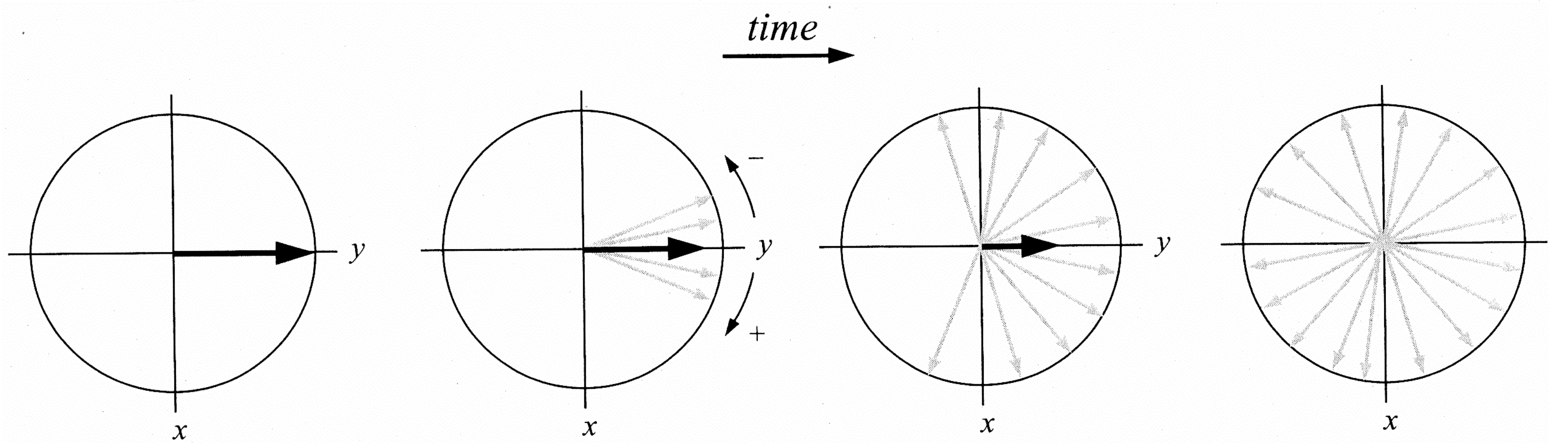
RELAXATION

Review: Transverse (Spin-Spin, T_2) Relaxation

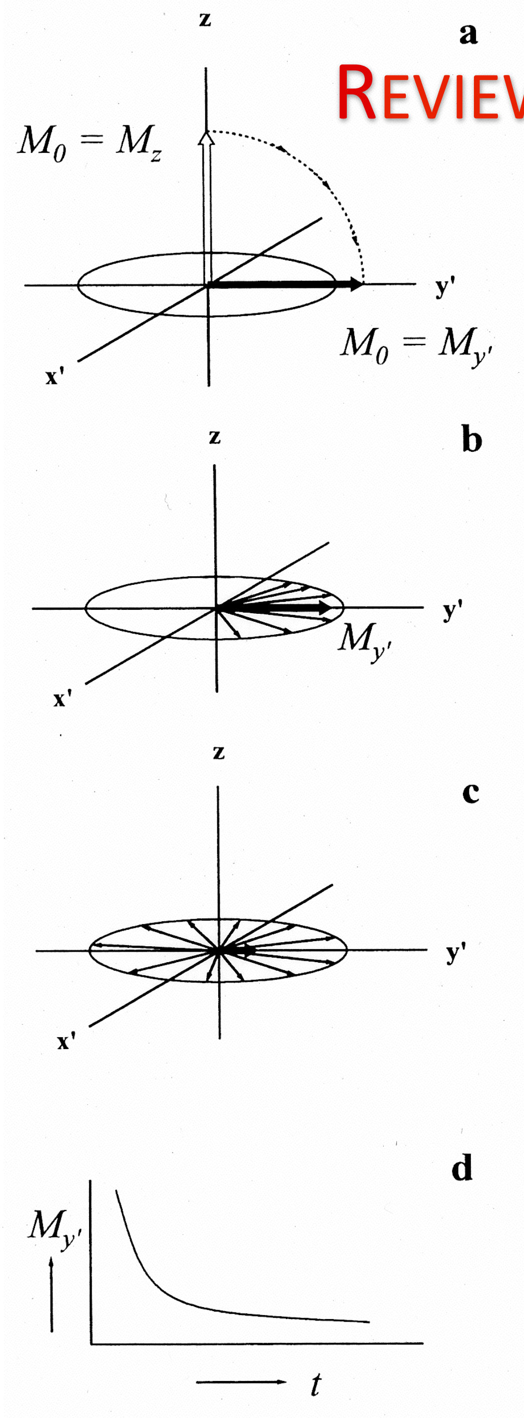
- Our oscillating signal decays as a function of time as the phase coherence between the precessing magnetic dipoles (vectors) is lost
- Transverse (spin-spin) or T_2 relaxation refers to loss of phase coherence in the transverse (x-y) plane
- This loss of coherence is due to local magnetic field differences experienced by the nuclei



- T_2 relaxation is characterized both by a component due to magnetic field inhomogeneity (the uninteresting component) and a by component arising from local fluctuating magnetic fields produced by the nuclei themselves

$$\frac{1}{T_2{}^*} = \frac{1}{T_2} + \frac{1}{T_{2(\Delta B_0)}}$$

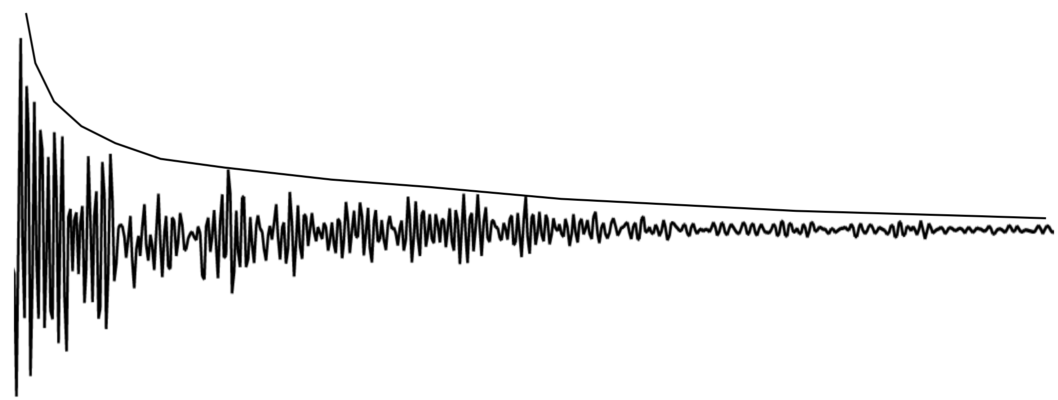
REVIEW: TRANSVERSE (T_2) RELAXATION



- The decay of signal due to T_2 relaxation is first order

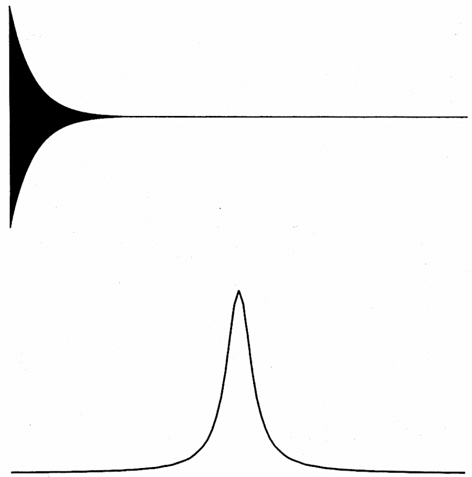
$$\frac{dM_y}{dt} = -\frac{M_y}{T_2{}^*}$$

$$M_y = M_{y_0} e^{(-t/T_2{}^*)}$$

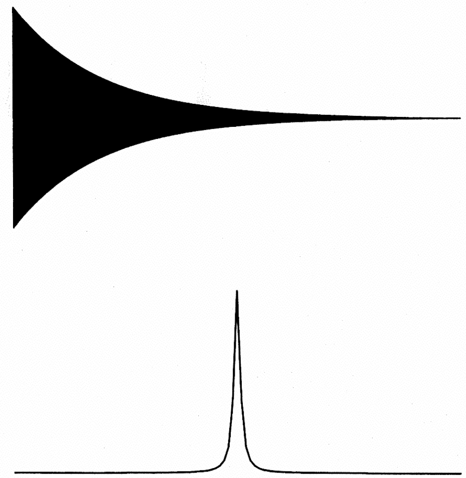


REVIEW: TRANSVERSE (T_2) RELAXATION

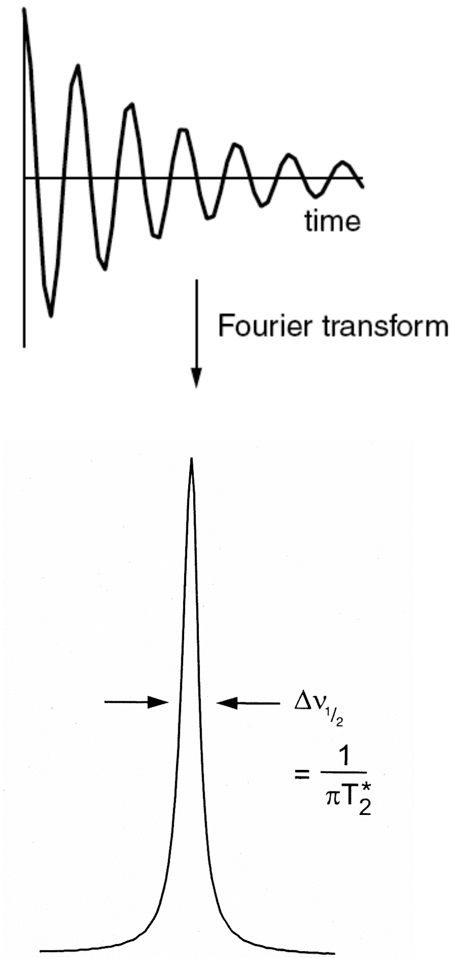
- Short T_2 times lead to broad lines (undesirable, lower signal-to-noise)
- A poorly shimmed magnet leads to short T_2 times and broad peaks



short T_2
fast relaxation



long T_2
slow relaxation

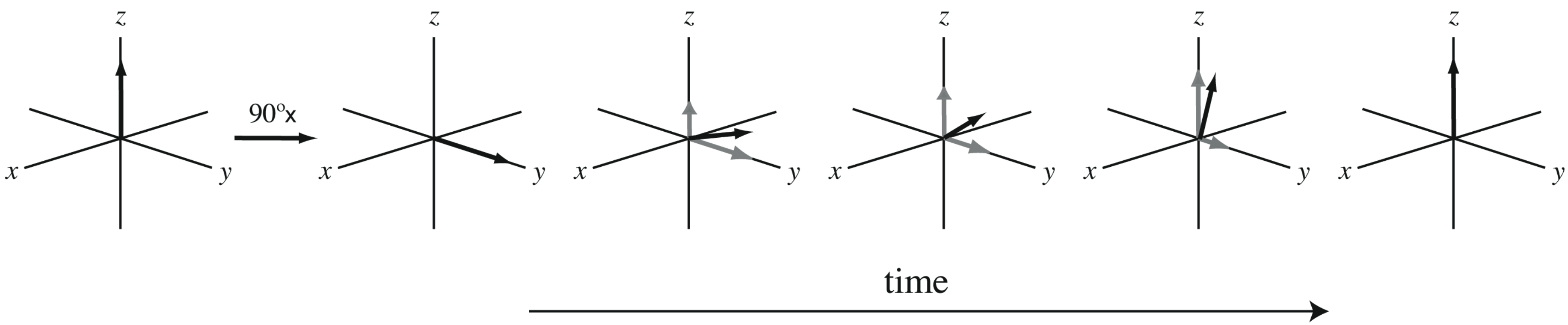


- The Fourier transform of a decaying exponential function ($\cos\omega t \bullet e^{(-t/T_2)}$) gives a **Lorentzian** line shape
- The width of a Lorenztian peak at 1/2 of the maximum height is $1/(\pi\, T_2{}^*)$

$$\Delta\upsilon_{1/2} = \frac{1}{\pi T_2{}^*} = \frac{1}{\pi T_2} + \gamma\Delta B_0$$

REVIEW: LONGITUDINAL (SPIN-LATTICE, T_1) RELAXATION

- Longitudinal (spin-lattice) or T_1 relaxation refers to return to thermal equilibrium (along the z axis) of the spin populations (return to equilibrium N_α and N_β values) following perturbation
- T_1 relaxation represents a loss of energy (heat) from the spins to the surroundings (i.e. the "lattice": neighboring molecules, solvent, inpurities, NMR tube walls, etc.)



