微基分(一) HW I　　　　I2B54 賴聿豐

Mina Queri　GRACE BUSINES SERIES BOOK

1. $f(x)=x^3+5x-5$ & $g(x)=e^{(3x-5)}$

(a) $(g^{(-1)}\circ f^{(-1)})(1)=\frac{5}{3}$ #

$f(x)=x^3+5x-5=1$

$\Rightarrow x^3+5x-6=0$

$\Rightarrow x=1$ 代入 $g(x)$

$g(x)=e^{(3x-5)}=1$

$\Rightarrow 3x-5=0$

$x=\frac{5}{3}$ #

(b) $(f^{(-1)}\circ g^{(-1)})(\frac{1}{e^2})=1$ #

$g(x)=e^{(3x-5)}=\frac{1}{e^2}$

$\Rightarrow 3x-5=-2$

$\Rightarrow x=\frac{3}{3}=1$

$f(x)=x^3+5x-5=1$

$\Rightarrow x^3+5x-6=0$

$\Rightarrow x=1$ #

2.

(a) $\sin(\arctan(\frac{7}{24}))$, I : $\frac{7}{25}$ #

$\frac{opp.}{adj.}=\frac{7}{24}$, $hyp=\sqrt{24^2+7^2}=25$

$\arctan\frac{7}{24}=\theta$, $\tan\theta=\frac{7}{24}$

$\sin\theta=\frac{opp}{hyp.}=\frac{7}{25}$

在 I $\sin>0$ $\therefore$ ans $=\frac{7}{25}$ #

(b) $\sec(\arcsin(\frac{5}{17}))$, II : $\frac{-17\sqrt{66}}{132}$ #

$\frac{opp}{hyp}=\frac{5}{17}$　$adj=\sqrt{17^2-5^2}=\sqrt{264}=2\sqrt{66}$

$\arcsin\frac{5}{17}=\theta$, $\sin\theta=\frac{5}{17}$

$\sec=\frac{hyp}{adj}=\frac{17}{2\sqrt{66}}=\frac{17\cdot\sqrt{66}}{2\sqrt{66}\cdot\sqrt{66}}=\frac{17\sqrt{66}}{132}$

$\because$ II $\sec<0$ $\therefore$ ans $=\frac{-17\sqrt{66}}{132}$

3.

(a) $7^{(x+5)}=e^{(x+2)}$: $x=\frac{5\ln 7-2}{1-\ln 7}$

$(x+5)\ln 7=\ln(e^{(x+2)})$

$(x+5)\ln 7=x+2$

$x\ln 7+5\ln 7=x+2$

$5\ln 7-2=x-x\ln 7$

$5\ln 7-2=x(1-\ln 7)$

$\frac{5\ln 7-2}{1-\ln 7}=x$ #

(b) $\arctan(5x)=\arcsin(3x)$: $x=\pm\frac{4}{15}$ #

$\tan\theta=5x$, $\sin\theta=3x$

$opp:adj=5x:1$, $opp:hyp=3x:1$

$opp:adj:hyp=15x:3:5$

$(15x)^2=5^2-3^2$

$225x^2=16$

$x^2=\frac{16}{225}$

$x=\pm\frac{4}{15}$ #

MINA QUERI　Produced By C.CHING

(a) $\lim\limits_{x\to2}\left(\frac{x^3-7x+6}{x-2}\right) = 5$ #

$x=2$ 代入 $x^3-7x+6=0$

$x^3-7x+6 \div (x-2) = x^2+2x-3$

$\lim\limits_{x\to2} \frac{\cancel{(x-2)}(x^2+2x-3)}{\cancel{(x-2)}}$

$= \lim\limits_{x\to2} x^2+2x-3$

代入 $x=2$ 得

$4+4-3=5$

(b) $\lim\limits_{x\to5}\left(\frac{\sqrt{x+4}-3}{x-5}\right) = \frac{1}{6}$ #

$\lim\limits_{x\to5} \frac{(\sqrt{x+4}-3)(\sqrt{x+4}+3)}{(x-5)(\sqrt{x+4}+3)} =$

$= \lim\limits_{x\to5} \frac{\cancel{(x+4-9)}^{\,x-5}}{\cancel{(x-5)}(\sqrt{x+4}+3)}$

$= \lim\limits_{x\to5} \frac{1}{\sqrt{x+4}+3}$

代入 $x=5$ 得

$\frac{1}{\sqrt{5+4}+3} = \frac{1}{3+3} = \frac{1}{6}$ #

(c) $\lim\limits_{x\to0}\left(\frac{\sec(x)-1}{x}\right) = 0$ #

$\lim\limits_{x\to0}\left(\frac{\left(\frac{1}{\cos(x)}-1\right)\cdot\cos(x)}{x\cdot\cos x}\right)$

$= \lim\limits_{x\to0}\left(\frac{1-\cos(x)}{x}\cdot\frac{1}{\cos x}\right)$

代入 $x=0$ 得

$0\cdot1=0$ #

(d) $\lim\limits_{x\to0}\left(x\cos\left(\frac{1}{x}\right)\right) = 0$ #

$-1 \le \cos\left(\frac{1}{x}\right) \le 1$

$-|x| \le x\cos\left(\frac{1}{x}\right) \le |x|$

$\lim\limits_{x\to0} -|x| = \lim\limits_{x\to0} |x| = 0$

由夾擠定理可知:

$\lim\limits_{x\to0} x\cos\left(\frac{1}{x}\right) = 0$

(e) $\lim\limits_{x\to3}\left(\frac{|x-3|}{x-3}\right)$ = 不存在

$\lim\limits_{x\to3^+} \frac{\cancel{(x-3)}}{\cancel{(x-3)}} = 1$

$\lim\limits_{x\to3^-} \frac{-\cancel{(x-3)}}{\cancel{(x-3)}} = -1$

∵ 左極限≠右極限

∴ 極限不存在

(f) $\lim\limits_{x\to7^+}\left(\frac{[[(x-5)^2]]-4}{x-7}\right) = 4$ #

$\lim\limits_{x\to7^-} \frac{(x-6)^2-4}{x-7} < \lim\limits_{x\to7}\left(\frac{[[(x-5)^2]]-4}{x-7}\right) \le \lim\limits_{x\to7^+} \frac{(x-5)^2-4}{x-7}$

$\lim\limits_{x\to7^+} \frac{x^2-10x+25-4}{x-7} =$

$= \lim\limits_{x\to7^+} \frac{x^2-10x+21}{x-7}$

$= \lim\limits_{x\to7^+} \frac{\cancel{(x-7)}(x-3)}{\cancel{x-7}} = 4$

∵ $x\to7^+$ ∴ 不考慮 $x\to7^-$

$[[(x-5)^2]]$ 可直接

寫為 $(x-5)^2$

(g) $\lim\limits_{x\to0}\left(\frac{\sin(x^2)(1-\cos(x))}{\cos(3x)\tan^4(2x)}\right) = \frac{1}{32}$ #

$\lim\limits_{x\to0}\left(\frac{\sin x^2}{\sin x}\right)\times\left(\frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)}\right)\times\left(\frac{1}{\cos(3x)}\right)\times\left(\frac{\cos^4(2x)}{1}\right)\times\left(\frac{2x}{\sin(2x)}\right)^4\cdot\frac{1}{2^4}$

$= \lim\limits_{x\to0} \frac{\sin^2 x}{x^2(1+\cos x)} \times \frac{1}{2^4}$

$= \lim\limits_{x\to0} \frac{1}{1+\cos(x)} \times \frac{1}{16} = \frac{1}{32}$ #