105 期中

1. (a) $\lim_{x\to 0} \frac{\sin^2(5x)}{x^2}$

$= \lim_{x\to 0} \frac{\sin(5x)\cdot\sin(5x)}{5x\cdot 5x}\cdot 25$

$= \lim_{x\to 0} 25$

A: 25 #

(b) $\lim_{x\to -2} \frac{\sqrt{x+6}-2}{x+2}$

$= \lim_{x\to -2} \frac{(\sqrt{x+6}-2)(\sqrt{x+6}+2)}{(x+2)(\sqrt{x+6}+2)}$

$= \lim_{x\to -2} \frac{\cancel{(x+2)}}{\cancel{(x+2)}(\sqrt{x+6}+2)}$

$= \lim_{x\to -2} \frac{1}{4}$

A. $\frac{1}{4}$ #

(c) $\lim_{x\to 3^+} \frac{x^2+5x+6}{x^2-9}$

$= \lim_{x\to 3^+} \frac{(x+3)(x+2)}{(x+3)(x-3)}$

$= \lim_{x\to 3^+} \frac{(x+2)}{(x-3)}$

$= \lim_{x\to 3^+} \infty$

$\begin{cases} \lim_{x\to 3^+} (x+2) = 5 \\ \lim_{x\to 3^+} \frac{1}{(x-3)} = \infty \end{cases}$

$\infty\cdot 5 = \infty$

A: no limit #

(d) $\lim_{x\to 5^-} ([x] + 2\sqrt{5-[x]})$

$= \lim_{x\to 5^-} 4 + 2\sqrt{5-4}$

$= \lim_{x\to 5^-} 6$

A: 6 #

2. (a) $y = 3\sqrt[3]{x} - \frac{3}{\sqrt{x}} + \frac{2}{x^2}$

$\Rightarrow y = 3x^{\frac{1}{3}} - 3x^{-\frac{1}{2}} + 2x^{-2}$

$y' = x^{-\frac{2}{3}} + \frac{3}{2}x^{-\frac{3}{2}} - 4x^{-3}$

A. $x^{-\frac{2}{3}} + \frac{3}{2}x^{-\frac{3}{2}} - 4x^{-3}$ #

(b) $y = x^3 e^x \sin x$

$y' = 3x^2 e^x \sin x + x^3 e^x \sin x + x^3 e^x \cos x$

$y' = x^2 e^x (\sin x(3+x) + x\cos x)$

A: $x^2 e^x (\sin x(3+x) + x\cos x)$ #

(c) $y = (2x^3+3x^2+x)^{10}$

$y' = 10(2x^3+3x^2+x)^9(6x^2+6x+1)$

A. $10(2x^3+3x^2+x)^9(6x^2+6x+1)$ #

(d) $y = \frac{x^2-5}{x^5+2x}$

$y' = \frac{2x(x^5+2x)-(x^2-5)(5x^4+2)}{(x^5+2x)^2}$

$y' = \frac{2x^6+4x^2-5x^6+25x^4-x^2+10}{(x^5+2x)^2}$

$y' = \frac{-3x^6+25x^4+3x^2+10}{(x^5+2x)^2}$

A $\frac{-3x^6+25x^4+3x^2+10}{(x^5+2x)^2}$ #

(e) $y = \tan^2 x$

$y' = 2(\tan x)(\sec^2 x)$

A: $2(\tan x)(\sec^2 x)$

(f) $y = \ln|3x+1|$

$y' = \frac{3}{3x+1}$

$\therefore$ A: $\frac{3}{3x+1}$ #

3. $f(x) = 3x + \sqrt[3]{x^2} - 2$ tangent line when $x=-1 \Rightarrow m = f'(-1)$

$f'(x) = 3 + \frac{2}{3}x^{-\frac{1}{3}}$ $\quad f'(-1) = 3 - \frac{2}{3} = \frac{7}{3}$ $\quad f(-1) = -3+1-2 = -4$

$-4 = \frac{7}{3}(-1) + b$ $\quad -4 + \frac{7}{3} = b$ $\quad -\frac{5}{3} = b$

A: $y = \frac{7}{3}x - \frac{5}{3}$

4. $f(x) = x^4 - \frac{1}{2}x - 1 \quad [0, 1]$

$f'(x) = 4x^3 - \frac{1}{2}$

$4x^3 - \frac{1}{2} = 0$

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$

left endpoint : $f(0) = -1$

right endpoint : $f(1) = \frac{-1}{2}$

critical number : $f(\frac{1}{2}) = \frac{-19}{16}$

$f(\frac{1}{2}) = \frac{1}{16} - \frac{1}{4} - 1 = \frac{1-4-16}{16} = \frac{-19}{16}$