- The return to equilibrium is first order

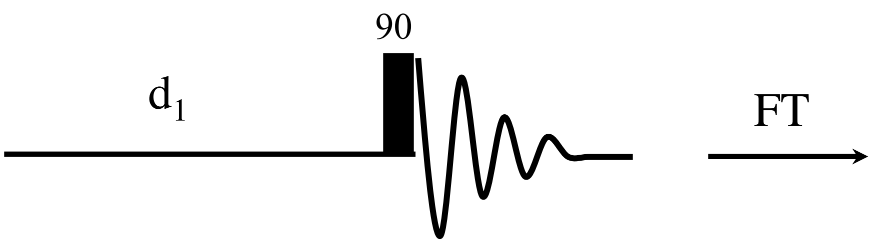
-after a 90° pulse, the return to equilibrium is described as shown below:

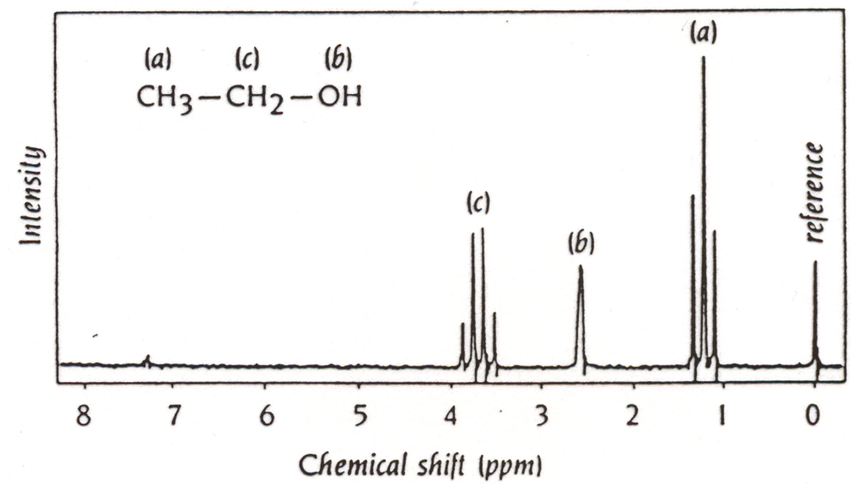
$$\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1} \qquad M_z = M_0(1 - e^{(-t/T_1)})$$

-after a 180° pulse.....

$$M_z = M_0(1 - 2e^{(-t/T_1)})$$

REVIEW: T_1 RELAXATION AND DATA ACQUISITON





- Need to wait for a time ("d_1") after acquisition of the FID in order for the spin populations to return to thermal equilibrium
  - -after a 90° pulse, should wait for ~$5T_1$
  - -thus, are limited in the number of "scans" that can be acquired in a given amount of time
- Important point: signal-to-noise (S/N) increases in proportion to the square root of the number of scans

$$S/N \propto \sqrt{NS}$$

- For instance, if 1 scan gives a particular S/N, in order to double the S/N, 4 scans are required (unfortunately)

REVIEW: OPTIMIZING PULSE WIDTH

What is the optimal pulse width/length/angle to use?

- a 90° pulse angle gives maximum S/N for one pulse, but the delay (d_1) between successive pulses must be long for recovery of equilibrium magnetization
- following a 90° pulse, if (d_1 + AQ) = 5 × T_1, then > 99% of equilibrium magnetization will be recovered before the next pulse
- thus, *for a given number of pulses* (without regards to time), 90° pulses will give maximum sensitivity (as long as (d_1 + AQ) ≥ 5 × T_1)

- other schemes, which permit faster pulsing (shorter d_1) combined with smaller pulse angles are possible
- for optimizing the S/N and total experimental time, the best compromise for the pulse width/angle is the ***Ernst Angle***
- Ernst Angle is in degrees

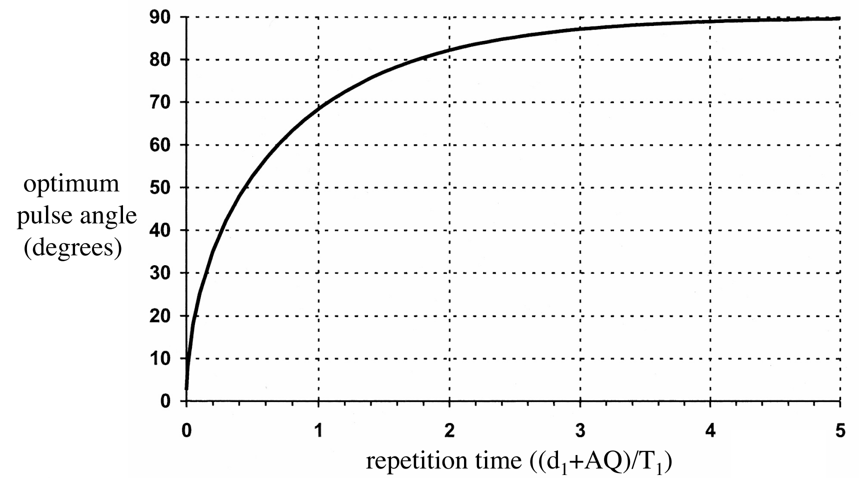
$$\cos \alpha_{\mathrm{Ernst}} = e^{-((\mathrm{d}_1 + \mathrm{AQ})/T_1)}$$

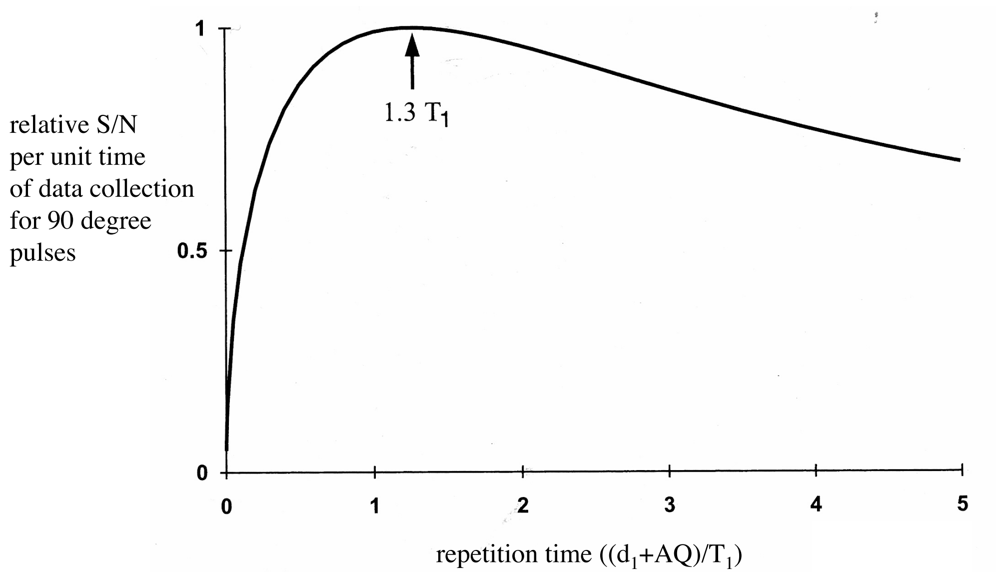
REVIEW: OPTIMIZING PULSE WIDTH

$$\cos\alpha_{\text{Ernst}} = e^{-((\mathrm{d}_1+\mathrm{AQ})/T_1)}$$

What is the optimal repetiton time (best S/N) for a given repetiton time?

- the optimal pulse angle is dependent on the repetition time
- as the repetition time shortens, so does the optimal pulse angle



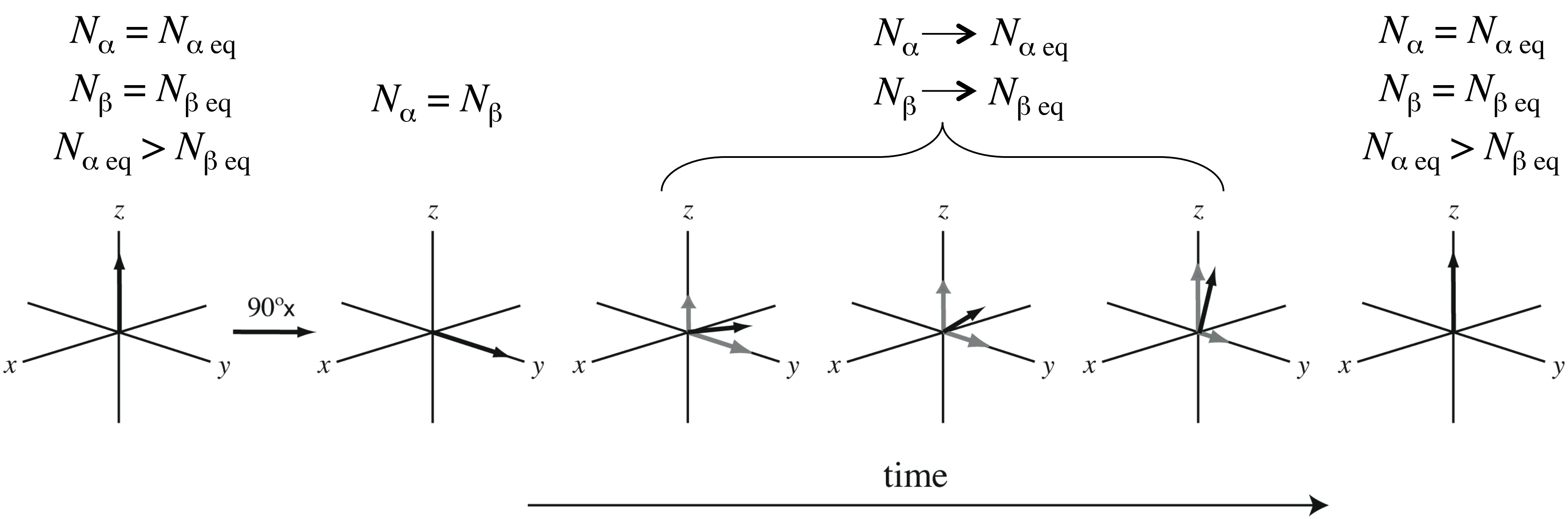


What is the optimal repetiton time (best S/N) for a given experiment time using 90 degree pulses?

