AER210

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Contents

1 Dimension Analysis

The most important idea is the **Buckingham Pi Theorem** defined as:

Number of π terms = (# of variables) - (minimum required # of primary dimensions)

Primary dimension is defined as the dimensions not formed from a combination of other dimensions, and secondary dimension is the combination of other primary dimensions.

What this says is that you can compile a function of F = func(A,B,C,D) into something simpler based on the dimensions: The step goes as below:

1. List all the variables
2. Express each variable in terms of their primary terms
3. Determine the required number of pi term
4. Form a pi term by multiplying of the non-selected variables by the product of the selected variables(The # of selected variables is determined by the number of primary terms).
5. Check if all pi terms are dimensionless.
6. Express a relationship among the pi terms

2 Fluid Mechanics

Definition: Fluid: A substance that deforms continuously under the application of a tangential(shear) force. No matter how small this tangential force may be.

Some key points we need to note for this course:

- A fluid at rest is at zero shear force state
- Assume No slip boundary condition
- use Continuum assumption

The last point is worth some discussion. By continuum assumption it is when the macroscopic length scale is much larger than the microscopic ones, and we need to verify the Knudsen number defined by:

$$Kn = \frac{\text{Microscopic length scale}}{\text{Macroscopic Length Scale}} << 1$$

Furthermore, to validate the assumption of continuum of fluid, the density goes as:

$$\rho = \lim_{\delta V -> \epsilon} \frac{\delta m}{\delta V}$$

where ϵ is a small section in the density diagram where the density is a constant.

2.1 Viscosity

Viscosity is the measure of fluid's internal resistance to deformation under shear stress.

The equation for shear stress in Newtonian Fluids is defined as:

$$\tau = \mu \frac{d\alpha}{dt} = \mu \frac{du}{dy}$$

u is the velocity of the flow and alpha and α is the rate of angular deformation of the element.

viscosity is dependent on various factors:

- Deformation (ex. Shear thinning/thickening fluid etc.)
- Temperature (ex. viscosity decreases as temperature increases)

a special form of viscosity is kinematic viscosity defined as:

$$\nu = \frac{\mu}{\rho}$$

2.2 Compressibility

The compressibility is defined by bulk modulus defined as:

$$E_v = -\frac{dP}{dV/V} = -\frac{dP}{d\rho/\rho}$$

dP is the differential change in pressure over a fractional change in volume.

3 Hydrostatics

the definition of hydrostatics is that:

> **Definition: Hydrostatics:**
> Study of fluids at rest or fluids undergoing a rigid body motion

hydrostatics have the following assumptions:

- Fluid has zero shear stress since no relative movement between adjacent particles.
- Pressure is only dependent on the relative height of the fluid.

3.1 Pressure

Consider the case where a gas act on a metal surface. The pressure force is going into the metal surface opposite to the normal of the surface. Therefore, for a pressure acted on a surface, the pressure equation is given by:

$$d\overrightarrow{F}_n = -PdA\hat{n}$$

and surprisingly, the pressure at a point in the fluid is independent of the direction, or isotropic! which is known as the **Pascal's Law**

3.1.1 Basic Equation for Pressure Field

Consider an infinitesimally small cube in fluid with a certain pressure field denoted by:

$$P(x_0, y_0 - \frac{\delta y}{2}, z_0) = P(x_0, y_0, z_0) + \frac{\partial P}{\partial y}(-\frac{\delta y}{2})$$

The sum of all the forces on all surfaces of the cube yield the net surface force:

$$F_{s,net} = -(\frac{\partial P}{\partial x}\hat{i} + \frac{\partial P}{\partial y}\hat{j} + \frac{\partial P}{\partial z}\hat{k})\delta x\delta y\delta z$$

The net body force, which is only the gravitation force is simply straightforward:

$$F_{b,net} = -\rho g\delta x\delta y\delta z\hat{k}$$

Eventually summing up the surface and the body force, the resulting net force of the fluid is:

$$-\overrightarrow{\triangledown}P - \rho\overrightarrow{g} = \rho\overrightarrow{a}$$

This equation is the hydrostatics equations for fluid under no shearing stresses.

