Speed Limits for Quantum Control of Local Spin Systems

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University of California, Berkeley

February 23, 2017

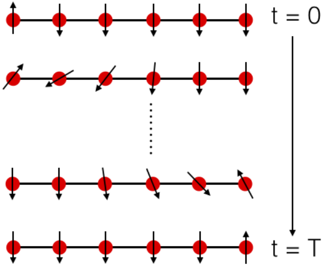
Outline

1. Motivation for bounds on optimal control
2. Brief review of Lieb-Robinson bounds
3. Bounds on state transfer and entanglement generation
4. Comparison to theory and numerics

Numerical upper bound on state transfer speed

[1] Caneva et al. arXiv:0902.4193, Murphy et al. arXiv: 1004.3445

Numerical upper bound on state transfer speed



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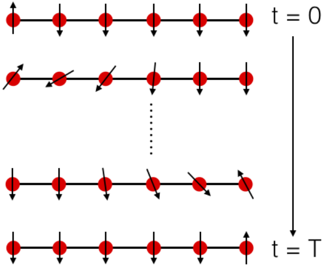

$$H = -\frac{J}{2}\sum_j \vec{\sigma}_j \cdot \vec{\sigma}_{j+1} + h_j(t)\sigma_j^z$$

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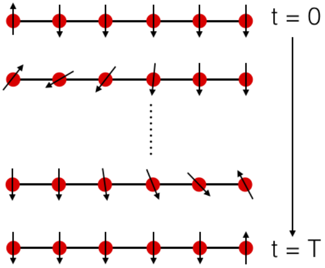

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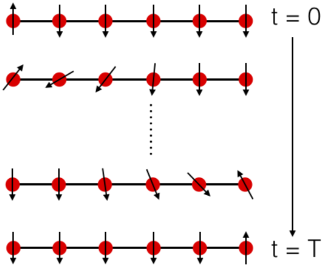

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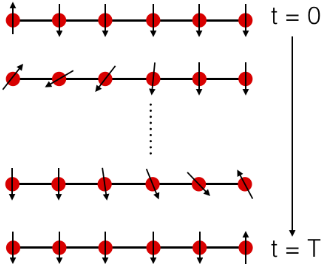

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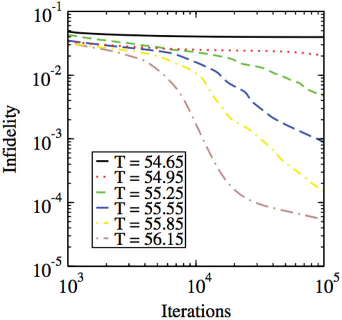

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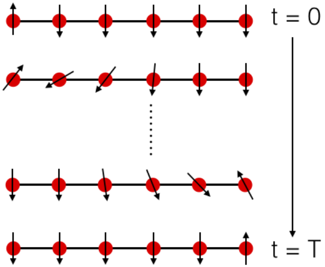

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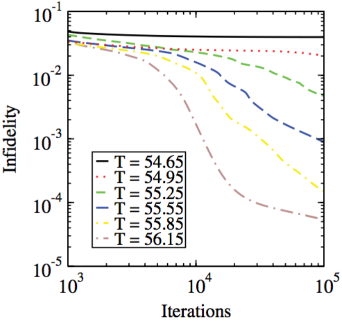

$$T_{\min} \approx R/2J$$

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Quantum Control in High and Low Dimension

$d = \dim(\mathcal{H}) \simeq 1$	$d \gg 1$

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These are different problems that require different techniques!

