第八組 演習作業(四)

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1.

(a) $\lim_{x\to 0^+}\left(x^{\ln(x+1)}\right)$ type 0^0

$\Rightarrow y=\lim_{x\to 0^+}\left(x^{\ln(x+1)}\right)$

$\ln y=\ln\left(\lim_{x\to 0^+}\left(x^{\ln(x+1)}\right)\right)$

$=\lim_{x\to 0^+}\ln\left(x^{\ln(x+1)}\right)$

$=\lim_{x\to 0^+}\ln(x+1)\ln x$ type: $0\cdot(-\infty)$

$=\lim_{x\to 0^+}\dfrac{\ln x}{\frac{1}{\ln(x+1)}}$ type: $\frac{-\infty}{\infty}$

$\underset{(\frac{-\infty}{\infty})}{\overset{L'}{=}}\lim_{x\to 0^+}\dfrac{\frac{1}{x}}{\frac{-2(\ln(x+1))}{(x+1)}}=0$

$\ln y=0\Rightarrow y=1$

A: 1 #

(b) $\lim_{x\to\infty}\left(1+\frac{8}{x}\right)^{6x}$ type 1^∞

$\Rightarrow y=\lim_{x\to\infty}\left(1+\frac{8}{x}\right)^{6x}$

$\ln y=\ln\left(\lim_{x\to\infty}\left(1+\frac{8}{x}\right)^{6x}\right)$

$=\lim_{x\to\infty}\left(\ln\left(1+\frac{8}{x}\right)^{6x}\right)$

$=\lim_{x\to\infty}6x\ln\left(1+\frac{8}{x}\right)$ type $\infty\cdot 0$

$=\lim_{x\to\infty}\dfrac{\ln\left(1+\frac{8}{x}\right)}{\frac{1}{6x}}$ type $\frac{0}{0}$

$\underset{(\frac{0}{0})}{\overset{L'}{=}}\lim_{x\to\infty}\dfrac{\frac{-8x^{-2}}{(1+\frac{8}{x})}}{-\frac{1}{6}x^{-2}}$

$=\lim_{x\to\infty}\dfrac{48}{1+\frac{8}{x}}=48$

$\ln y=48$ $y=e^{\ln 48}$

A: $e^{\ln 48}$ #

2. $\frac{d}{dx}\int_{2^x}^{\ln(x)}\left(\sin(2t^2)\right)dt$

$=\frac{d}{dx}\int_0^{\ln(x)}\left(\sin(2t^2)\right)dt-\frac{d}{dx}\int_0^{2^x}\left(\sin(2t^2)\right)dt$

$=\frac{d}{d\ln(x)}\int_0^{\ln(x)}\left(\sin(2t^2)\right)dt\cdot\frac{d\ln(x)}{dx}-\frac{d}{d2^x}\int_0^{2^x}\left(\sin(2t^2)\right)dt\cdot\frac{d2^x}{dx}$

$=\sin\left(2(\ln(x))^2\right)\left(\frac{1}{x}\right)-\sin\left(2(2^x)^2\right)2^x\cdot 1\cdot\ln 2$

$=\dfrac{\sin\left(2(\ln^2(x))\right)}{x}-\left(\sin\left(2^{2x+1}\right)\right)\cdot 2^x\ln 2$ #

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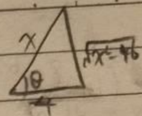
3. (a) $\int \left(\frac{3x}{\sqrt{x^2-16}}\right) dx$



Let $x = 4\sec\theta$

Then $dx = 4\sec\theta\tan\theta\, d\theta$

$= \int \left(\frac{dx}{\frac{1}{3x}\left(\sqrt{x^2-16}\right)} \right) dx$

$= \int \left(\frac{4\sec\theta\tan\theta}{\frac{\cos\theta}{12}\,|4\tan\theta|} \right) d\theta$

$= \int 12\sec^2\theta \frac{\tan\theta}{|\tan\theta|}\, d\theta$

For $x > 0$ $\quad 0 \le \theta < \frac{\pi}{2}$ $\quad \tan\theta > 0$

原式 $= \int 12\sec^2\theta \frac{\tan\theta}{\tan\theta}\, d\theta = 12\tan\theta = 12\tan\left(\sec^{-1}\frac{x}{4}\right) + C$

