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REFRACTION NEAR THE HORIZON

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ABSTRACT

We have studied the variation of astronomical refraction near the horizon. We have collected 144 measurements of refraction from seven sites by three techniques and have found that the variation of refraction on the horizon is substantially larger than has previously been realized. The rms deviation of our observations is 0°16, while the individual measurements range from 0°234 to 1°678. At the 95% confidence level the total refraction should vary over a range of 0°64. This surprising result has five applications: First, the time of sunrise can only be predicted with an accuracy of 4 min, despite all the extreme accuracy of modern positional astronomy. Second, during a low-altitude solar eclipse (such as is the 1990 eclipse viewed from Finland), the size and shape of the edge of the umbra will vary in an unpredictable manner by perhaps several kilometers, so that attempts to measure the diameter of the Sun may have an unexpected accuracy limit. Contact times may vary by typically 0.1 sec. Third, refraction variation will set a fundamental limit on the accuracy of any claimed archaeoastronomical alignment. Fourth, methods for aligning the Great Pyramid of Cheops to an accuracy of 2′ cannot involve near-horizon observations. Fifth, the historically important claim by A. Thom that British megalithic sites were used as accurate lunar observatories is shown to be wrong because the needed accuracy is much greater than can be obtained for long averaging intervals.

Key words: atmospheric refraction–astrometry–archaeoastronomy

1. Introduction

Archaeoastronomy often concerns itself with the contention that some ancient monuments are aligned with the rising and setting of celestial objects. Two well-known illustrations of this paradigm are the alignment of the avenue at Stonehenge with the midsummer sunrise (Hawkins 1965) and the precise alignments of many British megalithic sites toward significant moonrise positions (Thom 1971). To make a connection between the monument and the sky, an archaeoastronomer must be able to translate an azimuth measured at the site into a declination on the sky. The light from any celestial object is bent by refraction as it enters the Earth's atmosphere; thus, the azimuth-to-declination calculation must include a correction for refraction.

Astronomical refraction has long been studied, with significant contributions from Ptolemy, Tycho, Kepler, Cassini, and Newton (Mahan 1962). Astronomers have extensively analyzed refraction because of its importance for the discipline of astrometry. However, astrometry is always done for stars well away from the horizon, and solutions to the refraction integral (Smart 1979) all diverge as the horizon is approached. Near the horizon the refraction can be determined from observations, simple models (Bruin 1981), or complicated models (Garfinkel 1967). In modern times Garfinkel's program has become a standard that is used in most situations where accuracy is required.

Garfinkel's program assumes the U.S. Standard Atmosphere, an assumption which is valid as an average. However, the real atmosphere is never an exact match to this ideal. Deviations from the ideal atmosphere (e.g., thermal inversions) are common in real life and will lead to refraction that is appreciably different from the calculations of Garfinkel.

Archaeoastronomers have considered the effects related to simple temperature and pressure changes in the atmosphere. Their analysis adopts some equation for the temperature and pressure dependence of refraction (e.g., Allen 1973; Thom 1971; or the *Astronomical Almanac*).

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Then, for reasonable assumptions of day-to-day or season-to-season variations, the uncertainty in refraction is concluded to be typically 1′ (Heggie 1981), a value which is negligible for practical purposes. Thom (1971, 1974) adopts a value of 0.′5 as the 1σ refraction error. Unfortunately, this method does not account for the possibility of refraction variations due to changes in the *shape* of the temperature profile of the atmosphere.

Schaefer (1989) has developed a computer program that is capable of calculating refraction for an arbitrary atmospheric temperature profile. When this program is applied with typical thermal inversion conditions, the refraction can change by a large factor for observations near the horizon. These large variations will substantially increase the uncertainty in the translation from azimuth to declination. This theoretical conclusion requires a fundamental change in the archaeoastronomy paradigm.

Any such far-reaching theoretical conclusion must be tested observationally. In this paper we report on 144 observations of refraction near the horizon from seven sites.

2. Data

2.1 *Sunset Timings*

Minnaert (1954, p. 40) pointed out that a series of accurate sunset timings could be used as a measure of refraction and its daily variation. These timings, made over an ocean horizon, are taken when the last gleam of the solar disk is visible. The *true* angular distance from the zenith (Z_t) to the upper limb of the Sun with no correction for the effects of the atmosphere can readily be calculated for the observed time of sunset from the *Astronomical Almanac*. The *apparent* angular distance from the zenith (Z_a) of the upper limb of the Sun at sunset is the same as that of the horizon, or 90° for a level horizon. The general expression for atmospheric refraction (R) is

$$R = Z_t - Z_a \ , \tag{1a}$$

but at the ideal horizon it reduces to

$$R_0 = Z_t - 90° \ . \tag{1b}$$

Thus, R_0 is easily measured by a simple timing.

We have collected 116 timings as presented in Table 1. The first column gives an assigned reference number for each observation. The next four columns list the site name, latitude (in degrees), longitude (in degrees east of the Greenwich meridian), and altitude (in meters above mean sea level). In the fifth column appear the initials of the observer(s), who are Roger Buck, Drake Deming, William Liller, Bradley E. Schaefer, Elly Schaefer, Martha W. Schaefer, Ralph E. Schaefer, Rick Tucker, and Maria Zuber. The next two columns tabulate the date and Universal Time of the last gleam of sunset. The column labeled "*" refers to comments at the bottom of the table, while in the tenth column "GF" indicates whether a green flash was visible at the time of last gleam; if a green flash was seen, the note might give either the duration or intensity. The next column gives the calculated Z_t value in degrees for the upper limb (with a parallax correction), and the last column lists the value of R_0 in degrees after the correction for the horizon dip (see below).

