FIN 4500
Chapter 1
Analytical Skills I

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What's new?

- FIN 3000
- FIN 4100
- FIN 4500

Risk Clearly Matters!

- We face all sorts of risks. Use your school work and future job as an example
  - Not getting a decent grade
  - Finding a job; finding a great job
  - The company I work for may go bankrupt; not doing well
  - The risk of recession; the risk of high inflation
- Stay safe

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Concepts you need to Know

- Random variable (r.v.)
- Probability distributions
  - Discrete vs. continuous random variables
  - Objective vs. subjective
  - Where do probability distributions come from?
- Characteristics of probability distributions
  - Expected value
  - Standard deviation and variance
  - Skewness
  - Percentile values
    - Maximum probable loss
    - Value at risk

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Outline of Concepts you need to Know

Risk Management & Probability Theory

- To study how to manage risk, we need some tools
  - Relationship between two random variables
    - Covariance
    - Correlation coefficients
  - Stock return correlations?
  - Probability distributions for sums of random variables
    - Sum of normal distributions is a normal distribution
    - Expected value, variance, and standard deviation
      - of a constant times a random variable
      - of a sum of random variables

from probability theory

- Risk exists when we do not know exactly what will happen,
  - i.e., there is uncertainty about the outcome of some variable
  - Examples
  - We call these variables <u>random variables</u>

Random Variables

- A Random Variable is
    - A variable whose outcome is not known with certainty
    - Variables that are not random variables are called constants
    - Two types of random variables:
        - Discrete –
            - can count the number of outcomes
        - Continuous –
            - outcomes lie on the real line

Probability Distributions

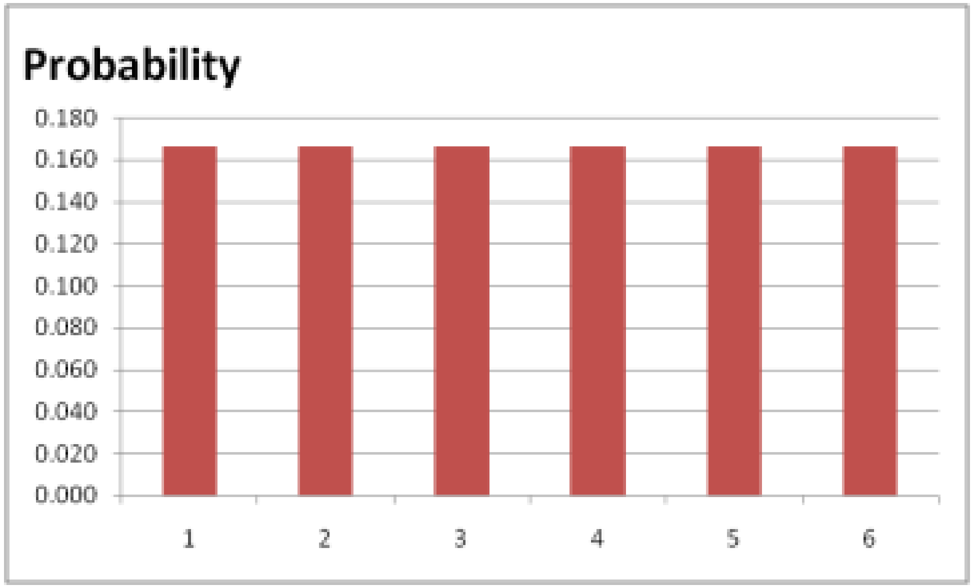
- Probability distribution:
  - Outcomes of a random variable
  - associated probabilities

Presenting Probability Distributions

- Example: roll a die:

  Outcome Prob 1 1/6 2 1/6 3 1/6 4 1/6 5 1/6 6 1/6

  Sum of probabilities =1



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Given the Probability Density Function (PDF), what is the Cumulative Distribution Function (CDF)?

Continuous Probability Distributions

- We will present distributions for continuous random variables graphically:

0.45

0.4

0.35

0.3

0.25

0.2

0.15

0.1

0.05

0

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

The area under the curve between two points gives the probability of the outcome falling in the interval defined by the two points

The area under the entire curve must equal one

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Normal Distribution (approximately)

What is the probability of this random variable equals 6?

