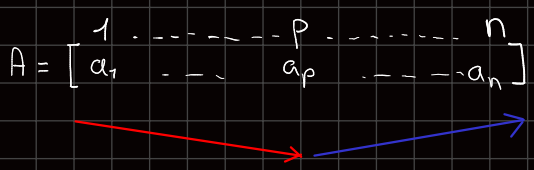
Exercise 4 (4 points). We will say an array $A[1..n]$ is a *chasm* If there exists an index p, called the *bottom* such that $A[1..p]$ is a decreasing sequence, and $A[p..n]$ is an increasing sequence.

For example, the array $B = [5; 3; 3; 2; 2; 1; 4; 7]$ is a chasm. It's bottom is the index $p = 6$. Consider the following problem

Input: a chasm array $A[1..n]$ of integers. It is given that its bottom is unique
Output: The bottom p.

- Design (the pseudocode of) a divide-and-conquer algorithm with complexity $O(\lg n)$.
- Write the recurrence for the execution time and solve it using the master theorem

Chasm:



$a_1 \geq \dots \geq a_p \quad a_p \leq \dots \leq a_n$

return index p

```
CHASM-INDEX (A, l, r):
1   if (l == r)
2       return l
3   m = l - ⌊(r-l)/2⌋
4   if A[m] > A[m+1] and m < r
5       return CHASM-INDEX (A, m+1, r)
6   else if A[m] > A[m-1] and m > l
7       return CHASM-INDEX (A, l, m)
8   else
9       return m
```

Casos esquina:

- Lista creciente →

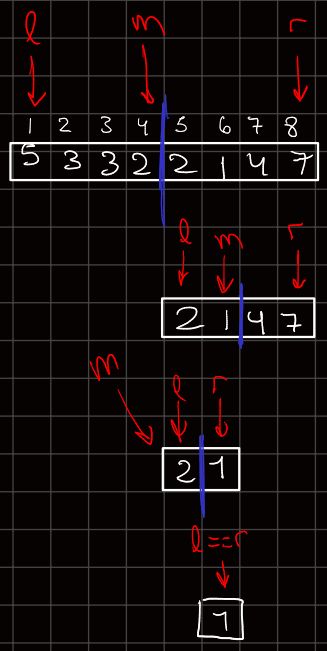
  $a_1 \leq a_2 \leq \dots \leq a_n$

  $p = 1$

- Lista decreciente ↘

  $a_1 \geq a_2 \geq \dots \geq a_n$

  $p = n$



$p = 6$

$A[4] \geq A[5] \quad 2 \geq 2$ (True)

$A[6] \geq A[7] \quad 1 \geq 4$ (false)

$A[5] \geq A[6] \quad 2 \geq 1$ (true)