Layer Temperature Failure $\tau = 10^{-1}$



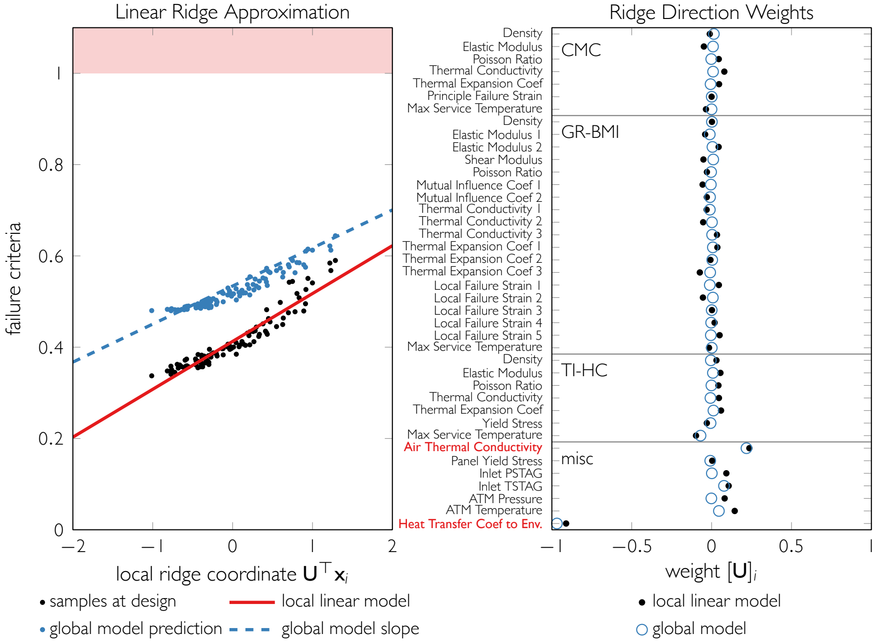
Thermal Layer Temperature Failure $\tau = 10^{-6}$



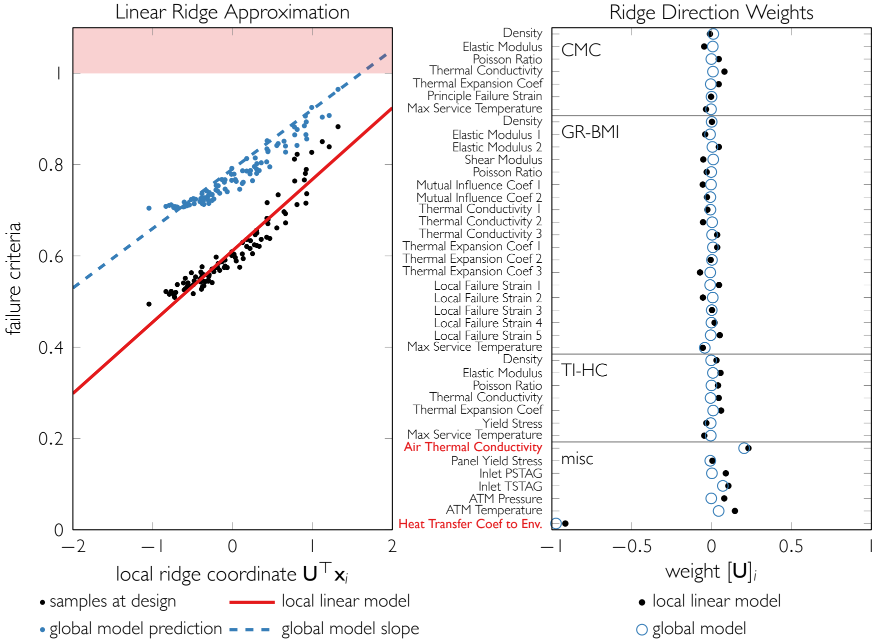
Inside Load Layer Temperature Failure $\tau = 10^{-6}$



