e) $T(n) = 7T(\lfloor n/2 \rfloor) + n^2$

$T(1) = 1$
$T(0) = 0$

$T(n) - 7T(\lfloor n/2 \rfloor) = n^2$

$n = 2^m$

$$\left(T(2^m) - 7T(2^{m-1}) = 2^{2m} \right) 7^0$$
$$\left(T(2^{m-1}) - 7T(2^{m-2}) = 2^{2(m-1)} \right) 7^1$$
$$\left(T(2^{m-2}) - 7T(2^{m-3}) = 2^{2(m-2)} \right) 7^2$$

$$\left(\underbrace{T(2^k)}_{a_k} - \underbrace{7T(2^{k-2})}_{a_{k-1}} = 2^{2k} \right) 7^{m-k}$$

$$a_k = 7^{m-k}\, T(2^k)$$

$$\sum_{k=1}^{m} a_k - a_{k-1} = \sum_{k=1}^{m} 2^{2k} \cdot 7^{m-k}$$

$a_m - a_0$

$$= \sum_{k=1}^{m} 2^{2k} \cdot \frac{7^m}{7^k} = 7^m \sum_{k=1}^{m} \frac{2^{2k}}{7^k} = 7^m \sum_{k=1}^{m} \left(\frac{2^2}{7}\right)^k$$

$$\frac{1 - \left(\frac{4}{7}\right)^m}{1 - \left(\frac{4}{7}\right)}$$

$$\sum_{k=0}^{m-1} \left(\frac{4}{7}\right)^{k+1}$$

$$= 7^m \left[\frac{\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}}{1 - \left(\frac{4}{7}\right)} \right] = 7^m \left[\frac{\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1}}{\frac{3}{7}} \right]$$

$$= 7^m \left(\frac{7}{3} \left[\left(\frac{4}{7}\right) - \left(\frac{4}{7}\right)^{m+1} \right] \right)$$

$$= 7^m \left(\frac{4}{3} - \left(\frac{7}{3}\right)\left(\frac{4}{7}\right)^{m+1} \right)$$

$$= \frac{4}{3} \cdot 7^m - \left(\frac{7}{3}\right)\left(\frac{4}{7}\right)^{m+1} \cdot 7^m$$

$$= \frac{4}{3} \cdot 7^m - \left(\frac{\cancel{7}}{3}\right)\left(\frac{4}{\cancel{7}}\right)\left(\frac{4}{7}\right)^m \cdot 7^m$$

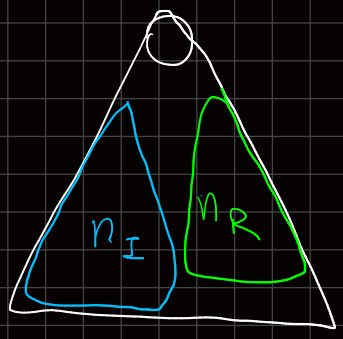
$$T(2^m) - T(1) = \frac{4}{3} 7^m - \frac{4}{3} \cdot \left(\frac{4}{7}\right)^m \cdot 7^m = \frac{4}{3} \cdot \frac{4^m}{\cancel{7^m}} \cdot \cancel{7^m}$$

$$T(2^m) - 7^m = \frac{4}{3} 7^m - \frac{4}{3} \cdot 4^m = \frac{4}{3} \cdot 7^m - \frac{4^{m+1}}{3} + 7^m = 7^m\left(\frac{4}{3} + 1\right) - \frac{4^{m+1}}{3}$$

$$T(2^m) = 7^m \left(\frac{7}{3}\right) - \frac{4^{m+1}}{3} = \frac{7^{m+1} - 4^{m+1}}{3}$$

$$T(n) = \frac{7}{3} n^{\lg 7} - \frac{4}{3} n^2$$

MAX- HEAPIFY es $\lg n$



$$T(n) = C + T\left(\underbrace{\max\{n_I, n_R\}}_{\text{¿Es } \frac{n}{2}\text{?}}\right)$$

$n_I \leq \frac{2n+2}{3}$ $\qquad 2^h \leq n \leq 2^{h+1}$ $\qquad T(n) = c + T'(n_I)$

$0{,}8n$

Built-max-heap

- Se puede reestructurar la complejidad de Max-heapify

$\rightarrow T(n) = O(\lg n)$ y $h = \Theta(\lg n) \Rightarrow T(n) = O(h)$

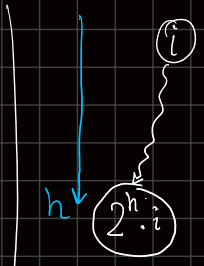
<u>Lemma</u>: Dado un heap con n nodos, el # de nodos de altura h está acotado por $\left\lceil \frac{n}{2^{h+1}} \right\rceil$

hoja

$n < 2i$

$\frac{n}{2} < i \leq n$

$i = \left\lceil \frac{n}{2} \right\rceil$



nodo de altura h debe tener al menos un hijo izquierdo

$2^h i \leq n$ y $n < 2^{h+1} i$

$$\boxed{\frac{n}{2^{h+1}} < i \leq \frac{n}{2^h}}$$

height(n) $\approx \Theta(\lg n)$

$$T(n) = \sum_{i=1}^{\lfloor n/2 \rfloor} T(A_i) = \sum_{h=1}^{H} \#\text{ nodos de altura } h \cdot \Theta(h)$$

$$\leq \sum_{h=1}^{H} \left\lceil \frac{n}{2^{h+1}} \right\rceil \cdot \Theta(h) < \Theta\left(\sum_{h=1}^{H} \left(\frac{n}{2^{h+1}} + 1\right) h\right)$$

$$< n \sum_{h=1}^{H} \frac{h}{2^{h+1}} + \sum_{h=1}^{H} h \leq \Theta\left(nC + H^2\right) = \Theta(n + \lg^2 n) = \Theta(n)$$