3.1.2 Pressure Variation at Rest

For a fluid at rest a = 0, the hydrostatics equation reduces to:

$$\frac{dP}{dz} = -\rho g$$

which shows that the pressure is only dependent on the vertical position(remember the previous section about how pressure is only relative height dependent)

3.1.3 Incompressible Fluids at Rest

For an incompressible fluids, ρg is a constant, so the equation in previous section can be integrated and expressed by:

$$P_1 - P_2 = \rho g(z_2 - z_1)$$

3.1.4 Compressible Fluids at Rest

Compressible fluid means density is no longer a constant. However, it is hard to model the density in general. Therefore, an approach for variation in heights are large is to consider the equation of ideal gas:

$$P = \rho RT$$

subbing this density term into the original equation we can obtain the relationship of pressure:

$$P_2 = P_1 e^{[-g\frac{z_2 - z_1}{RT_0}]}$$

3.1.5 Measurement Instrument

There are a few instrument to note

- Barometer: One vertical tube
- Piezometer: one turn vertical tube
- Manometer: U-Tube

They are all governed by the hydrostatic equation so don't worry about the calculation that much lol.

However here is a tip to follow:

- Any two points at the same height of the same liquid have the same pressure.
- Pressure increases as one goes down, decreases otherwise.
- Neglect the pressure changes due to the columns of gas.

Part 2

3.2 Hydrostatic Forces on submerged Surfaces

For any item submerged by fluid, the resultant pressure force can be obtained by:

$$\overrightarrow{F_R} = \int\int_{Area} \overrightarrow{dF}$$

3.3 Hydrostatic Forces on Planes

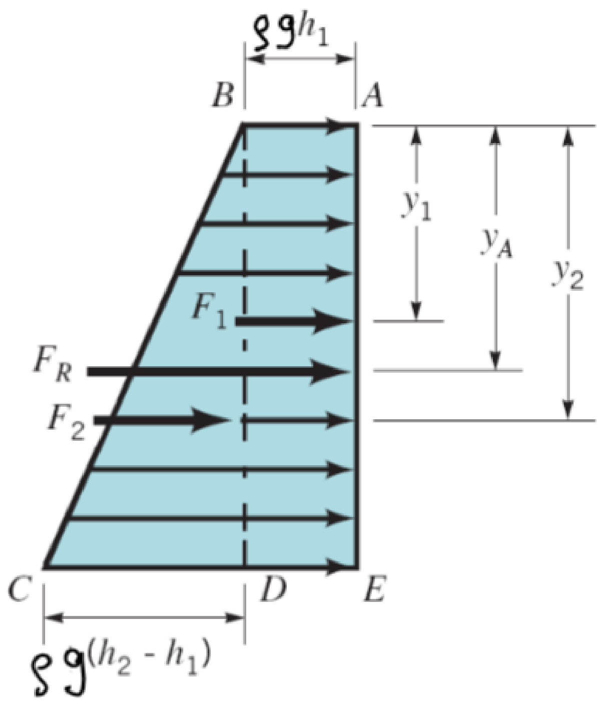
There are two ways to account for hydrostatic forces on plane surface. The first method is by integration and the second method is by finding the average of the force and centroid.

3.3.1 Method 1

as described above in previous section, the force can be accounted by a double integration, where we use the difference in depth as the independent variable.

