Tradeoffs and Comparison Complexity*

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Abstract

This paper develops a theory of how tradeoffs govern comparison complexity, and how this complexity generates systematic mistakes in choice. In our model, options are easier to compare when they involve less pronounced tradeoffs, in particular when they are 1) more similar feature-by-feature and 2) closer to dominance. These two postulates yield tractable measures of comparison complexity in the domains of multiattribute, lottery, and intertemporal choice. We then show how behavioral regularities in choice and valuation, such as context effects, preference reversals, and apparent probability weighting and hyperbolic discounting in valuations, can be understood as responses to comparison complexity. We test our model experimentally by varying the strength and nature of tradeoffs. First, we show that our complexity measures predict choice errors, choice inconsistency, and cognitive uncertainty in binary choice data across all three domains. Second, we document that manipulations of comparison complexity can reverse classic behavioral regularities, in line with the predictions of the theory.

Keywords: Complexity, multi-attribute choice, choice under risk, intertemporal choice, experiments

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1 Introduction

We often face difficult tradeoffs when comparing choice options, and standard economic models abstract from this difficulty. In choosing utility services, households must trade off fixed fees and variable usage fees. In shopping for mortgages, homebuyers weigh brokers' fees and closing costs against interest rates. In selecting health insurance plans, individuals balance premium and deductible costs as well other financial and non-financial features of coverage. In each of these settings, choosing optimally requires the decision-maker to engage in a cognitively challenging process in the presence of tradeoffs: they must both assess the relative values of different option features, and aggregate value across features.

Whether we consider multidimensional pricing schemes, intertemporal payments, or insurance plans, tradeoffs drive comparison complexity across a wide range of choice domains. Formalizing the relationship between tradeoffs and complexity in a way that can be applied across domains is important for at least two reasons. First, existing theoretical work on complexity tends to be domain-specific, which limits researchers' ability to organize the growing body of evidence suggesting that many behavioral regularities from different domains are driven by complexity. Second, a formal model provides a basis for understanding complexity-driven mistakes in the field: both their consequences for individual consumers, and their potential to shape firms' strategic considerations.

This paper has four objectives. First, we develop a theory of comparison complexity in the domains of multiattribute, lottery, and intertemporal choice. Across these domains, our theory formalizes the underlying intuition that comparisons are more difficult when they involve pronounced tradeoffs across option features. Second, we study the behavioral implications of tradeoff-based comparison complexity by embedding our theory of complexity in a choice model. Our model rationalizes a range of documented behavioral regularities — including context effects, preference reversals, and apparent probability weighting and hyperbolic discounting in valuations — and also produces novel predictions on how these regularities can be reversed by manipulating the nature of tradeoffs. Third, we bring experimental evidence to bear on our theory. We use rich experimental data in all three domains to test the novel implications of our theory for choice and valuation. Finally, we apply our model to draw out implications for strategic obfuscation by firms in product markets.

Theory of comparison complexity. We model a decision-maker who is uncertain about the values of two options, x and y, and chooses based on a noisy signal on how these values compare. The precision of this signal, τ_{xy}, captures the ease of comparison between x and

(a) Multiattribute		(b) Lottery		(c) Intertemporal	
x :	\$11/month, \$3.35/GB	x :	\$27 w.p. 25%, \$3 w.p. 75%	x :	\$60 in 61 days
x' :	\$32/month, \$1.6/GB	x' :	\$9 for sure	x' :	\$100 in 3 years
y :	\$10.5/month, \$4.45/GB	y :	\$20 w.p. 20%, \$3.2 w.p. 80%	y :	\$40 in 60 days

Figure 1: Choice Domains. Comparisons between a) phone plans characterized by a monthly and data use fee, b) monetary lotteries, and c) payoff flows.

y, and governs the decision-maker's likelihood of choosing the higher-valued option. We develop a theory of how τ depends on the features of choice options in the domains of multiattribute, lottery, and intertemporal choice.

Our theory is motivated by the observation that decision-makers struggle to aggregate tradeoffs across option features, and that not all comparisons require the same degree of aggregation. Tradeoffs complicate comparisons for at least two reasons: decision-makers may be uncertain over the relative importance of utility-relevant features, and even absent preference uncertainty, aggregating advantages and disadvantages across option features may be difficult. To illustrate, consider the three choice environments in Figure 1. Notice that across these domains, the comparison between x and y is simple — there is little need to make tradeoffs across option features to see that x is better than y. On the other hand, the comparison between x' and y is less obvious, as the DM now must engage with non-trivial tradeoffs across option features: 1a) involves a tradeoff between the monthly fee vs. usage fee, 1b) involves trading off a higher maximum payout against a lower payout probability, and 1c) involves a tradeoff between payout amounts and delays.

Our theory is built on two formal principles that capture this notion of tradeoff complexity: similarity and dominance. First, we posit that holding fixed their value difference, options are easier to compare if they are more *similar* — that is, that the ease of comparison is an increasing transformation H of the *value-dissimilarity* ratio:

$$\tau_{xy} = H\left(\frac{|v_x - v_y|}{d(x,y)}\right),$$

where the numerator contains the value difference between the two options and the denominator is a distance metric measuring their dissimilarity. Intuitively, similar options require less aggregation of tradeoffs to compare, as the DM can divert attention from features that are similar across options and so more easily assess differences. This intuition echoes work in psychology and economics (Tversky and Russo, 1969; Rubinstein, 1988) which has stressed the role of similarity in governing the ease of comparison, and we follow recent

Domain	Representation for τ_{xy}	Distance Metric
Multiattribute $U(x) = \sum_k \beta_k x_k$	$\tau_{xy} = H\left(\frac{\lvert U(x) - U(y)\rvert}{d_{L1}(x,y)}\right)$	$d_{L1}(x,y) = \sum_k \lvert\beta_k(x_k - y_k)\rvert$
Lottery $EU(x) = \sum_w u(w) f_x(w)$	$\tau_{xy} = H\left(\frac{\lvert EU(x) - EU(y)\rvert}{d_{CDF}(x,y)}\right)$	$d_{CDF}(x,y) = \int_0^1 \lvert u(F_x^{-1}(q)) - u(F_y^{-1}(q))\rvert\, dq$
Intertemporal $PV(x) = \sum_t \delta^t x_t$	$\tau_{xy} = H\left(\frac{\lvert PV(x) - PV(y)\rvert}{d_{CPF}(x,y)}\right)$	$d_{CPF}(x,y) = \ln(1/\delta) \int_0^\infty \lvert M_x(t) - M_y(t)\rvert \delta^t\, dt$

Table 1: Complexity Measures. $F_x^{-1}(q) = \inf\{w \in \mathbb{R} : q \leq F_x(w)\}$ denotes the quantile function of a lottery x. $M_x(t) = \sum_{t'<t} x_{t'}$ denotes the cumulative payoff function of a payoff flow x.

work in stochastic choice (He and Natenzon, 2023a,b) in our formalism.

To pin down the specific dissimilarity measure, we appeal to our second principle: that options are maximally easy to compare in the presence of *dominance*. As dominance eliminates the need to make tradeoffs, comparisons involving dominance should be more accurate than comparisons that do not. The relevant dominance notions in each of our domains — attribute-wise dominance in multiattribute choice, first-order stochastic dominance in lottery choice, and temporal dominance in intertemporal choice[1] — give rise to the appropriate dissimilarity measures in each domain, summarized in Table 1; for each of these measures, options are maximally easy to compare when they have a dominance relationship.

The postulates of similarity and dominance are not only satisfied by our representations, but also are key in characterizing them; we show that axioms on binary choice behavior corresponding to the postulates of similarity and dominance, in tandem with other easily understood axioms, characterize our representations for τ_{xy} in each domain.

Behavioral implications of comparison complexity. To study the behavioral implications of tradeoff-based comparison complexity beyond binary choice, we embed our theory of complexity in a multinomial choice model. In the model, the decision-maker faces a menu of options and chooses based on noisy signals on the ordinal value comparison between each pair of options, where the precision of these signals is governed by the ease of comparison.

This model generates two key implications. First, in binary choice, comparison complex-

[1]A payoff flow x *temporally dominates* y if at any point in time, x will have paid off more in total than y.

ity leads to noisy, but unbiased choice. Second, in larger menus, comparison complexity generates systematic distortions: a) context effects in multinomial choice, which occur when competing alternatives are hard to compare to each other but differ in their comparability to other options in the menu, and b) “pull-to-center" effects in the valuation of choice options – that is, when the option being valued is hard to compare to the price list, valuations are compressed towards the center of the price list.

To build intuition, consider the options $x \succ z^1 \succ z^2 \succ ... \succ z^n$, where $\{z^1, z^2 ..., z^n\}$ are themselves perfectly comparable, but hard to compare to x; this situation might arise, for instance, when the z^i are ordered by dominance but do not share a dominance relationship with x. Figure 2 illustrates the choice environments of interest. First, consider the binary menu in panel a). As the DM receives only an imprecise signal on how x and z^1 compare, she may err in her choice, but is more likely to choose the better option x. Next, consider the addition of z^2 to the menu in panel b). The DM is uncertain how x compares to either z^1 or z^2, but is certain that z^1 is superior to z^2: this additional information improves the DM's assessment of z^1 relative to x, and so distorts choice toward z^1, producing a context effect. Note that this information also worsens the DM's assessment of z^2 relative to x, and so pulls her assessment of x's value between those of the unambiguously ranked options z^1 and z^2. This logic generalizes to valuation tasks as in panel c), where the DM values x against $\{z^1, z^2, .., z^n\}$ in a multiple price list: the difficulty of comparing x to the unambiguously ranked items in the price list pulls her valuation of x toward the center of that ranking.

These forces rationalize a range of documented behavioral regularities in choice and valuation. In multiattribute choice, the context effects predicted by our model straightforwardly generate familiar decoy/asymmetric dominance effects. Given our lottery and intertemporal complexity measures, which predict which risky and intertemporal prospects are hard to compare to prices, our model also rationalizes documented regularities in valuation: preference reversals and apparent probability weighting and hyperbolic discounting, specifically because these patterns reflect pull-to-center biases.

(a) Binary Choice

$$x \quad \text{vs.} \quad z^1$$

(b) Context Effects

$$x \quad \text{vs.} \quad \begin{array}{c} z^1 \\ \curlyvee \\ z^2 \end{array}$$

(c) Valuations

$$x \quad \text{vs.} \quad \begin{array}{c} z^1 \\ \curlyvee \\ z^2 \\ \curlyvee \\ z^3 \\ \vdots \\ z^n \end{array}$$

Figure 2: Choice environments. $z^1, z^2, ..., z^n$ are easy to compare to each other, but hard to compare to x.

To illustrate, consider the preference reversal phenomenon: given lottery x which pays a large sum with a low chance, and y which pays a modest sum with a high chance, e.g.

$$x: \quad \$23.50 \text{ with } 19\% \text{ chance}$$
$$y: \quad \$4.75 \text{ with } 94\% \text{ chance}$$

subjects assess higher certainty equivalents for x, yet are more likely to choose y in direct choice. As x is harder to compare to money than y under our complexity notion, its valuation is biased upward toward the middle of the range of undominated prices, whereas the valuations of y are relatively accurate, and if anything are biased downward. Crucially, these pull-to-center distortions result only from the difficulty of valuing lotteries against money and so do not affect direct choice, thus rationalizing the reversal. As this effect operates through the differential ease of comparing prospects to prices, our model generates similar reversals in intertemporal choice, and also predicts that one can *reverse* the direction of these valuation distortions by manipulating the ease of comparing each prospect to prices: specifically by changing the numeraire against which options are valued. For instance, our model predicts that eliciting valuations of x and y in terms of probability equivalents — the probability p that makes each lottery indifferent to \$24 with p% chance — reverses the direction of the valuation distortions as now x is easier to compare to prices.

The same pull-to-center distortions that produce preference reversals in our model also generate the familiar patterns of probability weighting and hyperbolic discounting in standard valuation tasks. To illustrate, consider the valuation of a binary lottery l with payout probability p. The difficulty of comparing l to certain payments when the payout probability is away from the dominance points of $p = 0$ or $p = 1$ produces pull-to-center distortions: certainty equivalents will be compressed upward for small probabilities and downward for large probabilities, generating inverse-S probability weighting. The same logic generates apparent hyperbolic discounting in the valuation of delayed payments. Importantly, our model predicts that these pull-to-center distortions are not generic, but instead arise from the difficulty of comparing prospects to prices in canonical valuation tasks. In particular, our model generates the novel prediction that one can reverse the direction of probability-weighting and hyperbolic discounting through similar manipulations of tradeoffs in the valuation task.

Experimental tests. We conduct two sets of experiments. First, we test the validity of our proposed complexity measures using three large-scale binary choice datasets corresponding to our domains of interest: choice between induced-values multiattribute goods, lotteries, and time-dated payoff streams. In line with the theory, our complexity measures are strongly predictive of choice inconsistency, choice errors, and subjective uncertainty over choices in all three datasets. When a choice problem involves a lower value-dissimilarity ratio, subjects are more likely to make inconsistent choices across repeat instances of that problem, and are more likely to make choice "errors", where we define an error as a) choosing the lower-value option in multiattribute choice and b) choosing the less-preferred option according to a best-fit model of risk and time preferences in lottery and intertemporal choice, respectively. These relationships are quantitatively large: multiattribute choice inconsistency and error rates are more than four times as large for choice problems with the lowest versus highest value-dissimilarity ratio, with similarly pronounced relationships in our other domains. Furthermore, our complexity measures predict not only choice difficulty as revealed in behavior, but also the subjective difficulty of comparisons: the value-dissimilarity ratio is strongly negatively correlated with the uncertainty subjects report over their choices.

Second, we run a series of experiments to demonstrate that both preference reversals and classic distortions in lottery and intertemporal valuation are in part outgrowths of comparison complexity – specifically by showing that these patterns can be eliminated or reversed by manipulating the nature of tradeoffs as our model predicts.

In our preference reversals experiments, we 1) reproduce classic reversals in lottery choice and establish the presence of similar reversals in intertemporal choice, and 2) document that these reversals can be eliminated by manipulating the relative ease of comparing options to the price list. When valuing options against money, subjects have higher valuations for high-risk vs. low-risk lotteries (high-delay vs. low-delay intertemporal payments) — yet exhibit the opposite preference when choosing directly between options. In line with model predictions, however, the apparent inconsistency between valuations and direct choice disappears when subjects instead value lotteries (intertemporal payments) in terms of probability equivalents (time equivalents).

Finally, we document that two classic valuation patterns, probability-weighting and hyperbolic discounting, can be reversed by manipulating tradeoffs in the valuation task. In line with model predictions, we find that using probability equivalents rather than certainty equivalents to estimate the probability weighting function results in apparent *underweighting* of small probabilities and *overweighting* of large probabilities; similarly, relative to the discounting revealed by their present value equivalents, subjects' time equivalents exhibit *overvaluation* for delays close to the present and *undervaluation* for longer delays.

Benchmarking predictive power. Using our binary choice datasets, we demonstrate the predictive power of our theory. We structurally estimate a model in which choice rates depend only on the value-dissimilarity ratio, and compare its performance to leading benchmarks. In all three domains, our model explains a substantial share of variation in choice rates not captured by existing models, and in lottery and intertemporal choice, our model substantially outperforms leading models; our model explains 10–28% more variation than the leading models of cumulative prospect theory and hyperbolic discounting in these domains.

Application to strategic obfuscation. We use our model to study a Bertrand pricing game in which firms sell products of identical value to consumers, and can influence how comparable their products are to their competitors through choice of how value is distributed across product attributes. We find that in a symmetric environment, firms design dissimilar products in order to minimize the ease of comparison to their competitors and therefore soften price competition, which leads to spurious differentiation and positive equilibrium markups. On the other hand, when there are vertical cost differences between firms, firms with a sufficiently large cost advantage have incentives to design products that are easy to compare to high-cost competitors, and compete directly on price. As a result, our model predicts that an increase in competitive pressure brought on by lower costs can actually increase prices and obfuscation in equilibrium. We connect our predictions to field evidence, and discuss the extent to which our model of complexity-driven mistakes can be distinguished from alternative accounts of product proliferation, such as preference heterogeneity.

Contribution and relation to prior work. This paper builds on a growing literature on complexity in choice. While a growing body of experimental evidence has documented that a number of choice patterns across disparate domains are driven by mistakes or confusion (e.g. Oprea, 2022; Enke and Graeber, 2023; Enke et al., 2023), formal accounts of complexity typically focus on a single choice domain (e.g. Puri, 2023; Hu, 2023; Sanjurjo, 2023). Our paper proposes measures of comparison complexity in three distinct choice domains based on the same formal principles, and shows how a range of behavioral regularities in valuation and choice can be rationalized using the logic of comparison complexity.

Our theory of comparison complexity builds on a recent stochastic choice literature (Natenzon, 2019; He and Natenzon, 2023a,b) which formalizes the notion that choice depends not only on the value difference between options but also their similarity. Specifically, our model belongs to a class of *moderate utility models* axiomatized in He and Natenzon (2023a), wherein binary choice probabilities are a function of the value difference between two options normalized by a distance metric. We make three contributions to this literature. First, we propose specific distance metrics in three distinct choice domains, and show how these measures arise from a common set of cognitively grounded properties reflecting tradeoff complexity. Second, we bring experimental evidence to bear on this literature, quantifying the tight relationship between our proposed complexity measure and cognitive uncertainty, choice inconsistency, and choice errors. Third, we draw a connection between this literature and the growing body of experimental evidence on complexity. Specifically, we show how our theory of binary comparisons can be extended to a multinomial choice model that accounts for a range of documented choice patterns, and use the model to develop new testable predictions in these settings.

Section 2 develops the measure of comparison complexity. Section 3 develops the multinomial choice model, and applies the it to study context effects and valuations. Section 4 describes the experimental tests of the model. Section 5 discusses our application to strategic firm obfuscation. Section 6 discusses relationships between our theory and existing models and concludes. Appendix C contains proofs of all results stated in the main text.

2 Theory of Comparison Complexity

Let X denote the set of options, and let v_x denote the value of each $x \in X$. We consider a decision-maker (DM) who is uncertain about the value of each option, and when faced with a binary choice $\{x, y\}$, chooses based on a noisy signal on how v_x and v_y compare. In particular, the DM has continuous, i.i.d. priors over v_x for all $x \in X$ distributed according to a symmetric distribution Q, and observes a signal s_{xy} on the ordinal value comparison between x and y, given by

$$s_{xy} = \operatorname{sgn}(v_x - v_y) + \frac{1}{\sqrt{\tau_{xy}}}\epsilon_{xy},$$
$$\epsilon_{xy} \sim N(0,1)$$

and chooses the option with the highest posterior expected value. Here, the precision parameter τ_{xy} governs the *ease of comparison* between x and y. Letting $\rho(x, y)$ denote the likelihood[2] of choosing x over y conditional on v_x and v_y, we have $\rho(x, y) = \Phi(\operatorname{sgn}(v_x - v_y)\sqrt{\tau_{xy}})$, for Φ the standard normal CDF. That is, the decision-maker's likelihood of choosing the higher-valued option is increasing in the ease of comparison τ_{xy}. Let $\tau : \{(x, y) \in X^2 : x \neq y\} \to \mathbb{R}^+$ denote the associated mapping from each pair of options to their ease

[2]In particular, $\rho(x, y) = \mathbb{P}(\mathbb{E}[v_x|s_{xy}] > \mathbb{E}[v_y|s_{xy}] \,|\, v)$, where the DM randomizes if $\mathbb{E}[v_x|s_{xy}] = \mathbb{E}[v_y|s_{xy}]$.

of comparison. In what follows, we propose a theory of how τ depends on the structure of choice options in the domains of multiattribute, lottery, and intertemporal choice.

2.1 Comparison Complexity: General Principles

Our theory of τ formalizes the intuition that the difficulty of a comparison is governed by the degree to which it requires the DM to aggregate tradeoffs. The theory is grounded in two principles that capture this intuition: similarity and dominance.

First, we posit that holding fixed the value difference, options are easier to compare when they are more *similar*. Echoing work in psychology and economics (Tversky and Russo, 1969; Rubinstein, 1988; He and Natenzon, 2023a), this property reflects the intuition that if options are more similar, the DM can divert attention from the features that are similar across the options and so more easily assess the differences between them, thereby reducing the need to aggregate tradeoffs. Formally, we posit that the ease of comparison τ_{xy} is an increasing transformation H of the *value-dissimilarity* ratio:

$$\tau_{xy} = H\left(\frac{|v_x - v_y|}{d(x,y)}\right),$$

where the numerator contains the value difference between the two options and the denominator is a distance metric measuring their dissimilarity.

Second, we posit that options are maximally easy to compare when they have a *dominance* relationship, i.e. when there are no tradeoffs. As formalized below, this principle gives rise to specific distance metrics given the domain-relevant dominance notion.

2.2 Multiattribute Choice

Consider the domain of multiattribute choice, where each option $x \in X_1 \times \ldots \times X_n$ is defined on n real-valued attributes, i.e. $X_i = \mathbb{R}$. Utility is linear in attributes, where the value of each option x is given by $v_x = U(x) = \sum_k \beta_k x_k$ for attribute weights $\beta \in \mathbb{R}^n$. We propose that the ease of comparison in this domain is governed by the following representation:

Definition 1. *Say that τ has an L_1-complexity representation, denoted τ^{L1}, if there exists $\beta \in \mathbb{R}^n$, $\beta_k \neq 0$ for all k, such that*

$$\tau_{xy} = H\left(\frac{|U(x) - U(y)|}{d_{L1}(x,y)}\right)$$

for H continuous, strictly increasing with $H(0)=0$, where $d_{L1}(x,y)=\sum_k |\beta_k(x_k-y_k)|$ is the L_1 distance between x and y in value-transformed attribute space.

In words, under the L_1-complexity representation, the ease of comparison between two options is governed by their *value-dissimilarity ratio*: their *aggregated* value difference, normalized by a distance metric equal to the summed *feature-by-feature* value differences.

Note that this complexity representation satisfies the properties of *similarity* and *dominance*: holding fixed the value difference between two options, the ease of comparison is increasing in the similarity between x and y as measured by a distance metric, where this metric is chosen so that τ_{xy} achieves its maximal value in the presence of a dominance relationship. Specifically, if there is an attribute-wise dominance relationship between x and y; i.e. $\beta_k x_k \geq \beta_k y_k$ for all k, the ease of comparison τ_{xy} takes on its maximal value of $H(1)$.[3]

L_1-complexity also satisfies a *simplification* property, wherein reducing the number of attributes along which there is a value difference increases the ease of comparison. To take an example, suppose $n=3$, $\beta=(1,1,1)$, and consider the following comparisons:

$$\begin{array}{llcll} & (x,y) & \qquad & & (x',y) \\ x & =(10,7,9) & & x' & =(3,14,9) \\ y & =(3,15,5) & & y & =(3,15,5) \end{array}$$

Note that (x',y) is formed by eliminating the value difference along the first attribute and redistributing that value to the second attribute. Our complexity representation predicts that the DM finds (x',y) easier to compare than (x,y), i.e. $\tau_{x'y} > \tau_{xy}$. More generally, our model predicts that eliminating a value difference along some attribute i and redistributing it to another attribute j makes options easier to compare.[4] This property again reflects tradeoff complexity: if an individual finds it difficult to aggregate tradeoffs across features, we might expect that a simplification operation of the kind above, where some of that aggregation is done for the individual, makes the comparison easier.

2.2.1 Axiomatic Foundations

We now show that the above properties are not only satisfied by our complexity representation, but are also key properties in its characterization: specifically, L_1-complexity is characterized by axioms on binary choice behavior corresponding to the properties of similarity,

[3]To see this, note that by the triangle inequality we have $|\sum_k \beta_k(x_k-y_k)| \leq \sum_k |\beta_k(x_k-y_k)|$, and $|\sum_k \beta_k(x_k-y_k)| = \sum_k |\beta_k(x_k-y_k)|$ if and only if x and y have a dominace relationship.

[4]That is, given any x,y, for x' satisfying $x'_i=y_i$, $x'_k=x_k$ for all $k\neq i,j$, with $v_{x'}=v_x$, we have $\tau_{x'y}\geq\tau_{xy}$.

dominance, and simplification, along with two other easily understood axioms.

Let $\mathscr{D} = \{(x,y) \in \mathbb{R}^n \times \mathbb{R}^n : x \neq y\}$ denote the set of all pairs of distinct options. Consider a *binary choice rule* $\rho : \mathscr{D} \to [0,1]$ satisfying $\rho(x,y) = 1-\rho(y,x)$ for all x,y; here, $\rho(x,y)$ denotes the likelihood of choosing x over y. In our binary choice framework, τ has an L_1-Complexity representation if and only if binary choice probabilities take the form below, wherein the likelihood of choosing x over y is an increasing function of the signed value difference between the two options, normalized by their L_1 distance.

Definition 2. *Say that a binary choice rule ρ has an L_1-Complexity Representation if there exists $\beta \in \mathbb{R}^n$, $\beta \neq 0$, such that*

$$\rho(x,y) = G\left(\frac{U(x)-U(y)}{d_{L1}(x,y)}\right)$$

for some G continuous, strictly increasing, satisfying $G(r) = 1 - G(-r)$.[5]

Let $x_{\{k\}}y$ denote the option obtained by replacing the kth attribute of y with x_k[6]. Say that x *dominates* y, written $x >_D y$, if $\rho(x_{\{k\}}y, y) \geq 1/2$ for all k with a strict inequality for at least one k. Say that attribute k is *null* if $\rho(x_{\{k\}}z, y_{\{k\}}z) = 1/2$ for all $x,y,z \in X$. Consider the following axioms on ρ:

M1. **Continuity:** $\rho(x,y)$ is continuous on its domain.

M2. **Linearity:** $\rho(x,y) = \rho(\alpha x + (1-\alpha)z, \alpha y + (1-\alpha)z)$.

M3. **Moderate Transitivity:** If $\rho(x,y) \geq 1/2$ and $\rho(y,z) \geq 1/2$, then either $\rho(x,z) > \min\{\rho(x,y), \rho(y,z)\}$ or $\rho(x,z) = \rho(x,y) = \rho(y,z)$.

M4. **Dominance**: If $x >_D y$, then $\rho(x,y) \geq \rho(w,z)$ for any $w,z \in X$, where the inequality is strict if $w \not>_D z$.

M5. **Simplification**: For any $x, x', y \in X$, satisfying for $i \neq j$

$$x'_k = \begin{cases} y_i & k = i \\ x'_j & k = j \\ x_k & \text{otherwise} \end{cases}$$

for some $x'_j \in X_j$: if $\rho(x,y) \geq 1/2$ and $\rho(x',x) = 1/2$, then $\rho(x',y) \geq \rho(x,y)$.

[5]There is a one-to-one correspondence between G and H, which maps the value-dissimilarity ratio into signal precisions τ_{xy}. In particular, for $r \in [0,1]$, $H(r) = (\Phi^{-1}(G(r))^2$, where Φ is the standard normal CDF.

[6]That is $(x_{\{k\}}y)_k = x_k$ and $(x_{\{k\}}y)_j = y_j$ for all $j \neq k$.

Continuity and Linearity are standard axioms, and the latter reflects the fact that both utility and the L_1 distance are linear in attributes.[7] Moderate Transitivity is a stochastic transitivity condition that allows for choice probabilities to depend on a notion of the similarity between the options being compared, in addition to just their value difference.[8]

Dominance and Simplification are the exact counterparts of the properties of τ^{L1} discussed earlier, stated in terms of choice probabilities. Dominance says that if x is revealed better than y on every attribute, then the likelihood of correctly choosing x takes on its maximal value. Simplification says that eliminating the value difference between x and y along some attribute and redistributing that value to another attribute increases the DM's likelihood of correctly choosing x.[9]

When there are 3 or more attributes, M1–M5 characterize the behavioral implications of our representation for binary choice data. Moreover, the parameters (G, β) of our representation can be identified from binary choice data.[10]

Theorem 1. *Suppose that all attributes are non-null and $n > 2$. $\rho(x, y)$ has a L_1-complexity representation (G, β) if and only if it satisfies M1–M5. Moreover, if $\rho(x, y)$ also has a L_1-complexity representation (G', β'), then $G' = G$ and $\beta' = C\beta$ for $C > 0$.*

2.3 Risky and Intertemporal Choice

2.3.1 Lotteries

Consider the lottery domain, where each option x is a finite-support lottery over $\mathbb{R}$; that is each x is described by the mass function $f_x : \mathbb{R} \to [0, 1]$ where $f_x(w) > 0$ for finitely many w. Let F_x and F_x^{-1} denote the CDF and quantile function of x. Tastes are given by expected utility, with $v_x = EU(x) = \sum_w u(w) f_x(w)$ for u strictly increasing.

Definition 3. *τ has an CDF-complexity representation, denoted τ^{CDF}, if for u strictly increasing,*

$$\tau_{xy} = H\left(\frac{|EU(x) - EU(y)|}{d_{CDF}(x, y)}\right)$$

[7] In Appendix B.1, we extend Theorem 1 to allow for non-linearity in the utility function.

[8] He and Natenzon (2023) show that in a finite domain, ρ satisfies Moderate Transitivity if and only if $\rho(x, y)$ is increasing in the value difference between x and y, normalized by *some* distance metric $d(x, y)$. While this result does not apply to our domain as it is not finite, our other axioms can be thought of as adding structure to this distance metric.

[9] Hammond et al. (1998) argue that precisely this operation can be used to rationally simplify tradeoffs between multi-dimensional options.

[10] In Appendix B.1, we extend Theorem 1 to the two-attribute case.

for H continuous, strictly increasing with $H(0) = 0$, where the CDF distance d_{CDF} is given by

$$d_{CDF}(x, y) = \int_0^1 |u(F_x^{-1}(q)) - u(F_y^{-1}(q))| \, dq$$

As with the L_1-complexity representation, the ease of comparison between two options under the CDF-complexity representation is governed by their value-dissimilarity ratio — that is, the value difference between the two options normalized by a measure of their dissimilarity. The specific dissimilarity measure in our representation, $d_{CDF}(x, y)$, is a metric equal to the area between the utility-valued CDFs of x and y, and so captures how similarly the payoffs in x and y are distributed.[11]

This measure provides a formal foundation for empirical work which has documented a tight connection between the CDF distance and choice rates. In particular, Enke and Shubatt (2023) and Erev et al. (2008) show that the performance of lottery choice models markedly improves when choice noise is allowed to vary with the a special case of d_{CDF} in which u is linear. Fishburn (1978) proposes and axiomatizes a closely related model of binary choice over lotteries in which choice probabilities are increasing in the CDF ratio.[12] As discussed in Section 2.3.4 and Appendix B.1, we provide an alternative axiomatic foundation for this choice model using properties analogous to our multiattribute axioms. Later, we also extend each of our models to multinomial choice.

2.3.2 Intertemporal Payoff Flows

Now consider the intertemporal domain, where each option x is a finite payoff stream described by the *payoff function* $m_x : [0, \infty) \to \mathbb{R}$, where $m_x(t) \neq 0$ for finitely many t; $m_x(t)$ describes how much x pays off at time t. Let $M_x(t) = \sum_{t' \leq t} m_x(t')$ denote the *cumulative payoff function* of x, which describes how much money x pays in total up to time t. Utility is given by exponential discounting, with $v_x = PV(x) = \sum_t \delta^t m_x(t)$, for $\delta < 1$.[13]

Definition 4. *τ has an CPF-complexity representation, denoted τ^{CPF}, if there exists $\delta < 1$ such that*

$$\tau_{xy} = H\left(\frac{|PV(x) - PV(y)|}{d_{CPF}(x, y)}\right)$$

[11] d_{CDF} is a special case of the Wasserstein metric, a distance notion defined on probability distributions over a metric space.

[12] We thank Paulo Natenzon for bringing our attention to this paper.

[13] In Appendix B.1 we show how the theory can be generalized to allow for a general decreasing discount function $d : [0, \infty) \to \mathbb{R}^+$, and provide an axiomatic characterization of this generalized model.

for H continuous, strictly increasing with $H(0) = 0$, where the CPF distance d_{CPF} is given by

$$d_{CPF}(x, y) = \ln(1/\delta) \int_0^\infty |M_x(t) - M_y(t)| \cdot \delta^t \, dt.$$

As with our previous complexity measures, the ease of comparison between two options under the CPF-complexity representation is governed by their value-dissimilarity ratio, where the dissimilarity measure $d_{CPF}(x, y)$ is a metric that is proportional to the present value of the difference between the cumulative payoff functions of x and y, and captures how similarly x and y distribute their payoffs across time.

