

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(\lfloor \frac{n}{2} \rfloor) + n & \text{otherwise} \end{cases}$$

Prove by induction that $T(n) = \mathcal{O}(n^2)$

$$\exists c, n_0 > 0 \text{ tq } \forall n \geq n_0 \quad 0 \leq T(n) \leq cn^2$$

Usando las siguiente $c = \underline{2}$, $n_0 = \underline{1}$

C.B. $n=1$

$$T(1) = 1 \leq 2 \cdot 1^2 = 2$$

H.I. $P(1) \dots P(k-1)$ son T $k > 1$

\hat{P}_k ?

Tenemos: $T(k) = 2T(\lfloor \frac{k}{2} \rfloor) + k$

Como $k \geq 2$ $\lfloor \frac{k}{2} \rfloor \geq 1$ y $\lfloor \frac{k}{2} \rfloor < k \rightarrow P(\lfloor \frac{k}{2} \rfloor) = T$

$$T(k) = 2T(\lfloor \frac{k}{2} \rfloor) + k \leq 2 \left[2 \left(\lfloor \frac{k}{2} \rfloor \right)^2 \right] + k$$

$$\leq 4 \left(\lfloor \frac{k}{2} \rfloor \right)^2 + k \leq 4 \left(\frac{k}{2} \right)^2 + k$$

$$= k^2 + k \leq k^2 + k^2 = 2k^2$$

$$P(n) : T(n) \geq cn^2$$

$$P(k) : T(k) \geq ck^2$$

Prove by induction that $T(n) = \Omega(n \lg n)$

$$P(n): T(n) \geq c n \lg n$$

$$k \geq 4 \rightarrow \lfloor \frac{k}{2} \rfloor < k \rightarrow P(\lfloor \frac{k}{2} \rfloor) = T$$

$$T(k) = 2T(\lfloor \frac{k}{2} \rfloor) + k$$

$$\geq 2 \left(c \lfloor \frac{k}{2} \rfloor \lg \lfloor \frac{k}{2} \rfloor \right) + k$$

$$\geq 2c \left(\frac{k}{2} - 1 \right) \lg \left(\frac{k}{2} - 1 \right) + k$$

$$\geq 2c \left(\frac{k}{2} - 1 \right) \lg \left(\frac{k}{4} \right) + k \quad \begin{matrix} k \geq 4 \\ \lfloor \frac{k}{2} \rfloor \geq \frac{k}{2} - 1 \geq \frac{k}{4} \end{matrix}$$

$$= c(k-2)(\lg k - \lg 4) + k$$

$$= c(k-2)(\lg k - 2) + k$$

$$= c(k \lg k - 2 \lg k - 2k + 4) + k$$

$$= c k \lg k + k + 4 - 2 \lg k - 2k \geq c k \lg k$$

$$\lg k \leq k$$

Esto último cumple cuando $c \leq \frac{1}{8}$

Sea: $n_0 = 1$ y

$$c = \min \left\{ \frac{1}{8}, \frac{T(2)}{2 \lg 2}, \frac{T(3)}{3 \lg 3} \right\}$$

C.B.

$$\left\{ \begin{array}{l} T(1) \geq c \cdot 0 \\ T(2) \geq c \cdot 2 \lg 2 \\ T(3) \geq c \cdot 3 \lg 3 \end{array} \right.$$

Master theorem

$$9T(n/4) + n^2 - n$$

$$\log_4 9 \approx 1.5$$

$$4^{1.5} = 9 \rightarrow \begin{matrix} 1 \\ 2 \end{matrix}$$

$$T(n) \begin{cases} n & 1 \leq n \leq 2 \\ 6T(\lfloor \frac{n}{3} \rfloor) + n^2 - n & \text{cc.} \end{cases}$$

$$\begin{aligned} n &= 3^k \\ m=k \\ (T(3^k) - 6T(3^{k-1})) &= 3^{2k} - 3^k \quad \times 6^{k-m} \quad \leftarrow 3^k \cdot 2 \cdot 3 \quad 3^{k+1} \cdot 2 \\ (T(3^{k-1}) - 6T(3^{k-2})) &= 3^{2(k-1)} - 3^{k-1} \quad \times 6^{k-(k-1)} \rightarrow 3^{2(k-1)} \cdot 3^{k-1} \cdot 2^{k-1} \\ (T(3^{k-2}) - 6T(3^{k-3})) &= 3^{2(k-2)} - 3^{k-2} \quad \times 6^{k-(k-2)} \rightarrow 3^{2(k-1)} \cdot 2^{k-1} \\ &\vdots \\ m=1 \quad (T(3) - 6T(1)) &= 3^2 - 3 \quad \times 6^{k-1} \end{aligned}$$

$$6^0 T(3^k) - 6^k T(1) = \sum_{i=1}^k (3^3 \cdot 2)^{k-i} - 3^{i+1} \cdot 2$$

$$n = 3^m$$

$$T(3^k) = 6T(3^{k-1}) + (3^k)^2 - 3^k$$

$$a_k = 6^{m-k} T(3^k)$$

$$6^{m-k} [T(3^k) - 6T(3^{k-1}) = 9^k - 3^k]$$

$$\sum_{k=1}^m a_k - a_{k-1} = \sum_{k=1}^m (9^k - 3^k) \cdot 6^{m-k}$$

$$a_m - a_0 = \sum_{k=1}^m 9^k \cdot 6^{m-k} - \sum_{k=1}^m 3^k \cdot 6^{m-k}$$

$$= \sum_{k=1}^m 6^m \cdot \left(\frac{9}{6}\right)^k - \sum_{k=1}^m 6^m \cdot \left(\frac{3}{6}\right)^k$$

$$\alpha = \frac{9}{6} \quad \beta = \frac{3}{6}$$

$$= 6^m \left(\sum_{k=1}^m \alpha^k - \sum_{k=1}^m \beta^k \right)$$

$$= 6^m \left(\frac{\alpha^{m+1} - \alpha}{\alpha - 1} - \frac{\beta^{m+1} - \beta}{\beta - 1} \right)$$

$$= 6^m \left(\frac{\left(\frac{3}{2}\right)^{m+1} - \left(\frac{3}{2}\right)}{\frac{3}{2} - 1} - \frac{\left(\frac{1}{2}\right)^{m+1} - \left(\frac{1}{2}\right)}{\frac{1}{2} - 1} \right)$$

$$= 6^m \left(2 \left[\left(\frac{3}{2}\right)^{m+1} - \left(\frac{3}{2}\right) \right] + 2 \left[\left(\frac{1}{2}\right)^{m+1} - \left(\frac{1}{2}\right) \right] \right)$$

$$= 2 \left(\frac{3}{2}\right) \cdot 6^m \left(\frac{3}{2}\right)^m - 2 \cdot 6^m \left(\frac{3}{2}\right) + 6^m \cdot 2 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^m - 6^m \cdot 2 \cdot \left(\frac{1}{2}\right)$$

$$= 3 \cdot 9^m - 3 \cdot 6^m + 3^m - 6^m$$

induction

$$\sum_{i=0}^n s^{2i} = \frac{s^{2(n+1)} - s^0}{s^2 - 1} = \frac{s^{2n+2} - 1}{s^2 - 1}$$

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$$\sum_{i=0}^n s^{2i} = \frac{s^{2(n+1)} - 1}{s^2 - 1} = \frac{s^{2n+2} - 1}{s^2 - 1}$$