The altitude of the center of the Sun (h_c) for the exact time of sunset is easily calculated with the Floppy Almanac. The tabulated Z_t value is then

$$Z_t = 90° - h_c + \pi - \theta_s \ . \tag{2}$$

The angular radius of the Sun (θ_s) and the solar parallax (π) both had to be looked up in the printed version of the *Astronomical Almanac*. The Z_a value will not be exactly 90°, because the observer's eye is above sea level. The correction will be close to the geometrical horizon dip,

$$D = \cos^{-1}(1/[1 + A/6378 \text{ km}]) \ , \tag{3}$$

where A is the altitude of the site. The measured refraction then becomes

$$R_0 = Z_t - 90° - D \ . \tag{4}$$

Thus, all measured refractions have been corrected to sea level.

The Sun is not the only usable celestial object; both the Moon and Venus are bright enough to be visible all the way to the horizon. Two Moon and four Venus timings are given in the table. For one of the moonsets the lower limb was used since the upper limb was not illuminated. For these observations the sign and magnitude of the corrections in equation (2) had to be modified.

The uncertainties in the timing arise from several sources. First, there is the possibility that the Sun did not set over the ocean but actually set over a distant cloud bank. In such cases the Sun will appear to set sooner, and the calculated refraction will be less than without the cloud bank. However, in practice, this case is usually easily recognized, although even with binoculars a very low and distant cloud bank may go undetected. For such cases the error in refraction will be sufficiently small as to be of little importance for the conclusions of this paper. Second, the brightness of the Sun fades away so that the exact time of disappearance may be ill defined, but in almost all cases the time of sunset was easily identified to within a second or so. Most exceptions occurred when a long-duration green flash was seen, where the tabulated time is for the end of the green flash. Third, there will be some error in measuring the sunset time. The procedure for all the observations was to measure the time on a digital wristwatch, for which the measurement error will be roughly one second. Fourth, the correction from watch time to Universal Time will have some error. This correction was always measured by a comparison of the watch time with time signals, so that the uncertainty in the correction to UT is roughly a second. In summary, the

TABLE 1

Sunset Times

#	SITE	LAT (°)	LONG (°)	A(m)	OBSERVER	DATE	UT	*	GF	Z (°)	Ro (°)
1	Honolulu	21.278	-157.830	5	ES, RES	1989 Feb 5	4:23:31		No	90.560	0.488
2	Kaanapali	20.932	-156.696	40	MZ	1988 Sep 16	4:32:21		No	91.317	1.114
3	Kailua	19.628	-155.992	3	RB	1989 Oct 7	4:07:10		No	90.466	0.410
4	Keauhou	19.563	-155.965	3	RT	1989 Jan 3	3:57:33		No	90.524	0.468
5	Keokea	19.418	-155.884	280	ES, RES	1989 Feb 2	4:18:47		1 sec	91.085	0.548
6	Saddle Road	19.936	-155.690	800	MWS, BES	1988 Dec 4	3:46:53		No	91.474	0.567
7	Mauna Kea	19.823	-155.472	4205	MWS, BES	1988 Nov 30	3:49:40		1 sec	92.351	0.271
8	Mauna Kea	19.823	-155.472	4205	DD	1989 Jan 28	4:20:20		No	92.572	0.492
9	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 19	22:22:25		No	90.906	0.452
10	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 20	22:23:40		No	91.027	0.573
11	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 26	22:27:00	a	No	90.911	0.457
12	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 27	22:27:46		No	90.936	0.482
13	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 28	22:28:38		No	90.982	0.528
14	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 29	22:29:08		Bright	90.952	0.498
15	Vina del Mar	-32.950	-71.550	200	WL	1988 Aug 31	22:30:32		Faint	90.977	0.523
16	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 1	22:31:30	a	No	91.043	0.589
17	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 5	22:33:39		Faint	90.960	0.506
18	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 6	22:34:20		Faint	90.968	0.514
19	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 7	22:34:58		Bright	90.968	0.514
20	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 9	22:36:09	a	No	90.948	0.494
21	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 13	22:38:50	a	Yes	90.971	0.517
22	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 14	22:39:15	a	Yes	90.924	0.470
23	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 17	22:41:20		No	90.954	0.500
24	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 18	22:41:54		No	90.937	0.483
25	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 22	22:44:34		No	90.947	0.493
26	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 23	22:45:00	a	No	90.900	0.446
27	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 27	22:48:07		No	90.993	0.539
28	Vina del Mar	-32.950	-71.550	200	WL	1988 Sep 30	22:49:57		Yes	90.948	0.494
29	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 1	22:50:35		Yes	90.934	0.480
30	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 10	22:56:56		Yes	90.908	0.454
31	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 12	22:59:01		No	91.026	0.572
32	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 19	23:04:07		Faint	90.948	0.494
33	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 20	23:05:04		Faint	90.977	0.523
34	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 21	23:05:56		Bright	90.988	0.534
35	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 22	23:06:45		Bright	90.987	0.533
36	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 23	23:07:39		Faint	91.002	0.548
37	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 24	23:08:20		Faint	90.972	0.518
38	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 25	23:09:10		No	90.969	0.515
39	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 26	23:10:10		No	90.999	0.545
40	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 30	23:13:31		No	90.977	0.523
41	Vina del Mar	-32.950	-71.550	200	WL	1988 Oct 31	23:15:07		No	91.121	0.667
42	Vina del Mar	-32.950	-71.550	200	WL	1988 Nov 5	23:19:07		No	91.020	0.566
43	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 5	23:18:33		Bright	90.969	0.515
44	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 6	23:17:04	a	No	90.921	0.467
45	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 8	23:14:46		No	90.971	0.517
46	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 22	22:56:21		Faint	90.945	0.491
47	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 24	22:53:45		No	90.955	0.501
48	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 28	22:48:29		Bright	90.957	0.503
49	Vina del Mar	-32.950	-71.550	200	WL	1989 Mar 30	22:46:03		Faint	90.998	0.544
50	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 3	22:40:36		No	90.943	0.489
51	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 4	22:39:23		Faint	90.956	0.502
52	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 5	22:38:13		No	90.979	0.525
53	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 9	22:33:04		Faint	90.957	0.503
54	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 12	22:29:34		No	90.999	0.545
55	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 16	22:24:32	a	No	90.947	0.493
56	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 17	22:23:28	a	No	90.969	0.515
57	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 18	22:22:40		No	91.042	0.588
58	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 24	22:14:15	a	No	90.688	0.234
59	Vina del Mar	-32.950	-71.550	200	WL	1989 Apr 29	22:10:24		No	90.950	0.496
60	Vina del Mar	-32.950	-71.550	200	WL	1989 Jul 29	22:08:21		Yes	90.927	0.473

