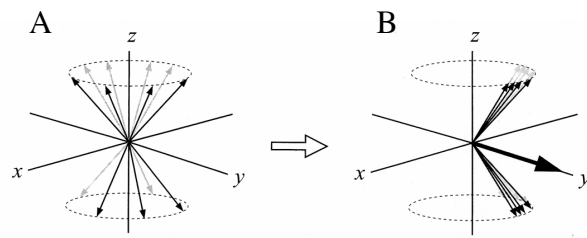


**Exam 1: CHEM/BCMB 4190/6190/8189 (107 points) Thursday, 10 September, 2020**

1). At thermal equilibrium, the microscopic view of an ensemble of spins in a magnetic field can be illustrated by individual magnetic dipoles precessing about the axis corresponding to the magnetic field ( $B_0$ ) direction ( $z$  axis), as shown in the figure (panel A).



a. The result in panel B of the figure is achieved by applying an electromagnetic pulse to the nuclear ensemble in panel A at the Larmor frequency of the nuclei. What was the pulse angle used to obtain this result. You will have to fully explain your answer for credit. **(6 points)**

*The pulse angle was probably 90 degrees. At thermal equilibrium (panel A), we know that the bulk magnetization vector ( $M_0$ ) is aligned along the  $+z$  axis (not shown here), and that an electromagnetic pulse (at the Larmor frequency) will rotate the bulk vector clockwise about the axis along which it was applied. Thus, if the pulse was applied along the  $+x$  axis, the bulk magnetization vector will rotate towards the  $+y$  axis. If the pulse angle were 90 degrees, the bulk vector would then be aligned along  $+y$  (by definition), as is shown in panel B. This is the simplest explanation, although there are also other possibilities (i.e., a 270 degree pulse applied along the  $-x$  axis).*

b. What is relationship between  $N_\alpha$  and  $N_\beta$  in panel B? For credit, you will have to explain how you know this based on the macroscopic result shown in panel B? **(4 points)**

*It is clear from panel B that the bulk magnetization vector has components only in the  $x$ - $y$  plane (no net component along the  $z$  axis). Thus, the number of spins in the  $\alpha$  state ( $N_\alpha$ ) must be equal to the number of spins in the  $\beta$  state ( $N_\beta$ ) in order for the sum of all of the nuclear dipole vectors ( $+z$  and  $-z$  components) to give a net result of zero along  $z$ .*

2). For an electromagnetic pulse applied for 5  $\mu\text{s}$  (at the Larmor frequency for a particular nucleus), the pulse angle is 45 degrees ( $\pi/4$  radians). The magnitude of the  $B_1$  field used to apply the pulse is 0.00062379 T.

a. Calculate the value of the gyromagnetic ratio for this nucleus? You will need to show your work for credit. **(6 points)**

*We know that the pulse angle ( $\Theta$ , in radians) is related to the gyromagnetic ratio ( $\gamma$ ), the pulse length ( $\tau_p$ ) and the applied electromagnetic field strength ( $B_1$ ) as shown:*

$$\Theta = \gamma B_1 \tau_p$$

Thus,

$$\gamma = \frac{\Theta}{B_1 \tau_p} = \frac{\frac{\pi}{4} \text{ rad}}{0.00062379 \text{ T} \times 0.000005 \text{ s}} = 25.1815 \times 10^7 \text{ rad T}^{-1} \text{ s}^{-1}$$

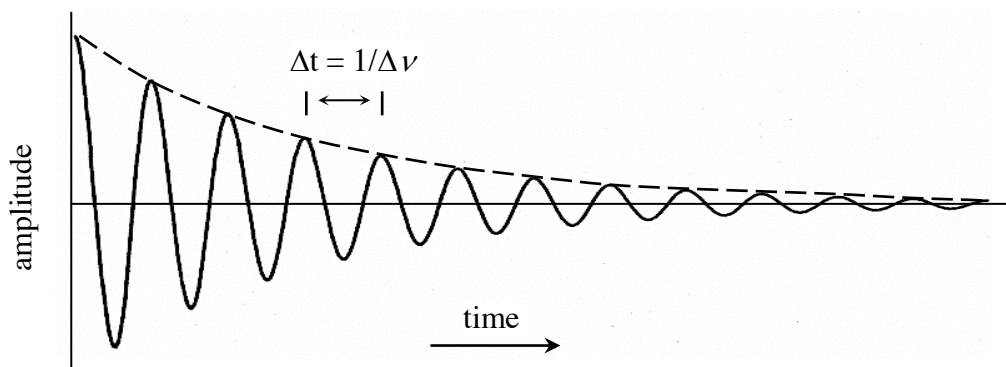
b. What is the identity of the nucleus from 'a'. You will have to explain how you arrived at your answer for credit. **(4 points)**