2.3.3 Shared Properties and Axiomatic Foundations

As with our multiattribute complexity measure, τ^{CDF} and τ^{CPF} satisfy our core properties of similarity and dominance. Holding fixed the value difference, τ^{CDF} and τ^{CPF} are increasing in the similarity between between options as measured by a distance metric — where the metric is chosen so that τ takes on its maximal value when the options have a dominance relationship. In particular, say that a lottery x *first-order stochastically dominates* y when $F_x(w) \leq F_y(w)$ for all w, and say that a payoff flow x *temporally dominates* y if if $M_x(t) \geq M_y(t)$ for all t — that is, if x will have paid out more in total than y at any point in time. τ_{xy}^{CDF} takes on its maximal value of $H(1)$ when there is a first-order stochastic dominance relationship between lotteries x and y, and τ_{xy}^{CPF} takes on their maximal value of $H(1)$ when there is a temporal dominance relationship between payoff flows x and y.

τ^{CDF} and τ^{CPF} also satisfy analogs of the simplification property for τ^{L1}, which says that aggregating value differences across different features into the same feature makes options easier to compare. As formally stated in Appendix B.1, concentrating value differences from different regions in the distribution of two lotteries into the same region increases the ease of comparison under to τ^{CDF}, and concentrating value differences from different time periods of two payoff flows into the same period increases the ease of comparison under τ^{CPF}.

The binary choice behavior implied by τ^{CDF} and τ^{CPF} can be characterized using axioms analogous to M1—M5. In Appendix B.1, we show how the binary choice rules corresponding to τ^{CDF} and τ^{CPF} are characterized by five axioms: direct translations of Continuity, Linearity, Moderate Transitivity, and Dominance, and an analog of Simplification.

Our complexity measures for risk and time not only share the same properties of L_1 complexity, but can also be seen as an extensions of the complexity notion. In Appendix B.2, we show that the CDF (CPF) complexity between two lotteries (payoff flows) is equivalent to the L_1 complexity computed over a common attribute representation of those choice options — specifically, the attribute representation that maximizes their ease of comparison. This suggests the following two-stage cognitive interpretation of our CDF and CPF complexity measures: in a “representation stage", the DM first represents the options using a common attribute structure to make them easier to compare, and then compares the options along these attributes in an “evaluation stage".

2.4 Parameterizing the Model

In each domain, choice probabilities under our model take the form

$$\rho(x,y) = G\left(\frac{U(x)-U(y)}{d(x,y)}\right),$$

where the signed value-dissimilarity ratio $\left(\frac{U(x)-U(y)}{d(x,y)}\right)$ is specified in each domain according to Definitions 1,3, and 4, and G is a strictly increasing transformation satisfying $G(r) = 1-G(-r)$. To obtain quantitative predictions, the analyst needs to specify the preference parameters that enter the value-dissimilarity ratio — the attribute weights β in multiattribute choice, the Bernoulli utility u in lottery choice, and the discount factor δ in intertemporal choice — as well as the transformation G. Each of these objects can be identified from binary choice data, as stated in Theorem 1 for multiattribute choice, and in Appendix B.1 for our other two domains.

Our preferred specification of G is given by the two-parameter functional form

$$G(r) = \begin{cases} (1-\kappa)-(0.5-\kappa)(1-r)^{\gamma} & r \geq 0 \\ \kappa + (0.5-\kappa)(1+r)^{\gamma} & r < 0 \end{cases} \tag{1}$$

Here κ is a tremble parameter that governs the error rates at dominance, and γ governs the curvature in the relationship between choice rates and the value-dissimilarity ratio. [14]

3 Multinomial Choice

We have thus far focused on comparison complexity in binary choice. We now extend our model to multinomial choice and show how comparison complexity can rationalize a range

[14] In Appendix E we also consider a three-parameter functional form that nests (1), wherein the additional parameter governs the sensitivity of choice rates to r around indifference.

of documented behavioral regularities, including preference reversals and apparent probability weighting and hyperbolic discounting in valuation tasks. We develop novel predictions on how manipulating tradeoffs can cause these patterns to disappear or even reverse.

3.1 Multinomial Choice Extension

Consider the same setting as in our binary choice framework. There is a set of options X, and the DM has continuous, i.i.d. priors over v_z for all $z \in X$, distributed according to a symmetric distribution Q. Let $\mathscr{M}$ denote the collection of finite subsets of X, and let $\mathscr{A} = \{A \in \mathscr{M} : |A| \geq 2\}$ denote the set of finite *menus*. The DM faces a *choice problem* $(A, C) \in \mathscr{A} \times \mathscr{M}$, comprised of a *menu* of options A and a *choice context* C – a set of options the DM observes but cannot choose, i.e. *phantom options*. The DM chooses from A based on signals on how each pair of options in $A \cup C$ compare.

In particular, for each pair of distinct options $x, y \in A \cup C$, the DM observes the signal

$$s_{xy} = \text{sgn}(v_x - v_y) + \frac{1}{\sqrt{\tau_{xy}}}\epsilon_{xy},$$
$$\epsilon_{xy} \sim N(0,1)$$

Letting s denote the collection of these signals, the DM chooses the option $x \in A$ with the maximal posterior expected value $\mathbb{E}[v_x|s]$. We are interested in the resulting choice probabilities for a choice problem, which are given by[15]

$$\rho(x, A|C) = \mathbb{P}(\{s : \mathbb{E}[v_x|s] > \mathbb{E}[v_y|s] \,\forall\, y \in A/\{x\}\} \,|\, v)$$

Note that on the set of binary choice problems, i.e. (A, C) such that $|A| = 2$ and $C = \varnothing$, this model is exactly the binary choice model studied in Section 2. With some abuse of notation, we let $\rho(x, y) = \rho(x, \{x, y\}|\varnothing)$ denote binary choice probabilities, and let $\rho(x, y|C) = \rho(x, \{x, y\}|C)$ denote binary choice probabilities given a choice context C.

To apply the choice model, the analyst needs to specify the ranking of the choice options according to v_z and the precision parameters τ_{xy}. While these parameters can be identified using binary choice behavior (see Appendix B.4), our approach will be to discipline the model using our theory of comparison complexity, which pins down v_x and τ_{xy} in our domains of interest.

[15]This formulation for $\rho(x, A|C)$ holds when ties in posterior expectations occur with probability 0. In the case of ties, we assume a symmetric tiebreaking rule. See Appendix B.3 for details.

3.2 Model Properties

This model has two key implications. First, it predicts that comparison complexity leads to noisy but unbiased choice in binary menus — the DM may err, but is more likely to pick the higher-value option. Second, it predicts systematic biases in choice from larger menus: when facing hard-to-compare alternatives, the DM relies on information from comparisons to other options, which can distort choice. This generates a) context effects when options are hard to compare to each other but differ in their ease of comparison to other options in the choice context, and b) systematic pull-to-center distortions in valuations as a function of how hard the option being valued is to compare to the numeraire.

3.2.1 Context Effects

To illustrate how the model generates systematic context effects, consider an example where $X = \{x, y, z\}$, with $v_x > v_y > v_z$ and where $\tau_{xy} = \tau_{xz} = 0$, $\tau_{yz} = \infty$. That is, the DM has no idea how x compares to y and z, but knows y is better than z. Here, the model predicts that $\rho(y, x|\{z\}) = 1$: the presence of z in the choice context provides additional information that rules out posterior beliefs over (v_x, v_y, v_z) in which $v_y < v_z$, thus distorting the the DM's choice in favor of the inferior option y. This is generalized in the following proposition, which says that if x and y are sufficiently hard to compare, the presence of an inferior option z that is easier to compare to y distorts choice in favor of that option.[16]

Proposition 1. *Let $v_x, v_y > v_z$. If $\tau_{yz} > \tau_{xz}$, there exists $\epsilon > 0$ such that if $\tau_{xy} < \epsilon$, $\rho(y, x|\{z\}) > 1/2$.*

When combined with our theory of comparison complexity in multiattribute choice, this result rationalizes familiar decoy and asymmetric dominance effects.

Corollary 1.1. *Consider options from $X = \mathbb{R}^n$, with $v_x = \sum \beta_k x_k$ and let τ have an L_1-complexity representation. Let $v_x, v_y > v_z$.*

(i) If $v_x = v_y$, then $d_{L1}(x,z) > d_{L1}(y,z)$ implies $\rho(y, x|\{z\}) > 1/2$.

(ii) For any value difference $\Delta = |v_x - v_y|$, there exists $\underline{d} \in \mathbb{R}^+$ such that if $d_{L1}(x, y) > \underline{d}$, there exists $z \in X$ with $d_{L1}(x,z) > d_{L1}(y,z)$ such that $\rho(y, x|\{z\}) > 1/2$.

[16] In Appendix C, we also consider the analogous result that the addition of a superior option z to the choice context can bias choice in favor of x if $\tau_{yz} > \tau_{xz}$.

Part (i) says that if x and y are indifferent, then introducing an inferior phantom option z that is more similar to y than x distorts choice in favor of y. Part (ii) says that if x and y are sufficiently dissimilar relative to their value difference, there exists a decoy that distorts choice in favor of y. In Appendix B.5, we discuss how our model can rationalize a range of documented decoy effects that other explanations cannot capture.

This model belongs to class of menu-dependent learning models in which the DM chooses based on a signal that depends on the menu of options (e.g. Safonov, 2022; Natenzon, 2019); as in our model, these menu-dependent signals in these models can generate context affects. In Section 6, we discuss how this model relates to others in this class.

3.2.2 Compression Effects in Valuation

We model the valuation of a choice option as a sequence of choices structured as a multiple price list, a workhorse experimental procedure for eliciting valuations. There is an option $x \in X$ to be valued against a *price list* $Z = \{z^1, z^2, .., z^n\} \subseteq X$: a set of options for which the ranking $v_{z^1} > v_{z^2} > ... > v_{z^n}$ is unambiguous, i.e. $\tau_{z^i z^j} = \infty$ for all $z^i, z^j \in Z$. For each price $z^k \in Z$, the DM chooses between z^k and x, revealing her valuation of x in terms of Z.

To capture this setting, we extend our multinomial choice framework as follows. The DM now faces a finite *menu sequence* $A^1, A^2, ..., A^n \in \mathscr{A}$ in a choice context C, generates a set of signals s for each pairwise comparison in $A^1 \cup A^2 \cup ... A^n \cup C$ and chooses the option from each menu with the highest posterior expected value, yielding the joint choice frequencies[17]

$$\rho((x^1, ... x^n), (A^1, ..., A^n)|C) = \mathbb{P}\left(\bigcap_{i=1}^{n} \{s : \mathbb{E}[v_{x^i}|s] > \mathbb{E}[v_y|s] \, \forall \, y \in A^i / \{x^i\}\} \,\Big|\, v\right)$$

where $\rho((x^1, ... x^n), (A^1, ..., A^n)|C)$ records the frequency of choosing $x^i \in A^i$ for $i = 1, ..., n$. Given an option x and a price list Z, a *valuation task* (x, Z) is simply the binary menu sequence $A^1, ..., A^n = \{x, z^1\}, ..., \{x, z^n\}$.

Since the DM learns the ranking of prices in Z, this procedure yields a single switching point: for any signal realization, there is an index $R \in \{1, ..., n, n+1\}$ for which the DM chooses the option $x \in A^k$ for all $k \geq R$, and the price $z^k \in A^k$ for all $k < R$. R reveals where the DM believes the object x falls within the ranking of prices, i.e. the subject's valuation in terms of Z. We will be interested in the distribution over R induced by

[17]As before, this formulation for choice probabilities holds when ties in posterior expectations occur with probability 0. In the case of ties, we assume a symmetric tiebreaking rule; See Appendix B.3 for details.

$\rho((x^1, ...x^n), (A^1, ..., A^n)|Z)$, which we denote by $R(x, Z)$.[18] With a slight abuse of notation, we denote $v_k \equiv v_{z^k}$ and $\tau_{xk} \equiv \tau_{xz^k}$.

Our model predicts that when x is hard to compare to prices, valuations will exhibit be systematically compressed towards the center of the price list. Let $R^*(x, z) \in \{1, ..., n, n+1\}$ denote the true ranking of x relative to Z.[19]

Proposition 2. *Given a valuation task (x, Z), where $\tau_{xk} = \tau$ and $v_x \neq v_k$ for all $k = 1, ..., n$, we have the following:*

(i) If $\tau = 0$, $\mathbb{E}[R(x, Z)] = (n+2)/2$.

(ii) As $\tau \to \infty$, $R(x, Z)$ converges in distribution to $\delta_{R^*(x,Z)}$.

Proposition 2 says i) when x is incomparable to prices, valuations exhibit "pull-to-center" effects, and ii) as x becomes more comparable to prices, valuations converge to the truth. Intuitively, if $\tau = 0$, the DM receives no information on where x falls within the ranking of prices – her posterior puts equal probability on each possible ranking, so she values x in the middle of the price list. As τ increases, valuation of x becomes increasingly accurate. Crucially, this "pull-to-center" force depends on how difficult x is to compare to prices. When combined with our theory of comparison complexity, this force rationalizes documented preference reversals and biases in valuation.

3.3 Preference Reversals

Consider the classic preference reversal phenomenon in risky choice. Lottery x pays a high amount with a low probability, while y pays a modest sum with a high probability, e.g.

$$x: \quad \$23.50 \text{ with } 19\%$$
$$y: \quad \$4.75 \text{ with } 94\%$$

Most subjects choose y over x in direct choice, yet state a higher certainty equivalent for x. A number of explanations for these reversals have been put forth, such as intransitive preferences or independence violations (see Seidl (2002) for a review). Our model rationalizes these reversals as the consequence of the differential ease of comparing options to

[18] Given a signal s, the DM's posterior switching point R is computed by calculating $\mathbb{E}[v_x|s]$ and $\mathbb{E}[v_j|s]$ for all $j \in \{1, 2, ..., n\}$, and finding the unique index R such that $\mathbb{E}[v_x|s] < \mathbb{E}[v_{R-1}|s]$ and $\mathbb{E}[v_x|s] > \mathbb{E}[v_R|s]$ (in the case of ties, we assume the DM randomizes as described in Appendix B.3).

[19] That is, $v_x > v_k$ if $k \geq R^*(x, Z)$ and $v_x < v_k$ otherwise. For ease of exposition, we assume x is not indifferent to any price in Z.

money. Under our complexity notion, y is easier to compare to money than x, which results in differential pull-to-center distortions. As this force distorts valuations but not direct choice, reversals can occur. As we show below, this logic not only generates reversals in lottery choice; it predicts that similar reversals occur in intertemporal choice, and that these reversals can be eliminated by manipulating the ease of comparing each option to prices.

3.3.1 Lottery Reversals

Consider the lottery domain, where $v_x = \sum_w u(w) f_x(w)$ for u strictly increasing, and where τ has a CDF-complexity representation $\tau^{CDF}_{xy} = H\left(\frac{EU(x)-EU(y)}{d_{CDF}(x,y)}\right)$ for which $H(1) = \infty$; that is, the DM perfectly learns the ranking between lotteries with a dominance relationship. Call $l = (w_l, p_l)$ a simple lottery if it pays out w_l with probability p_l, and nothing otherwise.

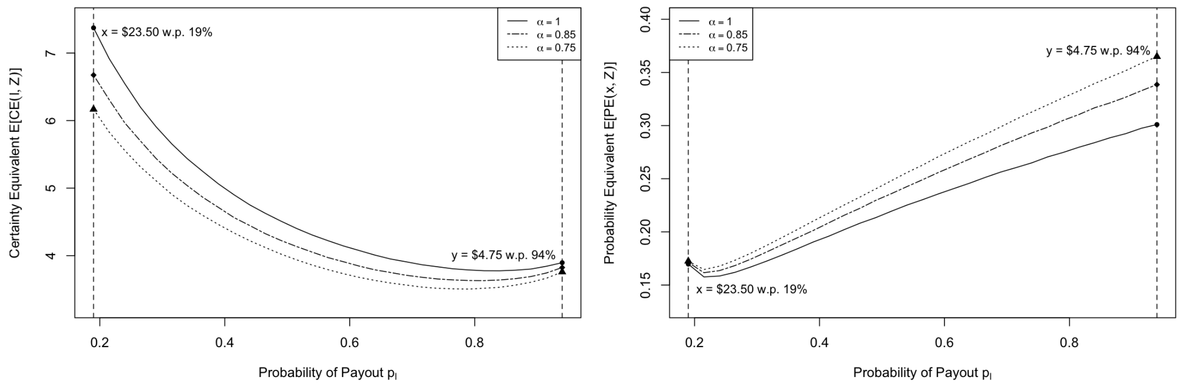
Example 1. (Classic preference reversals). Suppose the DM is weakly risk-averse, i.e. u is concave, and consider again the simple lotteries $x = (\$23.5, 0.19)$, $y = (\$4.75, 0.94)$. First, consider binary choice between the (equal expected value) lotteries. Since any risk-averse DM weakly prefers y to x, our model predicts that the DM is more likely to choose y.

Now consider a DM tasked with assessing certainty equivalents of the lotteries. Formally, the DM faces a valuation task (l, Z) in which the simple lottery $l = (w_l, p_l)$ is valued against a price list $Z = \{z^1, ..., z^n\}$, where each $z^k = (w_k, 1)$ is a sure payment. We work with "adapted" price lists, where Z is *adapted* to l if Z contains equal-sized steps and contains the minimal and maximal support points of l (i.e. $w_k - w_{k+1}$ is constant in k, and $w_n = 0$, $w_1 = w_l$). Recall that each valuation task (l, Z) produces a distribution of switching points $R(l, Z)$, and denote $CE(l, Z) = 1/2\left[w_{R(l,Z)-1} + w_{R(l,Z)}\right]$ the resulting distribution over certainty equivalents.

Figure 3a plots the expected certainty equivalents $\mathbb{E}[CE(l, Z)]$ simulated from our model for simple lotteries l with the same expected value as x and y. Notice that the high-risk lottery x is valued higher than the low-risk lottery y on average, despite the fact that y is weakly preferred to x for our risk-averse DM. Intuitively, x is dissimilar to and therefore difficult to compare to money, so its valuation is inflated towards the midpoint of the undominated range of prices $[0, w_x]$. On the other hand, y is easier to compare to money, so its valuation is less distorted, and if anything is pulled *downwards* towards the midpoint of undominated prices $[0, w_y]$. We have a reversal: $\rho(y, x) \geq 1/2$ and yet $\mathbb{E}[CE(x, Z)] > \mathbb{E}[CE(y, Z)]$.

The rest of the figure traces our model's predictions for preference reversals in general: for a high-risk lottery x' and a low-risk lottery y', we have $\mathbb{E}[CE(x', Z)] > \mathbb{E}[CE(y', Z)]$ — even though y' is preferred to x', and so $\rho(y', x') \geq 1/2$.

In our model, reversals result from the differential ease of comparing lotteries to money.



(a) Certainty Equivalents $\mathbb{E}[CE(l,z)]$ (b) Probability Equivalents $\mathbb{E}[PE(l,z)]$

Figure 3: Simulated average certainty equivalents and probability equivalents for simple lotteries $l = (w_l, p_l)$ with expected value equal to that of $x = (23.50, 0.19)$ as a function of p_l. Z is adapted to l and we set $|Z| = 15$. τ has a CDF-complexity representation parameterized by $u(w) = w^\alpha$ and $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0,1]$.

This echoes past work suggesting that the difficulty of valuing lotteries against a incongruent response scale may generate preference reversals (Tversky et al., 1990; Butler and Loomes, 2007). Unlike previous work, however, we provide a formal account of both what makes lotteries hard to value, and how this difficulty distorts valuation. As such, our model generates novel predictions: in particular, that one can eliminate these reversals by manipulating the ease of comparing each lottery to prices — specifically, by changing the units against which the lotteries are valued.

Example 2. (Reversals with probability equivalents). Consider the same lotteries x and y from Example 1. Instead of valuing x and y against money, suppose the DM assesses their *probability-equivalents*: the probability p that makes the lottery $z = (\$24, p)$ indifferent to each. Whereas y was easier to compare to money, x is easier to compare to this new numeraire, which is more similar to x than to y. Our model predicts that this change in numeraire *reverses* the distortions in the valuation of x and y.

Formally, the DM now values $l = (w_l, p_l)$ against a *probability list*: a price list of lotteries $Z = \{z^1, ..., z^n\}$, where each $z^k = (24, p_k)$. Analogous to Example 1, we work with *adapted* probability lists, for which those containing equal-sized steps with $p_1 = p_l$, $p_n = 0$. Denote by $PE(l,Z) = 1/2[p_{R(l,Z)-1} + p_{R(l,Z)}]$ the distribution over the DM's probability equivalents.

Figure 3b plots the simulated probability equivalents $\mathbb{E}[PE(l,Z)]$ for the same set of simple lotteries l as in Figure 3a. Notice that the valuation distortions in certainty equivalents *reverse* when the lotteries are valued using probability equivalents. Intuitively, y is harder to compare to the z^k, so its valuation is compressed upward towards the middle of the range of undominated probabilities, whereas x is easier to compare, so its valuation is closer to the truth. Thus, $\mathbb{E}[PE(y,Z)] > \mathbb{E}[PE(x,Z)]$ and so the reversal disappears.

Note that while valuation using probability equivalents eliminates the reversal in this example, our model does *not* predict that probability equivalents are more accurate than certainty equivalents. To see this, focus on the case of risk neutral preferences. Here, *both* methods of valuation are subject to bias: x and y are indifferent in truth, and yet we have $\mathbb{E}[CE(x,Z)] > \mathbb{E}[CE(y,Z)]$ and $\mathbb{E}[PE(x,Z)] < \mathbb{E}[PE(y,Z)]$.

3.3.2 Intertemporal Reversals

As preference reversals result the difficulty of comparing options to prices in our model, it predicts that this phenomenon is not limited to lottery choice. Consider the intertemporal domain, where $v_x = \sum_t \delta^t m_x(t)$ and $\tau_{xy} = \tau_{xy}^{CPF} = H\left(\frac{PV(x)-PV(y)}{d_{CPF}(x,y)}\right)$, with $H(1) = \infty$.

Example 3. (Intertemporal reversals). We have the following delayed payments:

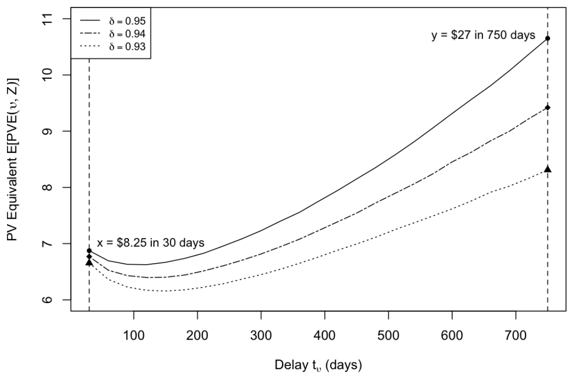
$$
\begin{aligned}
x &: \quad \$27 \text{ in } 750 \text{ days} \\
y &: \quad \$8.25 \text{ in } 30 \text{ days}
\end{aligned}
$$

Consider a DM with a monthly discount factor $\delta \leq 0.95$ choosing between these payments: since $v_x \geq v_y$ our model says the DM is more likely to choose y. Now suppose the DM must value each option in terms of dollars today. Under τ^{CPF}, the two payments differ in their ease of comparison to money today; following the same logic as in Example 1, the resulting distortions cause x to be valued higher than y.

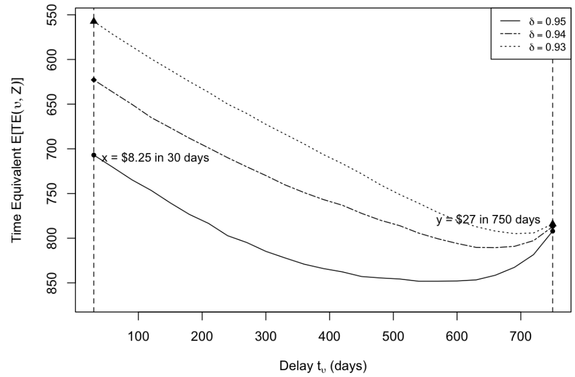
Formally, the DM faces a valuation task (υ, Z), where $\upsilon = (m_\upsilon, t_\upsilon)$ is a delayed payment that pays out $m_\upsilon > 0$ at time $t_\upsilon > 0$, valued against a price list $Z = \{z^1, ..., z^n\}$, where each $z^k = (m_k, 0)$ is an immediate payment. This setting is similar to those described in Examples 1 and 2, so we reserve further details for Appendix B.6. Figure 4a plots the present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ simulated from our model for delayed payments υ with the same present value as x and y (assuming a monthly discount factor $\delta = 0.95$). Here, the high-delay option x has a higher valuation than y, despite the fact that $\rho(y,x) \geq 1/2$.[20]

As we saw in lotteries, the direction of these distortions may be reversed by changing the units of valuation. Suppose the DM values options in terms of *time-equivalents*: the time t that makes the delayed payment $(\$27.50, t)$ indifferent to x or y. Formally, the DM values υ against a *time list* $Z = \{z^1, ..., z^n\}$, where each $z^k = (\$27.50, t_k)$. As x is more similar to the

[20]Using choice vignettes, Tversky et al. (1990) document similar intertemporal preference reversals.



(a) Present Value Equivalents $\mathbb{E}[PVE(l,z)]$



(b) Time Equivalents $\mathbb{E}[T(l,z)]$

Figure 4: Simulated average present value equivalents and time equivalents for delayed payments $\upsilon = (m_\upsilon, t_\upsilon)$ with present value equal to that of $x = (8.25, 30)$ under $\delta = 0.95$. For PVEs, Z is adapted to υ with $|Z| = 15$. For TEs, $Z = \{z^1, ..., z^n\}$, where $z^k = (27.5, t_\upsilon + t_k)$, for $(t_1, ..., t_n) = (0, 7, 30, 60, 120, 180, 240, 360, 480, 600, 720, 900, 1080, 1260, 1440)$ days. τ has a CPF-complexity representation with $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0,1]$.

numeraire under this valuation mode, the model predicts the distortions in time equivalents will be flipped relative to present value equivalents, as Figure 4b illustrates. Specifically, we have $\mathbb{E}[T(y,Z)] < \mathbb{E}[T(x,Z)]$ and so the "reversal" relative to direct choice disappears.

3.4 Biases in Valuation of Risk and Time

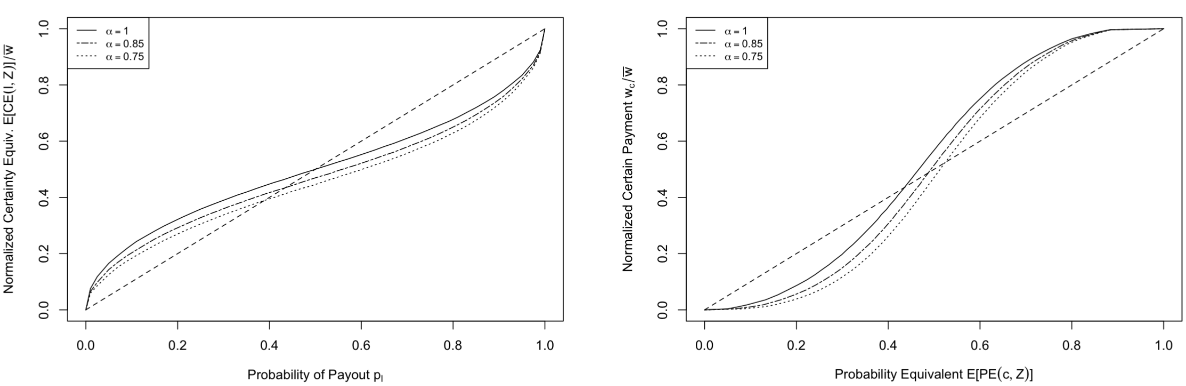
The same pull-to-center effects that produce preference reversals in our model can also generate classical patterns in the valuation of risk and time: probability-weighting and hyperbolic discounting. The same logic underlies both domains: consider a simple lottery with payout probability p, or a payoff flow that pays out with delay t. The presence of tradeoffs can make both options hard to compare to money, generating pull-to-center distortions: valuations of small (large) payoff probabilities are compressed upward (downward), generating apparent probability weighting; likewise, valuations of small (large) delays are compressed downward (upward), generating apparent hyperbolic discounting. Furthermore, our model predicts that these biases can be *reversed* by inverting the role of the numeraire.

3.4.1 Probability-Weighting in Lottery Valuation

Consider the standard paradigm used to estimate the probability weighting function, in which the DM provides certainty equivalents of simple lotteries $l = (\overline{w}, p_l)$. Previously, we showed that our model predicts pull-to-center effects which lead to overvaluation of low-probability lotteries and undervaluation of high-probability lotteries. Thus, if we estimated the agent's probability weighting function based on her valuations, we would conclude the agent is *overweighting* small probabilities and *underweighting* large probabilities. Figure 5a plots our model's predicted normalized certainty equivalents $\mathbb{E}[CE(l,Z)]/\overline{w}$ as a function of p_l for a DM with CRRA utility. The weights implied by these certainty equivalents reproduce the familiar inverse S-shaped pattern of probability weighting.

Importantly, our model does *not* predict that probability weighting occurs generically in choice; instead, it results from the difficulty of comparing a lottery to a price list of certain payments. This is important for two reasons. First, our model predicts probability weighting in valuation even absent such distortions in binary choice, which is consistent with evidence that the inverse S-shaped probability weighting function is far more prominent in valuation tasks than in direct choice (Harbaugh et al., 2010; Bouchouicha et al., 2023). Second, we predict that these patterns of probability weighting will be sensitive to the specific valuation paradigm. In particular, we predict that it is possible to *reverse* the pattern of apparent probability-weighting with an appropriate choice of price list currency.

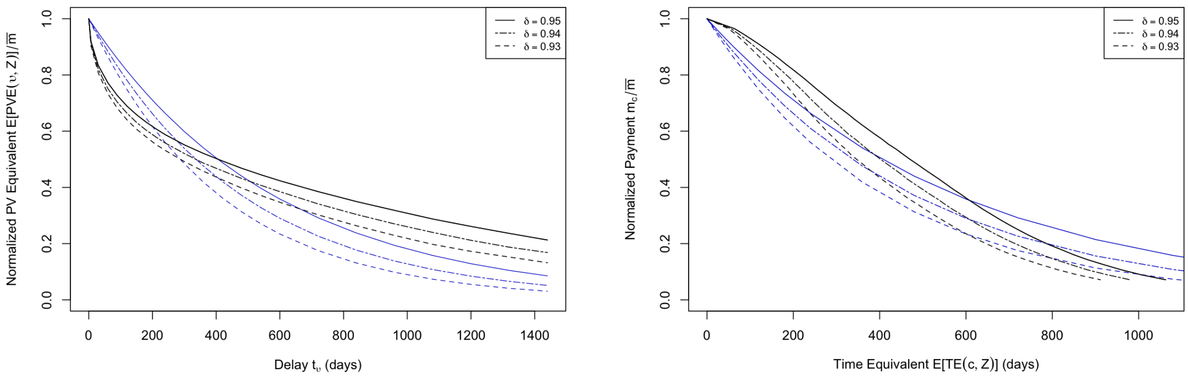
Consider an alternative paradigm for estimating the probability weighting function, in which the DM provides probability equivalents of a certain payment: the probability p that makes the lottery $z = (\overline{w}, p)$ indifferent to $c = (w_c, 1)$. Tracing out the probability equivalents p as a function of the normalized certain payments $w_c/\overline{w}$ should — absent complexity-driven distortions — reveal the same preferences as the certainty equivalents discussed above. Figure 5b plots the predicted relationship between the certain payment amount $w_c/\overline{w}$ (y-axis) and the associated probability equivalent $\mathbb{E}[PE(c,Z)]$ (x-axis), in



(a) Simulated normalized average certainty equivalents $\mathbb{E}[CE(l,Z)]/\overline{w}$ for simple lotteries $l = (\overline{w}, p_l)$ as a function of p_l.

(b) Simulated average probability equivalents $\mathbb{E}[PE(c,Z)]$ for certain payments $c = (w_c, p_l)$ and normalized payment amounts $w_c/\overline{w}$.

Figure 5: Probability weighting functions implied by valuations in certainty equivalents (left) and probability equivalents (right). Z is adapted to l with $|Z| = 15$. τ has a CDF-complexity representation with $u(w) = w^\alpha$ and $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0,1]$.



(a) Simulated average present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ (in black) for delayed payments $\upsilon = (\overline{m}, t_\upsilon)$ as a function of t_υ.