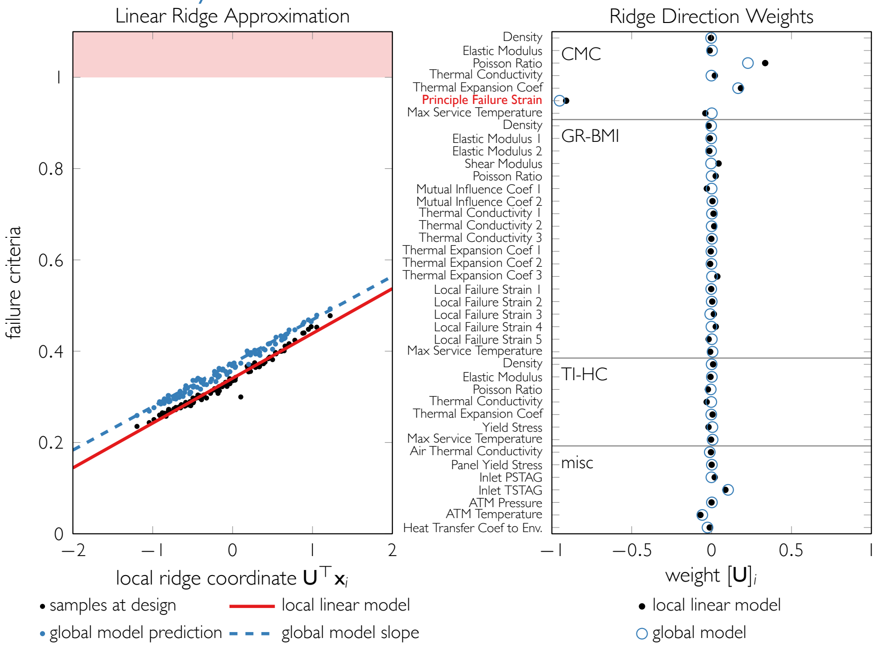
Middle Load Layer Temperature Failure $\tau = 10^{-6}$



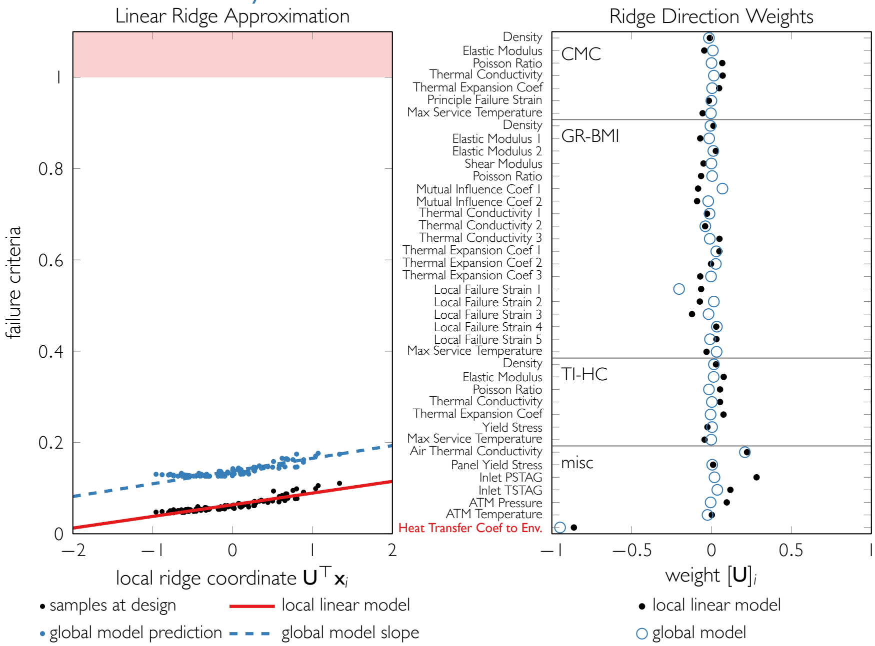
Outside Load Layer Temperature Failure $\tau = 10^{-6}$



