Jeffrey Qian, Math 2210

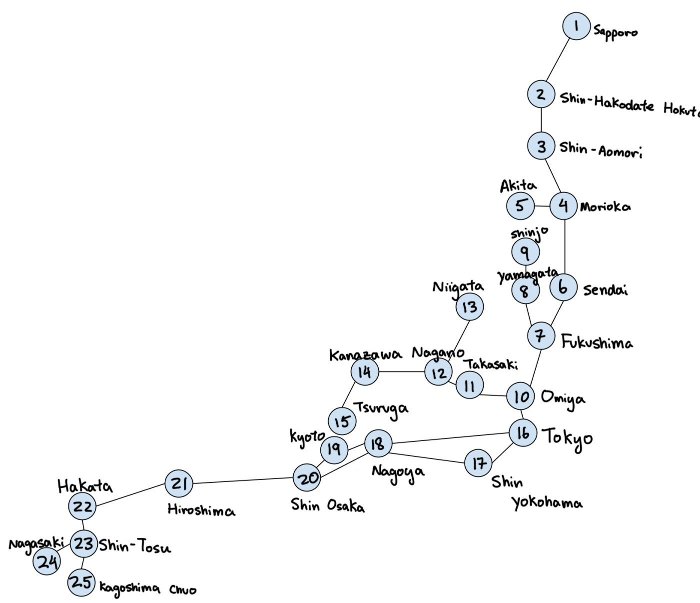
Project 2 - Japan's High Speed Train Network

Introduction

This project will study the connectedness between high-speed train stations in Japan. This project will see how the fielder set reveals sections of train stations that are more split and isolated from the rest of the network. Note: I took the railway stations and tracks from real-world data, but the network analysis, from my knowledge, is unique.

Network Description and Laplacian Matrix

The network in this project is the Shinkansen high-speed train network (also known as the Bullet Train network). The infrastructure spans the entire length of Japan, connecting multiple key cities. Each node in the network represents a train station in a city, and each edge in the network represents a railroad connecting two nodes. I referenced a map of the network when constructing my graph (source 1). There are 25 nodes in the original network and 26 edges. The graph is shown on the right with the node IDs and the cities that each node ID represents. The first 10 rows and columns of the Laplacian matrix are shown below.



$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 3 \end{bmatrix}$$

Fiedler Eigenvector, Eigenvalue, and Fiedler Set

The network is one connected component, meaning that the multiplicity of the eigenvalue 0 is one and that the Fiedler value λ_2 is positive. Thus, I do not need to add any more edges.

Using MATLAB, I computed the Fiedler eigenvalue to be $\lambda_2 = 0.0424$, greater than 0. I also calculated the Fiedler vector $\mathbf{v_2}$, which is listed below. I will present it as 5-row vectors, each containing 5 entries of $\mathbf{v_2}$ to fit this text better:

$$\begin{aligned} v_2(1:5) &= [-0.3013 \quad -0.2885 \quad -0.2635 \quad -0.2273 \quad -0.2374] \\ v_2(6:10) &= [-0.1715 \quad -0.1084 \quad -0.1187 \quad -0.1239 \quad -0.0304] \\ v_2(11:15) &= [-0.0413 \quad -0.0504 \quad -0.0527 \quad -0.0552 \quad -0.0577] \end{aligned}$$

$$v_2(16:20) = [0.0598 \quad 0.0903 \quad 0.1170 \quad 0.1453 \quad 0.1675]$$
$$v_2(21:25) = [0.2331 \quad 0.2888 \quad 0.3323 \quad 0.3470 \quad 0.3470]$$

Taking the index of all the positive values, I see that the Fiedler Set is F = {16, 17, 18, 19, 20, 21, 22, 23, 24, 25}, which is, surprisingly, a considerable section of the network.

Analysis and Discussion

Visually looking at the graph of the train station, we can see that the nodes are “sparsely” connected. Most nodes only have 2 edges, meaning that there is usually only one path to get from one node to another. Near the southern part of the graph, there are more interconnected nodes (16 through 20). For the passenger, this might mean that they do not have much choice if they want to travel between cities.

There are 10 nodes in the Fiedler set, so the split is uneven (15 nodes are not in the Fiedler set). These 10 nodes make up the southern and south-western parts of the network. There is 1 edge connecting the nodes in the Fiedler set to the rest of the graph. A consequence of this is that the lower half of the Shinkansen network is isolated from the rest of the network. In the future, if travel between the lower half and the upper half becomes popular, there might be a chance for congestion and delays.

It is interesting to see how eigenvalues and eigenvectors can be used to anticipate real-life consequences, though a more in-depth analysis is required to make further conclusions.

Sources:

1. Map of Japanese Bullet Train Network: https://www.japanstation.com/map-of-shinkansen-high-speed-train-network-in-japan/
2. Additional Reading on Laplacian Matrices: https://en.wikipedia.org/wiki/Laplacian_matrix

Additional:

1. My code and full results for this project: https://github.com/jeffreyqdd/LinAlg-NetworkAnalysis