3.3.2 Method 2

Recalling the dam example, the average pressure can be calculated as $\frac{\rho g h}{2}$ and the force can be estimated by the volume of pressure prism $\frac{\rho g h}{2} * A$ and when calculating the moment caused by such force, we assume this force passes through the centroid of the object.
Even when the prism is no longer a regular shape, the same approach can still be implemented



y_1, y_2 are the respective centroid of the two geometries BCD and ABDE.

$$F_r = F_{BCD} + F_{ABCE}$$

$$F_r y_A = F_{BCD} y_1 + F_{ABCE} y_2$$

where $y_1 = \frac{h_2 - h_1}{2}, y_2 = \frac{2(h_2 - h_1)}{3}$

3.4 Hydrostatics on Curved Surface

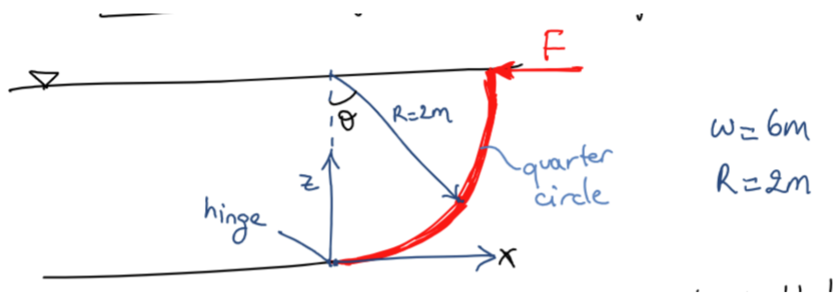
To account for hydrostatic pressure on a curved surface, there are 3 methods we can use

3.4.1 Method 1: Integration

For this case we can approximate:

$$dM = x\,dF_z + z\,dF_x$$

$$M = \int (x\,dF_z + z\,dF_x)$$

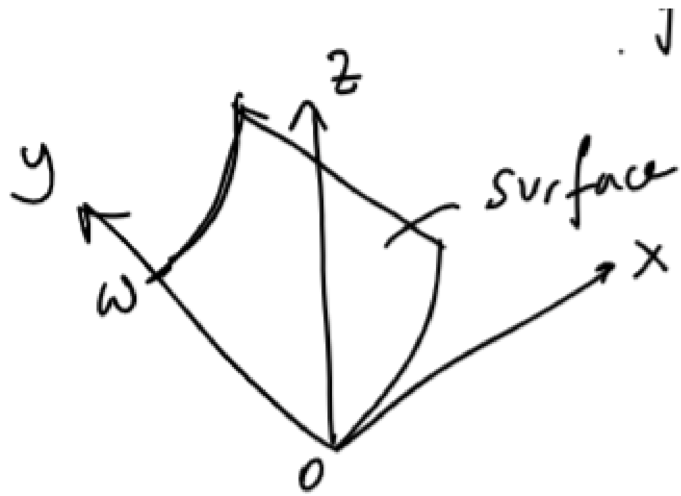


$$dF = p\,dA = \rho\,\omega\,ds$$

$$\Rightarrow \begin{cases} dF_x = \rho\omega\,\sin\theta\,ds, = \rho\,\omega\,\sin\theta\,Rd\theta \\ dF_z = -\rho\,\omega\,\cos\theta\,ds = -\rho\,\omega\,\cos\theta\,Rd\theta \end{cases}$$

3.4.2 Method 2: Parametrization

an alternative method is by parametrization which is necessary if the surface is 3 dimensional varying surface.



For this case we can first parametrize the equation:

$$\begin{aligned} &x = R\sin\theta, \\ &z = R - R\cos\theta, \quad \text{where} \quad 0 \le \theta \le \frac{\pi}{2}, \quad 0 \le y \le \omega \\ &y = y \end{aligned}$$

The vector equation for the surface is:

$$\boldsymbol{r}(\theta, y) = R\sin\theta\,\boldsymbol{i} + y\,\boldsymbol{j} + (R - R\cos\theta)\,\boldsymbol{k}$$

$$\boldsymbol{r}_\theta = R\cos\theta\,\boldsymbol{i} + R\sin\theta\,\boldsymbol{k}$$

$$\boldsymbol{r}_y = \boldsymbol{j}$$

$$d\boldsymbol{F}_p = -p\,\boldsymbol{n}\,dS \quad \text{where} \quad dS \text{ is the area element.}$$

