e) $T(n) = 7T(\lfloor n/2 \rfloor) + n^2$

$T(1) = 1$
$T(0) = 0$

$T(n) - 7T(\lfloor n/2 \rfloor) = n^2$

$n = 2^m$

$(\ T(2^m) - 7T(2^{m-1}) = 2^{2m}\ )\ 7^0$

$(\ T(2^{m-1}) - 7T(2^{m-2}) = 2^{2(m-1)}\ )\ 7^1$

$(\ T(2^{m-2}) - 7T(2^{m-3}) = 2^{2(m-2)}\ )\ 7^2$

$a_k = 7^{m-k}\, T(2^k)$

$(\ \underbrace{T(2^k)}_{a_k} - \underbrace{7T(2^{k-1})}_{a_{k-1}} = 2^{2k}\ )\ 7^{m-k}$

$$\frac{1-\left(\frac{4}{7}\right)^m}{1-\left(\frac{4}{7}\right)}$$

$$\sum_{k=0}^{m-1} \left(\frac{4}{7}\right)^{k+1}$$

$$\sum_{k=1}^{m} a_k - a_{k-1} = \sum_{k=1}^{m} 2^{2k} \cdot 7^{m-k}$$

$$a_m - a_0 = \sum_{k=1}^{m} 2^{2k} \cdot \frac{7^m}{7^k} = 7^m \sum_{k=1}^{m} \frac{2^{2k}}{7^k} = 7^m \sum_{k=1}^{m} \left(\frac{2^2}{7}\right)^k$$

$$= 7^m \left[\frac{\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}}{1 - \left(\frac{4}{7}\right)}\right] = 7^m \left[\frac{\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}}{\frac{3}{7}}\right]$$

$$= 7^m \left(\frac{7}{3}\left[\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}\right]\right)$$

$$= 7^m \left(\frac{4}{3} - \left(\frac{7}{3}\right)\left(\frac{4}{7}\right)^{m+1}\right)$$

$$= \frac{4}{3} \cdot 7^m - \left(\frac{7}{3}\right)\left(\frac{4}{7}\right)^{m+1} \cdot 7^m$$

$$= \frac{4}{3} \cdot 7^m - \left(\frac{\cancel{7}}{3}\right)\left(\frac{4}{\cancel{7}}\right)\left(\frac{4}{7}\right)^m \cdot 7^m$$

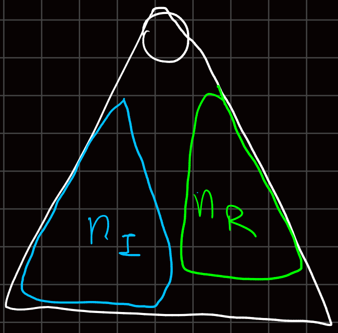
$$T(2^m) - T(1) = \frac{4}{3} 7^m - \frac{4}{3} \cdot \left(\frac{4}{7}\right)^m \cdot 7^m = \frac{4}{3} \cdot \frac{4^m}{\cancel{7^m}} \cancel{7^m}$$

$$T(2^m) - 7^m = \frac{4}{3} 7^m - \frac{4}{3} \cdot 4^m = \frac{4}{3} \cdot 7^m - \frac{4^{m+1}}{3} + 7^m = 7^m\left(\frac{4}{3} + 1\right) - \frac{4^{m+1}}{3}$$

$$T(2^m) = 7^m \left(\frac{7}{3}\right) - \frac{4^{m+1}}{3} = \frac{7^{m+1} - 4^{m+1}}{3}$$

$$T(n) = \frac{7}{3} n^{\lg 7} - \frac{4}{3} n^2$$

MAX-HEAPIFY es lgn



$$T(n) = C + T\left(\underbrace{\max\{n_I, n_R\}}\right)$$

¿Es $\frac{n}{2}$?

$$n_I \le \frac{2n+1}{3} \qquad 2^h \le n \le 2^{h+1} \qquad T(n) = C + T'(\underbrace{n_I})$$

$0{,}8n$

Built-max-heap

- Se puede reestructurar la complejidad de Max-heapify

$\rightarrow T(n) = O(\lg n)$ y $h = \Theta(\lg n) \Rightarrow T(n) = O(h)$

<u>Lemma</u>: Dado un heap con n nodos, el # de nodos de altura h está acotado por $\left\lceil \frac{n}{2^{h+1}} \right\rceil$

i ← hoja

$\cancel{2i}$

$n < 2i$

$\frac{n}{2} < i \le n$

$i = \left\lceil \frac{n}{2} \right\rceil$

i ← nodo de altura h debe tener al menos un hijo izquierdo

h

$2^h \cdot i$

$2^h i \le n$ y $n < 2^{h+1} i$

$$\boxed{\frac{n}{2^{h+1}} < i \le \frac{n}{2^h}}$$

$\text{heigh}(n) \approx \Theta(\lg n)$

$$T(n) = \sum_{i=1}^{\lfloor n/2 \rfloor} T(A_i) = \sum_{h=1}^{H} \#\text{ nodos de altura } h \cdot \Theta(h)$$

$$\le \sum_{h=1}^{H} \left\lceil \frac{n}{2^{h+1}} \right\rceil \cdot \Theta(h) < \Theta\left(\sum_{h=1}^{H} \left(\frac{n}{2^{h+1}} + 1 \right) h \right)$$

$$< n \sum_{h=1}^{H} \frac{h}{2^{h+1}} + \sum_{h=1}^{H} h \le \Theta\left(nC + H^2 \right) = \Theta(n + \lg^2 n) = \Theta(n)$$