演習課作業(二)　I2B54 賴秉逵

$f(x)$	$\frac{d}{dx}(f(x))$
$2x$	2
$g(x)+h(x)$	$g'(x)+h'(x)$
$g(x)h(x)$	$g'(x)h(x)+g(x)h'(x)$
$\frac{g(x)}{h(x)}$, $(h(x)\neq 0)$	$\frac{g'(x)h(x)-g(x)h'(x)}{(h(x))^2}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec x\tan x$
$\csc(x)$	$-\csc x\cot x$
e^x	e^x
a^x, (a is constant)	$a^x\ln a$
$\ln(x)$	$\frac{1}{x}$, $x>0$
$\log_a(x)$, (a is constant)	$\frac{1}{(\ln a)x}$

1.

$$g(x)=\begin{cases} ax^2, & x<2 \\ x^4+b, & x\geq 2 \end{cases}$$

$\lim_{x\to 2^-} ax^2 = 4a$

$\lim_{x\to 2^+} x^4+b = 16+b$

$g'_-(2) = 2ax \Rightarrow x=2, =4a$

$g'_+(2) = 4x^3 \Rightarrow x=2, =32$

$\begin{cases} 4a=32 \\ 4a=16+b \end{cases}$

$a=8$

$b=16$

A: $a=8$, $b=16$ #

2.

(a) $y=2x^3+5x^2+\frac{8}{\sqrt[3]{x^2}}+5$

$\frac{dy}{dx}\cdot\frac{1}{dy}y = \frac{d}{dx}(2x^3+5x^2+8x^{\frac{-2}{3}}+5)$

$\frac{dy}{dx} = 6x^2+10x+(\frac{-2}{1})8x^{\frac{-5}{3}}+5$

$= 6x^2+10x-\frac{16}{3}x^{\frac{-5}{3}}+5$

A: $6x^2+10x-\frac{16}{3}x^{\frac{-5}{3}}+5$ #

(b) $y=2(x^2-2)^3+e^{(\sin(x))}$

$\frac{dy}{dx}\frac{1}{dy}y = \frac{d}{dx}(2(x^2-2)^3+e^{(\sin(x))})$

$\frac{dy}{dx} = 2(3(x^2-2)^2(2x))+\cos(x)e^{(\sin(x))}$

$\frac{dy}{dx} = 6x^5-24x^3+24x+\cos(x)e^{(\sin(x))}$

A: $6x^5-24x^3+24x+\cos(x)e^{(\sin(x))}$ #

(c) $y=\log(x)\sin(x)$

$\frac{dy}{dx}\frac{1}{dy}y = \frac{d}{dx}(\log(x)\sin(x))$

$\frac{dy}{dx} = \frac{1}{(\ln 10)x}\sin(x)+\log(x)\cos x$

$\frac{dy}{dx} = \sin(x)\frac{1}{(\ln 10)x}+\cos(x)\frac{\ln x}{\ln 10}$

$\frac{dy}{dx} = \frac{\sin(x)+x\cos(x)\ln x}{x\ln 10}$

A: $\frac{\sin(x)+x\cos(x)\ln x}{x\ln 10}$ #

(d) $y=(2^x)\ln(3x+5)$

$\frac{dy}{dx}\frac{1}{dy}y = \frac{d}{dx}((2^x)\ln(3x+5))$

$\frac{dy}{dx} = \ln 2\cdot 2^x\cdot\ln(3x+5)+2^x\cdot\frac{3}{3x+5}$

$\frac{dy}{dx} = 2^x(\ln 2\cdot\ln(3x+5)+\frac{3}{3x+5})$

A: $2^x(\ln 2\cdot\ln(3x+5)+\frac{3}{3x+5})$ #

(e) $y = \frac{\sin(x)}{3x}$

$\frac{dy}{dx}\frac{d}{dy}y = \frac{d}{dx}\left(\frac{\sin(x)}{3x}\right)$

$\frac{dy}{dx} = \frac{\cos(x)\cdot 3x - \sin(x)\cdot 3}{9x^2}$

$\frac{dy}{dx} = \frac{x\cos(x) - \sin(x)}{3x^2}$

A: $\frac{x\cos(x) - \sin(x)}{3x^2}$ ✻

(f) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\frac{dy}{dx}\frac{d}{dy}y = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$

$\frac{dy}{dx} = -\frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{e^{2x} + 2e^x e^{-x} + e^{-2x}}$

$\frac{dy}{dx} = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{e^{2x} + e^{-2x}}$

$\frac{dy}{dx} = \frac{4}{e^{2x} + e^{-2x}}$

A: $\frac{4}{e^{2x} + e^{-2x}}$ ✻

(g) $y = \left(e^{(2x\sin(x))} + 3e^x\sin(x)\right)^2$

$\frac{dy}{dx}\cdot\frac{d}{dy}\cdot y = \frac{d}{dx}\left(e^{(2x\sin(x))} + 3e^x\sin(x)\right)^2$

$\frac{dy}{dx} = 2\left(e^{(2x\sin(x))} + 3e^x\sin(x)\right)\left(2\sin(x) + 2x\cos(x)\right)e^{(2x\sin(x))} + 3e^x\sin(x) + 3e^x\cos(x)$

$\frac{dy}{dx} = 2\left(e^{(2x\sin(x))} + 3e^x\sin(x)\right)\left(2(\sin(x) + x\cos(x))\right)e^{(2x\sin(x))} + 3e^x(\sin(x) + \cos(x))$

A: $2\left(e^{(2x\sin(x))} + 3e^x\sin(x)\right)\left(2(\sin(x) + x\cos(x))\right)e^{(2x\sin(x))} + 3e^x(\sin(x) + \cos(x))$

3.

(a) $x^2 + 5y^2 = e^{(xy)}$

$\frac{d}{dx}(x^2) + \frac{dy}{dx}\cdot\frac{d}{dy}\cdot 5y^2 = \frac{d}{dx}\left(e^{(xy)}\right)$

$2x + \frac{dy}{dx}\cdot 10y = \frac{dy}{dx}\cdot\frac{d}{dy}\cdot e^{xy}$

$2x + \frac{dy}{dx}\cdot 10y = \frac{dy}{dx}\cdot xe^{xy}$

$2x = \frac{dy}{dx}(xe^{(xy)} - 10y)$

$\frac{dy}{dx} = \frac{2x}{xe^{(xy)} - 10y}$

A: $\frac{2x}{xe^{(xy)} - 10y}$ ✻

(b) $y = (x-2)^{(\ln(3x))}$

$\frac{dy}{dx} = (\ln(3x))(x-2)^{(\ln(3x)-1)}(1)$

A: $(\ln(3x))(x-2)^{(\ln(3x)-1)}$ ✻