TABLE 1 (Continued)

#	SITE	LAT (°)	LONG (°)	A(m)	OBSERVER	DATE	UT	*	GF	Z (°)	Ro (°)
61	Vina del Mar	-32.950	-71.550	200	WL	1989 Jul 30	22:09:10		Yes	90.957	0.503
62	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 3	22:11:46		Bright	90.943	0.489
63	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 4	22:12:40		Yes	90.988	0.534
64	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 5	22:13:23		Long	90.977	0.543
65	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 9	22:16:18		No	91.045	0.591
66	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 12	22:17:51		No	90.953	0.499
67	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 13	22:18:39		Faint	90.980	0.526
68	Vina del Mar	-32.950	-71.550	200	WL	1989 Aug 14	22:19:18		No	90.978	0.524
69	Vina del Mar	-32.950	-71.550	200	WL	1989 Sep 6	22:34:11		No	90.965	0.511
70	Vina del Mar	-32.950	-71.550	200	WL	1989 Sep 12	22:37:56		No	90.948	0.494
71	Vina del Mar	-32.950	-71.550	200	WL	1989 Oct 4	22:52:40		No	90.962	0.508
72	Vina del Mar	-32.950	-71.550	200	WL	1989 Oct 23	23:07:11		Bright	90.948	0.494
73	Vina del Mar	-32.950	-71.550	200	WL	1989 Oct 25	23:08:48		Faint	90.937	0.483
74	Vina del Mar	-32.950	-71.550	200	WL	1989 Oct 28	23:11:22	a	No	90.937	0.483
75	Vina del Mar	-32.950	-71.550	200	WL	1989 Nov 2	23:15:53		Faint	90.958	0.504
76	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 12	23:51:08		No	90.959	0.503
77	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 13	23:51:50		Bright	90.960	0.506
78	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 16	23:53:34		Yes	90.923	0.469
79	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 18	23:54:45	a	No	90.923	0.469
80	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 23	23:57:39		No	90.990	0.536
81	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 24	23:58:05		Faint	90.990	0.536
82	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 25	23:58:25		Faint	90.977	0.523
83	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec26	23:58:51		Faint	90.987	0.533
84	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 27	23:59:09		No	90.978	0.524
85	Vina del Mar	-32.950	-71.550	200	WL	1989 Dec 28	23:59:30		No	90.983	0.529
86	La Serena	-29.900	-71.300	300	BES	1988 Jul 7	22:00:32		No	90.802	0.246
87	La Serena	-29.900	-71.300	300	BES	1988 Jul 8	22:01:20		No	90.867	0.311
88	La Serena	-29.900	-71.300	300	BES	1988 Jul 9	22:02:26		No	90.988	0.432
89	La Serena	-29.900	-71.300	300	BES	1989 Jul 23	22:08:54		2 sec	90.820	0.264
90	Cerro Tololo	-30.165	-70.815	2215	BES	1987 Jun 24	22:02:09		No	92.608	1.098
91	Cerro Tololo	-30.165	-70.815	2215	BES	1988 May 4	22:12:33		No	92.115	0.605
92	Cerro Tololo	-30.165	-70.815	2215	BES	1988 May 9	22:08:33		No	92.062	0.552
93	Cerro Tololo	-30.165	-70.815	2215	BES	1988 Jun 27	22:01:04		2 sec	92.176	0.666
94	Cerro Tololo	-30.165	-70.815	2215	BES	1988 Jun 29	22:02:17		6 sec	92.282	0.772
95	Cerro Tololo	-30.165	-70.815	2215	BES	1988 Jun 30	22:02:32		14 sec	92.258	0.748
96	Cerro Tololo	-30.165	-70.815	2215	BES	1988 Jul 2	22:01:56		No	91.990	0.480
97	Cerro Tololo	-30.165	-70.815	2215	BES	1988 Jul 4	22:03:46		11 sec	92.187	0.677
98	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 7	22:06:49		No	92.547	1.037
99	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 8	22:04:14		No	91.948	0.438
100	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 10	22:06:39		1 sec	92.237	0.727
101	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 11	22:07:08		No	92.235	0.725
102	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 12	22:08:16		No	92.360	0.850
103	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 13	22:12:57		No	93.188	1.678
104	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 14	8:03:28	b	No	92.152	0.642
105	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 14	22:09:01		No	92.308	0.798
106	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 15	22:09:28		No	92.293	0.783
107	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 16	22:10:25		No	92.378	0.868
108	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 17	0:12:31	c	No	91.841	0.331
109	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 17	22:09:52		No	92.163	0.653
110	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 18	0:14:31	c	No	91.884	0.374
111	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 18	11:46:02	d	No	91.975	0.465
112	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 18	22:09:21		3 sec	91.953	0.443
113	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 19	0:16:09	c	No	91.853	0.343
114	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 19	22:11:07	e	No	92.197	0.687
115	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 20	0:18:55	c	No	92.059	0.549
116	Cerro Tololo	-30.165	-70.815	2215	BES	1989 Jul 20	22:11:46		No	92.218	0.708

