On the Decision-Relevance of Subjective Beliefs*

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Abstract

While a large literature documents that subjective expectations predict many economic decisions, the quantitative magnitude of these relationships is often attenuated relative to theoretical predictions. This paper assesses one explanation for these findings: that individuals may be uncertain over how to incorporate beliefs about a quantity into their decision-making. I develop a theoretical framework demonstrating how uncertainty over the belief-action map attenuates the relationship between beliefs and actions, weakens behavioral responses to information, and reduces incentives to learn about the quantity. I experimentally test these predictions by eliciting subjects' uncertainty over the belief-action map and manipulating this uncertainty. I find support for all three predictions: uncertainty over the belief-action map attenuates the relationship between return expectations and portfolio allocations, weakens the behavioral response to information about returns, and reduces demand for this information. I further show that reducing this uncertainty using an easy-to-deploy intervention increases subjects' responsiveness to their beliefs.

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1 Introduction

Subjective beliefs play a central role in the predictions of economic models. As a result, a burgeoning literature measures subjective expectations to explain choice behavior and discipline model predictions in contexts ranging from macro-finance to education and health. The first-order stylized fact from this literature is that expectations can be measured in rich ways and that they are often statistically significant and economically meaningful predictors of behavior.

At the same time, this literature has also repeatedly documented two empirical puzzles that jointly suggest that the effect of expectations on economic decisions is more complicated than envisioned in canonical models. First, the quantitative magnitude of the link between expectations and decisions is often attenuated relative to theoretical predictions. This observation has been made in various contexts: for instance, individuals' portfolio allocations are insensitive to their expectations of stock returns in the well documented "attenuation puzzle" (e.g. Giglio et al., 2021; Laudenbach et al., 2021; Beutel and Weber, 2022), the relationship between spending and inflation expectations is often weak or inconsistent with standard models (e.g. D'Acunto et al., 2023; Coibion et al., 2022, 2023; Duca-Radu et al., 2021), and subjects fail to best-respond to their beliefs about opponents' play in experimental games (e.g. Costa-Gomes and Weizsäcker, 2008). Second, consistent with a quantitatively weak transmission of beliefs to decisions, information interventions designed to correct mistaken beliefs tend to have large effects on stated expectations, but tend to have comparatively modest effects on downstream economic decisions. Informing individuals' expectations of home price growth produces quantitatively weak effects on their investment decisions (Armona et al., 2019), correcting students' beliefs about the earnings outcomes associated with college majors has muted effects on their education decisions (Zafar, 2013; Wiswall and Zafar, 2015a,b), and correcting misperceptions about the extent of racial or economic inequities has little effect on individuals' policy preferences (Haaland and Roth, 2021; Alesina et al., 2018; Kuziemko et al., 2015). In Appendix A.1, I present a literature review that discusses additional examples of such attenuated transmission patterns.

Understanding the mechanisms behind these patterns is important for at least two reasons. From the perspective of theory, most economic models – both neoclassical and behavioral – posit a seamless transmission of beliefs into decisions, raising the question whether economists should not only model distortions in the formation of beliefs but also in the transmission of beliefs to decisions. Second, from an applied and policy perspective, providing insight on why some information interventions fail to change behavior may help policy-makers understand the scope for specific interventions to be successful, and provide guidance on which sets of beliefs they should focus on correcting.

This paper proposes an explanation that can account for these patterns: that individuals find it cognitively difficult to determine how exactly to translate their expectations into decisions — that is, they may be uncertain over the *belief-action map*. For instance, in the context of portfolio choice, it may be unclear what the precise mapping should be between one's return expectations and optimal portfolio equity share. Individuals may be similarly uncertain over how exactly to optimally adjust their consumption given a change in their inflation expectations, or how to incorporate earnings expectations in human capital investment decisions. Various mechanisms could lead to uncertainty over the belief-action map – individuals may have uncertainty over their own preferences, or may find it difficult to determine their optimal decision as a function of their beliefs due to the complexity of the decision. Regardless of what the precise cognitive foundations are, the objective of this paper is to study how uncertainty over the belief-action map affects the transmission from beliefs to behavior and the formation of beliefs.

To address this question, I first develop a model to formalize this uncertainty notion and structure the empirical analysis. In the model, the decision-maker (DM) chooses how much information to acquire about a quantity (e.g., stock market returns) and subsequently takes a decision (e.g., a portfolio allocation). I depart from standard rational inattention models (e.g. Caplin and Dean, 2015; Gabaix, 2019; Maćkowiak et al., 2021) by allowing for the DM to be uncertain over a decision weight parameter that governs the optimal mapping between their beliefs and their decision – in particular, I assume that the DM only has access to a noisy signal about their true decision weight. This model predicts that higher uncertainty over the belief-action map produces greater attenuation in the relationship between beliefs and decisions, and weakens the behavioral response to information about the quantity. Moreover, the model predicts that higher uncertainty over the belief-action map leads to less information acquisition. Thus, the model ties together two salient features of the subjective beliefs literature: that people are often poorly informed about important quantities, and that the link between their beliefs and their decisions is relatively weak.

To test these predictions, I implement incentivized portfolio allocation experiments with a large and diverse online subject pool. Participants split an endowment between two assets: an investment account tied to the returns of the S&P 500, and a safe asset with a guaranteed return. Participants also state their subjective return expectations of the S&P 500 over the investment horizon and are given an opportunity to acquire information about returns. In these experiments, I study how uncertainty over the belief-action map impacts the transmission of beliefs to decisions, the behavioral response to information interventions, and information acquisition. To this effect, I both directly measure variation in uncertainty over the belief-action map (as it endogenously exists in the population) and implement exogenous experimental manipulations of this uncertainty.

To measure uncertainty about the belief-action map, I modify techniques from the lab experimental literature on complexity and cognitive noise (Enke and Graeber, 2022, 2023) by measuring subjects' cognitive uncertainty over the belief-action map. In doing so, I take care to ensure that I only measure my object of interest – uncertainty about the belief-action map – rather than other sources of uncertainty, motivated by the well-documented fact that individuals have uncertainty over the accuracy of their beliefs (Manski and Molinari, 2010; Giustinelli et al., 2022). My measure, explained in detail in Section 3, can be interpreted as an individual's certainty over how to translate their beliefs about a quantity into a decision, given that those beliefs are formed with certainty.

Using this measure, I document that the cross-sectional relationship between subjects' return expectations and portfolio allocations is attenuated when they exhibit higher uncertainty about the belief-action map. This attenuation is economically significant in magnitude: the estimated relationship between return expectations and portfolio equity shares is more than twice as large for subjects with low uncertainty relative to subjects with high uncertainty. Similarly, I document that the effect of randomized information interventions (an expert forecast of S&P 500 returns) on portfolio allocations is twice as large for subjects with low uncertainty about the belief-action map. These observed patterns in attenuation persist after I account for measurement error using repeated elicitation of return expectations, and cannot be rationalized if my uncertainty measure merely captures normative factors that predict a weaker relationship between return expectations and portfolio allocations, such as risk aversion and beliefs about return variability. To complement these correlational results, I exogenously vary uncertainty over the belief-action map by introducing a treatment that renders the mapping from beliefs to investment decisions more complicated. In line with the correlational results, I find that this treatment (and the resulting higher uncertainty over the belief-action map) produces a stronger attenuation effect in the transmission of beliefs and information interventions.

Having established that uncertainty over the belief-action map weakens the transmission of beliefs and information to behavior, I turn to testing the last prediction of my model: that this uncertainty reduces subjects' incentives to acquire information. I first document several pieces of correlational evidence for this relationship. First, subjects with greater uncertainty over the belief-action map also report greater uncertainty over the accuracy of their return expectations, and revise their expectations to a greater extent in response to receiving information about returns[1]. Second, such subjects report obtaining information about the stock market at a lower frequency, and when given the opportunity to acquire an expert forecast of S&P 500 returns prior to their investment decision, exhibit less information demand. Consistent with model predictions, I also find that an exogenous increase in subjects' uncertainty over the belief-action map, produced by the treatment variation discussed above, reduces subjects' demand for the expert forecast.

Taken together, my results suggest that individuals face frictions not only in forming accurate beliefs, but also in translating those beliefs into behavior. In light of the increasing interest in using information interventions as a policy lever to change behavior by correcting miscalibrated beliefs, a key implication of these findings is that such interventions may have limited effects if individuals are uncertain about the belief-action map. On the other hand, the existence of this friction points to the potential usefulness of an additional policy lever: interventions that reduce uncertainty over the belief-action map, rather than merely correct beliefs.

In a follow-up to the main experiment, I study one such intervention. Inspired by the psychology literature on joint-versus-separate evaluation effects (Hsee et al., 1999), I design an intervention that encourages subjects to construct a full mapping between beliefs and

[1]This pattern persists controlling for subjects' baseline return expectations

decisions. The rationale for this intervention is as follows: if individuals are only asked to assess their optimal equity share for their current set of beliefs – e.g. when their expected return is 5% – it may never occur to them to consider what they would do if they held different return expectations – e.g. 3% or 8% – hence producing only a weak relationship between beliefs and actions. Yet, if individuals were encouraged to actively consider how much they would invest for a range of different return expectations, this may increase their sensitivity to their beliefs. Based on this idea, I implement a treatment in which – before participants make their actual investment decisions – they are asked to indicate how they would invest if their return expectations were different, for a range of different expectations. I find that this treatment has a large effect on behavior: the responsiveness of decisions to subjects' actual beliefs, as well as their behavioral response to information about returns, becomes considerably stronger. The effects of this light-touch intervention suggest that the weak belief-action link may be subject to successful policy interventions, and that such interventions may complement the use of information interventions.

This paper sheds light on how individuals' uncertainty over how to incorporate economic quantities into their decision-making can jointly explain a set of puzzles surrounding the measurement and utilization of subjective beliefs in economics research: the weak link between beliefs and behavior and the inconsistent effects of information interventions on beliefs vs. behavior. Furthermore, my results demonstrate two additional implications of this friction: 1) that it reduces demand for information about the quantity, thus adding to our understanding of the factors that cause individuals to hold poorly-informed beliefs about economic quantities, and 2) that there is scope for interventions that improve understanding of the belief-action map, in addition to interventions that merely correct beliefs. Put simply, my results suggest that in certain contexts, people don't understand how to translate beliefs into decisions, and that as a result, they respond less to information and have lower demand for information. Furthermore, this paper highlights the potential usefulness of direct measures of this uncertainty — which can be easily deployed in the surveys designed to elicit subjective beliefs — in facilitating the interpretation of subjective beliefs data, as well as the potential effectiveness of interventions aimed at improving subjects' understanding of the belief-action map.

The remainder of the paper is organized as follows. Section 1.1 reviews related literature. Section 2 develops the theoretical framework and derives the key predictions that structures the subsequent experimental design and analysis. Section 3 presents the design of the main experiment, and Section 4 discusses the experimental results. Section 5 discusses the extension to the main experiment. Section 6 concludes.

1.1 Related Literature

The main contribution of this paper is to propose a new explanation for the quantitatively weak transmission of beliefs and information on decision-relevant quantities to behavior, that simultaneously accounts for the low levels of information acquisition typically observed for such quantities. As such, this paper sheds light on patterns documented in a large and varied literature concerned with measuring subjective expectations and relating them to behavior (see Appendix A.1 for a review of attenuation patterns documented in this literature). This paper also connects to work studying the effects of information interventions (for reviews, see Haaland et al., 2023; Fuster and Zafar, 2023), and proposes a mechanism that can account for the often weak downstream effects of information interventions on behavior documented in this literature (Haaland et al., 2023). Appendix A.1 reviews specific evidence from this literature.

