第八組 演習作業(四)

組員: I2B43 許祖豪
NO. I2B54 趙秉豐
DATE I2B58 張肯維

1.

(a) $\lim_{x\to 0^+}\left(x^{\ln(x+1)}\right)$ type 0^0

$\Rightarrow y=\lim_{x\to 0^+}\left(x^{\ln(x+1)}\right)$

$\ln y=\ln\left(\lim_{x\to 0^+}\left(x^{\ln(x+1)}\right)\right)$ since $\ln()$ is continuous

$=\lim_{x\to 0^+}\ln\left(x^{\ln(x+1)}\right)$

$=\lim_{x\to 0^+}\ln(x+1)\ln x$

type: $0\cdot(-\infty)$

$=\lim_{x\to 0^+}\dfrac{\ln x}{\frac{1}{\ln(x+1)}}$

type: $\frac{-\infty}{\infty}$

$\overset{L'}{\underset{(\frac{\infty}{\infty})}{=}}\lim_{x\to 0^+}\dfrac{\frac{1}{x}}{\frac{1}{x\ln^2(x+1)}}=\ln^2(x+1)=0$

$\ln y=0\Rightarrow y=1$

A: 1 ※

(b) $\lim_{x\to\infty}\left(1+\frac{8}{x}\right)^{6x}$ type 1^∞

$\Rightarrow y=\lim_{x\to\infty}\left(1+\frac{8}{x}\right)^{6x}$

$\ln y=\ln\left(\lim_{x\to\infty}\left(1+\frac{8}{x}\right)^{6x}\right)$

$=\lim_{x\to\infty}\left(\ln\left(1+\frac{8}{x}\right)^{6x}\right)$

$=\lim_{x\to\infty}6x\ln\left(1+\frac{8}{x}\right)$ type $\infty\cdot 0$

$=\lim_{x\to\infty}\dfrac{\ln\left(1+\frac{8}{x}\right)}{\frac{1}{6x}}$ type $\frac{0}{0}$

$\overset{L'}{\underset{(\frac{0}{0})}{=}}\lim_{x\to\infty}\dfrac{\frac{-8x^{-2}}{\left(1+\frac{8}{x}\right)}}{-\frac{1}{6}x^{-2}}$

$=\lim_{x\to\infty}\dfrac{48}{1+\frac{8}{x}}=48$

$\ln y=48 \quad y=e^{48}$

A: e^{48} ※

2. $\dfrac{d}{dx}\displaystyle\int_{2^x}^{\ln(x)}\left(\sin(2t^2)\right)dt$

$=\dfrac{d}{dx}\displaystyle\int_0^{\ln(x)}\left(\sin(2t^2)\right)dt-\dfrac{d}{dx}\int_0^{2^x}\left(\sin(2t^2)\right)dt$

$=\dfrac{d}{d\ln(x)}\displaystyle\int_0^{\ln(x)}\left(\sin(2t^2)\right)dt\cdot\dfrac{d\ln(x)}{dx}-\dfrac{d}{d2^x}\int_0^{2^x}\left(\sin(2t^2)\right)dt\cdot\dfrac{d2^x}{dx}$

$=\sin\left(2(\ln(x))^2\right)\left(\frac{1}{x}\right)-\sin\left(2(2^x)^2\right)2^x\cdot 1\cdot\ln 2$

$=\dfrac{\sin\left(2\left(\ln^2(x)\right)\right)}{x}-\left(\sin\left(2^{2x+1}\right)\right)\cdot 2^x\ln 2$ ※

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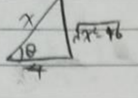
3. (a) $\int \left(\frac{3x}{\sqrt{x^2-16}}\right) dx$



let $x = 4\sec\theta$

Then $dx = 4\sec\theta\tan\theta\, d\theta$

$= \int \left(\frac{dx}{\frac{1}{3x}(\sqrt{x^2-16})}\right) dx$

$= \int \left(\frac{4\sec\theta\tan\theta}{\frac{\cos\theta}{12}\,|4\tan\theta|}\right) d\theta$