* Notes on times:
a-Uncertainties in times are from 10 to 30 seconds
b-Bottom limb of moon, uncertainty is 20 seconds
c-Venus
d-Top limb of moon, uncertainty is 20 seconds
e-Sunset occurred after this time (by perhaps 20 seconds)

average uncertainty in a timing is perhaps two or three seconds. Since an error of 2 seconds in the time corresponds to an error of 20 arc seconds (0°.006) for a typical observation, we conclude that the timing errors in our refraction measurements are negligible.

The measured R_0 values range from 0°.234 to 1°.678. The rms deviation observed at the two coastal Chilean sites ranges from 0°.048 to 0°.084. We can think of no reason that explains why these virtually identical sites apparently have a different mean R_0. The other two sites have an rms deviation of a quarter of a degree.

Minnaert (1954, p. 62) suggested that someone should compare the intensity of the green flash with the amount of refraction as deduced from sunset timings. The theory of the green flash (Menat 1980) implies that the visibility is affected by two competing influences. First, as the refraction increases the pathlength traversed by the light increases so that the green light is more absorbed. Second, as the refraction increases the *differential* refraction increases proportionately so that the green light is separated farther from the red light. It is not clear which effect should dominate. The green-flash visibility reported in Table 1 shows no obvious correlations with R_0.

We should note that during the last minute or so before sunset the upper limb of the Sun is frequently tinged with green, and when isolated temperature inversions nibble off pieces of the upper edge the separated strip can be quite intensely green. Similar effects have been described by Minnaert (1954) and others. The long enduring green flashes seen infrequently by us would seem to be extreme cases of this phenomenon where the separated piece remains above the horizon for many seconds.

The impression that one of us received in Vina del Mar is that, if the sky is near perfect, there will be a green flash. Only a very few exceptions were noted. It would seem that, at low elevations, the atmosphere does a more thorough job of separating out the green image from those of shorter and longer wavelengths. Jay Frogel has noted to us that during his several-year tenure in Chile he frequently saw green flashes from coastal La Serena, but rarely from the heights of Cerro Tololo.

2.2 *Sextant*

In 1968 Ken Seidelmann, R. L. Duncombe, Ralph F. Haupt, and W. A. Scott made a series of low-Sun observations of Z_a timed to the nearest second (K. Seidelmann 1989, private communication). Therefore, Z_t and R_0 could be calculated as in the previous section.

The altitude measurements were made using a marine sextant mounted on a tripod. Observations were collected on two mornings from the pier in Nag's Head, North Carolina, while the evening data were taken on two occasions at the west end of Kitty Hawk, North Carolina. The lines of sight were over open water and the altitude was measured with respect to the sea horizon. The data were obtained to look for the effects of irradiation, so the procedure was to alternate altitude measurements of the upper and lower limb.

The marine sextant was read out to a precision of 6″. A 180″ index correction for the sextant was applied to all observations. When observing the Sun with a marine sextant, the brightness of the Sun compared to the sky and the brightness of the sky compared to the sea will apparently raise the altitude of the upper limb of the Sun. This irradiation effect has been estimated to be as large as 60″, but will vary from observer to observer by typically 30″. For a given observer the irradiation will result in a constant offset to all measurements.

The individual measurements of R as a function of Z_a are plotted in Figure 1 and reveal some interesting properties. While the general shape of the curves is similar to the predictions in any standard reference (e.g., the *Astronomical Almanac*), there are large deviations from the standard curve.

The first type of deviation is a random variation about the smoothed curve. That is, there are variations of typically 0°.04 about the curve. In one case the refraction changed by an eighth of a degree between two observations taken 58 seconds apart. It might be possible that this one point is in error, perhaps a reading error, although there is no evidence to support this possibility other than the fact that a large change was recorded. But there are

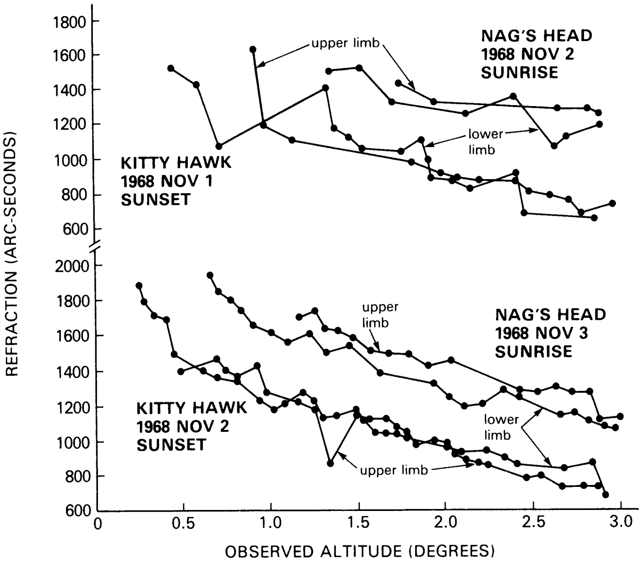

FIG. 1–Seidelmann's refraction data. United States Naval Observatory personnel used a marine sextant to measure the apparent altitude of the upper and lower limb of the Sun for two sunrises and two sunsets. The unrefracted altitude could be calculated since the time of each measurement was recorded; the difference in calculated and apparent altitude is the refraction. The general shape of the curves shows the refraction increasing as the altitude decreases, but there are significant and large variations around the smoothed curve, which ranges as high as 0°.12. The morning and evening curves show a systematic difference of 40%, a value much too high to be explained by standard theory, irradiation, or observational error.