To find a normal vector, we need $\boldsymbol{r}_\theta \times \boldsymbol{r}_y$:

$$\boldsymbol{r}_\theta \times \boldsymbol{r}_y = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ R\cos\theta & 0 & R\sin\theta \\ 0 & 1 & 0 \end{vmatrix} = -R\sin\theta\,\boldsymbol{i} + R\cos\theta\,\boldsymbol{k}$$

Since $0 \leq \theta \leq \dfrac{\pi}{2} \implies \cos\theta > 0$, so:

$$\boldsymbol{n} = \frac{\boldsymbol{r}_\theta \times \boldsymbol{r}_y}{\|\boldsymbol{r}_\theta \times \boldsymbol{r}_y\|}$$

$\boldsymbol{n}$ is in the direction of -d$\boldsymbol{F}_p$.

$$\begin{aligned} d\boldsymbol{F}_p &= -p\,\boldsymbol{n}\,dS \\ &= -p\frac{\boldsymbol{r}_\theta \times \boldsymbol{r}_y}{\|\boldsymbol{r}_\theta \times \boldsymbol{r}_y\|}\|\boldsymbol{r}_\theta \times \boldsymbol{r}_y\| d\theta\,dy \\ &= -p(\boldsymbol{r}_\theta \times \boldsymbol{r}_y) d\theta\,dy \\ &= -p\left[-R\sin\theta\,\boldsymbol{i} + R\cos\theta\,\boldsymbol{k}\right] d\theta\,dy \\ d\boldsymbol{F}_p &= (pR\sin\theta\,\boldsymbol{i} - pR\cos\theta\,\boldsymbol{k})\,d\theta\,dy \\ dF_x = pR\sin\theta\,d\theta\,dy, &\quad dF_z = -pR\cos\theta\,d\theta\,dy \end{aligned}$$

The moment is given by:

$$M_{\text{opening}} = \iint (x\,|dF_z| + z\,dF_x)$$

$$= \int_0^{\pi/2}\int_0^{\omega} [R\sin\theta(pR\cos\theta)]\,dy\,d\theta + \int_0^{\pi/2}\int_0^{\omega} [(R - R\cos\theta)(pR\sin\theta)]\,dy\,d\theta$$

$$= \int_0^{\pi/2} R^2\sin\theta\cos\theta\,\rho\,\omega\,d\theta + \int_0^{\pi/2}(R - R\cos\theta)\rho\,\omega R\sin\theta\,d\theta$$

which is exactly the same as before.

3.5 FLuid in Rigid Body Motion

3.5.1 Linear Rigid Body Motion

Consider a container carrying a fluid traveling in a constantly accelerating vehicle.

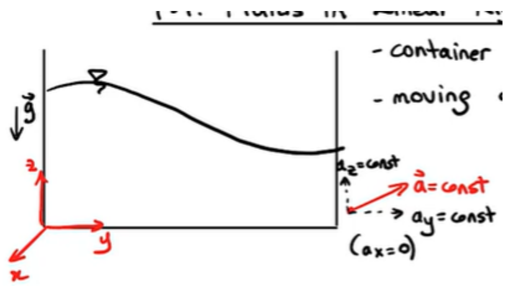
and we would like to find $z_{surface}$ THe equations are

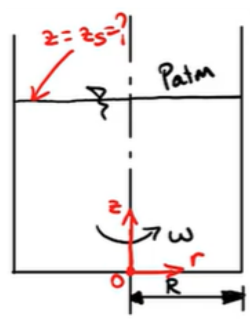
$$z_s = C_1 - \frac{a_y}{g + a_z}y \tag{1}$$

and

$$C_1 = H + \frac{a_y}{g + a_z}L$$

H is the height of the liquid at rest and L is the length of the container.





