

Question 1. Part A).

The values of x that lead to most inaccurate result in terms of relative error.

For `my_log1p(x)` the values of x which leads to the most inaccurate result are values which greater than 1. This is due to the fact that the *degree* of each term grows with `n`, which results us taking `x^n`. When x is greater than 1, the term `x^n` diverges as n grows. Consequently, the Taylor expansion fails to converge to an accurate value for `log(1+x)`.

Hence, an input of x greater than 1 will result in the function diverging and returning an error due to overflow.

Identify/describe the adv/limitations of using the above expansion

The advantages:

- This implementation of `log1p` is able to compute values which are much smaller than machine epsilon very accurately to the reference inbuilt `np.log1p(x)`.
- The output converge to an acceptable output within an acceptable time for x between 0 and 1.

The disadvantages:

- None linear complexity compare to the other inbuilt algorithms. (See figure 1)
- Can only be used for $x \in [0, 1]$.
- It's less accurate than the inbuilt `log(1+x)` for range of x between machine eps and 1

Specify the value of x for which you would use the expansion over the inbuilt.

Summarising the above advantages and disadvantages. We can conclude that the only usable range for `my_log1p(x)` is for $x \in [0, \epsilon]$, where $\epsilon = 2.22044604925 * 10^{-16}$.

Observe the graph below showing the relative errors. The function `my_log1p` produces good results even when the input are much less than machine eps. However, the inbuilt `log` function was unable to handle the addition `x+1` for x being less than machine eps. Consequently, the huge error spike in green towards 0.

Furthermore, `my_log1p` runtime tends to be much slower compare to the inbuilt `log` function as x grows. Hence, rendering it less desirable compare to quicker and more accurate alternatives.

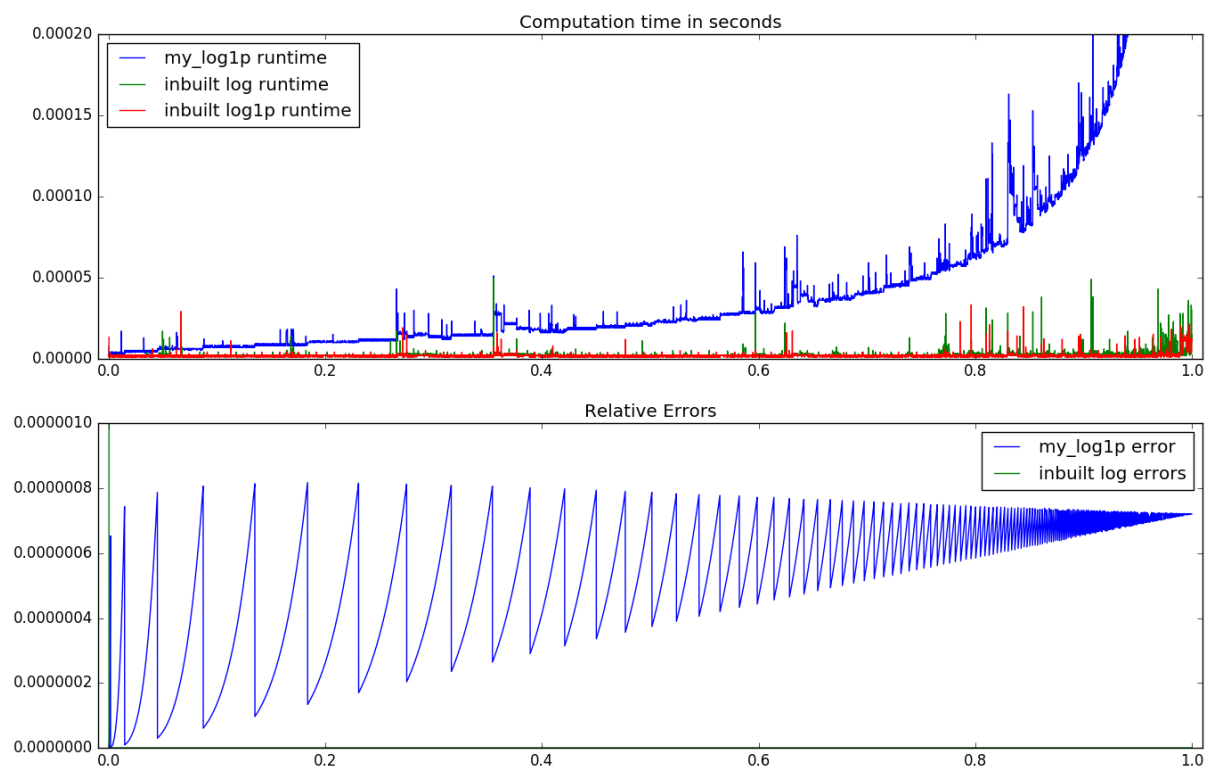


Figure 1: (top) Computation time and (Bottom) relative error of the algorithms.