*From the table at the end of the exam, we see that the nucleus with a gyromagnetic ratio that most closely matches that calculated in part 'c' is  $^{19}\text{F}$ . So, the identity of the nucleus is most likely  $^{19}\text{F}$ .*

c. Define "precession frequency" as it applies to NMR. Use complete sentences and no symbols, equations or drawings. **(4 points)**

*In a static magnetic field, the angular momentum of a spin/nucleus orients itself with the field. The associated magnetic moment or nuclear dipole is also oriented at a specific angle with respect to the static field. The dipole rotates about the static field with an axis of rotation parallel to the field. In analogy to a spinning top, this motion is referred to as precession. The angular velocity or speed (radians/sec) of this rotation, or precession, is therefore referred to as the precession frequency. This is also referred to as the Larmor frequency.*

3). The NMR signal (the observed FID) in the plot below results from a sample of a compound that has a single  $^1\text{H}$  nucleus ( $\text{CHCl}_3$ , for instance). The length of the period ( $\Delta t$ ) of the cosine-modulated signal is equal to  $1/\Delta\nu$ , as is shown on the plot.



a. What is the frequency observed in the FID for this nucleus if  $\Delta t$  is 0.0011 s? (2 points)

$$\nu \text{ (or } \Delta\nu) = 1/\Delta t = 1/0.0011 \text{ s} = 909 \text{ Hz}$$

b. What is the difference, in Hz, between the Larmor precession frequency for this nucleus and the frequency of the  $B_1$  field (pulse) used to get the signal. You will have to explain your reasoning for credit. (4 points)

*The difference is 909 Hz. The frequency of the  $B_1$  field is used as the reference frequency in NMR experiments. All observed signals in the observed FID are differences between Larmor precession frequencies of nuclei and the reference frequency. So, because the observed frequency is 909 Hz, this is also the difference between the Larmor frequency of the nucleus and the frequency at which the  $B_1$  field was applied.*

c. The dashed line follows the decay of the amplitude of the signal with time. If, after 1 second, the amplitude of the signal is 1/2 what it was initially, what is the effective  $T_2$  (i.e.  $T_2^*$ )? (4 points)

$$M_y = M_0 e^{-t/T_2} \quad \frac{M_y}{M_0} = e^{-t/T_2} \quad \ln\left(\frac{M_y}{M_0}\right) = -t/T_2 \quad T_2 = \frac{-t}{\ln\left(\frac{M_y}{M_0}\right)} = \frac{-1 \text{ s}}{\ln(0.5)} = 1.44 \text{ s}$$

d. After Fourier transformation of the signal, calculate the width of the frequency-domain signal (in Hz) at half of its maximum amplitude? You will have to show your work and arrive at an answer for credit. **(4 points)**

$$\Delta\nu_{1/2} = \frac{1}{\pi T_2^*} = \frac{1}{\pi \cdot 1.44 \text{ s}} = 0.22 \text{ Hz}$$

e. If the  $^1\text{H}$  signal was acquired using a 7 T magnet ( $B_0$  field 7 Tesla) what will be the approximate resonance frequency (in MHz) of the  $^{13}\text{C}$  nuclei in  $\text{CHCl}_3$  if the  $^{13}\text{C}$  spectrum were acquired using the same magnet. You must do a calculation and show your work for credit. **(4 points)**