3.5.2 Rotational Rigid Body Motion

the equation for the surface is characterized by

$$z_s = \frac{\omega^2}{2g} r^2 + C_1 \tag{2}$$

where C_1 is $H - \dfrac{\omega^2 R^2}{4g}$

4 FLowing Fluids

4.1 Conservation laws Mass & Energy

The volume flow rate of an area A can be defined by

$$\iint \overrightarrow{V} \cdot \overrightarrow{n} dA$$

this quantity is conserved under incompressible situation.

4.2 Conservation of Energy

Some assumptions of Bernoulli equation and Euler's equation are that the flow is

- Steady FLow(time-independence)
- Incompressible Fluid
- Frictional Losses Negligible

The euler's equation reads as below:

$$VdV + gdz + \frac{1}{\rho}dp = 0$$

however this is valid for compressible flow along a streamline that is not viscous. In the case of streamline is in a circular streamline, the equation goes to:

$$\frac{dP}{\rho} + \frac{V^2}{R}dn + gdz = 0$$

for a straight streamlines, R goes to infinity and the equation becomes:

$$\frac{dP}{\rho} + gdz = 0$$

integrating this from $z_1 \text{to} z_2$ the final outcomes is the equation:

$$P_1 - P_2 = \rho g(z_2 - z_1)$$

4.3 Momentum Equation

Control volume (eulerian concept) a control volume is an amount of volume passing through a control surface.
On the contrary the system approach is to focus on a set of particles with constant mass such that the object moves with the flow.
Reynolds Transport Theorem is an approach to convert equation in the system approach into the control volume approach

The essence of RTT is the equation

$$B = mb$$

b is the parameter per unit mass and B is parameter proportional with the mass.

$$B_{\text{sys}} = \int_{\text{sys}} b\,\rho\,dV \quad \Rightarrow \quad \frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{sys}} b\,\rho\,dV$$

$$B_{\text{cv}} = \int_{\text{cv}} b\,\rho\,dV \quad \Rightarrow \quad \frac{dB_{\text{cv}}}{dt} = \frac{d}{dt}\int_{\text{cv}} b\,\rho\,dV$$

yet, the only important part of the RTT is the equation:

$$\frac{dB_{\text{sys}}}{dt} = \frac{dB_{\text{cv}}}{dt} + \dot{B}_{\text{out}} - \dot{B}_{\text{in}}$$

- $\dfrac{dB_{\text{sys}}}{dt}$: Time rate of change of B for a system.
- $\dfrac{dB_{\text{cv}}}{dt}$: Time rate of change of B inside the control volume (CV).
- $\dot{B}_{\text{out}}$: The rate at which B goes out of the CV.
- $\dot{B}_{\text{in}}$: The rate at which B goes into the CV.

A more general form to the RTT is that:

$$\text{Lagrangian:} \quad \frac{dB_{\text{sys}}}{dt} = \frac{dB_{\text{cv}}}{dt} + \iint_{\text{cs}} \rho\, b\, \boldsymbol{v}\cdot d\boldsymbol{A}$$

$$\sum \boldsymbol{F}_{\text{cv}} = \frac{d}{dt}\iiint_{\text{cv}} \boldsymbol{v}\,(\rho\,dV) + \iint_{\text{cs}} \boldsymbol{v}\,(\rho\,\boldsymbol{v}\cdot d\boldsymbol{A})$$

speical cases for

- Steady flow: rate of change of momentum inside is 0: $F = \iint_{\text{cs}} \boldsymbol{v}\,(\rho\,\boldsymbol{v}\cdot d\boldsymbol{A}$
- Steady, 1 dimensional:$\sum \boldsymbol{F}_{\text{cv}} = \sum \dot{m}_{\text{out}}\boldsymbol{v}_{\text{out}} - \sum \dot{m}_{\text{in}}\boldsymbol{v}_{\text{in}}$
- steady, 1 dimensional, 1 inlet 1 outlet: $\sum \boldsymbol{F}_{\text{cv}} = \dot{m}(\boldsymbol{v}_{\text{out}} - \boldsymbol{v}_{\text{in}})$

5 General Form of Continuity Equation

5.1 Substantial Derivative

The substantial derivative is so called as material or total derivative of a quantity in a system. In this case we have the substantial derivative for density of a controlled mass:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} \tag{3}$$

where u v and w are the velocities along x,y, and z directions.