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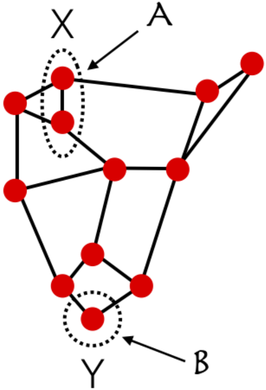
Local Spin Systems and Lieb-Robinson Bounds



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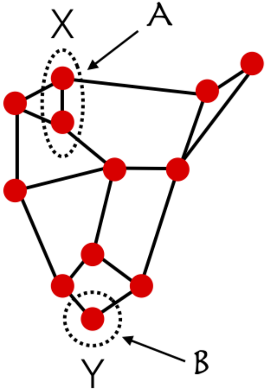

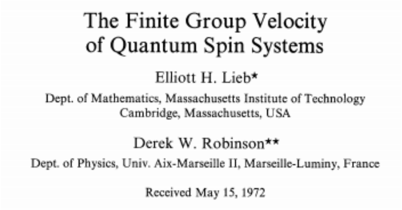
The Finite Group Velocity
of Quantum Spin Systems

Elliott H. Lieb*

Dept. of Mathematics, Massachusetts Institute of Technology
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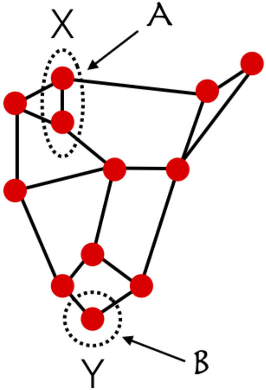

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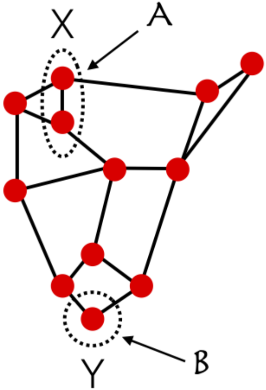

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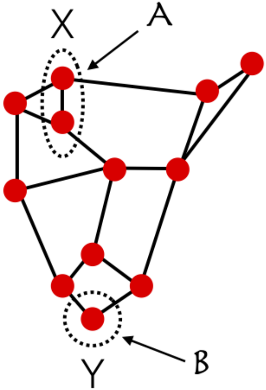

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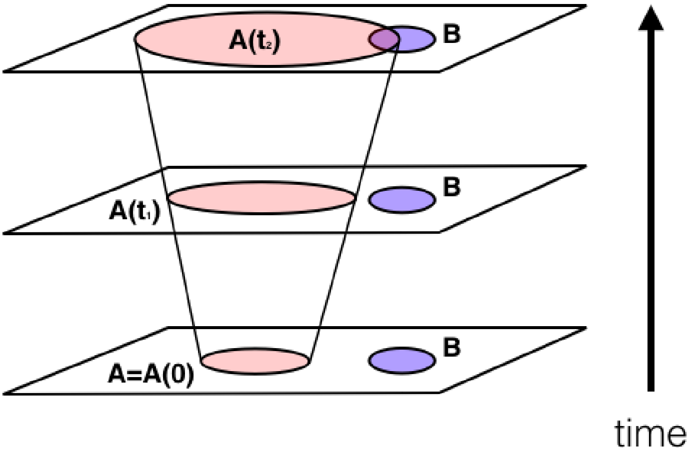
$$c_t(X, Y) \leq \begin{cases} c \min\left(|X|, |Y|\right) e^{-\frac{L - vt}{\xi}} \\ 2 \sum_{n=1}^{\infty} \frac{(2Jt)^n}{n!} N(n) \end{cases}$$

Emergent Lieb-Robinson Lightcone



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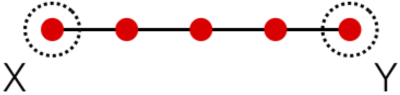

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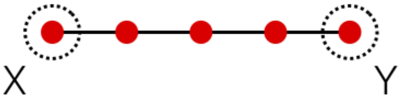

$$c_t(X, Y) \leq 2 I_L(4Jt)$$

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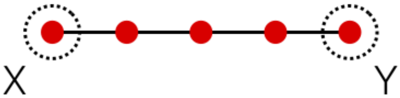

