


$$b) \left\lceil \frac{n+1}{16} \right\rceil = \left\lfloor \frac{n}{16} \right\rfloor + 1 \quad (T)$$

$$n = 16k + r \quad \left\lceil \frac{n+1}{16} \right\rceil \leq \left\lceil \frac{n+1}{16} \right\rceil \leq \left\lceil k+1 \right\rceil \quad \text{cumple}$$

$1 \leq r \leq 15$  

$$n = 16k \quad \left\lceil \frac{n+1}{16} \right\rceil = \left\lceil \frac{n}{16} \right\rceil + 1$$

$k+1$  cumple

$$c) 2n^2 - 200n + 25 = \omega(n)$$

Dado  $c > 0 \rightarrow$  Proponemos  $n_c = \underline{200+c}$  tq  $n > n_c \rightarrow 2n^2 - 200n + 25 > cn$

$$n > 200 + c$$

|             |               |              |   |                    |
|-------------|---------------|--------------|---|--------------------|
| • $n > 200$ | $\rightarrow$ | $n^2 > 200n$ |  | $\forall n \geq 1$ |
| • $n > c$   | $\rightarrow$ | $n^2 > cn$   |   |                    |

$$\Rightarrow \underline{2n^2 - 200n + 25} > \underline{cn + 25} > \underline{cn}$$

(3)

$$T(n) = \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor$$

Proponemos por Inducción que  $T(n) \leq c n \lg n$

Caso Base:

$$T(1) \leq c \lg 1? \rightarrow 0 \leq c \cdot 0$$

$$S: T(k) \leq c k \lg k \quad \forall 1 \leq k \leq n-1 \quad \dot{?} T(n) \leq c n \lg n?$$

$$\begin{aligned} T(n) &= \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} k \lg k \\ &\leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} n \lg n = \lg n + (n-1) \lg n \leq c n \lg n \end{aligned}$$

④ Algo(A, l, r)

if (l == r)

if (A[l] < l+2)

return l+2

else A[l] ≥ l+2

return l+1

$$m = \left\lfloor \frac{l+r}{2} \right\rfloor$$

if (A[m] ≤ m+1) ←

return Algo(A, m+1, r)

else

return Algo(A, l, m)

|   |   |   |   |   |   |   |    |
|---|---|---|---|---|---|---|----|
| 0 | 0 | 0 | 1 | 1 | 2 | 2 | 3  |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8  |
| 1 | 2 | 3 | 5 | 6 | 8 | 9 | 11 |

Case here

|         |   |   |
|---------|---|---|
| A = [1] | → | 3 |
| [2]     | → | 3 |
| [3]     | → | 2 |
| [4]     | → | 2 |

⑤

Algo(A, l, r)

if (l == r)

return A[l]

$$m = \left\lfloor \frac{r+l}{2} \right\rfloor$$

x = Algo(A, l, m)

y = Algo(A, m+1, r)

cx = cy = 0

For i = l to r

if (Ai == x) cx++

else (Ai == y) cy++

if (cx < cy)

return x

else

return y

⑥



$$1 \leq i \leq k$$

Para  $k+1$ ?

$$3i-1 \geq n+1$$

$$i \geq \left\lceil \frac{n+2}{3} \right\rceil$$

$$\left\lceil \frac{n+2}{3} \right\rceil \leq i \leq n$$

$$n - \left\lceil \frac{n+2}{3} \right\rceil + 1$$

$$n = 3k$$

$$3k - \left\lceil \frac{k+2}{3} \right\rceil + 1$$

$$3k - k - 2 + 1$$

$$2k - 1$$

$$n = 3k+1$$

$$3k+1 - \left\lceil \frac{3k+1+2}{3} \right\rceil + 1$$

$$3k+1 - \left\lceil k+1 \right\rceil + 1$$

$$3k+1 - k - 1 + 1$$

$$2k+1$$

$$n = 3k+2$$

$$3k+2 - \left\lceil \frac{3k+2+2}{3} \right\rceil + 1$$

$$3k+2 - \left\lceil k + \frac{4}{3} \right\rceil + 1$$

$$3k+2 - k - 1 + 1$$

$$2k+2$$