$= \int 12\sec^2\theta \frac{\tan\theta}{|\tan\theta|}\, d\theta$

For $x > 0$ $\quad 0 \le \theta < \frac{\pi}{2}$ $\quad \tan\theta > 0$

原式 $= \int 12\sec^2\theta \frac{\tan\theta}{\tan\theta} d\theta = 12\tan\theta = 12\tan(\sec^{-1}\frac{x}{4}) + C$

For $x < 0$, $\frac{\pi}{2} < \theta \le \pi$, $\sec\theta < 0$

原式 $= \int 12\sec^2\theta \frac{-\tan\theta}{|-\tan\theta|} d\theta = \int -12\sec^2\theta\, d\theta = 12\tan(-\theta) = 12\tan(\sec^{-1}\frac{x}{4}) + C$

A: $12\tan(\sec^{-1}\frac{|x|}{4}) + C$

方(二)

$\int \frac{3x}{\sqrt{x^2-16}} dx$ $\quad$ let $u = x^2+16$

$du = 2x\,dx$

$= \frac{3}{2}\int u^{-\frac{1}{2}}\, du$

$= \frac{3}{2} \cdot 2 u^{\frac{1}{2}} + C$

$= 3\sqrt{x^2-16} + C$ #

(b) $\int (x^2 \sin(2x))\, dx$ $\quad$ let $u = x^2$, $dv = \sin 2x$

$= \frac{-x^2}{2}\cos 2x - \int -x\cos 2x\, dx$ $\quad$ Then $du = 2x\,dx$, $v = \frac{-\cos 2x}{2}$

$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \int \frac{-1}{2}\sin 2x\, dx$ $\quad$ let $u = x$, $dv = -\cos 2x$

then $du = dx$, $v = \frac{-1}{2}\sin 2x$

$= \frac{-1}{2}(x^2\cos 2x) + \frac{1}{2}(x\sin 2x) + \frac{1}{4}\cos 2x + C$

A: $\frac{-1}{2}(x^2\cos 2x) + \frac{1}{2}(x\sin 2x) + \frac{1}{4}\cos 2x + C$ #

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(c) $\int \tan^3(x)\sec^9(x)\,dx$

Let $u = \sec(x)$

Then $du = \sec(x)\tan(x)\,dx$

$= \int \sec(x)\tan(x)\,(\sec^2(x)-1)\sec^8(x)\,dx$

$= \int u^8(u^2-1)\,du$

$= \int u^{10} - u^8\,du$

$= \frac{u^{11}}{11} - \frac{u^9}{9} + C$

$= \frac{\sec^{11}(x)}{11} - \frac{\sec^9(x)}{9} + C$

A: $\frac{\sec^{11}(x)}{11} - \frac{\sec^9(x)}{9} + C$ #

(d) $\int (\arctan(3x))\,dx$

Let $u = \arctan(3x)$ $\quad dv = dx$

$du = \frac{3}{1+9x^2}$ $\quad v = \int dx = x$

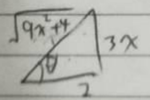
$= x\arctan(3x) - \int x \frac{3}{1+9x^2}\,dx$

$= x\arctan(3x) - \frac{1}{6}\int \frac{18x}{1+9x^2}\,dx$

$= x\arctan(3x) - \frac{1}{6}(\ln|1+9x^2|) + C$

A: $x\arctan(3x) - \frac{1}{6}(\ln|1+9x^2|) + C$

(e) $\int \left(\frac{3x^2}{\sqrt{9x^2+4}}\right)dx$



Let $3x = 2\tan\theta$ $\quad x = \frac{2}{3}\tan\theta$

Then $dx = \frac{2}{3}\sec^2\theta\,d\theta$

$= \int \frac{2\tan\theta \cdot \frac{2}{3}\tan\theta \cdot \frac{2}{3}\sec^2\theta\,d\theta}{2\sec\theta}$