(b) Relationship between simulated average time equivalents $\mathbb{E}[TE(c, Z)]$ (in black) for immediate payment $c = (m_c, 0)$ and normalized amount $m_c/\overline{m}$.

Figure 6: Discount functions implied by present value equivalents (left) and time equivalents (right). For PVEs, Z is adapted to υ with $|Z| = 15$. For TEs, $Z = \{z^1, ..., z^n\}$, where $z^k = (\overline{m}, t_k)$, for $(t_1, ..., t_n) = (0, 7, 30, 60, 120, 180, 240, 360, 480, 600, 720, 900, 1080, 1260, 1440)$ days. Blue curves plot distortion-free discount functions given the true discount rate δ. τ has a CPF-complexity representation parameterized by $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0, 1]$.

which the certain payment $c = (w_c, 1)$ is valued against a probability list $Z = \{z^1, ..., z^n\}$, for $z^k = (\overline{w}, p_k)$. Here we see a reversal of the inverse S-shaped pattern: the difficulty of comparing sure payments against the numeraire good causes probability equivalents to be compressed towards the middle of the price list, generating apparent *underweighting* of small probabilities and *overweighting* over large probabilities.[21]

3.4.2 Hyperbolic Discounting in Intertemporal Valuation

Now, consider a standard paradigm used to estimate time discounting: the DM values delayed payments $\upsilon = (\overline{m}, t_\upsilon)$ in terms of money today. Following the same logic as in lottery valuation, pull-to-center effects result in payments dated close to the present to be *undervalued* and payments further in the future to be *overvalued*. This produces a pattern of complexity-driven hyperbolic discounting, consistent with evidence from Enke et al. (2023). We further predict that this pattern persists when front-end delays are incorporated, since the apparent hyperbolicity predicted by our model is not driven by a preference for the present. This is also supported by evidence from Enke et al. (2023).

Figure 6a plots the normalized valuations $\mathbb{E}[PVE(\upsilon, Z)]/\overline{m}$ as a function of the delay t_υ, where υ is valued against a price list $Z = \{z^1, ..., z^n\}$ of immediate payments adapted to

[21]While Sprenger (2015) shows that a model of stochastic reference points can predict a larger degree of risk aversion in probability vs. certainty equivalents in this setting, this account cannot explain the simultaneous apparent overweighting of large probabilities that our model predicts in probability equivalents.

v. As the figure shows, the DM's valuations display apparent hyperbolicity: she undervalues payments close to the present and overvalues payments with longer delays.

Now consider an alternative valuation paradigm in which the DM assesses the *time equivalents* of an immediate payment: the delay t that makes the delayed payment $(\overline{m}, t)$ indifferent to the immediate payment $c = (m_c, 0)$. Figure 6b plots the predicted relationship between the immediate payment amount $m_c/\overline{m}$ (y-axis) and the associated time equivalents $\mathbb{E}[TE(c, Z)]$ (x-axis). Here, the model predicts an apparent reversal of hyperbolic discounting: the difficulty of comparing immediate payments to the numeraire good causes time equivalents to compressed towards the middle of the price list, generating overvaluation of payments close to the present and undervaluation of payments with longer delays.

We do not claim that observed hyperbolic discounting is purely a complexity-driven distortion; rather, we provide a mechanism for why canonical valuation tasks may overstate the degree of hyperbolic discounting present in direct choice. In Appendix B.7, we repeat the simulation exercise in Figure 6 for a DM with a hyperbolic discount function, and show how comparison complexity magnifies the extent of true hyperbolic discounting in present value equivalents, and reduces it in time equivalents.

4 Experimental Tests

We turn to testing the key predictions of our model. We assess whether 1) in binary choice, our complexity measures predict choice noise and errors; 2) classic preference reversals can be eliminated by manipulating the ease of comparing options to the price list; and 3) probability weighting and hyperbolic discounting can be reversed by eliciting valuations of money in terms of probability and time equivalents. We then quantify the predictive power of our binary choice model relative to benchmark models.

4.1 Tests of Complexity Measures

We test the validity of our proposed measures of comparison complexity against data from three binary choice experiments in multiattribute, intertemporal, and lottery choice. Below, we provide an overview of the goals and design features shared across the three experiments. We then present domain-specific details and results.

The purpose of these experiments is two-fold. First, we show that our proposed complexity measures indeed capture the difficulty of comparisons. We consider three natural indicators of choice complexity: choice errors, choice inconsistency, and subjective uncertainty, and test whether they are decreasing in the value-dissimilarity ratio, as our theory predicts. Second, we show that our model captures quantitatively important features of choice not explained by standard models. In each domain, we structurally estimate our model and compare its performance against leading behavioral models.

We carry out these analyses in three parallel binary choice datasets. We run new experiments in multiattribute and intertemporal choice and compile existing data from Enke and Shubatt (2023) and Peterson et al. (2021) to study lottery choice. In our experiments, we recruit participants through an online survey platform to make 50 incentivized binary choice problems. For each problem, we elicit participants' subjective certainty in the optimality of their decision. In order to measure choice consistency, 10 of these problems are randomly repeated throughout the survey. We collect an average of 37 choices for each of 662 multiattribute choice problems and 1,100 intertemporal problems – a total of more than 66,000 individual decisions. The compiled risk dataset includes nearly 10,000 binary choice problems (over 1 million decisions) and includes similar measures of cognitive uncertainty and choice consistency.[22]

4.1.1 Multiattribute Choice

Participants make binary choices between hypothetical phone plans characterized by either two, three, or four attributes, which include a device cost, a monthly flat fee, a data usage fee, and a quarterly wi-fi fee. Importantly, each choice problem has an objectively payoff-maximizing answer, which allows us to observe choice errors: subjects choose on behalf of a hypothetical consumer with a known budget and data usage, and are incentivized to choose the plan that will save the consumer the most money over one year. If a participant is selected to earn a bonus (1 in 2 chance), we select one of their choices at random and pay them based on the money saved. Across all choice problems, choosing the more expensive plan results in an average bonus of $4.2, and choosing the cheaper plan results in an average bonus of $8.98. For more detail on the design and pre-registration, see Appendix D.

Figures 7a, 7b, and 7c show binned scatterplots relating the L_1 ratio (x-axis) to choice errors, cognitive uncertainty, and inconsistency. All three indicators of complexity are strongly decreasing in the ratio. The average error rate, only around 5% for problems near-dominance, increases five-fold for problems with the lowest values of the L_1 ratio ($R^2 = 0.32$). We take this as strong evidence that (1) the ratio indeed captures cognitive complexity, and (2)

[22]Cognitive uncertainty is elicited only for the 500 problems from Enke and Shubatt (2023). Problems are only repeated in the Peterson et al. (2021) experiment. Unlike our experiments, subjects see these repeated problems immediately after giving an initial response.

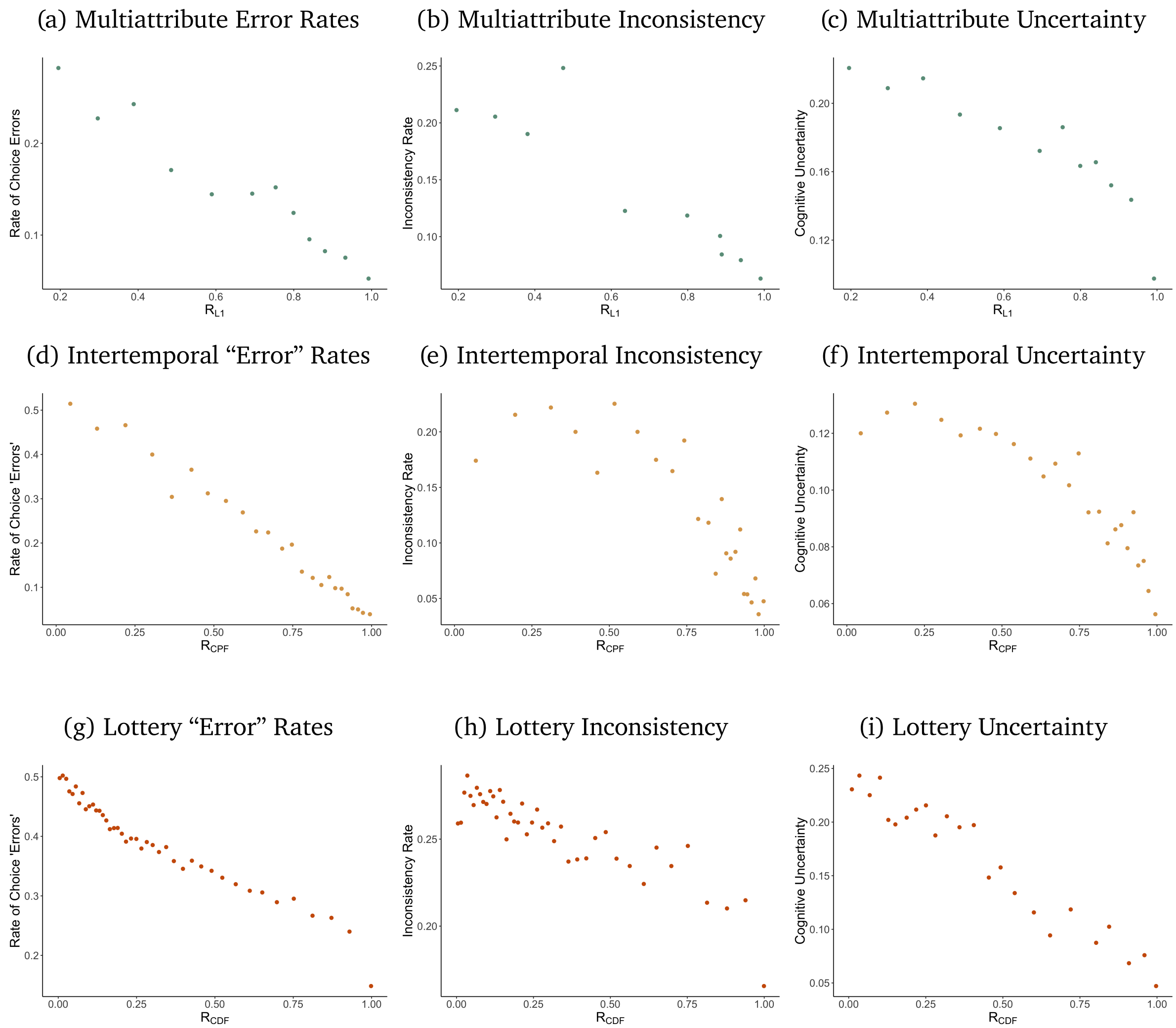


Figure 7: Binscatter of problem-level error rates, choice inconsistency, and cognitive uncertainty versus the value-dissimilarity ratio. Problem-level error rates in panels (a), (d), and (g) are constructed as the rate of choosing (a) the lower-value option, (d) the less-preferred option according to the best-fit exponential discounted utility model, and (g) the less-preferred option according to the best fit expected utility model (see Appendix E for estimation details). Problem-level inconsistency rates in panels (b), (e), and (h) are constructed as the percent chance subjects choose differently in any repeated instance of the choice problem. Problem-level cognitive uncertainty in panels (c), (f), (i) is constructed as the average subjective likelihood that subjects assign to making an error in that choice problem, i.e. choosing the less-preferred option. The value-dissimilarity ratios R_{L1}, R_{CPF}, and R_{CDF} follow Definitions 1, 4, and 3, respectively, where the preference parameters β, δ, and u are given by the known attribute weights in multiattribute choice, the best-fit exponential discount rate in intertemporal choice, and the best-fit expected utility preferences in lottery choice, respectively.

decision-makers respond to this complexity by making more random choices. Importantly, these relationships are not driven by variation in value difference alone: they are quantitatively unchanged when controlling for value differences, as the regression analysis in Appendix Table 3 shows.[23]

4.1.2 Intertemporal Choice

We ask participants to make binary choices between time-dated payoff streams. Each option has one or two payoffs, ranging between $1 and $40, to be received at delays ranging between the present and 2 years in the future. If a participant is selected to win a bonus (1 in 5 chance), we select a decision at random and pay out the chosen payment stream on the specified dates. For more details on the design and pre-registration, see Appendix D.

Unlike in multiattribute choice, we cannot directly observe choice errors: the definition of an "error" depends on the decision-maker's discount function. The CPF ratio also depends on this discount function, so we proceed by estimating a representative-agent exponential discount factor δ from the choice data (monthly $\hat{\delta} = 0.94$). For details on the specification, see Appendix E.1. Importantly, the discounting estimated from our choice data, which involves choices over money, should *not* be interpreted as subjects' "pure" time preferences (Cohen et al., 2020): it could reflect, for instance, access to outside credit or beliefs about repayment risk. Rather than identifying time preferences, we are interested in studying which problems are hard given subjects' required rate of return in our experimental setting.

Figures 7d, 7e, and 7f relate the CPF ratio (x-axis) to our indicators of choice complexity, coding a choice as an "error" if the subject chooses the option with lower value according to $\hat{\delta}$. Again, we find strong relationships between the ratio and choice "errors," cognitive uncertainty, and inconsistency. The average "error" rate ranges from around 5% for problems with the highest value of the CPF ratio to 50% for problems with the lowest value of the CPF ratio ($R^2 = 0.6$).[24] As in multiattribute choice, these relationships are unchanged when controlling for the value difference (see Appendix Table 6). This further indicates that the value-dissimilarity *ratio*, rather than the value difference alone, drives these relationships.

[23]We pre-registered analyses restricting to the 407 subjects (82.5% of the full sample) who do not report using a calculator in the experiment. Appendix Table 4 reports these analyses. Quantitative relationships are virtually unchanged.

[24]We also run analyses in which errors are classified using individually estimated discount factors $\hat{\delta}_i$ with similar results. See Appendix Figure 14 and Table 7.

4.1.3 Lottery Choice

In both lottery choice experiments, participants are asked to choose between two lotteries which pay off different amounts with known probabilities. If selected to receive a bonus, they receive the outcome of a lottery they chose in a randomly selected decision. As in intertemporal choice, both the CDF ratio and our notion of choice "errors" depend on an unknown preference object – the Bernoulli utility function. We proceed by estimating a representative-agent CRRA utility function with additive logit noise (see Appendix E.3 for details), and code "errors" as departures from this estimated model.[25] Figures 7g, 7h, and 7i show the results. Once again, all three outcomes are strongly decreasing in the ratio; in particular, the CDF ratio achieves an R^2 of 0.45 in variance explained over error rates. Consistent with our results in our other two domains, these relationships are driven by the value-dissimilarity ratio, as opposed to the value difference alone (see Appendix Table 9).

4.2 Tests of Preference Reversal Predictions

We run experiments mirroring the simulation exercises in Section 3.3 to show that preference reversals are an outgrowth of comparison complexity. Specifically, we establish the presence of reversals in lottery and intertemporal choice, and test the prediction that these reversals can be eliminated by manipulating the ease of comparing options to the price list.

In each domain, we construct two sets of "base" options: 3 high-risk and 3 low-risk lotteries, and 3 high-delay and 3 low-delay payoff flows (see Table 2). We elicit binary between these sets of options, as well as multiple price list valuations of each option. In these valuation tasks, we experimentally vary the ease of comparing each option to the price list: for lotteries, we elicit both certainty equivalents as well as probability equivalents using a yardstick lottery that pays \$24 with p% chance; for intertemporal choice, we elicit both present value equivalents as well as time equivalents using a yardstick option that pays \$27.5 in t days. We elicit choices and valuations for both the base options as well as a "scaled-up" version of each option, where we multiply payouts by a scale factor of 1.6.

Recall the predictions developed in Section 3.3. First, binary choice probabilities will favor low-risk (low-delay) options over high-risk (high-delay) options.[26] Second, valuations

[25]We also run all analyses in which errors are classified using individually estimated risk aversion parameters. We restrict this analysis to the Enke and Shubatt (2023) data since the data in Peterson et al. (2021) contains a low amount of unique choice problems for each subject (75% of subjects face $\leq$ 14 unique problems). Results are similar – see Appendix Figure 15 and Table 10.

[26]These choice options were calibrated so that given the preferences estimated from our binary choice data, subjects would prefer the low-risk/low-delay options.

	Lottery		*Intertemporal*
Low-risk/delay	\$4.50 w.p. 98% \$4.75 w.p. 94% \$5.00 w.p. 90%		\$8.25 in 30 days \$9.50 in 90 days \$11.00 in 150 days
High-risk/delay	\$19.50 w.p. 23% \$21.20 w.p. 21% \$23.50 w.p. 19%		\$24.00 in 650 days \$25.50 in 690 days \$27.00 in 750 days

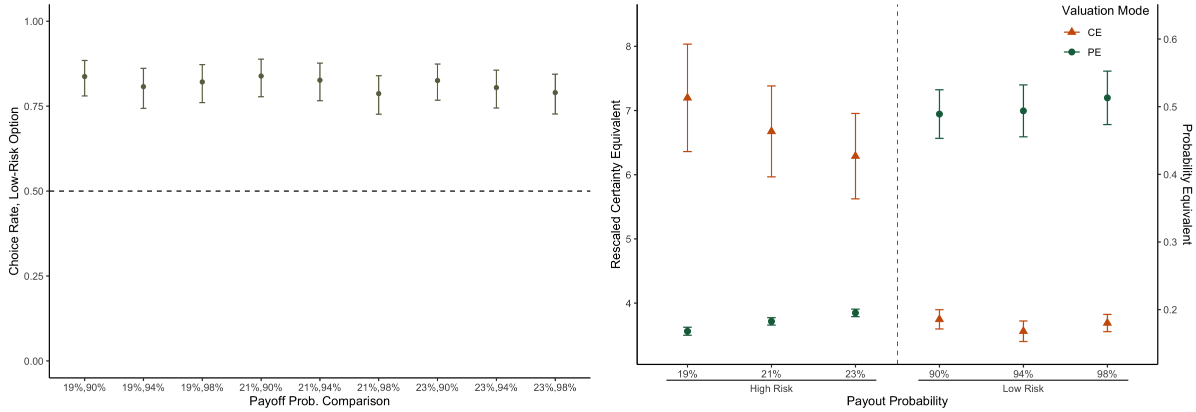
Table 2: Base options used in reversal experiments. We consider every possible pairing of a low-risk, high-risk option for lotteries; and every possible pairing of a low-delay, high-delay option for delayed payments.

will be higher for high-risk (high-delay) options compared to low-risk (low delay) options when they are valued in terms of money. Third, these relative valuations will flip when options are instead valued using probability equivalents (time equivalents).

We run separate incentivized lottery and intertemporal choice experiments on an online survey platform with 151 and 152 subjects, respectively. All subjects complete two parts, in random order: *Binary Choice* and *Valuation*. In *Binary choice*, subjects make 16 direct choices between option pairs.[27] In *Valuation*, participants value all 12 options (6 base options, at 2 scale factors) using multiple price lists corresponding to one of two randomly assigned valuation modes: certainty equivalents or probability equivalents in the lottery experiment, and present value equivalents or time equivalents in the intertemporal experiment. See Appendix D.3 for instructions and additional design details.

Figures 8 and 9 present results from our lottery and intertemporal choice experiment, aggregating across scale factors. Figures 8a and 9a show that binary choice rates favor the low-risk and low-delay options, with choice rates for those options well above 50%. In contrast, Figures 8b and 9b show that there is an apparent *reversal* in preference when the options are valued via certainty equivalents and present value equivalents: subjects assign higher valuations to high-risk and high-delay options on average. In these same figures, however, we see the predicted *flipping* of these valuation patterns when we manipulate the ease of comparing each option to the price list. In particular, when options are instead valued via probability equivalents and time equivalents, subjects instead assign higher valuations to the low-risk and low-delay options; thus eliminating the apparent reversal.

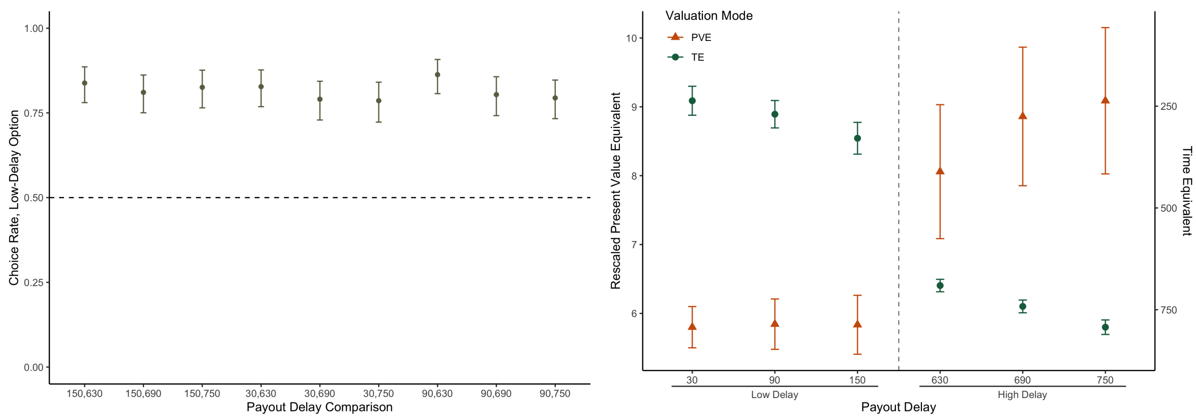
[27] 12 of these choices are drawn from the 18 high/low comparisons of direct interest, and 4 are filler problems included to limit repetition. These filler problems are not analyzed.



(a) Binary choice probabilities

(b) Certainty and Probability Equivalents

Figure 8: Preference reversal experiments for simple lotteries, aggregated across scale factor. Panel (a) presents binary choice rates for each high/low risk lottery comparison. Panel (b) presents average rescaled certainty equivalents, computed by dividing each certainty equivalent by the scale factor (scale on left axis), and average probability equivalents (scale on right axis). Error bars reflect 95% confidence intervals.



(a) Binary choice probabilities

(b) Present Value and Time Equivalents

Figure 9: Preference reversal experiments for delayed payments, aggregated across scale factor. Panel (a) presents binary choice rates for each high/low delay comparison. Panel (b) presents average rescaled present value equivalents, computed by deviding each present value equivalent by the scale factor (scale on left axis) and average time equivalents (scale on right axis). The direction of the time equivalent axis is inverted, so that valuations are increasing in the vertical axis. Error bars reflect 95% confidence intervals.

4.3 Tests of Valuation Predictions

We run a multiple price list experiment to 1) replicate familiar patterns of inverse-S shaped probability weighting and hyperbolic discounting in certainty equivalents and present value equivalents respectively; and 2) test the prediction, developed in Section 3.4, that these biases can be reversed by manipulating the units of valuation. Specifically, our model predicts that relative to the certainty equivalents of binary lotteries, probability equivalents of certain payments should exhibit underweighting of low probabilities and overweighting of high probabilities. Likewise, relative to the discounting revealed by the present value equivalents of delayed payments, time equivalents of immediate payments should exhibit overvaluation of delays close to the present and undervaluation of delays far in the future.

To test these predictions, we recruit 300 subjects through an online survey platform who each complete 24 incentivized multiple price lists: 12 for lotteries, and 12 for intertemporal payments. Each subject is randomly assigned to a valuation mode for each domain: certainty or probability equivalents for lotteries, and present value or time equivalents for intertemporal payments; the order of the domains is randomized. As these valuation tasks mirror the simulation exercises in Section 3.4, we relegate design details to Appendix D.3.

Lottery Valuations. We elicit certainty equivalents $CE(l)$ for a simple lottery $l = (\overline{w}, p_l)$, and relate the normalized valuations $CE(l)/\overline{w}$ to payout probabilities p_l. We also elicit probability equivalents $PE(c)$ of a certain payment $c = (w_c, 1)$ against a yardstick lottery $(\overline{w}, p)$, and relate the normalized payouts $w_c/\overline{w}$ to probability equivalents $PE(c)$. In our experiment, we draw $\overline{w}$ from $\{\$9, \$18, \$27\}$. For certainty equivalents, we draw p_l from $\{0.03, 0.05, 0.10, 0.25, 0.5, 0.75, 0.90, 0.95, 0.97\}$. For probability equivalents, we draw w_c so that $w_c/\overline{w} \in \{0.033, 0.056, 0.11, 0.25, 0.5, 0.75, 0.89, 0.944, 0.967\}$.

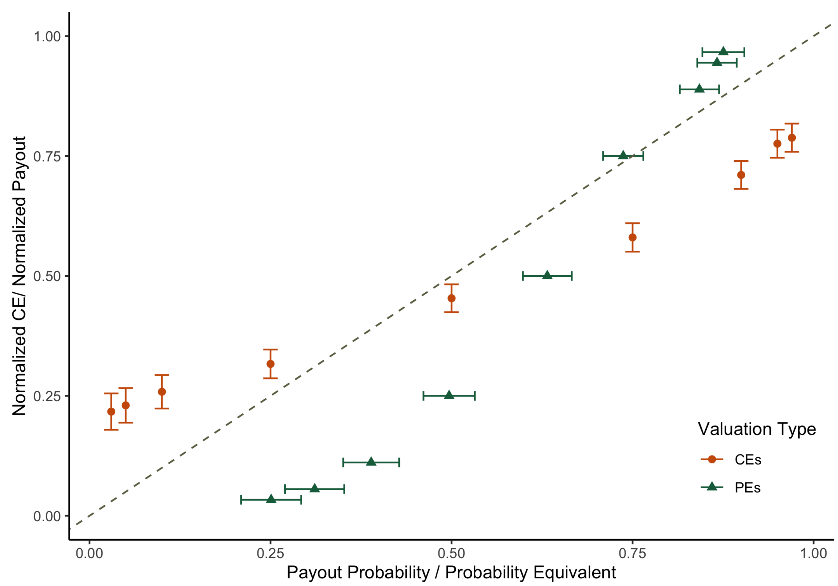


Figure 10: Probability weighting functions implied by lottery valuation experimental tasks. Orange dots plot the payout probability p_l against the average normalized certainty equivalent $CE(l)/\overline{w}$; turquoise triangles plot average probability equivalents $PE(c)$ against the normalized payout $w_c/\overline{w}$. The dashed black line represents linear probability weighting. Error bars reflect 95% confidence intervals.

Results for lotteries are presented in Figure 10. The probability weighting function implied by certainty equivalents follows the familiar pattern of overweighting small probabilities and underweighting large probabilities. Consistent with the model's predictions, we see the pattern *reverses* for probability equivalents. Here, small probabilities are instead *underweighted*, and large probabilities are *overweighted*.

Intertemporal Valuations. We elicit present value equivalents $PVE(v)$ for a delayed payment $v = (\overline{w}, t_v)$, and relate the normalized valuations $PVE(v)/\overline{m}$ to payout delays t_v. We also elicit time equivalents $TE(c)$ of an immediate payment $c = (m_c, 0)$ against a yardstick delayed payment $(\overline{m}, t)$, and relate normalized payouts $m_c/\overline{m}$ to time equivalents $TE(c)$. In our experiment, we draw $\overline{m}$ drawn from $\{25, 30, 35\}$. For present value equivalents, we draw t_v from $\{7, 30, 60, 120, 240, 360, 480, 720, 1080\}$ (in days). For time equivalents, we draw m_c so that $m_c/\overline{m} \in \{0.20, 0.35, 0.50, 0.65, 0.75, 0.85, 0.90, 0.95, 0.97\}$.

Results for intertemporal payment flows are presented in Figure 11. Focusing first on the discount function implied by present value equivalents, we document the familiar pattern of short-run impatience and long-run patience (i.e. hyperbolicity). We also see that this hyperbolicity is *more extreme* than the hyperbolic discount function estimated from our binary choice experiment (traced in the dotted line; see Appendix E.2 for estimation details).

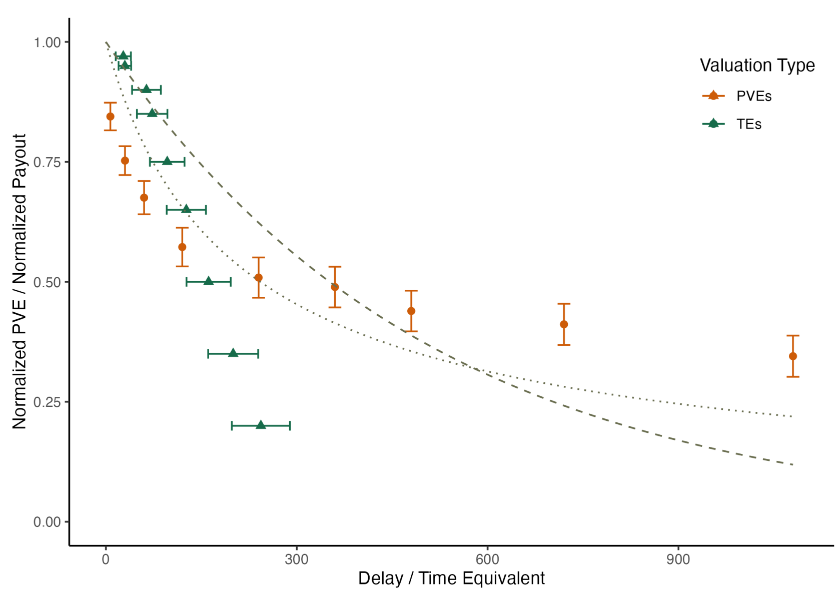


Figure 11: Discount functions implied by intertemporal valuation experimental tasks. Orange dots plot the payout delay t_v against the average normalized present value equivalent $PVE(l)/\overline{m}$; turquoise triangles plot average time equivalents $TE(c)$ against the normalized payout $m_c/\overline{m}$. The dashed line traces the exponential discount function estimated from binary choice data; the dotted line, the estimated hyperbolic discount function. Error bars reflect 95% confidence intervals.

This is consistent with the interpretation that the true hyperbolicity in temporal preferences, as revealed by subjects' binary choices, is exaggerated when measured through present value equivalents. However, when instead we elicit valuations using time equivalents, the pattern of hyperbolicity *reverses*: relative to both the hyperbolic discounting estimated from our binary choice data, as well as the discounting implied by their present value equivalents, subjects exhibit short-run patience and long-run impatience.

4.4 Benchmarking Model Performance

Whereas the predominant approach in behavioral economics is to model non-standard elements in the DM's objective function, this paper seeks to model an orthogonal feature of choice: how difficult the DM finds a choice, given her objectives. To study whether our theory indeed explains variation in choice patterns that are not captured by existing models, we compare the explanatory power of these modeling approaches in our binary choice data.

First, we structurally estimate our binary choice model in each domain, which is parameterized by a transformation G that maps the value-dissimilarity ratio into choice probabilities and (in the case of lottery and intertemporal choice) domain-specific preference parameters. Across all domains, we use the same specification of G developed in Section 2.4. We then compare the performance of our model against a set of models specifying the DM's objective function, which we fit to our data using logit errors. In particular, we estimate a "standard" benchmark model: a distortion-free logit model in multiattribute choice; exponential discounted utility in intertemporal choice; and expected utility in lottery choice. We also estimate a set of behavioral models: salience-weighting (Bordalo et al., 2013), focusing (Kőszegi and Szeidl, 2013), and relative thinking (Bushong et al., 2021) in multiattribute choice; hyperbolic discounting in intertemporal choice; and cumulative prospect theory in lottery choice. See Appendix E for details on each specification.

Figure 12 reports the variance explained over problem-level choice rates in each of our experimental datasets of 1) the standard benchmark model, 2) the leading behavioral model, and 3) our comparison complexity model. To characterize the extent to which our model captures orthogonal variation in choice relative to existing models, the final column of each figure includes the variance explained achieved by combining the predicted choice rates of the leading behavioral model and our comparison complexity model.[28] Appendix Tables 5, 8, and 11 report the full estimation results.

[28] Specifically, we report the variance explained of an OLS regression of choice rates against the predicted choices rates of the two models.

In multiattribute choice, we estimate versions of our L_1-complexity model corresponding to the two- and three-parameter specifications of G developed in Section 2.4. Both the two- and three-parameter models (R^2 of 0.29 and 0.32, respectively) are comparable to the leading behavioral model of relative thinking ($R^2 = 0.35$).[29] Importantly, L_1-complexity explains a substantial amount of variation in choices not captured by the relative thinking model. Combining the three-parameter L_1-complexity model and relative thinking results in an R^2 of 0.41, a 19% increase in variance explained compared to relative thinking alone.

