Construct a truth table (真值表) for each of these compound propositions(複合命題).
1. $p \rightarrow (\neg q \lor r)$
2. $(p \oplus q) \rightarrow (p \oplus \neg q)$

3. Determine (判定) whether $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are equivalent (邏輯上同義). Explain your answer.

Translate each of these statements into logical implications $p \rightarrow q$. Use the underlying characters to denote the propositions.
4. You can access the websites only if you pay a subscription fee (註冊費).
5. It snows whenever the wind blows from the northeast (吹東北風).
6. I will send you the video only if you send me an e-mail message.
7. It rains if it is a weekend day (週末).
8. To be a citizen (公民) of this country, it is sufficient(足夠的) that you were born in the United States.
9. It is necessary for you to be over 40 years old to be elected as President of this country.
10. You cannot ride rollercoaster (雲霄飛車) unless you are over 120cm tall.

Let *p, q*, and *r* be the propositions:
  p: The user enters a valid password.   *q*: Access is granted (准許讀取).
  r: The user has paid the subscription (訂閱) fee.

Write these propositions using *p, q*, and *r* and logical connectives.
11. The user has paid the subscription fee, but does not enter a valid password.
12. Access is granted whenever the user has paid the subscription fee and enters a valid password.
13. Access is denied (not granted) if the user has not paid the subscription fee.

1. P → (¬S ∨ r)

P	S	r	¬S	¬S ∨ r	P → (¬S ∨ r)
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

//

2. (P ⊕ S) → (P ⊕ ¬S)

P	S	P ⊕ S	¬S	P ⊕ ¬S	(P ⊕ S) → (P ⊕ ¬S)
T	T	F	F	T	T
T	F	T	T	F	F
F	T	T	F	F	F
F	F	F	T	T	T

//

4. $w → f$ 5. $w → s$

6. $v → m$

7. $w → r$

8. $b → c$

9. $p → o$

10. $¬t → ¬r$

(OR, $r → t$)

4. You can access the websites only if you pay a subscription fee (註冊費).
5. It snows whenever the wind blows from the northeast (吹東北風).
6. I will send you the video only if you send me an e-mail message.
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10. You cannot ride rollercoaster (雲霄飛車) unless you are over 120cm tall.

3. $(p \to q) \to r$ vs. $p \to (q \to r)$

p	q	r	$p \to q$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	**F**	F	**T**
F	F	T	T	T	T	T
F	F	F	T	**F**	T	T

← 不同 $p=F, q=T, r=F$

$(p \to q) \to r \equiv F$

$p \to (q \to r) \equiv T$

p: The user enters a valid password.   q: Access is granted (准許讀取).
r: The user has paid the subscription (訂閱) fee.

11. The user has paid the subscription fee, but does not enter a valid password.
12. Access is granted whenever the user has paid the subscription fee and enters a valid password.
13. Access is denied (not granted) if the user has not paid the subscription fee.

11. $r \wedge \neg p$

12. $r \wedge p \to q$

13. $\neg r \to \neg q$ (OR: $q \to r$)