For $x < 0$, $\frac{\pi}{2} < \theta \le \pi$, $\sec\theta < 0$

原式 $= \int 12\sec^2\theta \frac{-\tan\theta}{|-\tan\theta|}\, d\theta = \int -12\sec^2\theta\, d\theta = 12\tan(-\theta) = 12\tan\left(\sec^{-1}\frac{-x}{4}\right) + C$

A: $12\tan\left(\sec^{-1}\frac{|x|}{4}\right) + C$

(b) $\int \left(x^2\sin(2x)\right) dx$ $\quad$ Let $u = x^2$, $dv = \sin 2x$

$= \frac{-x^2\cos 2x}{2} - \int \frac{-\cos 2x}{2} \cdot 2x\, dx$ $\quad$ Then $du = 2x\, dx$, $v = \frac{-\cos 2x}{2}$

$= \frac{-x^2\cos 2x}{2} - 2x\sin 2x - \int 2\sin 2x\, dx$ $\quad$ Let $u = x$, $dv = -\cos 2x$

Then $du = dx$, $v = 2\sin 2x$

$= \frac{-x^2\cos 2x}{2} - 2x\sin 2x + \cos 2x + C$

A: $\frac{-x^2\cos 2x}{2} - 2x\sin 2x + \cos 2x + C$ #

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(c) $\int \tan^3(x)\sec^9(x)\,dx$

Let $u = \sec(x)$

Then $du = \sec(x)\tan(x)\,dx$

$= \int \sec(x)\tan(x)\,(\sec^2(x)-1)\sec^8(x)\,dx$

$= \int u^8(u^2-1)\,du$

$= \int u^{10} - u^8\,du$

$= \frac{u^9}{10} - \frac{u^7}{8} + C$

$= \frac{\sec^9(x)}{10} - \frac{\sec^7(x)}{8} + C$

A: $\frac{\sec^9(x)}{10} - \frac{\sec^7(x)}{8} + C$ ##

(d) $\int (\arctan(3x))\,dx$

Let $u = \arctan(3x)$ $\quad dv = dx$

$du = \frac{3}{1+x^2}\,dx$ $\quad v = \int dx = x$

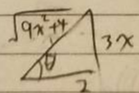
$= x\arctan(3x) - \int x \frac{3}{1+x^2}\,dx$

$= x\arctan(3x) - \frac{3}{2}\int \frac{2x}{1+x^2}\,dx$

$= x\arctan(3x) - \frac{3\ln(1+x^2)}{2} + C$

A: $x\arctan(3x) - \frac{3\ln(1+x^2)}{2} + C$

(e) $\int \left(\frac{3x^2}{\sqrt{9x^2+4}}\right)dx$



Let $3x = 2\tan\theta$ $\quad x = \frac{2}{3}\tan\theta$

Then $dx = \frac{2}{3}\sec^2\theta\,d\theta$

$= \int \frac{2\tan\theta \cdot \frac{2}{3}\tan\theta \cdot \frac{2}{3}\sec^2\theta\,d\theta}{2\sec\theta}$

$= \int \frac{4}{9}\tan^2\theta\sec\theta\,d\theta$

$= \int \frac{4}{9}(\sec^2\theta - 1)\sec\theta\,d\theta$

$= \frac{4}{9}\left(\int \sec^3\theta\,d\theta - \int \sec\theta\,d\theta\right)$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \tan\theta\sec\theta\tan\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$2\int \sec^3\theta\,d\theta = \sec\theta\tan\theta + \int \sec\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \frac{1}{2}(\sec\theta\tan\theta) + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