- the optimal pulse angle is 1.3 x T_1 for 90 degree pulses for a pre-determined total experiment time

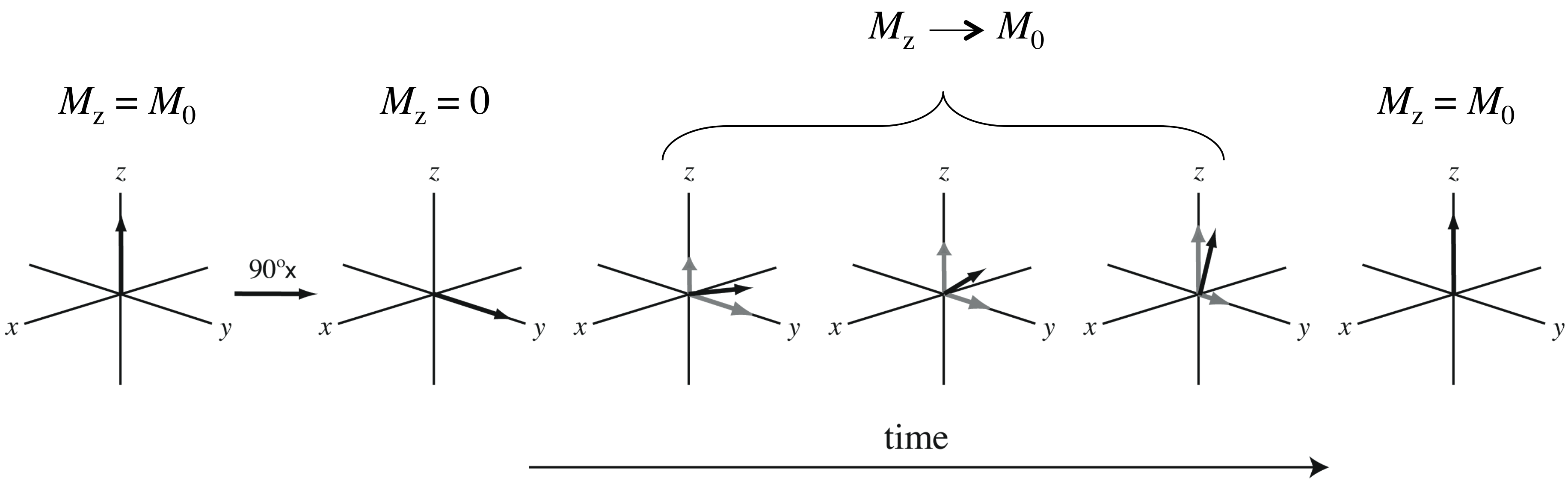
MEASURING T_1 RELAXATION

- Longitudinal (spin-lattice) or T_1 relaxation refers to return to thermal equilibrium (along the z axis) of the spin populations (return to equilibrium N_α and N_β values) following perturbation
- The equilibrium values of N_α and N_β are perturbed following a pulse
  - following a 90° pulse, $N_\alpha = N_\beta$, and N_α and N_β return to equilibrium values with time



MEASURING T_1 RELAXATION

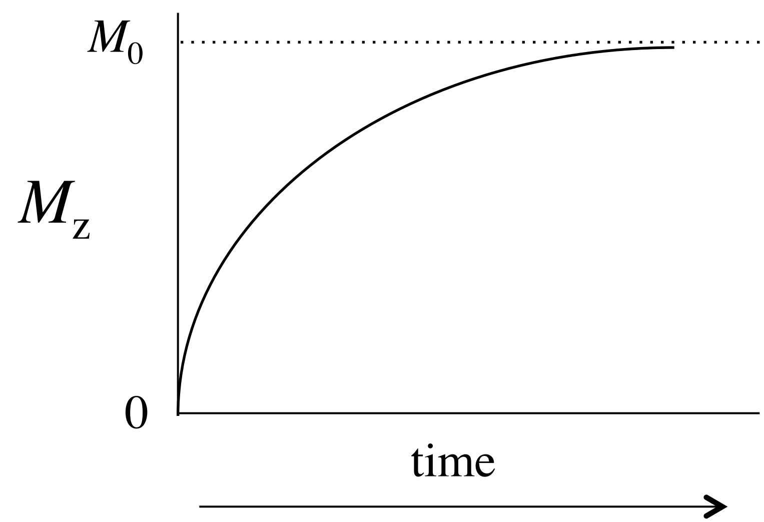
- If M_z could be measured with time following a **90°** pulse, T_1 could be determined easily



- The return to equilibrium is first order
  -after a 90° pulse, the return to equilibrium is described as shown below:

$$\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1}$$

$$M_z = M_0(1 - e^{(-t/T_1)})$$

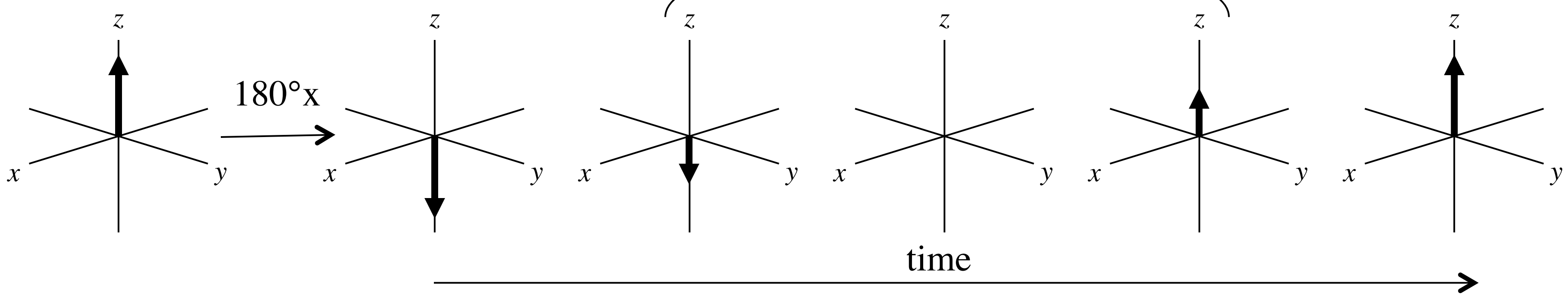


MEASURING T_1 RELAXATION

- If M_z could be measured with time following a **180°** pulse, T_1 could be determined easily

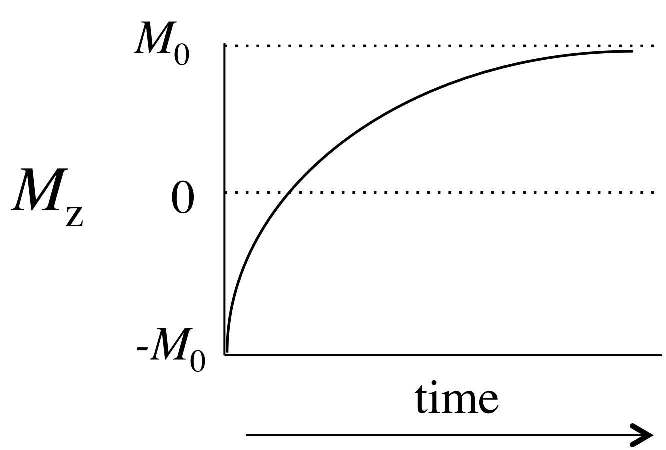
$M_z = M_0$ | $M_z = -M_0$ | $M_z \rightarrow M_0$ | $M_z = M_0$

$N_\alpha = N_{\alpha\,eq}$, $N_\beta = N_{\beta\,eq}$, $N_{\alpha\,eq} > N_{\beta\,eq}$ | $N_\alpha = N_{\beta\,eq}$, $N_\beta = N_{\alpha\,eq}$ | $N_\alpha \rightarrow N_{\alpha\,eq}$, $N_\beta \rightarrow N_{\beta\,eq}$ | $N_\alpha = N_{\alpha\,eq}$, $N_\beta = N_{\beta\,eq}$, $N_{\alpha\,eq} > N_{\beta\,eq}$



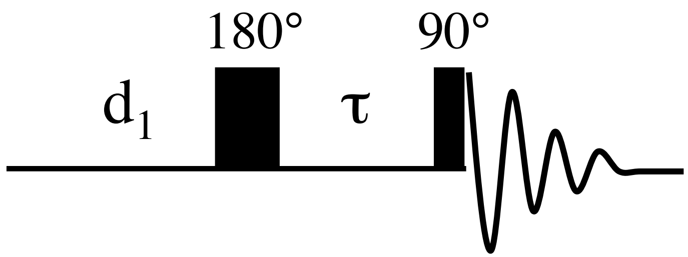
$$M_z = M_0(1 - \mathbf{2}e^{(-t/T_1)})$$

note !



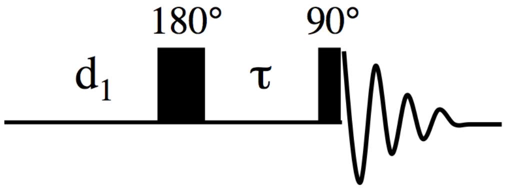
Measuring T_1 Relaxation: Inversion Recovery

- Problem: **can't measure M_z directly**. Can only measure bulk magnetization in transverse plane
- Solution: ***inversion recovery experiment***
- The 180° pulse places the bulk magnetization vector along the -*z* axis (NO components in the transverse plane)