In both intertemporal and lottery choice, our model captures significantly more variation in choice rates than leading behavioral models. In intertemporal choice, our CPF-complexity model ($R^2 = 0.88$) delivers a 13% improvement in variance explained over hyperbolic discounting ($R^2 = 0.78$), using the same number of parameters. In lottery choice, we estimate two versions of the CDF-complexity model — one that assumes risk-neutral preferences, and one that allows for utility curvature. Despite having far fewer free parameters, the risk-neutral CDF-complexity model ($R^2 = 0.66$) delivers a 12% improvement over cumulative prospect theory ($R^2 = 0.59$). This is in line with results from Enke and Shubatt (2023), who find that allowing complexity to enter the noise term of a logit choice model substantially improves performance over standard models.[30] Adding an additional parameter to capture

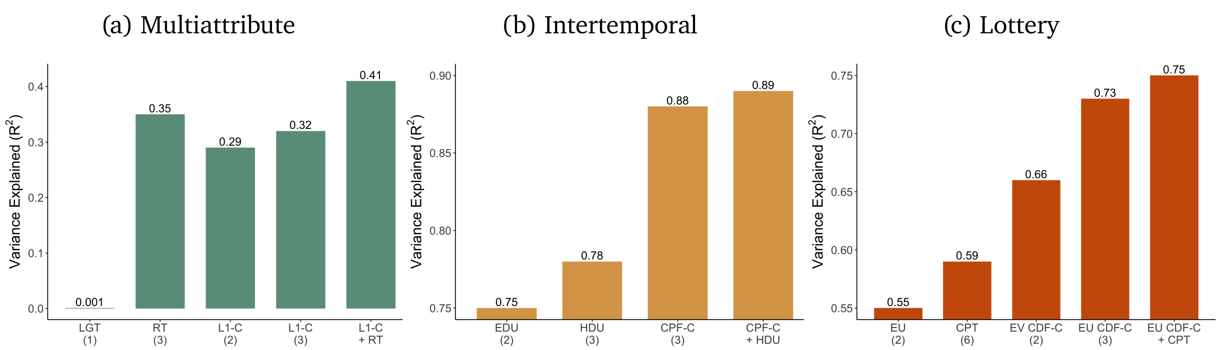


Figure 12: Variance explained of estimated models. Number of estimated model parameters in parentheses. “LGT”, “RT”, “L1-C” refer to the Distortion-Free, Relative Thinking, and L_1-complexity models described in Appendix E.1. “EDU”, “HDU”, and “CPF-C” refer to the Exponential Discounting, Hyperbolic Discounting, and CPF-complexity models desdribed in Appendix E.2. ‘EU", “CPT", “EV CDF-C", and “EU CDF-C" refer to the Expected Utility, Cumulative Prospect Theory, risk-neutral CDF-complexity, and expected utility CDF-complexity models described in Appendix E.3.

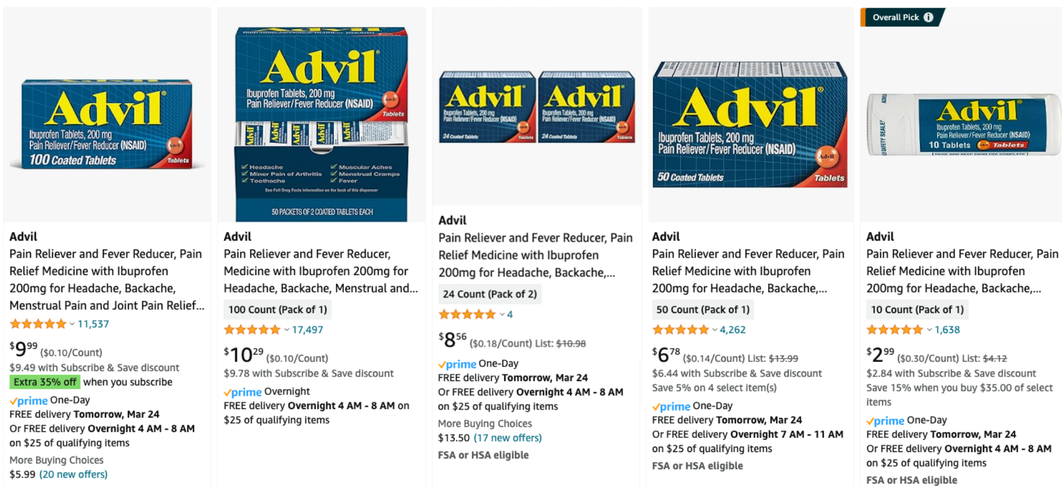
[29]Both the salience-weighting and focusing models deliver an R^2 identical to that of the distortion-free benchmark of 0.001.

[30]The “complexity index” developed by Enke and Shubatt loads heavily on “excess dissimilarity,” which is connected to a special case of our ratio: it is equal to the denominator of the CDF ratio minus the numerator, assuming risk-neutral preferences.

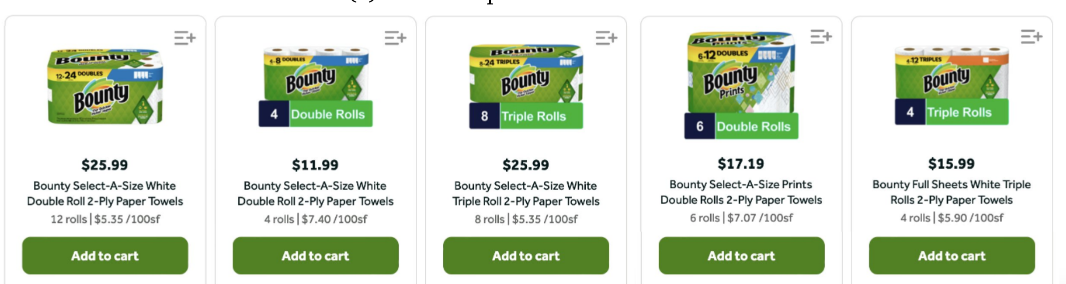
utility curvature in our model yields an R^2 of 0.73 – a 24% improvement in variance explained over cumulative prospect theory.

5 Strategic Obfuscation

In this section, we apply our model to study strategic obfuscation in markets. Our application is motivated by the practice of product proliferation, in which firms market an otherwise undifferentiated good in a large number of different packaging varieties in which quantities and other peripheral characteristics differ.



(a) Advil Ibuprofen tablet varieties



(b) Bounty paper towel varieties

Figure 13: Examples of product proliferation.

To illustrate, consider the Figure 13a, which shows a subset of Amazon search results for Advil tablets. Notice that across these listings it is not simply the total quantity that varies — quantities are expressed multidimensionally. For instance, the first two listings offer the same total quantity, marketed as 1 package of 100 tablets one case and 50 packages of 2 tablets in the other. The third and fourth listings market 2 packages of 24 tablets vs. 1 package of 50 tablets, again for a similar total quantity. Analogous multidimensional product proliferation can be found in a number of markets, such as for paper towels, where the size and number of rolls vary (see Figure 13b), or for soft drinks, where the size and number of cans in a package vary.

A standard explanation for this practice is that firms are catering to consumers with heterogenous preferences over how quantities are packaged. However, an alternative explanation has been discussed in the literature: product proliferation may be a strategic choice to make comparison shopping difficult, thus reducing consumers' price sensitivity (Ellison and Ellison, 2018; Richards et al., 2020). Writing on the Ibuprofen market, Ellison and Ellison (2018) remark "Sorting on price would be meaningless because these products, all with a chemically identical active ingredient, differ on so many dimensions, such as strength of tablet, size of package, and type of packaging." In the following section, we apply our model to formally study how comparison complexity can generate this kind of spurious differentiation in equilibrium; in Section 5.2, we return to the question of how the analyst may distinguish between consumer heterogeneity and confusion.

5.1 Duopoly with Strategic Obfuscation

5.1.1 Symmetric Duopoly

We consider a two-stage game where identical firms first choose a multidimensional product quantity vector, and then play a simultaneous Bertrand pricing game. We begin by analyzing the second stage game in which firms take quantities as given.

Pricing Game. Each firm $i \in \{a, b\}$ sells a good with multidimensional quantity $q^i = (q^i_1, q^i_2)$. We abstract away from vertical quantity differentiation and assume that firms sell the same total quantity, i.e. $q^i_1 + q^i_2 = Q$ for all $i = a, b$. Firms face identical marginal costs c for producing a unit of quantity Q. Each firm takes quantities as given and simultaneously chooses a price p^k; let $i = (p^i, q^i_1, q^i_2)$ denote firm i's offering.

There is a unit mass of identical consumers with unit demand, with utility given by $U(i) = q^i_1 + q^i_2 - p^i$. Consumers behave according to the L_1-complexity model, wherein choice probabilities are given by

$$\rho(i, -i) = G\left(\frac{U(i) - U(-i)}{d_{L1}(i, -i)}\right)$$

for G strictly increasing, where we additionally assume that $G(1) = G(-1) = 1$, so that consumers do not err in the face of dominance. Each firm's profits are thereby given by

$$\Pi(i,-i) = (p^i - c)G\left(\frac{U(i) - U(-i)}{d_{L1}(i,-i)}\right)$$

We analyze the pure-strategy Nash equilibria of the simultaneous pricing game.

Proposition 3. *If G is linear, there is a unique pure-strategy equilibrium, in which $p^a = p^b = c + \Delta q$, where $\Delta q = |q_1^a - q_1^b| + |q_2^a - q_2^b|$ denotes the dissimilarity between the firms' quantity profiles. More generally, if a symmetric pure-strategy equilibrium exists, it is unique and satisfies $p^a = p^b = c + \frac{1}{2G'(0)}\Delta q$.*

Note that in the frictionless Bertrand benchmark in which consumers choose the offering that maximizes their utility, firms price at marginal cost and earn zero profits in equilibrium. Proposition 3 states that due to complexity-driven errors, markups can be sustained in equilibrium so long as firms offer dissimilar quantity profiles, where the magnitude of the markups is increasing in the degree of dissimilarity.[31] Richards et al. (2020) provide suggestive evidence for this prediction in the soft drink market: focusing on the different packaging varieties sold by retailers (chiefly characterized by the number of cans and the size of each can), they find that within a soft drink brand, package varieties that are unique to a single retailer are sold at higher markups.[32]

Endogenous Quantities. We now consider the first-stage game, where firms choose their quantity profiles. In particular, consider a setting where each firm simultaneously[33] chooses a quantity profile along a segment of an indifference curve in quantity space: $(q_1, q_2) \in \{[\underline{q}, \overline{q}]^2 : q_1 + q_2 = Q\}$, where $\overline{q} + \underline{q} = Q$.

Assume that G is linear. Since in the subsequent pricing game, each firm's profits are given by $\frac{1}{2}\Delta q$, firms will want to locate as far apart from each other as possible:

Corollary 3.1. *Suppose G is linear. A pure-strategy equilibrium of the first-stage game exists, and in any such equilibrium, the equilibrium quantity dissimilarity is $\Delta q = 2(\overline{q} - \underline{q})$.*

This echoes the basic result from the classic Hotelling model: firms are driven to differentiate from their competitors to soften price competition. The key difference lies in the

[31] Of course, a number of well-studied oligopoly models predict equilibrium markups with frictionless consumers (e.g. Kreps and Scheinkman, 1983). We focus here on a Bertrand game to isolate the effect of comparison complexity from these other mechanisms.

[32] Using a structural model, the authors argue that a number of standard explanations, such as preference heterogeneity, cannot account for their findings, and these findings as strategic obfuscation by retailers.

[33] The equilibria in Corollary 3.1 are unchanged if instead firms move sequentially in first-stage game.

welfare interpretation of this differentiation: in the Hotelling model, differentiation caters to real taste differences between consumers, and so the efficient level of differentiation is in general positive. In our setting, however, differentiation is entirely spurious, and results purely in a transfer from firms to consumers.

5.1.2 Imitation vs. Obfuscation

As shown above, in a symmetric environment firms are driven to design products that are difficult to compare to their competitors. Below, we show that allowing for vertical cost differences between firms leads to a richer set of equilibrium predictions, in which firms may imitate their competition to increase price sensitivity.

Consider a setting identical to the one studied above, except that firms have different costs. In particular, let c^a and c^b denote the marginal costs for the two firms, and assume that a is the lower cost firm; Let $\Delta c = c^b - c^a$ denote a's cost advantage.

Rather than a simultaneous-move first-stage game, we consider a sequential first-stage game in which firm b first announces a location decision in $\{(q_1, q_2) \in [\underline{q}, \overline{q}]^2 : q_1 + q_2 = Q\}$, followed by firm a.[34] As before, profits are determined by a simulataneous-move pricing game played after the first-stage game. To ensure that pure-strategy equilibria are defined, we assume that if $q^a = q^b$ and $p^a = p^b$, $\rho(a,b) = \lim_{p \uparrow p^a} \rho((p, q^a), b)$ – that is, ties are broken in favor of the lower-cost firm if both firms sell identical products at identical prices.

As before let $\Delta q = |q_1^a - q_1^b| + |q_2^a - q_2^b|$ denote the quantity dissimilarity between the firms' offerings, and let $\overline{\Delta q} = 2(\overline{q} - \underline{q})$ denote the maximal quantity dissimilarity that can be implemented.

Proposition 4. *Suppose G is linear, and restrict to pure-strategy equilibria.*

1. *If $\Delta c > 2\overline{\Delta q}$, firm a imitates, i.e. $\Delta q = 0$ in equilibrium. The unique equilibrium prices satisfy $p^a = p^b = c^b$, and b's equilibrium market share is 0.*

2. *If $\Delta c < 2\overline{\Delta q}$, firm a obfuscates, i.e. $\Delta q = \overline{\Delta q}$ in equilibrium. The unique equilibrium prices satisfy $p^a = c^b + \overline{\Delta q}$, $p^b > p^a$. Furthermore, b's equilibrium market share is positive and strictly increasing in $\overline{\Delta q}$.*

The intuition is as follows. Given the high-cost firm's product choice, the low cost firm can pursue two strategies: they can either imitate their competitor, designing an identical product and using their cost advantage to capture the entire market, or they can obfuscate,

[34]In a simultaneous first-stage game, pure-strategy equilibria will not exist if a wishes to imitate, following the logic of hide-and-seek games.

designing a product that is difficult to compare and pricing high.[35] The relative attractiveness of these strategies depends on how large the firm's cost advantage is relative to the extent to which obfuscation is possible. Here, obfuscation generates market inefficiency: whereas imitation leads to the efficient allocation in which the low-cost firm serves the entire market, obfuscation allows the high-cost firm to sustain positive market share.

Note one key comparative static prediction: as a's cost advantage decreases, obfuscation becomes a more attractive strategy. As such, the model predicts that an increase in competitive pressure brought on by a decrease in b's costs can actually lead to an *increase* in equilibrium prices and obfuscation.

5.2 Confusion vs. Heterogeneity

A standard explanation for product proliferation is that it simply reflects heterogeneous tastes over packaging varieties. Indeed, the binary choice probabilities in the L_1-complexity model are observationally equivalent to the choice shares generated by a population of agents endowed with a distribution over linear attribute weights β, where each agent chooses correctly.[36] These two interpretations of the data have dramatically different welfare and policy implications — for instance, policies that standardize product attributes may make sense if low price sensitivity reflects confusion, but not if it reflects heterogeneous tastes. As such, it is important to understand the relative importance of each in any given application. Here, we outline two ways in which these accounts may be distinguished.

First, individual-level choice data can be used to distinguish between heterogeneity and confusion. If population-level choice data reflects only taste heterogeneity, we should not expect within-individual choices to be noisy or inconsistent. In particular, Theorem 1 implies that with sufficient within-individual choice data, the analyst can identify both the individual's utility (parameterized by β), as well as the extent to which the individual choices are prone to complexity driven errors (parameterized by G). In principle, this allows the analyst to jointly estimate the degree of true preference heterogeneity and confusion within a population. As likely few observational datasets are sufficiently rich for such an exercise, this points to the potential importance of survey data. In particular, one might adapt conjoint surveys, which are traditionally used by researchers and practitioners to study consumer

[35]If instead the low-cost firm a moves first in the first-stage game, the minimum dissimilarity a can implement is $1/2\overline{\Delta q}$. In this case, equilibria will either involve full obfuscation ($\Delta q = \overline{\Delta q}$) as before, or partial obfuscation ($\Delta q = 1/2\overline{\Delta q}$), with some threshold c^* such we have full obfuscation for $c < c^*$ and partial obfuscation otherwise.

[36]Formally, the L_1-Complexity model shares an intersection with pure characteristics models (Berry and Pakes, 2007); see Appendix B.8.

preferences, to also shed light on the degree of complexity-driven confusion.

Second, the observational equivalence between L_1-Complexity and taste heterogeneity holds only for binary choice data. As discussed in Section 3.2.1, our multinomial choice model predicts violations of regularity, wherein the addition of choice options can reduce the choice share of existing options. This feature can not only be used to distinguish between complexity and preference heterogeneity, but itself generates novel predictions on market outcomes, as the following example illustrates.

Example 4. (Three Firms.) Consider the duopoly pricing game analyzed in Section 5.1.1: we have firms a, b with marginal costs c selling the same total quantity; as before, let Δq denote the quantity dissimilarity between the two firms. Consider the addition of a third firm s, which has identical quantity attributes as firm a: $(q_1^a, q_2^a) = (q_1^s, q_2^s)$, but with costs $c^s > c$. Binary choice probabilities are as described in the L_1 complexity model: $\rho(i, j) = G\left(\frac{U(i)-U(j)}{d_{L1}(i,j)}\right)$, where we also assume that G is linear, i.e. $G(r) = \frac{1}{2} + \frac{1}{2}r$. We analyze the pure-strategy equilibria of the simultaneous-move pricing game.

First, consider the case where $c^s > c + \Delta q$: that is, firm s would be priced out by a under the equilibrium duopoly prices $p^a = p^b = c + \Delta q$ (see Proposition 3). Note that if binary choice probabilities reflect preference heterogeneity, the addition of s would have no impact on the equilibrium market outcome: a and b maintain their duopoly prices, and s is priced out of the market and so has no impact on demand. Under our multinomial choice model however, the addition of s will have an impact on demand, even if it is priced out by a: in particular, the fact that s is dominated by a but not by b increases a's market share (see Proposition 1), which then affects equilibrium prices.

To facilitate closed-form analysis of this equilibrium, we consider a binary-signal version of our multinomial choice model: for each firm i, j, consumers receive a signal that reveals the ordinal ranking between i, j with probability

$$\tau_{ij} = H\left(\frac{|U(i) - U(j)|}{d_{L1}(i,j)}\right)$$

and contains no information otherwise.[37] Let $\rho(p^i, \{p^i, p^j, p^k\})$ denote the demand for i's offering's as a function of the prices of the three firms. To ensure that pure strategy equilibria exist, we assume the same tiebreaking rule as in Section 5.1.2: in the case where firms a and s provide an identical offering, we break ties in favor of a: $\rho(p^a, \{p^a, p^b, p^s\}) =$

[37]It can be shown that the binary-signal version of the model is observationally equivalent to the Gaussian-signal model in binary choice, and that all formal results and qualitative simulation results in Section 3 hold in this version of the model.

$\lim_{p \uparrow p^a} \rho(p, \{p, p^b, p^c\})$ whenever $p^a = p^s$.

Proposition 5. *Suppose H is linear, i.e. $H(r) = r$.*

1. *A pure-strategy equilibrium exists, and in any such equilibrium, $p^b < p^a$.*
2. *If $\Delta q \geq c_s - c$, there is a unique pure-strategy equilibrium in which s prices at cost, and b's profits in this equilibrium are decreasing in Δq.*

The intuition is as follows: since the comparison to s makes a's offering more attractive than b, a is able to charge a higher price than b in equilibrium. Furthermore, since b has the best offering on the market, offering a highly dissimilar product from a may now be detrimental to b as it makes b's superior value harder for consumers to see. Firm b may therefore benefit from being located closer to a than the dupoly equilibrium dissimilarity. These predictions yield novel policy implications. For instance, suppose that s is a state-operated firm. The analysis above shows that by merely offering a product with fixed attributes, the state-operated firm may be able to engage in "standard-setting" and induce subsequent lower-cost entrants to design their products to be similar to the state-provided product, thereby reducing prices and the level of obfuscation in the market.

6 Discussion

This paper presents a theory of tradeoff-based comparison complexity in multiattribute, lottery, and intertemporal choice, in which comparisons are *easier* when options are more similar along their features, holding fixed their value difference fixed; and *easiest* when two options have a dominance relationship. We show how comparison complexity can rationalize documented patterns of systematically biased choices and valuations, and predicts that these patterns can be eliminated or even reversed by manipulating the nature of tradeoffs, and we experimentally confirm these predictions. Finally, we apply our model to develop insights on strategic obfuscation by firms in a pricing game. We close by discussing relationships to related models.

6.1 Relationship to Existing Models

Relationship to Linear Differentiation. He and Natenzon (2023b) propose a *Linear Differentiation Model* (LDM) defined on the same multiattribute domain as our L_1-complexity model, where binary choice probabilities are given by

$$\rho(x,y) = G\left(\frac{U(x) - U(y)}{\sqrt{(x-y)'\Sigma(x-y)}}\right)$$

for linear utility $U(x) = \sum_k \beta_k x_k$, an $n \times n$ positive definite matrix Σ, and a continuous, strictly increasing G. Note that this model is identical to the L_1-complexity model save for the choice of distance metric in the denominator: in the LDM, dissimilarity, is measured by the generalized Euclidean distance in contrast to the L_1 distance. Below, we outline two key formal dimensions along which the models differ.

Dominance. As Theorem 1 implies, the L_1-Complexity model respects dominance: the likelihood the DM chooses the superior option x over the inferior option y is maximized when x attribute-wise dominates y (written $x >_D y$). The LDM violates this property. For example, consider a parameterization of the LDM with $n = 3$, $\beta = (1,1,1)$, and $\Sigma = I$, i.e. the distance in the ratio is Euclidean: $d_{L2}(x,y) = \sqrt{\sum_k (x_k - y_k)^2}$. Consider the choice options $x = (4,0,0), x' = (-1,2,3), y = (0,0,0)$. In this case, the LDM predicts a dominance violation $\rho(x',y) > \rho(x,y)$: while both comparisons involve the same value difference, we have $d_{L2}(x,y) > d_{L2}(x',y)$.[38] In Appendix B.9, we show that such dominance violations arise for any parameterization of the LDM when there are 3 or more attributes.

Monotonicity. The L_1-Complexity model also satisfies a monotonicity property, wherein $x' >_D x$ implies $\rho(x',y) \geq \rho(x,y)$: that is, improving a choice option along each attribute cannot lead to a decrease in its probability of being chosen.[39] Just as the LDM generates dominance violations, the LDM also violates monotonicity. For example, take the the parameterization of LDM discussed above and consider the options $x = (5,5,0)$, $x' = (10,5,0)$, and $y = (0,0,0)$. The LDM predicts the monotonicity violation $\rho(x',y) < \rho(x,y)$. In Appendix B.9, we show that such monotonicity violations arise generically in the LDM.

This discussion highlights a key difference in how the analyst should interpret the attributes within each model, and the types of applications appropriate for each model. As L_1-complexity model satisfies dominance and monotonicity, it describes settings where the ranking within each attribute is relatively unambiguous to the DM, and where choice rates

[38]Intuitively, the Euclidean metric penalizes the concentrated attribute difference in the comparison (x,y) more heavily than the less concentrated attribute differences in (x',y).

[39]See Proposition 11 in the Appendix. More generally, each of L_1, CDF, and CPF representations satisfy monotonicity with respect to the appropriate dominance notions; see Lemma 1 in the Appendix.

principally reflect the difficulty of making tradeoffs across attributes. On the other hand, since the LDM violates dominance and monotonicity, it is better suited to applications where choice rates also reflect difficulty in processing within-attribute differences, or alternatively, reflect disagreement in a population over the valence of attributes.

Relationship to Bayesian Probit. Our multinomial choice model shares similarities with the Bayesian Probit model developed in Natenzon (2019). In this model, the DM has i.i.d. Gaussian priors over v_x, and chooses based on signals $s_x = v_x + \frac{1}{p}\epsilon_x$ received for each option in the menu, where $\epsilon_x \sim N(0,1)$ are jointly normal across options. The pairwise correlations of (ϵ_x, ϵ_y) allow the model to capture a notion of the ease of comparison between options, where x, y are more comparable if (ϵ_x, ϵ_y) are more highly correlated.

Recall that in our model, the DM only receives information on *ordinal* value comparisons. In Bayesian Probit, the DM learns about the *cardinal* value differences between choice options, which rules out certain intuitive choice patterns. For instance, consider the the following choice options in the multiattribute domain:

$$x = (2,0)$$
$$y = (0,1)$$
$$z = (0,0)$$

Since z is dominated by both x and y, we might expect that $\rho(x,z) = \rho(y,z) = 1$ and also $\rho(x,y) < 1$; that is, the DM does not err in the presence of dominance but finds tradeoffs across attributes difficult.[40] Bayesian Probit cannot rationalize this choice data; $\rho(x,z) = \rho(y,z) = 1$ implies that $\mathrm{Cor}(\epsilon_x, \epsilon_z) = \mathrm{Cor}(\epsilon_y, \epsilon_z) = 1$,[41] which implies $\mathrm{Cor}(\epsilon_x, \epsilon_y) = 1$. This yields the counterfactual prediction $\rho(x,y) = 1$.[42]

Intuitively, in Bayesian Probit the DM receives information on the *cardinal* value difference between choice options. As such, when the Bayesian Probit DM perfectly learns the cardinal value differences $v_x - v_z$ and $v_y - v_z$, they also learn $v_x - v_y$. In our model, on the other hand, the DM only receives information on the ordinal value comparison between choice options, which allows for situations in which the DM perfectly learns that $v_x > v_z$

[40]The L_1 complexity model generates the choice probabilities for $\beta_1, \beta_2 > 0$, and $G(-1) = G(1) = 1$.

[41]This analysis assumes that the global precision parameter $p < \infty$. If instead $p = \infty$, we will also have the prediction that $\rho(x,y) = 1$.

[42]More generally, when $\rho(x,z), \rho(y,z)$ are close but not necessary equal to 1, the Bayesian Probit model places lower bounds on the choice probability $\rho(x,y)$. As Appendix B.10 discusses, these bounds imply that there are binary choice rules rationalizable by L_1, CDF, and CPF complexity but not by Bayesian Probit.

and $v_y > v_z$, yet remains uncertain regarding the ranking between v_x and v_y.

Importantly, this feature of Bayesian Probit means it cannot capture key choice patterns in several of the applications considered in this paper. For instance, in the application to lottery valuation, we model settings where the DM finds a given lottery trivial to compare against certain options in the price list and difficult to compare to others — for instance, the DM is certain that the lottery that pays \$10 with 80% chance is worse than \$10 for sure, yet finds the same lottery difficult to compare to \$5 for sure — and yet perfectly understands the ranking within the price list, i.e. that \$10 is better than \$5. Bayesian Probit cannot accomodate these choice patterns, whereas our choice model can.

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APPENDIX

A Appendix: Tables and Figures

Table 3: Complexity Responses vs. L_1 Ratio

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
L_1 Ratio	-0.26^{***}	-0.26^{***}	-0.19^{***}	-0.19^{***}	-0.12^{***}	-0.12^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
Global Value Difference		0.00		−0.01		−0.00
		(0.00)		(0.01)		(0.00)
(Intercept)	0.32^{***}	0.32^{***}	0.26^{***}	0.28^{***}	0.25^{***}	0.27^{***}
	(0.01)	(0.02)	(0.01)	(0.03)	(0.00)	(0.01)
R^2	0.32	0.32	0.03	0.03	0.30	0.31
Adj. R^2	0.32	0.32	0.03	0.03	0.30	0.31
Num. obs.	662	662	4880	4880	662	662

OLS Estimates. Standard errors (in parentheses) are robust. "L_1 Ratio" and "Global Value Difference" are the L_1 ratio and the monetary value difference for each choice problem.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 4: Complexity Responses vs. L_1 Ratio, No Calculator Users

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
L_1 Ratio	-0.30^{***}	-0.29^{***}	-0.20^{***}	-0.20^{***}	-0.12^{***}	-0.12^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
Global Value Difference		−0.00		−0.01		-0.00^{*}
		(0.00)		(0.01)		(0.00)
(Intercept)	0.36^{***}	0.36^{***}	0.28^{***}	0.32^{***}	0.26^{***}	0.28^{***}
	(0.01)	(0.03)	(0.01)	(0.03)	(0.01)	(0.01)
R^2	0.32	0.32	0.03	0.03	0.30	0.30
Adj. R^2	0.32	0.32	0.03	0.03	0.30	0.30
Num. obs.	662	662	4020	4020	662	662

OLS Estimates. Standard errors (in parentheses) are robust. "L_1 Ratio" and "Global Value Difference" are the L_1 ratio and the monetary value difference for each choice problem.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 5: Structural Estimates: Multiattribute Choice

	DF	BGS	Focus	RT	L_1-C 2P	L_1-C, 3P
Parameter Estimates						
δ		1				
θ			0			
ω				0.84		
ξ				1.62		
κ					0.09	0.04
γ					2.36	0.86
ψ						0.55
η	0.37	0.37	0.37	1.54		
R^2	0.001	0.001	0.001	0.345	0.289	0.32

"DF", "BGS", "Focus", and "RT" refer to the Distortion-Free, Salience, Focusing, and Relative Thinking models described in Appendix E.1. "L_1-C, 2P" and "L_1-C, 3P" refer to the 2 and 3 parameter L_1-Complexity models described in Appendix E.1.

Table 6: Complexity Responses vs. CPF Ratio

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
Global CPF Ratio	-0.51^{***}	-0.51^{***}	-0.20^{***}	-0.17^{***}	-0.07^{***}	-0.07^{***}
	(0.01)	(0.01)	(0.01)	(0.02)	(0.00)	(0.00)
Global Value Difference		-0.00		-0.01^{***}		-0.00
		(0.00)		(0.00)		(0.00)
(Intercept)	0.55^{***}	0.55^{***}	0.27^{***}	0.28^{***}	0.15^{***}	0.15^{***}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
R^2	0.59	0.59	0.02	0.03	0.22	0.22
Adj. R^2	0.59	0.59	0.02	0.03	0.22	0.22
Num. obs.	1097	1097	8290	8290	1097	1097

OLS estimates. Standard errors (in parentheses) are robust. "Global CPF" Ratio" and "Global Value Difference" are the representative-agent CPF ratio and value difference for each choice problem, computed using the value of δ estimated in the Exponential Discounting model described in Appendix E.2.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 7: Individual-Level Error Rates vs. CPF Ratio

	Dependent Variable: Binary Error (Indiv. $\hat{\delta}$)			
	(1)	(2)	(3)	(4)
Global CPF Ratio	-0.18^{***}	-0.17^{***}		
	(0.01)	(0.01)		
Indiv. CPF Ratio			-0.38^{***}	-0.34^{***}
			(0.01)	(0.01)
Indiv. Value Difference		-0.01^{***}		-0.01^{***}
		(0.00)		(0.00)
(Intercept)	0.25^{***}	0.31^{***}	0.40^{***}	0.40^{***}
	(0.00)	(0.01)	(0.01)	(0.01)
R^2	0.02	0.06	0.10	0.11
Adj. R^2	0.02	0.06	0.10	0.11
Num. obs.	41450	41450	41450	41450

OLS estimates. Standard errors (in parentheses) are robust. "Global CPF" Ratio" is the representative-agent CPF ratio for each subject-choice problem, computed using the value of δ estimated in the Exponential Discounting model described in Appendix E.2. "Indiv. CPF" Ratio" and "Indiv. Value Difference" are the individual-level CPF ratio and value difference for each subject-choice problem, computed using the individual-level δ_i estimates under the same model.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 8: Structural Estimates: Intertemporal Choice

	EDU	HDU	CPF-C
Parameter Estimates			
δ	0.95		0.96
ι		0.16	
ζ		0.12	
κ			0.03
γ			0.85
η	0.35	0.43	
R^2	0.75	0.78	0.88

"EDU", "HDU", and "CPF-C" refer to the Exponential Discounting, Hyperbolic Discounting, and CPF Complexity models described in Appendix E.2. For these estimates, each time period is 24 days.

