b) $\left\lceil \frac{n+2}{16} \right\rceil = \left\lfloor \frac{n}{16} \right\rfloor + 1$ (T)

$n = 16k + r$ $\lceil k + v \rceil \leq \left\lceil \frac{n+2}{16} \right\rceil \leq \lceil k+1 \rceil$ cumple

$1 \leq r \leq 15$

$n = 16k$ $\left\lceil \frac{n+1}{16} \right\rceil = \left\lceil \frac{n}{16} \right\rceil + 1$

       cumple

     $k+1$

c) $2n^2 - 200n + 25 = \omega(n)$

Dado $c > 0 \longrightarrow$ Proponemos $n_c = 200 + c$ tq $n > n_c \rightarrow 2n^2 - 200n + 25 > cn$

$n > 200 + c$ $\bullet \, n > 200 \rightarrow n^2 > 200n$ $\forall n \geq 1$

                    $\bullet \, n > c \rightarrow n^2 > cn$

$\Rightarrow 2n^2 - 200n + 25 > cn + 25 > cn$

③

$T(n) = \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor$

Probaremos por Inducción que $T(n) \leq cn \lg n$

Caso Base:

¿ $T(1) \leq c \cdot 1 \lg 1$? $\longrightarrow 0 \leq c \cdot 0$

Si $T(k) \leq ck \lg k$ $\forall \, 1 \leq k \leq n-1$ ¿ $T(n) \leq cn \lg n$?

$T(n) = \left\lfloor \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \right\rfloor \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} T(k) \leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} k \lg k$

$\leq \lg n + \frac{1}{n} \sum_{k=1}^{n-1} n \lg n = \lg n + (n-1) \lg n \leq cn \lg n$

4

```
Algo (A, l, r)
    if (l == r)
        if (A[l] < l+2)
            return  l+2
        else A[l] ≥ l+2
            return  l+1

    m = ⌊ (l+r)/2 ⌋

    if (A[m] ≤ m+2)   ←
        return  Algo (A, m+1, r)

    else
        return  Algo (A, l, m)
```


$$\begin{array}{cccccccc} 0 & 0 & 0 & 1 & 1 & 2 & 2 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$

| 1 | 2 | 3 | 5 | 6 | 8 | 9 | 11 |

Casos base

$$A = \begin{array}{l} [1] \longrightarrow 3 \\ [2] \longrightarrow 3 \\ [3] \longrightarrow 2 \\ [4] \longrightarrow 2 \end{array}$$

5

```
Algo (A, l, r)
    if (l == r)
        return  A[l]

    m = ⌊ (r+l)/2 ⌋

    x = Algo (A, l, m)

    y = Algo (A, m+1, r)

    cx = cy = 0

    For  i = l  to  r

        if (Ai == x)    cx ++
        else (Ai == y)  cy ++

    if (cx < cy)
        return x
    else
        return y
```

⑥

$1 \le i \le k$

$3i-1 \ge n+1$

$i \ge \left\lceil \dfrac{n+2}{3} \right\rceil$

$\left\lceil \dfrac{n+2}{3} \right\rceil \le i \le n$

$$n - \left\lceil \dfrac{n+2}{3} \right\rceil + 1$$

$n = 3k$

$3k - \lceil k+2 \rceil + 1$
$3k - k - 2 + 1$
$2k - 1$

$n = 3k+1$

$3k+1 - \left\lceil \dfrac{3k+1)+2}{3} \right\rceil + 1$
$3k+1 - \lceil k+1 \rceil + 2$
$3k+1 - k - 1 + 1$
$2k+1$

$n = 3k+2$

$3k+2 - \left\lceil \dfrac{3k+2+2}{3} \right\rceil + 1$
$3k+2 - \left\lceil k + \dfrac{4}{3} \right\rceil + 1$
$3k+2 - k - 1 + 1$
$2k+2$

Para k+1?