*The Larmor frequency is directly proportional to magnetic field strength, as shown below, and we can calculate the frequency directly, in Hz, knowing the value for the gyromagnetic ratio of  $^{13}\text{C}$ :*

$$\nu_L = \gamma B_0 / (2\pi) = \omega_0 / (2\pi) = 6.7283 \times 10^7 \text{ rad T}^{-1} \text{ s}^{-1} \times 7 \text{ T} / 2\pi = 75 \text{ MHz}$$

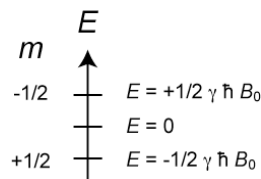
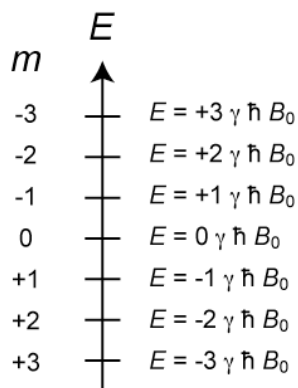
f. For a sample of  $\text{CHCl}_3$ , explain how you would determine experimentally what the  $270^\circ$  pulse width/length is for the single  $^1\text{H}$  resonance in your sample? **(4 points)**

*We know that only transverse magnetization is detected by the receiver of our NMR instrumentation. A perfect  $90^\circ$  pulse creates maximum transverse magnetization and a signal of maximum amplitude, whereas a perfect  $180^\circ$  pulse creates no transverse magnetization, so gives no detectable signal. Like a  $90^\circ$  pulse, a  $270^\circ$  pulse gives a signal of maximum amplitude with phase opposite to that of the  $90^\circ$  pulse. So, in order to determine the  $270^\circ$  pulse width, one simply slowly increases the pulse width from zero through a maximum ( $90^\circ$  pulse), then through a minimum (zero intensity,  $180^\circ$  pulse) and then to the next maximum with opposite sign. This should be the  $270^\circ$  pulse width and should be approximately three times the  $90^\circ$  pulse width.*

4). You record an  $^{15}\text{N}$  NMR spectrum of your compound using 100 scans. You realize that you need to improve the sensitivity (signal-to-noise) by 5-fold. How many scans will be required to record a spectrum with 5 times the signal-to-noise? **(4 points)**

*Signal-to-noise ( $S/N$ ) increases as the square root of the number of scans ( $S/N \propto N^{1/2}$ ). In order to improve  $S/N$  by  $5\times$ , one would have to acquire  $5^2$  or  $25\times$  the number of scans. So, if 100 scans were used initially,  $25 \times 100 = 2500$  scans would be required for a  $5\times$   $S/N$  improvement.*

5). A diagram of the Zeeman levels/states for  $^{13}\text{C}$  is shown to the right. Draw the Zeeman diagram for  $^{10}\text{B}$ . **(4 points)**



6). If we directly compare  $^{10}\text{B}$  spins and  $^{13}\text{C}$  spins;

a. What is the ratio of the bulk/macroscopic magnetization ( $M_{0\ 10\text{B}}/M_{0\ 13\text{C}}$ ) produced for equal numbers of nuclei? **(6 points)**

$$\text{Remember, } M_0 = \frac{N\gamma^2\hbar^2 B_0 I(I+1)}{3k_B T}$$

$$\frac{M_{0\ 10\text{B}}}{M_{0\ 13\text{C}}} = \frac{\frac{N\gamma_{10\text{B}}^2\hbar^2 B_0 I_{10\text{B}}(I_{10\text{B}}+1)}{3k_B T}}{\frac{N\gamma_{13\text{C}}^2\hbar^2 B_0 I_{13\text{C}}(I_{13\text{C}}+1)}{3k_B T}} = \frac{\gamma_{10\text{B}}^2 \times 3(3+1)}{\gamma_{13\text{C}}^2 \times \frac{1}{2}\left(\frac{1}{2}+1\right)} = \frac{(2.8747 \times 10^7 \text{ rad/Ts})^2 \times 12}{(6.7283 \times 10^7 \text{ rad/Ts})^2 \times \frac{3}{4}} = 2.9207$$

So, the bulk magnetization is about 2.9 times as large for  $^{10}\text{B}$  as for  $^{13}\text{C}$ .

b. How does the sensitivity of the NMR signal compare for  $^{10}\text{B}$  and  $^{13}\text{C}$  spins? **(6 points)**