Table 9: Complexity Responses vs. CDF Ratio

	Dependent Variable: Error Rate		*Dependent Variable*: Inconsistency Rate		*Dependent Variable*: CU	
	(1)	(2)	(3)	(4)	(5)	(6)
Global CDF Ratio	-0.32^{***}	-0.28^{***}	-0.10^{***}	-0.11^{***}	-0.19^{***}	-0.21^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Global Value Difference		-0.01^{***}		0.00^{***}		0.01^{***}
		(0.00)		(0.00)		(0.00)
(Intercept)	0.48^{***}	0.51^{***}	0.28^{***}	0.28^{***}	0.25^{***}	0.22^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
R^2	0.45	0.47	0.10	0.11	0.31	0.35
Adj. R^2	0.45	0.47	0.10	0.11	0.31	0.35
Num. obs.	10920	10920	10420	10420	500	500

OLS estimates. Standard errors (in parentheses) are robust. "Global CDF" Ratio" and "Global Value Difference" are the representative-agent CDF ratio and value difference for each choice problem, computed using the value of α estimated in the Expected Utility model described in Appendix E.3.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 10: Individual-Level Error Rates vs. CDF Ratio

	Dependent Variable: Binary Error (Indiv. $\hat{\alpha}$)			
	(1)	(2)	(3)	(4)
Global CDF Ratio	-0.27^{***}	-0.27^{***}		
	(0.01)	(0.01)		
Indiv. CDF Ratio			-0.35^{***}	-0.35^{***}
			(0.01)	(0.01)
Indiv. Value Difference		-0.00		0.00
		(0.00)		(0.00)
(Intercept)	0.36^{***}	0.36^{***}	0.41^{***}	0.41^{***}
	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.04	0.04	0.07	0.07
Adj. R^2	0.04	0.04	0.07	0.07
Num. obs.	12500	12500	12500	12500

OLS estimates. Standard errors (in parentheses) are robust. "Global CDF" Ratio" is the representative-agent CDF ratio for each subject-choice problem, computed using the value of α estimated in the Expected Utility model described in Appendix E.3. "Indiv. CDF" Ratio" and "Indiv. Value Difference" are the individual-level CPF ratio and value difference for each subject-choice problem, computed using the individual-level α_i estimates under the same model.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 11: Structural Estimates: Lottery Choice

	EU	RDEU	CPT	EV CDF-C	EU CDF-C
Parameter Estimates					
α	0.85	0.83	0.75		0.59
β		0.77	0.78		
λ		1.07	0.79		
χ			1.06		
ν			0.83		
κ				0.15	0.15
γ				0.77	0.71
η	0.22	0.24	0.33		
R^2	0.55	0.56	0.59	0.66	0.73

“EU", “RDEU", and “CPT" refer to the Expected Utility, Reference-Dependent Expected Utility, and Cumulative Prospect Theory models described in Appendix E.3. “EV CDF-C" and “EU CDF-C" refer to the risk-neutral and expected utility CDF complexity models described in Appendix E.3.

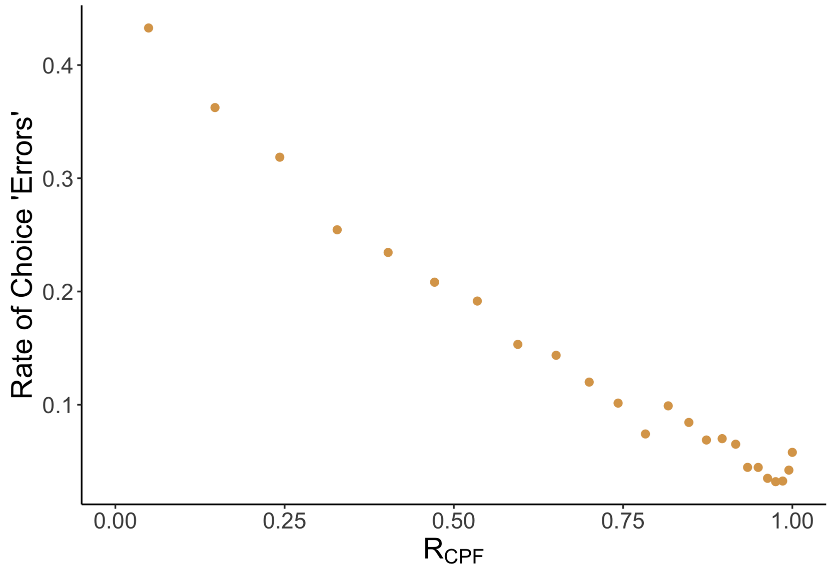


Figure 14: Binscatter of individual-level error dummies against individual-level CPF ratios. A choice is coded as an "error" if the individual chooses the lower-value option, according to the best-fit discount function estimated on their 50 experimental choices. We use the same 2-parameter structural model as in estimating global temporal preferences, which features exponential discounting and logit noise. The estimating equation is provided in Appendix E.2

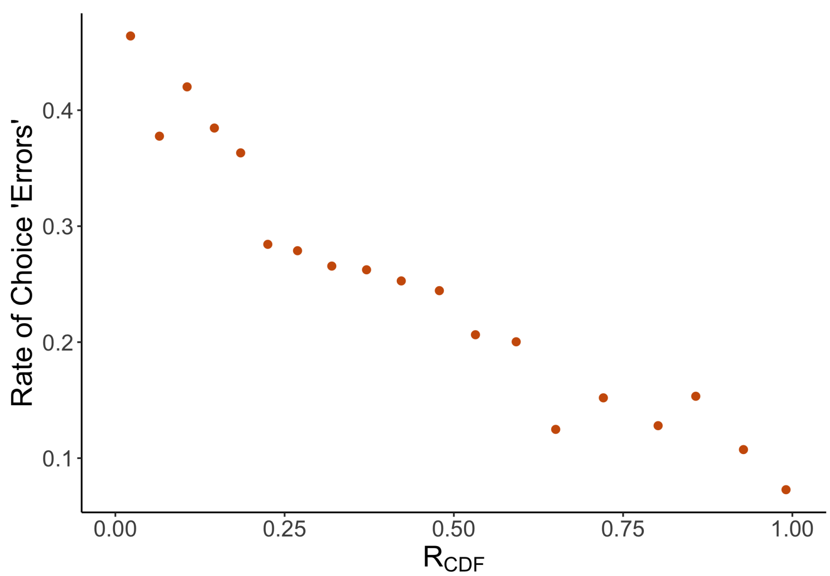


Figure 15: Binscatter of individual-level error dummies against individual-level CDF ratios. Analogously to time, a choice is coded as an "error" if the individual chooses the lower-value option, according to the utility function estimated on their 50 experimental choices. We use a 2-parameter structural model of risk preferences, which features symmetric CRRA utility and logit noise. The estimating equation is provided in Appendix E.3; we employ the parameter restriction $\alpha_i \geq 0.35$ in our estimations.

B Appendix: Additional Theoretical Results

Proofs of all results stated in this Appendix are compiled in Online Appendix F.

B.1 Characterization Results

B.1.1 Linear Multiattribute Choice

We state our characterization theorem in the case where $n \geq 2$. The $n = 2$ case requires an additional axiom. Let $x_{\{k\}} = x_{\{k\}}\vec{0}$.

M6. **Exchangeability**: If $\rho(x_{\{i\}}, x'_{\{j\}}) = 1/2$ and $\rho(x_{\{j\}}, x'_{\{i\}}) = 1/2$, with $x_k = x'_k = 0$ for all $k \neq i, j$, then $\rho(x, 0) = \rho(x', 0)$.

Exchangeability states that swapping attribute labels (adjusting for attribute weights) will not affect choice, and arises from the fact in our theory, the similarity in the denominator is defined over the same value-transformed attributes that govern preferences.

Theorem 2. *Suppose that all attributes are non-null. A binary choice rule ρ satisfies M1–M6 iff it has an L_1-complexity representation (G, β). If $n > 2$, ρ satisfies M1–M5 iff it has an L_1 complexity representation (G, β). Furthermore, if ρ also has an L_1-complexity representation (G', β') then $G' = G$ and there exists $C > 0$ such that $\beta' = C\beta$.*

B.1.2 Additively Separable Multiattribute Choice

We consider an extended multiattribute domain where each option in $X \equiv X_1 \times X_2 \times \ldots \times X_n$ is defined on n attribute dimensions, where each X_i is a connected and separable topological space. Preferences are additively separable in each attribute, where the value of each x is given by $U(x) = \sum_k u_k(x_k)$ for u_k continuous. Say that u_k is *non-trivial* if there exist $x_k, x'_k \in X_k$ such that $u_k(x_k) \neq u_k(x'_k)$. We propose that the ease of comparison in this domain is governed by the following representation:

Definition 5. *τ has an additively separable L_1-complexity representation if there exist continuous, non-trivial $u_i : X_i \to \mathbb{R}$ such that for $U(x) = \sum_k u_k(x_k)$ and $d_{L1}(x, y) = \sum_k |u_k(x_k) - u_k(y_k)|$, whenever $d_{L1}(x, y) \neq 0$,*

$$\tau_{xy} = H\left(\frac{|U(x) - U(y)|}{d_{L1}(x, y)}\right)$$

for H continuous, increasing with $H(0) = 0$, and $\tau_{xy} = 0$ otherwise. Similarly, a binary choice rule ρ has an additively separable L_1-complexity representation if there exist continuous, non-trivial $u_i : X_i \to \mathbb{R}$ such that whenever $d_{L1}(x, y) \neq 0$,

$$\rho(x, y) = G\left(\frac{U(x) - U(y)}{d_{L1}(x, y)}\right).$$

for G continuous, strictly increasing, and $\rho(x, y) = 1/2$ otherwise.

Note that this representation for τ satisfies the same principles of similarity, dominance, and simplification as the linear representation introduced in Section 2.1, and the corresponding representation for $\rho(x, y)$ satisfies axioms M1 and M3–M5. We provide an axiomatic characterization for this representation, in which Linearity is relaxed and replaced with two axioms.

First some definitions. For $E \subseteq I$, let $x_E y$ denote the option that replaces the value of option y along attributes $k \in E$ with x_k. Say that comparisons $(x, y), (w, z) \in \mathscr{D}$ are *congruent* if for all $i \in I$, either $\rho(x_{\{i\}}y, y) \geq 1/2$ and $\rho(w_{\{i\}}z, z) \geq 1/2$, or $\rho(x_{\{i\}}y, y) \leq 1/2$ and $\rho(w_{\{i\}}z, z) \leq 1/2$. That is, if (x, y) and (w, z) are congruent, the advantages and disadvantages in the two comparisons are located in the same attributes.

M7. **Separability:** $\rho(x_E z, y_E z) = \rho(x_E z', y_E z')$ for all $x, y, z, z' \in X$, $E \subseteq I$.

M8. **Tradeoff Congruence**. Suppose that (x, y) is congruent to (y, z), and $\rho(x, y), \rho(y, z) \geq 1/2$. Then $\rho(x, z) \leq \max\{\rho(x, y), \rho(y, z)\}$.

Separability is the stochastic analog of the familiar coordinate independence axiom in deterministic choice, which says that $x_E z \succeq y_E z \implies x_E z' \succeq y_E z'$ for all $E \subseteq I$, $x, y, z, z' \in X$. The interpretation of Tradeoff Congruence is as follows: consider the attribute-wise tradeoffs involved in comparing z to y and x to y, where x is in fact better than y, and y is better than z. If replacing y with x in the first comparison and replacing y with z in the second only increases the magnitude of these tradeoffs – i.e. if (x, y) and (y, z) are congruent – then (x, z) cannot be an easier comparison than both of the intermediate comparisons (x, y) and (y, z). Intuitively, both of these replacements only increase the size of the tradeoffs the DM must contend with, and so as revealed by choice probabilities, the DM cannot find the comparison (x, z) easier than both (x, y) and (y, z).

The following result states that Continuity, Moderate Transitivity, Dominance, Simplification, Separability, and Tradeoff Congruence characterize the additively separable representation, and that its primitives are identified from choice data.

Theorem 3. *Suppose that $n > 2$ and that all attributes are non-null. Then a binary choice rule ρ satisfies M1,M3–M5, M7–M8 if and only if it has an additively separable L_1-complexity representation. Morever, suppose that at least two attributes are non-null. If ρ has additively separable L_1 complexity representations $((u_i)_{i=1}^n, G)$ and $((u'_i)_{i=1}^n, G')$, then there exists $C > 0$, $b_i \in \mathbb{R}$ such that $u'_i = Cu_i + b_i$ for all i, and $G' = G$.*

B.1.3 Lottery Choice

Consider the lottery choice domain, where X is the set of finite state lotteries over $\mathbb{R}$. Note that the CDF-complexity representation for τ implies the following binary choice representation:

Definition 6. *A binary choice rule ρ has a CDF-Complexity representation if there exists $u : \mathbb{R} \to \mathbb{R}$ strictly increasing such that*

$$\rho(x, y) = G\left(\frac{EU(x) - EU(y)}{d_{CDF}(x, y)}\right)$$

for G continuous, strictly increasing.

Let $\geq$ denote the partial order on X corresponding to first-order stochastic dominance. Let $S_x = \{w \in \mathbb{R} : f_x(w) > 0\}$ denote the support of lottery x. Consider the following axioms:

L1. **Continuity:** $\rho(x, y)$ is continuous on its domain.

L2. **Independence:** $\rho(x, y) = \rho(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)z)$ for $\lambda \in (0, 1)$.

L3. **Moderate Transitivity:** If $\rho(x, y) \geq 1/2$, $\rho(y, z) \geq 1/2$, then either $\rho(x, z) > \min\{\rho(x, y), \rho(y, z)\}$ or $\rho(x, z) = \rho(x, y) = \rho(y, z)$.

L4. **Dominance:** $x \geq y$, then $\rho(x, y) \geq \rho(w, z)$ for any $w, z \in L(S)$, where the inequality is strict if $w \not\geq z$.

L5. **Simplification**. For any $x, y \in X$ and $w^* \in S_x \cup S_y$, consider x' with support in $S_x \cup S_y$ satisfying $F_{x'}(w^*) = F_y(w^*)$, and $F_{x'}(w) \neq F_x(w)$ for at most one $w \in S_x \cup S_y / \{w^*\}$. If $\rho(x, y) \geq 1/2$ and $\rho(x', x) = 1/2$, then $\rho(x', y) \geq \rho(x, y)$.

Axioms L1–L4 are the direct analogs of M1–M4 in the characterization of L_1 complexity. Axiom L5 says that concentrating value differences between lotteries in the same region of the distribution of the lotteries makes them easier to compare, and is an analog of the Simplification property (Axiom M5) for L_1 complexity. Axioms L1–L5 exhaust the behavioral content of CDF complexity.

Theorem 4. *A binary choice rule ρ satisfies L1-L5 if and only if it has a CDF-Complexity representation (G,u). Moreover, if (G',u') also represents ρ, then $G' = G$ and there exists $C > 0, b \in \mathbb{R}$ such that $u' = Cu + b$.*

B.1.4 Intertemporal Choice

Consider the intertemporal choice domain, where X is the set of finite payoff streams. For a payoff flow $x \in X$, let $T_x = \{t : m_x(t) \neq 0\}$ denote the *support* of x, and for $x, y \in X$ let $T_{xy} = T_x \cup T_y \cup \{0, \infty\}$ denote the *joint support* of x and y. We consider the following extension of our CPF complexity measure to general time discounting. Call $d : \mathbb{R}^+ \cup \{+\infty\} \to \mathbb{R}^+$ a *discount function* if d is strictly decreasing and $d(\infty) = 0$. We will consider discounted utility preferences of the form $DU(x) = \sum_t d(t)m_x(t)$. Note that d need not be continuous, and so our generalization can capture discontinuous time preferences such as quasi-hyperbolic discounting. Recall that $M_x(t) = \sum_{t' \leq t} m_x(t)$ is the cumulative payoff function of a payoff flow x.

Definition 7. *τ has a generalized CPF complexity representation if there exists a discount function d such that*

$$\rho(x,y) = G\left(\frac{DU(x) - DU(y)}{d_{CPF}(x,y)}\right)$$

for H continuous, strictly increasing with $H(0) = 0$, where $d_{CPF}(x,y) = \sum_{k=0}^{n-1} |M_x(t_k) - M_y(t_k)| \cdot (d(t_k) - d(t_{k+1}))$ for $t_0 < t_1 < \ldots < t_n$ enumerating T_{xy} is the generalized CPF distance. Similarly, a binary choice rule ρ has a generalized CPF complexity representation if

$$\rho(x,y) = G\left(\frac{DU(x) - DU(y)}{d_{CPF}(x,y)}\right)$$

for some continuous, strictly increasing G.

Note that if d is differentiable, d_{CPF} can be more conveniently rewritten as $d_{CPF}(x,y) = \int_0^\infty |M_x(t) - M_y(t)| \cdot (-d'(t))\, dt$. In the case where $d(t) = \delta^t$, generalized CPF complexity reduces to Definition 4. Let $\geq$ denote the partial order X corresponding to temporal dominance (i.e., $x \geq y$ iff at every time $t \in \mathbb{R}^+ \cup \{+\infty\}$, $M_x(t) \geq M_Y(t)$). Consider the following axioms:

T1. **Continuity:** $\rho(x,y)$ is continuous on its domain.

T2. **Linearity:** $\rho(x,y)=\rho(\lambda x+(1-\lambda)z,\lambda y+(1-\lambda)z)$ for $\lambda\in(0,1)$.

T3. **Moderate Transitivity:** If $\rho(x,y)\geq 1/2$, $\rho(y,z)\geq 1/2$, then either $\rho(x,z)>\min\{\rho(x,y),\rho(y,z)\}$ or $\rho(x,z)=\rho(x,y)=\rho(y,z)$.

T4. **Dominance:** $x\geq y$, then $\rho(x,y)\geq\rho(w,z)$ for any $w,z\in X$, where the inequality is strict if $w\not\geq z$.

T5. **Simplification**. For any $x,y\in X$ and $t^*\in T_x\cup T_y$, consider x' with support in $T_x\cup T_y$ satisfying $M_{x'}(t^*)=M_y(t^*)$, and $M_{x'}(t)\neq M_x(t)$ for at most one $t\in T_x\cup T_y/\{t^*\}$. If $\rho(x,y)\geq 1/2$ and $\rho(x',x)=1/2$, then $\rho(x',y)\geq\rho(x,y)$.

Axioms T1–T4 are the direct analogs of M1–M4 in the characterization of L_1 complexity. Axiom T5 says that concentrating value differences between payoff flows in the same region of time makes them easier to compare, and is an analog of the Simplification property (Axiom M5) for L_1 complexity. Axioms L1–L5 exhaust the behavioral content of CPF complexity.

Theorem 5. *A binary choice rule ρ satisfies T1 – T5 iff it has a generalized CPF-Complexity Representation (G,d). Moreover, if ρ is also represented by (G',d'), then $G'=G$, and there exists $C>0$ such that $d'=Cd$.*

To characterize CPF complexity with exponential discounting preferences, an additional standard stationarity axiom is needed.

T6. **Stationarity**. If $\rho(x,y)>1/2$, then for $x',y',k>0$ such that $m_{x'}(t)=m_x(t-k)$, $m_{y'}(t)=m_y(t-k)$ for all $t\geq k$ and $m_{x'}(t)=m_{y'}(t)=0$ for all $t<k$, $\rho(x',y')\geq 1/2$.

B.2 Relationship between CDF/CPF and L_1 Complexity

We formalize a connection between our complexity measures for lottery and intertemporal choice and our multiattribute complexity measure. In particular, we show that the CDF (CPF) complexity between two lotteries (payoff flows) is equivalent to the L_1 complexity computed over a common attribute representation of those choice options — specifically, the common attribute representation that maximizes their ease of comparison.

CPF Complexity and $\mathbf{L_1}$ Complexity. In the lottery domain, consider the set of couplings of lotteries x, y — that is, the set $\Gamma(x,y)$ of joint distributions $g(w_x, w_y)$ over payoffs such that $\sum_{w_y} g(w, w_y) = f_x(w)$ and $\sum_{w_x} g(w_x, w) = f_y(w)$ for all w. Note that each coupling g induces an attribute representation of x and y, in which the attributes are given by the set of joint utility-transformed payoff realizations in the support of g, weighted by the likelihoods of those payoff realizations.[43]

To take an example, consider a lottery x which pays \$18 w.p. 20%, and y which pays \$12 w.p. 25%, and consider the attribute structures induced by two different couplings:

	60%	20%	5%	15%
x	$u(0)$	$u(0)$	$u(18)$	$u(18)$
y	$u(0)$	$u(12)$	$u(12)$	$u(0)$

	75%	5%	20%
x	$u(0)$	$u(0)$	$u(18)$
y	$u(0)$	$u(12)$	$u(12)$

The attribute structure on the left corresponds to a coupling in which the lotteries are uncorrelated, and the attribute structure on the right corresponds to a coupling that imposes positive correlation between the lotteries. For each attribute structure induced by g, we can compute the ease of comparison under L_1 complexity, given by[44]

$$\tau_{xy}^{L1}(g) \equiv H\left(\frac{|\sum_{w_x,w_y} g(w_x,w_y)(u(w_x)-u(w_y))|}{\sum_{w_x,w_y} |g(w_x,w_y)(u(w_x)-u(w_y))|}\right).$$

Proposition 6 says that the attribute structure g that maximizes the ease of comparison according to $\tau_{xy}^{L1}(g)$ gives rise to exactly the CDF complexity representation.

Proposition 6. $\max_{g\in\Gamma(x,y)} \tau_{xy}^{L1}(g) = H\left(\frac{EU(x)-EU(y)}{d_{CDF}(x,y)}\right)$.

Proposition 6 points to the following two-stage cognitive interpretation of our CDF complexity measure: in a “representation stage", the DM first represents the lotteries using a common set of attributes — specifically, the attribute structure that makes the lotteries maximally easy to compare — and then compares the lotteries along these attributes in an “evaluation stage".

CDF Complexity and $\mathbf{L_1}$ Complexity The CPF complexity measure can similarly be inter-

[43] Such attribute representations of lotteries have been used in previous work, such as Bordalo, et al. (2012).

[44] For example, in the risk neutral case where $u(w) = w$, the ease of comparison is given by $H\left(\frac{0.05\cdot(6)+0.15\cdot(18)-0.2\cdot(12)}{0.05\cdot(6)+0.15\cdot(18)+0.2\cdot(12)}\right) = H(0.11)$ for the leftmost attribute representation and $H\left(\frac{0.20\cdot(6)-0.05\cdot(12)}{0.2\cdot(14)+0.6\cdot(4)}\right) = H(0.33)$ for the rightmost attribute representation

preted through this two-stage procedure. In what follows, we restrict attention to positively-valued payoff flows; i.e. $x \in X$ such that $m_x \geq 0$.

Consider a common attribute representation of payoff flows (x, y) in which the attributes are the discounted-delays of payoffs in (x, y), weighted by the payoff amount at each delay. Formally, let $B(x, y)$ denote the set of *joint payoff functions* $b : \mathbb{R}_+ \cup \{+\infty\} \times \mathbb{R}_+ \cup \{+\infty\} \to \mathbb{R}$ that map joint delays of x and y into payoff amounts, where $b(\infty, \infty) = 0$ and where $m_x(t) = \sum_{t_y} b(t, t_y)$ and $m_y(t) = \sum_{t_x} b(t_x, t)$ for all $t < \infty$; that is, the marginal payoff functions induced by b agree with those of x and y. For each attribute representation given by $b \in B(x, y)$, the ease of comparison under L_1 complexity is given by

$$\tau_{xy}^{L1}(b) \equiv H\left(\frac{|\sum_{t_x,t_y} b(t_x, t_y)(d(t_x) - d(t_y)|}{\sum_{t_x,t_y} |b(t_x, t_y)(d(t_x) - d(t_y))|}\right).$$

The following proposition states that the attribute structure g that maximizes the ease of comparison according to $\tau_{xy}^{L1}(b)$ gives rise to the CPF complexity representation.

Proposition 7. *For positively-valued payoff flows x, y, we have $\max\limits_{b \in B(x,y)} \tau_{xy}^{L1}(b) = H\left(\frac{DU(x) - DU(y)}{d_{CPF}(x,y)}\right)$, where d_{CPF} is the generalized CPF distance.*

B.3 Tiebreaking in Multinomial Choice

Fix any choice problem (A, C). In the event that a signal s induces a tie among options that maximize posterior expected value, we assume a symmetric tiebreaking rule in which the DM randomizes between the maximal options. In particular, for any option $x \in A$ and signal realization s, let $\mathcal{N}(x, s) \equiv |\{y \in A : \mathbb{E}[v_y|s] = \mathbb{E}[v_x|s]\}|$ denote the number of options in A with the same posterior expected value as x, and define the random variable

$$\mathscr{C}(x, s) \equiv \begin{cases} 1/\mathcal{N}(x, s) & \mathbb{E}[v_x|s] \geq \mathbb{E}[v_y|s] \,\forall\, y \in A \\ 0 & \text{otherwise} \end{cases}$$

Choice probabilities are given by

$$\rho(x, A|C) = \mathbb{E}[\mathscr{C}(x, s) \,|\, v].$$

We assume the same tiebreaking rule in our extension to menu sequences wherein the DM independently randomizes between the maximal options in each menu. In particular, fix a choice problem $((A^1, \ldots, A^n), C)$. For any option $x \in A^i$ and signal realization s, let

$\mathcal{N}^i(x,s) \equiv |\{y \in A^i : \mathbb{E}[v_y|s] = \mathbb{E}[v_x|s]\}|$ denote the number of options in A^i with the same posterior expected value as x, and define

$$\mathscr{C}^i(x,s) \equiv \begin{cases} 1/\mathcal{N}^i(x,s) & \mathbb{E}[v_x|s] \geq \mathbb{E}[v_y|s]\, \forall\, y \in A^i \\ 0 & \text{otherwise} \end{cases}$$

Choice probabilities are given by

$$\rho((x^1,...,x^n),(A^1,...,A^n)|C) = \mathbb{E}\left[\prod_{i=1}^{n} \mathscr{C}^i(x^i,s)\,\middle|\, v\right].$$

B.4 Identification in Multinomial Choice

Call $\rho : X \times \mathscr{A} \times \mathscr{M} \to [0,1]$ a multinomial choice rule if $\sum_{x \in A} \rho(x,A|C) = 1$ for all $(A,C) \in \mathscr{A} \times \mathscr{M}$. Our multinomial choice model is parameterized by the prior distribution Q, the value function $v : X \to \mathbb{R}$, and the signal precisisons $\tau : \mathscr{D} \to \mathbb{R}^+$, where we make the additional assumption that $\tau(x,y) = 0$ if $v(x) = v(y)$. The following result states that v is ordinally identified and τ is exactly identified.

Proposition 8. *Suppose that a multinomial choice rule ρ is represented by (Q,v,τ) and (Q',v',τ'). Then $\tau' = \tau$ and there exists $\phi : \mathbb{R} \to \mathbb{R}$ strictly increasing such that $v' = \phi \circ v$.*

This identification result relies only on binary choice data, from which the prior distribution Q cannot be identified. We conjecture that Q can be identified using choice data from larger menus.

B.5 Decoy Effects

Example 5. (Classic decoy effects). Consider a setting where options have two attributes, where $\beta = (1,1)$, and where τ has an L_1 complexity representation. Consider two indifferent choice options $x = (1,2)$, $y = (2,1)$, and consider the effect of including a phantom option on choice shares between x and y.

Case 1: $z = (1.8, 0.8)$. Since $d_{L1}(x,z) < d_{L1}(y,z)$, we have $\rho(y,x|\{z\}) > 0.5$. We recover the classic asymmetric dominance effect: the addition of an option that is dominated by the target option y but not by the competitor x distorts choice in favor of y.

Case 2: $z' = (1.5, 1.1)$. We again have $d_{L1}(x, z') < d_{L1}(y, z')$, we have $\rho(y, x|\{z'\}) > 0.5$. Here, the model predicts a "good deal" effect – z' is not dominated by either x or y, but its proximity to y makes the target option seem like a "good deal" relative to z, whereas its distance to x prevents the DM from drawing the same inference about the competing option.

Case 3: $z'' = (0.8, 0.5)$. Here, $d_{L1}(x, z'') = d_{L1}(y, z'')$, and so Corollary 1.1 implies that $\rho(y, x|\{z''\}) = 0.5$. That is, the model predicts that the addition of a mutually dominated option does not affect choice shares.

Comparison to other context-dependent models. Though each of the choice patterns above can be explained by familiar models, our model is distinct in simultaneously explaining all three. The salience (Bordalo et al., 2013) and focusing models (Kőszegi and Szeidl, 2013) cannot rationalize the decoy effects in Cases 1 and 2. The relative thinking model (Bushong et al., 2021), in which the DM weighs a given change along an attribute by less when there is a larger range of values along that attribute, can rationalize the decoy effect in Case 1 as a result of option z extending the range of attribute 2 more than attribute 1, but not Case 2, where z' has no effect on attribute ranges. The pairwise normalization model (Landry and Webb, 2021) predicts that z increases the relative of y relative to x whenever z_1/z_2 is closer to y_1/y_2 than it is to x_1/x_2, and so can rationalize the decoy effects in both Cases 1 and 2, but also delivers the counterfactual prediction that the addition of a mutually dominated option z'' will also distort choice in favor of x. Furthermore, all of these models are formulated in multi-attribute choice, which means they cannot easily explain documented decoy effects in lottery choice (Soltani et al., 2012) or other domains. Our choice framework straightforwardly applies to lottery and intertemporal choice.

We also make an important conceptual distinction from these existing models. In our model, biased choice does *not* arise from a behavioral bias, but instead as a rational response to imperfect comparability. We only expect decoy options to distort choice between x and y when their binary comparison is challenging. This is consistent with the fact that the attraction effect is muted when consumers face familiar choice contexts or have clear prior preferences (Huber et al., 2014) – an empirical phenomenon that is not well-explained by existing models.

B.6 Intertemporal Preference Reversals: Details

Present Value Equivalents. The DM faces a valuation task (v, Z), where $v = (m_v, t_v)$ is a delayed payment that pays out $m_v > 0$ at time $t_v > 0$, to be valued against a price list

$Z = \{z^1, ..., z^n\}$, where each $z^k = (m_k, 0)$ is an immediate payment. Call Z *adapted* to a delayed payment υ if $m_k - m_{k+1}$ is constant in k and $m_1 = m_\upsilon$, $m_n = 0$; we restrict attention to adapted price lists. Let $PVE(\upsilon, Z) = 1/2[m_{R(\upsilon,Z)-1} + m_{R(\upsilon,Z)}]$ denote the distribution over the DM's present value equivalents obtained from assigning each switching point to a valuation at the midpoint of the adjacent prices.

Time Equivalents. The DM faces a valuation task (υ, Z), where $\upsilon = (m_\upsilon, t_\upsilon)$ is a delayed payment to be valued against a price list $Z = \{z^1, ..., z^n\}$, where now each $z^k = (27.5, t_\upsilon + t_k)$ is an delayed payment; we restrict attention to time lists with $t_1 = 0$. We let

$$TE(c, Z) = \begin{cases} 1/2[t_{R(c,Z)-1} + t_{R(c,Z)}] & R(c, Z) < n + 1 \\ t_n + 1/2(t_n - t_{n-1}) & R(x, Z) = n + 1 \end{cases}$$

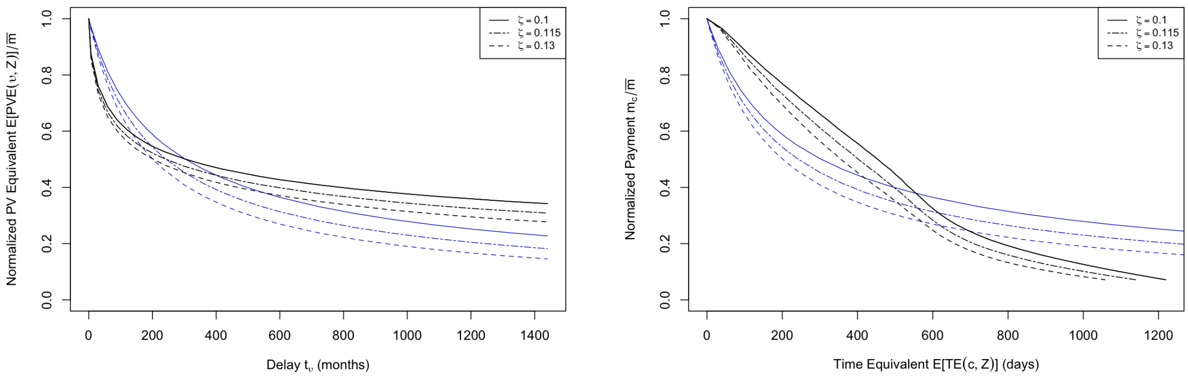
denote the distribution over the DM's time equivalents obtained from assigning each switching point to a valuation at the midpoint of the adjacent delays.