A: max: $\frac{-1}{2}$

min: $\frac{-19}{16}$ #

5. $f(x) = (1-9x^2)^{\frac{2}{3}}$

(a)

$f'(x) = \frac{2}{3}(1-9x^2)^{\frac{-1}{3}}(18x)$

$x = 0, \frac{1}{3}, \frac{-1}{3}$

$9x^2 = 1$

$x^2 = \frac{1}{9}$

$x = \frac{\pm 1}{3}$

$f(0) = 1$

$f(\frac{1}{3}) = 0$

$f(\frac{-1}{3}) = 0$

A: (a) max = 1

(b)

x	-1	$\frac{-1}{3}$	$\frac{-1}{5}$	0	$\frac{1}{5}$	$\frac{1}{3}$	1
$f'(x)$	$+$	0	$-$	0	$+$	0	$-$

$f'(-1) = + \cdot - \cdot - = +$

$f'(-\frac{1}{3}) = 0$

$f'(\frac{-1}{5}) = + \cdot + \cdot - = -$

$f(\frac{1}{5}) = + \cdot + \cdot + = +$

$f(\frac{1}{3}) = 0$

$f(1) = + \cdot - \cdot + = -$

(b)

increasing domain : $(-\infty, \frac{-1}{3}), (0, \frac{1}{5})$

decreasing domain : $(\frac{-1}{3}, 0), (\frac{-1}{5}, 0), (\frac{1}{3}, \infty)$

6. $f(x) = -3x^5 + 40x^3$

(a)

$f'(x) = -15x^4 + 120x^2$

$= -15x^2(x^2 - 8)$

$f(2) = 240$

$f(-2) = 240$

$f(0) = 0$

A: $(0, 0), (2, 240), (2, 240)$ #

(b)

$f''(x) = -60x^3 + 240x \quad f''(x) = -60x(x^2 - 4)$

$f''(x) = -60x(x-2)(x+2)$

$f''(0) = 0, \ x = 2, -2, 0$

x	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
$f''(x)$	$+$	0 $-$	$+$	$-$

$f''(-3) = + \cdot - \cdot - = +$

$f''(-1) = + \cdot - \cdot + = -$

$f''(1) = - \cdot - \cdot + = +$

$f''(3) = - \cdot + \cdot + = -$

A: concave upward : $(-\infty, 2), (0, 2)$

concave downward : $(-2, 0), (2, \infty)$

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7.

(a) $y=(3x)^{2x}$

$\ln y = 2x\ln 3x$

$\frac{d}{dy}\frac{dy}{dx}\ln y = \frac{d}{dx}(2x\ln 3x)$

$\frac{dy}{dx}\left(\frac{1}{y}\right) = 2\ln 3x + 2x\frac{\cancel{3}}{\cancel{3}x}$

$\frac{dy}{dx} = (2x\ln 3x)(2\ln 3x + 2)$

A: $(2x\ln 3x)(2\ln 3x+2)$

(b) $e^{x+y} = y^2 + \sqrt{\frac{x}{y}}$

$e^{x+y} = y^2 + x^{\frac{1}{2}}y^{-\frac{1}{2}}$

$\left(1+\frac{dy}{dx}\right)e^{x+y} = \frac{dy}{dx}2y + \frac{1}{2}x^{-\frac{1}{2}}y^{-\frac{1}{2}} + x^{\frac{1}{2}}\frac{dy}{dx}\frac{-1}{2}y^{-\frac{3}{2}}$

$e^{x+y} - \frac{1}{2\sqrt{xy}} = \frac{dy}{dx}\left(-e^{x+y} + 2y - \frac{x^{\frac{1}{2}}y^{-\frac{3}{2}}}{2}\right)$

$\frac{dy}{dx} = \frac{e^{x+y} - \frac{1}{2\sqrt{xy}}}{2y - e^{x+y} - \frac{\sqrt{x}}{2\sqrt{y^3}}}$

A: $\frac{e^{x+y} - \frac{1}{2\sqrt{xy}}}{2y - e^{x+y} - \frac{\sqrt{x}}{2\sqrt{y^3}}}$

106 期中 (A)

1.