*Sensitivity is proportional to the electromotive force ( $\varepsilon$ ) induced in the receiver coil by the bulk magnetic moment. The magnitude of  $\varepsilon$  is proportional to the rate of change in the magnetic moment ( $\varepsilon \propto dM/dt = \gamma M_0 B$ ):*

$$\varepsilon \propto \gamma M_0 B = \frac{N\gamma^3\hbar^2 B_0^2 I(I+1)}{3k_B T}$$

*So, whereas the magnitude of the bulk magnetization was dependent on  $\gamma^2$ , the sensitivity is dependent on  $\gamma^3$ .*

$$\frac{\varepsilon_{10\text{B}}}{\varepsilon_{13\text{C}}} = \frac{\frac{N\gamma_{10\text{B}}^3\hbar^2 B_0^2 I_{10\text{B}}(I_{10\text{B}}+1)}{3k_B T}}{\frac{N\gamma_{13\text{C}}^3\hbar^2 B_0^2 I_{13\text{C}}(I_{13\text{C}}+1)}{3k_B T}} = \frac{\gamma_{10\text{B}}^3 I_{10\text{B}}(I_{10\text{B}}+1)}{\gamma_{13\text{C}}^3 I_{13\text{C}}(I_{13\text{C}}+1)} = \frac{(2.8747 \times 10^7 \text{ rad/Ts})^3 \times 12}{(6.7283 \times 10^7 \text{ rad/Ts})^3 \times \frac{3}{4}} = 1.248$$

So,  $^{10}\text{B}$  nuclei are about 1.25 times more sensitive than  $^{13}\text{C}$  nuclei.

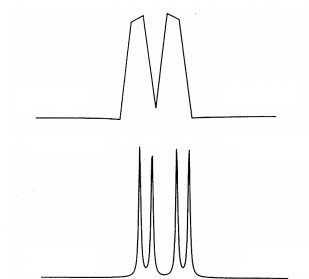
7). Fill in the blanks to make the following statements true:

- a). In NMR, the applied ( $B_1$ ) field is applied perpendicular to the static ( $B_0$ ) field. (2 points)
- b). At equilibrium, for an ensemble of spin  $\frac{1}{2}$  nuclei in a magnetic field the number of alpha ( $\alpha$ ) spins ( $N_\alpha$ ) = X, and the number of beta ( $\beta$ ) spins ( $N_\beta$ ) = Y. Immediately following a  $90^\circ$  pulse along the y axis, the number of  $\alpha$  spins =  $(X+Y)/2$  and the number of  $\beta$  spins =  $(X+Y)/2$ . (2 points)
- c). The number of Zeeman energy levels/states for  $^{17}\text{O}$  is 6. (2 points)
- d). For a spin  $\frac{1}{2}$  nucleus at equilibrium in a magnetic field, the alpha ( $\alpha$ ) state has a magnetic quantum number of  $1/2$ , its energy is less than that of the beta ( $\beta$ ) state, and the z component of its associated magnetic moment,  $\mu_z$ , is aligned parallel to the magnetic field. (3 points)

8). For a sample of  $\text{CHCl}_3$ , you apply a very long (seconds), low power pulse at the  $^1\text{H}$  frequency of the single  $^1\text{H}$  nucleus in the molecule. Immediately after this pulse, you apply a normal, high power  $90^\circ$  pulse at the same frequency and then record the FID. Sketch what you would expect the FID to look like. Also, sketch the spectrum you would expect to observe following Fourier transformation of the FID. You must thoroughly explain your answers for credit. (6 points).

*A long pulse leads to equilibration of the  $\alpha$  and  $\beta$  spin populations of the nucleus. In such an instance, there is no net, bulk (macroscopic) magnetization in either the longitudinal direction or transverse plane. If some longitudinal component were present, the  $90^\circ$  degree pulse would rotate that component into the transverse plane and it could then be detected. Because there is no such component, the  $90^\circ$  degree pulse does nothing, and there is no detectable signal. So, the FID will just be noise, and there will be no observable signals in the transformed spectrum.*

9). When you record the  $^1\text{H}$  NMR spectrum of a particular compound, you observe the spectrum shown above (right). However, you know from published data that the spectrum should look like that shown below (right). Both spectra were recorded at the same magnetic field strength (400 MHz).



