# 演習作業(三)

J2B 賴秉瑩

NO.

DATE

1.

(a) $\int \left(2x^3+3x^2+\frac{1}{2\sqrt{x}}+\frac{3}{x}\right)dx$

$= \int \left(2x^3+3x^2+\frac{1}{2}x^{-\frac{1}{2}}+3x^{-1}\right)dx$

$= \boxed{\frac{1}{2}x^4+x^3+x^{\frac{1}{2}}+3\ln x+C}$ #

(b) $\int \left(\frac{x^3-2x-4}{x-2}\right)dx$

$= \int \frac{(x-2)(x^2+2x+2)}{(x-2)}dx$

$= \boxed{\frac{1}{3}x^3+x^2+2x+C}$ #

(c) $\int \left(e^x(2e^x+4)((e^x+2)^2+5)^3\right)dx$ Let $u=e^x+2$ $du=e^x\,dx$

$\int 2u\cdot(u^2+5)^3\,du$

$= \frac{1}{4}(u^2+5)^4+C$

$= \boxed{\frac{1}{4}((e^x+2)^2+5)^4+C}$ #

(d)

$\int \left(\frac{5}{2+\sqrt{5x}}\right)dx$ Let $u=\sqrt{5x}$ $du=\frac{5}{2}\cdot\frac{1}{\sqrt{5x}}dx=\frac{5}{2u}dx$

$\int \left(\frac{5}{2+u}\right)\cdot\frac{2u}{5}du$

$= \int\left(\frac{-4}{u+2}+2\right)du$

$= \boxed{-4\ln(\sqrt{5x}+2)+2\sqrt{5x}}$ #

2. (a)

$\int_0^{\frac{\pi}{3}}\left(\frac{-\cos(x)}{1-\sin(x)}\right)dx$ let $u=\sin x$ $du=\cos x\,dx$ $x=\frac{\pi}{3}$, $u=\frac{\sqrt{3}}{2}$

$x=0$, $u=0$

$\int_1^{\frac{\sqrt{3}}{2}}\left(\frac{-1}{1-u}\right)du$

$= \int_1^{\frac{\sqrt{3}}{2}}(u-1)^{-1}du$

$= \left[\ln(u-1)\right]_0^{\frac{\sqrt{3}}{2}}$

$= \boxed{\ln\left(\frac{\sqrt{3}}{2}-1\right)}$ #

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2.(b)

$$\int_4^7 (|x^2+9x-90|)\,dx$$

$$= \int_4^7 (|(x+15)(x-6)|)\,dx$$

$$= \int_6^7 (x^2+9x-90)\,dx + \int_4^6 -(x^2+9x-90)\,dx$$

$$= \left[\tfrac{1}{3}x^3+\tfrac{9}{2}x^2-90x\right]_6^7 - \left[\tfrac{1}{3}x^3+\tfrac{9}{2}x^2-90x\right]_4^6$$

$$= \frac{343}{3}+\frac{441}{2}-630-72-162+540-72-162+540$$

$$+\frac{64}{3}+72-360 = \boxed{\frac{301}{6}}$$

(c) $\int_{-1}^{2\pi} (f(x))\,dx$, when $f(x) = \begin{cases} 1-e^{(2x)}, & x<0 \\ 4x\cos(2x^2), & x\geq 0 \end{cases}$

$$\int_{-1}^{0} \left(1-e^{(2x)}\right) dx$$

$$= \left[x-\frac{e^{2x}}{2}\right]_{-1}^{0}$$

$$= -\frac{1}{2} - \left(-1-\frac{1}{2e^2}\right)$$

$$= \frac{1}{2}+\frac{1}{2e^2}$$

$$\int_0^{2\pi} (4x\cos(2x^2))\,dx$$

$$= \left[\sin(2x^2)\right]_0^{2\pi}$$

$$= \left[\sin(2(2\pi)^2)\right] - \left[\sin 0\right]$$

$$= \sin(8\pi^2)$$

A: $\frac{1}{2}+\frac{1}{2e^2}+\sin(8\pi^2)$

3.(b) $F(x) = \int_{e^x}^{\ln(x)} (\sin(4t^2))\,dt$

$$= \int_0^{\ln(x)} (\sin(4t^2))\,dt - \int_0^{e^x} (\sin(4t^2))\,dt$$

$$F'(x) = \frac{d}{dx}\int_0^{\ln(x)} (\sin(4t^2))\,dt - \frac{d}{dx}\int_0^{e^x} (\sin(4t^2))\,dt$$

$$= \frac{d}{d(\ln(x))}\int_0^{\ln(x)} (\sin(4t^2))\,dt \cdot \frac{d\ln(x)}{dx} - \frac{d}{de^x}\int_0^{e^x} (\sin(4t^2))\,dt \,\frac{de^x}{dx}$$

$$= \boxed{\sin(4(\ln x)^2)\cdot\frac{1}{x} - \sin(4e^{2x})\cdot e^x}$$

3.(a) $F(x) = \int_x^{\cos(x)} \left(\sqrt{3\sin(t)+4}\right) dt$

$$= \int_0^{\cos(x)} \left(\sqrt{3\sin(t)+4}\right) dt - \int_0^{x} \left(\sqrt{3\sin(t)+4}\right) dt$$

$$F'(x) = \frac{d}{dx}\int_0^{\cos(x)} \left(\sqrt{3\sin(t)+4}\right) dt - \frac{d}{dx}\int_0^{x} \left(\sqrt{3\sin(t)+4}\right) dt$$

$$= \left[\frac{d}{d\cos(x)}\int_0^{\cos(x)} \left(\sqrt{3\sin(t)+4}\right) dt \cdot \frac{d\cos(x)}{dx}\right] - \sqrt{3\sin(x)+4}$$

$$= \boxed{\sqrt{3\sin(\cos(x))+4}\cdot(-\sin(x)) - \sqrt{3\sin(x)+4}}$$