

**Ejercicio 1.** Let  $X$  be a random variable that represents the number of heads in two tosses of a fair coin. What is the value of  $E[X^2]$ ? What is the value of  $E[X]^2$ ?

$\Omega$	$\times$	$\mathbb{Z}_0^+$	$(\cdot)^2$	$\mathbb{Z}_0^+$
cc		2		4
cs		1		1
sc		1		1
ss		0		0

$$\begin{aligned}
 E[X^2] &= \sum_{c=0}^{\infty} c P(X^2=c) = \cancel{0 P(X^2=0)} + 1 P(X^2=1) + 4 P(X^2=2) \\
 &= P(X^2=1) + 4 P(X^2=4) \\
 &= P((X=-1) \cup (X=1)) \\
 &= P(X=1) + 4 P(X=2) \\
 &= \frac{2}{4} + 4 \left( \frac{1}{4} \right) = 1,5
 \end{aligned}$$

Con variable indicadora:

$X = \#$  caras al lanzar 2 monedas

$X_1 = \#$  caras de la moneda 1

$$X = X_1 + X_2$$

$X_2 = \#$  " " " " 2

$$\begin{aligned}
 E[X^2] &= E[(X_1 + X_2)^2] = E[X_1^2 + X_1 X_2 + X_2 X_1 + X_2^2] = E[X_1^2] + 2E[X_1 X_2] + E[X_2^2] \\
 &= P(X_1^2=1) + 2P(X_1 X_2=1) + P(X_2^2=1) \\
 &= P(X_1=1) + 2P((X_1=1) \cap (X_2=1)) + P(X_2=1) \\
 &= P(X_1=1) + 2P(X_1=1)P(X_2=1) + P(X_2=1) \\
 &= \frac{1}{2} + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{2}
 \end{aligned}$$

$$E[X] = E[X_1 + X_2] = E[X_1] + E[X_2] = P(X_1=1) + P(X_2=1) = \frac{1}{2} + \frac{1}{2} = 1$$

$$E[X]^2 = 1^2 = 1$$

$$\text{Var}(X) = E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - E[2XE[X]] + E[E[X]^2]$$

$$= E[X^2] - 2E[X] \cdot E[X] + E[X]^2$$

$$= E[X^2] - 2E[X]^2 + E[X]^2$$

$$= E[X^2] - E[X]^2 \geq 0$$

$E[E[X]^2]$  : el promedio de constantes es igual a la constante

número

**Ejercicio 2.** Let  $X$  be a random variable that represents the sum of the results in the roll of  $n$  dice. What is the value of  $E[X]$ ?

$$\Omega = \{1, \dots, 6\}^n$$

$$|\Omega| = 6^n$$

$X$  = Suma de los  $n$  caras

¿ $E[X]$ ?

$$E[X] = \sum_{c=0}^{\infty} cP(X=c)$$

→ No es indicador, pero es más sencilla

•  $X_i$  = La cara del dado  $i$

$$X = X_1 + X_2 + \dots + X_n = \sum_{i=1}^n X_i$$

$$E[X] = \sum_{i=1}^n E[X_i]$$

$$E[X_i] = \sum_{c=0}^{\infty} cP(X_i=c) \rightarrow \sum_{c=1}^6 cP(X_i=c)$$

$$= \sum_{c=1}^6 c \cdot \frac{1}{6} = \frac{6(6+1)}{2} \cdot \frac{1}{6} = \frac{7}{2}$$

$$E[X] = \sum_{i=1}^n \frac{7}{2} = 3.5n$$