(a) $\lim_{x\to 0} \frac{\sin^3(3x)}{x^3}$

$\Rightarrow \lim_{x\to 0} \frac{\sin(3x)\sin(3x)\sin(3x)}{3x\cdot 3x\cdot 3x}\cdot 27$

$\Rightarrow \lim_{x\to 0} 27$

A: 27 #

(b) $\lim_{x\to 0} \frac{x}{\sqrt{4+2x}-\sqrt{4-2x}}$

$\Rightarrow \lim_{x\to 0} \frac{x(\sqrt{4+2x}+\sqrt{4-2x})}{(\sqrt{4+2x}-\sqrt{4-2x})(\sqrt{4+2x}+\sqrt{4-2x})}$

$\Rightarrow \lim_{x\to 0} \frac{\cancel{x}(\sqrt{4+2x}+\sqrt{4-2x})}{4\cancel{x}}$

$x=0$ 代入 $\Rightarrow \frac{4}{4}=1$ A: 1 #

2.

(a) $y=3x^{\frac{1}{5}}-3x^{\frac{1}{2}}+4x^{-2}$

$y'=\frac{3}{5}x^{\frac{-4}{5}}-\frac{3}{2}x^{\frac{-1}{2}}-8x$

A: $\frac{3}{5}x^{\frac{-4}{5}}-\frac{3}{2}x^{\frac{-1}{2}}-8x$ #

(b) $y=x^6e^x\cos x$

$y'=6x^5e^x\cos x+x^6e^x\cos x+x^6e^x(-\sin x)$

$y'=x^5e^x(6\cos x+x\cos x-x\sin x)$

$y'=x^5e^x(\cos x(6+x)-x\sin x)$

A: $x^5e^x(\cos x(6+x)-x\sin x)$ #

(c) $y=(3x^3-2x^2+4)^{12}$

$y'=12(3x^3-2x^2+4)^{11}(6x^2-4x)$

$y'=24(3x^3-2x^2+4)^{11}(3x^2-2x)$

A. $24(3x^3-2x^2+4)^{11}(3x^2-2x)$ #

(d) $y=\frac{x^3-6}{x^6+3x}$

$y'=\frac{3x^2(x^6+3x)-(x^3-6)(6x^5+3)}{(x^6+3x)^2}$

$y'=\frac{3(x^8+3x^3-2x^8+12x^5-x^3+6)}{(x^6+3x)^2}$

$y'=\frac{3(-x^8+12x^5+2x^3+6)}{(x^6+3x)^2}$

A: $\frac{3(-x^8+12x^5+2x^3+6)}{(x^6+3x)^2}$ #

3. $f(x)=\ln(x^2+1)$

$f'(x)=\frac{2}{x^2+1}$

$f'(-1)=1$

$f(-1)=\ln 2$

$y=mx+b$ $y=x+\ln 2+1$

$\ln 2=-1+b$

$b=\ln 2+1$

A: $y=x+\ln 2+1$ #

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4. $f(x)=\frac{x}{2}-\sin x \quad [0, 2\pi]$

$f'(x)=0$

$\Rightarrow \frac{1}{2}-\cos x=0$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3}$

$f(0)=0$

$f(\frac{\pi}{3})=\frac{\pi}{6}-\frac{\sqrt{3}}{2}=\frac{\pi-\sqrt{27}}{6}<0$

$f(2\pi)=\pi$

A: max: π

min: $\frac{\pi-3\sqrt{3}}{6}$ #

5. $f(x)=-3x^5+5x^3+5$

(a)(b)

$f'(x)=-15x^4+15x^2=-15x^2(x^2-1)$

$f'(x)=0 \quad x=1, -1, 0$

$f(1)=7$

$f(-1)=3$

$f(0)=5$

x	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
$f'(x)$	$-$	$+$	$+$	$-$

$f'(-2)=-\cdot+=\ominus$

$f(\frac{-1}{2})=-\cdot-=\oplus$

$f'(\frac{1}{2})=-\cdot-=\oplus$

$f(2)=-\cdot+=\ominus$

A: (a) max $=7$

(b) increasing: $(-1,0), (0,1)$

decreasing: $(-\infty,-1), (1,\infty)$ #

6. $f(x)=e^{\frac{-1}{8}(x-2)^2}$

(a)(b)