Using the same magnetic field strength (400 MHz), suggest two changes that you might make in how you acquire the  $^1\text{H}$  NMR spectrum of this compound so that the spectrum looks more like the bottom spectrum shown (right). You must justify/explain your answers for credit. **(4 points)**

*The spectrum shown at the top is diagnostic of poor digital resolution (DR). The number of points used to define the individual peaks is inadequate to do so, resulting in the inability to properly distinguish the peaks from one another. To improve the digital resolution (i.e. decrease the value of DR, which is the number of Hz per point), the acquisition time (AQ) or the number of points (NP) can be increased, or the spectral width can be decreased.*

$$\text{DR} = \frac{2 \times \text{SW}}{\text{NP}} = \frac{1}{\text{AQ}}$$

*It may also be that the magnetic field homogeneity is poor. This results in broadening of peaks. Proper shimming of the magnetic field would be indicated. Sample spinning may also help.*

10). You decide to record a  $^1\text{H}$  spectrum using only 90 degree pulses. You decide that you will collect 4 scans. After each pulse, you decide to wait long enough for 99.9% of the magnetization to return to thermal equilibrium before applying the next pulse. If  $T_1$  for your sample is 5 seconds, how long will the time between pulses be? **(6 points)**

*We know that the return to thermal equilibrium following a 90 degree pulse is governed by the first order equation shown below:*

$$M_z = M_0(1 - e^{-t/T_1}) \quad \frac{M_z}{M_0} = (1 - e^{-t/T_1}) \quad 1 - \frac{M_z}{M_0} = e^{-t/T_1} \quad \ln\left(1 - \frac{M_z}{M_0}\right) = -t/T_1$$

$$-T_1 \times \ln\left(1 - \frac{M_z}{M_0}\right) = t \quad -5 \times \ln(1 - 0.999) = 34.5 \text{ s}$$



**11).** Paramagnetic compounds have an unpaired electron that can promote very efficient and fast NMR relaxation. In cases where such compounds interact with other molecules in solution, they can increase substantially the relaxation rates of nuclei in those molecules.

a. Why might it be important to make sure that NMR samples, in general, are clean and do not contain significant amounts of impurities that are paramagnetic? Please explain for credit. **(4 points)**

*If  $T_2$  relaxation rates are increased substantially, this can lead to very broad lines and significant decreases in signal-to-noise. Under circumstances where very narrow lines and good signal-to-noise are paramount, it is important to make sure that contamination by paramagnetic impurities is minimized.*

b. It is a reasonably common practice to sometimes deliberately add certain paramagnetic compounds to NMR samples for the purpose of decreasing  $T_1$  relaxation times. Why might this be useful? Please explain for credit. **(4 points)**

*When  $T_1$  relaxation times are long, the relaxation delay ( $d_1$ ) must often also be long to allow the spin state populations to re-establish before the next pulse. Thus, the total experimental time can be very long if multiple scans are collected. Paramagnetic relaxation agents can significantly decrease  $T_1$ , allowing fast and efficient relaxation and re-establishment of equilibrium spin state populations. This may be useful when  $T_1$  values are very long, and when broadening of signals and signal-to-noise are not limiting.*

**12).** There are two contributors to the NMR relaxation parameter  $T_2^*$ . Please identify these contributors and their origins and how they contribute, microscopically, to relaxation in the transverse plane. **(8 points)**

*Magnetic field inhomogeneity is the first contributor. For bulk magnetization in the transverse plane, the precession frequencies of the individual magnetic dipoles that comprise the ensemble are dependent on the magnitude of the field strength that each experiences. As the static magnetic field becomes less homogeneous, the range of precession frequencies increases, and the individual magnetic dipoles diverge from one another faster, destroying the phase coherence in the transverse plane and decreasing the amplitude of the observed signal.*

*The second contributor is spin-spin relaxation or the “true”  $T_2$  component. The individual nuclear dipoles reorient randomly creating local electromagnetic fields that also result in loss of phase coherence in the transverse plane.*

Name \_\_\_\_\_

Name \_\_\_\_\_

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You may find some of the information below useful:

**Table 1-1.**

Properties of some nuclides of importance in NMR spectroscopy.