The last three terms are the Convective Derivative, defined as the time rate of change due to the movement of the fluid element to where the flow properties are spatially different. The first time is the local derivative, which is the time rate of change at a fixed point due to unsteady fluctuations of the property(ρ in this case). Alternatively, the equation can be written in the following form:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + (\mathbf{V} \cdot \nabla \rho) \tag{4}$$

5.2 Divergence of Velocity

Physically the meaning of divergence of velocity is the time rate of change of a infinitesimal moving volume per unit volume.

$$\nabla \cdot \mathbf{V} = \frac{1}{\delta\Omega}\frac{D(\delta\Omega)}{Dt} \tag{5}$$

and for incompressible flow, you can see that the rate of change of volume should be 0 and therefore, the divergence of velocity is 0 as well. This also means that u,v,w are dependent on each other such that the sum of their rate of change goes to 0.

5.3 Finite Control Volume

This is the first fluid model following the central principle of:

- Mass can be neither created nor destroyed.
- Net mass flow rate out of the CV is equivalent to the time rate of decrease of mass inside the system.

$$\frac{\partial}{\partial t}\iiint_{\Omega} \rho \, d\Omega + \iint_{S} \rho \mathbf{V} \cdot d\mathbf{S} = 0 \tag{6}$$

5.4 Finite System of Constant Mass

This method focuses on a chunk of mass flowing with the flow. This model follows a few principles other than the mass cannot be destroyed principle:

- Mass inside of the system is constant

$$\frac{D \text{ mass}}{Dt} = 0$$

5.5 Infinitesimal Control Volume

Similar to a finite control volume system, but this time the control volume is infinitesimally small such that we can express a more general differential form. The net mass flow rate out of Control volume is then:

$$\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] dx\, dy\, dz \tag{7}$$

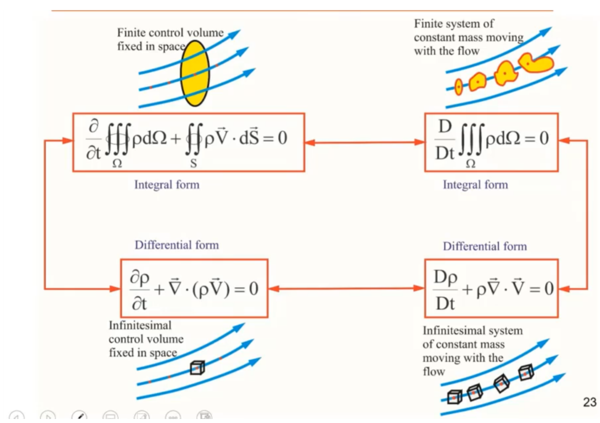
and the most relationship is the conservation of mass flow rate in the system:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{8}$$

5.6 Infinitesimal System of Fluid

This is a Lagrangian interpretation of the control mass system, where the rate of change of mass in the system is 0.

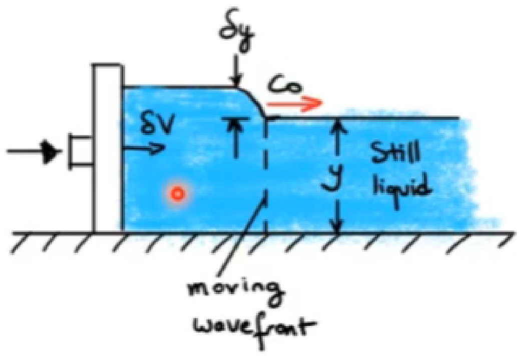
$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{V} = 0 \tag{9}$$



all four equations are equivalent.
For a steady flow condition, the properties in the system is no longer time dependent. Therefore, all the models just remove the time dependent part. However, the total derivative of a function is not removable since it is also depending on other spatial terms.
For Incompressible situation, the density is simply a constant.

