b) $\left\lceil \frac{n+2}{16} \right\rceil = \left\lfloor \frac{n}{16} \right\rfloor + 1$ (T)

$n = 16k + r$ $\quad \lceil k + u \rceil \leq \left\lceil \frac{n+2}{16} \right\rceil \leq \lceil k+1 \rceil$ cumple

$1 \leq r \leq 15$

$n = 16k$ $\quad \left\lceil \frac{n+1}{16} \right\rceil = \left\lceil \frac{n}{16} \right\rceil + 1$

cumple

$k+1$

c) $2n^2 - 200n + 25 = \omega(n)$

Dado $c > 0 \longrightarrow$ Proponemos $n_c = 200+c$ tq $n > n_c \rightarrow 2n^2 - 200n + 25 > cn$

$n > 200 + c$

$\bullet\ n > 200 \rightarrow n^2 > 200n$ $\quad \forall n \geq 1$

$\bullet\ n > c \rightarrow n^2 > cn$

$\Rightarrow 2n^2 - 200n + 25 > cn + 25 > cn$

(3)

$$T(n) = \left\lfloor \lg n + \frac{1}{n}\sum_{k=1}^{n-1} T(k) \right\rfloor$$

Probaremos por Inducción que $T(n) \leq c n \lg n$

Caso Base:

¿$T(1) \leq c 1 \lg 1$? $\longrightarrow 0 \leq c.0$

Si $T(k) \leq c k \lg k$ $\quad \forall\ 1 \leq k \leq n-1$ $\quad$ ¿$T(n) \leq c n \lg n$?

$$T(n) = \left\lfloor \lg n + \frac{1}{n}\sum_{k=1}^{n-1} T(k) \right\rfloor \leq \lg n + \frac{1}{n}\sum_{k=1}^{n-1} T(k) \leq \lg n + \frac{1}{n}\sum_{k=1}^{n-1} k \lg k$$

$$\leq \lg n + \frac{1}{\cancel{n}}\sum_{k=1}^{n-1} \cancel{n} \lg n = \lg n + (n-1)\lg n \leq c n \lg n$$

④

```
Algo(A, l, r)
    if (l == r)
        if (A[l] < l+2)
            return l+2
        else A[l] ≥ l+2
            return l+1

    m = ⌊(l+r)/2⌋

    if (A[m] ≤ m+1)
        return Algo(A, m+1, r)
    else
        return Algo(A, l, m)
```

0	0	0	1	1	2	2	3
1	2	3	4	5	6	7	8
1	2	3	5	6	8	9	11

Caso base

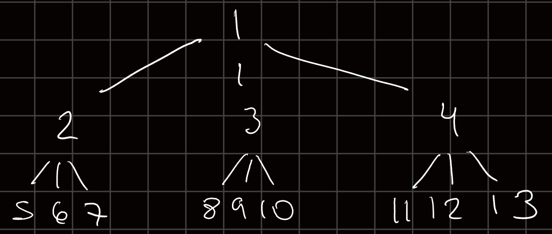
A = [1] ⟶ 3
[2] ⟶ 3
[3] ⟶ 2
[4] ⟶ 2

---

⑤

```
Algo(A, l, r)
    if (l == r)
        return A[l]

    m = ⌊(r+l)/2⌋

    x = Algo(A, l, m)

    y = Algo(A, m+1, r)

    cx = cy = 0

    For i = l to r
        if (Ai == x)  cx++
        else (Ai == y)  cy++

    if (cx < cy)
        return x
    else
        return y
```

⑥



$$i \to 3i-1, \quad i \to 3i, \quad i \to 3i+1 \qquad 1 \leq i \leq k$$

Para k+1?

$$3i - 1 \geq n+1$$

$$i \geq \left\lceil \frac{n+2}{3} \right\rceil$$

$$\left\lceil \frac{n+2}{3} \right\rceil \leq i \leq n$$

$$\underline{n - \left\lceil \frac{n+2}{3} \right\rceil + 1}$$

$n = 3k$

$$3k - \lceil k+2 \rceil + 1$$
$$3k - k - 2 + 1$$
$$2k - 1$$

$n = 3k+1$

$$3k+1 - \left\lceil \frac{3k+1+2}{3} \right\rceil + 1$$
$$3k+1 - \lceil k+1 \rceil + 1$$
$$3k+1 - k - \cancel{1} \cancel{+1}$$
$$2k+1$$

$n = 3k+2$

$$3k+2 - \left\lceil \frac{3k+2+2}{3} \right\rceil + 1$$
$$3k+2 - \left\lceil k + \frac{4}{3} \right\rceil + 1$$
$$3k+2 - k - \cancel{1} \cancel{+1}$$
$$2k+2$$