

$$b) \left\lceil \frac{n+1}{16} \right\rceil = \left\lfloor \frac{n}{16} \right\rfloor + 1 \quad (T)$$

$$n = 16k + r \quad \left\lceil \frac{n+1}{16} \right\rceil \leq \left\lceil \frac{n+1}{16} \right\rceil \leq \left\lceil k+1 \right\rceil \quad \text{cumple}$$

$1 \leq r \leq 15$ 

$$n = 16k \quad \left\lceil \frac{n+1}{16} \right\rceil = \left\lfloor \frac{n}{16} \right\rfloor + 1$$

$k+1$ cumple

$$c) 2n^2 - 200n + 25 = \omega(n)$$

Dado $c > 0 \rightarrow$ Proponemos $n_c = \underline{200+c}$ tq $n > n_c \rightarrow 2n^2 - 200n + 25 > cn$

$$n > 200 + c$$

• $n > 200$	\rightarrow	$n^2 > 200n$	\downarrow	$\forall n \geq 1$
• $n > c$	\rightarrow	$n^2 > cn$		

$$\Rightarrow \underline{2n^2 - 200n + 25} > \underline{cn + 25} > \underline{cn}$$

(3)

$$T(n) = \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor$$

Proponemos por Inducción que $T(n) \leq c n \lg n$

Caso Base:

$$T(1) \leq c \lg 1? \rightarrow 0 \leq c \cdot 0$$

$$S: T(k) \leq c k \lg k \quad \forall 1 \leq k \leq n-1 \quad \dot{?} T(n) \leq c n \lg n?$$

$$\begin{aligned} T(n) &= \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} k \lg k \\ &\leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} n \lg n = \lg n + (n-1) \lg n \leq c n \lg n \end{aligned}$$

④ Algo (A, l, r)

if (l == r)

if (A[l] < l+2)

return l+2

else A[l] ≥ l+2

return l+1

$$m = \left\lfloor \frac{l+r}{2} \right\rfloor$$

if (A[m] ≤ m+1) ←

return Algo (A, m+1, r)

else

return Algo (A, l, m)

0	0	0	1	1	2	2	3
1	2	3	4	5	6	7	8
1	2	3	5	6	8	9	11

Case here

A = [1]	→	3
[2]	→	3
[3]	→	2
[4]	→	2

⑤

Algo (A, l, r)

if (l == r)

return A[l]

$$m = \left\lfloor \frac{r+l}{2} \right\rfloor$$

x = Algo (A, l, m)

y = Algo (A, m+1, r)

cx = cy = 0

For i = l to r

if (A[i] == x) cx ++

else (A[i] == y) cy ++

if (cx < cy)

return x

else

return y

⑥



$$1 \leq i \leq k$$

Para $k+1$?

$$3i-1 \geq n+1$$

$$i \geq \left\lceil \frac{n+2}{3} \right\rceil$$

$$\left\lceil \frac{n+2}{3} \right\rceil \leq i \leq n$$

$$n - \left\lceil \frac{n+2}{3} \right\rceil + 1$$

$$n = 3k$$

$$\begin{aligned} 3k - \left\lceil \frac{k+2}{3} \right\rceil + 1 \\ 3k - k - 2 + 1 \\ 2k - 1 \end{aligned}$$

$$n = 3k+1$$

$$\begin{aligned} 3k+1 - \left\lceil \frac{3k+1+2}{3} \right\rceil + 1 \\ 3k+1 - \left\lceil k+1 \right\rceil + 1 \\ 3k+1 - k - 1 + 1 \\ 2k+1 \end{aligned}$$

$$n = 3k+2$$

$$\begin{aligned} 3k+2 - \left\lceil \frac{3k+2+2}{3} \right\rceil + 1 \\ 3k+2 - \left\lceil k + \frac{4}{3} \right\rceil + 1 \\ 3k+2 - k - 1 + 1 \\ 2k+2 \end{aligned}$$