FIT3139: Lab questions for week 8

1. Write a Monte Carlo simulation script for the stochastic version of the epidemic Susceptibles-Infectives-Recovered (SIR) model discussed in Week 7. (Refer to the Gillespie's algorithm – see Week 7 lecture slides – and use initial values provided on slide 27 for your simulation).

2. Revise the Buffon's needle problem handled in Week 7 where we computed the axiomatic probability of a needle of length $r < 1$ will land crossing some line, given a floor with equally spaced parallel lines, where each pair of lines are a unit distance apart.

   (a) Using the axiomatic probability expression we derived during the lecture, write a script that finds a Monte Carlo estimate of π for some given value of $r < 1$.

   (b) Derive the expression of the axiomatic probability if the needle had a length $r > 1$?

   (c) Use the Monte carlo method to find the probability of the needle crossing some line when $r > 1$.

3. This question motivates the rejection sampling approach (see slide 24 in week 7 lecture slides) to sample random variables from arbitrary distributions. In this exercise, we want to sample from a distribution characterized as $\hat{f}(x) = \sqrt{\frac{2}{\pi}} \exp(-x^2/2)$ when $x \geq 0$.

   (a) Before anything, plot and visualize this distribution for varying values of $x \geq 0$.

   (b) To apply rejection sampling on this distribution, choose the negative exponential distribution as the proposal distribution. You will have to make a decision on the values for the constant c and the rate parameter λ of the exponential distribution you have considered for the proposal distribution, such that $c \times p(x) \geq \hat{f}(x)$. After making this choice, plot your $c \times p(x)$ against the plot of $\hat{f}(x)$.

(c) After this, implement the rejection sampling approach to sample randomly from $\hat{f}(x)$. One way to check your sampling approach is consistent is to plot a histogram of sampled values (say, in intervals of 0.5 of the sampled points) and compare them against the plot of $\hat{f}(x)$.

4. Consider the following scenario:

   A deadly virus has affected some people on a large secluded island nation. (By secluded, you should assume that there is no traffic to or out of the island.) Any person on the island can contract the virus *immediately* upon contact with a person who is already carrying the virus. Once the virus is contracted, the person evenually dies. These deaths are proportionate to the population of those who are carrying the virus. Assume that each carrier of the virus comes in contact with a constant portion of those on the island who are not already carriers. Assume also that there are no births or deaths (due to causes other than contracting the virus itself).

   (a) Construct a mathematical model of the above scenario.

   (b) Simulate in the lab the spread of the virus. It is assumed here that you will test a variety of values for the parameters of your model.

   (c) Identify (if any) the variables in your mathematical model which are not coupled with the remainder of the variables.

   (d) What are the steady-state solutions of the model?

   (e) Compute and plot the phase-plane trajectories (as described in your lecture) of variables that are coupled in this model.

5. Consider the following change to the above scenario. Take into account that the babies born (all of which are free from the virus) add to the population of the island and that the birth rate is proportional to the existing population which is free from the virus.

(a) Revise the mathematical model to account for the changed scenario.

(b) Simulate in the lab the spread of the virus. Compare this with the previous model over the same parameters that are common between models.

(c) What are the steady state solutions of this model?

(d) Compute and plot the phase-plane trajectories (as above) for this revised model.