$f'(x)=\frac{d}{dx}\left(e^{\frac{-1}{8}(x-2)^2}\right)$

$f'(x)=\frac{-1}{8}(2(x-2)(1))\left(e^{\frac{-1}{8}(x-2)^2}\right)$

$f'(x)=\frac{-1}{4}(x-2)\left(e^{\frac{-1}{8}(x-2)^2}\right)$

$f(0)=e^{-\frac{1}{8}\cdot 4}=e^{-\frac{1}{2}}$

$f''(x)=\frac{-1}{4}(1)\left(e^{-\frac{1}{8}(x-2)^2}\right)$

$-\frac{1}{4}(x-2)\left(\frac{-1}{4}(x-2)\left(e^{\frac{-1}{8}(x-2)^2}\right)\right)$

$=\frac{-1}{4}e^{\frac{-1}{8}(x-2)^2}\left(1-\frac{(x-2)^2}{4}\right)$

$f''(x)=0, \ x=0$

$f(-1)=-\cdot-=\oplus$

$f(1)=-\cdot+=\ominus$

x	$(-\infty,0)$	$(0,\infty)$
$f''(x)$	$+$	$-$

A: (a) $(0, e^{\frac{-1}{2}})$ (b) concave upward $(-\infty,0)$

concave downward $(0,\infty)$

7.

(a) $y=\tan^6 e^{4x}$

$\frac{dy}{dx}=6\tan^5(\sec^2)e^{4x}+\tan^6 4e^{4x}$

$\frac{dy}{dx}=\tan^5 e^{4x}(6\sec^2+4\tan^6)$

$\frac{dy}{dx}=2\tan^5 e^{4x}(3\sec^2+2\tan^6)$

A: $2\tan^5 e^{4x}(3\sec^2+2\tan^6)$ #

(b) $y=x^{2x}$

$\ln y=2x\ln x$

$\frac{dy}{dx}\frac{1}{y}=2\ln x+2x\cdot\frac{1}{x}$

$\frac{dy}{dx}=2x^{2x}(\ln x+1)$

A: $2^{2x}(\ln x+1)$ #

(c) $e^{xy}-\sqrt{x^2+y^2}=x^3+6$

$\frac{dy}{dx}xe^{xy}+ye^{xy}=3x^2+\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x+\frac{dy}{dx}2y)$

$\frac{dy}{dx}\left(\frac{x}{\sqrt{x^2+y^2}}-xe^{xy}\right)=ye^{xy}-3x^2-\frac{x}{\sqrt{x^2+y^2}}$

$\frac{dy}{dx}=\frac{ye^{xy}-3x^2-\frac{x}{\sqrt{x^2+y^2}}}{\frac{x}{\sqrt{x^2+y}}-xe^{xy}}$

A: $\frac{ye^{xy}-3x^2-\frac{x}{\sqrt{x^2+y^2}}}{\frac{\cancel{x}}{\sqrt{x^2+y^2}}-xe^{xy}}$ #

2.
(e)
$y = \ln|5x+6|$

$\frac{dy}{dx} = \frac{5}{5x+6}$

A: $\frac{5}{5x+6}$ #

(f)
$y = 2^x + \log_2 x$

$\frac{dy}{dx} = \ln 2 \cdot 2^x + \frac{1}{\ln 2\, x}$

A: $\ln 2 \cdot 2^x + \frac{1}{\ln 2\, x}$ #

4.
$y = -6x+4$ tangent line of $f(x) = k - x^2$ 求 k

$f'(x) = -2x$

$-2x = -6 \quad x = 3$

$y = -18+4 = -14$

$-14 = k-9$

$-5 = k$

A: $k = -5$ #

107 期中 (A)

1.

(a)

$$\lim_{x\to 0}\left(\frac{\sin 3x}{\tan x}+\frac{\tan 5x}{\sin 2x}\right)$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{\sin 3x}{3x}\cdot\frac{\cos x\cdot x}{\sin x}\cdot\frac{3x}{x}+\frac{2x}{\sin 2x}\cdot\frac{\sin 5x}{\cos 5x\cdot 5x}\cdot\frac{5x}{2x}\right)$$

$$\Rightarrow \lim_{x\to 0} 3+\frac{5}{2}=\frac{11}{2}$$

A: $\frac{11}{2}$ #

(b)

$$\lim_{x\to -4}\frac{\sqrt{13+x}-3}{x+4}$$

$$\Rightarrow \lim_{x\to -4}\frac{(\sqrt{13+x}-3)(\sqrt{13+x}+3)}{(x+4)(\sqrt{13+x}+3)}$$