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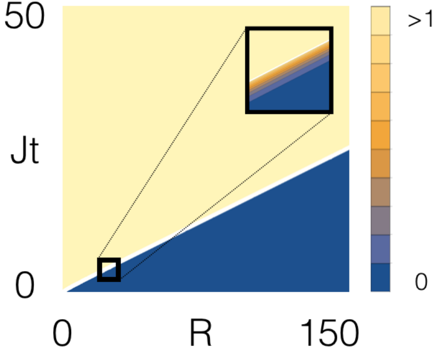

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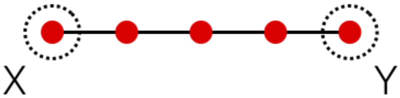

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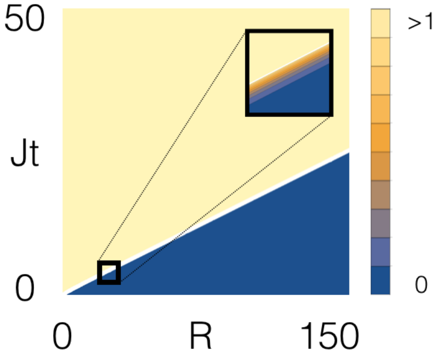

$v_{\mathrm{LR}} \leq 6J$

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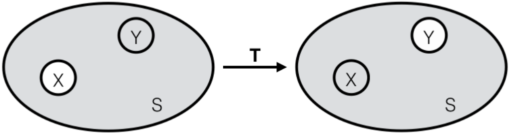

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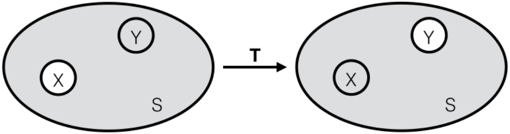

T : unitary accounting for passive dynamics + control

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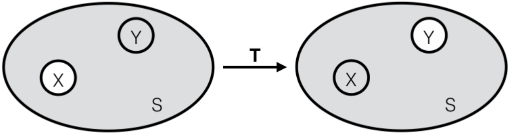

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Goal: $\forall \rho_X \;\; F(\rho_X, \left(T \rho T^\dagger\right)_Y) \approx 1$

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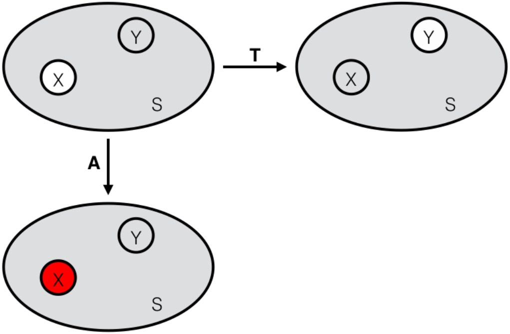

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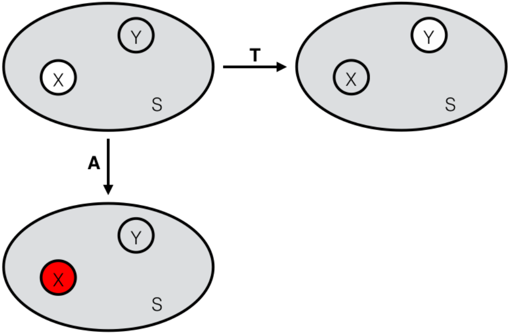

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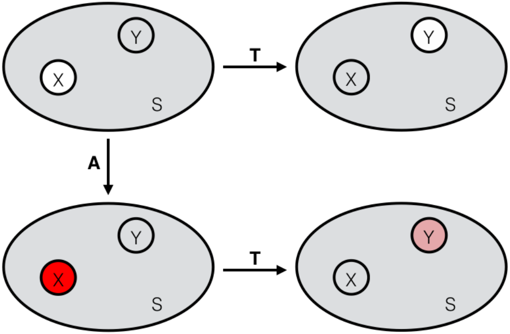