TABLE 2

Seidelmann's Refraction Observations

DATE	SITE	TIME	T(°C)	P(mmHg)	R(2°)	R(1°)	R_0
1968 Nov 1	Kitty Hawk	Sunset	16	30.08	0.25°	0.35°	0.49°
1968 Nov 2	Nag's Head	Sunrise	14	30.02	0.36°	...	0.64°
1968 Nov 2	Kitty Hawk	Sunset	19	29.98	0.27°	0.34°	0.50°
1968 Nov 3	Nag's Head	Sunrise	14	30.02	0.38°	0.47°	0.66°

eleven cases of changes by greater than 0°.06 between adjacent pairs of measurements. It is highly unlikely that all these cases are errors. Hence, we conclude that the refraction of the Sun can vary by typically 0°.04 and up to 0°.12 on a time scale of one minute.

The second type of variation is the offset between the smoothed curves for sunrise and sunset (see Table 2). This difference amounts to a 400″ (roughly 40%) change. This large systematic change cannot be caused by irradiation effects, as these are smaller than roughly 60″. Nor can the morning/evening difference be caused by temperature and pressure effects, since the observed values (see Table 2) lead to variations smaller than 2% according to standard models. In fact, even a 26°.67 C temperature change will result in only a 16% change with the model of Garfinkel (1967). Our preferred explanation is that the low-altitude thermal structure of the atmosphere is different for a sight line over the ocean in the morning when compared to over the sound in the evening.

The refraction as a function of Z_a can be extrapolated reasonably accurately to zero altitude by fitting the data to a standard curve. The corresponding values of R_0 are presented in Table 2.

2.3 *Temperature Profile*

A third method for establishing R_0 is to measure the temperature of the atmosphere as a function of height. This temperature profile can be used to calculate R_0 directly by means of the program given by Schaefer (1989).

To look for variations in R_0, temperature profiles taken on a number of days are needed for the site. We have been able to locate adequate profiles for only one site, O'Neill, Nebraska (Lettau and Davidson 1957). From 1953 August 1 to September 8 the site was continuously monitored during seven several-day observing periods as part of the Great Plains Turbulence Field Program. During each observing period the air temperature was continuously recorded from ground level to a height of 2000 meters.

We have selected from their data all temperature profiles that were taken near times of local sunrise, sunset, and midnight. The temperature profile above 2000 meters was assumed to fall off at the standard lapse rate. This is likely to be a valid assumption since all of the temperature profiles had already steadied down before the altitude had reached 2000 meters. Any deviation from this assumption will only increase the variance of the refraction.

These temperature profiles were then used in the program of Schaefer (1989) to calculate R_0. The program idealizes the atmosphere as thin layers of uniform density, where the density in each layer is determined from the assumed temperature profile. The calculation is a ray tracing of a beam of light as it traverses the atmospheric layers. The program results for the temperature profiles from O'Neill are listed in Table 3.

The R_0 value can vary from 0°.406 to 0°.660. The rms deviation for sunrise, sunset, and midnight are 0°.06, 0°.05, and 0°.05, respectively. For all data from O'Neill the rms deviation is 0°.056.

The program can also be used to examine the variation as a function of altitude above the horizon (that is, 90° −

TABLE 3

Refraction From Temperature Profiles

DATE	TIME	R_0
1953 Aug 8	Sunset	0.463°
1953 Aug 8	Midnight	0.557°
1953 Aug 9	Sunrise	0.609°
1953 Aug 9	Sunset	0.528°
1953 Aug 13	Sunrise	0.507°
1953 Aug 13	Sunset	0.530°
1953 Aug 13	Midnight	0.611°
1953 Aug 18	Sunset	0.561°
1953 Aug 18	Midnight	0.660°
1953 Aug 19	Sunrise	0.635°
1953 Aug 19	Sunset	0.548°
1953 Aug 22	Sunrise	0.629°
1953 Aug 24	Sunset	0.406°
1953 Aug 24	Midnight	0.556°
1953 Aug 25	Sunrise	0.547°
1953 Aug 25	Sunset	0.561°
1953 Aug 31	Sunrise	0.496°
1953 Aug 31	Sunset	0.531°
1953 Aug 31	Midnight	0.565°
1953 Sep 1	Sunrise	0.611°
1953 Sep 7	Sunrise	0.606°
1953 Sep 7	Sunset	0.543°
1953 Sep 7	Midnight	0.530°
1953 Sep 8	Sunrise	0.654°

Z_a). We have calculated the refraction for various cases with and without thermal inversions. The details of the differences will depend on the specific case considered, but in general, the differences caused by thermal inversions are proportional to the total amount of refraction. This means that the variation of refraction at an altitude of 10° ($Z_a = 80°$) will be roughly 15% of the variation on the horizon.