Nuclide	Spin $I$	Electric quadrupole moment <sup>a)</sup> [ $eQ$ ] [ $10^{-28} \text{ m}^2$ ]	Natural abundance) [%]	Relative sensitivity <sup>b)</sup>	Gyromagnetic ratio $\gamma^a)$ [ $10^7 \text{ rad T}^{-1} \text{ s}^{-1}$ ]	NMR frequency [MHz] <sup>b)</sup> ( $B_0 = 2.3488 \text{ T}$ )
$^1\text{H}$	1/2	–	99.985	1.00	26.7519	100.0
$^2\text{H}$	1	$2.87 \times 10^{-3}$	0.015	$9.65 \times 10^3$	4.1066	15.351
$^3\text{H}^c)$	1/2	–	–	1.21	28.5350	106.664
$^6\text{Li}$	1	$-6.4 \times 10^{-4}$	7.42	$8.5 \times 10^{-3}$	3.9371	14.716
$^{10}\text{B}$	3	$8.5 \times 10^{-2}$	19.58	$1.99 \times 10^{-2}$	2.8747	10.746
$^{11}\text{B}$	3/2	$4.1 \times 10^{-2}$	80.42	0.17	8.5847	32.084
$^{12}\text{C}$	0	–	98.9	–	–	–
$^{13}\text{C}$	1/2	–	1.108	$1.59 \times 10^{-2}$	6.7283	25.144
$^{14}\text{N}$	1	$1.67 \times 10^{-2}$	99.63	$1.01 \times 10^{-3}$	1.9338	7.224
$^{15}\text{N}$	1/2	–	0.37	$1.04 \times 10^{-3}$	-2.7126	10.133
$^{16}\text{O}$	0	–	99.96	–	–	–
$^{17}\text{O}$	5/2	$-2.6 \times 10^{-2}$	0.037	$2.91 \times 10^{-2}$	-3.6280	13.557
$^{19}\text{F}$	1/2	–	100	0.83	25.1815	94.077
$^{23}\text{Na}$	3/2	0.1	100	$9.25 \times 10^{-2}$	7.0704	26.451
$^{25}\text{Mg}$	5/2	0.22	10.13	$2.67 \times 10^{-3}$	-1.6389	6.1195
$^{29}\text{Si}$	1/2	–	4.70	$7.84 \times 10^{-3}$	-5.3190	19.865
$^{31}\text{P}$	1/2	–	100	$6.63 \times 10^{-2}$	10.8394	40.481
$^{39}\text{K}$	3/2	$5.5 \times 10^{-2}$	93.1	$5.08 \times 10^{-4}$	1.2499	4.667
$^{43}\text{Ca}$	7/2	$-5.0 \times 10^{-2}$	0.145	$6.40 \times 10^{-3}$	-1.8028	6.728
$^{57}\text{Fe}$	1/2	–	2.19	$3.37 \times 10^{-5}$	0.8687	3.231
$^{59}\text{Co}$	7/2	0.42	100	0.28	6.3015	23.614
$^{119}\text{Sn}$	1/2	–	8.58	$5.18 \times 10^{-2}$	-10.0318	37.272
$^{133}\text{Cs}$	7/2	$-3.0 \times 10^{-3}$	100	$4.74 \times 10^{-2}$	3.5339	13.117
$^{195}\text{Pt}$	1/2	–	33.8	$9.94 \times 10^{-3}$	5.8383	21.499

