FIN 4500
Chapter 10
Market Risk

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- **Market risk (measured by <u>value at risk</u>)** is the risk related to the uncertainty of a financial institution (FI)'s **earnings** on its trading portfolio <u>caused by changes in market conditions</u>
  - **Related** to systematic risk; but **not** exactly same as market risk -- looks at the downside only

- Risks such as <u>interest rate risk, credit risk, liquidity risk and foreign exchange risk</u> affect market risk

- It <u>can be measured over periods</u> as short as one day

- Usually measured in terms of dollar exposure amount or as a relative amount against some benchmark

Market Risk Measures

We discuss different ways to measure market risk in this lecture

- RiskMetrics
- Historic or back simulation
- Monte Carlo simulation
- Expected shortfall
- Regulatory models

History of RiskMetrics Model

- Developed by J.P. Morgan (JPM) in 1992
  - A daily report measuring a financial institution’s market risk

- JPM had a large portfolio of assets in 1994
  - 14 active trading locations
  - 120 independent units trading
    - Fixed income securities; Foreign exchange; Commodities
    - Derivatives; Emerging market securities; Proprietary assets

- It is an independent commercial product
  - 2008, listed on New York Stock Exchange
  - 2010, acquired by MSCI for $1.55 billion

RiskMetrics is on VaR

- Value at Risk (*VaR*) again
  - Definition: The **worst** loss over a given target holding period at a given confidence level under normal market condition

- What question does *VaR* address?

Q1: Given a certain market change or shock, how much could your portfolio suffer?

Q2: If it turns out to be a bad day tomorrow, what is the worst loss of your investment?

Examples for a "Bad Day"

- To define a "bad day" in such a way that the day is so bad, or the loss is so severe that such a day (or loss) occurs under normal market condition only once out of every **20** trading days, put another way, the chance of the occurrence (loss) is only **5%**

- If it turns out to be a bad day tomorrow, what will be the worst loss of your investment, given that day happens once out of every 20 trading days?

Percentile Values for the Normal Distribution (Review)

- If X (*loss*) is normally distributed with
  - mean = m
  - standard deviation = s

- Then
  - Probability (X > **m + 1.645 s**) = 0.05
  - Probability (X > **m + 1.96 s**) = 0.025
  - Probability (X > **m + 2.330 s**) = 0.01

Key Elements of Calculating VaR

- **Holding period**: target time horizon during which you hold your investment position, also target measuring period
  - i.e., for how long?

- **Confidence level**
  - How bad it goes?

- Probability **distribution** of return

DEAR and N-Day VaR

- One-day VaR, or Daily VaR is usually calculated, especially in the case of Riskmetrics model. It is usually termed **Daily Earning at Risk, DEAR**

- **More-than-one-day** VaR can be derived from DEAR from following formula (under the assumption that market volatility is constant over time):

$$\text{N-Day VaR} = \text{DEAR} \times \sqrt{N}$$

VaR through a Risk Factor

Market risk = **Estimated potential loss** under adverse circumstances

VaR = \$ value of the position × Price volatility of the position **(for other assets)**

$$\text{VaR} = \begin{array}{c}\text{\$ Value}\\ \text{of the}\\ \text{position}\end{array} \times \begin{array}{c}\text{Price}\\ \text{sensitivity of}\\ \text{the position}\end{array} \times \begin{array}{c}\text{Potential}\\ \text{adverse move}\\ \text{in yield}\end{array}$$ **(for bonds)**

Details of the RiskMetrics model

- Normal distribution is assumed for market change
- One-day holding period
- 99% confidence level
- Benchmark of market risk management

Basic Methodology and Process

- Calculating DEAR figures for **each** of the business lines (risk factors) – **Standalone** risk
  - Fixed-income securities
  - Foreign Exchange (FX)
  - Equity
  - …

- Portfolio **Aggregation** – Portfolio Risk
  - Different trading positions aggregated
  - Different risk factors aggregated
  - Correlation effect considered