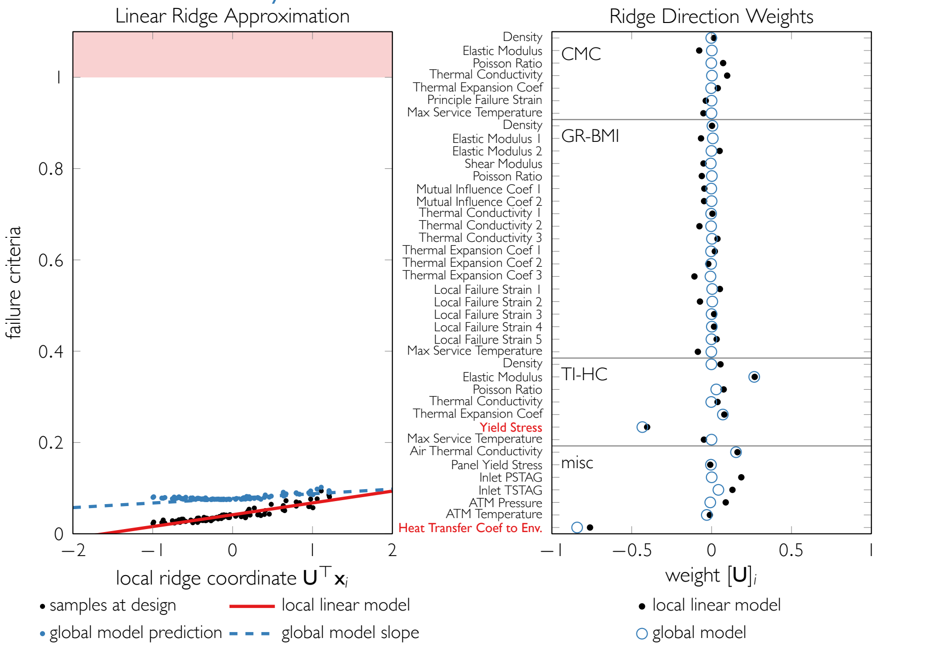
Thermal Layer Structural Failure $\tau = 10^{-6}$



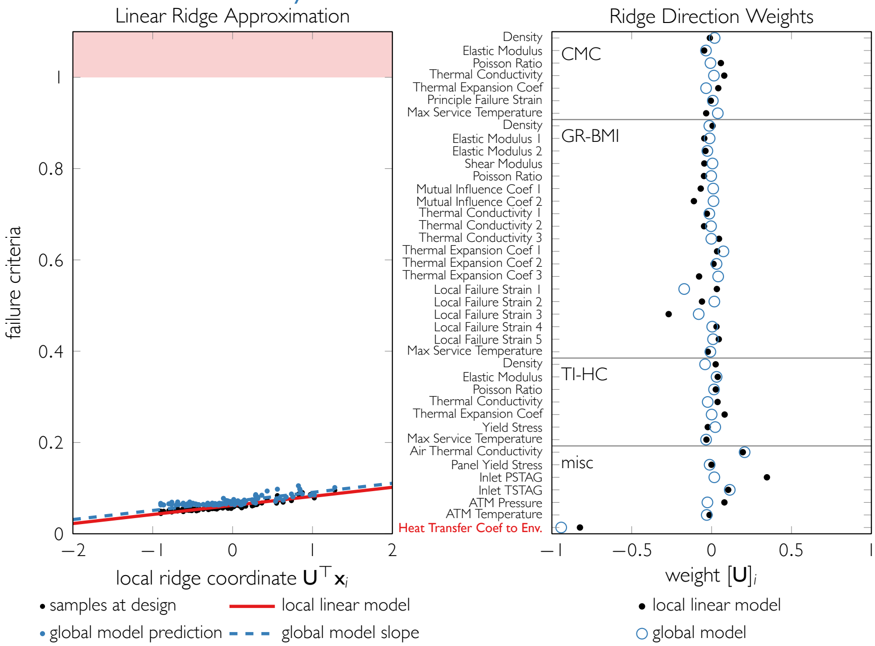
Inside Load Layer Structural Failure $\tau = 10^{-6}$