0.45

0.4

0.35

0.3

0.25

0.2

0.15

0.1

0.05

0

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

Normal Distribution (approximately)

What is the probability of this random variable being between 6 and 7?

0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

Probabilities with Continuous Distributions

- Find the probability that cash flows < $2,000
- Find the probability that $2,000 < cash flows < $5,000

Probability

Possible Cash

$2,000 $5,000

Flows

Where do Probability Distributions Come From?

- In many situations, **past history** provides information that can help formulate a probability distribution
  - One of the main roles of actuaries is to provide managers information about probability distributions
- Often there is a substantial amount of **<u>subjectivity</u>** involved in formulating probability distributions

Where do Probability Distributions Come From?

- Highlight Our Perspective:
  - We are looking **forward**
  - The future is unknown
  - We model the future as random variables
  - Probability distributions contain all of the information

about the random variables

Use of Discrete Distributions

- Discrete random variables are easier to use for numerical problems (do not need calculus)

- Therefore, we will primarily use discrete random variables as approximations of continuous random variables

Key Points

- Probability distributions describe the **possible outcomes** for a random variable and **probability** of the outcome occurring.

- There are **discrete** random variables and **continuous** random variables

- There is often a **subjective** component when modeling probability distributions

- Often decisions must be made without full information about outcomes and probabilities

Example

Suppose you have the following loss distribution:
(<u>Try to plot the Loss distribution with/without insurance</u>)

20,000 prob = .01
10,000 prob = .02
Loss = 5,000 prob = .05
1,000 prob = .07
0 prob = .85

• What happens to the probability distribution if you purchase insurance with a $1,000 deductible for a premium = $800

Cost	Prob
1,800	.01
1,800	.02
1,800	.05
1,800	.07
800	.85

Characteristics of Probability Distributions

- Instead of comparing entire distributions, managers often work with characteristics of probability distributions:

- Expected value

- Standard deviation or variance
- Skewness
- Percentile values

- How does Risk Management (RM) affect these

characteristics?

Expected Value

- The expected value is the weighted average of the outcomes

<u>Outcome</u> <u>Probability</u>

x1 p1 x2 p2 x3 p3

- Discrete r.v.: X =
  - E(X) = x1*p1 + x2*p2+x3*p3
- Continuous r.v.:
  - Calculating expected value requires integral calculus • But should be able to identify Expected Value visually given the graph of a probability distribution

Standard Deviation and Variance

- Standard deviation indicates the expected magnitude of the error from using the expected value as a predictor of the outcome

- Variance = standard deviation squared

- Formula for variance for X (r.v. from previous page)

- Let E(X) = expected value of X

$$Var(X) = [x1 - E(X)]^2 * p1 + [x2 - E(X)]^2 * p2 + [x3 - E(X)]^2 * p3$$

Example

Suppose you have the following loss

distribution: 20,000 prob = .01

10,000 prob = .02

Loss = 5,000 prob = .05 1,000

prob = .07

0 prob = .85

Expected loss = 720

Standard deviation = 2607.99

What are the expected loss and standard deviation of the distribution?

- What happens to the probability distribution if you purchase insurance with a $1,000 deductible for a premium = $800 <u>Cost</u> <u>Prob</u>

1,800 .01

1,800 .02 1,800 .05 1,800 .07 800 .85

Expected cost = 950
Standard deviation = 357.07

Again, what are the expected cost and standard deviation of the distribution?

