b) $\left\lceil \frac{n+2}{16} \right\rceil = \left\lfloor \frac{n}{16} \right\rfloor + 1$ (T)

$n = 16k + r$ $\quad \lceil k + 0 \rceil \leq \left\lceil \frac{n+2}{16} \right\rceil \leq \lceil k+1 \rceil$ cumple

$1 \leq r \leq 15$

$n = 16k$ $\quad \left\lceil \frac{n+1}{16} \right\rceil = \left\lceil \frac{n}{16} \right\rceil + 1$

Cumple

$k+1$

c) $2n^2 - 200n + 25 = \omega(n)$

Dado $c > 0 \longrightarrow$ Proponemos $n_c = \underline{200+c}$ tq $n > n_c \rightarrow 2n^2 - 200n + 25 > cn$

$n > 200 + c$

- $n > 200 \rightarrow n^2 > 200n$ $\quad \forall n \geq 1$
- $n > c \rightarrow n^2 > cn$

$\Rightarrow \underline{2n^2 - 200n + 25} > cn + 25 > \underline{cn}$

(3)

$$T(n) = \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor$$

Probaremos por Inducción que $T(n) \leq cn\lg n$

<u>Caso Base</u>:

¿$T(1) \leq c1\lg 1$? $\longrightarrow 0 \leq c.0$

Si $T(k) \leq ck\lg k$ $\quad \forall\ 1 \leq k \leq n-1$ $\quad$ ¿$T(n) \leq cn\lg n$?

$$T(n) = \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} k\lg k$$

$$\leq \lg n + \frac{1}{\cancel{n}} \sum_{k=1}^{n-1} \cancel{n}\lg n = \lg n + (n-1)\lg n \leq cn\lg n$$

④ Algo (A, l, r)

```
if (l == r)
    if (A[l] < l+2)
        return l+2
    else A[l] >= l+2
        return l+1

m = ⌊(l+r)/2⌋

if (A[m] <= m+1)
    return Algo (A, m+1, r)
else
    return Algo (A, l, m)
```

0	0	0	1	1	2	2	3
1	2	3	4	5	6	7	8
1	2	3	5	6	8	9	11

Caso base

$A = [1] \longrightarrow 3$

$[2] \longrightarrow 3$

$[3] \longrightarrow 2$

$[4] \longrightarrow 2$

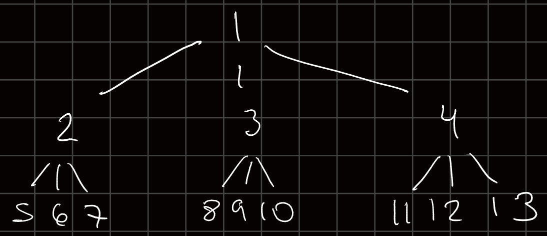
---

⑤

Algo (A, l, r)

```
if (l == r)
    return A[l]

m = ⌊(r+l)/2⌋

x = Algo (A, l, m)

y = Algo (A, m+1, r)

cx = cy = 0

For i = l to r

    if (Ai == x)    cx ++
    else (Ai == y)  cy ++

if (cx < cy)
    return x
else
    return y
```

6



$$i \begin{cases} \to 3i-1 \\ \to 3i \\ \to 3i+1 \end{cases}$$

$1 \le i \le k$

Para k+1?

$3i-1 \ge n+1$

$i \ge \left\lceil \frac{n+2}{3} \right\rceil$

$\left\lceil \frac{n+2}{3} \right\rceil \le i \le n$

$\underline{n - \left\lceil \frac{n+2}{3} \right\rceil + 1}$

$n = 3k$

$3k - \lceil k+2 \rceil + 1$

$3k - k - 2 + 1$

$2k - 1$

$n = 3k+1$

$3k+1 - \left\lceil \frac{3k+1+2}{3} \right\rceil + 1$

$3k+1 - \lceil k+1 \rceil + 1$

$3k+1 - k - \cancel{1} + \cancel{1}$

$2k+1$

$n = 3k+2$

$3k+2 - \left\lceil \frac{3k+2+2}{3} \right\rceil + 1$

$3k+2 - \left\lceil k + \frac{4}{3} \right\rceil + 1$

$3k+2 - k - \cancel{1} + \cancel{1}$

$2k+2$