1. Market Risk of Fixed-Income Securities

$$\text{VaR} = \begin{array}{c}\text{\$ value}\\ \text{of the}\\ \text{position}\end{array} \times \begin{array}{c}\text{price}\\ \text{sensitivity of}\\ \text{the position}\end{array} \times \begin{array}{c}\text{Potential}\\ \text{adverse move}\\ \text{in yield}\end{array}$$

This is equivalent to: $dP = P \times (-\frac{D}{1+r}) \times dr$

- P: bond price or the dollar value of the position
- D: duration of a bond
  - Duration is a measure of the sensitivity of the price of a bond or other debt instrument to a change in interest rates.
- -D/(1+r): price sensitivity of the position per unit of interest rate change (modified duration)
- dr: change in interest rate

$$dP = P \times (-\frac{D}{1+r}) \times dr$$

<u>The price change of the bond (dP) is caused by dr.</u>

VaR here is about the change in market interest rate

D is the duration of the bond; P and r are the price and yield of the bond.

1. An increase in interest rate leads to a lower price
2. We try to figure out the price decline when interest rate is abnormally high

The above expression is equivalent to DEAR:

$$\textit{Dollar value of position} \times \textit{price sensitivity} \times \textit{potential adverse move in yield}$$

Example:

Suppose a bank has a $1 million market value position in zero-coupon bonds of 7 years to maturity with a face value of $1,631,483. Today's yield on these bonds is **7.243%** per annum. (suppose compounded annually)

$$dP = P \times \left(-\frac{D}{1+r}\right) \times dr$$

1. An increase in interest rate leads to a lower price
2. We try to figure out the price decline when interest rate is abnormally high

Note: 1,631,483/1.07243^7 = $1 million

Defining Extreme Negative Change

- Define **bad yield changes** such that there is only a **1 percent** chance that yield changes will exceed this amount in either direction
- A yield increase
- **Assume** bond yield changes follow a normal distribution
  - **Mean 0**
  - **Standard deviation is 10 bps**

Percentile Values for the Normal Distribution

- If X (loss) is normally distributed with
  - mean = m
  - standard deviation = s

- Then
  - Probability (X > **m + 1.645 s**) = 0.05
  - Probability (X > **m + 1.96 s**) = 0.025
  - Probability (X > **m + 2.330 s**) = 0.01

  - Under the normal distribution, there is a 1% chance that the interest rate increases more than 2.33*10bp = **23.3 bp**

DEAR for Bond Value

$$dP = P \times \left(-\frac{D}{1+r}\right) \times dr$$

A negative price change means a loss to the bond:

- For the 7-year zero coupon bond,

DEAR = *Dollar value of position* × *price sensitivity* × *potential adverse move in yield*

= \$1,000,000 × (-7/1.07243) × 0.00233

= -\$15,210

- On average, <u>in 99 trading days out of 100</u>, the loss on this position will not exceed \$15,210 over a one-day horizon.

VAR for **N days** of the Bond Holding

- The variance for yield changes over N days = N s^2
  - Daily yield changes are independent from other
- The standard deviation for yield change over N days = $\sqrt{N}s$
- To calculate the potential loss for more than one day:

  Market value at risk (VAR) = DEAR × $\sqrt{N}$

- Example:

  For a **10-day period**, VAR = \$15,210 × $\sqrt{10}$= \$48,098

- At 1% VaR, the firm may lose \$48,098 in 10 days

2. Market Risk of Foreign Exchange

Suppose the bank has a €800,000 (Euro) trading position in spot market. If the exchange rate is €1 = $1.25 then:

- Dollar amount of position = 800,000×1.25 = $1 mil.

Suppose that the daily changes in the $/€ exchange rate over the past year have a mean 0 and standard deviation (s) 0.00565, then:

- 99% of time, adverse moves of the exchange rate will not exceed 2.33 s, i.e., 0.0131645 (=2.33*0.00565)

- Thus, DEAR = $1 million × 0.0131645 = $13,164.