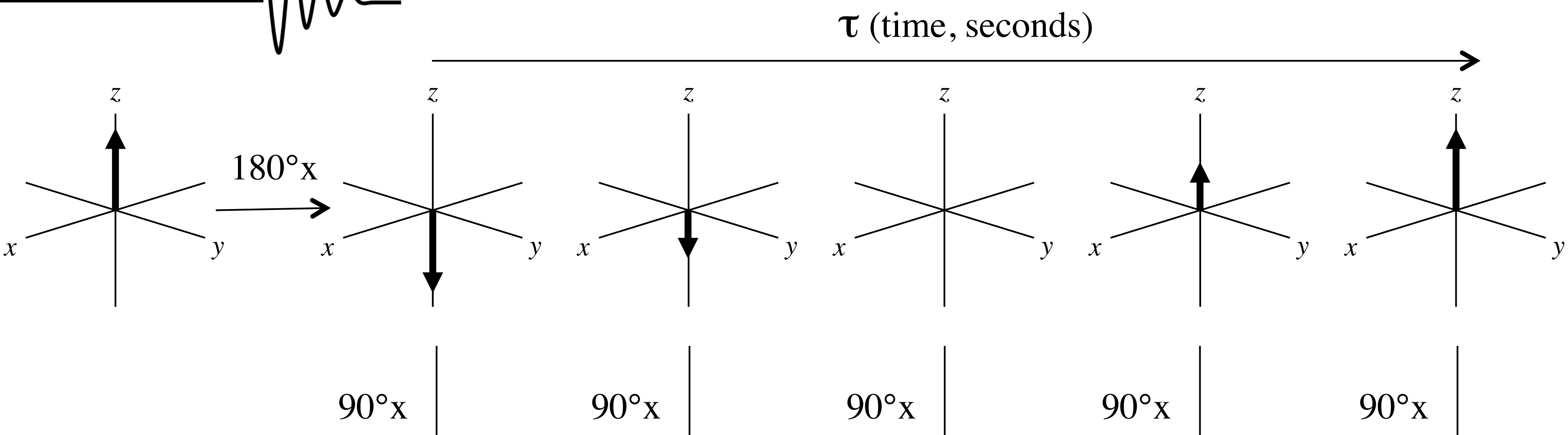


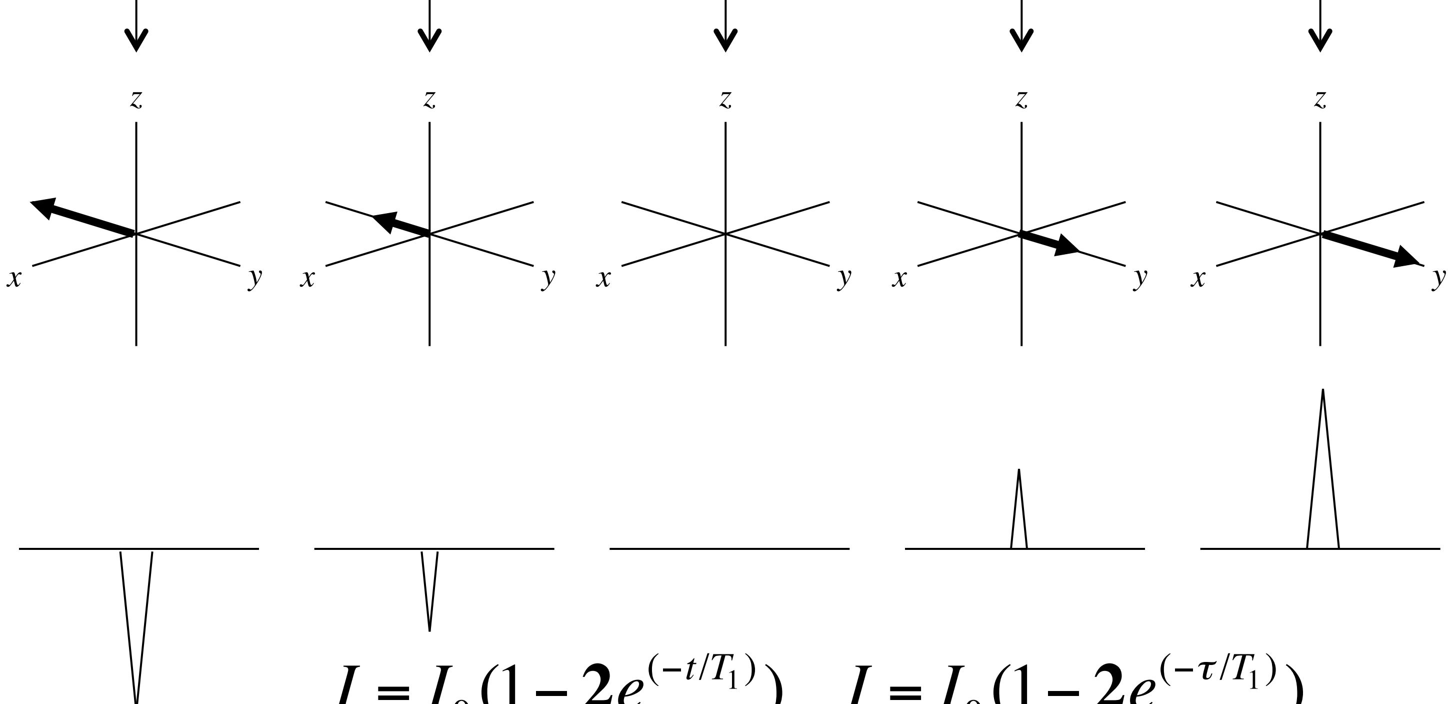
- During the τ delay period, as the α and β spin populations return towards their equilibrium values (i.e. T_1 relaxation), the absolute magnitude of the bulk magnetization vector along -*z* decreases initially (short τ values), becomes zero, and then increases along +*z* (long τ values) until it equals M_0 (return to thermal equilibrium)
- The 90° "read" pulse places the magnetization in the transverse (*x*-*y*) plane, and thus the magnitude of the bulk magnetization vector in the transverse plane is proportional to the magnitude of the bulk vector when it was along *z*
- ***The magnitude of the Fourier transformed signal produced, therefore, is proportional to the magnitude of the bulk vector when it was along z***
- So, the intensity of the signal as a function of τ, where I_0 is the intensity when $\tau = \infty$, allows determination of T_1

$$M_z = M_0(1 - 2e^{(-t/T_1)}) \quad I = I_0(1 - 2e^{(-t/T_1)}) \quad I = I_0(1 - 2e^{(-\tau/T_1)})$$



INVERSION RECOVERY





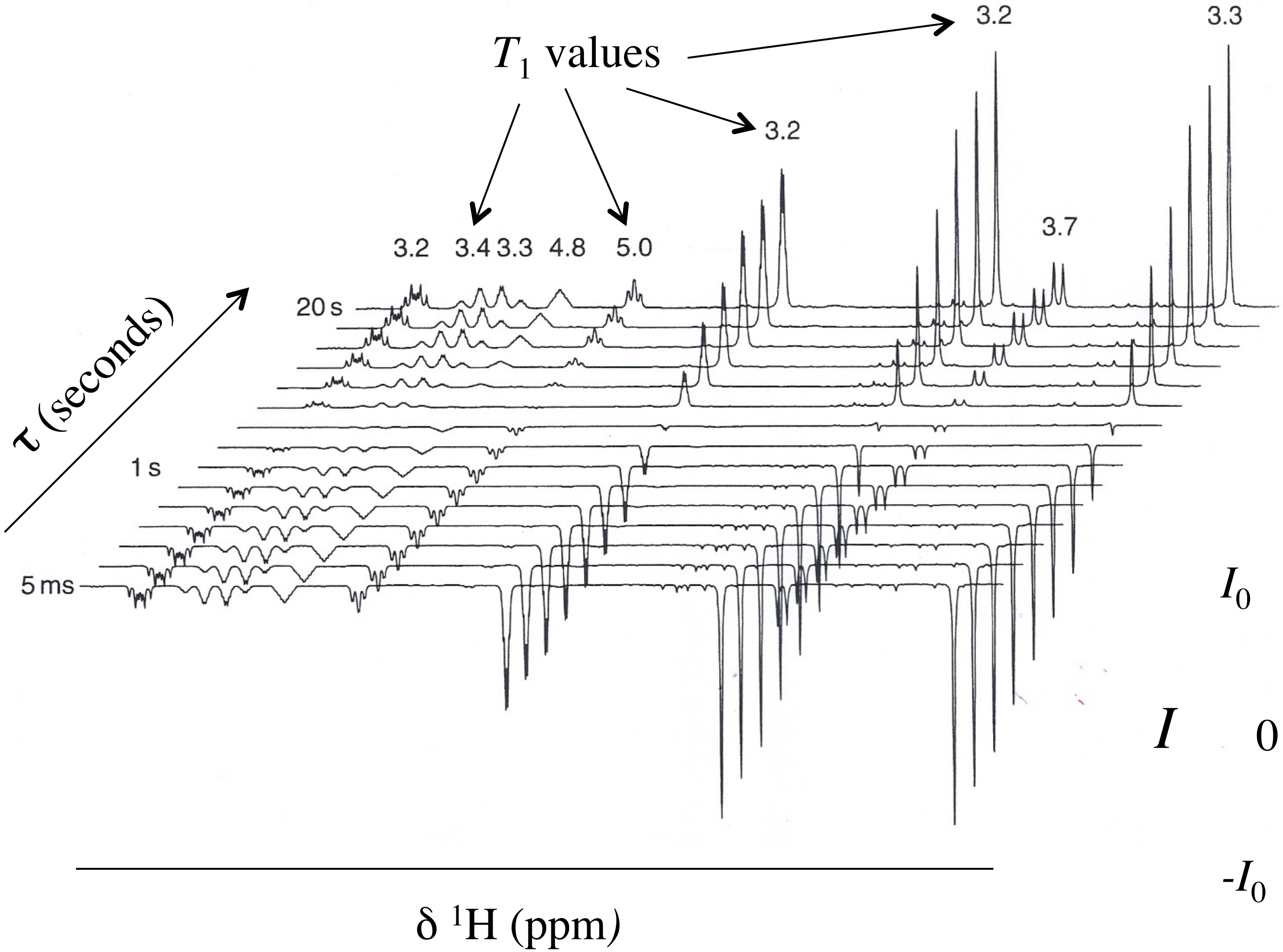
here, the magnitude of bulk magnetization vector in transverse plane is proportional to the magnitude of the bulk magnetization vector along $\pm z$ after the τ period/delay

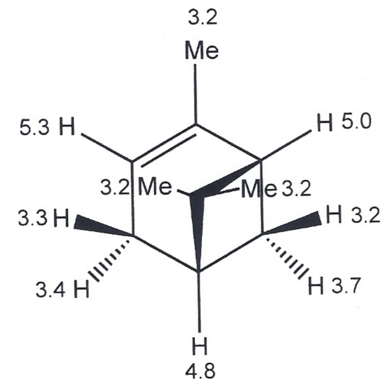
so, ***signal intensity*** after Fourier transform now is proportional to the magnitude of the bulk magnetization vector along $\pm z$ after the τ period/delay

$$I = I_0(1-2e^{(-t/T_1)}) \quad I = I_0(1-2e^{(-\tau/T_1)})$$

Measuring T_1 Relaxation: Inversion Recovery

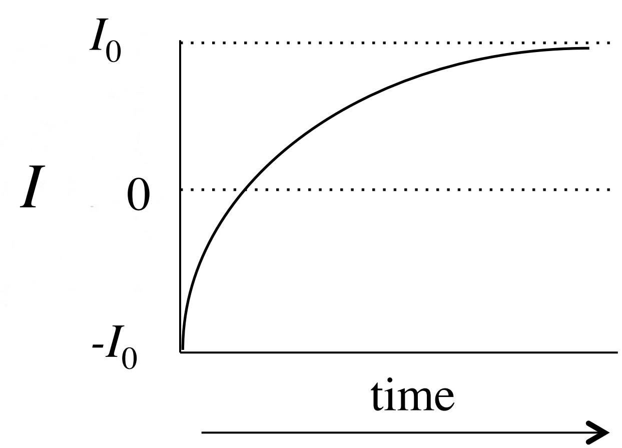
- For well resolved signals, can determine all T_1 values in a single inversion recovery experiment (i.e. spectra acquired at many different τ values)
- For each signal, data (I versus "t" or "τ") are fit to equation shown to get T_1 values





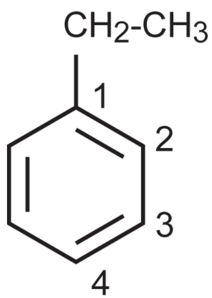
2.1

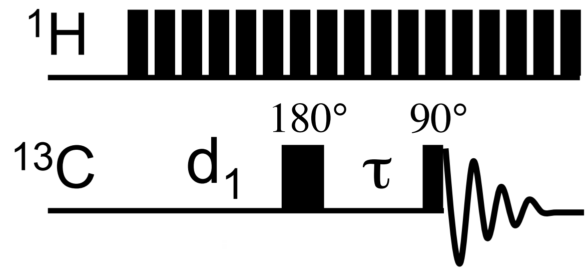
$$I = I_0\left(1 - 2e^{(-t/T_1)}\right)$$