6 Open Channel Flows

It is a study of how surface wave behave under disturbance. To solve this problem we have to set



a control volume that is moving with the wave front. So from the perspective, the flow is coming into the system with c_0

Here are some assumptions:

- Uniform flow across any cross-section
- Neglect Friction at the bottom surface of the CV and the air drag at the top.

$$c_0\delta y = \delta V(y+\delta y) \implies \delta V = c_0\frac{\delta y}{y+\delta y} \tag{1}$$

$$\text{Momentum equation:} \quad \sum F_{\text{CV}} = \dot{m}_{\text{out}}\mathbf{V}_{\text{out}} - \dot{m}_{\text{in}}\mathbf{V}_{\text{in}} \quad \text{(steady flow)} \tag{10}$$

$$\text{In } x\text{-direction:} \quad F_2 - F_1 = \dot{m}(-V_2) - \dot{m}(-V_1) \tag{11}$$

$$\frac{1}{2}\rho g(y+\delta y)^2 b - \frac{1}{2}\rho g y^2 b = \rho c_0 y b(-c_0\delta V) - \rho c_0 y b(-c_0) \tag{12}$$

$$\frac{1}{2}\rho g b(y^2 + 2y\delta y + \delta y^2 - y^2) = \rho c_0 y b \delta V \tag{13}$$

$$g\left(1+\frac{\delta y}{2y}\right)\delta y = c_0\delta V \tag{2}$$

and for very small surface wave $\delta y << y$ so the speed of the wave can be expressed by:

$$c_0 = \sqrt{gy}$$

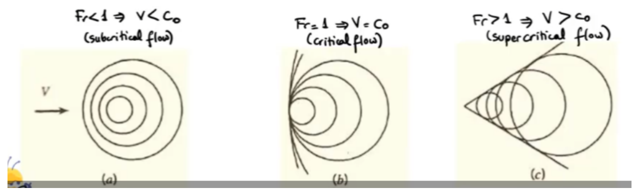
6.1 Froude Number

there is also a very important dimensionless value called froude value.

$$F_r = \frac{V}{\sqrt{gy}}$$

- The Froude number governs the character of the flow in open channels:

$$\begin{aligned} \text{Fr} < 1 &\implies \text{subcritical flow} \\ \text{Fr} = 1 &\implies \text{critical flow} \\ \text{Fr} > 1 &\implies \text{supercritical flow} \end{aligned}$$


Fr<1 ⇒ V< c_0
(subcritical flow)
V
(a)
Fr= 1 ⇒ V = c_0
(critical flow)
(b)
Fr>1 ⇒ V > c_0
(supercritical flow)
(c)

7 Compressible Flow

The speed of sound is governed by the equation

$$c^2 = (\frac{\partial P}{\partial \rho})_s$$

For isentropic flow of ideal gas, the velocity can be simply written as:

$$c = \sqrt{\gamma RT}$$

More generally for fluid, we can find the speed of sound by bulk modulus.

$$E_v = \rho \frac{dP}{d\rho}$$

such that

$$c = \sqrt{\frac{E_v}{\rho}} \text{ For any fluid}$$

7.1 Mach Number

The mach number is an important metric for compressible flow.

$$M = \frac{V}{c}$$

where V is the velocity of the flow and c is the speed of sound.

1. $\mathrm{M}_\infty \leq 0.3 \implies$ flow is incompressible
2. $\mathrm{M}_\infty > 0.3 \implies$ flow is compressible

Also it can be used to determine something else

$$\begin{aligned} \mathrm{M}_\infty < 1 &\implies \text{Subsonic flow} \\ \mathrm{M}_\infty = 1 &\implies \text{Sonic flow} \\ \mathrm{M}_\infty > 1 &\implies \text{Supersonic flow} \end{aligned}$$

Also

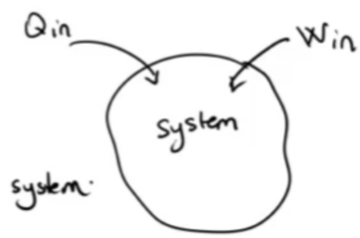
$$\begin{aligned} 0.8 \leq \mathrm{M}_\infty \leq 1 &\implies \text{Transonic flow} \\ \mathrm{M}_\infty \geq 5 &\implies \text{Hypersonic flow} \end{aligned}$$

7.2 Steady Isentropic Flow of 1-dimensional compressible flow

Continuity Equation

under steady state, the mass flow rate should be a constant.

