FIT3139: Lab questions for week 3

1. Write the following set of equations in matrix form:
   $50 = 5x_3 - 6x_2$
   $2x_2 + 7x_3 + 30 = 0$
   $x_1 - 7x_3 = 50 - 3x_2 + 5x_1$

   Use MATLAB to solve for the unknowns. In addition, use it to compute the transpose and the inverse of the coefficient matrix.

2. Develop, debug and test your own MATLAB script to switch the rows of a matrix using a permutation matrix. Include error traps for erroneous input (think about the possibilities).

3. (a) Develop a MATLAB script that implements naive Gaussian elimination on square matrices. Test on randomly generated matrices.

   (b) Extend the above to include partial pivoting and test it on randomly generated matrices.

   (c) Use the above two script to factorize A into factors LU. Test on random matrices.

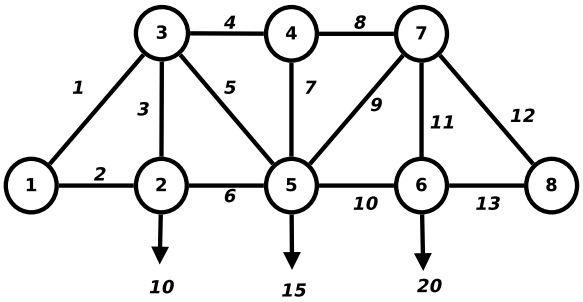
   (d) Use all of the above to solve a general linear equation $Ax = LUx = b$. Think about how you want to test your script?

4. (a) Using MATLAB commands, solve $Ax = b$, where:
   $$A = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{pmatrix} \quad b = \begin{pmatrix} 0.1 \\ 0.3 \\ 0.5 \end{pmatrix}$$

   (b) Compute the determinant of A by hand. Compare the determinant of A using MATLAB built-in. Can you reason the discrepancy?

5. Model and solve the structural engineering problem below using system of linear equations:

   The following diagram depicts a plane truss having 13 members (the numbered lines) connected by 8 joints (the numbered circles). The indicated loads, in tons, are applied at joints 2, 5 and 6, and we wish to determine the resulting force on each member of the truss.



   For the truss to be in static equilibrium, there must be no net force, horizontally or vertically at any joint. Thus, we can determine the member forces to the left and right at each joint and equate them, and similarly equate the vertical forces upward and downward at each joint to form a system of linear equations. For the eight joints, this would give 16 equations, which is more than the 13 unknown forces to be determined. For there to be a unique solution of unknown forces, treat joint 1 to be rigidly fixed both horizontally and vertically, and that joint 8 is fixed vertically. Doing this you have a square system to solve by resolving the member forces into horizontal and vertical components. Assume that all triangles your see are 45-45-90 degree triangles.

   For example, the equations corresponding to member forces on joint 2 are given by:
   $f_2 = f_6$
   $f_3 = 10$

   Use MATLAB to find forces on each member of the truss.

6. Write a script to find the best least-squares line through a given set of points of the form (x, y). Test your script on the following data
   $(0.0, 2.9)$
   $(0.5, 2.7)$
   $(1.0, 4.8)$

$(1.5, 5.3)$
$(2.0, 7.1)$
$(2.5, 7.6)$
$(3.0, 7.7)$
$(3.5, 7.6)$
$(4.0, 9.4)$
$(4.5, 9.0)$
$(5.0, 9.6)$
$(5.5, 10.0)$
$(6.0, 10.2)$

Plot the points and the line in MATLAB. Output the slope and the intercept of the line. Also output the total least-squares error, as well as the individual residual (error) of each point to the line.

7. Write a script to find the best parameters of the general curve $y = ax^2 + bx + c$ that best fits the following data in the least square sense:
$(0.0, 2.9)$
$(0.5, 2.7)$
$(1.0, 4.8)$
$(1.5, 5.3)$
$(2.0, 7.1)$
$(2.5, 7.6)$
$(3.0, 7.7)$
$(3.5, 7.6)$
$(4.0, 9.4)$
$(4.5, 9.0)$
$(5.0, 9.6)$
$(5.5, 10.0)$
$(6.0, 10.2)$
$(6.5, 9.7)$
$(7.0, 8.3)$
$(7.5, 8.4)$
$(8.0, 9.0)$
$(8.5, 8.3)$
$(9.0, 6.6)$
$(9.5, 6.7)$
$(10.0, 4.1)$

Plot the data as well as the best parameterize curve. Output the parameters and the total least-square error.