FIT3139: Lab questions for week 5

1. A model of insect populations (in which all adults are assumed to die before next breeeding) leads to the difference equation

   $N_{k+1} = \frac{\lambda N_k}{1+aN_k}$
   where λ and a are positive constants.

   (a) Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the growth rate.

   (b) What is the general shape of the graph of $R(N_k)$?

   (c) Express the unrestricted growth rate r and the carrying capacity K, for this model, in terms of the parameters a and λ.

2. Another alternative model for restricted population growth is given by: $N_{k+1} = N_k e^{a(1-N_k/K)}$,
   where K is the carrying capacity and a is a positive constant.

   (a) Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the variable growth rate $R(N_k)$.

   (b) Sketch $R(N_k)$ and compare it with $R(N_k)$ of a discrete logistic equation.

   (c) Show that $a = \log(r + 1)$ where r, the unrestricted growth rate, is defined as the limit of $R(N_k)$ as $N_k \to 0$.

   (d) Use a computer to iterate numerically the above model when $K = 1000$, $N_0 = 100$ and you should use your own choice of values of a. Describe the types of behaviour which appear for different values of a. In particular:

i. When do 2-cycles (oscillations with a periodicity of 2) first appear?

ii. When do 2-cycles become 4-cycles?

iii. Does the model exhibit chaos?

3. A model which has been used to analyse insect populations, a modification of the one introduced in Question 1, is:
$N_{k+1} = \frac{\lambda N_k}{(1+N_k)^b}$
for the insect population N_k. Using a computer, sketch solutions for the following values of the parameters:

	λ	b
Moth	1.3	0.1
Mosquito	10.6	1.9
Potato Beetle	75.0	3.4

Test the above with your own choices of N_0. Observe the behaviour when you vary N_0.

4. For the discrete logistic equation with $K = 1000$ examine N_{100} as you vary N_0 slightly in each of the following cases:

   (a) $r = 0.5$

   (b) $r = 2.3$

   (c) $r = 3$

   Start with $N_0 = 100$ and then try $N_0 = 101$ and $N_0 = 99$. What happens for each value of r?

5. In a host-parasite system, a parasite searches for a host on which to deposit its eggs. Define:

   - N_k = Number of host species in kth breeding season
   - P_k = Number of parasite species in kth breeding season
   - f = fraction of hosts not parasitized
   - c = average number of eggs laid by parasite which survive
   - λ = host rate, given that all adults die before their offspring can breed.

(a) Show that N_k and P_k satisfy:
$N_{k+1} = \lambda f N_k$
and
$P_{k+1} = cN_k[1 - f]$

(b) If $f = e^{-\gamma P_k}$ numerically simulate this model for $\gamma = 0.068$, $c = 1$ and $\lambda = 2$. What observations can you gather from this simulation?