Thus far, the literature has discussed two broad potential reasons for the weak transmission of beliefs into actions. First, researchers have proposed that transaction costs or similar frictions could drive attenuation patterns (e.g. Duffie and Sun, 1990; Peng and Xiong, 2006; Giglio et al., 2021). While there exists evidence for such a mechanism, it is orthogonal to the friction studied in this paper and moreover cannot account the results in my experiment, in which subjects face no transaction costs. Second, researchers have often attributed the attenuated beliefs-actions link as the result of classical measurement error. In spite of this, subsequent contributions showed that even after correcting for measurement error, much attenuation remains (e.g. Giglio et al., 2021; Constantin et al., 2023). Moreover, potential measurement error does not explain why exogenous information interventions often only have quantitatively weak effects on behavior, or why uncertainty about the belief-action link is correlated with low information acquisition.

Two prior contributions have studied the potential effect of cognitive limitations on the weak beliefs-actions link. D'Acunto et al. (2023) provide complementary field evidence by showing that there is an economically meaningful relationship between inflation expectations and planned consumption among high IQ individuals, and no such relationship along low IQ individuals.[2] This finding is consistent with my model and evidence, but could also reflect heterogeneity in measurement error. Constantin et al. (2023) provide evidence for an attenuated link between subjective expectations and asset valuations in a lab experiment, and argue that it reflects cognitive noise.[3] Compared to their paper, my contribution is (i) to provide a formal model of uncertainty about the belief-action map, (ii) to directly measure and exogenously manipulate this uncertainty, (iii) to explain not just the beliefs-action link but to tie it together with the weak effects of information interventions and low information acquisition and (iv) to propose an easy-to-deploy intervention that increases the transmission of beliefs to decisions.

More generally, my model borrows from the techniques in the recent literatures on rational inattention (e.g. Caplin and Dean, 2015; Gabaix, 2019; Maćkowiak et al., 2021), which studies how costs of information acquisition and processing may limit the extent to which decision-makers attend to decision-relevant quantities, and cognitive noise (Woodford, 2020; Khaw et al., 2021; Enke and Graeber, 2023, 2022), which studies the implications of imprecision

[2] See also D'Acunto et al. (2023) for related evidence.

[3] In particular, Constantin et al. (2023) implement a treatment manipulation in which the payoff distribution of the experimental asset is explicitly given to subjects, and find less insensitivity in this treatment relative to the baseline treatment, in which subjects form and state beliefs about the payoff distribution. The effects of this treatment could be driven by measurement error in subjects' stated beliefs about asset payoffs, rather than cognitive noise.

in the optimization process for behavior. As such, my model shares a similar approach to Illut and Valchev (2023), who study a dynamic general equilibrium model in which agents exhibit uncertainty over the mapping between beliefs and decisions, and can endogenously learn about this mapping.

2 Theoretical Framework

In what follows, I develop the theoretical framework that will guide the subsequent empirical analysis. The key ingredients of the model are as follows: there is an economic quantity θ (e.g. the expected return of the stock market or expected inflation), which the decision-maker can learn about at a cost. The quantity is payoff-relevant to an action a taken by the decision-maker; in particular, the optimal mapping between the quantity and actions is given by a decision weight β. Rather than perfectly observing β, as in standard models, the decision-maker has uncertainty over β. In what follows, I characterize how this uncertainty affects the relationship between the decision-maker's beliefs about θ and their actions, and how much the decision-maker learns about θ. Derivations of predictions are included in Appendix A.2.

2.1 Setup and Information Structure

There is an economic quantity θ that is relevant to the payoffs of the DM's action a, which are given by

$$u(a,\theta) = -\left(a - \beta\theta\right)^2 \tag{1}$$

where β is the normative decision weight. To take a concrete example, consider an application to an investment decision where θ is the expected return of the equity market portfolio and a is the DM's portfolio equity share. Here, β reflects all factors that affect the normative mapping between expected returns and the portfolio equity share, such as the DM's level of risk aversion or the DM's beliefs about the variability of equity returns.

The DM holds priors over θ distributed according to $N(\overline{\theta}, \sigma_\theta^2)$. The DM can generate information about θ at a cost, which reflects both costs of information acquisition and information processing. Specifically, the DM chooses the precision τ of a signal $s_\theta \sim N(\theta, 1/\tau)$, where τ represents the DM's level of effort in learning about θ. I impose some structure on these costs: I assume setting a given value of τ comes at a linear cost c.

Rather than perfectly observing the decision weight β as in a standard rational inattention framework, the DM instead observes the decision weight with noise. In particular, I assume that the DM holds priors $N\left(0, \sigma_\beta^2\right)$ over β, independent of her priors over θ, and observes the cognitive signal $s_\beta \sim N\left(\beta, \sigma_\zeta^2\right)$, independent of s_θ; this cognitive signal can be interpreted as the result of a cognitive sampling or deliberation process, and greater cognitive noise σ_ζ^2 corresponds to a less precise cognitive signal. Note that here, the assumption that the DM has mean 0 priors over β is substantive, and drives the key relationships between uncertainty over β and both behavior and information acquisition derived later in the section. One interpretation of this assumption is that in the absence of any cognitive signals — that is, if the DM is completely ignorant over the belief-action map — the DM by default does not use θ in her decision-making.

The timing of the model is as follows: The DM first observes s_β and forms estimates of the β. The DM then chooses the signal precision τ, receives the signals s_β, and forms estimates of θ. Finally, the DM takes an action a. This results in the optimization problem

$$\max_{\tau}\left\{E\left(\max_{a} E\left(-\left(a-\beta\theta\right)^{2}|s_\beta, s_\theta\right)|s_\beta\right)-c\tau\right\} \tag{2}$$

which I refer to as the decision-maker's *first-stage problem*. Below, I discuss the interpretation of the primitives in the framework, and how they relate to applications of interest.

Interpretation of uncertainty over θ. Following the rational inattention literature, I interpret uncertainty over θ as resulting from the possibility that the DM may not process all available information about θ at the time of her decision, which stems from information acquisition and information processing costs. For example, in the case where θ is the expected return of the market portfolio, uncertainty over θ reflects the fact that the DM may not attend to all relevant information about financial markets, such as news or expert forecasts, or otherwise may not incorporate this information into her estimate of expected returns.[4] As such, the DM may be uncertain over the accuracy of her beliefs over θ. Such subjective uncertainty over expectations is well documented (Manski and Molinari, 2010; Giustinelli et al., 2022; Giglio et al., 2021).

Interpretation of uncertainty over β. The model is agnostic to the possible sources of uncertainty over the decision weight β. Here, I briefly discuss several candidate sources of uncertainty. One possible source of uncertainty is preference uncertainty. To illustrate, consider again the case where θ is the expected return of the market portfolio and a is the DM's portfolio equity share. The DM may be uncertain over the degree to which she is risk-averse, and so is uncertain over her optimal portfolio equity allocation should respond to her beliefs over expected returns. Another possible source of uncertainty is uncertainty over other decision-relevant quantities – for example, the DM may be uncertain over the variability of equity returns, which results in further uncertainty over how to translate beliefs over expected returns into a portfolio allocation. Finally, uncertainty may arise due to difficulties the DM faces in the process of optimization – for example, to arrive at a portfolio equity share, the DM must correctly combine knowledge of her preferences and decision-relevant quantities, a process she may find cognitively difficult.

[4] In applications where θ is the moment of a distribution, such as the expected return of the market portfolio, note that this framework draws a distinction between uncertainty over θ (i.e. the DM's uncertainty over the quality of her estimate of expected returns) and the DM's beliefs about the variability of the quantity. Section 3 discusses how both notions of uncertainty are elicited in the experimental design.

2.2 Model Predictions

Actions in the First Stage Problem. Conditional on her imprecise signal of the decision weight s_β, the DM's posterior belief over β is given by $N\left((1-\lambda)s_\beta, \tilde{\sigma}_\beta^2\right)$, where the *attenuation factor* λ is given by $\lambda = \sigma_\zeta^2 / \left(\sigma_\zeta^2 + \sigma_\beta^2\right)$. Given a posterior expectation over θ given by $\hat{\theta}$, the DM's optimal action is

$$a^* = (1-\lambda)s_\beta\hat{\theta} \tag{3}$$

and so the DM's average action, conditional on the true β and on her beliefs over the quantity, is given by

$$E[a^*|\beta, \hat{\theta}] = (1-\lambda)\beta\hat{\theta} \tag{4}$$

Notice that uncertainty over the belief-action map σ_ζ^2 attenuates the the relationship between the DM's beliefs about θ and her actions relative to the normative benchmark, under which $a^* = \beta\hat{\theta}$; note also that as uncertainty over the belief-action map approaches zero, λ converges to 0, recovering the normative benchmark.[5]

Information Acquisition in the First Stage Problem. I now turn to characterizing the DM's decision to acquire information about θ, that is, their choice of τ. At an interior solution, the DM's posterior belief about θ is distributed according to $\theta|s_\theta \sim N\left(\hat{\theta}, \hat{\sigma}_\theta^2\right)$, where $\hat{\theta} = \alpha\overline{\theta} + (1-\alpha)s_\theta$, for $\alpha = \frac{\sqrt{c}}{\sigma^2(1-\lambda)|s_\beta|}$. In particular, the DM chooses the signal precision τ^* and has posterior uncertainty over the quantity $\hat{\sigma}_\theta^2$ given by

$$\tau^* = \frac{(1-\lambda)|s_\beta|}{\sqrt{c}} - \frac{1}{(\sigma_\theta)^2} \tag{5}$$

$$\hat{\sigma}_\theta^2 = \frac{\sqrt{c}}{(1-\lambda)|s_\beta|} \tag{6}$$

Notice that the intensity of the DM's information acquisition (posterior uncertainty over θ) is decreasing (increasing) in the attenuation factor λ, which is in turn increasing in the DM's uncertainty over the quantity action map σ_ζ^2. This reflects the intuition that if the DM is uncertain over how to incorporate a quantity in her decision-making, she will have less incentives to expend costly effort to learn about that quantity.

Information Interventions. I model an information intervention as a signal about the quantity $\phi \sim N(\theta, 1/\tau_\phi)$ observed after the first-stage problem; for simplicity I assume that the DM does not anticipate receiving the information intervention in the first-stage problem. The change in beliefs induced by the information is given by

$$\Delta\hat{\theta} = \frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_\theta^2 + 1/\tau_\phi}(\phi - \hat{\theta}) \tag{7}$$

[5] An alternative interpretation of expression (4) is that with probability $1-\lambda$, the DM deliberates and correctly incorporates her beliefs $\hat{\theta}$ into her decision, and with probability λ, the DM does not incorporate her beliefs into her decision. In Appendix A.2, I show that analogs of the subsequent predictions hold in this random choice account.

which is increasing in $\hat{\sigma}_\theta^2$, the DM's uncertainty over θ. Since $\hat{\sigma}_\theta^2$ is correlated with uncertainty over the belief-action map β, greater uncertainty over the quantity action map predicts greater responsiveness of *beliefs* to the information intervention. As a function of this change in beliefs, however, the average change in action induced by the information is given by

$$E[\Delta a^*|\beta, \Delta\hat{\theta}, \phi] = (1-\lambda)\beta\Delta\hat{\theta} \tag{8}$$

Therefore, while uncertainty over the belief-action map β *increases* the responsiveness of the DM's beliefs to information, it simultaneously *mutes* the effect of information on the DM's actions, for a given change in beliefs. The intuition for these patterns are as follows: greater uncertainty over the belief-action map causes the DM to acquire less information about θ in the first-stage problem, and so all else equal, the DM's beliefs will be more responsive to the information. At the same time, greater uncertainty over the belief-action map attenuates the relationship between the DM's beliefs about θ and their actions, which weakens the DM's behavioral response to the information for a given change in beliefs.