$$\Rightarrow \lim_{x\to -4}\frac{(x+4)}{(x+4)(\sqrt{13+x}+3)}=\frac{1}{6}$$

A: $\frac{1}{6}$ #

(c)

$$\lim_{x\to\infty}\frac{\sqrt{x^2+x}}{-2x}$$

$$\Rightarrow \lim_{x\to\infty}\frac{\sqrt{x^2+x}\cdot\left(\frac{1}{x}\right)}{-2x\left(\frac{1}{x}\right)}$$

$$\Rightarrow \lim_{x\to\infty}\frac{\sqrt{(x^2+x)\left(\frac{1}{x^2}\right)}}{-2}$$

$$\Rightarrow \lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x}}}{-2}=\frac{-1}{2}$$

A: $\frac{-1}{2}$ #

(d)

$$\lim_{x\to 5^-}\left(4\left[\!\left[\frac{x+2}{2}\right]\!\right]-x\right)$$

$$\Rightarrow \lim_{x\to 5^-}\left(4\left[\!\left[\frac{4.9+2}{2}\right]\!\right]-x\right)$$

$$\Rightarrow \lim_{x\to 5^-}\left(4\left[\!\left[3.45\right]\!\right]-x\right)$$

$$\Rightarrow \lim_{x\to 5^-}(4\cdot 3-4)=8$$

A: 8 #

2.

(a)

$$y=\sqrt[5]{x}-\frac{2}{\sqrt[3]{x}}+\frac{3}{x^2}$$

$$y=x^{\frac{1}{5}}-2x^{-\frac{1}{3}}+3x^{-2}$$

$$y'=\frac{1}{5}x^{-\frac{4}{5}}+\frac{2}{3}x^{-\frac{4}{3}}-6x^{-3}$$

A: $\frac{1}{5}x^{-\frac{4}{5}}+\frac{2}{3}x^{-\frac{4}{3}}-6x^{-3}$

(b)

$$y=x\sin(\ln(x))$$

$$y'=\sin(\ln(x))+x\cos(\ln(x))\left(\frac{1}{x}\right)$$

$$y'=\sin(\ln(x))+\cos(\ln(x))$$

A: $\sin(\ln(x))+\cos(\ln(x))$ #

(c)

$$y=(x+\sqrt{1+x})^{10}$$

$$y'=10(x+\sqrt{1+x})^{9}\left(1+\frac{1}{2}(1+x)^{-\frac{1}{2}}(1)\right)$$

$$y'=10(x+\sqrt{1+x})^{9}\left(1+\frac{1}{2\sqrt{1+x}}\right)$$

A: $10(x+\sqrt{1+x})^{9}\left(1+\frac{1}{2\sqrt{1+x}}\right)$ #

(d)

$$y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$$

$$y'=\frac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$$

$$y'=\frac{-4}{e^{2x}+e^{-2x}-2}$$

A: $\frac{4}{e^{2x}+e^{-2x}-2}$ #

(e)

$y = \sec^2(x)$

$y' = 2(\sec(x))(\sec(x))(\tan(x))$

$y' = 2\sec^2(x)\tan(x)$

A: $2\sec^2(x)\tan(x)$ ✗

3. $f(x) = xe^x$, $x = -2$

$f'(x) = e^x + xe^x = (x+1)e^x$

$f'(-2) = -e^{-2}$ $f(-2) = -2e^{-2}$

$-2e^{-2} = -e^{-2}(-2) + b$

$-4e^{-2} = b$ A: $y = -e^{-2}x - 4e^{-2}$ ✗

4.

$f(x) = x^5 + \frac{10}{3}x^3 + 5x - 7$ $[0,1]$

$f'(x) = 5x^4 + 10x^2 + 5 = 5(x^4 + 2x^2 + 1)$

$f'(x) > 0$ $x \in \mathbb{R}$

$f(1) = 1 + \frac{10}{3} + 5 - 7 = \frac{28 - 21}{3} = \frac{7}{3}$

$f(0) = -7$

A: max: $\frac{7}{3}$
min: -7

5. $f(x) = x^{\frac{1}{3}}(x-3)^{\frac{2}{3}}$

(a)

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}(x-3)^{\frac{2}{3}} + x^{\frac{1}{3}} \cdot \frac{2}{3}(x-3)^{-\frac{1}{3}}(1)$