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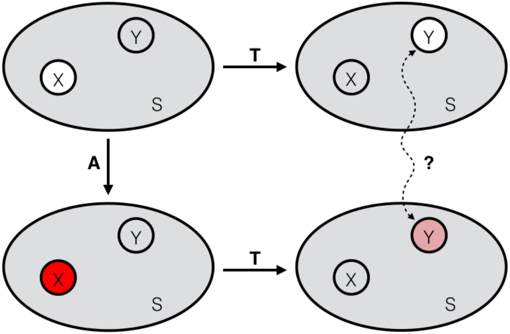

T : unitary accounting for passive dynamics + control
A : bounded operator on X accounting for state preparation

Goal: $\forall \rho_X \;\; F(\rho_X, \left(T\rho T^\dagger\right)_Y) \approx 1$
Necessary condition: $\exists A \;\; F((T\rho T^\dagger)_Y, (TA\rho A^\dagger T^\dagger)_Y) \approx 0$

Algebraic Bound on State Transfer

[2] JME, K. Birgitta Whaley 2016 arXiv:1612.04767

Algebraic Bound on State Transfer

Let X and Y be disjoint subsystems of a system S in the initial state ρ.

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Then the fidelity between the reduced states of subsystem Y given the overall states $T\rho T^\dagger$ and $TA\rho A^\dagger T^\dagger$ satisfies:

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$$F \geq 1 - c_T(X, Y) \|A\| .[2] \tag{2}$$

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Figure of Merit for Entanglement Generation

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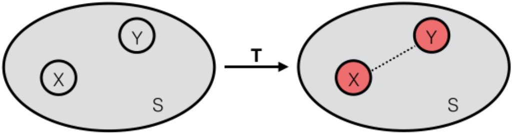

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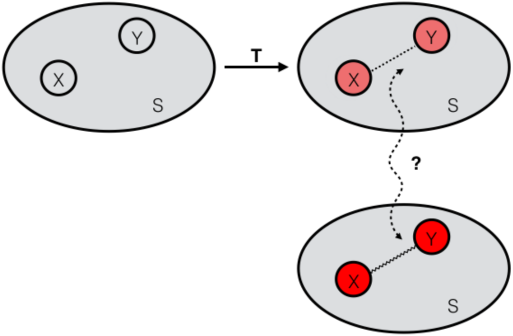

$\max \{F(\rho, \psi) : \psi \text{ maximally entangled}\}$

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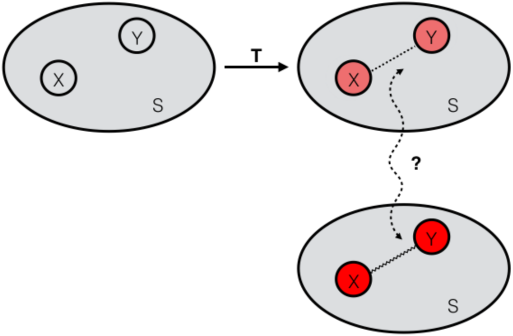

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Intuition:

large entanglement $\longrightarrow$ large correlations

small entanglement $\longleftarrow$ small correlations

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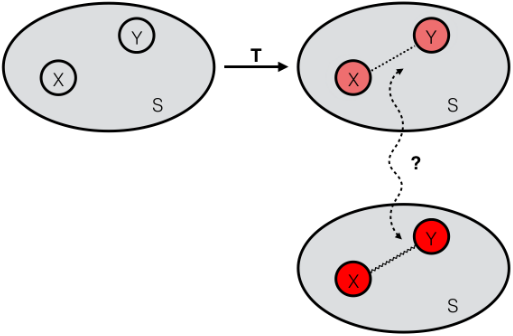

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connected correlator: $\langle AB\rangle_c = \langle AB\rangle - \langle A\rangle\langle B\rangle$