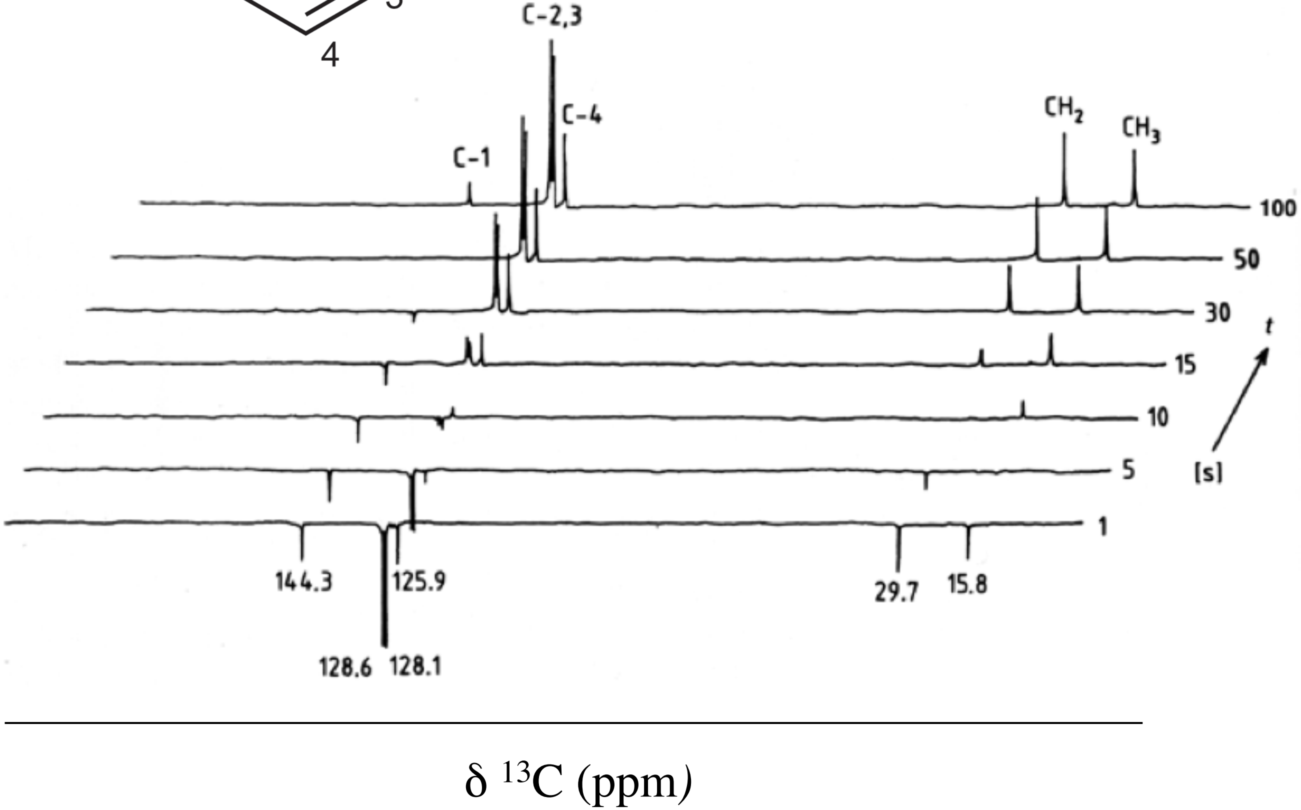


Measuring T_1 Relaxation: Inversion Recovery

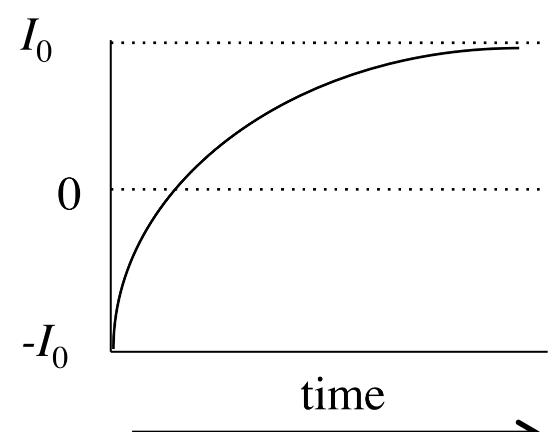
- Measuring T_1 for ^{13}C is no different
- Broadband ^{1}H-decoupling used usually to improve signal-to-noise







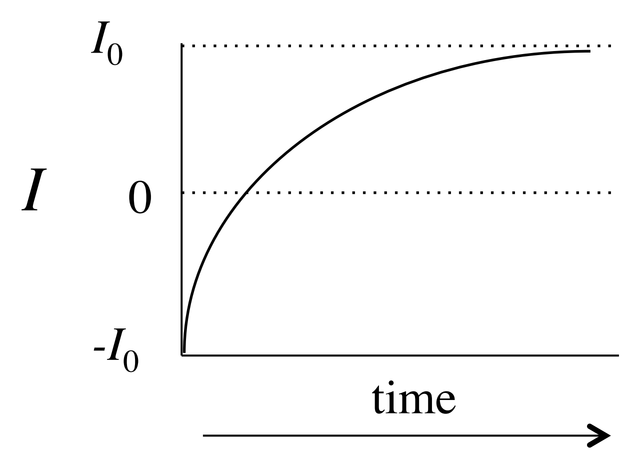
$$I = I_0(1 - \mathbf{2}e^{(-t/T_1)})$$



MEASURING T_1 RELAXATION: INVERSION RECOVERY

- Measure I versus time (t) or "τ", fit to the equation
- Can make an *estimate* without fitting by determining τ when the signal intensity is 0 (i.e. when $I = 0$)

$$I = I_0(1 - \mathbf{2}e^{(-t/T_1)})$$



$$0 = I_0(1 - \mathbf{2}e^{(-t/T_1)}) = I_0 - I_0\mathbf{2}e^{(-t/T_1)}$$

$$I_0 = I_0\mathbf{2}e^{(-t/T_1)} \quad 1 = \mathbf{2}e^{(-t/T_1)} \quad 1/\mathbf{2} = e^{(-t/T_1)}$$

$$\ln(1/\mathbf{2}) = -t/T_1 \quad \ln(\mathbf{2}) = t/T_1 \quad T_1 = \frac{t}{\ln(\mathbf{2})}$$

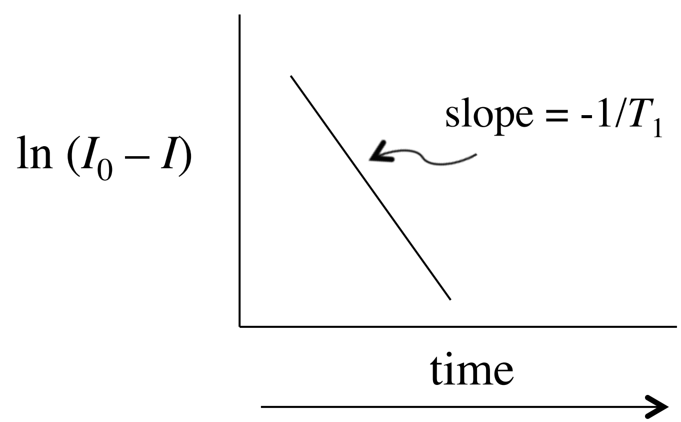
$$\text{usually written } T_1 = \frac{\tau_{\text{zero}}}{\ln(\mathbf{2})}$$

- Typically, it is not advisable to linearize the equation
- However, the equation can be linearized and T_1 determined from the slope of the appropriate plot

$$I = I_0(1 - \mathbf{2}e^{(-t/T_1)}) = I_0 - I_0\mathbf{2}e^{(-t/T_1)}$$

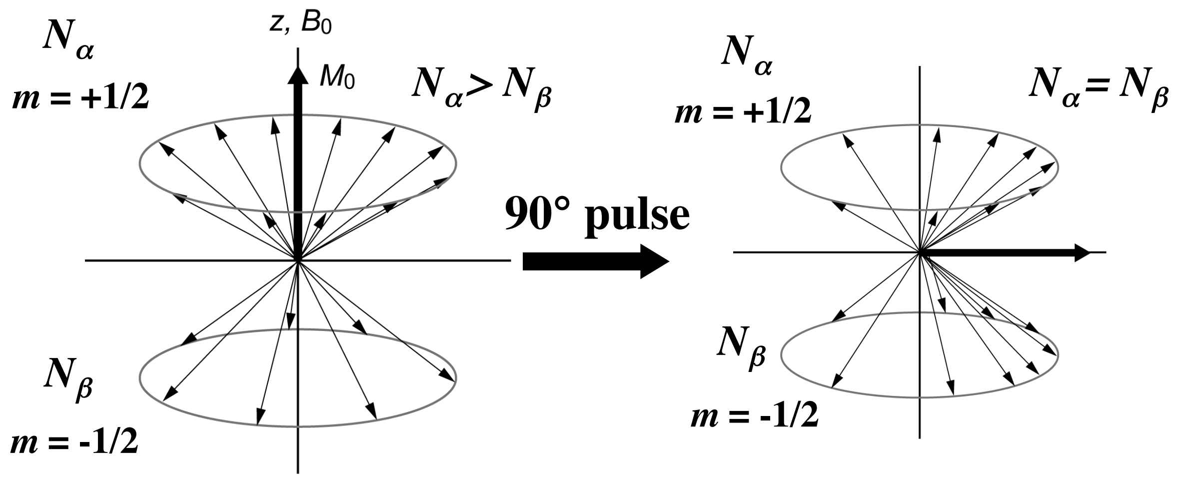
$$I_0 - I = I_0\mathbf{2}e^{(-t/T_1)}$$

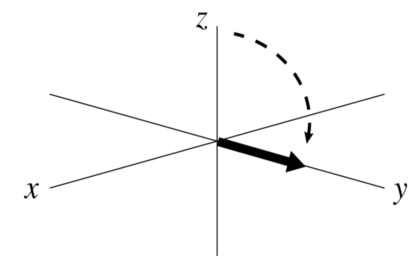
$$\ln(I_0 - I) = \ln 2I_0 - t/T_1 \quad (\text{slope} = -1/T_1)$$



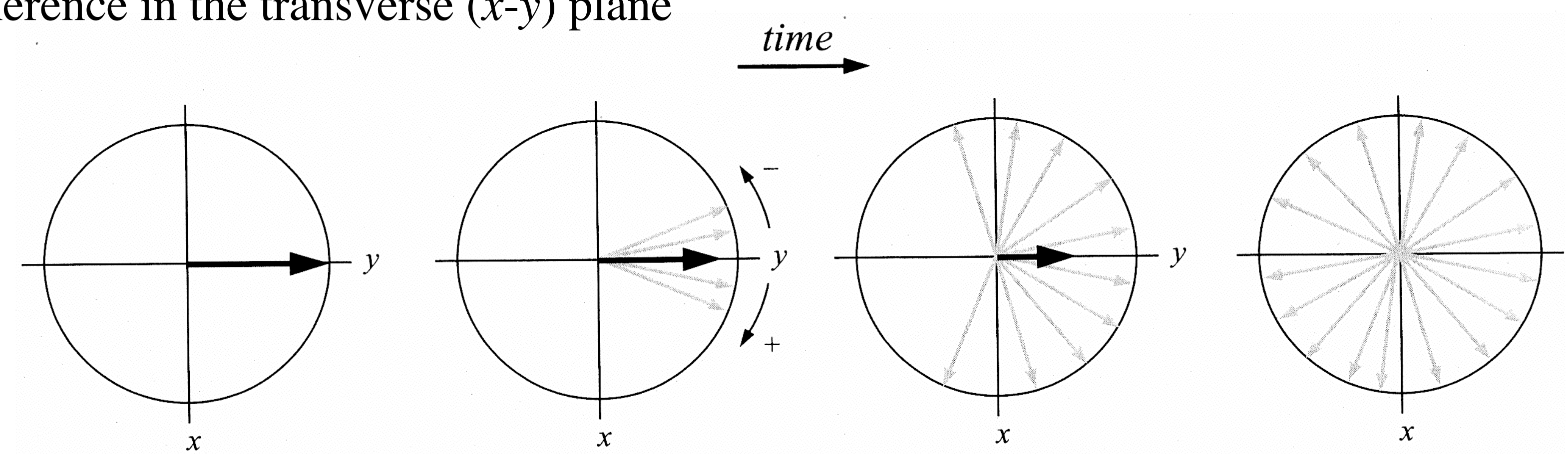
MEASURING T_2 RELAXATION

- A 90° (π/2) pulse equilibrates N_α and N_β and promotes "**phase coherence**" among the nuclear magnetic dipoles, creating ***transverse*** magnetization.