B.7 Intertemporal Valuations with Hyperbolic Preferences

We consider the intertemporal domain with hyperbolic discounting, where $v_x = DU(x) \equiv \sum_t d(t)m_x(t)$ for $d(t) = (1 + \iota t/24)^{-\zeta/\iota}$ and $\tau_{xy} = \tau_{xy}^{CPF} = H\left(\frac{DU(x)-DU(y)}{d_{CPF}(x,y)}\right)$, where d_{CPF} is the generalized CPF distance in Definition 7. Again, $H(1) = \infty$, meaning the DM perfectly learns the ranking between prospects with a temporal dominance relationship.

Figure 16 conducts the same simulation exercise as Figure 6 in the main text, except now the DM's true time preferences are given by hyperbolic discounting instead of exponential discounting. In particular, Figure 16a plots the normalized plots the normalized present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]/\overline{m}$ of a delayed payment $\upsilon = (\overline{m}, t_\upsilon)$ as a function of the delay t_υ, where the delayed payment $\upsilon = (\overline{m}, t_\upsilon)$ is valued against a price list $Z = \{z^1, ..., z^n\}$ of immediate payments adapted to υ. As the figure shows, the DM's valuations exaggerates the hyperbolicity inherent to her preferences: relative to her true discount rate (blue lines), she undervalues payments close to the present and overvalues payments with longer delays.

On the other hand, Figure 16b plots the time equivalents of an immediate payment $c = (m_c, 0)$: the predicted relationship between the immediate payment amount $m_c/\overline{m}$ (y-axis) and the associated time equivalents $\mathbb{E}[TE(c, Z)]$ (x-axis). Here, the model predicts overvaluation of payments close to the present and undervaluation of payments with longer delays relative to the DM's hyperbolic discount function.



(a) Simulated average present value equivalents $\mathbb{E}[PVE(\upsilon, Z)]$ (in black) for delayed payments $\upsilon = (\overline{m}, t_\upsilon)$ as a function of t_υ.

(b) Relationship between simulated average time equivalents $\mathbb{E}[TE(c, Z)]$ (in black) for immediate payment $c = (m_c, 0)$ and normalized amount $m_c/\overline{m}$.

Figure 16: Discount functions implied by present value equivalents (left) and time equivalents (right). For PVEs, Z is adapted to υ with $|Z| = 15$. For TEs, $Z = \{z^1, ..., z^n\}$, where $z^k = (\overline{m}, t_k)$, for $(t_1, ..., t_n) = (0, 7, 30, 60, 120, 180, 240, 360, 480, 600, 720, 900, 1080, 1260, 1440)$ days. Blue curves plot distortion-free discount functions given the true hyperbolic parameters (ι, ζ), for $\iota = 0.159$, ζ varying. τ has a generalized CPF-complexity representation parameterized by $H(r) = (\Phi^{-1}(G(r)))^2$, for G given by (1) with $\kappa = 0, \gamma = 0.5$. Priors are distributed $Q \sim U[0, 1]$.

B.8 Relationship to Random Utility

The choice probabilities generated by our L_1-complexity model shares the following intersection with random utility models:

Proposition 9. *Suppose ρ has an L_1-complexity representation (β, G), where $G(1) = G(-1) = 1$. Then, there exists an n-dimensional random vector $\tilde{\beta}$ such that for all $(x, y) \in \mathscr{D}$, $\rho(x, y) = \mathbb{P}\left\{\sum_k \tilde{\beta}_k x_k \geq \sum_k \tilde{\beta}_k y_k\right\}$, where for all k, $\mathbb{P}\left\{sgn(\tilde{\beta}_k) = sgn(\beta_k)\right\} = 1$.*

This result suggests two additional interpretations to the binary choice probabilities produced by the L_1-complexity model. First, we can interpret L_1-complexity as describing the binary choices of an agent who correctly understands the valence of each attribute – that is, the sign of each attribute weight – but has only a noisy perception over the exact attribute weights – in other words, the DM understands dominance but errs when making tradeoffs across attributes. Second, if ρ describes population-level data, we can interpret L_1-complexity as describing the choices of a population of agents who agree on the valence of each attribute, but disagree on the relative weights of attributes.

B.9 Relationship to Linear Differentiation Model

In this section, we contrast the L_1-Complexity model (Definition 2) against the Linear Differentiation Model (LDM) proposed in He and Natenzon (2023b), also defined on the domain of $\mathbb{R}^n$.

Definition 8. *(He and Natenzon, 2023b). A binary choice rule ρ has a linear differentiation representation if there exists $\beta \in \mathbb{R}^n$, an $n \times n$ symmetric positive-definite matrix Σ, and a continuous, strictly increasing function G, such that*

$$\rho(x, y) = G\left(\frac{\beta'(x-y)}{\sqrt{(x-y)'\Sigma(x-y)}}\right).$$

From Definitions 2 and 8, it is immediate that the intersection between L_1-Complexity and the LDM is trivial. Below, we discuss two axes along which the models differ.

Dominance. As Theorem 1 implies, the L_1-Complexity model satisfies a dominance property (M4), wherein if x attribute-wise dominates y (written $x >_D y$), the choice probability $\rho(x, y)$ is maximal. Consider a weaker dominance notion, which requires only that if $x >_D y$ and $w \not>_D z$, then $\rho(x, y) \geq \rho(w, z)$. When there are three or more attributes, the LDM fails this dominance notion[45].

Proposition 10. *Suppose ρ has a linear differentiation representation and at least 3 attributes are non-null. There exists $x, y, w, z \in \mathbb{R}$, such that $x >_D y$ and $w \not>_D z$, such that $\rho(x, y) < \rho(w, z)$.*

Monotonicity. Say that a binary choice rule ρ is *weakly monotonic* if $x' >_D x$ implies $\rho(x', y) \geq \rho(x, y)$ for any $y \in \mathbb{R}^n$: that is improving a choice option along each attribute cannot lead to a decrease in its probability of being chosen over some other choice option. The L_1-complexity model satisfies such a monotonicity property, whereas in general, the LDM violates monotonicity.

Proposition 11. *If ρ has an L_1-complexity representation, ρ is weakly monotonic. If instead ρ has a linear differentiation representation, and at least 2 attributes are non-null, then ρ is not weakly monotonic: there exists $x, x', y \in \mathbb{R}^n$ with $x' >_D x$ such that $\rho(x, y) > \rho(x', y)$.*

The intuition for the monotonicity violations produced by the LDM stem stems from a property of the model formalized in Proposition 1 from He and Natenzon (2023b), which

[45]When there are two attributes, there are certain parameterizations of the LDM that satisfy the dominance notion.

states that any x that solves $\max_x \rho(x,y)$ lies along along the same one-dimensional subset of $\mathbb{R}^n$ containing y. An immediate implication of this property is that for any x' that dominates x, we will have $\rho(x',y) < \rho(x,y)$ so long as x' does not also lie in that subset.

B.10 Relationship to Bayesian Probit

Consider the same example as in Section 6: we have $x = (2,0), y = (0,1), z = (0,0)$, where $v_x = 2$, $v_y = 1$, $v_z = 0$, and $q \equiv \rho(x,z) = \rho(y,z)$. We are interested in the lower bound that the Bayesian Probit model places on $\rho(x,y)$.

Let $\sigma_{ij} = \mathrm{Cor}(\epsilon_i, \epsilon_j)$ and let Σ denote the correlation matrix between $(\epsilon_x, \epsilon_y, \epsilon_z)$. In Bayesian Probit, binary choice probabilities are given by $\rho(i,j) = \Phi\left(\frac{\sqrt{p}}{\sqrt{2}} \cdot (v_i - v_j) \cdot \frac{1}{\sqrt{1-\sigma_{ij}}}\right)$. The conditions on binary choice $\rho(x,z) = \rho(y,z) = q$, as well as the restriction that $\det(\Sigma)$ is positive, yields the set of restrictions

$$\begin{aligned}
\sigma_{xz} &= 1 - \frac{(v_x - v_z)^2 p}{2\Phi^{-1}(q)} \\
\sigma_{yz} &= 1 - \frac{(v_y - v_z)^2 p}{2\Phi^{-1}(q)} \\
0 &\leq 1 + 2\sigma_{xy}\sigma_{xz}\sigma_{yz} - \sigma_{xy}^2 \sigma_{yz}^2 \sigma_{xz}^2.
\end{aligned}$$

For any given q, p, we can numerically solve for the minimum level σ_{xy}^* satisfying the above restrictions, which in turn yields a lower bound on the choice probability $\rho(x,y)$ given by $\rho^*(x,y) = \Phi\left(\frac{\sqrt{p}}{\sqrt{2}} \cdot (v_x - v_y) \cdot \frac{1}{\sqrt{1-\sigma_{xy}^*}}\right)$, since $\rho(x,y)$ is increasing in σ_{xy}. Table 12 lists these bounds as a function of q, for a range of global precision parameters p; we see that the model cannot accomodate $\rho(x,y)$ close to 1/2. Note that in contrast, the L_1-complexity model (with $\beta = (1,1)$) can accomodate any $\rho(x,y) \in (1/2, q)$. As such, there are binary choice rules rationalizable by L_1-complexity, and therefore our multinomial extension, that cannot be rationalized by Bayesian Probit. More generally, by considering similar examples where x and y dominate z but do not themselves have a dominance relationship in the domains of lottery and intertemporal choice, one can show that there are binary choice rules rationalizable by CDF and CPF complexity but not by Bayesian Probit.

Table 12: Bayesian Probit Bounds

	$q=0.9$	$q=0.92$	$q=0.94$	$q=0.96$	$q=0.98$	$q=0.99$
$p=0.01$	0.666	0.680	0.698	0.720	0.753	0.781
$p=0.1$	0.668	0.682	0.700	0.722	0.754	0.782
$p=0.2$	0.670	0.685	0.702	0.724	0.756	0.783
$p=0.4$	0.676	0.690	0.706	0.727	0.759	0.785
$p=0.7$	0.686	0.698	0.713	0.733	0.763	0.789
$p=1$	0.698	0.708	0.721	0.739	0.767	0.792

Minimal $\rho(x,y)$ under Bayesian Probit for $v_x=2, v_y=1, v_z=0$ given $q\equiv\rho(x,z)=\rho(y,z)$ and p.

C Appendix: Proofs of Main Text Results

C.1 Characterization of L_1-Complexity

Begin with some basic definitions and observations. Let X be a space of options, and let $\mathscr{D}=\{(x,y)\in X\times X : x\neq y\}$. Say $\rho:\mathscr{D}\to[0,1]$ is a *binary choice rule* on X if $\rho(x,y)=1-\rho(y,x)$.

Call a (complete) binary relation $\succeq$ on X the *stochastic order* induced by a binary choice rule ρ if for all $x\neq y$, $x\succeq y$ if $\rho(x,y)\geq 1/2$, and for all $x\in X$, $x\succeq x$. Say that a binary choice rule ρ satisfies *moderate transitivity* if for $\rho(x,y),\rho(y,z)\geq 1/2$, then $\rho(x,z)>\min\{\rho(x,y),\rho(y,z)\}$ or $\rho(x,z)=\rho(x,y)=\rho(y,z)$. Say that a binary choice rule ρ satisfies *weak transitivity* if for $\rho(x,y),\rho(y,z)\geq 1/2$, $\rho(x,y)\geq 1/2$. Consider a partial order $\geq_X$ on X. Say that a binary choice rule ρ satisfies *monotonicity* with respect to $\geq_X$ if $x'\geq_X x$ implies $\rho(x',y)\geq\rho(x,y)$, where the inequality is strict whenever $x\not\geq_X x'$, $x\not\geq_X y$ and $y\not\geq_X x$. Say that ρ satisfies *dominance* with respect $\geq_X$ if whenever $x\geq_X y$, we have $\rho(x,y)\geq\rho(w,z)$ for all $w,z\in X$, where the inequality is strict if $w\not\geq_X z$.

Lemma 1. *If ρ defined on X satisfies moderate transitivity and dominance with respect to a partial order $\geq_X$, then it satisfies monotonicity with respect to $\geq_X$.*

Proof. Take any options x,y, and suppose $x'\geq_X x$. If $x\geq_X x'$, then $x'=x$ since $\geq_X$ is a partial order and is therefore antisymmetric, and we are done. Now consider the case where $x\not\geq_X x'$. Note that if $x\geq_X y$, since $\geq_X$ is transitive we also have $x'\geq_X y$, and so Dominance implies that $\rho(x',y)\geq\rho(x,y)$ and we are done.

Now consider the case where $x\not\geq_X y$. Let $\succeq$ denote the stochastic order induced by ρ; since ρ satisfies MST, $\succeq$ is complete and transitive. By dominance, we have $\rho(x',x)>\rho(x,x')\implies\rho(x',x)>1/2$ and so $x'\succ x$. There are three cases to consider:

Case 1: $x' \succeq x \succeq y$. By moderate transitivity, $\rho(x', y) > \min\{\rho(x', x), \rho(x, y)\}$ or $\rho(x', y) = \rho(x', x) = \rho(x, y)$. But since $\rho(x', x) > \rho(x, y)$ by dominance, it must be the case that $\rho(x', y) > \rho(x, y)$.

Case 2: $x' \succeq y \succeq x$. By definition of $\succeq$, $\rho(x', y) \geq 1/2 \geq \rho(x, y)$. Also, since $x' \succ x$, we must have one of $x' \succ y$ or $y \succ x$, and so by definition of $\succeq$ we must have one of $\rho(x', y) > 1/2$ or $1/2 > \rho(x, y)$, which implies $\rho(x', y) > \rho(x, y)$.

Case 3: $y \succeq x' \succeq x$. Toward a contradiction, suppose that $\rho(y, x') > \rho(y, x)$. By moderate transitivity, we have $\rho(y, x) > \min\{\rho(y, x'), \rho(x', x)\}$ which implies $\rho(y, x) > \rho(x', x)$, which contradicts dominance and so $\rho(y, x') \leq \rho(y, x) \implies \rho(x', y) \geq \rho(x, y)$.

All that remains is to show that $\rho(x', y) > \rho(x, y)$ when $x \not\geq_X y$ and $y \not\geq_X x$. We have already shown that the inequality is strict in Cases 1 and 2; all that remains is to show that the inequality is strict in Case 3. Suppose $y \succeq x' \succeq x$. Toward a contradiction, suppose that $\rho(y, x') \geq \rho(y, x)$. Moderate transitivity then implies that either (i) $\rho(y, x) > \min\{\rho(y, x'), \rho(x', x)\}$ or (ii) $\rho(y, x) = \rho(y, x') = \rho(x', x)$. As we saw above, it cannot be the case that (i) holds. If (ii) holds, then dominance implies that $y \geq x$, a contradiction. $\square$

For the following result, we consider the case where X is a convex set. Say ρ is *linear* if $\rho(x, y) = \rho(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)z)$ for all $x, y, z \in X$, $\lambda \in (0, 1)$. Say ρ is *superadditive* if for any x, y, x', y' with $\rho(x, y), \rho(x', y') \geq 1/2$, for any $\lambda \in [0, 1]$ we have $\rho(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y') \geq \min\{\rho(x, y), \rho(x', y')\}$.

Lemma 2. *Let X be a vector space. If ρ defined on X satisfies moderate transitivity and linearity, then ρ is superadditive.*

Proof. Since X is a vector space and so contains additive inverses, linearity implies $\rho(x, y) = \rho(Cx, Cy)$ and $\rho(x, y) = \rho(x - z, y - z)$ for any for any $C > 0, x, y, z \in X$. Now consider x, y, x', y' with $\rho(x, y), \rho(x', y') \geq 1/2$, and $\lambda \in [0, 1]$. The above implies that

$$\rho(\lambda(x - y), 0) = \rho(x, y) \geq 1/2$$
$$\rho(0, -(1-\lambda)(x' - y')) = \rho(x', y') \geq 1/2$$
$$\rho(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y') = \rho(\lambda(x - y), -(1-\lambda)(x' - y'))$$

This, in conjunction with moderate transitivity, implies that

$$\begin{aligned}\rho(\lambda x+(1-\lambda)x',\lambda y+(1-\lambda)y') &= \rho(\lambda(x-y),-(1-\lambda)(x'-y'))\\ &\geq \min\{\rho(\lambda(x-y),0),\rho(0,-(1-\lambda)(x'-y'))\}\\ &= \min\{\rho(x,y),\rho(x',y')\}\end{aligned}$$

□

Proof of Theorem 1.

Necessity of the axioms is immediate from the definition. We now show sufficiency.

Assume that M1–M5 holds. Let $\succeq$ denote the stochastic order on $\mathbb{R}^n$ induced by ρ. By weak transitivity, $\succeq$ is transitive. Since ρ satisfies Continuity and Linearity, $\succeq$ satisfies axioms D1–D3 of Theorem 9.1 of Gilboa (2009). Invoking an intermediate step in the proof of this theorem, we conclude that there exists weights $\beta \in \mathbb{R}^n$ such that $U(x)=\sum_k \beta_k x_k$ represents $\succeq$. Since all attributes are non-null, we have that $\beta_k \neq 0$ for all k. For the remainder of the proof, we henceforth identify each option x with its weighted attribute values, so that $U(x)=\sum_k x_k$. Since ρ satisfies Dominance and MST, Lemma 1 implies that ρ satisfies monotonicity with respect to the component-wise dominance relation on $\mathbb{R}^n$.

For $z \in \mathbb{R}^n$, Let $d^+(z)=\sum_{k:z_k\geq 0} z_k$ and $d^-(x)=\sum_{k:z_k<0}|z_k|$ denote the summed advantages and disadvantages in the comparison between z and 0. Say that z has *no dominance relationship* if $d^+(z), d^-(z)>0$.

Claim 1. For any $z\in\mathbb{R}^n$ satisfying $\sum_k z_k \geq 0$, $\rho(z,0)=\rho(d^+(z)e_1-d^-(z)e_2,0)$.

Proof. For $i,j\in\{1,\dots,n\}$, $i\neq j$, define $z^{ij}\in\mathbb{R}^n$ satisfying

$$z_k^{ij}=\begin{cases} d^+(z) & k=i\\ -d^-(z) & k=j\\ 0 & \text{otherwise}\end{cases}$$

Note that because we have normalized utility weights 1, for all $i\neq j$, $l\neq m$, we have $U(z^{ij})=d^+(z)-d^-(z)=U(z^{lm})$, and so $z^{ij}\sim z^{lm}$. We will first show that $\rho(z^{ij},0)=\rho(z^{lm},0)$ for all $i\neq j, l\neq m$. It is sufficient to show that for all i,j $\rho(z^{ij},0)=\rho(z^{12},0)$. There are two cases to consider:

Case 1: $j > i$. Since $z^{1j} \sim z^{ij}$, and since $z_i^{1j} = 0$, $z_k^{1j} = z_k^{ij}$ for all $k \neq i, 1$, Simplification implies that $\rho(z^{1j}, 0) \geq \rho(z^{ij}, 0)$. Also, since $z_1^{ij} = 0$, and $z_k^{ij} = z_k^{1j}$ for all $k \neq 1, i$, Simplification implies $\rho(z^{1j}, 0) \leq \rho(z^{ij}, 0)$, and so $\rho(z^{1j}, 0) = \rho(z^{ij}, 0)$. A analogous argument yields $\rho(z^{12}, 0) = \rho(z^{1j}, 0)$, and so $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$.

Case 2: $j < i$. By analogous arguments as above, we have $\rho(z^{ij}, 0) = \rho(z^{nj}, 0)$, $\rho(z^{nj}, 0) = \rho(z^{n2}, 0)$ and $\rho(z^{n2}, 0) = \rho(z^{12}, 0)$, and so $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$ as desired.

Let $K^+ = \{i \in \{1, 2, ..., n\} : z_i \geq 0\}$ and $K^- = \{i \in \{1, 2, ..., n\} : z_i < 0\}$. Defining $\lambda_i = \frac{z_i}{\sum_{k \in K^+} z_k}$ for $i \in K^+$, and $\gamma_j = \frac{z_j}{\sum_{k \in K^-} z_k}$ for $j \in K^-$, note that $z = \sum_{i \in K^+} \sum_{j \in K^-} \lambda_i \gamma_j z^{ij}$, and so z can be expressed as a mixture of z^{ij}'s. Since ρ satisfies superadditivity by Lemma 2, by inductive application of superadditivity, we have $\rho(z, 0) \geq \rho(z^{ij}, 0)$ for all $i \neq j$, which in turn implies $\rho(z, 0) \geq \rho(z^{12}, 0)$.

Note that by repeated application of Simplification, we have $\rho(z, 0) \leq \rho(z^{ij}, 0)$, for some i where $z_i \geq 0$, and some j where $z_j \leq 0$. Since $\rho(z^{ij}, 0) = \rho(z^{12}, 0)$, we have $\rho(z, 0) \leq \rho(z^{12}, 0)$, and so $\rho(z, 0) = \rho(z^{12}, 0)$ as desired. $\square$

Claim 2. For z with $\sum_k z_k \geq 0$, $\rho(z, 0) = \tilde{G}\left(\frac{d^+(z) - d^-(z)}{d^+(z) + d^-(z)}\right)$ for some strictly increasing, continuous $\tilde{G} : [0, 1] \to \mathbb{R}$.

Proof. Fix any z with $\sum_k z_k \geq 0$, and consider the case where z has no dominance relationship, that is $d^+(z) > 0$, $d^-(z) > 0$. By Claim 1, we have $\rho(z, 0) = \rho(d^+(z)e_1 - d^- e_2, 0)$. Define $F : [1, \infty) \to [1/2, 1)$ by $F(t) = \rho(te_1 - e_2, 0)$; by monotonicity, of ρ, F is strictly increasing. By Linearity, we have $\rho(d^+(z)e_1 - d^- e_2, 0) = \rho((d^+(z)/d^-(z))e_1 - e_2, 0) = F(d^+(z)/d^-(z))$.

Let $\varphi(z) = \frac{z-1}{z+1}$; and define $\tilde{G} : [0, 1) \to \mathbb{R}$ where $\tilde{G}(z) = F(\varphi^{-1}(z))$; since φ and F are strictly increasing, $\tilde{G}$ is strictly increasing. By construction, we have $F(z) = \tilde{G}\left(\frac{z-1}{z+1}\right)$, and so $\rho(z, 0) = \rho(d^+(z)e_1 - d^- e_2, 0) = \tilde{G}\left(\frac{d^+(z) - d^-(z)}{d^+(z) + d^-(z)}\right)$. Since ρ is continuous, $\tilde{G}$ is continuous on its domain $[0, 1)$, and in particular is uniformly continuous since it is increasing and bounded. Take the continuous extension of $\tilde{G}$ to $[0, 1]$.

Now consider the case where z has a dominance relationship; that is $d^+(z) > 0$, $d^-(z) = 0$. By Dominance, $\rho(z, 0) = \rho(d^+(z)e_1 - d^- e_2, 0)$ takes on some constant value q such that $q > \rho(z', 0)$ for all z' without a dominance relationship, which implies that $q > \tilde{G}(t)$ for all $t \in [0, 1)$. Since ρ is continuous, it must be the case that $q = \tilde{G}(1)$. $\square$

Now, let $G : [-1,1] \to \mathbb{R}$ be the symmetric extension of $\tilde{G}$ satisfying

$$G(z) = \begin{cases} \tilde{G}(z) & z \geq 0 \\ 1 - \tilde{G}(-z) & z < 0 \end{cases}$$

Claim 3. For any z, $\rho(z,0) = G\left(\frac{d^+(z)-d^-(z)}{d^+(z)+d^-(z)}\right)$.

Proof. Claim 1 implies that $\rho(z,0) = G\left(\frac{d^+(z)-d^-(z)}{d^+(z)+d^-(z)}\right)$ whenever $\sum_k z_k \geq 0$. Now consider the case where $\sum_k z_k < 0$. Note that

$$\begin{aligned}
\rho(z,0) &= 1 - \rho(-z,0) \\
&= 1 - \tilde{G}\left(\frac{d^-(z) - d^+(z)}{d^+(z) + d^-(z)}\right) \\
&= G\left(\frac{d^+(z) - d^-(z)}{d^+(z) + d^-(z)}\right)
\end{aligned}$$

as desired, where the first equality uses symmetry and Linearity of ρ and the second equality uses Claim 2. □

Take any x, y, and let $z = x - y$. Due to linearity, we have

$$\begin{aligned}
\rho(x,y) &= \rho(z,0) \\
&= G\left(\frac{d^+(z) - d^-(z)}{d^+(z) + d^-(z)}\right) \\
&= G\left(\frac{\sum_k z_k}{\sum_k |z_k|}\right) \\
&= G\left(\frac{U(x) - U(y)}{d_{L1}(x,y)}\right)
\end{aligned}$$

as desired.

Finally, to show uniqueness, suppose (G,β) and (G',β') both represent ρ. Define the stochastic preference relation $\succeq$ as before. Since G and G' are both strictly increasing and symmetric around 0, $U(x) = \sum_k \beta_k x_k$ and $U'(x) = \sum_k \beta'_k x_k$ both represent $\succeq$, and so there exists $C > 0$ such that $\beta_k' = C\beta_k$. This in turn implies that for all $z \in \mathbb{R}^n$, we have

$$G\left(\frac{\sum_k \beta_k z_k}{\sum_k |\beta_k z_k|}\right) = G'\left(\frac{\sum_k \beta_k' z_k}{\sum_k |\beta_k' z_k|}\right)$$
$$= G'\left(\frac{\sum_k \beta_k z_k}{\sum_k |\beta_k z_k|}\right)$$

Let $z = \alpha/\beta_1 e_1 + \gamma/\beta_2 e_2$. Note that for any $r \in [-1,1]$, there exists α, γ such that $\frac{\sum_k \beta_k z_k}{\sum_k |\beta_k z_k|} = \frac{\alpha-\gamma}{|\alpha+\gamma|} = r$, and so $G'(r) = G(r)$ for all $r \in [-1,1]$.

□

C.2 Multinomial Choice Results

We will prove our results for a more general signal structure, where

$$s_{xy} = \text{sgn}(v_x - v_y) + \frac{1}{\sqrt{\tau_{xy}}} e_{xy}$$

where the e_{xy} are distributed according to a continuous distribution with density g that is symmetric around 0 and satisfies the monotone likelihood ratio property: that is $\frac{\partial}{\partial x}\frac{g(x-t)}{g(x)} > 0$ for all $t > 0$. We begin with the following basic observation:

Lemma 3. *Consider a continuous distribution with density g that is symmetric around 0 and satisfies the monotone likelihood ratio property. The function g then has the following properties:*

1. $g'(x-t)g(x) - g(x-t)g'(x) > 0$ *for all* $t > 0$, x
2. g *is unimodal; that is* $g'(x) = -g'(-x) \leq 0$ *for all* $x > 0$
3. $g(t-x) > g(-t-x)$ *for any* $t, x > 0$

Proof. 1) follows directly from the definition of MLRP. To see 2), towards a contradiction suppose $g'(x) > 0$ for some $x > 0$. Then for any $t > 0$, 1) implies

$$g'(x-t) \geq \frac{g(x-t)g'(x)}{g(x)} \geq 0$$

So for any $y > x$, $g'(y) > 0$. Symmetry implies that for any $y < -x$, $g'(y) < 0$, and so g is not integrable, a contradiction. To see 3), note that by symmetry, $\frac{g(t-x)}{g(-t-x)} = 1$ for $x = 0$. MLRP of g implies that $\frac{g(t-x)}{g(-t-x)} > 1$ for all $x > 0$ as desired. □

The following observations pertain to finite set of options A. Enumerate A by $1,2,...,N$ and let $s=(s_{ij})_{i<j}$ collect all pairwise signals in A. Let $X^N_{(k)}$ denote the kth order statistic among N draws from the prior distribution q. Let $V^N_{(k)}=\mathbb{E}[X_{(N+1-k)}]$, that is, $V^N_{(k)}$ gives the expected value of an option if it is ranked kth. Let $\pi:A\to A$ denote a permutation function; let Π denote the set of permuation functions on A. With some abuse of notation, associate each π with the event that the v_i's are ordered according to π: that is $\pi(i)=n$ means that option i is ranked nth in the ordering. The posterior expected value of an option i given signal s is then given by

$$\mathbb{E}[v_i|s]=\sum_{n=1}^{N}V^N_{(n)}\cdot Pr(\pi(i)=n|s)$$

where

$$Pr(\pi(i)=n|s)\propto\sum_{\pi\in\Pi:\pi(i)=n}\prod_{k=1}^{N}\prod_{j<k}g\left(\sqrt{\tau_{jk}}(s_{jk}-\operatorname{sgn}(\pi(k)-\pi(j))\right)$$

Lemma 4. *Take any permutation π satisfying $\pi(i)<\pi(j)$. Then $\frac{\partial}{\partial e_{ij}}Pr(\pi|s)>0$.*

Proof. We have

$$\begin{aligned}Pr(\pi|s)&=\frac{\prod_{k<l}g\left(\sqrt{\tau_{kl}}(s_{kl}-\operatorname{sgn}(\pi(l)-\pi(k)))\right)}{\sum_{\pi'\in\Pi}\prod_{k<l}g\left(\sqrt{\tau_{kl}}(s_{kl}-\operatorname{sgn}(\pi(l)-\pi(k)))\right)}\\&=\frac{\lambda g\left(\sqrt{\tau_{ij}}s_{ij}-\sqrt{\tau_{ij}}\right)}{\alpha g\left(\sqrt{\tau_{ij}}s_{ij}-\sqrt{\tau_{ij}}\right)+\beta g\left(\sqrt{\tau_{ij}}s_{ij}+\sqrt{\tau_{ij}}\right)}\\&=\frac{\lambda g\left(e_{ij}+\eta-\sqrt{\tau_{ij}}\right)}{\alpha g\left(e_{ij}+\eta-\sqrt{\tau_{ij}}\right)+\beta g\left(e_{ij}+\eta+\sqrt{\tau_{ij}}\right)}\end{aligned}$$

where the $\lambda,\alpha,\beta,\eta$ are non-negative and do not depend on e_{ij}. This implies that

$$\frac{\partial}{\partial e_{ij}}Pr(\pi|s)=\frac{\partial}{\partial e_{ij}}\left(\frac{\lambda}{\alpha+\beta\frac{g\left(e_{ij}+\eta+\sqrt{\tau_{ij}}\right)}{g\left(e_{ij}+\eta-\sqrt{\tau_{ij}}\right)}}\right)>0$$

by MLRP of g. □

Lemma 5. $1\{\mathbb{E}[v_i|s]>\mathbb{E}[v_j|s]\}$ *is increasing in e_{ij}.*

Proof. We show the stronger result that $\mathbb{E}[v_i|s]-\mathbb{E}[v_j|s]$ is increasing in s_{ij}. Note that

$$\begin{aligned}\mathbb{E}[v_i|s]-\mathbb{E}[v_j|s] &= \sum_{\pi\in\Pi}\left(V^N_{(\pi(i))}-V^N_{(\pi(j))}\right)Pr(\pi|s)\\ &= \sum_{\pi\in\Pi:\pi(i)<pi(j)}\left(V^N_{(\pi(i))}-V^N_{(\pi(j))}\right)Pr(\pi|s)+\sum_{\pi\in\Pi:\pi(i)>pi(j)}\left(V^N_{(\pi(i))}-V^N_{(\pi(j))}\right)Pr(\pi|s)\end{aligned}$$

Since $V^N_{(\pi(i))}-V^N_{(\pi(j))}>0$ if $\pi(i)<\pi(k)$ and $V^N_{(\pi(i))}-V^N_{(\pi(j))}<0$ otherwise, Lemma 4 implies that $\frac{\partial}{\partial e_{ij}}\left[\mathbb{E}[v_i|s]-\mathbb{E}[v_j|s]\right]>0$. □

We now observe a result that will be useful for the proof of Proposition 1.