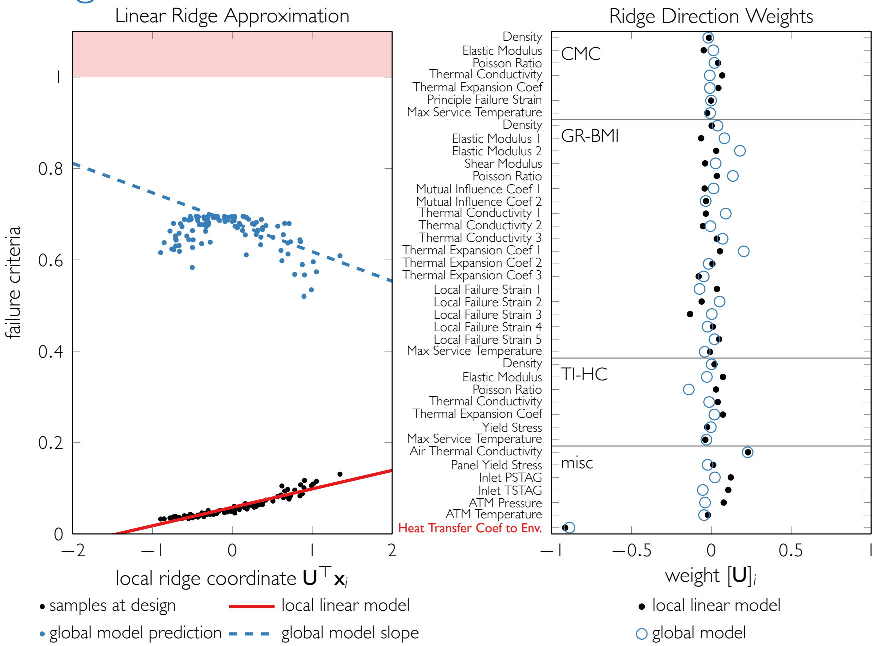
Middle Load Layer Structural Failure $\tau = 10^{-6}$



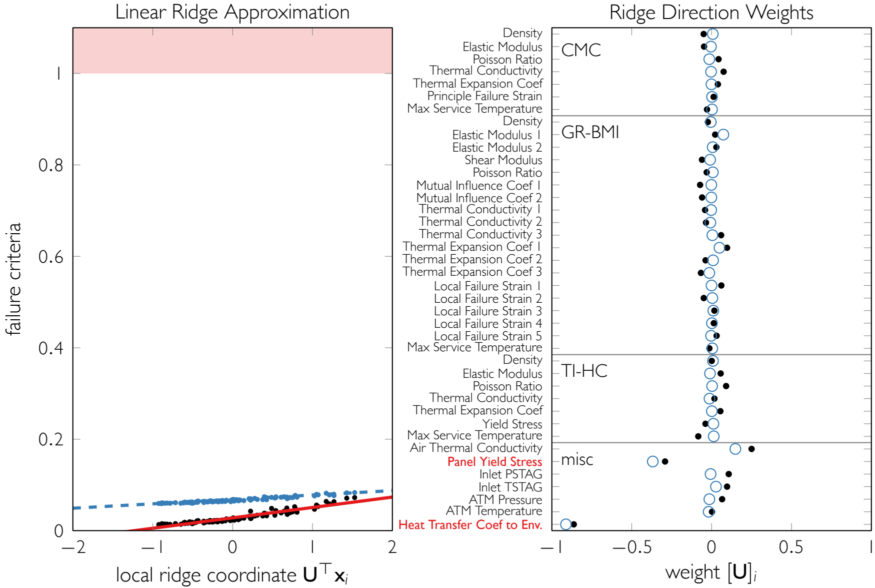
Outside Load Layer Structural Failure $\tau = 10^{-6}$