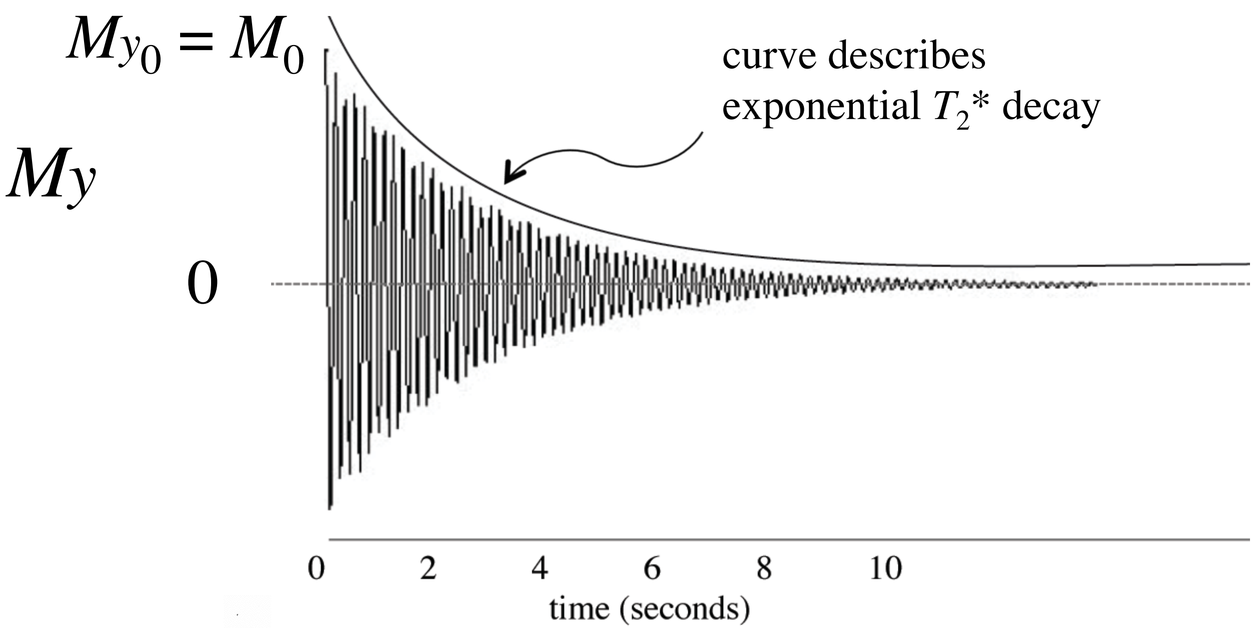
- Transverse (spin-spin) or T_2 relaxation refers to the time-dependent loss of phase coherence in the transverse (x-y) plane



Measuring T_2 Relaxation

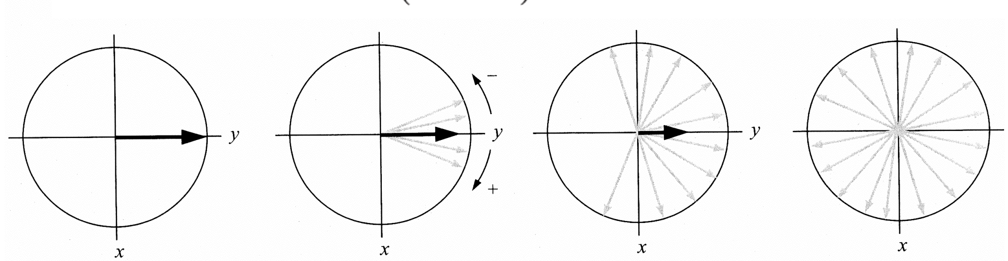
- T_2 relaxation is characterized both by a component due to magnetic field inhomogeneity (the *uninteresting* component) and a by component arising from local fluctuating magnetic fields produced by the nuclei themselves (the *interesting* component)
- The combination of these components is called T_2*
- T_2* is EASY to measure…….

$$\frac{1}{T_2*} = \frac{1}{T_2} + \frac{1}{T_{2(\Delta B_0)}}$$



$$\frac{dM_y}{dt} = -\frac{M_y}{T_2*}$$

$$M_y = M_{y_0} e^{(-t/T_2*)}$$



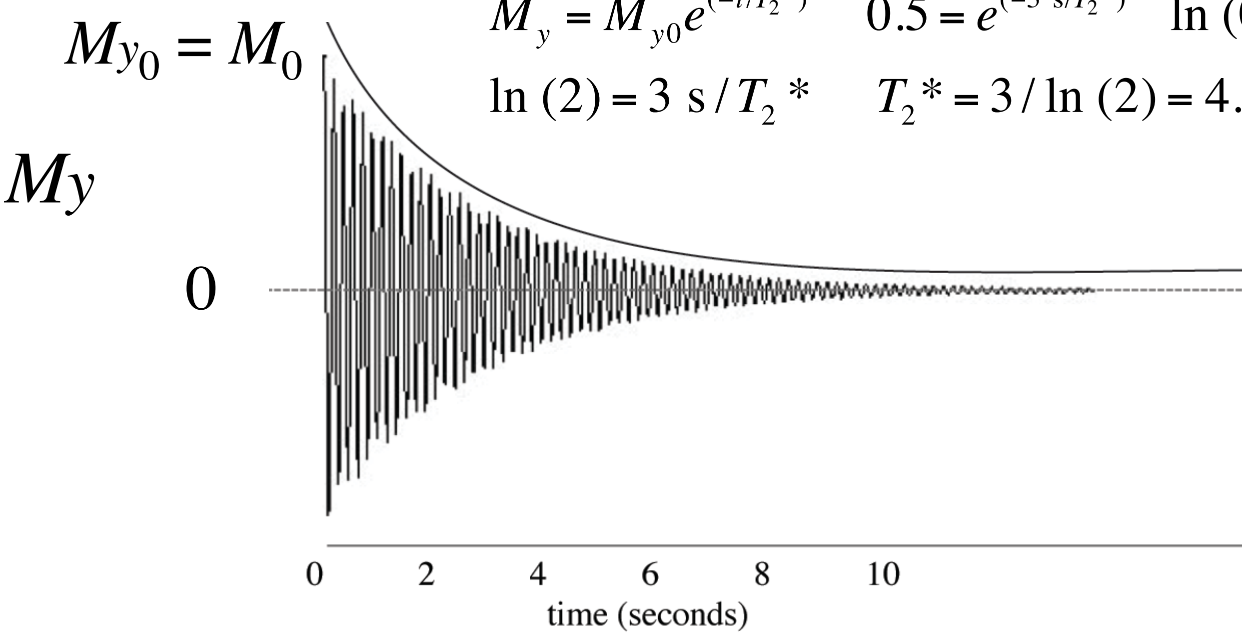
Measuring T_2 Relaxation

- T_2* is EASY to *estimate* also
- Estimate M_y/M_{y0} at a particular time

  - estimate M_y/M_{y0} is approximately 0.5 at 3 seconds, so

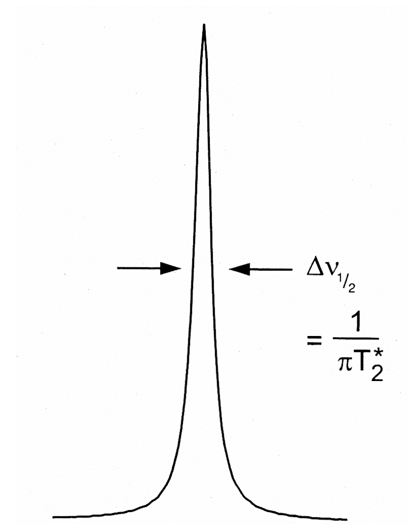
$$M_y = M_{y0} e^{(-t/T_2*)} \qquad 0.5 = e^{(-3\text{ s}/T_2*)} \qquad \ln(0.5) = -3\text{ s}/T_2*$$

$$\ln(2) = 3\text{ s}/T_2* \qquad T_2* = 3/\ln(2) = 4.3\text{ s}$$



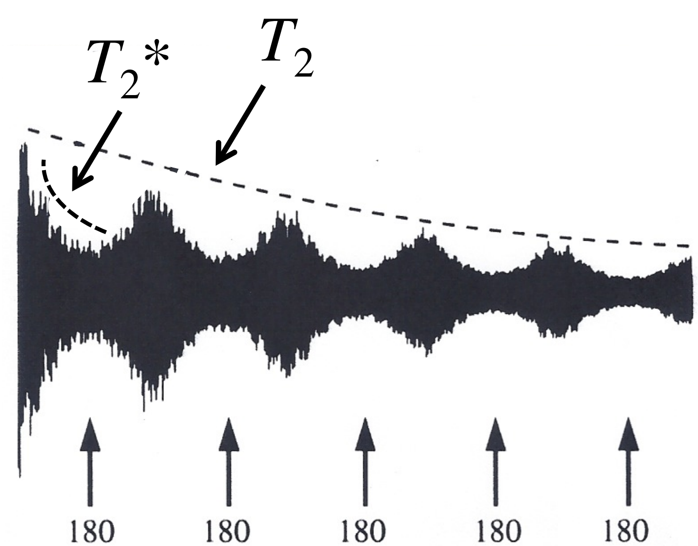
- T_2* can also be estimated from width of the Lorentzian NMR signal at ½ of its maximum height

$$\Delta\upsilon_{1/2} = \frac{1}{\pi T_2*}$$



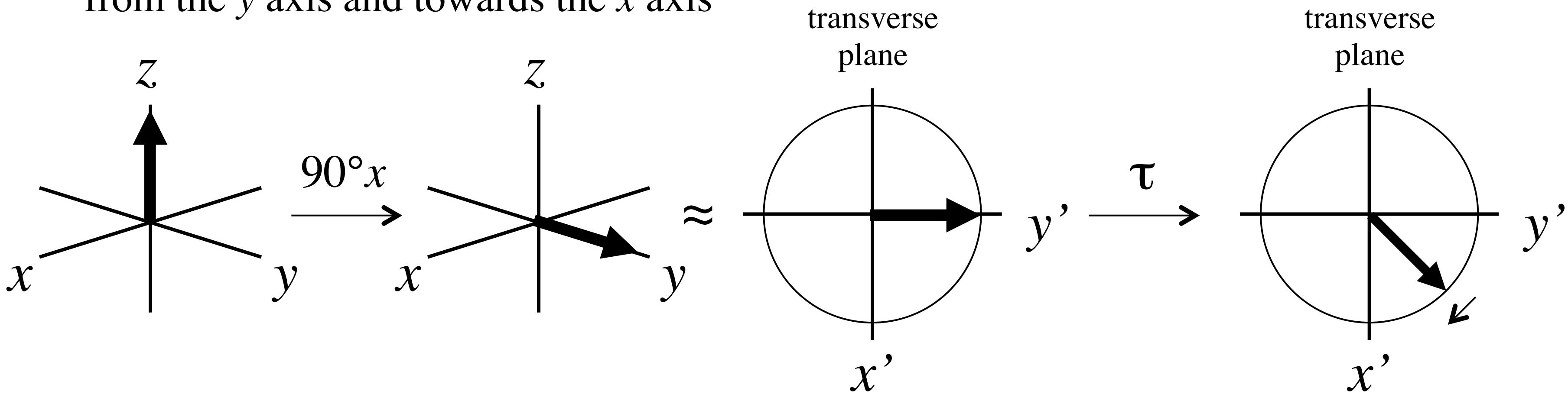
MEASURING T_2 RELAXATION: SPIN-ECHO EXPERIMENT

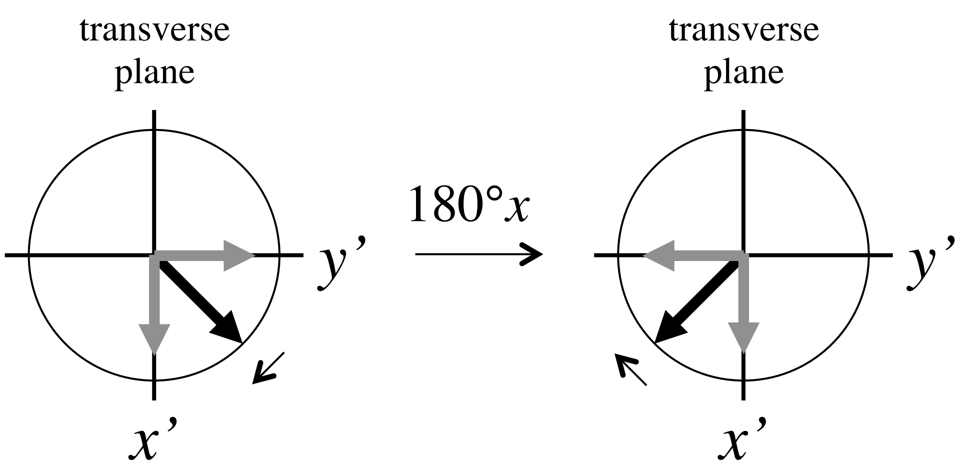
- The natural component of T_2^* contributed by the local fluctuating magnetic fields of the nuclei themselves - the interesting component that we'll call T_2 - is more difficult to measure than T_2^*
- The contribution from magnetic field inhomogeneities has to be eliminated
- The most common way to do this is with a **spin-echo experiment**
- The spin echo experiment for measuring T_2 is based on the fact that the effects of magnetic field inhomogeneity can be **refocused**, and thus eliminated, leaving only the effects of the natural T_2 component to lead to loss of phase coherence