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Then

$$F(\rho, \psi) \leq \sqrt{\frac{79}{81} + \frac{2f}{27} - \frac{f^2}{18}} \tag{3}$$

for any maximally entangled state Ψ.[3]

[3] JME, K. Birgitta Whaley 2016 arXiv:1612.04767

Bound on Correlations

[4] JME, K. Birgitta Whaley 2016 arXiv:1612.04767

Bound on Correlations

Let a system S be initialized in the state ρ with the property that for any disjoint subsystems $X, Y \subset S$ and any A and B with $\|A\|, \|B\| \leq 1$ acting on X and Y, respectively, $|\langle AB_c \rangle| \leq f_0(X, Y)$.

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Then for some unitary T:

$$\begin{aligned} |\langle A_T B_T \rangle_c| \leq\ & f_0(Z, \bar{Z}) \\ & + 2\left((c_T(X, \bar{Z}) + 1)(c_T(Z, Y) + 1)\right) - 1 \end{aligned} \tag{4}$$

for $X \subseteq Z \subseteq S \setminus Y$.[4]

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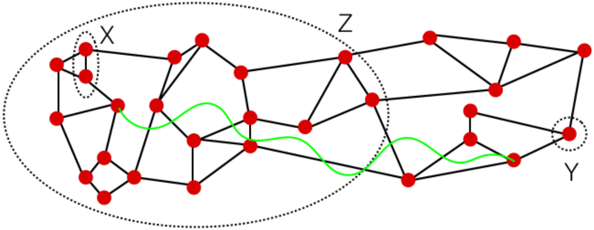

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[5] Calabrese, Cardy 2006 arXiv:0601225

State Transfer v. Entanglement Generation

state transfer:

requires $c_T(X, Y) \gtrsim 0$

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State Transfer v. Entanglement Generation

state transfer:

requires $c_T(X, Y) \gtrsim 0$

entanglement generation:

requires $(c_T(X, \bar{Z}) + 1)(c_T(Z, Y) + 1) - 1 \gtrsim 0$

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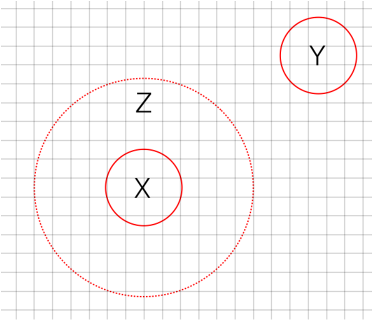
State Transfer v. Entanglement Generation

state transfer:

requires $c_T(X, Y) \gtrsim 0$

entanglement generation:

requires $(c_T(X, \bar{Z}) + 1)(c_T(Z, Y) + 1) - 1 \gtrsim 0$



[5] Calabrese, Cardy 2006 arXiv:0601225

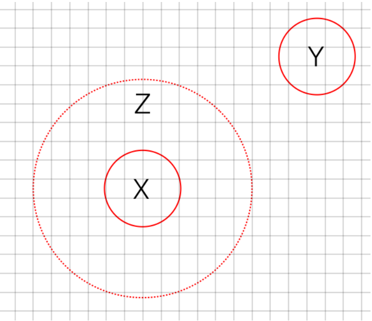
State Transfer v. Entanglement Generation

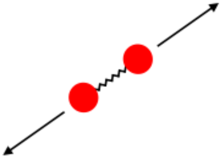
state transfer:

requires $c_T(X, Y) \gtrsim 0$

entanglement generation:

requires $(c_T(X, \bar{Z}) + 1)(c_T(Z, Y) + 1) - 1 \gtrsim 0$





"entanglement carriers"[5]

[5] Calabrese, Cardy 2006 arXiv:0601225

Is the Gap Real?