Stringers Structural Failure $\tau = 10^{-6}$



Baffle 1 Structural Failure $\tau = 10^{-6}$



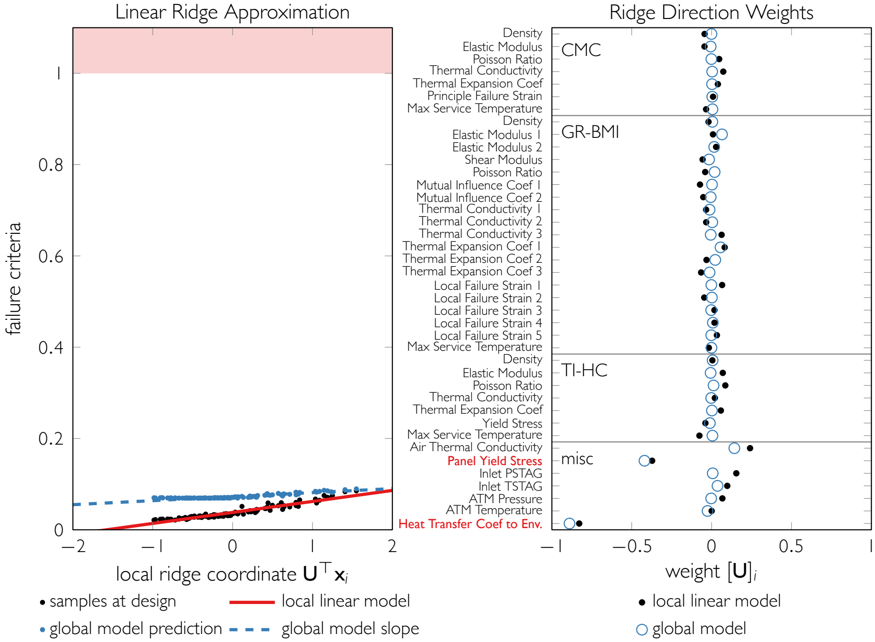
• samples at design — local linear model

