

## Quiz 2 [1.4-1.7]

3/25/2020

1-8: 7 points each, 9-11: 6 points each

Translate each of these statements into logical expressions. Use the underlying characters to denote your predicates. *The domain consists of all animals in the world.*

1. No monkey can speak French.
2. Some old dogs can learn new tricks.
3. There exists a pig that can swim and catch fish.

Express (表達) the following mathematical statements using logical symbols (Domain: all real numbers)

4. The average (算術平均) of two positive numbers is positive.
5. The difference of two positive numbers is not always positive (不一定為正).

$L(x,y)$  "x loves y." Translate the following into logic expressions with predicate and quantifiers. (domain: all students in your class)

6. Everybody loves Jerry.
7. There is somebody whom no one loves.
8. There is somebody whom Linda does not love.

Determine (判斷) the truth value of questions 9-11. The domain of each variable consists of all real numbers. **Explain your answers.**

9.  $\forall x \exists y (x^2 = y)$
10.  $\exists x \forall y (xy = 0)$
11.  $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

12. Show  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are logically equivalent (邏輯上同義), by developing a series of logical equivalencies, without using the truth table (不可用真值表) [10 points]

*Prove or disprove* (by a counterexample, 反例) (13-14): [10 points]

13. The difference of two positive integers is positive.
14. If  $x + y \geq 2$  then  $x \geq 1$  or  $y \geq 1$ , where  $x$  and  $y$  are real numbers.

15. Proof by contradiction: [6 points]

There does not exist two positive real numbers  $x, y$ , such that  $\frac{1}{5} + \frac{x}{y} = \frac{1+x}{5+y}$ .

答題參考：

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \quad \neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r) \quad (p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$$

證明題的重點

先決定證明的策略 (direct, indirect, contradiction) 後,  
寫出符合邏輯推論的敘述, 以說明所要證明的命題為真

1.  $M(x)$ :  $x$  is a monkey "所有的猴子都不会说法语".

$F(x)$ :  $x$  can speak French.  $\forall x (M(x) \rightarrow \neg F(x))$

或: 不存在一只会说法语的猴子.

$$\neg \exists x (M(x) \wedge F(x)) \equiv \forall x \neg (M(x) \wedge F(x))$$

$$\equiv \forall x (\neg M(x) \vee \neg F(x))$$

$$\equiv \forall x (M(x) \rightarrow \neg F(x))$$

2.  $\exists x (D(x) \wedge T(x))$

3.  $\exists x (P(x) \wedge S(x) \wedge F(x))$

4.  $\forall x \forall y ((x > 0) \wedge (y > 0) \rightarrow (\frac{x+y}{2} > 0))$

5.  $\exists x \exists y ((x > 0) \wedge (y > 0) \wedge (x - y \leq 0))$

6.  $\forall x L(x, \text{Jerry})$

7.  $\exists x \forall y \neg L(y, x)$

8.  $\exists x \neg L(\text{Linda}, x)$

9. True (奇数之平方仍为奇数)

10. True ( $x=0$ ,  $\forall y (xy=0)$ )

11. false (不存在比数... 所有非零数来横着)

6 each.  
no explain  $\rightarrow 0$   
wrong reason  $\uparrow$

12.  $\neg(p \vee (\neg p \wedge q))$

$$\equiv \neg p \wedge \neg(\neg p \wedge q)$$

$$\equiv \neg p \wedge (\neg \neg p \vee \neg q)$$

$$\equiv \neg p \wedge (p \vee \neg q)$$

$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$$

$$\equiv F \vee (\neg p \wedge \neg q) \equiv \neg p \wedge \neg q$$

13. disprove, 反例  $3-5 = -2 < 0$

14. Indirect proof.

欲証  $x < 1 \wedge y < 1 \rightarrow x+y < 2$

根據數學公理 ( $小+小 < 大+大$ ). Q.E.D.

