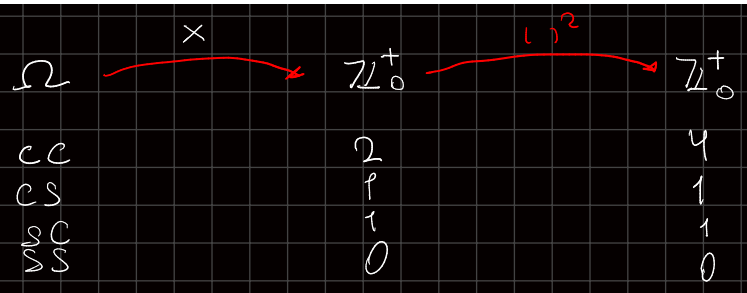
Ejercicio 1. Let X be a random variable that represents the number of heads in two tosses of a fair coin. What is the value of $E[X^2]$? What is the value of $E[X]^2$?



$$E[X^2] = \sum_{c=0}^{\infty} c\,P(X^2=c) = \cancel{0\,P(X^2=0)} + 1\,P(X^2=1) + 4\,P(X^2=2)$$

$$= P(X^2=1) + 4P(X^2=4)$$

$$= P((X=-1) \cup (X=1))$$

$$= P(X=1) + 4P(X=2)$$

$$= \frac{2}{4} + 4\left(\frac{1}{4}\right) = 1{,}5$$

Con variable indicadora:

X = # caras al lanzar 2 monedas

X_1 = # caras de la moneda 1 $\qquad X = X_1 + X_2$

X_2 = # " " " 2

$$E[X^2] = E\left[(X_1+X_2)^2\right] = E\left[X_1^2 + X_1X_2 + X_2X_1 + X_2^2\right] = E[X_1^2] + 2E[X_1X_2] + E[X_2^2]$$

$$= P(X_1^2=1) + 2P(X_1X_2=1) + P(X_2^2=1)$$

$$= P(X_1=1) + 2P((X_1=1)\cap(X_2=1)) + P(X_2=1)$$

$$= P(X_1=1) + 2P(X_1=1)\,P(X_2=1) + P(X_2=1)$$

$$= \frac{1}{2} + \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$E[X] = E[X_1+X_2] = E[X_1] + E[X_2] = P(X_1=1) + P(X_2=1) = \frac{1}{2} + \frac{1}{2} = 1$$

$$E[X]^2 = 1^2 = 1$$

$$\text{Var}(X) = E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - E[2XE[X]] + E[E[X]^2]$$

$$= E[X^2] - 2E[X] \cdot E[X] + E[X]^2$$

$$= E[X^2] - 2E[X]^2 + E[X]^2$$

$$= E[X^2] - E[X]^2 \geq 0$$

$E[E[X]^2]$: el promedio de constantes es igual a la constante

número

Ejercicio 2. Let X be a random variable that represents the sum of the results in the roll of n dice. What is the value of $E[X]$?

$\Omega = \{1, \dots, 6\}^n$ $\quad |\Omega| = 6^n$ $\quad X$ = Suma de las n caras $\quad$ ¿$E[X]$?

$$E[X] = \sum_{c=0}^{\infty} cP(X = c)$$

No es indicadora pero es más sencilla

- X_i = La cara del dado i

$$X = X_1 + X_2 + \dots + X_n = \sum_{i=1}^{n} X_i$$

$$E[X] = \sum_{i=1}^{n} E[X_i]$$

$$E[X_i] = \sum_{c=0}^{6} cP(X_i = c) \longrightarrow \sum_{c=1}^{6} cP(X_i = c)$$

$$= \sum_{c=1}^{6} c \cdot \frac{1}{6} = \frac{6(6+1)}{2} \cdot \frac{1}{6} = \frac{7}{2}$$

$$E[X] = \sum_{i=1}^{n} \frac{7}{2} = 3.5n$$