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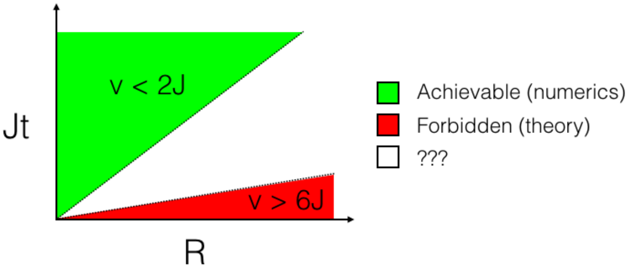
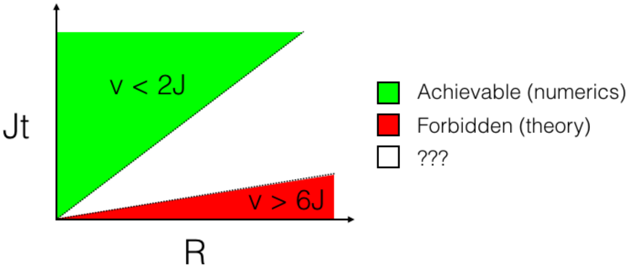
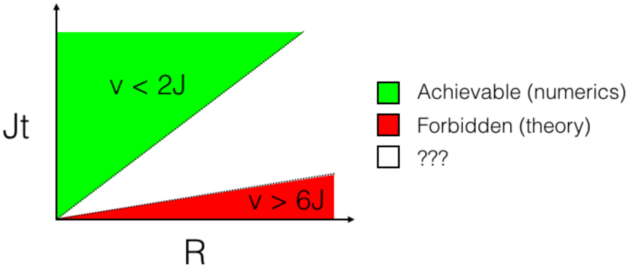


Is the Gap Real?



$2J$: group velocity of 1-excitation Heisenberg chain

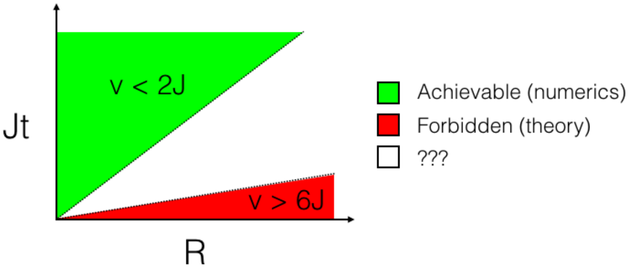
Is the Gap Real?



$2J$: group velocity of 1-excitation Heisenberg chain

Lieb-Robinson velocities can be significantly higher than group velocities

Is the Gap Real?



2J: group velocity of 1-excitation Heisenberg chain

Lieb-Robinson velocities can be significantly higher than group velocities

theoretical bound applies to arbitrary 2-local models

Thank you!

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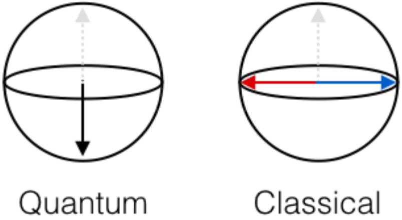
Quantum Speed Limits without Spatial Structure

Orthogonalization/distinguishability time:

$$\theta(\psi(t), \psi) \leq \frac{1}{2}\Delta_{\mathsf{max}} t \longrightarrow t_{\perp} = \pi/\Delta_{\mathsf{max}}$$

Information Transfer:



$$t_*^Q = \pi/\Delta_{\mathsf{max}} \qquad t_*^C = \pi/2\Delta_{\mathsf{max}}$$

Entanglement generation: $t_*^E = \frac{2}{\Delta_{\mathsf{max}}} \arccos d^{-1/2}$