$f'(x) = \frac{1}{3}(x-3)^{-\frac{1}{3}}(x)^{-\frac{2}{3}}(x - 3 + 2x)$

$f'(x) = (x-3)^{-\frac{1}{3}}(x)^{-\frac{2}{3}}(x-1)$

$f'(x) = 0$, $x = 3, 0, 1$

x	$(-\infty,0)$	$(0,1)$	$(1,3)$	$(3,\infty)$
$f'(x)$	+	+	−	+

$f'(-1) = - \cdot + \cdot - = +$

$f'(\frac{1}{2}) = - \cdot + \cdot - = +$

$f'(2) = - \cdot + \cdot + = -$

$f(4) = + \cdot + \cdot + = +$

A: (a) no relative extrema
(b) increasing: $(-\infty,0), (0,1)$
$(3,\infty)$
decreasing: $(1,3)$

6.

$f(x) = x^3 + 9x^2 + 33x - 8$

(a)

$f'(x) = 3x^2 + 18x + 33$

$f''(x) = 6x + 18$

$f''(x) = 0$ $x = -3$

$f(-3) = -9 + 81 - 99 - 8 = -35$

(b)

x	$(-\infty,-3)$	$(-3,\infty)$
$f''(x)$	−	+

$f''(-4) = -6$

$f''(0) = 18$

A: (a) $(-3, -35)$ (b) concave upward $(-3,\infty)$
concave downward $(-\infty,-3)$

7.

(a) $y = (x+1)^{\ln(x)}$

$\ln y = \ln(x)\ln(x+1)$

$\frac{dy}{dx} = (x+1)^{\ln(x)}\left(\frac{1}{x}\ln(x+1) + \ln(x)\left(\frac{1}{x+1}\right)\right)$

$\frac{dy}{dx} = (x+1)^{\ln(x)}\left(\frac{\ln(x+1)}{x} + \frac{\ln(x)}{x+1}\right)$

A: $(x+1)^{\ln(x)}\left(\frac{\ln(x+1)}{x} + \frac{\ln(x)}{x+1}\right)$ ✗

(b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 8$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}\frac{dy}{dx}y^{-\frac{1}{3}} = 0$

$\frac{dy}{dx} = \frac{-x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = -\sqrt[3]{\frac{y}{x}}$

$\frac{d^2y}{dx^2} = \frac{-1}{3}x^{-\frac{4}{3}}y^{\frac{-1}{3}} + x^{\frac{-1}{3}}\frac{dy}{dx}\left(\frac{-1}{3}\right)(y^{-\frac{4}{3}})$

$= \frac{-1}{3}(x^{-\frac{4}{3}})(y^{-\frac{4}{3}})\left(y + x\frac{-x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}}\right)$

$= \frac{-1}{3}(x^{-\frac{4}{3}})(y^{-\frac{4}{3}})\left(y + \frac{x^{\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)$

$= \frac{-1}{3}\left(\frac{1}{\sqrt[3]{x^4}}\right)\left(\frac{1}{\sqrt[3]{y^4}}\right)\left(y + \sqrt[3]{x^2y}\right)$

A: (a) $\sqrt[3]{\frac{-y}{x}}$ (b) $\frac{-1}{3}\left(\frac{1}{\sqrt[3]{x^4}}\right)\left(\frac{1}{\sqrt[3]{y^4}}\right)\left(y + \sqrt[3]{x^2y}\right)$ ✗

107 期中 (B)

1. (a)

$$\lim_{x\to 0} \frac{\sin(3x)}{\tan x}$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin(3x)}{3x} \frac{\cos x \cdot x}{\sin x} \cdot \frac{3x}{x} = 3$$

A: 3 #

2 (b) $y = x(\sin x)\ln(x)$

$$\frac{dy}{dx} = 1(\sin x)\ln(x) + x(\cos x)\ln(x) + x(\sin x)\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = \sin x \ln x + x\cos x \ln x + \sin x$$

A $\sin x \ln x + x\cos x \ln x + \sin x$ #

6.

$$y = \tan^8(e^{5x})$$

$$y = (\tan(e^{5x}))^8$$

$$\frac{dy}{dx} = 8(\tan e^{5x})^7(\sec^2 e^{5x})(5)$$

$$\frac{dy}{dx} = 40(\tan^7 e^{5x})(\sec^2 e^{5x})$$

A: $40(\tan^7 e^{5x})(\sec^2 e^{5x})$ #

7.