Lemma 6. *Consider any two options x,y with $\tau_{xy}=0$. Then, for any z with $v_z>\max\{v_y,v_x\}$ $\rho(x,y|\{z\})$ is decreasing in τ_{yz} and increasing in τ_{xz}. Likewise, if $v_z<\min\{v_y,v_x\}$ $\rho(x,y|\{z\})$ is increasing in τ_{yz} and decreasing in τ_{xz}.*

Proof. Suppose that $v_z>\max\{v_y,v_x\}$; the proof for the case where $v_z<\max\{v_y,v_x\}$ is identical. For options i,j,k let π_{ijk} denote the the permutation that ranks i first, j second, and k last. We have

$$\begin{aligned}Pr(\pi_{xyz}|s) &= g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}-2\sqrt{\tau_{yz}})/Pr(s)\\ Pr(\pi_{xzy}|s) &= g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz})/Pr(s)\\ Pr(\pi_{zxy}|s) &= g(e_{xz})g(e_{yz})/Pr(s)\\ Pr(\pi_{yxz}|s) &= g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}-2\sqrt{\tau_{yz}})/Pr(s)\\ Pr(\pi_{yzx}|s) &= g(e_{xz})g(e_{yz}-2\sqrt{\tau_{yz}})/Pr(s)\\ Pr(\pi_{zyx}|s) &= g(e_{xz})g(e_{yz})/Pr(s)\end{aligned}$$

and so we have

$$\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]=\left(V^3_{(1)}-V^3_{(3)}\right)(Pr(\pi_{yzx}|s)-Pr(\pi_{xzy}|s))$$

Lemma 4 then implies that $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]$ is strictly increasing in e_{yz} and decreasing in e_{xz}. This implies that for $e^*_{yz}(\tau_{xz},e_{xz})$ defined implicitly by

$$g(e_{xz}-2\sqrt{\tau_{xz}})g(e^*_{yz}(\tau_{xz},e_{xz}))=g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})$$

for any realization of e_{xz}, we have $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]=0$ when $e_{yz}=e^*_{yz}(\tau_{xz},e_{xz})$, and so $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]>0$ whenever $e_{yz}>e^*_{yz}(\tau_{xz},e_{xz})$, and $\mathbb{E}[v_y|s]-\mathbb{E}[v_x|s]\leq 0$ otherwise.

Here we note three properties of $e^*_{yz}(\tau_{xz}, e_{xz})$:

1. $e^*_{yz}(\tau_{xz}, e_{xz})$ is strictly increasing in e_{xz}.
2. $e^*_{yz}(\tau_{xz}, e_{xz})$ is decreasing in τ_{xz} whenever $e_{xz} \leq \sqrt{\tau_{xz}}$.
3. $e^*_{yz}\left(\tau_{xz}, \sqrt{\tau_{xz}}\right) = \sqrt{\tau_{yz}}$, and $e^*_{yz}(\tau_{xz}, e_{xz}) \leq \sqrt{\tau_{yz}}$ whenever $e_{xz} \leq \sqrt{\tau_{xz}}$.

Property follows 1 by implicitly differentiating the equality $\frac{g(e_{xz}-2\sqrt{\tau_{xz}})}{g(e_{xz})} = \frac{g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})}{g(e^*_{yz}(\tau_{xz},e_{xz}))}$ and MLRP. Property 2 follows from differentiating the same equality, MLRP, and part 2 of Lemma 3. Property 3 follows from symmetry of g and Property 1. We have

$$
\begin{aligned}
\rho(y;x|z) = &\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}} \int_{e_{yz}=-\infty}^{\infty} 1\left\{\mathbb{E}[v_y|s] - \mathbb{E}[v_x|s] \geq 0\right\} g(e_{xz})g(e_{yz})\, de_{yz} de_{xz} \\
&+ \int_{e_{xz}=\sqrt{\tau_{xz}}}^{\infty} \int_{e_{yz}=-\infty}^{\infty} 1\left\{\mathbb{E}[v_y|s] - \mathbb{E}[v_x|s] \geq 0\right\} g(e_{xz})g(e_{yz})\, de_{yz} de_{xz}
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\int_{e_{xz}=\sqrt{\tau_{xz}}}^{\infty} \int_{e_{yz}=-\infty}^{\infty} 1\left\{\mathbb{E}[v_y|s] - \mathbb{E}[v_x|s] \geq 0\right\} g(e_{xz})g(e_{yz})\, de_{yz} de_{xz} \\
&= \int_{e_{xz}=\sqrt{\tau_{xz}}}^{\infty} \int_{e_{yz}=-\infty}^{\infty} 1\left\{g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}) - g(e_{xz})g(e_{yz}-2\sqrt{\tau_{yz}}) \geq 0\right\} g(e_{xz})g(e_{yz})\, de_{yz} de_{xz} \\
&= \int_{e'_{xz}=-\infty}^{\sqrt{\tau_{xz}}} \int_{e'_{yz}=-\infty}^{\infty} 1\left\{g(e'_{xz})g(e'_{yz}-2\sqrt{\tau_{yz}}) - g(e'_{xz}-2\sqrt{\tau_{xz}})g(e'_{yz}) \geq 0\right\} g(e'_{xz}-2\sqrt{\tau_{xz}})g(e'_{yz}-2\sqrt{\tau_{yz}}) de'_{yz} de'_{xz} \\
&= \int_{e'_{xz}=-\infty}^{\sqrt{\tau_{xz}}} \int_{e'_{yz}=-\infty}^{\infty} 1\left\{\mathbb{E}[v_y|s] - \mathbb{E}[v_x|s] \leq 0\right\} g(e'_{xz}-2\sqrt{\tau_{xz}})g(e'_{yz}-2\sqrt{\tau_{yz}}) de'_{yz} de'_{xz}
\end{aligned}
$$

where the third line uses the change of variables $e'_{xz} = 2\sqrt{\tau_{xz}} - e_{xz}$, $e'_{yz} = 2\sqrt{\tau_{yz}} - e_{yz}$. This implies that

$$
\begin{aligned}
\rho(y;x|z) = &\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}} \int_{e_{yz}=e^*_{yz}(\tau_{xz},e_{xz})}^{\infty} g(e_{xz})g(e_{yz})\, de_{yz}de_{xz} \\
&+ \int_{e'_{xz}=-\infty}^{\sqrt{\tau_{xz}}} \int_{e_{yz}=-\infty}^{e^*_{yz}(\tau_{xz},e_{xz})} g(e_{xz}-2\sqrt{\tau_{xz}})g(e_{yz}-2\sqrt{\tau_{yz}})\, de_{yz}de_{xz} \\
= &\int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}} \int_{e_{yz}=e^*_{yz}(\tau_{xz},e_{xz})}^{\infty} g(e_{xz})g(e_{yz})\, de_{yz}de_{xz} \\
&+ \int_{e_{xz}=-\infty}^{-\sqrt{\tau_{xz}}} \int_{e_{yz}=-\infty}^{e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})} g(e_{xz})g(e_{yz}-2\sqrt{\tau_{yz}})\, de_{yz}de_{xz}
\end{aligned}
$$

and so

$$
\begin{aligned}
&\frac{\partial}{\partial \tau_{xz}}\rho(y;x|z) \\
&= \frac{1}{2\sqrt{\tau_{xz}}}\left[\int_{e_{yz}=e^*_{yz}(\tau_{xz},\sqrt{\tau_{xz}})}^{\infty} g\left(\sqrt{\tau_{xz}}\right) g(e_{yz})\, de_{yz} - \int_{e_{yz}=-\infty}^{e^*_{yz}(\tau_{xz},\sqrt{\tau_{xz}})} g\left(-\sqrt{\tau_{xz}}\right) g(e_{yz}-2\sqrt{\tau_{yz}})\, de_{yz}\right] \\
&\quad + \int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}} -\frac{\partial}{\partial \tau_{xz}} e^*_{yz}(\tau_{xz},e_{xz})g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}))\, de_{xz} \\
&\quad + \int_{e_{xz}=-\infty}^{-\sqrt{\tau_{xz}}} \left[\frac{\partial}{\partial \tau_{xz}} e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}}) + \frac{1}{\sqrt{\tau_{xz}}}\frac{\partial}{\partial e_{xz}} e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})\right] g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})-2\sqrt{\tau_{yz}})\, de_{xz} \\
&= \frac{g(\sqrt{\tau_{xz}})}{2\sqrt{\tau_{xz}}}\left[G(-\sqrt{\tau_{yz}}) - G\left(e^*_{yz}(\tau_{xz},\sqrt{\tau_{xz}}\right)-2\sqrt{\tau_{yz}})\right] \\
&\quad + \int_{e_{xz}=-\infty}^{\sqrt{\tau_{xz}}} -\frac{\partial}{\partial \tau_{xz}} e^*_{yz}(\tau_{xz},e_{xz})\left[g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz})) - g(e_{xz}-2\sqrt{\tau_{xz}})g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})\right] de_{xz} \\
&\quad + \int_{e_{xz}=-\infty}^{-\sqrt{\tau_{xz}}} \frac{1}{\sqrt{\tau_{xz}}}\frac{\partial}{\partial e_{xz}} e^*_{yz}(\tau_{xz},e_{xz})g(e_{xz})g(e^*_{yz}(\tau_{xz},e_{xz}+2\sqrt{\tau_{xz}})-2\sqrt{\tau_{yz}})\, de_{xz}
\end{aligned}
$$

The first term is equal to 0 since $e^*(\tau_{xz},\sqrt{\tau_{xz}}) = \sqrt{\tau_{yx}}$. To see that the second term is non-negative, note that on the domain of integration, $\frac{\partial}{\partial \tau_{xz}} e^*_{yz}(\tau_{xz},e_{xz}) \leq 0$ (Property 2), and $e^*_{yz}(\tau_{xz},e_{xz}) \leq \sqrt{\tau_{yz}}$ (Property 3) and so applying part 3) of Lemma 3, $g(e^*_{yz}(\tau_{xz},e_{xz})) \geq g(e^*_{yz}(\tau_{xz},e_{xz})-2\sqrt{\tau_{yz}})$ and $g(e_{xz}) > g(e_{xz}-2\sqrt{\tau_{xz}})$. To see that the third term is strictly positive, note that $\frac{\partial}{\partial e_{xz}} e^*_{yz}(\tau_{xz},e_{xz}) > 0$ (Property 1). We therefore have $\frac{\partial}{\partial \tau_{xz}}\rho(y,x|\{z\}) > 0$. A symmetric argument shows that $\frac{\partial}{\partial \tau_{yz}}\rho(y,x|\{z\}) < 0$. □

Proof of Proposition 1

Suppose $v_x, v_y > v_z$, and $\tau_{yz} > \tau_{xz}$. Lemma 6 implies that if $\tau_{xy} = 0$, $\rho(y,z|\{z\}) > 1/2$. The desired result then follows from the fact that $\rho(y,z|\{z\})$ is continuous in τ_{xy}.

□

Proof of Proposition 2

Let π_k denote the ordering over $x, z^1, ..., z^n$ in which x is ranked kth and the z^j are ordered correctly, and let $p_k(s)$ denote the DM's posterior belief over π_k given signal s, where $p(s) = (p_1(s), ..., p_{n+1}(s))$. Note that

$$\mathbb{E}[v_x|s] = \sum_{k=1}^{n+1} V_{(k)}^N p_k(s)$$

$$\mathbb{E}[v_j|s] = \left(\sum_{k=1}^{j} p_k(s)\right) V_{(j+1)}^N + \left(\sum_{k=j+1}^{n+1} p_k(s)\right) V_{(j)}^N \quad \forall j = 1, ..., n$$

where $V_{(k)}^N$ is the expectation of the kth order statistic from $N = n+1$ draws over the DM's prior of Q.

To show (i), consider the case where $\tau = 0$. We have that with probability 1, $p_k(s) = 1/n+1$ for all $k \in \{1, ..., n+1\}$. Let μ denote the expectation of Q. By symmetry of Q, we have $V_{(k)}^N = 2\mu - V_{(N+1-k)}^N$ for all $k = 1, ..., N$, and so $\mathbb{E}[v_x|s] = \mu$ with probability 1.

First consider the case where n is odd, and let $j^* = \frac{n+1}{2}$. We have $\mathbb{E}[v_{j^*}|s] = \frac{1}{2}V_{j^*}^N + \frac{1}{2}V_{j^*+1}^N = \mu$ with probability 1, and so $\mathbb{E}[v_x|s] = \mathbb{E}[v_{j^*}|s]$ and $\mathbb{E}[v_x|s] \neq \mathbb{E}[v_k|s]$ for any $k \neq j^*$ with probability 1. This implies that $\mathbb{P}(R(x,Z) = j^*) = \mathbb{P}(R(x,Z) = j^*+1) = 1/2$, and so $\mathbb{E}[R(x,Z)] = \frac{n+2}{2}$ as desired.

Now consider the case where n is even. Let $j^* = n/2$, $k^* = n/2+1$. Since $V_{(k)}^N = 2\mu - V_{(N+1-k)}^N$, we have $V_{(j^*)}^N > V_{(j^*+1)}^N = \mu = V_{(k^*)}^N > V_{(k^*+1)}^N$. This implies that with probability 1, $\mathbb{E}[v_{j^*}|s] = \frac{n/2}{n+1}V_{(j^*)+1}^N + \frac{n/2+1}{n+1}V_{(j^*)}^n > \mu$, and $\mathbb{E}[v_{k^*}|s] = \frac{n/2+1}{n+1}V_{(k^*)+1}^N + \frac{n/2}{n+1}V_{(k^*)}^n < \mu$, which in turn implies that $\mathbb{E}[v_{k^*}|s] < \mathbb{E}[v_x|s] < \mathbb{E}[v_{j^*}|s]$, and so we have $R(x,Z) = k^* = \frac{n+2}{2}$ with probability 1. This implies that $\mathbb{E}[R(x,Z)] = \frac{n+2}{2}$.

To show (2), Let $R(s)$ denote the DM's switching point given the signal s: that is, $R(s) = R$ if $\mathbb{E}[v_x|s] > \mathbb{E}[v_k|s]$ for all $k \geq R$ and $\mathbb{E}[v_x|s] < \mathbb{E}[v_k|s]$ for all $k < R$. Note that $R(s)$ is well defined for any $\tau > 0$ since ties in posterior expected values occur with probability 0 if $\tau > 0$. Note that there exists $\epsilon > 0$ such that whenever $p_k(s) > 1 - \epsilon$, $R(s) = k$. Since $p_{R^*(x,Z)}(s) \to_p 1$ as $\tau \to \infty$, $R(s) \to_p R^*(x,Z)$ as $\tau \to \infty$.

$\square$

C.3 Results on Strategic Obfuscation

We begin by proving an intermediate result.

Lemma 7. *Consider the second-stage game in Section 5.1.2, where $c^b \geq c^a$. If $\Delta q > 0$, there is a unique pure-strategy equilibrium, in which $p^a = c^b + \Delta q$ and $p^b = c^b + \frac{1}{4}\Delta q + \frac{3}{4}\sqrt{\Delta q^2 + \frac{8}{9}\Delta q \Delta c}$. Furthermore, if $\Delta c > 0$, b's equilibrium market share and equilibrium profits are strictly increasing in Δq.*

Proof. The profits of a and b are given by

$$\Pi_a(p^a, p^b) = (p^a - c^a)\left(\frac{1}{2} + \frac{1}{2} \cdot \frac{p^b - p^a}{\Delta q + |p^b - p^a|}\right)$$
$$\Pi_b(p^b, p^a) = (p^b - c^b)\left(\frac{1}{2} + \frac{1}{2} \cdot \frac{p^a - p^b}{\Delta q + |p^b - p^a|}\right)$$

and FOC's yield

$$\frac{\partial}{\partial p^a}\Pi_a(p^a, p^b) = \begin{cases} \dfrac{\frac{1}{2}\Delta q + p^b - p^a}{\Delta q + p^b - p^a} - \dfrac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)^2}(p^a - c^a) & p^b \geq p^a \\ \dfrac{\frac{1}{2}\Delta q}{\Delta q + p^a - p^b} - \dfrac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(p^a - c^a) & p^b < p^a \end{cases}$$
$$\frac{\partial}{\partial p^b}\Pi_b(p^b, p^a) = \begin{cases} \dfrac{\frac{1}{2}\Delta q}{\Delta q + p^b - p^a} - \dfrac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)^2}(p^b - c^b) & p^b \geq p^a \\ \dfrac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \dfrac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(p^b - c^b) & p^b < p^a \end{cases}$$

We first show that in a pure strategy equilibrium, it must be the case that $p^a = c^b + \Delta q$. First, consider the case where $p^a < c^b + \Delta q$. Note that if $p^a \leq p^b$, we have

$$\begin{aligned} \frac{\partial}{\partial p^b}\Pi_b(p^b, p^a) &= \frac{\frac{1}{2}\Delta q}{\Delta q + p^b - p^a} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)^2}(p^b - c^b) \\ &> \frac{\frac{1}{2}\Delta q}{\Delta q + p^b - p^a} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)(p^b - c^b)}(p^b - c^b) \\ &= 0 \end{aligned}$$

where the second inequality follows from the fact that $p^a < c^b + \Delta q$ implies that $\Delta q + p^b -$

$p^a > p^b - c^b$. Also, for $p^a > p^b$, we have

$$\begin{aligned}
\frac{\partial}{\partial p^b}\Pi_b(p^b,p^a) &= \frac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(p^b - c^b) \\
&> \frac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(\Delta q) \\
&> \frac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(\Delta q + p^a - p^b) \\
&= 0
\end{aligned}$$

where the second inequality follows from the fact that $p^b < p^a < c^b + \Delta q$ and the third inequality follows from the fact that $p^a - p^b > 0$. We therefore have that $\Pi_b(p^b, p^a)$ is strictly increasing for all $p^b > 0$, and so $p^a < c^b + \Delta q$ cannot be part of an equilibrium strategy.

Now consider the case where $p^a > c^b + \Delta q$. If $p^a \leq p^b$, we have

$$\begin{aligned}
\frac{\partial}{\partial p^b}\Pi_b(p^b,p^a) &= \frac{\frac{1}{2}\Delta q}{\Delta q + p^b - p^a} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)^2}(p^b - c^b) \\
&< \frac{\frac{1}{2}\Delta q}{\Delta q + p^b - p^a} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)(p^b - c^b)}(p^b - c^b) \\
&= 0
\end{aligned}$$

where the second inequality uses the fact that $p^a > c^b + \Delta q$ implies that $\Delta q + p^b - p^a < p^b - c^b$, and so $\frac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)^2} > \frac{\frac{1}{2}\Delta q}{(\Delta q + p^b - p^a)(p^b - c^b)}$. This implies that b's best response to $p^a > c^b + \Delta q$ must involve setting $p^b < p^a$. Toward a contradiction, suppose that $p^b < p^a$ is part of an equilibrium strategy. We have

$$\frac{\partial}{\partial p^a}\Pi_a(p^a,p^b) = \frac{\frac{1}{2}\Delta q}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(p^a - c^a)$$

This implies that for all $p^a > p^b$, $\frac{\partial}{\partial p^a}\Pi_a(p^a, p^b) > 0$ whenever $-c^a < \Delta q - p^b$, and $\frac{\partial}{\partial p^a}\Pi_a(p^a, p^b) < 0$ whenever $-c^a > \Delta q - p^b$. In equilibrium we must have $\frac{\partial}{\partial p^a}\Pi_a(p^a, p^b) = 0$ and so we therefore must have $-c^a = \Delta q - p^b \implies p^b = c^a + \Delta q$.

Since $p^a > p^b = c^b + \Delta q$, we have

$$\begin{aligned}
\frac{\partial}{\partial p^b}\Pi_b(p^b,p^a) &= \frac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(p^b - c^b) \\
&= \frac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p^b)^2}(\Delta q) \\
&> \frac{\frac{1}{2}\Delta q + p^a - p^b}{\Delta q + p^a - p^b} - \frac{\frac{1}{2}\Delta q}{\Delta q + p^a - p^b} \\
&> 0
\end{aligned}$$

and so $p^b = c^b + \Delta q$ cannot be a best response to p^a. We have a contradiction, and so it must be the case that $p^a = c^b + \Delta q_V + \Delta q$ in equilibrium.

Now we show that that $p^b \geq p^a$ in equilibrium. Suppose not and assume $p^b < p^a$: by the above argument, it must be the case that $p^b = c^a + \Delta q$. Again following the argument above, we have $\frac{\partial}{\partial p^b}\Pi_b(p^b,p^a) > 0$ for $p^b = c^b + \Delta q$, $p^a > p^b$, and so $p^b = c^b + \Delta q$ cannot be a best response to $p^a > p^b$. In both cases, we have a contradiction; we must have $p^b \geq p^a$ in equilibrium.

Since in equilibrium we must have $\frac{\partial}{\partial p^a}\Pi_a(c^b + \Delta q, p^b) = 0$, b's equilibrium price must solve

$$\frac{p^b - \left(c^b + \frac{1}{2}\Delta q\right)}{p^b - c^b} - \frac{\frac{1}{2}\Delta q}{(p^b - c^b)^2}(\Delta q + \Delta c) = 0$$

which yields $p^b = c^b + \frac{1}{4}\Delta q + \frac{3}{4}\sqrt{\Delta q^2 + \frac{8}{9}\Delta q \Delta c}$.

Finally, to verify that p^b is a best response to p^a, note that for $p^a = c^b + \Delta q$, $\Pi_b(p,p^a)$ is constant in p for all $p > p^a$, and moreover for $p < p^a$ we have

$$\begin{aligned}
\frac{\partial}{\partial p^b}\Pi_b(p^b,p^a) &= \frac{\frac{1}{2}\Delta q + p^a - p}{\Delta q + p^a - p} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p)^2}(p - c^b) \\
&> \frac{\frac{1}{2}\Delta q + p^a - p}{\Delta q + p^a - p} - \frac{\frac{1}{2}\Delta q}{(\Delta q + p^a - p)^2}(\Delta q) \\
&> 0.
\end{aligned}$$

where the second inequality follows from the fact that $p < p^a = c^b + \Delta q$.

Now suppose that $\Delta c > 0$. To show that b's equilibrium market share is increasing in

Δq, note that under the unique equilibrium prices, we have

$$\rho(p^b, p^a) = \frac{1}{2} + \frac{1}{2} \cdot \frac{\phi(\Delta q)}{\Delta q + \phi(\Delta q)}$$

where $\phi(\Delta q) = \frac{3}{4}\sqrt{\Delta q^2 + \frac{8}{9}\Delta q \Delta c} - \frac{3}{4}\Delta q$. Note that

$$\frac{\partial}{\partial \Delta q}\rho(p^b, p^a) = \frac{1}{2} \cdot \frac{\phi(\Delta q) - \Delta q \phi'(\Delta q)}{(\Delta q + \phi(\Delta q))^2}$$

All that remains to show is that $\Delta q \phi'(\Delta q) < \phi(\Delta q)$. Note that

$$\begin{aligned}
\Delta q \phi'(\Delta q) &= \frac{3}{4} \cdot \frac{2\Delta q^2 + \frac{8}{9}\Delta c \Delta q}{2\sqrt{\Delta q^2 + \frac{8}{9}\Delta q \Delta c}} - \frac{3}{4}\Delta q \\
&= \frac{3}{4}\sqrt{\Delta q^2 + \frac{8}{9}\Delta q \Delta c} - \frac{3}{4}\Delta q - \frac{3}{4} \cdot \frac{\frac{8}{9}\Delta c \Delta q}{2\sqrt{\Delta q^2 + \frac{8}{9}\Delta q \Delta c}} \\
&< \phi(\Delta q)
\end{aligned}$$

and so b's equilibrium market share is strictly increasing in Δq. Since b's equilibrium prices are increasing in Δq, b's equilibrium profits are also strictly increasing in Δq. □

Proof of Proposition 3

The case where G is linear follows directly from Lemma 7. To show the second statement, note that a necessary condition for a symmetric equilibrium is that $\frac{\partial}{\partial p^a}\Pi_a(p^a, p^b) = 0$ for $p^a = p^b$, which implies $G(0) - G'(0)\frac{\Delta q}{(\Delta q)^2}(p^a - c) = 0$, which in turn implies $p^a = p^b = c + \frac{1}{2G'(0)}\Delta q$, and that such an equilibrium is unique, if it exists.

□

Proof of Proposition 4

First, note that given location decisions of both firms leading to a quantity dissimilarity Δq, the equilibrium prices are as given in Lemma 7. Let $\Pi_a^*(\Delta q, \Delta c)$ and $\Pi_b^*(\Delta q, \Delta c)$ denote firm a and b's equilibrium profits as a function of Δq and Δc. We will show

1. $\Pi_a^*(0, \Delta c) = \Pi_a^*(\Delta c/2, \Delta c) = \Delta c$.
2. $\frac{\partial}{\partial \Delta q}\Pi_a^*(\Delta q, \Delta c) < 0$ for $\Delta q \in (0, \frac{1}{9}\Delta c)$, and $\frac{\partial}{\partial \Delta q}\Pi_a^*(\Delta q, \Delta c) > 0$ for $\Delta q > \frac{1}{9}\Delta c$.

3. $\Pi_b^*(\Delta q, \Delta c)$ is strictly increasing in Δq.

3) follows directly from Lemma 7. $\Pi_a^*(\Delta c/2, \Delta c) = \Delta c$ also follows from Lemma 7. When $\Delta q = 0$, it is straightforward to show that the unique equilibrium prices satisfy $p^a = p^b = c^b$, and so a captures the entire market; we have $\Pi_a^*(0, \Delta c) = \Delta c$ and so 1) holds.

To see that 2) holds, let $\lambda \equiv \Delta c/\Delta q$. We can rewrite $\Pi_q^*(\Delta q, \Delta c)$ as

$$\Pi_a^*(\Delta q, \Delta c) = \varphi(\lambda) \equiv (1+\lambda)\Delta c \frac{3\sqrt{\lambda^2 + 8/9\lambda} - \lambda}{3\sqrt{\lambda^2 + 8/9\lambda} + \lambda}$$

It can be shown that

$$\varphi'(\lambda) = \Delta c \frac{8(\lambda^2 - \lambda)}{(3\sqrt{\lambda^2 + \frac{8}{9}\lambda})^2} \cdot \left(1 - \frac{1}{3\sqrt{\lambda^2 + \frac{8}{9}\lambda}}\right)$$

This implies that $\varphi'(\lambda) < 0$ for $\lambda \in (0, 1/9)$, and $\varphi'(\lambda) > 0$ for $\lambda > \frac{1}{9}$, and so 2) holds.

Now solve for the location decisions of each firm via backward induction. First, fix a location decision made by b in the first-stage game and let $\widehat{\Delta q}$ denote the maximal quantity dissimilarity implementable by a. 1) and 2) imply that a's best response is to choose the same location as b whenever $\widehat{\Delta q} < \Delta c/2$, and to implement the maximal quantity dissimilarity $\widehat{\Delta q}$ whenever $\widehat{\Delta q} > \Delta c/2$.

Now consider b's location decision: If $\overline{\Delta q} > \Delta c/2$, 3) implies that b's equilibrium location decision must implement $\widehat{\Delta q} = \overline{\Delta q}$. In this case, the previous analysis implies a's location decision must implement $\Delta q = \overline{\Delta q}$, and so Lemma 7 implies that the unique equilibrium prices satisfy $p^a = c^b + \overline{\Delta q}$, $p^b > p^a$, and that b's equilibrium market share is positive and strictly increasing in $\overline{\Delta q}$.

If instead $\overline{\Delta q} < \Delta c/2$, then regardless of b's location decision, a chooses the same location as b and so b's market share is 0 by Lemma 7. b is therefore indifferent over any location, and in equilibrium $\Delta q = 0$. Lemma 7 then implies that the equilibrium prices satisfy, $p^a = c^b$ and $p^b > p^a$, and so b's equilibrium market share is 0.

□

Proof of Proposition 5

We first show that in any pure strategy equilibrium, it must be the case that $p^b \leq p^a \leq p^s$. First, note that in equilibrium we cannot have $p^a > p^s > c$, since p^a earns 0 profits whenever $p^a > p^s$ and earns strictly positive profits by setting $p^a = p^s$. Also, in equilibrium we cannot have $p^s \leq c$: if in this case $p^a \leq p^s$, then a earns strictly negative profits and so has a profitable deviation, and if $p^a > p^s$, then s earns strictly negative profits and so has a profitable deviation. We also cannot have $p^a \leq c$, since a has profits at most equal to 0 and so has a profitable deviation to $p^a = p^s > c$. It therefore must be the case that $p^s \geq p^a > c$ in equilibrium.

Note that in equilibrium we cannot have $p^b \leq c$, since b earns at most 0 profits, and so b has a profitable deviation to $p^b \in (c, p^a)$. It also cannot be the case that $p^b \geq p^s$, as since $p^s \geq p^a$, in this case b earns 0 profits. In equilibrium we therefore must have $p^s \geq p_a$, $p^s > p_b$, and $p_a, p_b > c$, with $p_s > c$.

Now it suffices to show we must have $p^b \leq p^a$ in equilibrium. Toward a contradiction, suppose not; the above implies that we have $p^s > p^b > p^a > c$ in equilibrium. In this case, the profits of a and b are given by

$$\begin{aligned}
\Pi_a &= (p^a - c)\left[\frac{p^b - p^a}{\Delta q + p^b - p^a} + \frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^b - p^a} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} + \frac{\Delta q}{\Delta q + p^b - p^a} \cdot \frac{\Delta q}{\Delta q + p^s - p^b}\right] \\
\Pi_b &= (p^b - c)\left[\frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^b - p^a} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b}\right]
\end{aligned}$$

First, note that it must be the case that $p^a < c + \Delta q$. To see this, suppose not; we have $p^a \geq c + \Delta q$, which in turn implies $\frac{p^b - c}{\Delta q + p^b - p^a} \geq 1$; this in turn implies that

$$\begin{aligned}
\frac{\partial}{\partial p^b}\Pi_b &\propto \frac{\Delta q)(p^s - p^b)}{(\Delta q + p^b - p^a)(\Delta q + p^s - p^b)} - (p^b - c)\left[\frac{\Delta q(p^s - p^b)}{(\Delta q + p^b - p^a)^2(\Delta q + p^s - p^b)} + \frac{\Delta q^2}{(\Delta q + p^b - p^a)(\Delta q + p^s - p^b)^2}\right] \\
&\leq \frac{\Delta q)(p^s - p^b)}{(\Delta q + p^b - p^a)(\Delta q + p^s - p^b)} - \frac{\Delta q(p^s - p^b)}{(\Delta q + p^b - p^a)(\Delta q + p^s - p^b)} - \frac{\Delta q^2}{(\Delta q + p^s - p^b)^2} \\
&< 0
\end{aligned}$$

and so for any $p^b > p^a$, b's profits are decreasing in p^b, which cannot be the case since we assumed that $p^b > p^a$ in equilibrium. It therefore must be the case that $p^a \geq c + \Delta q$. This in turn implies

$$\begin{aligned}
&(p^a - c)\left[\frac{\Delta q}{(\Delta q + p^b - p^a)^2} - \frac{1}{2} \cdot \frac{\Delta q}{(\Delta q + p^b - p^a)^2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^b - p^a)^2} \cdot \frac{\Delta q}{\Delta q + p^s - p^b}\right] \\
&< \Delta q\left[\frac{\Delta q}{(\Delta q + p^b - p^a)^2} - \frac{1}{2} \cdot \frac{\Delta q}{(\Delta q + p^b - p^a)^2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^b - p^a)^2} \cdot \frac{\Delta q}{\Delta q + p^s - p^b}\right] \\
&= \frac{\Delta q^2}{(\Delta q + p^b - p^a)^2} \cdot \frac{1}{2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} \\
&< \frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^b - p^a} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b}
\end{aligned}$$

which implies that $\Pi_a' > 0$ for any $p^a < p^b$, which cannot be the case since we assumed $p^a < p^b$ in equilibrium. We therefore have a contradiction: it must be the case that $p^b \leq p^a$ in equilibrium, which in turn implies that $p^s \geq p^a \geq p^b > c$ in equilibrium. In this case, the profits of a and b are given by

$$\Pi_a = (p^a - c)\left[\frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^a - p^b} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} + \frac{\Delta q}{\Delta q + p^a - p^b} \cdot \frac{\Delta q}{\Delta q + p^s - p^b}\right]$$
$$\Pi_b = (p^b - c)\left[\frac{p^a - p^b}{\Delta q + p^a - p^b} + \frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^a - p^b} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b}\right]$$

We now show that a unique pure strategy equilibrium in which $p^s = c^s$ exists. Start by showing that $\frac{\partial^2}{\partial (p^b)^2}\Pi_b < 0$ in this range. Note that

$$\begin{aligned}\frac{\partial^2}{\partial (p^b)^2}\Pi_b =& \frac{1}{2} \cdot \frac{\Delta q}{(\Delta q + p^a - p^b)^2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^a - p^b)^2} - \frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^a - p^b} \cdot \frac{p^s - p^b}{(\Delta q + p^s - p^b)^2} \\ &+ (p^b - c)\frac{\partial}{\partial p^b}\left[\frac{1}{2} \cdot \frac{\Delta q}{(\Delta q + p^a - p^b)^2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^a - p^b)^2} - \frac{1}{2} \cdot \frac{\Delta q}{\Delta q + p^a - p^b} \cdot \frac{p^s - p^b}{(\Delta q + p^s - p^b)^2}\right]\end{aligned}$$

Since $\frac{\partial}{\partial p^b}\left[\frac{\Delta q}{\Delta q + p^a - p^b} \cdot \frac{p^s - p^b}{(\Delta q + p^s - p^b)^2}\right] > 0$, $\frac{\Delta q}{(\Delta q + p^a - p^b)^2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^a - p^b)^2} < 0$, and

$$\begin{aligned}&\frac{\partial}{\partial p^b}\left[\frac{1}{2} \cdot \frac{\Delta q}{(\Delta q + p^a - p^b)^2} \cdot \frac{p^s - p^b}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^a - p^b)^2}\right] \\ &= \frac{\partial}{\partial p^b}\left[\frac{\Delta q}{(\Delta q + p^s - p^b)^2} \cdot \frac{-(\Delta q + \frac{1}{2}(p^s - p^b))}{\Delta q + p^s - p^b}\right] \\ &= -\frac{2\Delta q}{(\Delta q + p^s - p^b)^3} \cdot \frac{\Delta q + \frac{1}{2}(p^s - p^b)}{\Delta q + p^s - p^b} - \frac{\Delta q}{(\Delta q + p^s - p^b)^2} \cdot \frac{\frac{1}{2}\Delta q}{(\Delta q + p^s - p^b)^2}\end{aligned}$$

and so $\frac{\partial^2}{\partial (p^b)^2}\Pi_b < 0$.