Let $u = \sec\theta$ $\quad dv = \sec^2\theta\,d\theta$

Then $du = \sec\theta\tan\theta$ $\quad v = \tan\theta$

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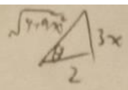


$$原式 = \frac{4}{9}\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\tan\theta+\sec\theta| - \ln|\tan\theta+\sec\theta|\right) + C$$

$$= \frac{2}{9}\sec\theta\tan\theta - \frac{2}{9}\ln|\tan\theta+\sec\theta| + C$$

$$= \frac{2}{9}\sec\left(\tan^{-1}\frac{3x}{2}\right)\tan\left(\tan^{-1}\frac{3x}{2}\right) - \frac{2}{9}\ln\left|\tan\left(\tan^{-1}\frac{3x}{2}\right)+\sec\left(\tan^{-1}\frac{3x}{2}\right)\right| + C$$

$$= \frac{x}{3}\sec\left(\tan^{-1}\frac{3x}{2}\right) - \frac{2}{9}\ln\left|\frac{3x}{2}+\sec\left(\tan^{-1}\frac{3x}{2}\right)\right| + C$$

A: $\frac{x}{3}\sec\left(\tan^{-1}\frac{3x}{2}\right) - \frac{2}{9}\ln\left|\frac{3x}{2}+\sec\left(\tan^{-1}\frac{3x}{2}\right)\right| + C$ ※

(f) $\int(\cos(8x)\cos(3x))\,dx$

$$= \frac{1}{2}\int(\cos(11x)+\cos(5x))\,dx$$

$$= \frac{1}{2}\int\left(\frac{1}{11}\cos(11x)\cdot 11 + \frac{1}{5}\cos(5x)\cdot 5\right)dx$$

$$= \frac{\sin(11x)}{22} + \frac{\sin(5x)}{10} + C$$

A: $\frac{\sin(11x)}{22} + \frac{\sin(5x)}{10} + C$ ※

(g) $\int\left(\frac{x^2+x+1}{x^3+x}\right)dx$

Let $\left(\frac{x^2+x+1}{x(x^2+1)}\right) = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$= \int\left(\frac{x^2+x+1}{x(x^2+1)}\right)dx$$

Then $x^2+x+1 = A(x^2+1)+(Bx+C)(x)$

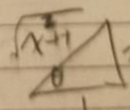
For $x=0$, $1=A(1) \Rightarrow A=1$

For $x=1$, $\frac{3}{2} = B+C$

For $x=-1$, $-\frac{1}{2} = -B+C$

$\Rightarrow 2C=1,\ C=\frac{1}{2},\ B=1$

$$原式 = \int\left(\frac{1}{x} + \frac{x}{x^2+1} + \frac{1}{2}\left(\frac{1}{x^2+1}\right)\right)dx$$



Let $x=\tan\theta$

Then $dx = \sec^2\theta\,d\theta$

$\frac{1}{2}\frac{\sec^2\theta}{\sec^2\theta}d\theta$

$= \int\frac{1}{2}d\theta$

$= \frac{1}{2}\theta$

$= \frac{1}{2}\tan^{-1}x$

$$= \ln|x| + \frac{1}{2}\ln|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

A: $\ln|x| + \frac{1}{2}\ln|x^2+1| + \frac{1}{2}\tan^{-1}x + C$ ※

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(h) $\int \left(\frac{1}{\sin(x)\cos(x)}\right) dx$

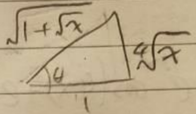
$= \int \left(\frac{2}{\sin(2x)}\right) dx$

$= \ln \sin(2x) + C$

A: $\ln \sin(2x) + C$

4.

(a) $\int_1^{25} \left(\frac{1}{\sqrt{x}(1+\sqrt{x})^3}\right) dx$



Let $\sqrt[4]{x} = \tan\theta$

$x = \tan^4\theta$

$dx = 4(\tan^4\theta)(\sec^2\theta)\,d\theta$

$\Rightarrow \int \frac{4(\tan^4\theta)(\sec^2\theta)}{\tan^2\theta \ \sec^6\theta}\, d\theta$

$= \int \frac{4\tan^2\theta}{\sec^4\theta}\, d\theta$

$= \int \frac{4\sec^2\theta - 4}{\sec^4\theta}\, d\theta$

$= 4\int \frac{1}{\sec^2\theta}\, d\theta - 4\int \frac{1}{\sec^4\theta}\, d\theta$