$B_0$ (Tesla, T)	Resonance frequencies (MHz)	
	$^1\text{H}$	$^{13}\text{C}$
9.4	400	100.6
11.74	500	125.7
14.09	600	150.9
18.79	800	201.2

$$\gamma_{^1\text{H}} = 26.7519 \times 10^7 \text{ rad/T/s}, I = 1/2$$

$$\gamma_{^{10}\text{B}} = 2.8747 \times 10^7 \text{ rad/T/s}, I = 3$$

$$\gamma_{^{11}\text{B}} = 8.5847 \times 10^7 \text{ rad/T/s}, I = 3/2$$

$$\gamma_{^{13}\text{C}} = 6.7283 \times 10^7 \text{ rad/T/s}, I = 1/2$$

$$\gamma_{^{15}\text{N}} = -2.7126 \times 10^7 \text{ rad/T/s}, I = 1/2$$

$$\gamma_{^{17}\text{O}} = -3.6280 \times 10^7 \text{ rad/T/s}, I = 5/2$$

**You may find some of the following information or equations useful:**

$$k_B = 1.381 \times 10^{-23} \text{ J/K}$$

$$\text{Avagadro's number} = 6.02214179 \times 10^{23} \text{ mol}^{-1}$$

$$h = 6.626 \times 10^{-34} \text{ Js}$$

$$\hbar = h/(2\pi)$$

$$P = \hbar \sqrt{I(I+1)}$$

$$P_z = m\hbar$$

$$\mu = \gamma P = \hbar \gamma \sqrt{I(I+1)}$$

$$\text{for } m = \frac{1}{2}, \cos(\theta) = \frac{m\hbar}{\hbar \sqrt{I(I+1)}} = \frac{m}{\sqrt{I(I+1)}}$$

$$\pi/2 \text{ radians} = 90^\circ$$

$$M_0 = \frac{N\gamma^2 \hbar^2 B_0 I(I+1)}{3k_B T}$$

$$B_2 = \frac{\Delta \nu \sqrt{J^2 - J_r^2}}{J_r} = \frac{J \Delta \nu}{J_r}$$

$$\varepsilon \propto dM/dt = \gamma M_0 B = \frac{N\gamma^3 \hbar^2 B_0^2 I(I+1)}{3k_B T}$$

$$\Delta \nu = \nu_{BS} - \nu_0 = \frac{B_2^2}{2(\Delta B)} = \frac{B_2^2}{2(\nu_0 - \nu_i)}$$

$$S/N \propto NS^{1/2} \text{ (signal-to-noise improves with (number of scans)}^{1/2})$$

$$m = (-I, -I+1, \dots, I-1, I)$$

$$\Theta = \gamma B_1 \tau_p$$

$$E = -\mu_z B_0 = -m\gamma \hbar B_0$$

$$\frac{N_\beta}{N_\alpha} \approx 1 - \left( \frac{\gamma \hbar B_0}{k_B T} \right)$$

$$\Delta E = \mu_z B_0 = \gamma \hbar B_0 = h\nu_L = h\nu_1$$

$$B_{eff} = B_0(1 - \sigma)$$

$$\nu_L = |\gamma/(2\pi)| B_0 = \omega_0/(2\pi)$$

$$\nu_L = \frac{\gamma}{2\pi} (1 - \sigma) B_0$$

$$\omega_0 = \gamma B_0$$

$$\Delta \delta = \frac{\Delta \nu}{\text{observe frequency}} \times 10^6$$

$$SW = 1/(2DW) = \text{Nyquist frequency } (\nu_{NQ})/2$$

$$M_y = M_0 e^{-t/T_2^*}$$

$$AQ = DW * TD$$

$$M_z = M_0(1 - e^{-t/T_1})$$

$$DR = 2SW/TD = 1/AQ$$

$$M_z = M_0(1 - 2e^{-t/T_1})$$

$$(TD \equiv NP)$$

$$\Delta \nu_{1/2} = \frac{1}{\pi T_2^*}$$

$$\cos \alpha_{\text{Ernst}} = e^{-((d_1 + AQ)/T_1)}$$

$$\pi/2 \text{ radians} = 90^\circ$$

$$\frac{1}{T_2^*} = \frac{\gamma \Delta B_0}{2} + \frac{1}{T_2} \quad \frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2(B_0)}$$

$$t_{\text{zero}} = T_1 \ln(2)$$

$$1 + \gamma_A/\gamma_X \quad 1 - \gamma_A/\gamma_X$$

$$\eta = \gamma_a / (2\gamma_x)$$

$$\text{multiplicity} = 2nI + 1$$

$$I = (1 + \eta) I_0$$

$$\Theta = 2\pi J \tau$$

$$I \propto 1/r^6$$