$= \int \frac{4}{9}\tan^2\theta\sec\theta\,d\theta$

$= \int \frac{4}{9}(\sec^2\theta - 1)\sec\theta\,d\theta$

$= \frac{4}{9}\left(\int \sec^3\theta\,d\theta - \int \sec\theta\,d\theta\right)$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \tan\theta\sec\theta\tan\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$2\int \sec^3\theta\,d\theta = \sec\theta\tan\theta + \int \sec\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \frac{1}{2}(\sec\theta\tan\theta) + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

Let $u = \sec\theta$ $\quad dv = \sec^2\theta\,d\theta$

Then $du = \sec\theta\tan\theta$ $\quad v = \tan\theta$

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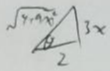


原式 $= \frac{4}{9}\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\tan\theta+\sec\theta| - \ln|\tan\theta+\sec\theta|\right)+C$

$= \frac{2}{9}\sec\theta\tan\theta - \frac{2}{9}\ln|\tan\theta+\sec\theta| + C$

$= \frac{2}{9}\sec\left(\tan^{-1}\frac{3x}{2}\right)\tan\left(\tan^{-1}\frac{3x}{2}\right) - \frac{2}{9}\ln\left|\tan\left(\tan^{-1}\frac{3x}{2}\right)+\sec\left(\tan^{-1}\frac{3x}{2}\right)\right|+C$

$= \frac{x}{3}\sec\left(\tan^{-1}\frac{3x}{2}\right) - \frac{2}{9}\ln\left|\frac{3x}{2}+\sec\left(\tan^{-1}\frac{3x}{2}\right)\right|+C$

A: $\frac{x}{3}\sec\left(\tan^{-1}\frac{3x}{2}\right) - \frac{2}{9}\ln\left|\frac{3x}{2}+\sec\left(\tan^{-1}\frac{3x}{2}\right)\right|+C$ #

(f) $\int(\cos(8x)\cos(3x))\,dx$

$= \frac{1}{2}\int(\cos(11x)+\cos(5x))\,dx$

$= \frac{1}{2}\int\left(\frac{1}{11}\cos(11x)\cdot 11 + \frac{1}{5}\cos(5x)\cdot 5\right)dx$

$= \frac{\sin(11x)}{22} + \frac{\sin(5x)}{10} + C$ A: $\frac{\sin(11x)}{22}+\frac{\sin(5x)}{10}+C$ #

(g) $\int\left(\frac{x^2+x+1}{x^3+x}\right)dx$

$= \int\left(\frac{x^2+x+1}{x(x^2+1)}\right)dx$

Let $\left(\frac{x^2+x+1}{x(x^2+1)}\right) = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

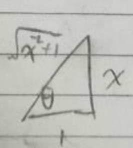
Then $x^2+x+1 = A(x^2+1) + (Bx+C)(x)$

For $x=0$, $1 = A(1) \Rightarrow A = 1$

For $x=1$, For $x=-1$, $\begin{cases} 1 = B+C \\ -1 = B-C \end{cases} \Rightarrow B=0 \quad C=1$

原式 $= \int\left(\frac{1}{x} + \frac{1}{x^2+1}\right)dx$

$= \ln|x| + \tan^{-1}x + C$



$x = \tan\theta$

$dx = \sec^2\theta\, d\theta = x^2+1$

$\theta = \tan^{-1}x$

$\frac{d\theta}{dx} = \frac{1}{\sec^2\theta} = \frac{1}{x^2+1}$

A: $\ln|x| + \tan^{-1}x + C$ #

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(h) $\int \left(\frac{1}{\sin(x)\cos(x)} \right) dx$

$= \int \left(\frac{2}{\sin(2x)} \right) dx$

$= \int \left(2\csc(2x) \right) dx = -\ln\left|\csc(2x) + \cot(2x)\right| + C$

A: $-\ln\left|\csc(2x) + \cot(2x)\right| + C$

4.