3. Market Risk of Equities

Suppose the bank holds a $1 million trading position in stocks that reflect a U.S. stock market index (e.g., the Wilshire 5000). **So here β=1.**

Suppose that the daily changes in U.S. stock market returns have a mean 0 and standard deviation 2%, then 99% of time, downward moves of stock returns will not exceed 2.33 s (i.e., 0.0466) over a one-day horizon.

- DEAR = $1,000,000 × 0.0466 = $46,600

Market Risk of Equities

- What is the DEAR if the a bank holds a $1 million equity portfolio with $\beta = 1.25$?
- DEAR = $\$1{,}000{,}000 \times 1.25 \times 2.33 \times 0.02 = \$58{,}250$

Aggregating DEAR Estimates

- **Cannot simply sum up individual DEARs**.
- In order to aggregate the DEARs from individual exposures we require the correlation matrix.

	7-year zero	$/Euro €	U.S. Stock index
7-year zero	------	-0.2	0.4
$/Euro €		-------	0.1
U.S. stock index			-------

- FX is for foreign exchanges
  - Corr(zero, FX) = -0.2
  - Corr(zero, index) = 0.4
  - Corr(FX, index)= 0.1

Calculation

- From previous examples, we already know that the daily earning at risk (DEAR) for zero-coupon bonds is $15,210, DEAR for FX is $13,164, DEAR for stocks (if β = 1) is $46,600.

- Then three-asset case:
- DEAR portfolio = $[DEAR_a^2 + DEAR_b^2 + DEAR_c^2 + 2r_{ab} \times DEAR_a \times DEAR_b + 2r_{ac} \times DEAR_a \times DEAR_c + 2r_{bc} \times DEAR_b \times DEAR_c]^{1/2}$

$= [15{,}210^2 + 13{,}164^2 + 46{,}600^2 + 2 \times (-0.2) \times 15{,}210 \times 13{,}164 + 2 \times 0.4 \times 15{,}210 \times 46{,}600 + 2 \times 0.1 \times 13{,}164 \times 46{,}600)]^{1/2}$

= $56,443

Criticisms and Shortcomings of RiskMetrics

There are events that assumption of a **symmetric normal distribution** for all asset returns. For some assets, such as options and short-term securities (bonds), this is highly questionable.

Other market risk measures:

i) Historic or Back Simulation

- Basic idea: Revalue portfolio based on actual prices (returns) on the assets that existed yesterday, the day before that, etc. (usually previous 500 days)
- Then calculate 1% worst-case (5^{th} lowest value of 500 days) outcomes
- Only 1% of the outcomes were lower

Example: VaR of historical distribution

Probability	A	Probability	B
50%	$ 100 m	50%	$ 100m
49%	$ 80 m	49%	$ 92m
1%	$-920 m	**0.25%**	$ -920m
		0.75%	$-1704m

What are VaRs of the two distributions at 1% level?

- $920m

ii) Monte Carlo Simulation VaR

To overcome problem of limited number of observations, synthesize additional observations

- Perhaps 10,000 synthetic observations

Employ historic correlations and the random number generator to synthesize observations

Order the synthetic data and calculate VaR

- VAR at the 1 percent confidence level is the 100th worst simulated loss out of 10,000 observations