$$y = e^8 + e^{8x} + e^{x+8} + e^{x^8} + e^{8^x} + 8^{x^e}$$

$$\frac{dy}{dx} = 0 + 8e^{8x} + e^{x+8} + x^7 e^{x^8} + \ln 8 \cdot 8^x e^{8^x} + \ln 8 \cdot 8^{x^e} x^{e-1}$$

A: $8e^{8x} + e^{x+8} + x^7 e^{x^8} + \ln 8 \cdot 8^x e^{8^x} + \ln 8 \cdot 8^{x^e} x^{e-1}$ #

9.

$$2x^2 + y^2 = 5$$

$$4x + \frac{dy}{dx}2y = 0$$

$$\frac{dy}{dx} = \frac{-2x}{y} \quad 代入\left(\frac{1}{\sqrt{2}}, -2\right)$$

$$\frac{d^2y}{dx^2} = \frac{-2y + 2x\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-2y - \frac{4x^2}{y}}{y^2} = \frac{-2y^2 - 4x^2}{y^3} \quad 代入\left(\frac{1}{\sqrt{2}}, -2\right)$$

$$\frac{dy}{dx} = \frac{\frac{-2}{\sqrt{2}}}{2} = \frac{-2}{2\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-8-2}{-8} = \frac{5}{4}$$

A ① $\frac{dy}{dx} = \frac{-2x}{y}$ ② $\frac{d^2y}{dx^2} = \frac{-2y^2 - 4x^2}{y^3}$ ③ $\frac{-\sqrt{2}}{2}$ ④ $\frac{5}{4}$ #

108 期中

1.

(a)

$$\lim_{x\to -5} \frac{x+5}{\sqrt{30+x}-5}$$

$$\Rightarrow \lim_{x\to -5} \frac{(x+5)(\sqrt{30+x}+5)}{(\sqrt{30+x}-5)(\sqrt{30+x}+5)}$$

$$= \lim_{x\to -5} \frac{\cancel{(x+5)}\ \sqrt{30+x}+5}{\cancel{(x+5)}} = 10$$

A 10 #

(b)

$$\lim_{x\to 0} \frac{\sin(4x)+\cos(x)-1}{\tan(3x)}$$

$$\Rightarrow \lim_{x\to 0} \left(\frac{\sin(4x)\ \cos(3x)\ \cdot 3x \cdot 4}{\sin(3x)\ 4x\ \cdot 3}\right) + \left(\frac{\cos 3x \ \cos 3x\ 3x}{\sin 3x \quad 3x}\right) - \left(\frac{3x\ \cos 3x}{\sin 3x \quad 3x}\right)$$

$$\Rightarrow \lim_{x\to 0} \quad \frac{4}{3} + \cancel{\frac{1}{3x}} - \cancel{\frac{1}{3x}} = \frac{4}{3}$$

A $\frac{4}{3}$ #

(c)

$$\lim_{x\to\infty} \frac{\sqrt{9x^2-6x}}{2x+10000}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\sqrt{9x^2-6x\left(\frac{1}{x^2}\right)}}{(2x+10000)\frac{1}{x}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\sqrt{9-\frac{6}{x}}}{2+\frac{10000}{x}} = \frac{3}{2}$$

A $\frac{3}{2}$ #

(d)

$$\lim_{x\to 5^+} \frac{[[x^2+x]]-30}{(x-5)^2}$$

$$\Rightarrow \lim_{x\to 5^+} \frac{[[(5.3)^2+5.3]]-30}{(x-5)^2} \Rightarrow$$ 25+5=30 與直接代 x=5 一樣

$$\Rightarrow \lim_{x\to 5^+} \frac{(x+6)\cancel{(x-5)}}{(x-5)^2} = \infty$$

A: doesn't exist

2.

(a)

$$y = \sqrt[5]{x^3} - \frac{2}{\sqrt[3]{x}} + \ln(7^2)$$

$$y = x^{\frac{3}{5}} - 2x^{-\frac{1}{3}} + \ln(7^2)$$

$$\frac{dy}{dx} = \frac{3}{5}x^{-\frac{2}{5}} + \frac{2}{3}x^{-\frac{4}{3}}$$

A: $\frac{3}{5}x^{-\frac{2}{5}} + \frac{2}{3}x^{-\frac{4}{3}}$ #

(b)

$$y = e^{x^2}\ln(\cos(x))$$

$$\frac{dy}{dx} = 2xe^{x^2}\ln(\cos(x)) + e^{x^2}\frac{-\sin x}{\cos x}$$

A: $2xe^{x^2}\ln(\cos(x)) - e^{x^2}\frac{\sin x}{\cos}$ #

(c)

$$y = \tan^5(7x)$$

$$\frac{dy}{dx} = 5(\tan 7x)^4(\sec^2 7x)(7)$$

$$\frac{dy}{dx} = 35(\tan 7x)^4(\sec 7x)^2$$

A: $35(\tan 7x)^4(\sec 7x)^2$

(d)