Demand for Additional Information. Now consider the DM's demand for the information ϕ analyzed above. Given the DM's choice of costly signal precision τ^* in first-stage problem, let $V_0^{\tau^*} \equiv \max_a E(u(a,\theta)|s_\beta, s_\theta)$ denote the DM's expected utility if no additional information is acquired, and let $V_\phi^{\tau^*} \equiv E[\max_a E(u(a,\theta)|s_\beta, s_\theta, \phi)\,|s_\beta, s_\theta]$ denote the DM's expected utility from observing the signal ϕ. The DM's willingness-to-pay for the signal is given by

$$WTP_\phi \equiv V_\phi^{\tau^*} - V_0^{\tau^*} = (1-\lambda)^2 s_\beta^2 \frac{\tau_\phi \hat{\sigma}_\theta^4}{1+\tau_\phi\hat{\sigma}_\theta^2} \tag{9}$$

That is, holding fixed the DM's posterior uncertainty over the quantity after the first-stage problem $\hat{\sigma}_\theta^2$, greater uncertainty over the belief-action map reduces the DM's valuation for the signal through the attenuation factor λ. The intuition is the same as for information acquisition in the first-stage problem: if the DM is uncertain over how to incorporate a quantity in her decision-making, she will have lower valuation for information regarding that quantity.[6]

Counterfactual Cognitive Uncertainty. I now define the primary measure of uncertainty over the belief-action map. Note that in the setting studied above, the DM's uncertainty over her optimal action comes from two sources: uncertainty over the quantity θ, and uncertainty over the belief-action map β. To isolate uncertainty this second source of uncertainty, consider a setting a where the quantity θ is known and equal to θ^{cf}; I refer to such a decision context as a *counterfactual elicitation.* Here, the DM's uncertainty over the optimal action reflects only uncertainty over β. In particular, the DM's posterior over the

[6] Note that (5) and (6) show that the DM's uncertainty over β also affects $\hat{\sigma}_\theta^2$ through the DM's information acquisition choice τ^* in the first-stage problem. This implies that correlationally, a relationship between the DM's uncertainty over β and the DM's willingness-to-pay for information may not hold. However, (9) indicates that an exogenous increase in the DM's uncertainty over β should decrease the DM's willingness-to-pay for information.

optimal action is given by $N(a_{cf}, \sigma_{cf}^2)$, where

$$a_{cf} = (1-\lambda)s_\beta\theta^{cf} \tag{10}$$

$$\sigma_{cf} = |\theta^{cf}|\frac{\sigma_\beta\sigma_\zeta}{\sqrt{\sigma_\beta^2+\sigma_\zeta^2}} = |\theta^{cf}|\sqrt{\lambda}\tilde{\sigma}_\beta \tag{11}$$

Note that the DM's posterior uncertainty σ_{cf}, which we will call the DM's counterfactual *cognitive uncertainty* (CU), is increasing in our object of interest, the DM's posterior uncertainty over the belief-action map $\tilde{\sigma}_\beta$. Under our maintained assumptions, this leads to the following predictions:

Predictions.

1. a) Individuals with higher counterfactual CU (higher σ_{cf}) exhibit greater attenuation in the cross-sectional relationship between their beliefs about the quantity and behavior (higher estimated λ).

   b) An exogenous increase in uncertainty over the belief-action map similarly results in greater cross-sectional attenuation.

2. a) Individuals with higher counterfactual CU exhibit a weaker behavioral response to information about θ (lower $E[\Delta a^*]$) for a given change in beliefs induced by the information.

   b) An exogenous increase in uncertainty over the belief-action map similarly reduces the behavioral response to information.

3. a) Individuals with higher counterfactual CU exhibit greater uncertainty in their beliefs about the quantity (higher $\hat{\sigma}_\theta^2$), report having acquired less information about the quantity (lower τ^*), and respond more in terms of beliefs to information about the quantity.

   b) An exogenous increase in uncertainty over the belief-action map leads to less demand for information about the quantity (lower WTP_ϕ).

3 Design: Main Experiment

I test the key predictions of the model in an experiment designed to relate subjects' beliefs about S&P 500 returns to their portfolio allocations in an investment task. To test these predictions, the experiment contains the following key components: i) an elicitation of subjects' beliefs at baseline over the expected returns of the S&P 500; ii) a baseline investment task in which subjects allocate money between assets tied to the performance of the S&P 500; iii) an elicitation of subjects' cognitive uncertainty over their decisions in a counterfactual investment task in which they are asked how they would invest given a hypothetical distribution of S&P 500 returns; iv) an information acquisition component in which subjects are given the opportunity to obtain an expert estimate over S&P 500 returns, after which subjects' return expectations and investment decisions are re-elicited; and v) a between-subjects treatment

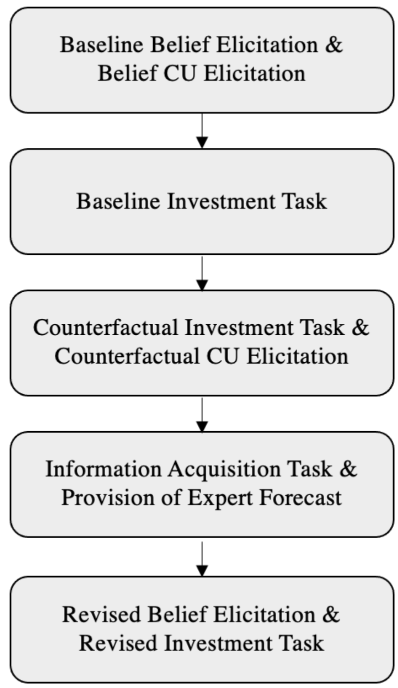

Figure 1: Sequence of experimental components.

manipulation that varies the complexity of the investment task.

Using this design, I test Prediction 1 by relating counterfactual CU to the cross-sectional relationship between return expectations and investment decisions, and by analyzing the effect of the complexity manipulation on this cross-sectional relationship. I test Prediction 2 by relating counterfactual CU to subjects' behavioral response to the expert estimate, and by studying how this behavioral response varies across treatment. I test Prediction 3 correlationally by relating counterfactual CU to subjects' confidence in their return expectations, the frequency with which they acquire information about the S&P 500 in the field, as well as the responsiveness of their beliefs to the receiving the expert estimate. Finally, I test Prediction 3 causally by studying the effect of the complexity manipulation on subjects' demand for the expert estimate.

Below, I describe the experimental design in more detail. Experimental instructions are included in Appendix A.7.

3.1 Belief Elicitations

The experiment begins by eliciting subjects' beliefs over the expected one-year return of the S&P 500. For the *baseline* expectation measure, beliefs are elicited via a two-step procedure: subjects are first asked whether they expect the S&P 500 to increase or decrease in value over the next 12 months, and are then asked to state the percentage increase/decrease they expect over that time period. Upon entering a percentage estimate, subjects are given real-time feedback on the value that a $100 investment in the S&P 500 would accrue in 12 months as implied by their estimate. This feedback is included to address the finding that survey expectations are sensitive to whether subjects are asked to forecast stock returns in percent or prices in units of currency (Glaser, Langer, Reynders and Weber, 2007; Glaser, Iliewa and Weber, 2019).

To elicit subjects' beliefs about the variability of one-year S&P 500 returns around their point estimates, subjects are asked to enter the probability with which they expect the one-year S&P 500 return to fall within each of five ranges of possible returns, so as to elicit their beliefs over the distribution of one-year returns. The ranges shown are determined by the subject's point estimate (see Appendix A.7 for details on this procedure). In addition, in order to address concerns surrounding measurement error, subjects' expectations of the one-year S&P 500 return are then re-elicited. For this *repeated* expectation measure, subjects are asked to enter the value, in dollars, that they expect a $100 investment in the S&P 500 to be worth after 12 months.

3.2 Baseline Investment Task

After baseline beliefs are elicited, subjects complete the *baseline* investment task, in which they are asked how they would allocate $1000 between an asset that generates an annual risk-free return of 2%, and an asset that tracks the return of the S&P 500.

The baseline investment task is the first of three investment tasks subjects complete in the experiment. Each investment task is incentivized according to a one-year horizon: there is a 10% chance that one of the three investment tasks the subject completes will be randomly selected for payment; in this event, the subject will receive the total value of their investment for that task in 12 months time, divided by 100. The remaining two investment tasks are described below.

3.3 Counterfactual Investment Task

In order to obtain a measure of subjects' uncertainty over the quantity action map, subjects complete a *counterfactual* investment task, in which they are asked how they would invest in the baseline investment task under a hypothetical distribution of S&P 500 returns. In particular, subjects are first shown a histogram corresponding to a hypothetical distribution of one-year S&P 500 returns, along with the expected return corresponding to this distribution. Subjects are then presented with the same investment decision as the baseline investment task, and asked how they would allocate the $1000 between the two assets if they knew that the one-year return of the S&P 500 would be drawn from the hypothetical distribution. In particular, the hypothetical distribution is a truncated normal distributions parameterized by a mean return of θ^{cf} and standard deviation of 15%, truncated at $[\theta^{cf} - 35, \theta^{cf} + 35]$.[7] The expected return of the hypothetical distribution θ^{cf} is itself randomized between subjects, taking on values $\theta^{cf} \in \{-10\%, -5\%, 5\%, 10\%, 15\%\}$. This allows me to obtain an estimate of

[7]The standard deviation of the hypothetical distributions was chosen to match the historical standard deviation of S&P 500 returns, which is approximately 15%.

the relationship between return expectations and investment behavior that is unconfounded by measurement error and endogeneity of return expectations, which, as discussed in Section 4, I use as an additional test of Prediction 1. The primary purpose of the counterfactual investment task, however, is to elicit subjects' uncertainty over the belief-action map, described in more detail below.

3.4 Measures of Cognitive Uncertainty

Uncertainty over the Belief-Action Map. As motivated by the theoretical framework, I measure subjects' uncertainty over the belief-action map by measuring their uncertainty over the optimality of their decision in the counterfactual investment task. Recall the rationale for such a procedure: my model predicts that eliciting cognitive uncertainty in a decision problem in which the quantity — that is, the expected return — is known to the subject, yields a measure of the subjects' uncertainty over the belief-action map that is unconfounded with the subjects' uncertainty over the quantity itself. Following Enke and Graeber (2023, 2022), cognitive uncertainty is elicited by asking subjects to report the subjective probability that their utility maximizing portfolio allocation is contained in a range around the allocation that they chose. In particular, in the screen after subjects make their decisions in counterfactual investment task, they are asked:

> *On the previous screen, you indicated that you would invest \$a of the \$1000 in the stock account if you knew the S&P 500 return was determined by the procedure we described. In this next question, we are interested in how certain you are in your decision.*
>
> *How certain are you that you would actually be best off investing between \$*$(a-20)$ *and \$*$(a+20)$ *in the stock account, given your own preferences and the available information?*

I interpret this question as capturing the subject's uncertainty over their utility-maximizing decision in the counterfactual investment task, corresponding to σ_{cf} in the theoretical framework. As discussed in Section 2.2, σ_{cf} is determined by the subject's uncertainty over the belief-action map — that is, the subject's uncertainty over how to arrive at the optimal portfolio allocation given the expected return of the S&P 500.