Measuring T_2 Relaxation: Spin-Echo Experiment

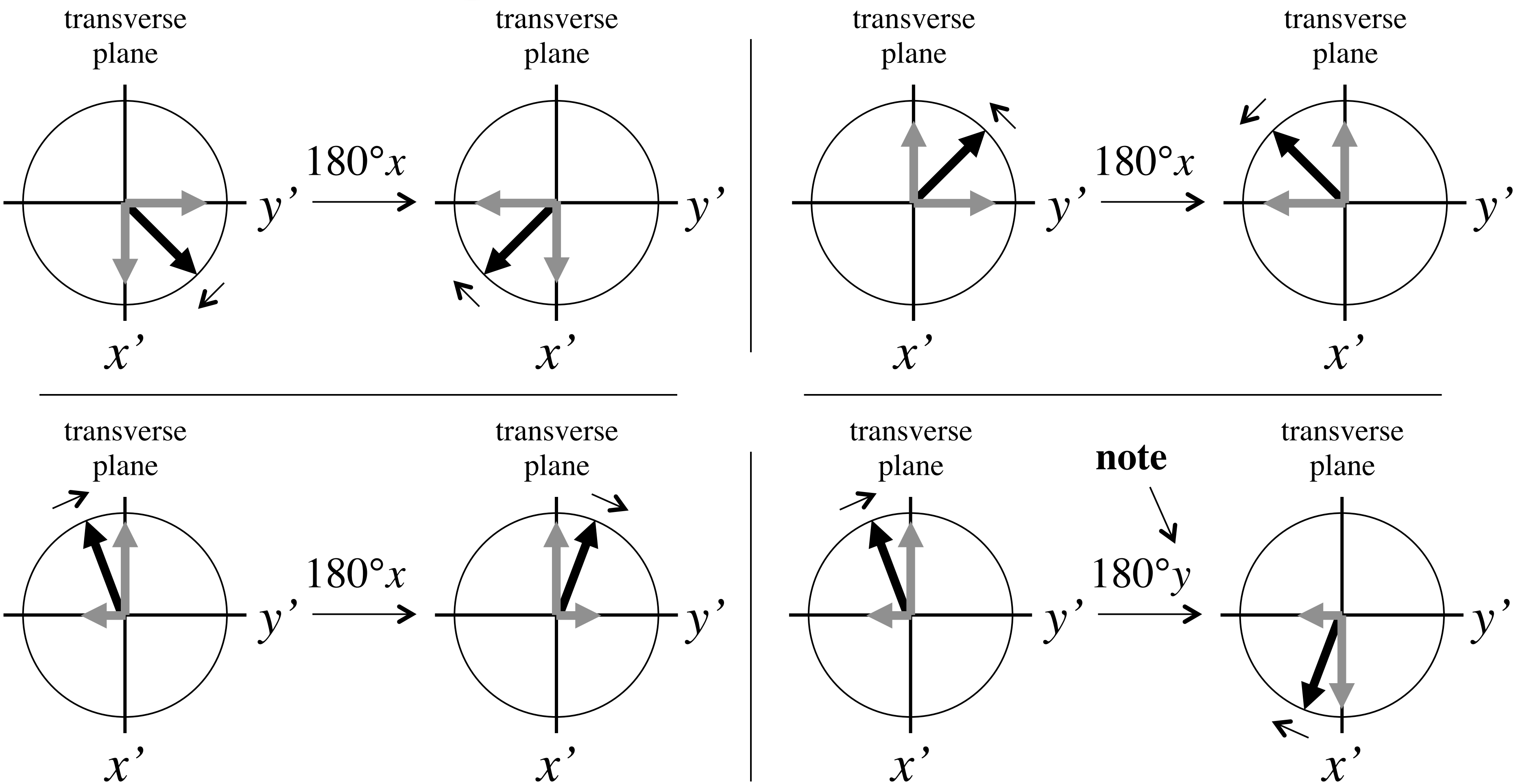
- In order to understand how the spin echo method works, we have to first consider how the 180° pulses affect magnetic dipole vectors in the transverse plane
- Following a 90° pulse, the bulk magnetization vector is in the *x*-*y* plane
- In the rotating frame, we'll assume that the Larmor frequency of the nucleus is slightly faster than the reference frequency, so, after a short time τ, the vector has moved away from the *y* axis and towards the *x* axis





- Consider the vector a sum of *x* and *y* components
- Consider the effect of the 180° pulse on each component separately
- Add the resulting x and y components to get the sum
- Relative frequency does not change

Measuring T_2 Relaxation: Spin-Echo Experiment

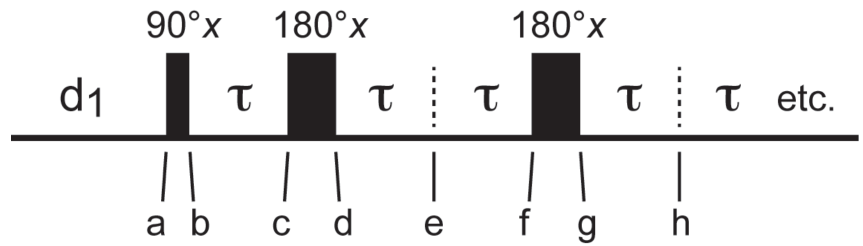


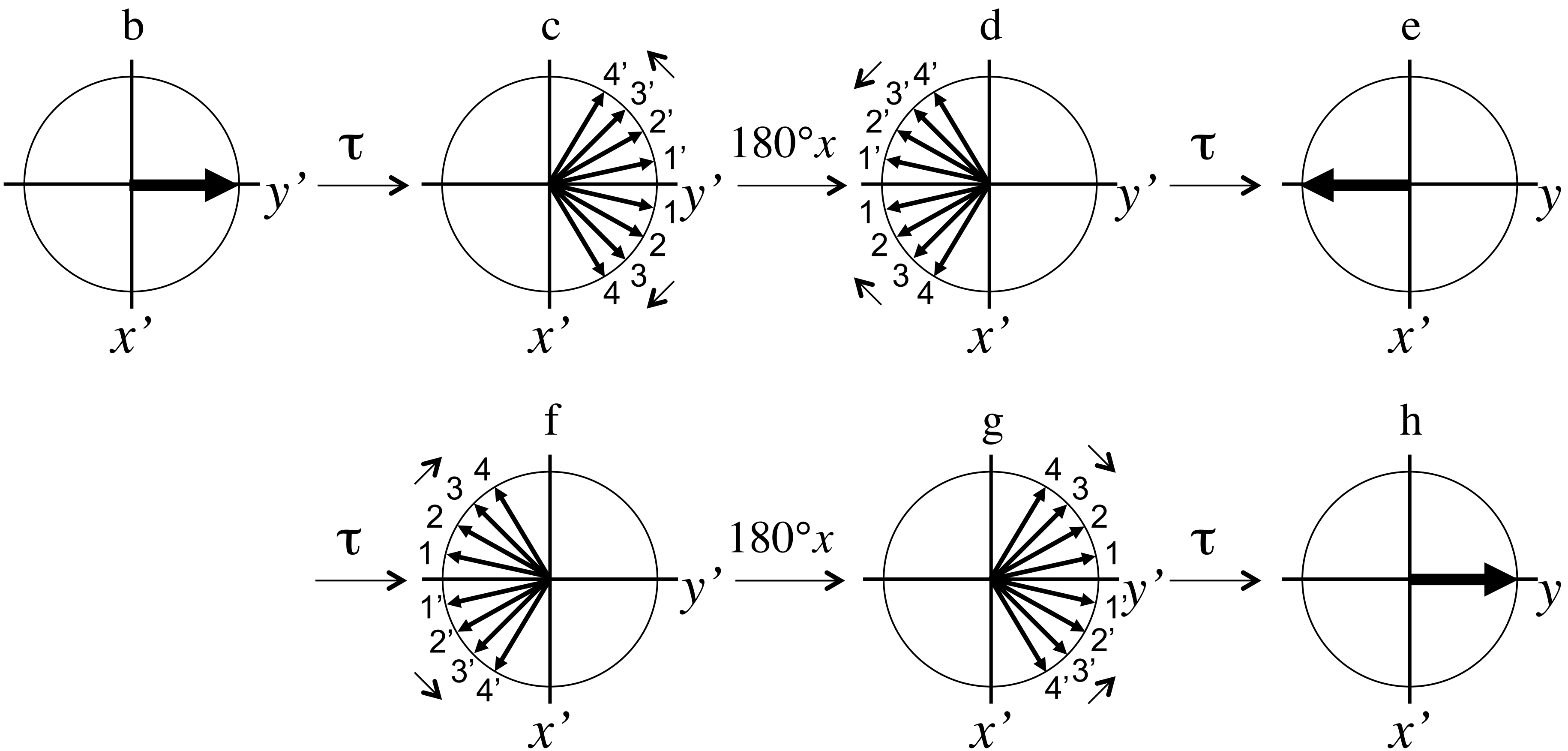
- So, the net result in all cases is simply ***reflection through the z-x plane for a 180°x pulse and reflection through the z-y plane for a 180°y pulse***
- So, this is a simple way to decide the effect of a 180° pulse without considering the effect on each of the component vectors

Measuring T_2 Relaxation: Spin-Echo Experiment

- In the rotating frame, in the first τ period, the individual dipoles lose phase coherence due to T_2* processes
- The 180°x pulse reflects the vectors through the z-x plane
- In the next τ period, the dipole vectors refocus, etc.