15. 矛盾証法: "假設所証為 false"

---即, 存在  $x, y \in \mathbb{R}^+$ , 使得  $\frac{1}{5} + \frac{y}{y} = \frac{1+x}{5+y}$

$$\text{左} = \frac{1}{5} + \frac{y}{y} = \frac{y+5x}{5y} = \frac{1+x}{5+y} = \text{右}.$$

$$(y+5x)(5+y) = 5y(1+x)$$

展開:

$$\cancel{5y} + \cancel{5xy} + 25x + y^2 = \cancel{5y} + \cancel{5xy}$$

$$25x + y^2 = 0$$

$$y^2 = -25x < 0 \quad (\because x \in \mathbb{R}^+)$$

But  $\because y \in \mathbb{R}^+, y^2 > 0$ , 矛盾.

$\therefore$  所証為 TRUE.

Q.E.D.



$$1. \quad \forall x (M(x) \rightarrow \neg F(x)) \quad \text{继续加} ( )$$

$$2. \quad \exists x (D(x) \wedge T(x))$$

$$4. \quad \forall x \forall y (x > 0 \wedge y > 0 \rightarrow \frac{x+y}{2} > 0)$$

$$5. \quad \exists x \exists y (x > 0 \wedge y > 0 \wedge x - y \leq 0)$$

$$6. \quad \forall x \quad \underline{L(x, \text{Jerry})}$$

$$7. \quad \exists x \quad \neg \forall y L(y, x) \quad x$$

$$\equiv \exists x \quad \exists y \neg L(y, x)$$

$$\exists x \forall y L(y, \neg x) \quad x$$

8. Prove that

$$\exists x \forall y (y \neq 0 \rightarrow xy = 1) \text{ is false.}$$

即, 解证 “不存在逆数”.

(用) 矛盾证法: 假设逆数存在  $x = c$

任意非零实数与其乘积皆为 1 ( $y \neq 0 \rightarrow xy = 1$ )

$$\text{取 } y_1 = 2, \quad y_2 = 3$$

$$\text{则 } 2c = 1, \quad 3c = 1$$

$$\Rightarrow 2c = 1 = 3c$$

$$\Rightarrow 2c = 3c$$

$$\Rightarrow 0 = 3c - 2c = c$$

$$\Rightarrow c = 0$$

但  $2c = 1$ .  $c$  不可等于 0. -- (矛盾!)

$\therefore$  根据矛盾证法:  $\therefore$  得 逆数不存在 Q.E.D.



## Quiz 2 [1.4-1.6]

3/11/2019

1-8: 5 points each, 9-11: 12 points each, 12-15: 6 points each

Translate each of these statements into logical expressions. Use the underlying characters to denote your predicates. *The domain consists of all animals in the world.*

1. No monkey can speak French.
2. Some old dogs can learn new tricks.
3. There exists a pig that can swim and catch fish.

$M(x)$   ~~$M(x)$~~   ~~$M(x)$~~   
 predicates: 大寫英文.

Express (表達) the following mathematical statements using logical symbols (Domain: all real numbers)

4. The average (算術平均) of two positive numbers is positive.
5. The difference of two positive numbers is not always positive (不一定為正).

$L(x,y)$  "x loves y." Translate the following into logic expressions with predicate and quantifiers. (domain: all students in your class)

6. Everybody loves Jerry.
7. There is somebody whom no one loves.
8. There is somebody whom Linda does not love.

$\exists x \forall y \neg L(y, x)$

For questions 9-11, 先轉換每句為邏輯敘述。再判斷推論是否正確。

9. *Every student in Tatung University has an Internet account. Tom does not have an Internet account. Therefore, Tom is not a student in Tatung University.*

Domain: all students in Taipei.  $T(x)$ : x is a student of Tatung University.  $I(x)$ : x has an Internet account.

10. *All parrots (鸚鵡) like fruit. My pet bird Mary is not a parrot. Therefore, Mary does not like fruit.*

Domain: all animals.  $P(x)$ : x is a parrot.  $F(x)$ : x likes fruit.

11. *All students in this class understand logic. Peter is a student in this class. Therefore, Peter understands logic.*

Domain: all students in school.  $C(x)$ : x is a student of this class.  $L(x)$ : x understands logic.

Determine (判斷) the truth value of questions 12-14. The domain of each variable consists of all real numbers. Explain your answers.

12.  $\forall x \exists y (x^2 = y)$

13.  $\exists x \forall y (xy = 0)$

14.  $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

15. Show  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  are logically equivalent (邏輯上同義), by developing a series of logical equivalencies, without using the truth table (不可用真值表)

參考規則

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$$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r) \quad (p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$$