Energy Equations



which is governed by the eequation:

$$\frac{d\left(E_{\text{TOT}}\right)_{\text{sys}}}{dt} = \dot{Q}_{\text{in}} + \dot{W}_{\text{in}} \tag{14}$$

where Q_{in} is 0 under adiabatic flows. The work is also only from the pressure differences.

$$\dot{W}_{pressure} = P_1 A_1 V_1 - P_2 A_2 V_2$$

$$\begin{aligned}
\frac{d\left(E_{\text{TOT}}\right)_{\text{sys}}}{dt} &= \frac{d\left(E_{\text{TOT}}\right)_{\text{cv}}}{dt} + \left(\dot{E}_{\text{TOT}}\right)_{\text{out}} - \left(\dot{E}_{\text{TOT}}\right)_{\text{in}} \\
\frac{d\left(E_{\text{TOT}}\right)_{\text{sys}}}{dt} &= \dot{m}\left(e_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(e_1 + \frac{V_1^2}{2} + gz_1\right) \\
P_1 A_1 V_1 - P_2 A_2 V_2 &= \dot{m}\left(e_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(e_1 + \frac{V_1^2}{2} + gz_1\right)
\end{aligned}$$

and we have the compressible Bernoulli's Equation:

$$\frac{P_2}{\rho_2} + e_2 + \frac{V_2^2}{2} + gz_2 = \frac{P_1}{\rho_1} + e_1 + \frac{V_1^2}{2} + gz_1 \tag{15}$$

e_2, e_1 are the internal energies at both states. We can rewrite this into

$$h_2 + \frac{v_2^2}{2} + gz_2 = h_1 + \frac{v_1^2}{2} + gz_1$$

For high speed flows, the potential energy is negligible. And for cases where the velocity decreases adiabetically, the excess kinetic energy would be converted into enthalpy and thus raising the pressure and temperature.

7.3 Stagnation Properties

Properties of a fluid at the stagnation state are called stagnation properties: we can express the stagnation temperature as a function of the static and dynamic temperature:

$$T_0 = T + \frac{v^2}{2c_p}$$

this is for the case when the velocity goes to 0 eventually and all the kinetic energy is transferred into stagnation temperature. This equation is derived by

$$h_1 - h_0 = c_p(T_1 - T_0)$$

Similarly at low speed, the dynamic temperature is negligible.

Now we can derive stagnation properties in terms of Mach Number.

- Temperature:
$$\frac{T_0}{T} = 1 + \frac{(\gamma - 1)}{2}M^2 \tag{16}$$

- Pressure:
$$\frac{P_0}{P} = \left[1 + \frac{(\gamma - 1)}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \tag{17}$$

- Density:
$$\frac{\rho_0}{\rho} = \left[1 + \frac{(\gamma - 1)}{2}M^2\right]^{\frac{1}{\gamma-1}} \tag{18}$$

When density changes are less than %5 we can assume incompressible by using the density equation above. or by finding the mach number rule of 0.3

Summary

The most important equations are simply the Continuity equation and the conservation of energy

$$\frac{dA}{A} = -\frac{dV}{V}(1 - M^2) \tag{19}$$

based on the equation you can see how the relationship between A and v changes for subsonic flow and supersonic flow.

- Subsonic: Area increase, velocity decrease
- Supersonic: Area decrease, velocity decrease
- Critical: Only when Area is at minimum or maximum at a converging-diverging duct.