Jeff 20/4/22

Generating correlated random number $\quad \rho$ = correlation

$$\mathbb{E}(dX_1\, dX_2) = \rho$$

$$S_1 = \mu_1 S_1\, dt + S_1 \sigma_1 \sqrt{dt}\, dX_1$$

$$S_2 = \mu_2 S_2\, dt + S_2 \sigma_2 \sqrt{dt}\, dX_2$$

Start with generating 2 independent random $(\varepsilon_1, \varepsilon_2)$ variable, let dX_2 be a linear combination

$$dX_1 = \varepsilon_1 \quad , \quad dX_2 = A\varepsilon_1 + B\varepsilon_2$$

Some properties

$\mathbb{E}(\varepsilon_1) = \mathbb{E}(\varepsilon_2) = 0$ (normal dist property)

$\mathbb{E}(\varepsilon_1^2) = \mathbb{E}(\varepsilon_2^2) = 1$

$\mathbb{E}(\varepsilon_1 \varepsilon_2) = 0$ (independent)

Using 2 constraints, solve 2 unknown A & B

$$\mathbb{E}(dX_1\, dX_2) = \rho = \mathbb{E}((\varepsilon_1)(A\varepsilon_1 + B\varepsilon_2))$$

$$= \mathbb{E}(A \cancelto{1}{\varepsilon_1^2} + B \cancelto{0}{\varepsilon_2 \varepsilon_1})$$

$$= \mathbb{E}(A) = \rho$$

$$\therefore A = \rho$$

$$\mathbb{E}(\varepsilon_2^2) = 1 = \mathbb{E}\left[(A\varepsilon_1 + B\varepsilon_2)^2\right]$$

$$= \mathbb{E}\left[A^2 \varepsilon_1^2 + 2AB\,\varepsilon_1 \varepsilon_2 + B^2 \varepsilon_2^2\right]$$

$$1 = \mathbb{E}\left[\cancelto{p^2}{A^2}\ \cancelto{1}{\varepsilon_1^2}\right] + 2AB\,\cancelto{0}{\mathbb{E}(\varepsilon_1\varepsilon_2)} + \mathbb{E}\left[B^2\ \cancelto{1}{\varepsilon_2^2}\right]$$

$$1 = p^2 + B^2$$

$$\sqrt{1-p^2} = B$$

hence

$$dX_1 = \varepsilon_1$$

$$dX_2 = p\,\varepsilon_1 + \sqrt{1-p^2}\ \varepsilon_2$$