One concern is that this uncertainty measure captures additional factors beyond subjects' uncertainty over the ex-ante optimal investment, such as the subjects' level of risk aversion or subjects' beliefs about the variability of the hypothetical return distribution. To the extent these factors affect the normative decision weight — that is, the relationship between return expectations and optimal investment decisions — this may confound correlational tests of the model predictions that rely on this cognitive uncertainty measure. As discussed in detail in Section 5, I address this potential confound in a number of ways. First, I show that after including controls for risk aversion and beliefs about the variability of S&P 500 returns, the two factors that govern the normative decision weight, residual variation in the cognitive uncertainty measure continues to predict attenuation and learning in a manner consistent with the predictions of the model. Second, I demonstrate that the observed relationship between counterfactual CU and attenuation in the transmission of beliefs to behavior cannot be rationalized if the counterfactual CU solely captures risk aversion or subjective return variability. Finally, I conduct causal tests for the model predictions that do not rely on the counterfactual CU measure using the complexity manipulation discussed below.

Uncertainty over Beliefs. To test Prediction 3a, which states that counterfactual CU should predict greater uncertainty over beliefs about the quantity, I also elicit subjects' cognitive uncertainty over their baseline return expectations. Following Enke and Graeber (2023), I elicit cognitive uncertainty over return expectations by asking subjects to report the subjective probability that their forecast of S&P 500 returns falls within a range around a consensus expert estimate of the return. This measure is elicited in the screen after subjects enter their return expectations. I interpret this question as capturing the subjects' uncertainty over the quality of their forecast of expected S&P 500 returns, corresponding to $\hat{\sigma}_{\theta}^2$ in the theoretical framework.

3.5 Expert Forecast and Revised Investment Task

To test Predictions 2 and 3, the experiment contains an information acquisition module designed to shed light on both subjects' responsiveness to information about S&P 500 returns, as well as their demand for that information.

In particular, following the counterfactual investment task, subjects complete a *revised* investment task, which is identical to the baseline investment task. Prior to making their allocations in the task, however, subjects have the opportunity to obtain information about S&P 500 returns: they are given a choice between receiving a consensus expert estimate of the one-year S&P 500 return, which consists of the average over estimates made by a sample of professional forecasters, or an additional bonus payment of 20 cents. A randomly selected 5% of subjects' choices are implemented, and the remaining subjects receive the information. To generate exogenous variation in the information, the estimate that subjects obtain is randomized to take on one of two values.[8] This procedure allows me to characterize subjects' responses to a randomized information intervention, as a subset of subjects receive the forecast regardless of their information acquisition decisions, while also eliciting an incentivized measure of subjects' demand for the information. Following this procedure, subjects' revised return expectations are elicited via the same process used for the baseline expectation measure, and subjects complete the revised investment task.

[8] The consensus expert estimates are formed by averaging S&P 500 forecasts obtained from a Reuter's poll conducted in August 23, 2022. To construct 12-month return estimates for each forecaster surveyed in the poll, I linearly interpolated the return implied by each forecaster's mid-year 2023 forecast and end-of-year 2023 forecast. The two consensus estimates were taken by averaging the 10 forecasters with the highest return forecasts and averaging the 10 forecasters with the lowest return forecasts.

3.6 Complexity Manipulation

To exogenously manipulate uncertainty over the belief-action map, I employ a between-subjects treatment aimed at increasing the complexity of the investment task while keeping the incentives of the task identical. In particular, while in the *standard* treatment, the two assets are described as a bank account that generates a risk-free return and a stock account tied to the value of the S&P 500, in the *complex treatment*, the two assets are instead represented as portfolios comprised of leveraged and inverse S&P 500 exchange-traded funds and interest accounts, constructed in such a way as to replicate the returns of the assets in the standard treatment. In particular, subjects in the complex treatment allocate money between Account A and Account B, where the accounts are described as follows:[9]

Account A

Portfolio Wt.	Fund Description
15%	-2x daily returns of S&P 500
35%	3x daily returns of S&P 500
25%	1x daily returns of S&P 500
25%	2x daily returns of S&P 500

Account B

Portfolio Wt.	Fund Description
25%	1x daily returns of S&P 500
15%	-3x daily returns of S&P 500
10%	2x daily returns of S&P 500
50%	4% annual return

This treatment manipulation keeps the incentives of the task unchanged, but increases the complexity of the mapping between the quantity of interest – the expected return of the S&P 500 – and the subjects' optimal portfolio allocation by obscuring the relationship between S&P 500 returns and the payoffs of the accounts. As such, this complexity manipulation is analogous to a treatment manipulation deployed in Enke and Graeber (2023), in which the authors manipulate the cognitive noise associated with a lottery choice task by re-describing lottery payouts as algebraic expressions.

3.7 Additional Survey Questions

After the two main components of the experiment, subjects are asked a set of unincentivized survey questions. In addition to standard demographic questions, subjects are asked about their financial decision-making. In particular, to measure the extent to which subjects have gathered information about the S&P 500 prior to the experiment (corresponding to τ^* in the theoretical framework), subjects are asked to report the frequency with which they gather information about the performance of the S&P 500. Subjects also report whether they

[9]To ensure the tasks in the complex treatment are as close to economically identical as possible to those in the standard treatment, subjects in the complex treatment are (truthfully) informed that in the task, the returns of funds in the portfolios will not be subject to management fees, and that the portfolios are re-balanced daily.

participate in the stock market, and if so, the share of their total wealth they invest in stocks. Finally, subjects complete the "Big Three" financial literacy questionnaire, a set of three questions designed to measure familiarity with basic personal finance concepts (Lusardi and Mitchell, 2011).

3.8 Logistics and Sample

The experiment was conducted on Prolific, an online worker platform. As pre-registered, a sample size of $N = 1200$ subjects were recruited from the population of U.S. Prolific workers with at least 500 completes and with a Prolific approval rating of at least 98%. 777 subjects were randomized into the standard treatment, and 423 subjects were randomized into the complex treatment.

Participants completed a comprehension check quiz consisting of four questions. Subjects were given two attempts to answer the comprehension check, and any participant who failed to answer all comprehension check questions correctly by the second attempt were excluded from the study (9% of subjects). I additionally implemented two attention checks throughout the course of the study, and exclude all participants who failed both attention checks ($< 1\%$ of subjects). As pre-registered, subjects who fail either the comprehension check or the attention checks are not counted towards the sample size of $N = 1200$. Subjects earned $2 in base payment for completion of the study, and as described above, had the opportunity to earn additional bonus payment.

The experiment was pre-registered at aspredicted.org. Predictions 1–3, as well as details involving sample restrictions and the treatment of outliers, are specified in the pre-registration.

4 Results: Main Experiment

Following a discussion of sample descriptives, this section is organized according to the three main sets of predictions of the model: first, that uncertainty over the belief-action map attenuates the cross-sectional relationship between beliefs about the corresponding quantity and behavior (Prediction 1); second, that this uncertainty weakens the behavioral response to information (Prediction 2); and third, that uncertainty over the belief-action map reduces the DM's incentives to learn about the quantity (Prediction 3).

As pre-registered, these analyses exclude subjects with baseline or revised return expectations that are greater than 30% or less than -30%, in order to limit influence of potential outliers. This excludes 33 subjects ($< 3\%$ of the total sample), leaving a main sample of 1,167 subjects. Appendix A.6 demonstrates that the results are robust to less stringent sample restrictions.

4.1 Descriptive Statistics

Demographics and Baseline Return Expectations. Appendix Table 6 reports sample demographics. Notably, the sample appears to be more financially sophisticated relative to the general U.S. population; 67% of subjects report participating in the stock market relative to estimated national average of 58% (Saad, Lydia and Jones, Jeffrey M., 2022), and the sample average score on the "Big Three" financial literacy questions is 2.66, significantly higher than the average of 1.79 found in a representative U.S. sample (Lusardi and Mitchell, 2011).

Appendix Table 7 reports baseline beliefs over S&P 500 returns. The average expected one-year S&P 500 return is 7.11%, with substantial heterogeneity across subjects: at the 10th percentile of the distribution, subjects reported an expected return of -10%, and at the 90th percentile, they reported an expected return of 20%. The average standard deviation of one-year S&P 500 returns implied by subjects' subjective return distributions[10] is 16.41%, which is close to the historical standard deviation of one-year S&P 500 returns of approximately 15%.

Cognitive Uncertainty. Appendix Figure 6 shows histograms of the CU measures collected in the experiment. Panel a) of the figure plots the distribution over the main CU measure of interest: subjects' CU over the counterfactual investment decision in the standard treatment. 90% of subjects report strictly positive CU for this measure, suggesting that a large share of subjects are indeed uncertain over how to incorporate knowledge of expected returns into their investment decisions. Appendix Table 8 reports correlates of CU over the counterfactual investment decision. The most consistent correlation is that males report lower cognitive uncertainty, consistent with existing work eliciting cognitive uncertainty (Enke and Graeber, 2023, 2022) as well as a large body of evidence studying how other measures of confidence vary by gender. In addition, CU is positively correlated with both risk aversion and beliefs about the variability of S&P 500 returns.

4.2 Attenuation in the Cross-Section

In this section, I discuss evidence for Prediction 1, which states that greater uncertainty over the belief-action map attenuates the transmission from beliefs to behavior.

4.2.1 Prediction 1a: Correlational Evidence on Cross-Sectional Attenuation

Focusing on subjects in the Standard treatment, I first provide evidence for Prediction 1a, which states that higher uncertainty over the belief-action map, as measured by counterfactual CU — is correlated with greater attenuation in the cross-sectional relationship between

[10] To construct the implied standard deviation from the distribution question, I first split each bucket into ranges of 5 percentage points. For each of these ranges, I compute the probability that a $N(\hat{\theta}, 15^2)$ distribution assigns to that range, where the standard deviation of this distribution was chosen to match that of historical S&P 500 returns, which is approximately 15%. I then weight these probabilities by the subjective probability of each bucket reported by the respondent. I finally calculate the standard deviation based on the mid-points of the narrower ranges, and their associated subjective probabilities.

their beliefs and behavior.

Panel (a) of Figure 2 plots a binscatter of the relationship between subjects' baseline portfolio allocations and their baseline return expectations, for the subsamples of subjects with above- and below-median counterfactual CU. We see that, as predicted, the portfolio allocations of subjects with above-median CU are less sensitive to baseline return expectations, relative to the below-median CU subsample.

Table 1 presents corresponding regression analyses. Column 1 regresses baseline portfolio allocations against baseline return expectations. These estimates indicate that a one percentage point increase in the expected one-year S&P 500 return is associated with a 0.76 percentage point increase in the portfolio share allocated to the stock account. Consistent with existing evidence on the "attenuation puzzle" (Giglio et al., 2021), this estimated slope is an order of magnitude smaller than what standard calibrations of benchmark models suggest.[11]

Column 2 investigates the extent to which this attenuation is driven by uncertainty over the belief-action map, regressing baseline portfolio allocations against baseline return expectations, an indicator for above-median counterfactual CU, and their interaction[12]. According to Prediction 1, the coefficient on this interaction term should be negative: the estimated coefficient on return expectations should be attenuated for high-CU subjects. Consistent with this prediction, I find that the coefficient on the interaction term is negative and large in magnitude: the estimated coefficient on return expectations for below-median CU subjects (1.15) is more than twice that of high-CU subjects (0.41). Column 3 shows that this result is robust to including controls for demographic variables, risk aversion, and beliefs about return volatility. Because standard asset pricing models predict both beliefs about return volatility and risk aversion are associated with attenuation, the specification in column 3 also controls for the interactions of these variables with baseline beliefs.

