FIT3139: Lab questions for week 1

1. The surface area of a sphere having radius r is given by

$$A(r) = 4\pi r^2.$$

The task is to explore how the surface area increases when the radius is increased by a tiny amount δr. The change in the surface area is given by:

$$\begin{aligned}\delta A &= 4\pi(r+\delta r)^2 - 4\pi r^2 &\quad (1)\\ \delta A &= 4\pi(2r+\delta r)\times\delta r &\quad (2)\\ \delta A &\approx 8\pi r\times\delta r &\quad (3)\end{aligned}$$

The equations 1 and 2 are exact, while equation 3 is approximate ignoring the term $(\delta r)^2$.

Write a script that takes the sphere radius r (in kilometers) and increase amount δr (in millimeters), and then displays the surface area increase (in square meters). What is the increase when the radius of a spherical Earth ($r = 6367$ kms) is increased by a few millimeters. Explore the answer to this question using each of the above formulae.

2. An *oblate spheroid* such as the Earth is obtained by revolving an ellipse about its minor axis. Informally, it is the shape of a slightly compressed ball. The Earth's equatorial radius about 20 kms longer than its polar radius.

The surface area of an oblate spheroid is given by

$$A(r_1, r_2) = 2\pi\left(r_1^2 + \frac{r_2^2}{\sin(\gamma)}\log\left(\frac{\cos(\gamma)}{1-\sin(\gamma)}\right)\right)$$

where r_1 is the equatorial radius and r_2 is the polar radius, and $\gamma = \arccos(\frac{r_2}{r_1})$.

We assume $r_2 < r_1$. Write a script that inputs the two radii and displays both $A(r_1, r_2)$ and the spherical approximation $4\pi(\frac{(r_1+r_2)}{2})^2$. Apply the script to varying values of the radii, including Earth data $(r_1, r_2) = (6378.137, 6356.752)$. Display enough digits so that the discrepancy is revealed.

3. A quadratic equation of the form $f(x) = x^2+bx+c$ attains its minimum value at the critical point $x_c = -b/2$. Computing the minimum value of f on an interval $[L, R]$ requires some checking. If x_c is in the interval, then $f(x_c)$ is the answer. Otherwise, f is minimized, for the values in the interval, at either L or R.

   Write a script that inputs real numbers $L, R, b,$ and,c and prints the minimum value of the quadratic function $f(x) = x^2 + bx + c$ on the interval $[L, R]$. The script should also display the value of x where the minimum occurs.

4. Extend the script above to plot the curve for the accepted values. Color the range $[L, R]$ and highlight the minima.

5. Write a script which accepts a positive integer value x and evaluates the boolean expression which is true if x is divisible by 2,5, and 7.

6. Write a script which accepts positive integer values $a, b,$ and,c and evaluates the boolean expression which is true if it is possible to form a Pythagorean triangle whose three sides have lengths given by these integers.

7. Assume that $\theta_1, \theta_2,$ and, θ_3 are initialized with positive integer values and that their sum is 180. Regard these as the three angles in a triangle. Write a script to output whether the triangle is Equilateral, Isosceles, or Scalene triangle, for the given input.

8. A *DNA sequence* is a string of characters drawn from the alphabet composed of four characters $\{A, T, G, C\}$. In DNA, 'A' complements with 'T' and vice versa; 'G' complements with 'C' and vice versa. A *complement* of a DNA sequence is another sequence in which each character is a complement of the corresponding character in the DNA sequence. A *reverse complement* is a sequence that is a reverse of the complementary string.

Write a script to accept a DNA sequence of arbitrary length from a file and output (to the screen) its complement as well as its reverse complement sequences.

9. The quotient 22/7 is the closest rational approximation of π with both the numerator and denominator less than 100. Write a script that inputs a positive integer M and prints the best approximation of π of the form p/q where p, q are integers that satisfy, $1 \leq p, q \leq M$.

10. The fibonacci numbers $f_0, f_1, \cdots$ are defined recursively as:

$$\begin{aligned} f_0 &= 0 \\ f_1 &= 1 \\ f_{n+1} &= f_n + f_{n-1}. \end{aligned}$$

It can be shown that the quotients $r_n = \frac{f_{n+1}}{f_n}$ converges to the *golden ratio*

$$\phi = \frac{(1 + \sqrt{(5)})}{2}.$$

Write a script that displays the values of $n, f_n, r_n,$ and$|\phi - r_n|$ for $1 \leq n \leq U$, where U is the smallest value of n such that $|r_n - r_{n+1}| \leq 10^{-15}$.