• global model prediction - - - global model slope

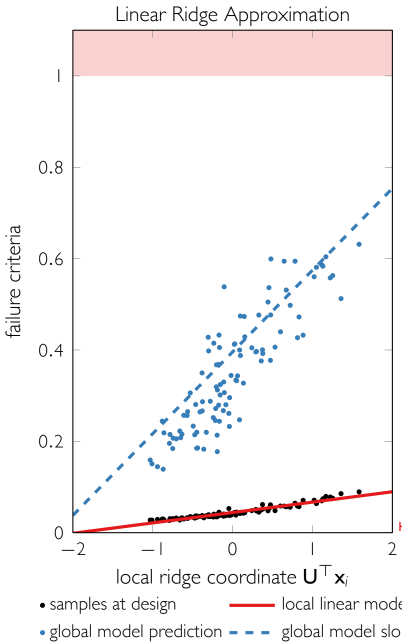
• local linear model

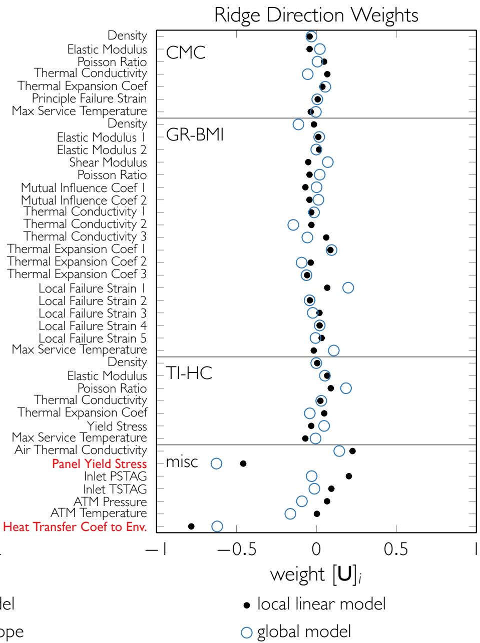
○ global model

Baffle 2 Structural Failure $\tau = 10^{-6}$

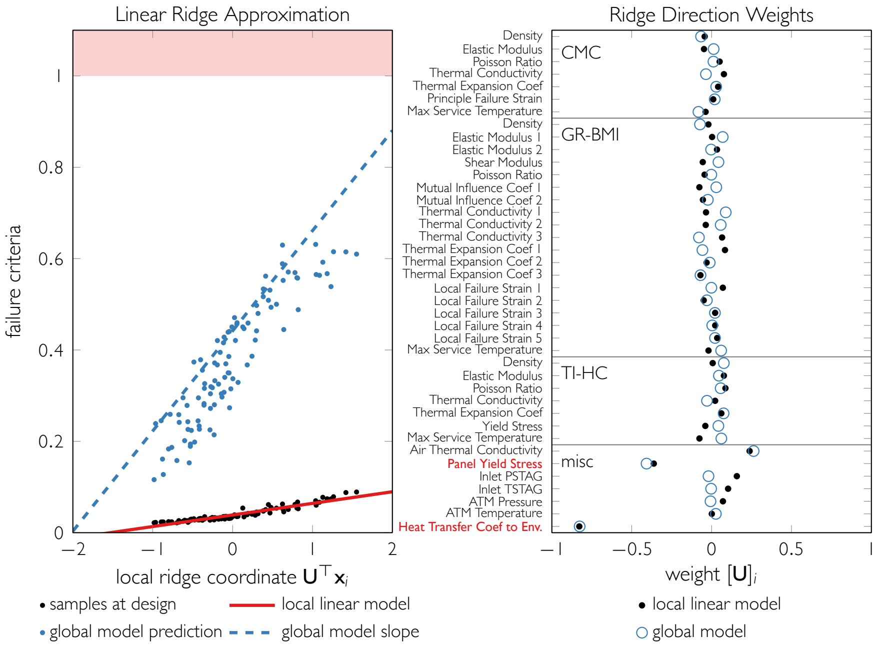


Baffle 3 Structural Failure $\tau = 10^{-6}$

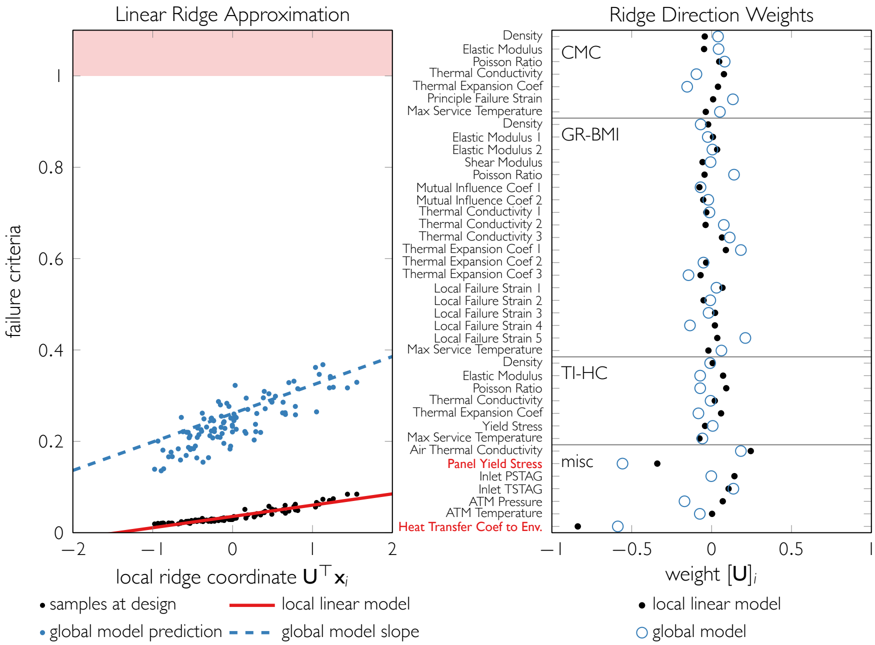




Baffle 4 Structural Failure $\tau = 10^{-6}$



Baffle 5 Structural Failure $\tau = 10^{-6}$



Summary

Using 1150 high fidelity samples we construct MULTI-F designs satisfying chance constraints to $5 \cdot 10^{-5}$

Future work:

- Improve sampling to refine model near DUU solution
- Use trust region techniques for convergence
- Use sup-norm polynomial ridge approximation:

$$\underset{\substack{\mathbf{U} \in \operatorname{Range} \mathbb{G}(n, \mathbb{R}^m) \\ g \in \mathcal{P}_p(\mathbb{R}^n)}}{\text{minimize}} \max_i |f(\mathbf{x}_i) - g(\mathbf{U}^\top \mathbf{x}_i)|$$