Because classical measurement error in subject's return expectations can also drive attenuation in the relationship between beliefs and portfolio allocations, I replicate the analyses in columns 1–3 using the repeated elicitation of return expectations collected in the experiment to reduce the bias from measurement error through an instrumental variables approach. In particular, I follow the *Obviously Related Instrumental Variables* (ORIV) approach proposed in Gillen et al. (2019), which includes both sets of belief elicitations as explanatory variables and as instruments; under the assumption that measurement error is uncorrelated between the two belief elicitations, ORIV eliminates the bias from classical measurement error.[13]

[11]The frictionless Merton (1969) model predicts that $\textit{S\&P 500 Share} = \frac{1}{\gamma}\frac{E(r)-r_f}{Var(r)}$ where $E[r]$ and $Var(r)$ is the expectation and variance of S&P 500 returns, respectively, r_f is the risk-free rate, and γ is the coefficient of relative risk aversion. Taking $\gamma = 8$ (within the upper end of experimental estimates of γ) and $Var(r) = 0.15^2$ (to match historical volatility), the predicted slope would be $\frac{1}{\gamma Var(r)} = 5.5$.

[12]Throughout this paper, I use median splits for CU variables for ease of interpretation. As Appendix A.5 demonstrates, all results continue to hold when using continuous CU measures.

[13]In particular, ORIV estimates a "stacked" IV model by appending to the original dataset one in which the baseline and repeated belief elicitations are swapped, and in this "stacked" dataset, using one belief

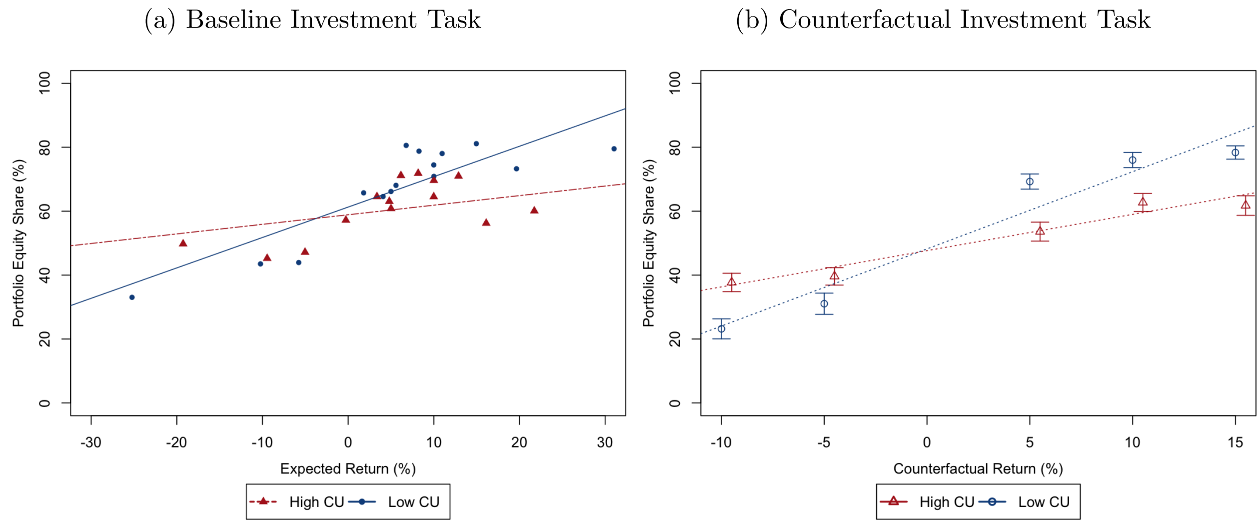

Figure 2: Panel (a) plots a binscatter of investment decisions in the baseline investment task against baseline return expectations, for subjects in the Standard treatment with above-median vs. below-median counterfactual CU. Panel (b) plots the average investment decisions in the counterfactual investment task against the counterfactual expected return, for subjects in the Standard treatment with above-median vs. below-median counterfactual CU. Whiskers show standard error bars.

Columns 4–6 report these estimates; we see that the ORIV estimates are quantitatively similar to the OLS estimates, and that in particular, the coefficient on the interaction term between CU and baseline beliefs remains negative and statistically significant.

While the above analysis suggests that as predicted, counterfactual CU is correlated with a weaker transmission from beliefs to behavior, the cross-sectional data are subject to a number of potential confounds. First, I cannot completely rule out the influence of measurement error in belief reports using ORIV if measurement error is correlated across repeat elicitations. Second, since return beliefs are not randomly assigned, I cannot rule out standard endogeneity concerns, which may bias the estimates reported above.

To better address these concerns, I exploit the design of the counterfactual investment tasks. Recall that in these tasks, the return of the stock account is not tied to the actual S&P 500 return but rather drawn from a distribution known to the subject, the expecation of which—which I will refer to as the *counterfactual expected return*—is itself randomized. As such, subjects' return expectations for this task are fixed (i.e. measured without error) and exogenously determined, addressing both of the confounds described above.

Panel (b) of Figure 2 plots the average portfolio share in this task against the counterfactual expected return, once again for the subsamples of subjects with above- and below-median counterfactual CU. As in Panel (a) of the figure, the relationship between expected returns and portfolio shares is attenuated for high-CU subjects. Appendix Table 9 presents

elicitation as an instrument for the other.

Table 1: Counterfactual CU vs. Attenuation in Baseline Investment Task

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.76***	1.15***	1.27***	0.86***	1.22***	1.32***
	(0.09)	(0.12)	(0.12)	(0.10)	(0.14)	(0.13)
High Cfact. CU		−1.82	0.32		−2.36	−0.07
		(2.01)	(1.99)		(2.07)	(2.11)
Return Beliefs × High Cfact. CU		−0.74***	−0.71***		−0.68***	−0.64***
		(0.17)	(0.17)		(0.19)	(0.18)
(Intercept)	59.70***	60.54***	43.11***	59.23***	60.30***	40.93***
	(1.03)	(1.35)	(4.36)	(1.05)	(1.42)	(4.89)
Controls	N	N	Y	N	N	Y
R^2	0.11	0.15	0.22	0.12	0.15	0.23
Num. obs.	755	755	755	1410	1410	1410

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

corresponding regression analyses.

Disentangling CU from Normative Drivers of Attenuation. One potential concern with the preceding results on attenuation is that the measure of cognitive uncertainty simply captures factors that normatively predict attenuation in the relationship between return expectations and portfolio allocation, which according to the Merton (1969) model, are the DM's risk aversion and beliefs about the volatility of stock returns. In particular, the Merton model predicts

$$\textit{S\&P 500 Share} = \frac{1}{\gamma} \cdot \frac{E(r) - r_f}{Var(r)}$$

where $E[r]$ and $Var(r)$ are the expectation and subjective variance of S&P 500 returns, respectively, r_f is the risk-free rate, and γ is the coefficient of relative risk aversion. Below, I discuss three approaches to disentangle cognitive uncertainty from γ and $Var(r)$, the normative drivers of attenuation.

First, note that cognitive uncertainty continues to predict attenuation both in cross-section and in response to information when controlling for γ and $Var(r)$, as discussed above. This suggests that cognitive uncertainty is not merely a proxy for either risk aversion or subjective return variance: holding either of these two factors constant, residual variation in cognitive uncertainty still strongly predicts attenuation between beliefs and behavior.

Second, note that the Merton model predicts that, conditional on return expectations, the share allocated to the risky asset must always be decreasing in risk aversion, and assuming agents are risk-averse, must always be decreasing in subjective return variance. In contrast, as both panels of Figure 2 suggest, when return expectations are low, subjects with high CU appear to allocate a *greater* portfolio share to the risky asset on average, whereas when return expectations are high, portfolio shares appear to be increasing in CU. Note that this "switching point" pattern cannot arise if cognitive uncertainty is a simple proxy for risk aversion or subjective return variance.[14]

4.2.2 Prediction 1b: Causal Evidence on Cross-Sectional Attenuation

I now discuss evidence for Prediction 1b, which states that an exogenous increase in uncertainty over the belief-action map results in greater attenuation in the relationship between their beliefs about the corresponding quantity and behavior. To test this prediction, I utilize the complexity manipulation discussed in Section 3. Appendix Figure 7 shows that this complexity manipulation did indeed increase CU over the belief-action map, as measured by counterfactual CU: subjects in the Complex treatment exhibit 32% higher counterfactual CU compared to subjects in the Standard treatment.

Figure 3 documents that as predicted, the complexity manipulation causes attenuation in the cross-section. Panel (a) reports a binscatter of portfolio equity shares against return expectations in the baseline task, whereas panel (b) plots the average portfolio equity share against expected returns in the counterfactual task. Both panels demonstrate that the complexity treatment flattens the transmission of beleifs to behavior. Appendix Tables 10 and 11 provide corroborating regression analyses.

4.3 Behavioral Responses to Information

This section discusses tests of Prediction 2, which states that greater uncertainty over the belief-action map reduces the responsiveness of behavior to information about the corresponding quantity, holding fixed the change in beliefs induced by the information.

4.3.1 Prediction 2a: Correlational Evidence on Responsiveness to Information

Recall that in the experiment, subjects have the opportunity to acquire information after completing the baseline investment task, and that revised beliefs and portfolio allocations are elicited afterwards. Because acquisition of the estimate is endogenous for subjects whose information acquisition choices were implemented, I restrict the analysis here to subjects whose information acquisition choices were not implemented; recall that all such subjects in this subsample received the estimate, which is randomized to either take on a higher or

[14] Appendix Table 13 investigates the switching point pattern using regression analyses, demonstrating that for subjects with expected returns less than or equal to the risk-free-rate, portfolio equity shares are significantly *higher* for high-CU subjects conditional on expected returns, whereas the opposite is true for subjects with expected returns higher than the risk-free-rate.

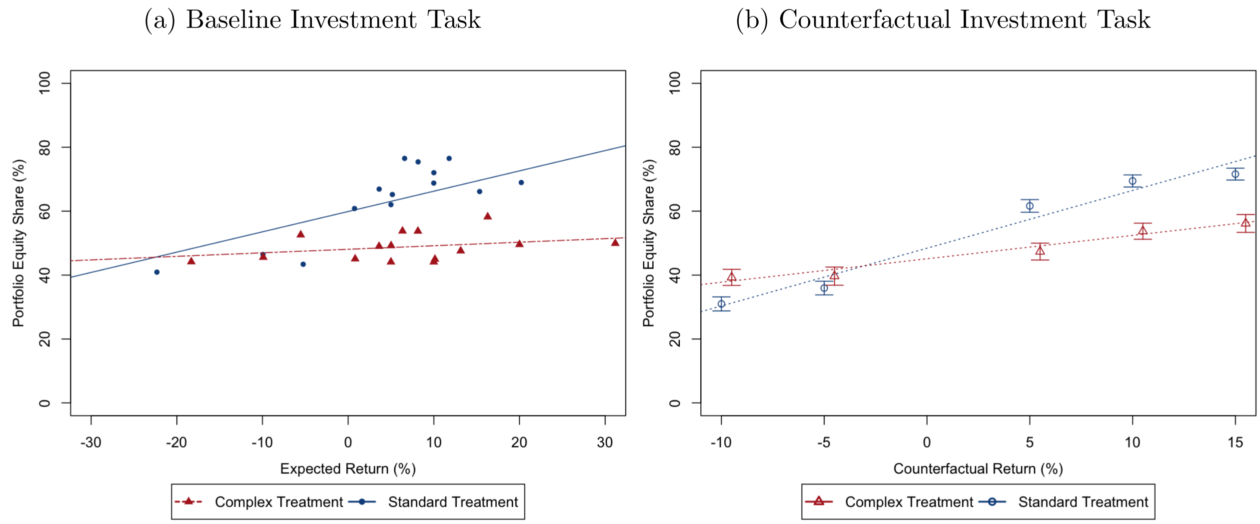

Figure 3: Panel (a) plots a binscatter of investment decisions in the baseline investment task against baseline return expectations, split by treatment. Panel (b) plots the average investment decisions in the counterfactual investment task against the counterfactual expected return, split by treatment. Whiskers show standard error bars.

lower value.