Proof of State Transfer Bound

X-localization of an operator: $[A]_X = \int_{U(\bar{X})} UAU^\dagger d\mu(U)$

small lemma: $\left\| U\rho U^\dagger - V\rho V^\dagger \right\|_1 \leq 2\left\| U - V \right\|$

$$
\begin{aligned}
&\left\| \mathrm{tr}_{S\setminus Y}\left(A_T\rho_T A_T^\dagger - \rho_T\right)\right\|_1 \\
&\quad \leq \left\| \mathrm{tr}_{S\setminus Y}\left(A_T\rho_T A_T^\dagger - [A_T]_{S\setminus Y}\,\rho_T \left[A_T^\dagger\right]_{S\setminus Y}\right)\right\|_1 \\
&\qquad + \left\| \mathrm{tr}_{S\setminus Y}\left([A_T]_{S\setminus Y}\,\rho_T \left[A_T^\dagger\right]_{S\setminus Y} - \rho_T\right)\right\|_1 \\
&\quad \leq \left\| A_T\rho_T A_T^\dagger - [A_T]_{S\setminus Y}\,\rho_T \left[A_T^\dagger\right]_{S\setminus Y}\right\|_1 \leq 2\left\| A_T - [A_T]_{S\setminus Y}\right\| \\
&\quad = 2\left\| A_T - \int UA_T U^\dagger d\mu(U)\right\| = 2\int \left\|[A_T, U]\right\| d\mu(U) \\
&\quad \leq 2c_T(X,Y)\left\|A\right\|
\end{aligned}
$$

Similar to analysis in [6]

[6] Bravyi, Hastings, Verstraete 2006 arXiv:0603121

Proof of Entanglement Bound

Let $|\Psi\rangle$ be maximally entangled, $\Delta = \rho - |\Psi\rangle\langle\Psi|$.

$$\begin{aligned}\langle AB\rangle_c &= \mathrm{Tr}(\rho AB) - \mathrm{Tr}(\rho A)\,\mathrm{Tr}(\rho B) \\ &= \langle\Psi| AB |\Psi\rangle - \langle\Psi| A |\Psi\rangle \langle\Psi| B |\Psi\rangle \\ &\quad + \mathrm{Tr}(\Delta AB) - \mathrm{Tr}(\Delta A)\,\mathrm{Tr}(\Delta B)\,.\end{aligned} \tag{5}$$

Rearranging and taking the modulus:

$$\begin{aligned}&|\langle AB\rangle_c - \langle\Psi| AB |\Psi\rangle + \langle\Psi| A |\Psi\rangle \langle\Psi| B |\Psi\rangle| \\ &\quad\leq |\mathrm{Tr}(\Delta AB)| + |\mathrm{Tr}(\Delta A)|\,|\mathrm{Tr}(\Delta B)| \\ &\quad\leq \|\Delta\|_1 \|A\| \|B\| + \|\Delta\|_1^2 \|A\| \|B\| \\ &\quad\leq \|\Delta\|_1 + \|\Delta\|_1^2 \leq 3\|\Delta\|_1\,.\end{aligned} \tag{6}$$

Reverse triangle inequality:

$$\begin{aligned}3\|\Delta\|_1 &\geq \big||\langle\Psi| AB |\Psi\rangle - \langle\Psi| A |\Psi\rangle \langle\Psi| B |\Psi\rangle| - |\langle AB\rangle_c|\big| \\ &= \Big||\langle AB\rangle_{c,\psi}| - |\langle AB\rangle_c|\Big|\end{aligned} \tag{7}$$

Proof of Entanglement Bound

For any maximally entangled state, there are A' and B' such that $\langle A'B' \rangle_c \geq 2/3$, so that

$$3 \|\Delta\|_1 \geq \left| \frac{2}{3} - \left| \langle A'B' \rangle_c \right| \right|. \tag{8}$$

Given $|\langle A'B' \rangle_c| \leq f$ (assuming w.l.o.g. $f < 2/3$)

$$3 \|\Delta\|_1 \geq \frac{2}{3} - f \tag{9}$$

$$F \leq \sqrt{1 - \frac{1}{2} \|\Delta\|_1^2} \leq \sqrt{\frac{79}{81} + \frac{2f}{27} - \frac{f^2}{18}} \tag{10}$$