大同大學 108 學年度第一學期期中考試試題

科目代號:G1011A,B,C,D,F,G,H,J,K,L　　科目名稱: 微積分 (一)
班級: **C1AB, E1AB, I1AB, M1AB, T1AB**
註: 本次考試不可參考書籍及筆記　　不可使用計算機　　命題老師: 張薰文、張建瑋、蔡援宗、廖漢雄

1. Find the limits. (20%)

   (a) $\lim_{x\to -5} \dfrac{x+5}{\sqrt{30+x}-5}$.

   (b) $\lim_{x\to 0} \dfrac{\sin(4x)+\cos(x)-1}{\tan(3x)}$.

   (c) $\lim_{x\to \infty} \dfrac{\sqrt{9x^2-6x}}{2x+10000}$.

   (d) $\lim_{x\to 5^+} \dfrac{[[x^2+x]]-30}{(x-5)^2}$, $[[x]]$ 爲高斯函數.

2. Find the derivative $\dfrac{dy}{dx}$ of the followings. (32%)

   (a) $y = \sqrt[5]{x^3} - \dfrac{2}{\sqrt[3]{x}} + \ln(7^2)$.

   (b) $y = e^{x^2}\ln(\cos(x))$.

   (c) $y = \tan^5(7x)$.

   (d) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ **(use the Quotient Rule)**.

3. Find an equation of the tangent line to the graph of $f(x) = e^{2x}\cos(x)$ when $x = \dfrac{\pi}{2}$. (8%)

4. Let $f(x) = \left(4-x^2\right)^{2/5}$. (10%)

   (a) Determine the open intervals on which $f(x)$ is increasing or decreasing.

   (b) Find the relative extrema of $f(x)$.

5. Let $f(x) = \dfrac{1}{2}x^4 + x^3 - 18x^2 + 5x + 135$ (10%)

   (a) Determine the open intervals on which the graph of $f(x)$ is concave upward or concave downward.

   (b) Find the points of inflection(反曲點).

6. Evaluate $\dfrac{dy}{dx}$ of the following (20%)

   (a) $y^{x^2} = x^{y^2}$.

   (b) $x^2y^3 + \sin(xy) + e^{x^2+y^2} = -1$.