2.4 *Thom's Data*

Thom (1958 and 1971) gives an extensive discussion of refraction near the horizon, based on his 600 theodolite measurements from a site 30 kilometers southwest of Glasgow, Scotland. He repeatedly measured the apparent altitude of various landmarks for which unrefracted altitudes could be determined from Ordnance Survey maps. This difference in apparent and true altitudes is the terrestrial refraction, r in arc seconds, a quantity related to the astronomical refraction relevant for the Sun and Moon. One obvious difference is that the amount of terrestrial refraction will depend on the path length of the ray, but Thom accounts for the varying distance to his landmarks by defining a refraction coefficient,

$$K = r\,T^2/LP \quad , \tag{5}$$

normalized by the path length L (in feet). The temperature T (in degrees Fahrenheit) and pressure P (in inches of mercury) are included so that the variations in these quantities are taken into account.

If the sight lines that Thom measured are typical for the atmosphere, then K will be proportional to the astronomical refraction. Thom's average K value is 7.5, which would correspond to R_0 equal to 0°57. However, the variance of R_0 will not be proportional to the corresponding variance of Thom's K measurements. Each portion of the path length from the observer to outside the atmosphere will have some typical variation. These variations will add up perhaps as a random walk or perhaps coherently, depending on the atmospheric structure. Thus, if only one out of N segments has the variation measured (as Thom did), then the total variation in the astronomical refraction will be between $N^{0.5}$ and N times larger, where N will be roughly the inverse of the fraction of bending occurring from the observer to the landmark. From the program of Schaefer (1989), N will be greater than 4 for Thom's sight lines. Hence, the observed variation in K must at least be doubled so as to get a lower limit on variations in R_0.

Unfortunately, Thom does not give any tabulation of individual observations, nor does he present a histogram showing the variation. Thom repeatedly refers to the great scatter among the observations of even a single landmark, but quotes the variation for only four out of 14 sight lines. When scaled to astronomical refraction, the standard deviation of the variation ranged between 0°05 and 0°09 with a weighted mean of 0°75. The abstract of Thom (1958) quotes an uncertainty of 15% (or 0°09) for an unspecified set of data. As discussed earlier, these variations are likely to be at least doubled when the line of sight extends to outside the atmosphere. Hence, Thom's data give a variation in R_0 of 0°18 or larger.

Thom (1971) reports that K varies from 5 to 13, a factor of 2°6, and that the average K values for different times of the day vary from 6.5 to 8.8, while seasonal effects change K by at least one unit. The variation about these means is not given, but since the daily range of variation is small (in a quadratic sense) compared to the total range of variations, the total range about the hourly means must be comparable to the total range of variation. A variation in K from 5 to 13 corresponds to a range of R_0 from 0°39 to 1°01. When extended from terrestrial to astronomical refraction, the corresponding range will be even larger than 0°62.

A complete analysis of Thom's data is not possible because the individual measurements are not given, but his observed variations are comparable to those obtained earlier in this paper. That is to say, for the conditions of Scotland, the astronomical refraction on the horizon has a one standard deviation of >0°18, while the range is >0°62.

3. Discussion

Figure 2 shows a histogram of all 144 measurements of R_0 for the seven sites. The average and the variation of R_0 for the seven sites is summarized in Table 4. The average R_0 for all observations is 0°551, a value close to that quoted by standard sources for 10° C (0°590 in Allen (1973), 0°569 in Garfinkel (1967), 0°563 in the *Astronomical Almanac*, 0°567 in Smart (1979), and 0°572 in Thom (1971)). We conclude, therefore, that the standard references are reliable guides to refraction *under average conditions*.

However, the variation of R_0 about the average is substantially larger than has been previously realized. The evidence for this conclusion is: The observed values of R_0 range from 0°234 to 1°678, while the overall rms deviation is 0°16. The sunrise/sunset variation for the Outer Banks of North Carolina is 40%, a difference that is over a factor

TABLE 4

Observation Summary

SITE	#OBS	#GF	$\langle R_0 \rangle$	σ_R
Hawaii	8	2	0.545°	0.248°
Vina del Mar	77	35	0.510°	0.048°
La Serena	4	1	0.313°	0.084°
Cerro Tololo	27	6	0.692°	0.275°
Nag's Head	2	...	0.65°	...
Kitty Hawk	2	...	0.50°	...
O'Neill	24	...	0.556°	0.057°
All	144	...	0.551°	0.160°

