FIT3139: Lab questions for week 4

1. Let
   $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$

   Work out the following on a paper. (A calculator is allowed if you need one.):

   (a) What is the characteristic polynomial of A?

   (b) Find its eigenvalues?

   (c) What are the corresponding eigenvectors?

2. A system is defined using the following relationship:
   $-0.5S_k = S_{k-1} - S_{k+1}$,
   where initial values are $S_0 = 0$ and $S_1 = 1$.

   (a) At what rate is the system growing?

   (b) Compute and plot the values of $S_0, S_1, \cdots S_{100}$

3. Implement the interval bisection method to find at least one root of the following equations:
   $x^3 - 2x - 5 = 0$,
   $e^{-x} = x$,
   $x \sin(x) = 1$,
   $x^3 - 3x^2 + 3x - 1 = 0$
   You should terminate the iterations when the bracket size falls below the tolerance of `tol` $= 10^{-5}$.

[Tips to addressing question 3: `You will need to handle a function symbolically in this exercise. There are several ways to do this in MATLAB. See http://ocw.usu.edu/Civil_and_Environmental_Engineering/Numerical_Methods_in_Civil_Engineering/FunctionsAsArguments.pdf`. Also, the lower and upper values of a bracket to the root which is binary searched in that interval can be passed to your script as arguments. The onus is on you to choose the bracket sensibly.]

4. Implement Newton and secant methods on the same set of equations as above to find at least one root. Terminate the iteration when the absolute difference between successive values, $|x_{k+1} - x_k|$, falls below a tolerance of 10^{-9}. (Note: Secant method will be explained in the first lecture of week 4.)

   [Tips to addressing question 4: `Again, you will need to handle a function symbolically in this exercise. Additionally you will have to deal with the differential of the function in symbolic terms. MATLAB has the function 'diff' to differentiate symbolic expressions: See http://www.mathworks.com/help/toolbox/symbolic/diff.html. Alternatively, you can work out (by hand) the symbolic expression of the differential of the functions whose root you have to evaluate and pass it as an argument to your script. The initial guess(es) required for Newton and Secant methods are also passed as arguments.`]

5. In neutron transport theory, the critical length of a fuel rod is determined by the roots of the equation $\cot(x) = (x^2 - 1)/(2x)$. Can you find the smallest positive root of this equation using one of your implementations of Bisection, Newton or Secant method? (Note: Secant method will be explained in the first lecture of week 4.)

6. Compute and list the first several iterations of Newton's method for solving each of the following equations, starting with the given initial guess.
   (a) $x^2 - 1 = 0, x_0 = 10^6$
   (b) $(x - 1)^4 = 0, x_0 = 10$

   Can you estimate the rate of convergence from the output?