

**Exercise 4 (4 points).** We will say an array  $A[1..n]$  is a *chasm* if there exists an index  $p$ , called the *bottom* such that  $A[1..p]$  is a decreasing sequence, and  $A[p..n]$  is an increasing sequence.

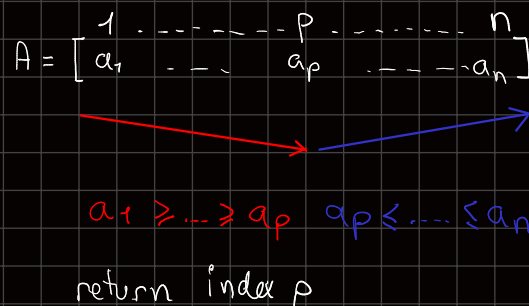
For example, the array  $B = [5; 3; 3; 2; 2; 1; 4; 7]$  is a chasm. It's bottom is the index  $p = 6$ . Consider the following problem

**Input:** a chasm array  $A[1..n]$  of integers. It is given that its bottom is unique

**Output:** The bottom  $p$ .

- Design (the pseudocode of) a divide-and-conquer algorithm with complexity  $O(\lg n)$ .
- Write the recurrence for the execution time and solve it using the master theorem

Chasm :



CHASM - INDEX ( $A, l, r$ ):

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1  if ( $l == r$ )
2      return  $l$ 
3   $m = l - \lfloor \frac{r-l}{2} \rfloor$ 
4  if  $A[m] > A[m+1]$  and  $m < r$ 
5      return CHASM-INDEX ( $A, m+1, r$ )
6  else if  $A[m] > A[m-1]$  and  $m > l$ 
7      return CHASM-INDEX ( $A, l, m$ )
8  else
9      return  $m$ 

```



$p=6$

Casos esquina:

- Lista creciente  $\rightarrow$   
 $a_1 \leq a_2 \leq \dots \leq a_n$   
 $p=1$
- Lista decreciente  $\rightarrow$   
 $a_1 \geq a_2 \geq \dots \geq a_n$   
 $p=n$

$$A[4] \geq A[5] \quad 2 \geq 2 \quad (\text{True})$$

$$A[6] \geq A[7] \quad 1 \geq 4 \quad (\text{False})$$

$$A[5] \geq A[6] \quad 2 \geq 1 \quad (\text{True})$$