Criticisms on VaR

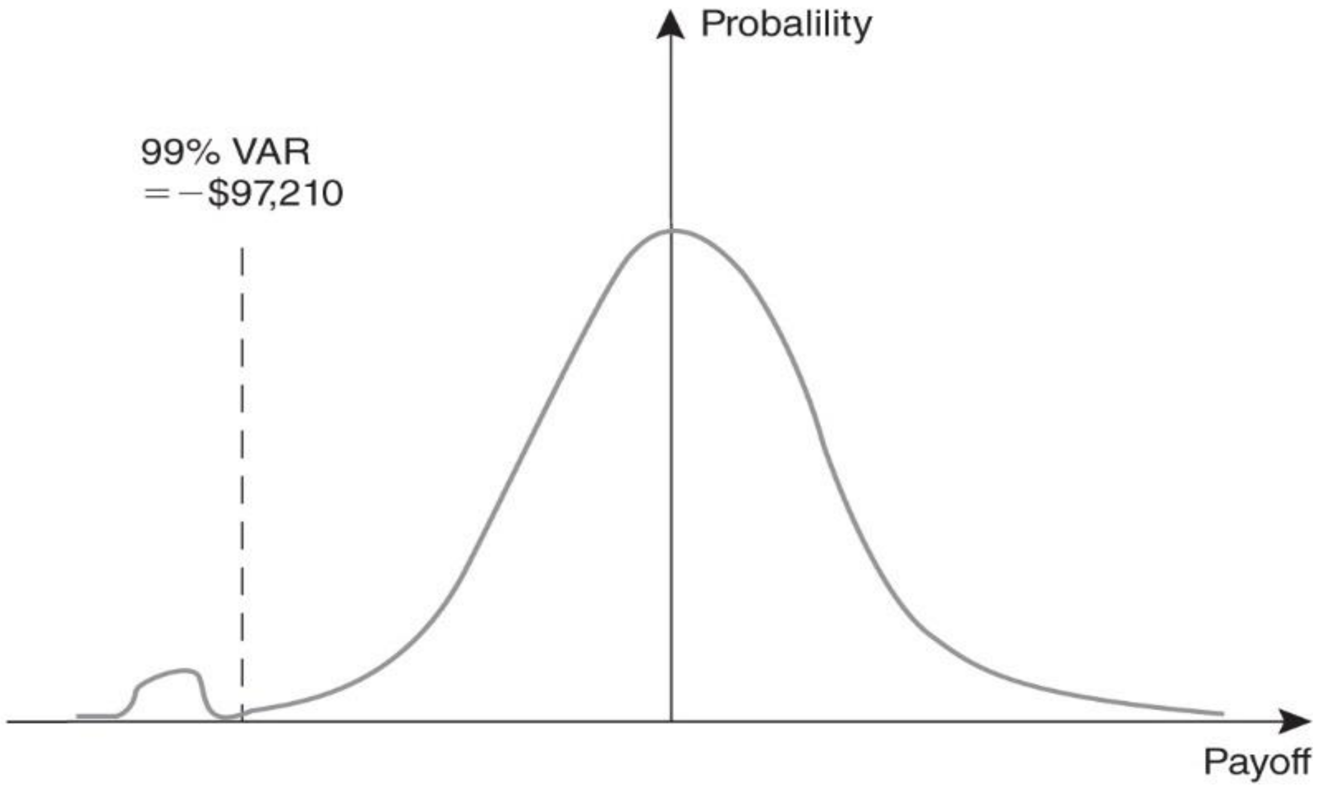
- It does not say **how much a position can lose** on the exceptional hundredth case (Beyond VaR).

- There **isn't enough uniformity** in VAR systems, so the numbers are not comparable between institutions.

- From January 2013, regulators have replaced VaR with the **expected shortfall** measure as the main measure of market risk.

iii) Expected Shortfall

- Expected Shortfall = Conditional VAR = Expected Tail Loss
- Average of the losses in the tail of the distribution beyond the 99th percentile



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Example: Expected Shortfall

Probability	A	Probability	B
50%	$ 100 m	50%	$ 100m
49%	$ 80 m	49%	$ 92m
1%	$-920 m	0.25%	$ -920m
		0.75%	$-1704m

What are expected shortfall of the two distributions at 1% level?

ES(A) = -920m

ES(B) = 0.25*(-920) + 0.75*(-1,704)

= -1,508m

iv) Market Risk Regulations Under Basel III

- Partial risk factor approach
  - 5 risk classes
  - 20 asset buckets
  - Risk weights
  - <u>Obtain capital requirement across different buckets</u>
- Fuller risk factor approach
  - Replace risk classes using risk factors
- Large bank internal models (**following BIS requirements**)
  - Adverse change in rates in 99% percentile
  - Minimum holding period of 10 days
  - Include a “stress” VaR – 10-day, 99% percentile VaR with model inputs incorporating historical data from a one-year period of significant financial stress