FIT3139: Additional lab exercise for Week 7

If you have completed the pharmaco-kinetics modelling and simulation exercise undertaken in Week 6 lab, attempt the following questions as additional work for this week's lab.

Numerical exercise

1. Write/Implement your own version of Runge Kutta second-order (RK2) that takes the values b and h as arguments,[1] in addition to the first-order ordinary differential equation (ODE) you are solving and the initial value for the problem.

2. Write/Implement Euler's method for the same.

3. Using the scripts you have written, solve the following ODE problems for varying values of $h = 0.1, 0.05, 0.025, 0.0125$:

   (a)
   $$\frac{dy}{dx} = x\sqrt{1 - y^2}, \qquad y(0) = 4,$$
   whose exact solution is $y(x) = \sin(x^2/2) + 4$.

   (b)
   $$\frac{dy}{dx} = y^3, \qquad y(0) = 0.$$

4. By plotting the numerical solutions for the above ODEs using your scripts (for RK2 and Euler), try and reason how the error is growing/varying and any other anamolies you may notice.

[1] Refer to the Week 6 lectures for what these variables represent in RK.

Build and simulate a continuous model for measles epidemic

A mathematical model of measles epidemic was discussed during one of the lectures in earlier weeks. The model was presented as a pair of coupled non-linear **difference** equations. The reason for the applicability of difference equations was the assumption that there was a significant latent period between catching the disease and becoming contagious. If this latent period is small (ideally zero) a model of an epidemic involving coupled **differential** equations can be formulated. Your goal is to attempt this and work out the steady-state and phase-plane analysis of such a continuous model of measles epidemic on paper. Further use what you have worked out to simulate the populations of the three groups in the model – susceptibles (S), infectives (I) and recovered (R) – using the following assumptions:

- The disease is transmitted by close proximity or contact between an infective and susceptible.
- A susceptible becomes an infective immediately ater transmission.
- Infectives eventually become removed.
- The population of susceptibles is not altered by emigration, immigration, births or deaths.
- Each infective infects a constant fraction of the susceptible population per unit time.
- The number of infectives removed is proportional to the number of infectives present.