Calculate Standard Deviation

- G = GDP growth for 2020

4% prob .4
G = 1% prob .4
-1% prob .2

$E(G) = 4\% * 0.4 + 1\% * 0.4 + (-1\%) * 0.2 = 1.8\%$

$Var(G) = 0.00376$

$Std(G) = 1.94\%$

Example: Compare standard deviations

• Compare standard deviation for 3 discrete r.v.'s

Distribution 1 Distribution 2 Distribution 3

Outcome Prob Outcome Prob Outcome Prob $250 0.33 $0 0.33 $0 0.4 $500 0.34 $500 0.34 $500 0.2 $750 0.33 $1000 0.33 $1000 0.4 • Means of distributions 1, 2, 3 are 500

- Std of 1 < Std of 2 b/c outcomes are farther from the mean
- Std of 2 < Std of 3 b/c extreme outcomes have greater probabilities

Compare Standard Deviation for A & B

Alternative Way to Define Variance/Standard Deviation

$$Var(X) = [x1 - E(X)]^2 p1 + [x2 - E(X)]^2 p2 + [x3 - E(X)]^2 p3$$

Assume $z_i = [x_i - E(x)]^2$; z_i is the deviation squared

$$Var(X) = z1*p1 + z2*p2 + z3*p3$$

- Variance of X becomes the expected value of Z

Skewness

- Skewness measures the symmetry of the distribution
  - No skewness => symmetric
  - Most loss distributions exhibit skewness
  - Is G skewed?

    Yes

4% prob 0.4
G = 1% prob 0.4 -1% prob
0.2

Percentile Values

- If Prob(Y < y) = p

→ y is the p^{th} percentile value

- Examples:

- Prob (profit < \$0) = 0.50

→ \$0 is the 50^{th} percentile value

- Prob(loss < \$1million) = 0.95

→ \$1 million is the 95^{th} percentile value

- Median, Tercile, Quartile (and Inter-quartile range), Quintile, Decile

Common Uses of Percentile Values

- Want a measure of how bad property losses can be, so you ask: losses will only be bigger than this number with probability 0.01.
  - Prob (property loss < \$100 million) = 0.99
  - \$100 million is called the Maximum Probable Loss at the 0.01 (or 0.99) level

Common Uses of Percentile Values

- Portfolio worth $500
- How much could we possibly lose over the coming day?
  - Find the amount such that we will lose more than this amount only 1% of the time
  - Write a probability statement for this quantity:
    - Prob(value of portfolio tomorrow < x) = 0.01
    - Suppose x = $400 ➔ Value at Risk = $100

Value at Risk (VaR)

- VaR is a measure of the potential loss in value on a portfolio over a specified time interval
  - Refer to ***<u>losses</u>***

- If Value at Risk at the 5% level for the next week is \$20 million, then

- Prob(loss in portfolio value ≥ \$20 million) = 0.05

- In words, there is 5% chance that the portfolio will lose more \$20 million over the next week

Why the Normal Distribution?

- The Normal Distribution has special properties that make it useful in many applications.
- Normal Distribution Video

- Even when random variables are not normally distributed, the central limit theorem tells us that using the normal distribution can be appropriate if we are analyzing a sum of independent identically distributed random variables.

Percentile Values for the Normal Distribution

- If X is normally distributed with
  - mean = m
  - standard deviation = s
- Then
  - Probability (X > m + 1.645 s) = 0.05

- Probability (X < m – 1.645 s) =0.05

- Probability (X > m + 1.96 s) = 0.025
- Probability (X < m – 1.96 s) =0.025

- Probability (X > m + 2.330 s) = 0.01
- Probability (X < m - 2.330 s) = 0.01

Important Properties of Normal Distribution • Probability

Area= .05

m+1.645s Value at time T

m

Value at time T

VaR - Example

- Portfolio worth $40 million today. What is the one-day VaR at the 5% level?

- Analysis indicates that the portfolio value 1 day from now is normally distributed with an expected value of \$40 million and a standard deviation of \$2 million

  - From the normal distribution:

$$\text{Prob[Value in 1 day} < \$40 - 1.645(\$2)] = 0.05$$

$$\text{Prob[Value in 1 day} < \$36.71] = 0.05$$

→ 1 day VaR at 5% level = \$3.29 million

VaR – Another Example

- Portfolio worth \$100 million today. What is the one-day VaR at the 1% level?

- Analysis indicates that the portfolio value 1 day from now is normally distributed with an expected value of \$100 million and a standard deviation of \$5 million

  - From the normal distribution:

Prob[Value in 1 day <] = 0.01

Prob[Value in 1 day <] = 0.01

→ 1 day VaR at 1% level = _____ million