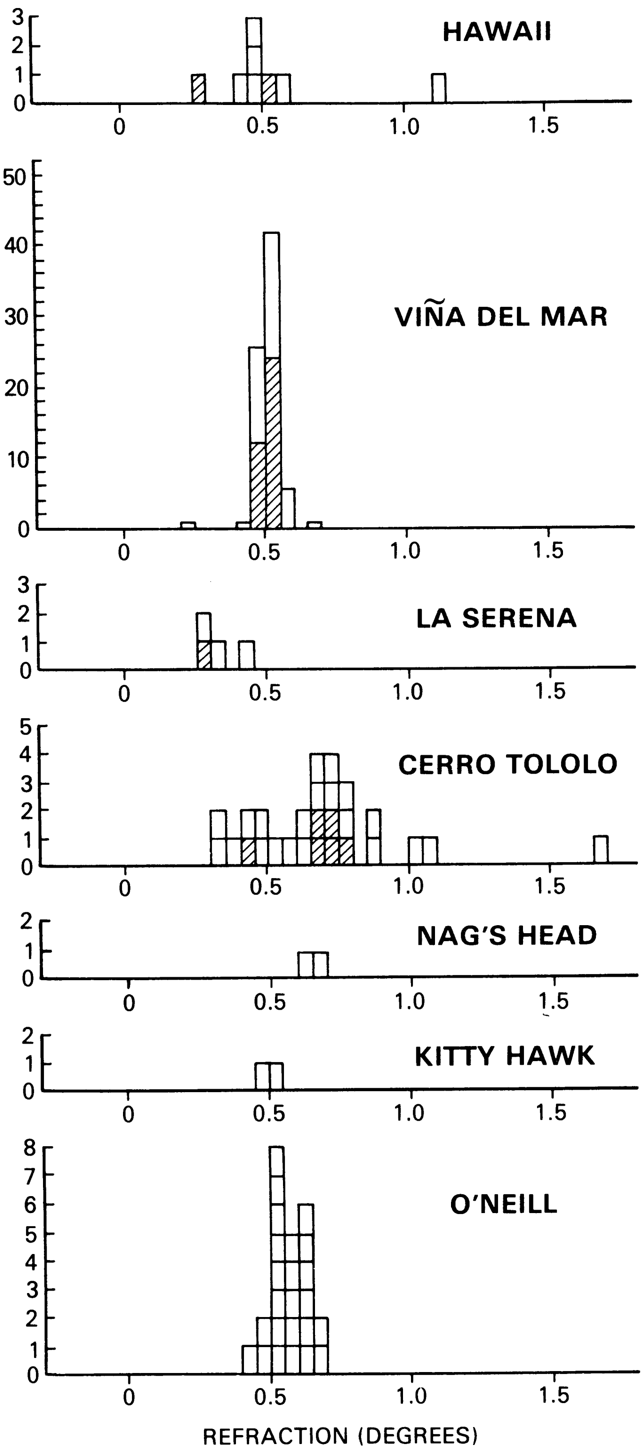

FIG. 2–Histogram of observed refraction. We have collected a total of 144 measurements of R_0 from seven sites using three different techniques. The total range is from 0°234 to 1°678, with an overall rms deviation of 0°16. 97% of our data fall within a range of 0°64 in size. The primary result of this paper is that this observed variation is significantly larger than has been previously realized. The shaded boxes represent sunsets where a green flash was seen.

of 20 times larger than allowed for with standard models. Furthermore, Seidelmann's data show typical variations in refraction from minute to minute of 0°04 even up to several degrees elevation and of 0°12 in 58 seconds at one-degree elevation. Thom's data imply a variation at least from 0°39 to 1°01 with a 1σ scatter of greater than 0°18. We conclude, then, that the day-to-day and minute-to-minute variation of R_0 is more than an order of magnitude larger than the variation adopted by many archaeoastronomers.

We note that, even though the average rms deviation is 0°16, the total expected range of variation will be several times larger. Only 68% of the time will the refraction fall in a range equal to twice the rms deviation. Even for a total range of four times the rms deviation, 5% of the observations should be outside the range. For our data, with rms deviation of 0°16, four out of 144 values are outside the range from 0°23 to 0°87. Thus, we see that a reasonable range for the variation of R_0 for an unknown site is about 0°64.

4. Applications

4.1 *Time of Sunrise*

Modern science has measured the position of the Moon with an accuracy of one meter and the direction to the Sun with an accuracy of approximately 0"01. So it came as a surprise to us that the time of sunrise and moonrise cannot be predicted with an accuracy of several minutes. The variation of refraction provides a fundamental lower limit on the accuracy of sunrise prediction. The uncertainty in the rise time will be

$$\sigma_t = (4 \text{ minutes/degree})\, \sigma_R/(\cos\delta \sin\alpha) \ , \qquad (6)$$

where δ is the declination of the object, σ_R is the uncertainty in the refraction, and α is the angle relative to the horizon of the declination circle of the object. The value for α can be evaluated from

$$\cos\alpha = (\sin\lambda - \sin\delta \cos Z_a)/(\cos\delta \sin Z_a) \ , \qquad (7)$$

where λ is the observer's latitude. For a typical case with $\lambda = 45°$, $\delta = 0°$, $Z_a = 90°$, and a 2σ range of R of 0°64, the uncertainty in the rise time will be 3.6 min.

4.2 *Solar Eclipse*

During a solar eclipse the cone shape of the umbra will be distorted by refraction within the Earth's atmosphere. The size of this refraction effect is significant only for elevations lower than several degrees, but there the minute-to-minute refraction variations will shift the size, shape, and position of the shadow edge in an unknowable manner. This will result in uncertainties in the path width and contact times. A slightly modified version of the program given by Schaefer (1989) suggests typical variations will shift the shadow edge by several kilometers and the contact times by 0.1 sec. Shifts of this size could doom, for example, the attempts to measure the radius of the Sun accurately with the "Solrad 90" experiment du ing the 1990 solar eclipse in Finland. In this experiment, a long row of school children, perhaps extending all the way across the zone of totality, will measure the duration of totality with wristwatches. A 3-kilometer uncertainty in the refraction correction will cause an uncertainty of 3.″2 in the solar diameter.

Fortunately, though, a geometrical peculiarity of all eclipse paths will greatly lessen the impact of refraction uncertainties on the width of the path of totality. This peculiarity is that the azimuth of the edge of the path is close to the azimuth of the rising Sun for locations where the shadow of the Moon first touches the Earth. The unknown effects of refraction will primarily shift the shadow along the line from the observer to the Sun which is nearly parallel with the path edge, so that the position of the edge will not change much. Although this geometric peculiarity will decrease the errors due to refraction, the errors may still be unacceptably large. So a detailed calculation for specific locations in Finland will be needed to determine if the Solrad 90 experiment can achieve the desired accuracy.

