

QS(A, l, r)

if (l < r):

q = PARTITION(A, l, r)

QS(A, l, q-1)

QS(A, q+1, r)

PARTITION(A, l, r)

x = A[r]

i = l-1

For j = l to r-1

if A[j] ≤ x

i = i+1

swap(A[i], A[j])

swap(A[i+1], x)

return i+1

1 2 3 4 5 6 7 8 9 10 11

**Ejercicio 3 (5 pts).** It is known that the possible number of comparisons between elements you can make if you execute the Quicksort algorithm in an array of 11 elements can range from 22 to 55.

a) • Prove why the minimum number of comparisons is exactly 22.

b) • Give an input example of an array that makes exactly 33 comparisons.

c) • How many swaps does Quicksort perform with your previous example?

a)

- Cada PARTITION(A, l, r) hace exactamente  $r-l$  comparaciones
- Si el array tiene tamaño  $t$ , entonces hacemos  $t-1$  comparaciones
- Cada QS(A, n) empieza haciendo  $n-1$  comparaciones y llama a otras Quicksorts ( $q-1$ ) y ( $n-q$ )
- Si  $T_n$ : min # de comparaciones de un array de tamaño  $n$ .

$$T_n = n-1 + \min_{q=1}^n (T_{q-1} + T_{n-q})$$

min

T	0	1	2	3	4	5	6	7	8	9	10	11
n	0	0	1	2	4	6	8	10	13	16	19	22

max

T	0	1	2	3	4	5	6	7	8	9	10	11
n	0	0	1	3	6	10	15	21	28	36	45	55

1 5 2 7 3 6 4

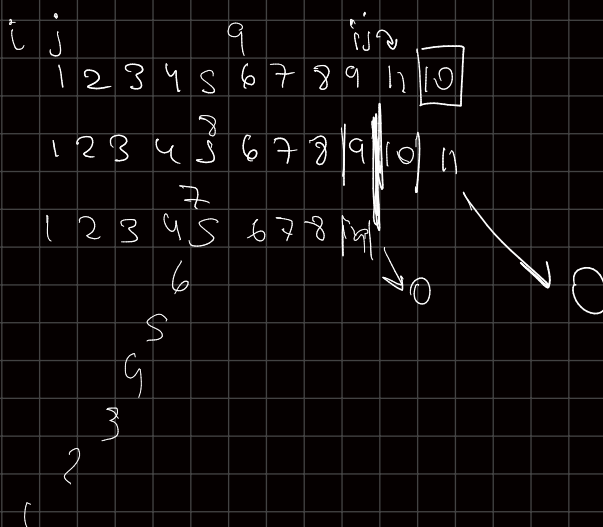
1 2 3 4 5 6 7

1 5 2 7 3 6 4

array de 33 ite

b) 1 2 3 4 5 6 7 8 9 10 11

1 10 6 3 5 2 8 9 11 7 4



Exr 9.7 Cada componente de  $A[1..n]$  e de  $B[1..n]$  vale 0 ou 1. Se  $A[j] = 1 = B[j]$  para algum  $j$ , o algoritmo devolve 1; caso contrário, devolve 0.

ELEMENTO-COMUM ( $A, B, n$ )

```

1   $j \leftarrow 1$ 
2  enquanto  $j \leq n$  e  $(A[j] \neq 1$  ou  $B[j] \neq 1)$ 
3      faça  $j \leftarrow j + 1$ 
4  se  $j \leq n$ 
5      então devolva 1
6  senão devolva 0
    
```

Suponha que o valor de cada componente de  $A[1..n]$  e  $B[1..n]$  é escolhida ao acaso e independentemente para ser 0 ou 1 com probabilidade  $1/2$ . Mostre que o consumo de tempo esperado do algoritmo é  $O(1)$ . (Dica:  $1/(1-x)^2 = 1 + 2x + 3x^2 + \dots$ )

$X_i$ : El índice  $j$  para un input

$$X_i = \begin{cases} 1 & \text{si } j=i \\ 0 & \text{si no} \end{cases}$$

$$X = \underbrace{1}_{\odot} \cdot \underbrace{(j=1)}_{\odot} + \underbrace{2}_{3} \cdot \underbrace{(j=2)}_{\odot} + \underbrace{3}_{\odot} \cdot \underbrace{(j=3)}_{\odot} + \dots + \underbrace{(n+1)}_{\odot} \cdot \underbrace{(j=n+1)}_{\odot}$$

$$1 \leq i \leq n \quad P(X_i = 1) =$$

$$\left[ \left( \frac{3}{4} \right)^n + \sum_{i=1}^{n-1} \left( \frac{3}{4} \right)^{i-1} \cdot \frac{1}{4} \right]$$