

$$e) T(n) = 7T(\lfloor n/2 \rfloor) + n^2$$

$$T(n) - 7T(\lfloor n/2 \rfloor) = n^2$$

$$T(1) = 1$$

$$T(0) = 0$$

$$(T(2^m) - 7T(2^{m-1}) = 2^{2m}) \quad 7^0$$

$$(T(2^{m-1}) - 7T(2^{m-2}) = 2^{2(m-1)}) \quad 7^1$$

$$(T(2^{m-2}) - 7T(2^{m-3}) = 2^{2(m-2)}) \quad 7^2$$

$$(T(2^k) - 7T(2^{k-1}) = 2^{2k}) \quad 7^{m-k}$$

$$\sum_{k=1}^m a_k - a_{k-1} = \sum_{k=1}^m 2^{2k} \cdot 7^{m-k}$$

$$\downarrow a_m - a_0 = \sum_{k=1}^m 2^{2k} \cdot \frac{7^m}{7^k} = 7^m \sum_{k=1}^m \frac{2^{2k}}{7^k} = 7^m \sum_{k=1}^m \left(\frac{2^2}{7}\right)^k$$

$$= 7^m \left[\frac{\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}}{1 - \left(\frac{4}{7}\right)} \right] = 7^m \left[\frac{\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}}{\frac{3}{7}} \right]$$

$$= 7^m \left(\frac{7}{3} \left[\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1} \right] \right)$$

$$= 7^m \left(\frac{4}{3} - \left(\frac{7}{3}\right) \left(\frac{4}{7}\right)^{m+1} \right)$$

$$= \frac{4}{3} \cdot 7^m - \left(\frac{7}{3}\right) \left(\frac{4}{7}\right)^{m+1} \cdot 7^m$$

$$= \frac{4}{3} \cdot 7^m - \left(\frac{7}{3}\right) \left(\frac{4}{7}\right) \left(\frac{4}{7}\right)^m \cdot 7^m$$

$$T(2^m) - T(1) = \frac{4}{3} 7^m - \frac{4}{3} \cdot \left(\frac{4}{7}\right)^m \cdot 7^m = \frac{4}{3} - \frac{4^m}{7^m} \cdot 7^m$$

$$T(2^m) - 7^m = \frac{4}{3} 7^m - \frac{4}{3} \cdot 4^m = \frac{4}{3} \cdot 7^m - \frac{4^{m+1}}{3} + 7^m = 7^m \left(\frac{4}{3} + 1 \right) - \frac{4^{m+1}}{3}$$

$$T(2^m) = 7^m \left(\frac{7}{3} \right) - \frac{4^{m+1}}{3} = \frac{7^{m+1} - 4^{m+1}}{3}$$

$$T(n) = \frac{7}{3} n^{\lg 7} - \frac{4}{3} n^2$$

$$a_k = 7^{m-k} T(2^k)$$

$$\frac{1 - \left(\frac{4}{7}\right)^m}{1 - \left(\frac{4}{7}\right)}$$

$$\sum_{k=0}^{m-1} \left(\frac{4}{7}\right)^{k+1}$$

$$\sum_{k=1}^m \left(\frac{2^2}{7}\right)^k$$

MAX-HEAPIFY $\Theta(\lg n)$



$$T(n) = C + T(\max\{n_L, n_R\})$$

$$i \in \frac{n}{2}?$$

$$n_L \leq \frac{2n+1}{3}$$

$$\Theta(\lg n)$$

$$2^h \leq n \leq 2^{h+1}$$

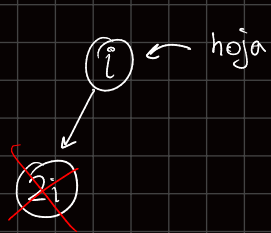
$$T(n) = C + T'(\frac{n}{2})$$

Built-max-heap

- Se puede reestructurar la complejidad de Max-heapify

$$\rightarrow T(n) = O(\lg n) \quad y \quad h = \Theta(\lg n) \Rightarrow T(n) = O(h)$$

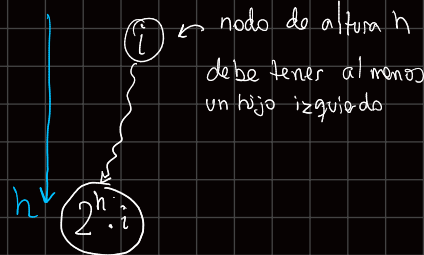
Lemma: Dado un heap con n nodos, el # de nodos de altura h está acotado por $\left\lceil \frac{n}{2^{h+1}} \right\rceil$



$$n < 2i$$

$$\frac{n}{2} < i \leq n$$

$$i = \left\lceil \frac{n}{2} \right\rceil$$



$$2^h i \leq n \quad y \quad n < 2^{h+1} i$$

$$\frac{n}{2^{h+1}} < i \leq \frac{n}{2^h}$$

$$\text{height}(n) \approx \Theta(\lg n)$$

$$T(n) = \sum_{i=1}^{\lfloor n/2 \rfloor} T(A_i) = \sum_{h=1}^H \# \text{ nodos de altura } h \cdot \Theta(h)$$

$$\leq \sum_{h=1}^H \left\lceil \frac{n}{2^{h+1}} \right\rceil \cdot \Theta(h) < \Theta \left(\sum_{h=1}^H \left(\frac{n}{2^{h+1}} + 1 \right) h \right)$$

$$< n \sum_{h=1}^H \frac{h}{2^{h+1}} + \sum_{h=1}^H h \leq \Theta(nc + H^2) = \Theta(n + \lg^2 n) = \Theta(n)$$