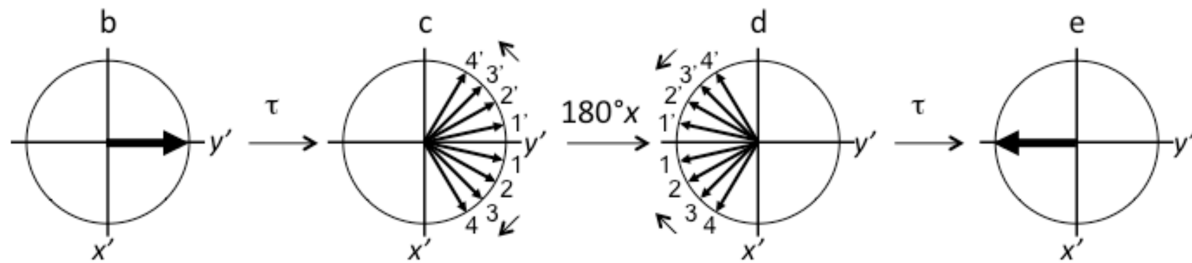
Carr-Purcell spin-echo experiment

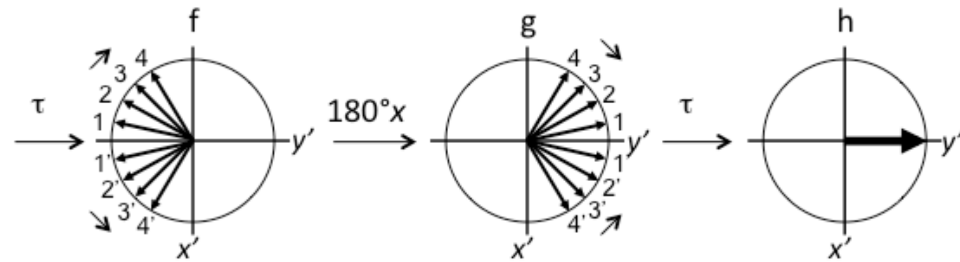


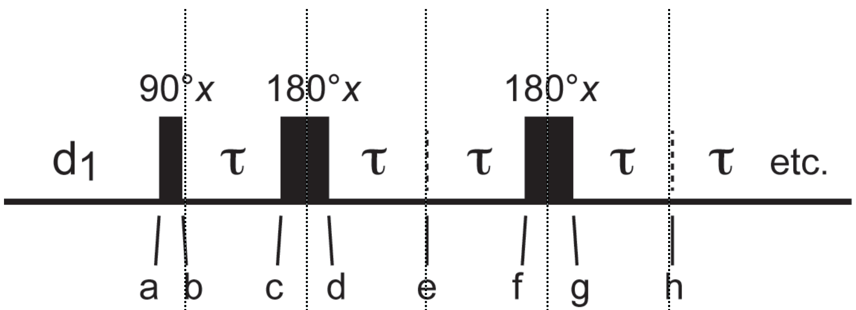


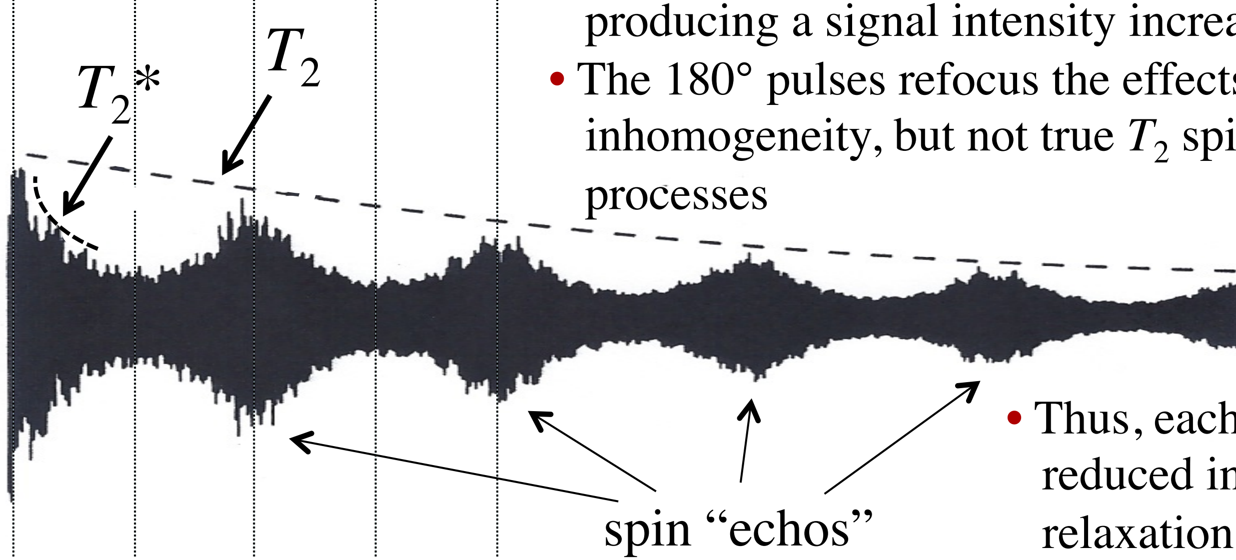
MEASURING T_2 RELAXATION: SPIN-ECHO EXPERIMENT

- So, a spin-echo is an increased signal intensity resulting from refocusing of nuclear spin dipoles





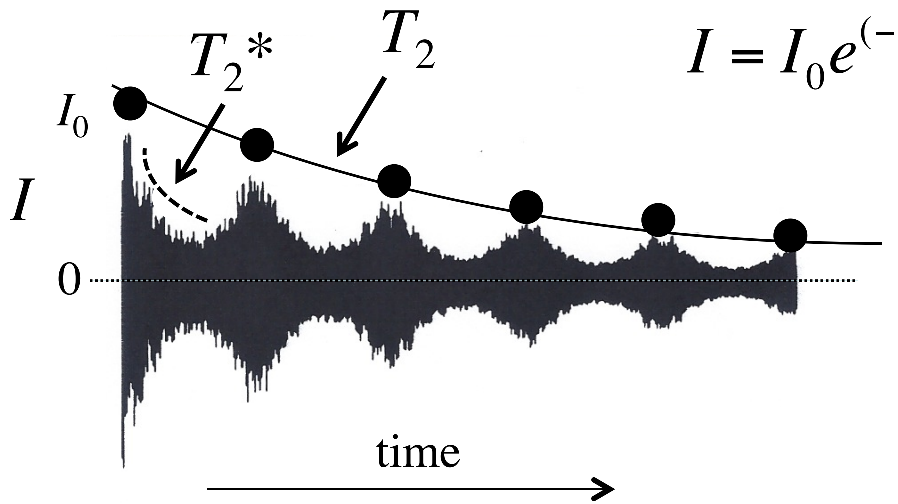




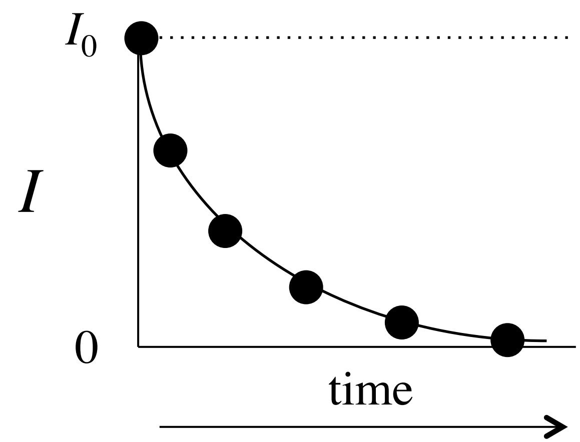
- Between each pair of 180° pulses the dipoles refocus, producing a signal intensity increase or "echo"
- The 180° pulses refocus the effects of magnetic field inhomogeneity, but not true T_2 spin-spin relaxation processes
- Thus, each successive echo has a reduced intensity due to true T_2 relaxation processes

MEASURING T_2 RELAXATION: SPIN-ECHO EXPERIMENT

- The intensities of the echos depend only on T_2, and not T_2*, as a simple exponential



$$I = I_0 e^{(-t/T_2)}$$

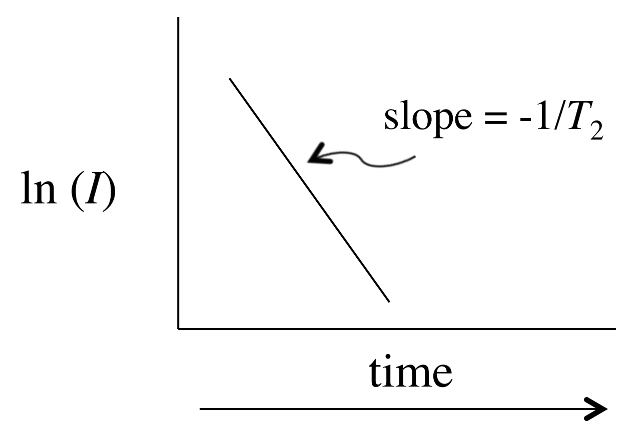


- As with T_2*, T_2 can be estimated by estimating I/I_0 at any given time

$$I = I_0 e^{(-t/T_2)} \qquad \frac{I}{I_0} = e^{(-t/T_2)} \qquad \ln\left(\frac{I}{I_0}\right) = \frac{-t}{T_2} \qquad T_2 = -t / \ln\left(\frac{I}{I_0}\right)$$

- The equation can also be linearized, but this is best used for rough estimates, and should not replace fitting to the exponential equation

$$I = I_0 e^{(-t/T_2)} \qquad \ln I = \ln (I_0) - t / T_2$$

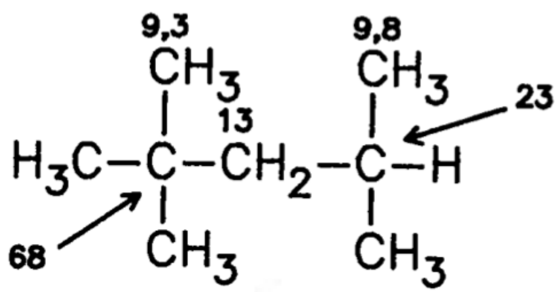


More on Relaxation…..

T_1 – Influence of Directly Attached Hydrogens

- The main contribution to spin-lattice (T_1) relaxation is dipole-dipole (through space) interactions
- For ^{13}C, the primary mechanism for relaxation is via the dipolar interaction with hydrogens
- Hydrogens attached directly to the ^{13}C nucleus contribute most significantly due to the close proximity between the hydrogens and ^{13}C nucleus

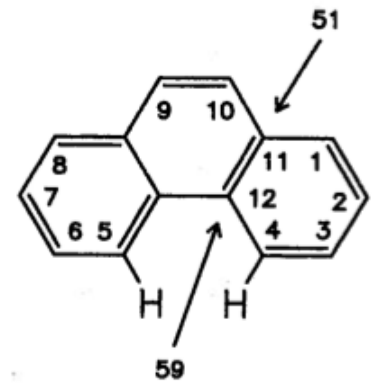
- T_1 values for ^{13}C nuclei decrease with increasing numbers of attached hydrogens



- The T_1 (-^{13}CH) ≈ 2 T_1 (-$^{13}CH_2$) …….. not perfect, but pretty close
- For -CH_3, fast rotation about the C-C bond leads to smaller τ_c and relatively larger T_1
- These depend on the magnetic moments (so, ^2H less efficient at promoting relaxation than ^1H

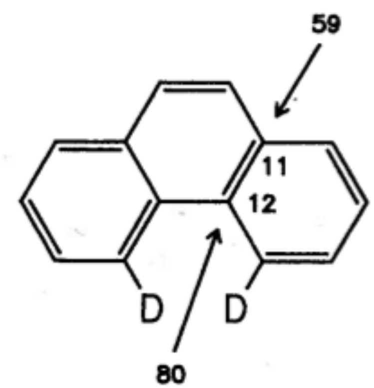
T_1 – Influence of Neighboring Hydrogens

- Through space, dipolar interactions promote relaxation
- These include interactions between nuclei not directly bonded to one another



3A

- The magnetic moment of ^{2}H (D) is less than the magnetic moment of ^{1}H
- Thus, ^{2}H (D) is much less efficient at promoting T_1 relaxation than ^{1}H



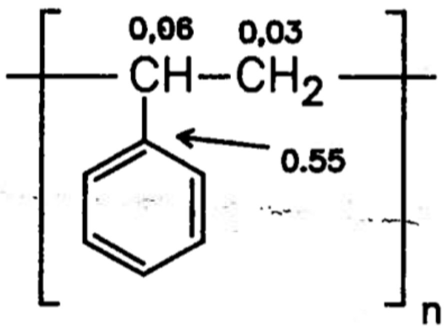
3B

T_1 – Influence of Molecular Size

- The rotational correlation time for a molecule, tc, is proportional to the inverse of the T_1 relaxation time constant

$$\tau_c \propto T_1^{-1}$$

polystyrene: molecular weight 100,000 – 400,000



- Relaxation times for large molecules are short (large τ_c)
- Relaxation times for small molecules are long (small τ_c)

T_1 – Segmental Mobilities

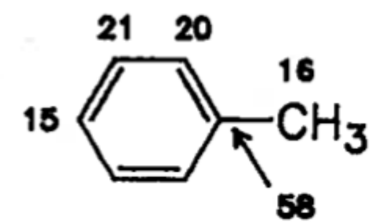
- Different parts of molecules have different mobilities, and, therefore, different effective correlation times
- This leads to variations in relaxation times due to these motional variations

$$\begin{array}{ccccccccccc} CH_3 & - & CH_2 & - & CH_2 & - & CH_2 & - & CH_2 & - & C_5H_{11} \\ 8.7 & & 6.6 & & 5.7 & & 5.0 & & 4.4 & & \end{array}$$

- In straight-chain polymers, for example, the central regions exhibit relatively less motional flexibility, larger effective correlation times, and shorter T_1 relaxation times
- The ends of such polymers exhibit relative more motional flexibility, smaller effective correlation times, and longer T_1 relaxation times

T_1 – Motional Anisotropy

- Molecular reorientation or rotation does not necessarily occur isotropically (i.e. the same or uniformly in all directions or about all axes)
- Anisotropic motion can lead to distinct differences in correlation times for nuclei within a molecule, and distinctly different T_1 relaxation times



- In some benzene derivatives, correlation times for nuclei on the axis of rotation may be much longer than those for groups not on the axis
- Thus, nuclei on the axis of rotation display shorter T_1 relaxation times