Panel (a) of Figure 4 plots the average change in portfolio equity shares as a function of whether the subject received a high vs. low estimate, for the subsamples of subjects with above-median and below-median counterfactual CU. Subjects who received the high estimate revise their portfolio equity shares upwards on average, but this revision is smaller in magnitude for high-CU subjects. Similarly, subjects who received the low estimate revise their equity shares downwards on average, with a less pronounced revision for high-CU subjects. In other words, the information has a muted impact on the portfolio allocations of subjects with high CU.

I now turn to regression evidence. The primary specification of interest is given by

$$\Delta Portfolio\ Share_i = \eta_0 + \eta_1 \Delta Exp.\ Returns_i + \eta_2 CU_i + \delta \Delta Exp.\ Returns_i \times CU_i + \epsilon_i \quad (12)$$

where $\Delta Portfolio\ Share_i$ is the change in subjects' portfolio equity shares across the baseline and revised investment tasks, $\Delta Exp.\ Returns_i$ is the change in subjects' return expectations across the baseline and revised belief elicitations, and CU_i is the subjects' counterfactual CU. Prediction 1 states that δ, the coefficient on the interaction term, should be negative—that is, a given revision in expected returns produced by information leads to a smaller effect on portfolio allocations for subjects with high CU.

I estimate this model using an instrumental variables approach, using an indicator for whether the subject received the high estimate, as well as its interaction with CU_i, as in-

(a) Behavioral Response to Information by CU (b) Behavioral Response to Information by Treatment

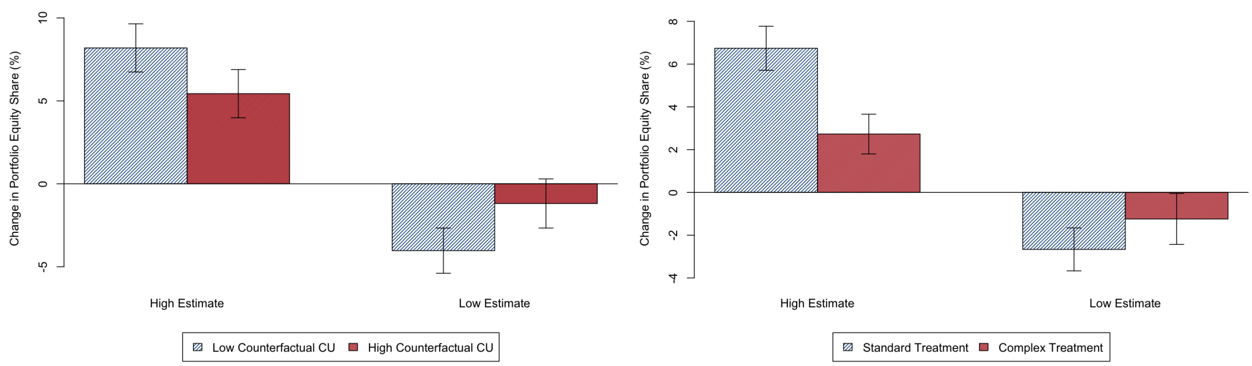

Figure 4: Panel (a) plots the average change in portfolio equity shares for subjects in the Standard treatment in response to receiving the expert estimate, split by median counterfactual CU. Panel (b) plots the average change in portfolio equity shares for subjects in response to receiving the expert estimate, split by treatment. Whiskers show standard error bars.

struments for ($\Delta Exp.\ Returns_i$, $\Delta Exp.\ Returns_i \times CU_i$).[15] Column 2 of Table 2 reports the corresponding estimates. Consistent with Prediction 1, the coefficient on the interaction term is negative: a one percentage point increase in the revision of return expectations is associated with a 2.21 percentage point increase in the revision of portfolio equity shares for low-CU subjects, and only a 0.81 percentage point increase in the revision of portfolio equity shares for high-CU subjects. Column 3 demonstrates that this result is robust to adding to controls for baseline beliefs, demographics, risk aversion, and beliefs about return volatility.

4.3.2 Prediction 2b: Causal Evidence on Responsiveness to Information

I now demonstrate that an exogenous increase in uncertainty over the belief-action map mutes the behavioral response to information. As with the correlational analysis, I restrict this analysis to subjects whose information acquisition decisions are not selected to be implemented. Panel (b) of Figure 4 documents that as predicted, the complexity manipulation results in attenuation in response to information: the information has a substantially weaker effect in the portfolio allocation of subjects in the Complex treatment relative to the Standard treatment. Columns 5 and 6 of Table 2 presents the corresponding regression analysis, in which I estimate the specification

$$\Delta Portfolio\ Share_i = \eta_1 + \eta_2 \Delta Exp.\ Returns_i + \eta_3 Complex_i + \delta \Delta Exp.\ Returns_i \times Complex_i + \epsilon_i \tag{13}$$

where $Complex_i$ is a treatment dummy. Analogous to the correlational analysis, I use an indicator for whether the subject received the high estimate as well as its interaction with

[15] As a test of Prediction 1, one could also regress the revised portfolio share against revised return expectations, CU, and their interaction, using the same set of instruments for revised return expectations. Appendix Table 12 reports estimates for this model; results are consistent with Prediction 1.

Table 2: Behavioral Response to Information

	Dependent Variable: Change in Equity Share					
	(1)	(2)	(3)	(4)	(5)	(6)
ΔBeliefs	1.37^{***}	2.21^{***}	2.02^{***}	1.06^{***}	1.37^{***}	1.29^{***}
	(0.21)	(0.37)	(0.34)	(0.15)	(0.21)	(0.20)
High Cfact. CU		1.17	0.63			
		(1.59)	(1.55)			
ΔBeliefs × High Cfact. CU		-1.40^{**}	-1.22^{**}			
		(0.45)	(0.43)			
Complex					0.44	0.33
					(1.18)	(1.23)
ΔBeliefs × Complex					-0.85^{**}	-0.77^{*}
					(0.30)	(0.32)
(Intercept)	−0.14	−0.51	−2.14	0.11	−0.14	−3.03
	(0.80)	(1.09)	(4.20)	(0.60)	(0.80)	(3.17)
Baseline Belief Controls	N	N	Y	N	N	Y
Demographic Controls	N	N	Y	N	N	Y
Num. obs.	723	723	723	1113	1113	1113

IV estimates instrumenting for the change in beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

$Complex_i$, as instruments for $(\Delta Exp.\ Returns_i,\ \Delta Exp.\ Returns_i \times Complex_i)$. As predicted, the coefficient on the interaction term δ is negative.

4.4 Evidence on Learning and Information Acquisition

4.4.1 Prediction 3a: Correlational Evidence on Learning

Focusing on subjects in the Standard treatment, I now provide evidence for Prediction 3a, which states that individuals with higher uncertainty over the belief-action map, as measured by counterfactual CU, exhibit greater uncertainty in their beliefs.

Columns 1 and 2 of Table 3 regresses the cognitive uncertainty associated with subjects' baseline return expectations, which we will refer to as *belief CU*, against counterfactual CU. Consistent with Prediction 3, there is a strong positive relationship between counterfactual CU and belief CU, with a Pearson correlation coefficient of 0.38 ($p < 0.001$).[16] Recall the

[16] Recall that the theoretical framework draws a distinction between belief CU and subjects' beliefs about return variability, and in particular predicts that counterfactual CU is positively correlated with the former

mechanism underlying the prediction above: subjects who are uncertain over the belief-action map have less motives to acquire information about the quantity in question. As such, we should expect subjects with higher counterfactual CU to report having acquired less information about S&P 500 returns. Columns 3 and 4 of Table 3 provide evidence for this mechanism: Column 1 regresses the reported frequency with which subjects gather information about the S&P 500[17] against counterfactual CU, and finds that subjects with higher CU gather information at a lower frequency; column 2 shows that the relationship persists after including demographic controls.

Recall that while the theoretical framework predicts that counterfactual CU is negatively correlated with subjects' information acquisition prior to the experiment (i.e. the subject's first-stage problem), the unconditional relationship between counterfactual CU and our experimental measure of information demand is theoretically ambiguous. As Section 2 discusses, while greater uncertainty over the belief-action map reduces the value of new information, the fact that high-CU subjects acquired less information prior to the experiment and therefore hold greater uncertainty over the quantity increases their demand for new information.[18] The framework does predict, however, that controlling for uncertainty over the quantity, higher counterfactual CU should be correlated with lower demand for information. Columns 5–7 of Table 3 report this analysis. Column 5 indicates that while counterfactual CU is associated with lower demand for information, the effect size is not statistically significant at conventional levels. As columns 6–7 indicate, however, after controlling for belief CU, the effect size doubles in magnitude and is statistically significant ($p < 0.05$): subjects who report complete uncertainty are 18 percentage points less likely to acquire the information that subjects who report no uncertainty.

Taken together, these results suggest that, in line with Prediction 3a, subjects who are uncertain over how to incorporate return expectations into their investment decisions have less incentives to form well-calibrated return expectations: they are both less certain over their return expectations and less likely to acquire information about expected returns, both in the field and in the lab. Indeed, as Appendix Table 15, demonstrates, counterfactual CU predicts two other signatures of poorly calibrated beliefs: high CU subjects exhibit lower across-elicitation consistency in their return expectations, as measured by whether subjects' baseline return expectations are consistent with their responses to the repeated elicitation, and also exhibit higher rates of rounding in their return expectations. This suggests that the miscalibrated beliefs about core economic quantities that are often documented in the survey literature may not be driven solely by information frictions, but also by the fact that respondents may be uncertain over how to incorporate such economic quantities into their

uncertainty measure but makes no predictions regarding correlation with the latter. It is nevertheless the case that counterfactual CU is correlated with subjective return variance, with a Pearson correlation coefficient of 0.13 ($p < 0.001$).

[17] In particular, subjects are asked "How frequently did you gather information about the performance of the S&P 500 over the stock market in the last three months, where answers are coded as 0—"Not at all", 1—"Once a month", 2—"Twice a month", 3—"Weekly", 4—"Several times a week", 5—"Daily".

[18] Appendix Table 14 shows that subjects with higher belief CU and subjects who report gathering less information about the quantity indeed exhibit higher demand for information in the experiment.

Table 3: Counterfactual CU vs. Information Acquisition

	Dep. Variable: Belief CU		*Dep. Variable*: Freq. Acquired Stock Info		*Dep. Variable*: Acquired Expert Estimate		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cfact. CU	0.36^{***}	0.34^{***}	-1.27^{***}	-0.85^{***}	-0.08	-0.18^{*}	-0.20^{**}
	(0.03)	(0.03)	(0.22)	(0.20)	(0.07)	(0.07)	(0.07)
Belief CU						0.27^{***}	0.26^{**}
						(0.08)	(0.08)
(Intercept)	0.44^{***}	0.62^{***}	1.88^{***}	-0.86^{***}	0.54^{***}	0.42^{***}	0.57^{***}
	(0.02)	(0.04)	(0.11)	(0.21)	(0.03)	(0.05)	(0.11)
Controls	N	Y	N	Y	N	N	Y
R^2	0.15	0.20	0.04	0.30	0.00	0.02	0.03
Num. obs.	755	755	755	755	755	755	755

OLS estimates, standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

decision-making, and therefore have little reason to invest in well-informed beliefs.

