演習作業(三)　　　　J2B 賴秉堃

NO.

DATE

1.

(a)

$\int \left(2x^3+3x^2+\frac{1}{2\sqrt{x}}+\frac{3}{x}\right)dx$

$=\int \left(2x^3+3x^2+2x^{-\frac{1}{2}}+3x^{-1}\right)dx$

$=\boxed{\frac{1}{2}x^4+x^3+4x^{\frac{1}{2}}+3\ln x+C}$ #

(b) $\int \left(\frac{x^3-2x-4}{x-2}\right)dx$

$=\int \frac{\cancel{(x-2)}(x^2+2x+2)}{\cancel{(x-2)}}dx$

$=\boxed{\frac{1}{3}x^3+x^2+2x+C}$ #

(c)

$\int \left(e^x(2e^x+4)\left((e^x+2)^2+5\right)^3\right)dx$ Let $u=e^x+2$ $du=e^x\,dx$

$\int 2u\cdot(u^2+5)^3\,du$

$=\frac{3}{4}(u^2+5)^4+C$

$=\boxed{\frac{3}{4}\left((e^x+2)^2+5\right)^4+C}$ #

(d)

$\int \left(\frac{5}{2+\sqrt{5x}}\right)dx$ Let $u=5x$ $du=5dx$

$\int \left(\frac{1}{2+u^{\frac{1}{2}}}\right)du$

$=\int (2+u^{\frac{1}{2}})^{-1}du$

$=\ln(2+u^{\frac{1}{2}})$ $=\boxed{\ln(2+\sqrt{5x})+C}$ #

2.(a)

$\int_0^{\frac{\pi}{3}} \left(\frac{-\cos(x)}{1-\sin(x)}\right)dx$ Let $u=\sin x$ $du=\cos x\,dx$ $x=\frac{\pi}{3}$, $u=\frac{\sqrt{3}}{2}$

$x=0$, $u=1$

$\int_1^{\frac{\sqrt{3}}{2}} \left(\frac{-1}{1-u}\right)du$

$=\int_1^{\frac{\sqrt{3}}{2}} (u-1)^{-1}du$

$=\left[\ln(u-1)\right]_1^{\frac{\sqrt{3}}{2}}$

$=\boxed{\ln(\frac{\sqrt{3}}{2}-1)-\ln 0}$ #

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2.(b)

$$\int_4^7 \left(\left|x^2+9x-90\right|\right)dx$$

$$=\left[\left|\frac{1}{3}x^3+\frac{9}{2}x^2-90x\right|\right]_4^7$$

$$=\frac{1}{3}(7)^3+\frac{9}{2}(7)^2-90(7)-\frac{1}{3}(4)^3-\frac{9}{2}(4)^2+90(4)$$

$$= 93 + 148.5 - 270$$

$$=\boxed{-28.5}\ \#$$

(c) $$\int_{-1}^{2\pi}(f(x))dx, \text{ when } f(x)=\begin{cases}1-e^{(2x)}, & x<0\\ 4x\cos(2x^2), & x\geq 0\end{cases}$$

$$\int_{-1}^{0}\left(1-e^{(2x)}\right)dx \qquad \int_0^{2\pi}\left(4x\cos(2x^2)\right)dx$$

$$=\left[x-\frac{e^{2x}}{2}\right]_{-1}^{0} \qquad =\left[\sin(2x^2)\right]_0^{2\pi}$$

$$=-\frac{1}{2}-\left(-1-\frac{1}{2e^2}\right) \qquad =0$$

$$=\frac{1}{2}+\frac{1}{2e^2}$$

A: $\boxed{\frac{1}{2}+\frac{1}{2e^2}}\ \#$

3.(a) $$F(x)=\int_x^{\cos(x)}\left(\sqrt{3\sin(t)+4}\right)dt$$

$$=\int_0^{\cos(x)}\left(\sqrt{3\sin(t)+4}\right)dt-\int_0^{x}\left(\sqrt{3\sin(t)+4}\right)dt$$

$$F'(x)=\frac{d}{dx}\int_0^{\cos(x)}\left(\sqrt{3\sin(t)+4}\right)dt-\frac{d}{dx}\int_0^{x}\left(\sqrt{3\sin(t)+4}\right)dt$$

$$=\left[\frac{d}{d\cos(x)}\int_0^{\cos(x)}\left(\sqrt{3\sin(t)+4}\right)dt\cdot\frac{d\cos(x)}{dx}\right]-\sqrt{3\sin(x)+4}$$

$$=\boxed{\sqrt{3\sin(\cos(x))+4}\cdot(-\sin(x))-\sqrt{3\sin(x)+4}}\ \#$$

3(b) $$F(x)=\int_{e^x}^{\ln(x)}\left(\sin(4t^2)\right)dt$$

$$=\int_0^{\ln(x)}\left(\sin(4t^2)\right)dt-\int_0^{e^x}\left(\sin(4t^2)\right)dt$$

$$F'(x)=\frac{d}{dx}\int_0^{\ln(x)}\left(\sin(4t^2)\right)dt-\frac{d}{dx}\int_0^{e^x}\left(\sin(4t^2)\right)dt$$

$$=\frac{d}{d\ln(x)}\int_0^{\ln(x)}\left(\sin(4t^2)\right)dt\cdot\frac{d\ln(x)}{dx}-\frac{d}{de^x}\int_0^{e^x}\left(\sin(4t^2)\right)dt\cdot\frac{de^x}{dx}$$

$$=\boxed{\sin(4(\ln x)^2)\cdot\frac{1}{x}-\sin(4e^{2x})\cdot e^x}\ \#$$