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{dy}{dx} = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{4}{e^{2x}+e^{-2x}+2}$$

A: $\frac{4}{e^{2x}+e^{-2x}+2}$ #

LIHIT LAB. rupo A4-S

3. $f(x)=e^{2x}\cos(x)$, $x=\frac{\pi}{2}$

$f'(x)=2e^{2x}\cos(x)-e^{2x}\sin(x)$

$f'(\frac{\pi}{2})=2e^{\pi}\cdot 0-e^{\pi}\cdot 1=e^{\pi}$

$f(\frac{\pi}{2})=0$

$(0)=e^{\pi}(\frac{\pi}{2})+b$

$b=-\frac{\pi}{2}e^{\pi}$

A: $y=e^{\pi}x-\frac{\pi}{2}e^{\pi}$ ✗

4. $f(x)=(4-x^2)^{\frac{2}{5}}$

(a) (b)

$f'(x)=\frac{2}{5}(4-x^2)^{\frac{-3}{5}}(-2x)$

$f'(x)=\frac{-4x}{5}(4-x^2)^{\frac{3}{5}}$

$f'(x)=0 \quad x=0,2,-2$

x	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
$f'(x)$	$-$	$+$	$-$	$+$

$f'(-3)=+\cdot-=\ominus$

$f'(-1)=+\cdot+=\oplus$

$f'(1)=-\cdot+=\ominus$

$f'(3)=-\cdot-=\oplus$

$f(-2)=0$

$f(2)=0$

A: (a) increasing $(-2,0)$ $(2,\infty)$

decreasing $(-\infty,-2)(0,2)$

(b) relative minima : 0

5. $f(x)=\frac{1}{2}x^4+x^3-18x^2+5x+135$

(a)(b)

$f'(x)=2x^3+3x^2-36x+5$

$f''(x)=6x^2+6x-36=6(x+3)(x-2)$

$f''(x)=0, \quad x=-3,2$

$f(-3)=\frac{1}{2}(81)+(-27)-18(9)-15+135=-28.5$

$f(2)=8+8-72+10+135=89$

x	$(-\infty,-3)$	$(-3,2)$	$(2,\infty)$
$f''(x)$	$+$	$-$	$+$

$f''(-4)=-\cdot-=\oplus$

$f''(0)=+\cdot-=\ominus$

$f''(3)=+\cdot+=\oplus$

A: (a) concave upward: $(-\infty,-3),(2,\infty)$ (b) $(-3,-28.5),(2,89)$ ✗

concave downward: $(-3,2)$

6.

(a) $y^{x^2}=x^{y^2}$

$x^2\ln y=y^2\ln x$

$2x\ln y+x^2\frac{dy}{dx}\frac{1}{y}=\frac{dy}{dx}2y\ln x+y^2\frac{1}{x}$

$\frac{dy}{dx}(\frac{x^2}{y}-2y\ln x)=\frac{y^2}{x}-2x\ln y$

$\frac{dy}{dx}=\frac{(\frac{y^2}{x}-2x\ln y)}{(\frac{x^2}{y}-2y\ln x)}$

A. $\frac{\frac{y^2}{x}-2x\ln y}{\frac{x^2}{y}-2y\ln x}$

(b) $x^2y^3+\sin(xy)+e^{x^2+y^2}=-1$

$2xy^3+x^2\frac{dy}{dx}3y^2+\cos(xy)(y+x\frac{dy}{dx})+(2x+\frac{dy}{dx}2y)e^{x^2+y^2}=0$

$\frac{dy}{dx}(3x^2y^2+x\cos(xy)+2ye^{x^2+y^2})=-2xy^3-y\cos(xy)-2xe^{x^2+y^2}$

$\frac{dy}{dx}=\frac{-2xy^3-y\cos(xy)-2xe^{x^2+y^2}}{3x^2y^2+x\cos(xy)-2ye^{x^2+y^2}}$

A: $\frac{-2xy^3-y\cos(xy)-2xe^{x^2+y^2}}{3x^2y^2+x\cos(xy)-2ye^{x^2+y^2}}$ ✗