Finally, the theoretical framework predicts a key behavioral consequence of the correlation between counterfactual CU and belief CU: the beliefs of individuals with higher counterfactual CU should be more responsive to information about S&P 500 returns. I measure responsiveness to information using the implied information weight, computed as

$$\textit{Info Wt.} = \frac{\textit{Revised Exp. Return} - \textit{Baseline Exp. Return}}{\textit{Return Estimate} - \textit{Baseline Exp. Return}} \times 100$$

which captures the extent to which beliefs move toward the return estimate. Table 4 reports the relationship between this measure and counterfactual CU. As pre-registered, we restrict this analysis to subjects with *Info Wt.* $\in [-5, 105]$ and whose information acquisition decisions were not implemented. Column 1 reports that as predicted, higher counterfactual CU is correlated with a higher implied information weight; the beliefs of subjects who report complete uncertainty are 77% more responsive to information compared to subjects who report no uncertainty. Columns 2 and 3 demonstrate that this effect is robust to controlling for baseline beliefs and demographics.

Taken together, this result and the results on attenuation documented in Section 4.1 demonstrates that as predicted, the beliefs of high-CU subjects are more responsive to information, and yet the behavior of high-CU subjects is *less* responsive to information, for a given change in beliefs. The return expectations are high-CU subjects are more responsive to information precisely because those subjects are uncertain over how to incorporate those beliefs in their investment decisions, and so have less incentives to form well-informed return expectations in the first place. This highlights a key distinction in the policy implications

Table 4: Implied Information Weight vs. CU Measures

	Dependent Variable: Implied Information Weight					
	(1)	(2)	(3)	(4)	(5)	(6)
Cfact. CU	0.35^{***}	0.35^{***}	0.28^{***}			
	(0.06)	(0.06)	(0.06)			
Belief CU				0.47^{***}	0.48^{***}	0.36^{***}
				(0.06)	(0.06)	(0.06)
(Intercept)	45.12^{***}	42.79^{***}	91.97^{***}	31.94^{***}	28.83^{***}	75.41^{***}
	(2.93)	(3.07)	(8.30)	(3.87)	(3.91)	(9.04)
Baseline Belief Controls	N	Y	Y	N	Y	Y
Demographic Controls	N	N	Y	N	N	Y
R^2	0.05	0.06	0.14	0.08	0.09	0.16
Num. obs.	638	638	638	638	638	638

OLS estimates, standard errors (in parentheses) are robust. Demographic controls include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

of miscalibrated beliefs generated by the account studied in this paper, relative to an account based solely on information frictions or rational inattention. Whereas the rational inattention account suggests that correcting miscalbrated beliefs will bring behavior in line to the rational benchmark, the results in this paper highlight that even if such information interventions are effective in correcting beliefs, they may still fail to correct behavior to the extent miscalibrated beliefs are the result of uncertainty over the belief-action map.

4.4.2 Prediction 3b: Causal Evidence on Learning

I now provide evidence that an exogenous increase in uncertainty over the belief-action map reduces demand for information about the corresponding quantity. In particular, I study how subjects' demand for information varies across the Standard and Complex treatments. Table 5 shows that, as predicted, subjects in the Complex treatment exhibit a lower demand for information: compared to subjects in the Standard treatment, they are 21.5% less likely to choose to obtain the expert estimate over the additional bonus payment. In line with the correlational evidence, subjects with greater uncertainty over the belief-action map appear to have less incentives to acquire information about the quantity.

5 Extension: Intervening on the Mapping

The results from the main experiment indicate that uncertainty over the belief-action map attenuates the relationship between beliefs and actions, and in particular weakens the behavioral response to information. In light of the increasing interest in deploying information interventions as a policy tool to effect change in behavior (see Appendix A.1 for related liter-

Table 5: Information Acquisition vs. Treatment

	Dependent Variable: Acquired Expert Estimate		
	(1)	(2)	(3)
Complex	-0.11^{***}	-0.11^{***}	-0.11^{***}
	(0.03)	(0.03)	(0.03)
Belief CU		0.16^{**}	0.16^{**}
		(0.06)	(0.06)
Freq. Acquired Stock Info		-0.03^{***}	-0.04^{***}
		(0.01)	(0.01)
(Intercept)	0.51^{***}	0.47^{***}	0.35^{***}
	(0.02)	(0.04)	(0.09)
Controls	N	N	Y
R^2	0.01	0.03	0.04
Num. obs.	1167	1167	1167

OLS estimates, standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

ature), a key implication of the friction studied in this paper is that interventions that merely correct individuals' beliefs are unlikely to be successful, to the extent individuals are uncertain over the belief-action map. For instance, central bank policies aimed at changing savings behavior by shifting inflation expectations may fail to have the desired effect if individuals are uncertain over how to incorporate their inflation expectations into their decision-making.

At the same time, the existence of this friction points to the potential usefulness of an additional policy lever: interventions aimed at reducing individuals' uncertainty over the belief-action map. In particular, my model predicts a complementarity between the two types of policy tools – that information interventions should be more effective when combined with interventions aimed at improving individuals' understanding of the belief-action map. To provide a proof of concept of this complementarity, I run an extension to the main experiment in which I deploy such an intervention and test whether it strengthens the behavioral response to information.

5.1 Extension: Design and Predictions

Subjects are assigned either to a *Control* or *Intervention* treatment, both of which follow the design of the main study. In the Control treatment, I first elicit subjects' baseline beliefs over the expected returns of S&P 500 returns, as well as their decisions in a baseline investment task identical to that in the main study.[19] Subjects are then provided with a

[19]That is, subjects choose how to invest an endowment between a stock account delivering the return of the S&P 500 and a bank account delivering a 2% return, and face a 1-year investment horizon.

consensus expert estimate of S&P 500 returns, randomized to take on either a high or a low value, as in the main study.[20] After receiving the expert estimate, subjects' revised return expectations are elicited, and subjects are again asked for their investment decisions in a revised investment task identical to the baseline investment task. One of the two investment tasks are randomly selected to be incentivized, and subjects are paid according to the same procedure employed in the baseline treatment.

The Intervention treatment is identical in structure and incentives to the Control treatment, except subjects additionally receive an intervention aimed at reducing their uncertainty over the belief-action map. In particular, prior to making their baseline investment decisions subjects in the intervention treatment are first asked to make a series of hypothetical investment decisions. In each of these decisions, subjects indicate how they would choose in the investment task if they held different expectations of S&P 500 returns; subjects are queried for a range of expected returns in $\{0\%, 5\%, 10\%, 15\%, 20\%\}$.[21] Subjects then have access to their hypothetical investment decisions when they complete both the baseline and revised investment task.

This treatment is motivated by work on joint vs. separate evaluation in the judgement and decision-making literature (see Hsee et al. (1999) for a review), which documents in a range of settings that when individuals consider a set of decisions jointly as opposed to in isolation, their behavior is more responsive to decision-relevant parameters that vary across the decisions. In particular, by encouraging subjects to jointly consider how they would invest for a range of return expectations in addition to their actual beliefs, the intervention treatment forces subjects to explicitly think through and construct a mapping between their return expectations and their portfolio allocations in the investment task. Note also that while the proposed intervention is low-touch and can be deployed in a range of decision-making contexts, we should *not* expect such an intervention to entirely eliminate frictions related to uncertainty over the belief-action map, either in the present context or in general. As such, this extension should be viewed as a proof-of-concept test of whether a relatively weak intervention can strengthen the behavioral response to information by reducing subjects' uncertainty over the belief-action map.

The main predictions of interest are as follows: subjects in the Intervention treatment should exhibit 1) a stronger behavioral response in their investment decisions to the expert estimate, and 2) a stronger relationship between their baseline return expectations and portfolio allocation decisions. These predictions, as well as details involving sample restrictions and the treatment of outliers, are specified in a pre-registration accessible accessed at aspredicted.org.

[20] Analogously to the main study, the consensus expert estimates are formed by averaging S&P 500 forecasts obtained from a Reuter's Poll conducted in November 21, 2023. To construct 12-month return estimates for each forecaster surveyed in the poll, I linearly interpolated the return implied by each forecaster's mid-year 2024 forecast and end-of-year 2024 forecast. Each of the two consensus estimates were taken by averaging 9 forecasters in this poll.

[21] To help subjects think through the uncertainty inherent in S&P 500 returns, the hypothetical scenarios include a range of likely returns given by the expected return $\pm 15\%$.

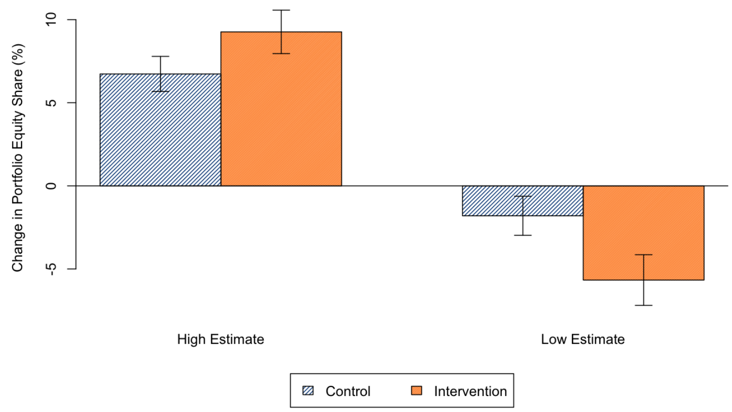

Figure 5: Average change in portfolio equity shares for subjects in response to receiving the expert estimate, split by treatment. Whiskers show standard error bars.

5.2 Extension: Results

The experiment was conducted on Prolific. $N = 800$ subjects were recruited from the population of U.S. Prolific workers with at least 500 completes and with a Prolific approval rating of at least 98%. 402 and 398 subjects were randomized into the control and intervention treatments, respectively. As in the main study, subjects who did not pass a comprehension and attention checks were excluded from the study. Subjects earned $2 in base payment for completion of the study, and had the opportunity to earn additional bonus payment. In the results below, I employ the same sample restrictions used in the main study.

Appendix Figure 8 shows that the Intervention treatment was successful in reducing subject's CU over their portfolio decisions in both their baseline and revised investment decisions. Having verified that our manipulation reduced subjects' subjective uncertainty over the belief action map, we test our main predictions of interest.

Behavioral Response to Information. Figure 5 plots the average change in portfolio equity shares as a function of whether subjects received a high vs. low estimate. As predicted, information is more effective in shifting behavior in the Intervention treatment relative to Control. Moreover, this effect is quantitatively large; indicated by regression analyses in Appendix Table 16, subjects in the Intervention treatment exhibit a behavioral response to information nearly twice as large as that of subjects in the Control treatment.

Baseline Attenuation. As predicted by the model, the Intervention treatment also reduces attenuation in subjects' belief-action relationship at baseline. As Appendix Table 17 documents, the cross-sectional relationship between return beliefs and portfolio equity shares is over 60% stronger in magnitude in the Intervention treatment relative to the Control treatment.

Analysis of subjects' hypothetical decisions sheds additional light on the mechanism underlying the Intervention treatment. For each subject in the Intervention treatment, I compute an individual-level slope coefficient by regressing their hypothetical decisions against the corresponding hypothetical expected returns. Appendix Figure 9 reports the distribution of these slope coefficients. For the median subject, a 1 percentage point change in the expected one-year S&P 500 return is associated with a 3.5 percentage point increase in hypothetical portfolio shares, far larger in magnitude than the coefficients estimated from Control subjects' actual beliefs and investment decisions, both in the cross section (0.62, Table 17) and in response to information (1.16, Table 16). Moreoever, treated subjects' actual decisions align with their hypothetical decisions; the correlation between subjects' actual allocations and the allocations implied by their hypothetical decisions and return expectations are 0.61 and 0.56 for the baseline and revised investment tasks, respectively.[22] In sum, being encouraged to consider how they would invest across a range of possible return expectations appears to reduce subjects' uncertainty over the belief-action map, and in turn, increase the responsiveness of their behavior to their actual beliefs.

