Parallel Methods for Neuron Network

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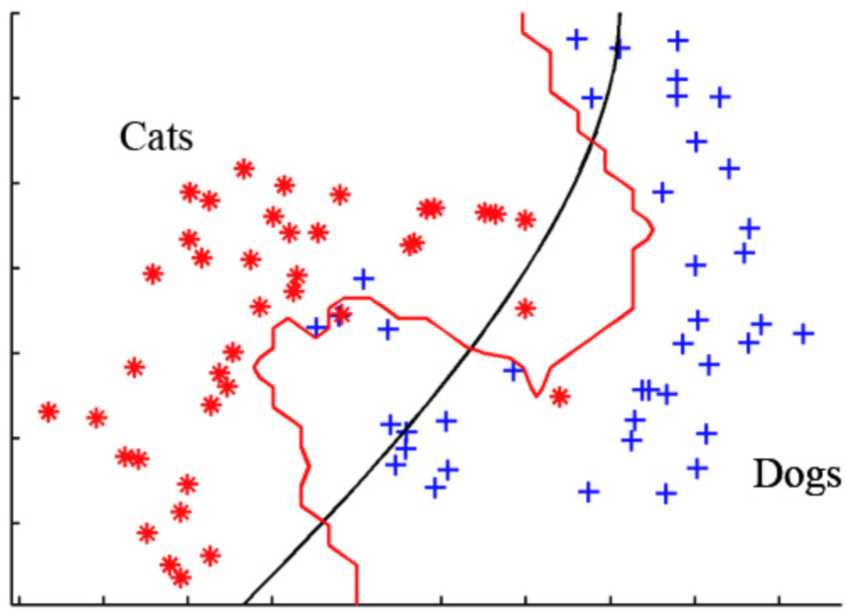
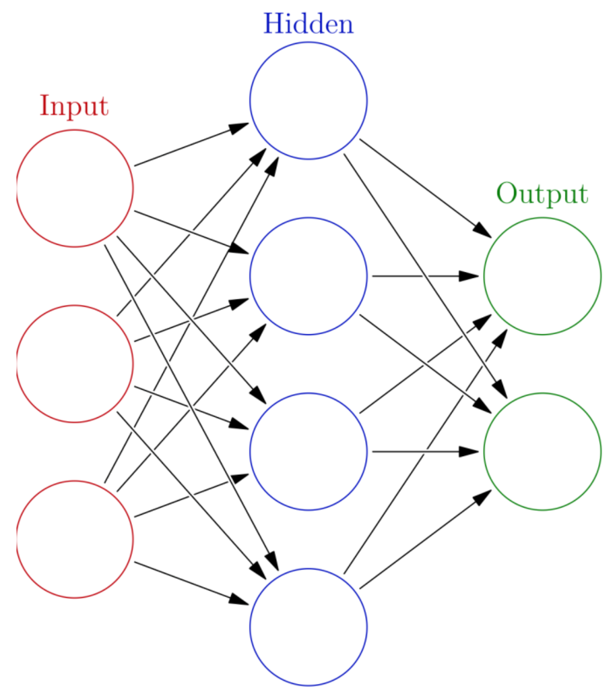
Machine Learning and Artificial Neuron Network





Problem of Interest: ANN

$$\min_{W,b} \frac{1}{n} \sum_{k=1}^{n} l(\phi_l(W_l \phi_{l-1}(\cdots \phi_1(W_1 x_k + b_1) \cdots + b_l)), y_k)$$

- W is the weight
- b is the bias
- ϕ is activation function
- x is feature
- y is label
- l is loss function

Drawback of SGD

- SGD is a first-order method, thus it converges slow.
- SGD suffers from vanishing gradient problem
- Most importantly, it is hard to parallelize SGD

$$\min_{W,b} \frac{1}{n} \sum_{k=1}^{n} l(\phi_l(W_l \phi_{l-1}(\cdots \phi_1(W_1 x_k + b_1) \cdots + b_l)), y_k)$$

- Equivalent Problem

$$\min_{W,b,z} \frac{1}{n} \sum_{k=1}^{n} l(\phi_l(z_l^k), y_k)$$

$$s.t. z_l^k = W_l \phi_{l-1}(z_{l-1}^k) + b_l$$

$$\cdots$$

$$z_2^k = W_2 \phi_1(z_1^k) + b_2$$

$$z_1^k = W_1 x^k + b_1$$

- Relaxed Problem

$$\min_{W,b,z} \frac{1}{n} \sum_{k=1}^{n} l(\phi_l(z_l^k), y_k) + \sum_{i=1}^{l} \mu_i \| z_i^k - W_i \phi_{i-1}(z_{i-1}^k) - b_i \|_2^2$$

$$\min_{W,b,z} \frac{1}{n} \sum_{k=1}^{n} l(\phi_l(z_l^k), y_k) + \sum_{i=1}^{l} \mu_i \| z_i^k - W_i \phi_{i-1}(z_{i-1}^k) - b_i \|_1$$

Alternating Minimization

- W-update

$$[W_i, b_i] = \left(\sum_k z_i^k [\phi_{i-1}(z_{i-1}^k; 1)]\right) \times \left(\sum_k [\phi_{i-1}(z_{i-1}^k; 1)][\phi_{i-1}(z_{i-1}^k; 1)]^T\right)^{-1}$$