(a) $\int_{1}^{25} \left(\frac{1}{\sqrt{x}(1+\sqrt{x})^3} \right) dx$

let $u = 1 + \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$x = 25, u = 6$

$x = 1, u = 2$

$= \int_{2}^{6} \left(2u^{-3} \right) du$

$= \left[-u^{-2} \right]_{2}^{6}$

$= \frac{-1}{36} + \frac{1}{4} = \frac{9-1}{36} = \frac{\cancel{8}^{2}}{\cancel{36}_{9}} = \frac{2}{9}$

A: $\frac{2}{9}$ #

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(b) $\int_0^\pi \left(e^{(2x)}\cos(4x)\right)dx$ Let $u=\cos(4x)$, $dv=e^{2x}dx$

Then $du=-4\sin(4x)dx$, $v=\frac{1}{2}e^{2x}$

$\Rightarrow \int\left(e^{(2x)}\cos(4x)\right)dx = \frac{1}{2}e^{2x}\cos(4x)+2\int e^{2x}\sin(4x)\,dx$

Let $u=\sin(4x)$ $dv=e^{2x}dx$

$du=4\cos(4x)dx$ $v=\frac{1}{2}e^{2x}$

$\int\left(e^{(2x)}\cos(4x)\right)dx = \frac{1}{2}e^{2x}\cos(4x)+2\left(\frac{1}{2}e^{2x}\sin(4x)-\int\frac{1}{2}e^{2x}\cdot4\cos(4x)dx\right)$

$\int\left(e^{(2x)}\cos(4x)\right)dx = -\frac{1}{2}e^{2x}\cos(4x)+e^{2x}\sin(4x)-4\int e^{2x}4\cos(4x)dx$

$5\int\left(e^{2x}\cos(4x)\right)dx = \frac{-1}{2}e^{2x}\cos(4x)+e^{2x}\sin(4x)+C$

$\int\left(e^{2x}\cos(4x)\right)dx = \frac{-1}{10}e^{2x}\cos(4x)+\frac{1}{5}e^{2x}\sin(4x)+C$

$\left[\frac{-1}{10}e^{2x}\cos(4x)+\frac{1}{5}e^{2x}\sin(4x)\right]_0^\pi = \frac{e^{2\pi}}{10}-\frac{1}{10}=\frac{e^{2\pi}-1}{10}$ A: $\frac{e^{2\pi}-1}{10}$ #

(c) $\int_0^4\left(|x^2+2x-3|\right)dx$

$\Rightarrow y=\int_0^4\left(|(x+3)(x-1)|\right)dx$ $x>1$ $y>0$

$x<1$ $y<0$

原式 $=\int_1^4(x^2+2x-3)dx-\int_0^1(x^2+2x-3)dx$

$=\left[\frac{1}{3}x^3+x^2-3x\right]_1^4-\left[\frac{1}{3}x^3+x^2-3x\right]_0^1$

$=\frac{64}{3}+16-12-2\left(\frac{1}{3}+1-3\right)$

$=\frac{62}{3}+8=\frac{86}{3}$ A: $\frac{86}{3}$ #

(d) $\int_0^{\frac{\pi}{2}} \left(\cos^5(x)\right)dx$

$\Rightarrow \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int \cos^3(x)\,dx$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int (1-\sin^2 x)\cos x\,dx$ Let $u=\sin x$ Then $du=\cos x\,dx$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int 1-u^2\,du$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\left(u - \frac{u^3}{3}\right)$

$= \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\left(\sin(x) - \frac{\sin^3(x)}{3}\right)$

$\left[\frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\left(\sin(x) - \frac{\sin^3(x)}{3}\right)\right]_0^{\frac{\pi}{2}}$

$= \frac{4}{5} - \frac{4}{15}$

$= \frac{12-4}{15}$

$= \frac{8}{15}$

A: $\frac{8}{15}$ #