

Exercise 4 (4 points). We will say an array $A[1..n]$ is a *chasm* if there exists an index p , called the *bottom* such that $A[1..p]$ is a decreasing sequence, and $A[p..n]$ is an increasing sequence.

For example, the array $B = [5; 3; 3; 2; 2; 1; 4; 7]$ is a chasm. It's bottom is the index $p = 6$. Consider the following problem

Input: a chasm array $A[1..n]$ of integers. It is given that its bottom is unique

Output: The bottom p .

- Design (the pseudocode of) a divide-and-conquer algorithm with complexity $O(\lg n)$.
- Write the recurrence for the execution time and solve it using the master theorem

Chasm :



CHASM - INDEX (A, l, r):

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1  if ( $l == r$ )
2      return  $l$ 
3   $m = l - \lfloor \frac{r-l}{2} \rfloor$ 
4  if  $A[m] > A[m+1]$  and  $m < r$ 
5      return CHASM-INDEX ( $A, m+1, r$ )
6  else if  $A[m] > A[m-1]$  and  $m > l$ 
7      return CHASM-INDEX ( $A, l, m$ )
8  else
9      return  $m$ 

```



$p=6$

Casos esquina:

- Lista creciente

$$a_1 \leq a_2 \leq \dots \leq a_n$$

$$p=1$$

- Lista decreciente

$$a_1 \geq a_2 \geq \dots \geq a_n$$

$$p=n$$

$$A[4] \geq A[5] \quad 2 \geq 2 \quad (\text{True})$$

$$A[6] \geq A[7] \quad 1 \geq 4 \quad (\text{False})$$

$$A[5] \geq A[6] \quad 2 \geq 1 \quad (\text{True})$$