$$\Omega \quad \xrightarrow{X} \quad \mathbb{Z}_0^+ \quad \xrightarrow{(\ )^2} \quad \mathbb{Z}_0^+$$

Ω	$\mathbb{Z}_0^+$	$\mathbb{Z}_0^+$
cc	2	4
cs	1	1
sc	1	1
ss	0	0

$$E[X^2] = \sum_{c=0}^{\infty} c\, P(X^2 = c) = \cancel{0\,P(X^2=0)} + 1\,P(X^2=1) + 4\,P(X^2=2)$$

$$= P(X^2 = 1) + 4\,P(X^2 = 4)$$

$$= P((X=-1) \cup (X=1))$$

$$= P(X=1) + 4\,P(X=2)$$

$$= \frac{2}{4} + 4\left(\frac{1}{4}\right) = 1,5$$

Con variable indicadora :

$X = \#$ caras al lanzar 2 monedas

$X_1 = \#$ caras de la moneda 1

$X_2 = \#$ " " " " 2 $\qquad\qquad X = X_1 + X_2$

$$E[X^2] = E[(X_1 + X_2)^2] = E[X_1^2 + X_1 X_2 + X_2 X_1 + X_2^2] = E[X_1^2] + 2E[X_1 X_2] + E[X_2^2]$$

$$= P(X_1^2 = 1) + 2P(X_1 X_2 = 1) + P(X_2^2 = 1)$$

$$= P(X_1 = 1) + 2P((X_1=1) \cap (X_2=1)) + P(X_2 = 1)$$

$$= P(X_1 = 1) + 2P(X_1=1)\,P(X_2=1) + P(X_2 = 1)$$

$$= \frac{1}{2} + \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$E[X] = E[X_1 + X_2] = E[X_1] + E[X_2] = P(X_1=1) + P(X_2=1) = \frac{1}{2} + \frac{1}{2} = 1$$

$$E[X]^2 = 1^2 = 1$$

$$\text{Var}(X) = E\left[(X - E[X])^2\right]$$
$$= E\left[X^2 - 2XE[X] + E[X]^2\right]$$
$$= E[X^2] - E[2XE[X]] + E[E[X]^2]$$
$$= E[X^2] - 2E[X] \cdot E[X] + E[X]^2$$
$$= E[X^2] - 2E[X]^2 + E[X]^2$$
$$= E[X^2] - E[X]^2 \geq 0$$

número

$E[E[x]^2]$: el promedio de constantes es igual a la constante

Ejercicio 2. Let X be a random variable that represents the sum of the results in the roll of n dice. What is the value of $E[X]$?

$$\Omega = \{1 - 6\}^n \qquad |\Omega| = 6^n \qquad X = \text{Suma de las } n \text{ caras} \qquad ¿E[x]?$$

$$E[x] = \sum_{c=0}^{\infty} cP(X=c)$$

No es indicadora pero es más sencilla

- $X_i = $ La cara del dado i

$$X = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i$$

$$E[X] = \sum_{i=1}^{\infty} E[X_i]$$

$$E[X_i] = \sum_{c=0}^{\infty} cP(X_i = c) \longrightarrow \sum_{c=1}^{6} cP(X_i = c)$$

$$= \sum_{c=1}^{6} c \cdot \frac{1}{6} = \frac{6(6+1)}{2} \cdot \frac{1}{6} = \frac{7}{2}$$

$$E[X] = \sum_{i=1}^{n} \frac{7}{2} = 3.5n$$

Ejercicio 5. Consider the following algorithm that determines the largest and smallest element of a vector $v[1\ldots n]$ with distinct positive numbers.

LARGESTSMALLEST(v, n)
1: $largest = v[1]$
2: $smallest = v[1]$ — siempre se ejecuta
3: **for** $i = 2$ **to** n
4:   **if** $v[i] > largest$
5:     $largest = v[i]$
6:   **else**
7:     **if** $v[i] < smallest$ ← la línea 7 se ejecuta siempre al else
8:       $smallest = v[i]$
9: **return** $largest, smallest$

Assume the input of the algorithm is a permutation of 1 to n chosen uniformly from all permutations of 1 to n. What is the expected number of comparisons executed in line 7 of the algorithm? What is the expected number of assignments performed in line 8 of the algorithm?

- $\Omega = \{$ permutaciones $(1,\ldots,n)\}$

- $X = \#$ comparaciones ejecutadas de la línea 7 al ejecutar el algoritmo

$$X_i = \begin{cases} 1 & : \text{ se ejecuta 7} \\ 0 & : \text{ c.c} \end{cases}$$

$X_i = \#$ comparaciones en la i-ésima iteración $\longrightarrow$ variable indicadora

$$X = X_2 + X_3 + \ldots + X_n = \sum_{i=2}^{n} X_i$$

$$E[X] = \sum_{i=2}^{n} E[X_i] = \sum_{i=2}^{n} P(X_i = 1) = \sum_{i=2}^{n} P(\text{En la i-ésima iteración ejecutamos la línea 7})$$

$= P($ En la i-ésima iteración ejecutamos la línea 7 $)$

$= P($ " " " " " línea 6 $)$ $P(A^c) = 1 - P(A)$

$= P($ " " " " la condicional de la línea 4 es falsa $)$

$= P($ " " " " $(V[i] > mayor) = F)$ **HIRING PROBLEM**

$= P($ " " " " $V[i] < \max(V[1:i-1]))$

$= P($ " " " " el $\underline{\max V[1:i]}$ NO esté en la posición i $)$

$= 1 - P($ " " " " el máx $V[1:i]$ si está en la posición i $)$

$= 1 - \dfrac{1}{i} = \dfrac{i-1}{i}$

[cloud: $i=1 \quad \dfrac{1-\frac{1}{1}}{1} = 0$]

$$E[X] = \sum_{i=2}^{n} E[X_i] = \sum_{i=2}^{n}\left(1 - \frac{1}{i}\right) = \sum_{i=1}^{n}\left(1 - \frac{1}{i}\right)$$

$$= \sum_{i=1}^{n} 1 - \sum_{i=1}^{n} \frac{1}{i} = n - \lg n$$

colocar el máximo en todos los espacios menos el último

los $i-1$ números restantes permutados

Por conteo $= \dfrac{(i-1) \times (i-1)!}{i!} = \dfrac{i-1}{i}$

i permutaciones