Now, define

$$\xi(p^s) \equiv \frac{p^s - (c + \Delta q)}{p^s - c} + \frac{1}{2}\frac{\Delta q(p^s - (c + \Delta q))}{(p^s - c)^2} - \Delta q\left[\frac{\Delta q}{(p^s - c)^2} - \frac{1}{2}\frac{\Delta q(p^s - (c + \Delta q))}{(p^s - c)^3} + \frac{1}{2}\frac{\Delta q^2}{(p^s - c)^3}\right]$$

we will show that for $p^s > c$, $\xi(p^s)$ has a unique root p^{s*}, where $p^{s*} > c + \Delta q$, and that $\xi(p^s) > 0$ for $p^s > p^{s*}$ and $\xi(p^s) < 0$ for $p^s < p^{s*}$. To see that ξ has such a root, note that $\xi(p^s) < 0$ for $p^s = c + \Delta q$ and $\xi(p^s) > 0$ for p^s large enough, and apply the intermediate value theorem. To show that this root is unique, it is sufficient to show that $(p^s - c)^2\xi(p^s)$ is strictly increasing for $p^s > c$; to see this, note that

$$\frac{\partial}{\partial p^s}(p^s - c)^2\xi(p^s) = 2p^s - 2c - \frac{1}{2}\Delta q + \frac{\Delta q^3}{(p^s - c)^2};$$

clearly $\frac{\partial}{\partial p^s}(p^s-c)^2\xi(p^s)>0$ if $p^s \geq c+\Delta q$, and if $p^s \in (c, c+\Delta q)$, we have

$$\begin{aligned}
\frac{\partial}{\partial p^s}(p^s-c)^2\xi(p^s) &= 2p^s-2c-\frac{1}{2}\Delta q+\frac{\Delta q^3}{(p^s-c)^3} \\
&> 2p^s-2c-\frac{1}{2}\Delta q+\frac{\Delta q^3}{\Delta q^2} \\
&= 2(p^s-c)+\frac{1}{2}\Delta q>0
\end{aligned}$$

And so $\frac{\partial}{\partial p^s}(p^s-c)^2\xi(p^s)>0$ for all $p^s<c$; this also implies that $\xi(p^s)<0$ for $p^s<p^{s*}$ and $\xi(p^s)>0$ for $p^s>p^{s*}$, and so the desired result obtains. We now consider two cases.

Case 1: $c^s \geq p^{s*}$. We first show that a unique equilibrium exists in which $p^s=c^s$ and $p^b=c+\Delta q$. In such an equilibrium, we must have $\frac{\partial}{\partial p^b}\Pi_b=0$ for $p^b=c+\Delta q$. Let

$$\begin{aligned}
\varphi(p^a)=&\frac{p^a-(c+\Delta q)}{p^a-c}+\frac{1}{2}\cdot\frac{\Delta q}{p^a-c}\cdot\frac{p^s-(c+\Delta q)}{p^s-c} \\
&-\Delta q\left[\frac{\Delta q}{(p^a-c)^2}-\frac{1}{2}\cdot\frac{\Delta q}{(p^a-c)^2}\cdot\frac{p^s-(c+\Delta q)}{(p^s-c)}+\frac{1}{2}\cdot\frac{\Delta q}{p^a-c}\cdot\frac{\Delta q}{(p^s-c)^2}\right]
\end{aligned}$$

denote $\frac{\partial}{\partial p^b}\Pi_b$ evaluated at $p^b=c+\Delta q$. Note that $(p^a-c)^2\varphi(p^a)$ takes on the quadratic form

$$(p^a)^2-p^a\left(2c+\Delta q-\frac{1}{2}\Delta q\cdot\frac{p^s-(c+\Delta q)}{(p^s-c)}+\frac{1}{2}\cdot\frac{\Delta q^3}{(p^s-c)^2}\right)+K$$

It suffices to show that $(p^a-c)^2\varphi(p^a)(p^a-c)^2$ has a unique root on $[c+\Delta q, p^s]$. Note the facts above imply that $\varphi(p^s)=\xi(p^s)\geq 0$, and that

$$\begin{aligned}
\varphi(c+\Delta q) &= \frac{1}{2}\frac{p^s-(c+\Delta q)}{p^s-c}-1+\frac{1}{2}\frac{p^s-(c+\Delta q)}{p^s-c}-\frac{1}{2}\frac{\Delta q^2}{(p^s-c)^2} \\
&= \frac{p^s-(c+\Delta q)}{p^s-c}-1-\frac{1}{2}\frac{\Delta q^2}{(p^s-c)^2}<0
\end{aligned}$$

and so by the intermediate value theorem φ has a root on $[c+\Delta q, p^s]$. To see that this root is unique, note that for $p^a \geq c+\Delta q$, we have

$$\begin{aligned}
\frac{\partial}{\partial p^a}(\varphi(p^a)/(p^a-c)^2) &\geq 2(c+\Delta q)-2c-\Delta q+\frac{1}{2}\Delta q\cdot\frac{p^s-(c+\Delta q)}{(p^s-c)}-\frac{1}{2}\cdot\frac{\Delta q^3}{(p^s-c)^2} \\
&= \Delta q\left(1-\frac{1}{2}\cdot\frac{\Delta q^2}{(p^s-c)^2}\right)+\frac{1}{2}\Delta q\cdot\frac{p^s-(c+\Delta q)}{(p^s-c)}>0
\end{aligned}$$

since $p^s \geq p^{s*} > c+\Delta q$. Therefore, there exists a unique $p^a \in [c+\Delta q+p^s]$ such that

$\frac{\partial}{\partial p^b}\Pi_b = 0$ for $p^b = c + \Delta q$. To verify that these prices constitute an equilibrium, note that b's second order conditions are satisfied, and so b has no profitable deviation; similarly, at $b = c + \Delta q$ we have $\frac{\partial}{\partial p^a}\Pi_a = 0$, and so a has no profitable deviation. Since $p^a \leq c^s$, s also has no profitable deviation.

To see that any equilibria for which $p^s = c^s$ must involves $p^b = c+\Delta q$, note that an equilibrium cannot contain $p^b > c + \Delta q$, since this would imply $\frac{\partial}{\partial p^a}\Pi_a < 0$ for all $p^a \in [p^b, p^s]$. Toward a contradiction, suppose $p^b < c + \Delta q$ in equilibrium. This implies $\frac{\partial}{\partial p^a}\Pi_a > 0$ for all $p^a \in [p^b, p^s]$, and so we must have $p^a = p^s = c^s$ in equilibrium. But note that at $p^b = c+\Delta q$ and $p^a = p^s$, we have $\frac{\partial}{\partial p^b}\Pi_b = \xi(p^s) > 0$, which due to the concavity of Π_b implies that b's best response satisfies $p^b > c + \Delta q$, a contradiction. We therefore conclude there is a unique equilibrium for which $p^s = c^s$ in this case.

Case 2: $c^s < p^{s*}$. Begin by showing that any equilibrium in which $p^s = c^s$ must involve $p^b < c+\Delta q$. By the previous argument, we cannot have $p^b > c+\Delta q$ in equilibrium. Toward a contradiction, suppose that $p^b = c + \Delta q$ in equilibrium; this means that $p^s = c^s > c + \Delta q$, and the above arguments imply that in equilibrium p^a must satisfy $(p^a - c)^2\varphi(p^a) = 0$. However, since $\varphi(p^s) = \xi(p^s) < 0$ and $\varphi(c+\Delta q) < 0$ and by the above arguments we have $\frac{\partial}{\partial p^a}((p^a - c)^2\varphi(p^a)) > 0$ for all $p^a \in [c+\Delta q, p^s]$, $\varphi(p^a)$ does not have a root on $[c+\Delta q, p^s]$, a contradiction.

We therefore must have $p^b < c + \Delta q$ in equilibrium. To see that such an equilibrium is unique, note that by the previous arguments, any such equilibrium must satisfy $p^a = p^s$. Due to the second-order conditions on Π_b, p^b has a unique best-response. All that remains is to verify that this best response is less than $c + \Delta q$. To see this, note that at $p^b = c + \Delta q$ and $p^a = p^s$, we have $\frac{\partial}{\partial p^b}\Pi_b = \xi(p^s) < 0$, which due to the concavity of Π_b implies that b's best response satisfies $p^b < c + \Delta q$.

Finally, to see that b's equilibrium profits are decreasing in Δq an equilibrium where $p^s = c^s < c + \Delta q$, note that b's profits in this equilibrium are given by

$$\Pi_b^*(\Delta q) = \max_{p^b}\left\{(p^b - c)\left[\frac{c^s - p^b}{\Delta q + c^s - p^b} + \frac{1}{2}\cdot\frac{\Delta q}{\Delta q + c^s - p^b}\cdot\frac{c^s - p^b}{\Delta q + c^s - p^b}\right]\right\}$$

applying the envelope theorem, we have

$$\Pi_b^{*'}(\Delta q) = \frac{p^b - c}{(\Delta q + p^s - p^b)^2}\left[-(p^s - p^b) + \frac{(p^s - p^b - \Delta q)(p^s - p^b)}{p^s - p^b + \Delta q}\right] < 0$$

since $\frac{p^s - p^b - \Delta q}{p^s - p^b + \Delta q} < 1$.

□

D Appendix: Experiments

Here we provide more details on the design and pre-registration of our choice and valuation experiments experiments. Screenshots of experimental instructions, comprehension checks, and sample choice interfaces for these experiments are compiled in Online Appendix G.

D.1 Multi-Attribute Binary Choice

Problem Selection. In our multiattribute choice experiments, we collected data on 662 choice problems in total: 582 problems in the *main* problem sample, and 80 problems in a *robustness* problem sample.

The main sample consists of 80 two-attribute problems, 432 three-attribute problems, and 104 four-attribute problems. The three-attribute choice options are characterized by a monthly fee, a per-GB usage rate (where the fictional consumer has a monthly usage of 6 GB), and an annual device cost; the two-attribute choice options consist only of a monthly fee and usage rate, and the four-attribute choice options additionally contain a quarterly wi-fi charge. The two-attribute problems are generated by drawing a value difference (in bonus payment terms) from one of two values in $\{\$3.84, \$5.76\}$ and an L_1-ratio from one of 12 values in $\{1.00, 0.94, 0.89, 0.84, 0.80, 0.76, 0.70, 0.59, 0.48, 0.39, 0.30, 0.20\}$.[46] The three- and four-attribute problems are generated by similarly drawing a value difference and L_1 ratio value, which determines the summed attribute-wise advantages and disadvantages in the comparison, and randomizing how the advantage and disadvantages are split across the attributes.

The robustness sample consists of 10 two-attribute problems, 60 three-attribute problems, and 10 four-attribute problems that are identical in structure to those main sample except for the attribute weights: in the robustnesss sample, the fictional consumer has a monthly usage of 12 GB. Each problem in the robustness sample is constructed to match the utility-weighted attribute values of a corresponding problem in the main sample.

Sample Collection and Screening. We collect choice data from the two problem samples in separate experiments. In the main experiment, each subject completes 50 choice problems

[46]Due to rounding in the attribute values, the actual L_1 ratios of the problems deviate slightly from these values.

in total: 30 randomly drawn unique three-attribute problems, 10 repeat problems drawn from these 30 unique problems, and 10 randomly drawn unique two- or four-attribute problems. Participants first complete the 40 three-attribute problems; for their last 10 problems, they will see either two- or four-attribute problems, with 30% of participants randomly assigned to the two-attribute problems and the remaining participants assigned to the four-attribute problems. The robustness experiment follows an identical structure, except that 50% of participants are randomly assigned to the two-attribute problems with the remaining participants assigned to the four-attribute problems.

Participants for both the main and robustness experiments were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. As pre-registered, data for both the main and robustness experiment were collected in waves to reach a pre-specified number of participants who did not report using a calculator in the experiment: 350 for the main experiment and 48 in the robustness experiment. In total, 428 subjects were recruited for the main experiment (357 non-calculator users) and 65 subjects were recruited for the robustness experiment (50 non-calculator users). The pre-registration for these expeirments can be accessed at `https://aspredicted.org/TNQ_XBQ`.

D.2 Intertemporal Binary Choice

Problem Selection. In our intertemporal choice experiments, we collected data on 1100 choice problems in total: 900 problems in the *broad* problem sample, and 200 problems in a *targeted* problem sample.

In the broad problem sample, choice options contain either one or two payouts; in total, there are 300 1-payout vs. 1-payout choice problems, 300 1-payout vs. 2-payout choice problems, and 300 2-payout vs. 2-payout choice problems. For each choice problem, the options are generated by sampling payout amounts and payout delays. The delays of each payout (in days) are drawn from {0, 12, 24, 48, 72, 108, 144, 180, 216, 264, 312, 360, 420, 480, 540, 600, 660, 720}, and the monetary amount of each payout is drawn from $\{\$0, \$0.50, ..., \$20\}$ for two-payout options and $\{\$0, \$0.50, ..., \$40\}$ for one-payout options. Rather than uniformly sampling from these ranges, we employ a sampling procedure that 1) undersamples dominance problems, 2) excludes problems involving very large value differences and problems near indifference, and 3) stratifies by CPF ratio and value difference (computed using a benchmark discount factor).

In the selected problem sample, problems are generated from sampling the same payout amounts and delays as for the broad problem sample, but are generated using a sampling procedure that holds fixed the threshold discount rate that makes the two options in the choice problem indifferent for a DM with exponential time preferences. In particular, 100 problems in the selected sample involve a threshold monthly discount rate of 1 (meaning that *any* individual with exponential time preferences should prefer the option that pays off earlier), and 100 problems involve a threshold monthly discount rate of 0.747. Within each of these subsamples, 50 problems involve 1-payout vs 2-payout options, and 50 problems involve 2-payout vs. 2-payout options. The sampling procedure for the selected problem sample was additionally designed to stratify by CPF ratio and to reduce variation in the value difference.

Sample Collection and Screening. In the main experiment, each subject completes 50 choice problems in total: 40 unique problems randomly drawn from the combined sample of 1100 problems, and 10 repeat problems randomly drawn from these 40 unique problems. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 829 subjects in total were recruited for the study. The pre-registration for this experiment can be accessed at `https://aspredicted.org/QCJ_S81`.

D.3 Preference Reversal and Valuation Experiments

D.3.1 Lottery Preference Reversals: Experimental Details

Price List and Choice Construction. The experiment concerns 12 lottery options: 6 "base" options consisting of 3 high-risk and 3 low-risk options as described in Table 2, and 6 "scaled-up" options constructed by multiplying the payments in the base options by a scale factor of 1.6. The experiment consists of two parts, which subjects complete in random order: *Binary choice* and *Valuation*.

In *Binary Choice*, subjects make 16 binary choices between lotteries, 12 of which are drawn from the 18 possible high/low risk lottery comparisons within the base and scaled-up options, and 6 of which are filler problems that are included to limit repetition. The order of problems are randomized, subject to the constraints that 1) no single choice option appears in consecutive choice problems, 2) no consecutive choice problems contain the same set of payoff probabilities, and 3) no consecutive choice problems are both filler problems.

In *Valuation*, subjects value all 12 options using one of two randomly assigned valuation modes: certainty equivalents and probability equivalents. In describing the price lists, we will denote each lottery being valued as $(\$\lambda w, p)$, where w is the base payoff amount, p is the payoff probability, and $\lambda \in \{1, 1.6\}$ is the scale factor. For certainty equivalent price lists, each lottery $(\$\lambda w, p)$ is valued against a price list $Z = (z^1, .., z^n)$ of certain payments with evenly-spaced payoff differences, i.e. $z^k = (\$\lambda[w-(k-1)d), 1)$ and $n = \lfloor w/d \rfloor$. We use $d = 0.25$ for the low-risk lotteries and $d = 1$ for the lottery $(\$\lambda \cdot 19.5, 0.23)$, and $d = 1.25$ for the remaining high-risk lotteries. For probability equivalents, each lottery $(\$\lambda w, p)$ is valued against a price list $Z = (z^1, .., z^n)$ of lotteries that pay off $\$\lambda \cdot 24$ with evenly-spaced payoff probability differences, i.e. $z^k = (\$\lambda{\cdot}24, p-(k-1)d)$ and $n = \lfloor p/d \rfloor$. We use $d = 0.05$ for the low-risk lotteries and $d = 0.01$ for the high-risk lotteries. For both valuation modes, the order of the valuation tasks is randomized subject to the constraint that no consecutive valuation tasks involve lotteries with the same payoff probability or scale factor.

If a participant is selected to win a bonus (1 in 5 chance), one part of the study (*Valuation* or *Binary Choice*) is selected at random. If *Binary Choice* is selected, subjects receive the option they chose in a randomly selected decision. If *Valuation* is selected, subjects receive the option they chose in a randomly selected decision within a randomly selected price list.

Sample Collection and Screening. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 151 subjects in total were recruited for the study. The median study completion time was 20 minutes. The pre-registration for this experiment can be accessed at `https://aspredicted.org/C62_GRK`.

D.3.2 Intertemporal Preference Reversals: Experimental Details

Price List and Choice Construction. The experiment concerns 12 delayed payment options: 6 "base" options consisting of 3 high-delay and 3 low-delay options as described in Table 2, and 6 "scaled-up" options constructed by multiplying the payments in the base options by a scale factor of 1.6. The experiment consists of two parts, which subjects complete in random order: *Binary choice* and *Valuation*.

In *Binary Choice*, subjects make 16 binary choices between delayed payments, 12 of which are drawn from the 18 possible high/low delay comparisons within the base and scaled-up options, and 6 of which are filler problems that are included to limit repetition. The order of problems are randomized, subject to the constraints that 1) no single choice option appears in consecutive choice problems, 2) no consecutive choice problems contain the same set of payoff delays, and 3) no consecutive choice problems are both filler problems.

In *Valuation*, subjects value all 12 options using one of two randomly assigned valuation modes: present value equivalents and time equivalents. In describing the price lists, we will denote each delayed payment being valued as $(\lambda m, t)$, where m is the base payoff amount, t is the payoff delay (in days), and $\lambda \in \{1, 1.6\}$ is the scale factor. For present value equivalent price lists, each option $(\$\lambda m, t)$ is valued against a price list $Z = (z^1, .., z^n)$ of immediate payments with evenly-spaced payoff differences, i.e. $z^k = (\$\lambda[w-(k-1)d), 0)$ and $n = \lfloor w/d \rfloor$. We use $d = 0.5$ for the low-delay options and $d = 1.25$ for the high-delay options. For time equivalents, each option $(\$\lambda m, t)$ is valued against a price list $Z = (z^1, .., z^n)$ of delayed payments $z^k = (\$\lambda \cdot 27.5, \tau_k)$ and where $\tau_k = t + d_k$, for $(d_1, ..., d_n) = (0, 7, 15, 30, 45, 60, 90, 120, 180, 240, 300, 360, 420, 480, 540, 600, 660, 720, 840, 960, 1080)$. For both valuation modes, the order of the valuation tasks is randomized subject to the constraint that no consecutive valuation tasks involve lotteries with the same payoff delay or scale factor.

If a participant is selected to win a bonus (1 in 10 chance), one part of the study (*Valuation* or *Binary Choice*) is selected at random. If *Binary Choice* is selected, subjects receive the option they chose in a randomly selected decision. If *Valuation* is selected, subjects receive the option they chose in a randomly selected decision within a randomly selected price list.

Sample Collection and Screening. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 152 subjects in total were recruited for the study. The median study completion time was 19 minutes. The pre-registration for this experiment can be accessed at `https://aspredicted.org/C62_GRK`.

D.3.3 Valuation Experiments: Experimental Details

Price List and Choice Construction. Subjects complete two parts of the experiment, in random order: *Risk* and *Time*. In *Risk*, subjects complete 12 multiple price list valuation tasks corresponding to one of two randomly selected valuation modes: certainty equivalents and probability equivalents. In *Time*, subjects complete 12 multiple price list valuation tasks corresponding to one of two randomly selected valuation modes: present value equivalents and time equivalents.

For certainty equivalents, subjects value a simple lottery $l = (\overline{w}, p_l)$, against a price list

$Z = \{z^1, ..., z^n\}$ of certain payments adapted to l, with $n = 19$. For probability equivalents, subjects value a certain payment $c = (w_c, 1)$ against a probability list $Z = \{z^1, ..., z^n\}$ of yardstick lotteries $z^k = (\overline{w}, p_k)$ adapted to c, with $n = 21$. We draw $\overline{w}$ from $\{\$9, \$18, \$27\}$. For certainty equivalents, we draw p_l from $\{0.03, 0.05, 0.10, 0.25, 0.5, 0.75, 0.90, 0.95, 0.97\}$. For probability equivalents, we draw w_c so that $w_c/\overline{w} \in \{0.033, 0.056, 0.11, 0.25, 0.5, 0.75, 0.89, 0.944, 0.967\}$. Subjects complete 12 price lists randomly selected from the 27 possible price lists; the order of the price lists are randomized, subject to the constraint that no consecutive price list contains the same payoff probability p_l (normalized payment $w_c/\overline{w}$) for certainty equivalents (probability equivalents).

For present value equivalents, subjects value a delayed payment $v = (\$\overline{m}, t_v)$, against a price list $Z = \{z^1, ..., z^n\}$ of immediate payments adapted to l, with $n = 21$. For time equivalents, subjects value a certain payment $c = (w_c, 1)$ against a probability list $Z = \{z^1, ..., z^n\}$ of yardstick delayed payments $z^k = (\overline{m}, t_k)$, with $(t_1, ..., t_n) = (0, 7, 15, 30, 45, 60, 90, 120, 150, 180, 240, 300, 360, 420, 480, 540, 600, 720, 840, 960, 1080)$. We draw $\overline{m}$ from $\{25, 30, 35\}$. For present value equivalents, we draw t_v from $\{7, 30, 60, 120, 240, 360, 480, 720, 1080\}$ (in days). For time equivalents, we draw m_c so that $m_c/\overline{m} \in \{0.20, 0.35, 0.50, 0.65, 0.75, 0.85, 0.90, 0.95, 0.97\}$. Subjects complete 12 price lists randomly selected from the 27 possible price lists; the order of the price lists are randomized, subject to the constraint that no consecutive price list contains the same payoff delay t_v (normalized payment $m_c/\overline{m}$) for present value equivalents (time equivalents).

If a participant is selected to win a bonus (1 in 8 chance), one part of the study (*Risk* or *Time*) is selected at random, and a price list within that part is randomly selected; Subjects receive the option they chose in a randomly selected decision within that price list.

Sample Collection and Screening. Participants were recruited from Prolific, screening for subjects based in U.S. with a Prolific approval rating greater than or equal to 98% and with 500 or more completes using Prolific's pre-screening tools. Participants who did not pass a comprehension check were screened out of the study. 302 subjects in total were recruited for the study. The median completion time is 24 minutes. The pre-registration for this experiment can be accessed at `https://aspredicted.org/D8R_552`.

E Appendix: Structural Specifications

We estimate several standard models of value in multi-attribute objects, intertemporal payoffs, and lotteries, assuming logit choice probabilities:

$$\rho(x,y) = \text{sgm}_\eta(V(x) - V(y))$$

where $\text{sgm}_\eta(t) = 1/(1+\exp(-\eta t))$ is the sigmoid function for $\eta \geq 0$. For each of these standard models, we jointly estimate a parameterized V function and the logit noise parameter η. We additionally estimate our parameterized model of similarity-based complexity from Section 2.4,

$$\rho(x,y) = G\left(\frac{U(x)-U(y)}{d(x,y)}\right),$$

$$G(r) = \begin{cases} (1-\kappa) - (0.5-\kappa)\dfrac{(1-r)^\gamma}{(r^\psi + (1-r)^\psi)^{1/\psi}} & r \geq 0 \\ \kappa + (0.5-\kappa)\dfrac{(1+r)^\gamma}{(r^\psi + (1-r)^\psi)^{1/\psi}} & r < 0 \end{cases}$$

for $\kappa \in [0, 0.5], \gamma, \psi > 0$. Unless stated otherwise, we will use the 2-parameter functional form of G in which we fix $\psi = 1$. In each domain, we jointly estimate the parameterized value-dissimilarity ratio and the G-function parameters κ and γ (and ψ, if applicable). Below we give the equations for each model estimated in the paper.

E.1 Multi-attribute Choice

In the following structural equations, we normalize utility the weights β_k to be equal to 1: that is, the true value of option x is given by $U(x) = \sum_k x_k$.

Distortion-Free Logit. Choice rates are given by logit noise applied to the value difference:

$$\rho(x,y) = \text{sgm}_\eta(U(x) - U(y)).$$

This model is parameterized by η.

Salience. We use the continuous salience-weighting model described in Appendix C of Bordalo et al. (2013), where

$$\rho(x,y) = \text{sgm}_\eta(V_{BGS}(x|\{x,y\}) - V_{BGS}(y|\{x,y\}))$$

$$V_{BGS}(x|\{x,y\}) \equiv \sum_k x_k \left(1 + \frac{|x_k - (x_k+y_k)/2|}{|x_k| + |(x_k+y_k)/2|}\right)^{1-\delta}$$

where $\delta \leq 1$. This model is parameterized by (η, δ).

Focusing. We use the power function parameterization described in Kőszegi and Szeidl (2013), where

$$\rho(x, y) = \text{sgm}_\eta(V_{KS}(x|\{x, y\}) - V_{KS}(y|\{x, y\}))$$
$$V_{KS}(x|\{x, y\}) = \sum_k x_k |x_k - y_k|^\theta$$

where $\theta \geq 0$. This model is parameterized by (η, θ).

Relative Thinking. We use the power function parameterization described in Bushong et al. (2021), where

$$\rho(x, y) = \text{sgm}_\eta(V_{BRS}(x|\{x, y\}) - V_{BRS}(y|\{x, y\}))$$
$$V_{BRS}(x|\{x, y\}) \equiv \sum_k x_k \left[(1-\omega) + \omega \frac{1}{|x_k - y_k| + \xi}\right]$$

where $\omega \in [0, 1]$, $\xi > 0$. This model is parameterized by (η, ω, ξ).

$\mathbf{L_1}$ Complexity. Choice probabilities in our model is given by

$$\rho(x, y) = G\left(\frac{U(x) - U(y)}{d_{L1}(x, y)}\right)$$

where d_{L1} is defined as in Definition 1. We estimate both the 2 and 3 parameter versions of G; our model is parameterized by (κ, γ) for the former and (κ, γ, ψ) for the latter.

E.2 Intertemporal Choice

Exponential Discounting. Choice probabilities are given by

$$\rho(x, y) = \text{sgm}_\eta(PV(x) - PV(y))$$
$$PV \equiv \sum_t \delta^t m_x(t)$$

The parameters of the model are given by (η, δ). This model is also used for the estimation of individual-level discount factors used in Figure 14 and Table 7.

Hyperbolic Discounting. We use the hyperbolic discount function proposed in Loewenstein and Prelec (1992):

$$\rho(x,y) = \text{sgm}_\eta(V_{hb}(x) - V_{hb}(Y))$$
$$V_{hb} \equiv \sum_t (1+\iota t)^{-\zeta/\iota} m_x(t)$$

for $\iota, \zeta > 0$. The parameters of this model are (η, ι, ζ).

CPF Complexity. Choice probabilities in our model are given by

$$\rho(x,y) = G\left(\frac{PV(x) - PV(y)}{d_{CPF}(x,y)}\right)$$

where $d_{CPF}(x,y)$ is defined as in Definition 4. Our model is parameterized by (δ, κ, γ).

E.3 Choice Under Risk

Expected Utility. To estimate the global preference parameters used in Figure 7 and Table 9, we assume agents have a Bernoulli utility function that exhibits constant relative risk aversion for both pure-gain and pure-loss lotteries:

$$\rho(x,y) = \text{sgm}_\eta(EU_{sym}(x) - EU_{sym}(y))$$
$$EU_{sym}(x) \equiv \sum_w f_x(w) u_{sym}(w)$$
$$u_{sym}(w) \equiv \begin{cases} w^\alpha & w \geq 0 \\ -(-w)^\alpha & w < 0 \end{cases}$$

for $\alpha > 0$. This model is parameterized by (η, α). This model is also used for the estimation of individual-level preferences used in Figure 15 and Table 10.

Reference-Dependence. The DM has expected utility preferences, where the (two parameter) Bernoulli utility function allows for separate curvature parameters for positive and negative payouts, with a loss-aversion parameter λ.

$$\rho(x,y) = \text{sgm}_\eta(EU_{rd}(x) - EU_{rd}(y))$$
$$EU_{rd}(x) \equiv \sum_w f_x(w) u_{rd}(w)$$
$$u_{rd}(w) \equiv \begin{cases} w^\alpha & w \geq 0 \\ -\lambda(-w)^\beta & w < 0 \end{cases}$$

for $\alpha, \beta > 0$. This model is paramaterized by $(\eta, \alpha, \beta, \lambda)$.

Cumulative Prospect Theory. We also estimate a model where the agent exhibits probability weighting and loss aversion, following Tversky and Kahneman (1992). We use the probability weighting function given by Gonzalez and Wu (1999). Let the distinct payoffs in a lottery x be ordered by $w_{-m}, \ldots, w_{-1}, w_0, w_1, \ldots, w_n$, where $w_{-m}, \ldots, w_0$ indicate negative payoffs and $w_0, \ldots, w_n$ indicate positive payoffs, with $p_{-m}, \ldots, p_n$ denoting the associated probabilities. The value of x is given by

$$U_{cpt}(x) = \sum_{k=-m}^{0} u_{pt}(w_k)\pi_k + \sum_{k=0}^{n} u_{pt}(w_k)\pi_k,$$
$$\pi_n = q(p_n),\ \pi_{-m} = q(p_{-m})$$
$$\pi_k = q(p_k + \ldots + p_n) - q(p_{k+1} + \ldots + p_n), \quad 0 \leq k < n$$
$$\pi_k = q(p_{-m} + \ldots + p_k) - q(p_{-m} + \ldots + p_{k-1}), \quad -m < k < 0$$
$$q(p) = \frac{\chi p^\nu}{\chi p^\nu + (1-p)^\nu}$$
$$u_{pt}(w) \equiv \begin{cases} w^\alpha & w \geq 0 \\ -\lambda(-w)^\beta & w < 0 \end{cases}$$

for $\alpha, \beta, \chi, \nu, \lambda > 0$. Choice probabilities are given by

$$\rho(x,y) = \text{sgm}_\eta(U_{cpt}(x) - U_{cpt}(y))$$

This model is parameterized by $(\eta, \alpha, \beta, \chi, \nu, \lambda)$.

CDF Complexity. We estimate two versions of our model: one that assumes risk neutrality, and one that allows for utility curvature. In the risk neutral model, choice probabilities are given by

$$\rho(x,y) = G\left(\frac{EU(x) - EU(y)}{d_{CDF}(x,y)}\right)$$

$EU(x) = \sum_w w f_x(w)$ and d_{CDF} are defined as in Definition 3 with the Bernoulli utility function u given by $u(x) = x$. This model is parameterized by (κ, γ).

In the model that allows for utility curvature, choice probabilities are given by

$$\rho(x,y) = G\left(\frac{EU(x) - EU(y)}{d_{CDF}(x,y)}\right)$$

$EU(x) = \sum_w u_{sym}(w) f_x(w)$ and d_{CDF} are defined as in Definition 3 with the Bernoulli utility function $u = u_{sym}$. This model is parameterized by (κ, γ, α).