4.3 *Archaeoastronomy*

Archaeoastronomers have greatly overestimated the conversion accuracy from azimuth to declination. The uncertainty in declination will be

$$\sigma_\delta = \sigma_R \cos \alpha \ . \tag{8}$$

For a typical case with $\alpha = 45°$ and a range in R of 0.°64, the range in the declination will be 0.°45. This uncertainty in the indicated declination sets a fundamental limit on the accuracy of any alignment. Thus, the paradigm of archaeoastronomy has an unexpectedly poor limit on its accuracy.

In some cases this limit may be small enough to ignore. First, for sites near the equator, α will be near 90° so that σ_δ will be small. Second, if the altitude of the horizon is high enough, σ_R may be sufficiently small so that σ_δ can be ignored. At an altitude of 10° or more above an ideal horizon, the range of R will be 0.°06 or smaller. Third, if the site has few strongly indicated sight lines, then the alignment to sunrise or moonrise may still be statistically significant despite a large σ_δ. This final case applies to Stonehenge, where the avenue is the one strongly indicated sight line.

4.4 *The Great Pyramid*

The Great Pyramid of Cheops is oriented north/south with the astonishing accuracy of roughly 0.°05 (Petrie 1940). This feat involves two separate tasks: First, the ancient Egyptians had to establish a north/south axis to high accuracy; then they had to construct the pyramid to follow the axis. The first task, that of locating north, can only be an astronomical problem.

We can think of only four possible methods for establishing a true north: First, they could have sighted toward a position in the sky that has no diurnal motion. This method is difficult because there was no naked-eye star near to the pole at the time the pyramids were built. Second, they could have observed the shadow of the Sun. Difficulties arise because the disk of the Sun creates a penumbral region at the edge of any shadow so that the resulting uncertainties will be too large. Third, they could directly observe star positions as they are eclipsed by a large gnomon, but this method also has logistical difficulties in achieving the needed accuracy. Finally, the azimuth of some celestial body can be observed both rising and setting with the true north being a bisector of the angle.

The primary difficulty with this final method is to be sure that both the rising and setting observations are made at the same apparent altitude. One method has been proposed by Haack (1984) where the altitude of some star is standardized by the time when it first becomes visible. However, Schaefer (1986) has shown that variations in the atmospheric clarity are much too large, so that this method cannot have the needed accuracy. Nevertheless, these difficulties can be overcome if a bright source (say the Sun) or an artificial horizon are used.

The refraction variations will provide another source of error. The uncertainty in the azimuth will be

$$\sigma_{az} = \sigma_R \cot \alpha \ . \tag{9}$$

For a latitude of 30°, α must be 60° or smaller so that $\sigma_R =$ 0.°16 translates to an azimuthal uncertainty of 0.°37. For a sight line 10° above an ideal horizon, the azimuthal uncertainty is 0.°06. The uncertainty in the true north direction will be 1.41 times σ_{az} because two measurements are involved. Thus, we see that any horizon-based method for orienting the pyramids has too large of an error merely because of variable refraction.

4.5 *Megalithic Lunar Observatories*

Most alignments in the archaeoastronomical literature do not require accuracies better than half a degree, so the results of this paper will have little impact on these claims. However, one of the premier results in archaeoastronomy claims an accuracy of half an arc minute. This result is the assertion by Thom (1971) that many British megalithic sites were used as accurate lunar observatories. He claims that these observatories were employed to measure the nutation of the orbit of the Moon, an effect which can cause a declination shift of 0.°15. He based this claim on a histogram of declination differences with peaks separated by 0.°15. For the histogram peaks to be significantly separated, the range of each peak should be less than 0.°05, a value comparable to the observed peak width. For Thom's sites with $\lambda \approx 55°$, $\delta \approx \pm 23°$, $Z_a \approx 90°$, and a range in R of greater than 0.°62, we find that the range of indicated declination is greater than 0.°55 for an individual observation. However, the daily movement of the Moon in declination is so great that Thom is forced to assume that neolithic man interpolated three individual observations to find the maximum declination for a given lunation. The error in the deduced declination will always be larger than the error for a single observation, so that even in the best case of three consecutive clear nights at the time of maximum declination the deduced declination will have an error 29% larger than that of an individual observation. Hence, the indicated declination will have an error with a range of 0°71. So the actual error of Thom's sight lines is 85 times his claimed error and is 14 times larger than allowable for a valid claim of nutation. As such, the results from this paper clearly refute one of the most historically significant results in archaeoastronomy.

This conclusion could be made invalid if neolithic man were able to average sufficiently large numbers of observations. Thom has already been forced to a similar conclusion by the variation in the horizontal parallax of the Moon. That is, the varying distance to the Moon will cause a shift in the apparent vertical position of the Moon (exactly like with refraction variations) with a range of 0°10. This effect by itself is sufficient to increase the errors in Thom's histogram to the point where the peaks lose their significance. Thom concludes that the neolithic builders had to average their observed azimuths over a long period of time. A similar dodge might be proposed to explain how neolithic man reduced the refraction error from 0°71 to 0°05. To improve the uncertainty by a factor of 14, a total of 14^2 or 196 observations need to be averaged. Observations are only possible once a month for times near the lunar standstill. The Moon is sufficiently near a standstill (within 0°05 of maximum declination) for only a time of nine months out of every 18.6 years. Therefore, the minimum time required to collect 196 observations is 410 years. With additional troubles due to parallax and missed measurements because of clouds, the needed averaging time is likely to be significantly longer than four centuries. Even if such an averaging time were plausible for an illiterate culture, the changing obliquity of the ecliptic (0°04 in four centuries) would wipe out any significant clustering of sight lines. In conclusion, we find that the variation of the refraction correction does indeed constitute a refutation of the megalithic lunar observatory idea.

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