- z-update

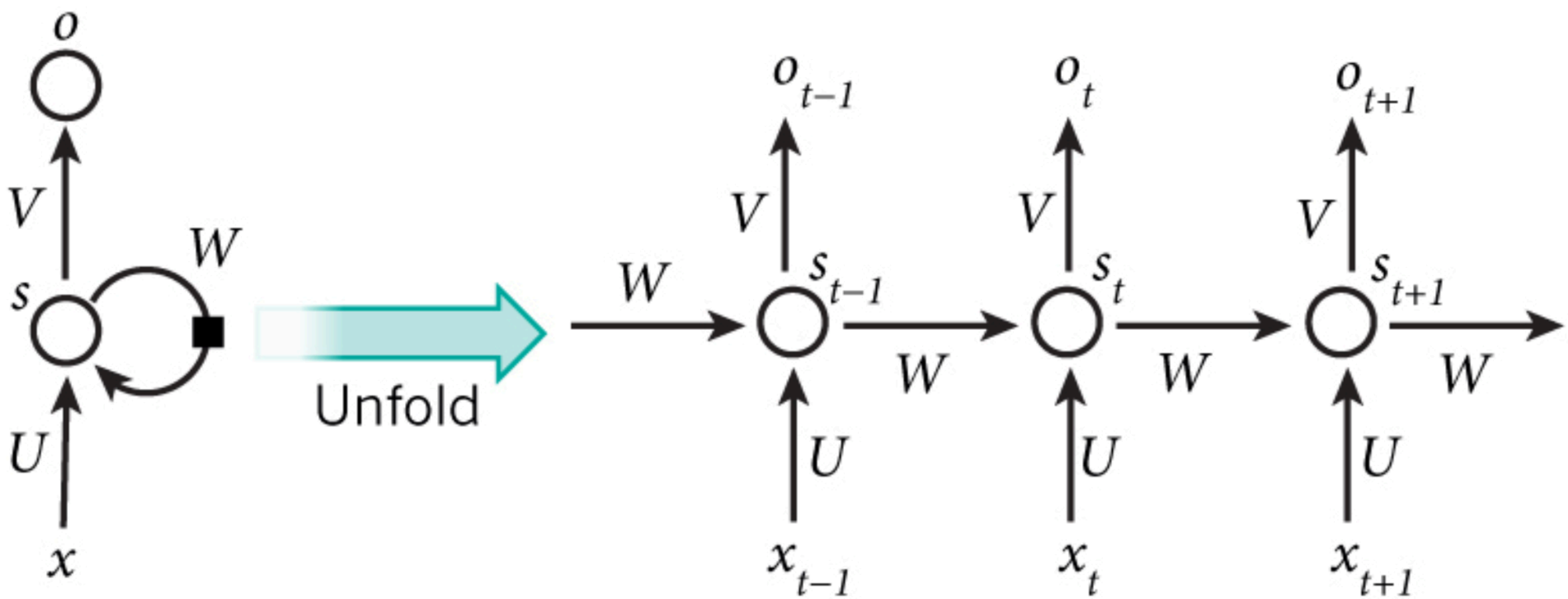
$$\min_z l(\phi_l(z_l^k), y_k) + \sum_{i=1}^{l} \mu_i \| z_i^k - W_i \phi_{i-1}(z_{i-1}^k) - b_i \|_2^2$$

one or more steps of damped Newton

cheap to compute Hessian

parallel computing

RNN



RNN

$$\min_{W,V,b} \frac{1}{n}\sum_{k=1}^{n} l(\phi_l(W_l\phi_{l-1}(\cdots\phi_1(W_1x_k + V_1s_0 + b_1)\cdots + V_ls_l + b_l)), y_k)$$

- Equivalent Problem

$$\begin{aligned}
&\min_{W,b,z} \frac{1}{n}\sum_{k=1}^{n} l(\phi_l(z_l^k), y_k) \\
&s.t. z_l^k = W_l\phi_{l-1}(z_{l-1}^k) + V_ls_{l-1}^k + b_l \\
&\qquad \cdots \\
&\qquad z_2^k = W_2\phi_1(z_1^k) + V_2s_1^k + b_2 \\
&\qquad z_1^k = W_1x^k + V_1s_0^k + b_1
\end{aligned}$$

- Relaxed Problem

$$\min_{W,b,z} \frac{1}{n}\sum_{k=1}^{n} l(\phi_l(z_l^k), y_k) + \sum_{i=1}^{l} \mu_i \| z_i^k - W_i\phi_{i-1}(z_{i-1}^k) - V_is_{i-1}^k - b_i\|_2^2$$

With L_2 Regulation

- W-update

$$[W_i, b_i] = \left(\sum_k z_i^k [\phi_{i-1}(z_{i-1}^k; 1)]\right) \times \left(\sum_k [\phi_{i-1}(z_{i-1}^k; 1)][\phi_{i-1}(z_{i-1}^k; 1)]^T + \frac{\lambda}{\mu_i} I\right)^{-1}$$

- z-update

$$\min_z l(\phi_l(z_l^k), y_k) + \sum_{i=1}^{l} \mu_i \| z_i^k - W_i \phi_{i-1}(z_{i-1}^k) - b_i \|_2^2$$

one or more steps of damped Newton

cheap to compute Hessian

parallel computing

Code

- neuralNetwork.jl
- https://github.com/Yuanchu/neuralNetwork

Numerical Test: Auto-Encoder