**Exercise 4 (4 points).** We will say an array $A[1..n]$ is a *chasm* If there exists an index $p$, called the *bottom* such that $A[1..p]$ is a decreasing sequence, and $A[p..n]$ is an increasing sequence.

For example, the array $B = [5; 3; 3; 2; 2; 1; 4; 7]$ is a chasm. It's bottom is the index $p = 6$. Consider the following problem

**Input:** a chasm array $A[1..n]$ of integers. It is given that its bottom is unique

**Output:** The bottom $p$.

- Design (the pseudocode of) a divide-and-conquer algorithm with complexity $O(\lg n)$.
- Write the recurrence for the execution time and solve it using the master theorem

Chasm :

$$\begin{array}{cccccc} & 1 & \cdots & p & \cdots & n \\ A = [ & a_1 & \cdots & a_p & \cdots & a_n ] \end{array}$$

$a_1 \geq \cdots \geq a_p \quad a_p \leq \cdots \leq a_n$

return index $p$

Casos esquina :

- Lista creciente

  $a_1 \leq a_2 \leq \cdots \leq a_n$

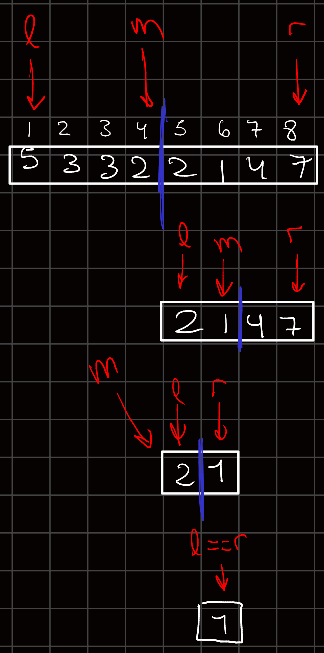
  $p = 1$

- Lista decreciente

  $a_1 \geq a_2 \geq \cdots \geq a_n$

  $p = n$

```
CHASM-INDEX (A, l, r):
    if ( l == r)
        return l
    m = l - ⌊(r-l)/2⌋
    if A[m] > A[m+1] and m < r
        return CHASM-INDEX (A, m+1, r)
    else if A[m] > A[m-1] and m > l
        return CHASM-INDEX (A, l, m)
    else
        return m
```




$p = 6$

$A[4] \geq A[5] \quad 2 \geq 2 \quad$ (True)

$A[6] \geq A[7] \quad 1 \geq 4 \quad$ (False)

$A[5] \geq A[6] \quad 2 \geq 1 \quad$ (True)