$= 4\int \cos^2(x)\, d\theta - 4\int \cos^4\theta\, d\theta$

$= 4\int \left(\frac{1+\cos 2x}{2}\right) d\theta - 4\int \left(\frac{1+\cos 2x}{2}\right)^2 d\theta$

$= 2\int (1+\cos 2x)\, d\theta - \int (1 + 2\cos 2x + \cos^2 2x)\, d\theta$

$= 2x + \sin 2x - x - \sin 2x + \frac{1}{2}\int (1-\cos 4x)\, d\theta$

$= x + \frac{x}{2} - \frac{1}{8}\sin 4x + C$

$= \frac{3}{2}x - \frac{1}{8}\sin 4x + C$

$\left[\frac{3}{2}x - \frac{1}{8}\sin 4x\right]_1^{25}$

$= \frac{75}{2} - \frac{1}{8}\sin 100 - \frac{3}{2} + \frac{1}{8}\sin 4$

$= 36 - \frac{1}{8}(\sin 100 - \sin 4)$

A: $36 - \frac{1}{8}(\sin 100 - \sin 4)$ #

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(b) $\int_0^{\pi}\left(e^{(2x)}\cos(4x)\right)dx$ Let $u=\cos(4x)$, $dv=e^{2x}dx$

Then $du=-4\sin(4x)dx$, $v=\frac{1}{2}e^{2x}$

$\Rightarrow \int\left(e^{(2x)}\cos(4x)\right)dx=\frac{1}{2}e^{2x}\cos(4x)+2\int e^{2x}\sin(4x)dx$

Let $u=\sin(4x)$ $dv=e^{2x}dx$

$du=4\cos(4x)$ $v=\frac{1}{2}e^{2x}$

$\int\left(e^{2x}\cos(4x)\right)dx=\frac{1}{2}e^{2x}\cos(4x)+e^{2x}\sin(4x)-2\int\left(e^{2x}\cos(4x)\right)dx$

$3\int\left(e^{2x}\cos(4x)\right)dx=\frac{1}{2}e^{2x}\cos(4x)+e^{2x}\sin(4x)$

$\int\left(e^{2x}\cos(4x)\right)dx=\frac{1}{6}e^{2x}\cos(4x)+\frac{1}{3}e^{2x}\sin(4x)$

$\left[\frac{1}{6}e^{2x}\cos(4x)+\frac{1}{3}e^{2x}\sin(4x)\right]_0^{\pi}$

$=\frac{e^{2\pi}}{6}-\frac{1}{6}=\frac{e^{2\pi}-1}{6}$ A: $\frac{e^{2\pi}-1}{6}$ #

(c) $\int_0^4\left(|x^2+2x-3|\right)dx$

$\Rightarrow y=\int\left(|(x+3)(x-1)|\right)dx$ $x>1$ $y>0$

$x<1$ $y<0$

原式 $=\int_1^4(x^2+2x-3)dx-\int_0^1(x^2+2x-3)dx$

$=\left[\frac{1}{3}x^3+x^2-3x\right]_1^4-\left[\frac{1}{3}x^3+x^2-3x\right]_0^1$

$=\frac{64}{3}+16-12-2\left(\frac{1}{3}+1-3\right)$ A: $\frac{86}{3}$ #

$=\frac{62}{3}+8=\frac{86}{3}$

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(d) $\int_0^{\frac{\pi}{2}} \left(\cos^5(x)\right) dx$

$\Rightarrow \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int \cos^3(x)\,dx$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int (1-\sin^2 x)\cos x\,dx$ Let $u=\sin x$ Then $du=\cos x\,dx$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int 1-u^2\,du$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\left(u - \frac{u^3}{3}\right)$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\left(\sin(x) - \frac{\sin^3(x)}{3}\right)$

$\left[\frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\left(\sin(x) - \frac{\sin^3(x)}{3}\right)\right]_0^{\frac{\pi}{2}}$

$= \frac{4}{5} - \frac{4}{15}$

$= \frac{12-4}{15}$

$= \frac{8}{15}$

A: $\frac{8}{15}$ ⌗