6 Discussion

This paper provides evidence that uncertainty over the belief-action map may be an important driver of a set of puzzling findings surrounding the measurement and utilization of subjective beliefs in economics research: the weak link between beliefs and behavior, and the inconsistent effects of information interventions on beliefs vs. behavior. In particular, in an incentivized survey experiment relating subjects' stock return expectations to their portfolio choices, this paper documents that uncertainty over the belief-action map attenuates the relationship between subjects' beliefs and behavior and reduces their behavioral response to information. In addition, this paper documents that uncertainty over the belief-action map reduces subjects' demand for information about the quantity, adding to our understanding of why individuals often hold miscalibrated beliefs about core economic quantities. Finally, this paper highlihgts the potential efficacy of interventions focused on reducing uncertainty over the belief-action map as a policy lever, and provides a proof-of-concept of how such interventions may be complementary with standard information interventions.

Importantly, this paper is *not* an argument against the relevance or productive use of subjective beliefs data in economics research, given the large body of evidence demonstrating that elicited beliefs are in fact economically meaningful predictors of behavior across many settings. Rather, this paper demonstrates that in certain decision-making contexts, uncertainty over belief-action map may be an important consideration in interpreting the quantitative relationship between beliefs and behavior, predicting the efficacy of information interventions, and understanding the belief formation process, and proposes a portable method to measure this uncertainty. Furthermore, this paper suggests a scope for models in which decision-makers face frictions not only in forming beliefs, but also in the transmission of beliefs into actions.

[22] Subjects' implied allocations are computed as $a^i_{imp} = \hat{\gamma}^i_0 + \hat{\gamma}^i_1 \hat{\theta}^i$, where $\hat{\theta}^i$ is subject i's return expectation and $(\hat{\gamma}^i_0, \hat{\gamma}^i_1)$ are level and slope estimates obtained from regressing subject i's hypothetical investment decisions against the corresponding hypothetical expected returns.

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Appendix

A.1 Evidence for Attenuation Between Beliefs and Behavior

Here, I discuss evidence for attenuation in the relationship between beliefs and behavior in various decision-making contexts.

Expected Asset Returns and Portfolio Allocations. A number of studies find that the relationship between individuals' beliefs about the expected returns of assets and their portfolio allocations are weaker than standard calibrations of the Merton (1969) model, which specifies that in a single-asset portfolio allocation problem with power-utility investors, $\phi = \frac{1}{\gamma}\frac{E[R_i]-R_f}{Var(R_i)}$, where ϕ is the portfolio weight allocated to the risky asset, $E[R_i]$ is the expected return of the risky asset, R_f is the risk-free rate, γ is the coefficient of relative risk aversion, and $Var(R_f)$ is the (subjective) variance of risky asset returns.

Using cross-sectional survey and administrative data on the equity return expectations and portfolio equity shares of Vanguard investors, Giglio et al. (2021) estimates that a one percentage point increase in expected equity returns is associated with a 0.7 percentage point increase in portfolio equity shares, an order of magnitude lower than what the Merton (1969) model would predict given standard calibrations of $Var[R_i]$ and γ; importantly, these estimates control for measurement error using a repeated elicitation of return expectations following Gillen et al. (2019). Using a different approach to correct for measurement error, Ameriks et al. (2020) also estimate a relationship between expected equity returns and portfolio equity shares in a sample of Vanguard investors that is an order of magnitude lower relative to the Merton model. Laudenbach et al. (2021) and Beutel and Weber (2022) estimate comparably weak cross-sectional relationships between equity return expectations and portfolio equity shares in samples of German investors, measuring the latter using administrative data on stockholdings and a hypothetical investment task, respectively. Using survey data on housing return expectations and responses to a hypothetical investment task collected from a representative U.S. sample, Armona et al. (2019) estimate cross-sectional relationships between housing return expectations and housing portfolio shares that are quantitatively similar to to those estimated in Giglio et al. (2021), which, coupled with the fact that subjective housing return volatilities measured in the former study are lower than subjective equity return volatilities measured in the latter, is further evidence that in this context, the return expectation–portfolio share relationship is substantially weaker than what the Merton model predicts. Using a similar survey methodology as Armona et al. (2019), Liu and Palmer (2021) estimate a similarly attenuated relationship between housing return expectations and housing portfolio shares.

While the above results correspond to cross-sectional relationships, a number of studies analyze the impact of information interventions on return expectations and portfolio allocation decisions. Armona et al. (2019) deploy a randomized information intervention to inform respondents' housing return expectations. Using this intervention to instrument for respondents' return expectations, they find that a 1 percentage point increase in housing return expectations corresponds to a 3.67 percentage point increase in the housing portfolio share. While this magnitude is larger than the corresponding cross-sectional relationships, given respondents' subjective housing return volatilities, the Merton model would require an implausibly large degree of risk aversion to rationalize such a magnitude.[23] In an incentivized lab experiment conducted on a sample of Master of Finance students, Andries et al. (2022) provide subjects with signals about the return of a risky asset, and elicit return expectations and decisions in an investment task in which subjects decide how much money to invest in the asset. Using these signals as an instrument for return expectations, they find that a 1 percentage point increase in return expectations corresponds to a 3.82 percentage point increase in equity shares — which given the return volatilities in their experiment, implies an implausibly large level of risk aversion under the Merton model.[24] Beutel and Weber (2022) also study the impacts of a randomized information intervention targeting respondents' beliefs about the returns of the German DAX; using a similar IV specification, they find that a 1 percentage point increase in return expectations corresponds to a 2.8 percentage point increase in portfolio equity shares, which given the historical volatility of the DAX, requires a degree of risk aversion near the upper bound of experimental estimates to rationalize.[25]

Inflation Expectation and Consumption Decisions. The intertemporal Euler equation specifies the relationship $E_t[c_{t+1}] - c_t = \sigma \log \beta + \sigma(r_t - E_t[\pi_{t+1}])$ where c_t denotes log consumption at period t, $E_t[\pi_{t+1}]$ and r_t denotes expected inflation and the nominal interest rate, respectively, and σ and β denotes the intertemporal elasticity of substitution (EIS) and a time-discounting factor, respectively. Importantly, $\sigma > 0$, indicating that that current consumption (consumption growth) should be increasing (decreasing) in inflation expectations. In particular, models calibrated to match aggregate data typically require σ close to 1.

A number of studies estimating the cross-sectional relationship between inflation expectations and consumption measures tend to estimate relationships that are either weak or inconsistent with the Euler equation. Using the Michigan Survey of Consumers, Bachmann et al. (2015) estimates the relationship between one-year inflation expectations and a qualitative measure of respondents' attitudes towards spending on durables and finds no significant relationship between the two measures. Notably, contrary to the predictions of the Euler equation, the paper reports a *negative* relationship between inflation expectation and durables spending attitudes inside the zero lower bound, though this effect is qualitatively weak, as the authors note: they find that respondents who expect a one percentage higher inflation rate are 0.5 percentage points more likely to report that it is a good time to spend on durables. Several follow-up studies, also relying on qualitative consumption measures, also find weak relationships between inflation expectations and consumption. In a survey of German consumers, Dräger and Nghiem (2021) estimates that a 1 percentage point increase in one-year inflation expectations is associated with higher likelihood of reporting a

[23]Given that Armona et al. (2019) find an average subjective return standard deviation of 5.6% in their sample, the estimated magnitude of 3.67 would require a CRRA parameter of $\gamma = \frac{1}{3.76 \cdot 0.056^2} = 84.8$ to rationalize.

[24]The authors report a median implied CRRA parameter of $\gamma = 20$ in their experiment.

[25]Given the historical standard deviation of DAX returns of 21%, the estimated magnitude of 2.8 would require a CRRA parameter of $\gamma = \frac{1}{2.8 \cdot 0.21^2} = 8.1$ to rationalize, near the upper bound of experimental estimates, which typically finds values of γ between 3 and 10.

planned increase in total expenditures of 1 percentage point, an effect that, while consistent with the Euler equation, is quantitatively weak and only statistically significant for one of the two survey waves analyzed in the study; the paper finds no significant relationship between inflation expectations and planned durables expenditures. Duca-Radu et al. (2021) find similarly small effect sizes in a large multi-country survey, and find that subjects who expect a 1 percentage point higher one-year inflation rate are 0.26 percentage points more likely to report that it is a good time to spend. While the above survey evidence relies on quantitative consumption measures, a number of studies also find similarly weak effects utilizing quantitative consumption measures. Using panel survey data to measure inflation expectations and actual spending in a U.S. sample, Burke and Ozdagli (2014) find a precisely estimated null relationship between inflation expectations and both non-durables and durables spending. Crump et al. (2022) utilize survey measures of respondents' expected one-year increase in monthly spending in a U.S. sample, and estimate that a 1 percentage point increase in inflation expectations is associated with a 0.18 percentage point increase in expected spending growth. While the authors interpret this relationship as consistent with the Euler equation, such an interpretation relies on respondents interpreting the spending growth question in nominal terms, which would require them to adjust their responses to the spending growth question, which is elicited in percentage terms, to be net of their own inflation expectations.[26] If, on the other hand, respondents interpret this question in real terms, which requires no such adjustment on the part of subjects, the estimated relationship is inconsistent with the Euler equation.

Research studying the impact of information interventions on inflation expectations and consumption decisions also tend to find weak or inconsistent relationships between inflation expectations and consumption. Coibion et al. (2023) deploys a randomized information intervention designed to inform the inflation expectations of a sample of Dutch households, and utilizes this intervention as an instrument for inflation expectations in estimating the relationship between inflation expectations and quantitative survey measures of spending. While there is a large first stage on inflation expectations, it does not translate to a statistically significant relationship between inflation expectations and total spending, and in contrast to the standard Euler equation, the paper estimates a negative relationship between inflation expectations and durables spending. Studying an information intervention in a U.S. sample, Coibion et al. (2022) finds that higher inflation expectations does translate to an economically meaningful increase in total spending, but estimates a negative relationship between inflation expectations and durables spending.

Notably, D'Acunto et al. (2023) provides evidence that the mixed findings in this literature may be in part driven by cognitive constraints: in a survey of Finnish men measuring inflation expectations and qualitative consumption plans, they find that there is a positive, economically meaningful relationship between inflation expectations and planned durable

[26] In particular, respondents are asked "12 months from now, I expect my overall monthly household spending to have [increased/decreased] by X%". If respondents interpret this question in nominal terms, then the estimated relationship implies that a 1 percentage point increase in inflation expectations is associated with $0.18 - 1 = -0.82$ percentage point change in real consumption growth, consistent with an EIS of $\sigma = 0.82$.

consumption among high (above median) IQ men, as prescribed by the Euler equation – in this subsample, individuals who believe the one-year inflation rate will increase are 4% more likely to state that it is a good time to spend on durables – whereas there is no such relationship along low IQ individuals. Bachmann et al. (2015), Duca-Radu et al. (2021), Dräger and Nghiem (2021), Burke and Ozdagli (2014), and Coibion et al. (2022) provide suggestive corroborating evidence of the role of cognitive constraints, finding that various proxies for lower cognitive constraints such as more accurate inflation expectations, higher financial literacy, and higher education tend to predict consumption responses to inflation expectations more in line with the Euler equation.

Earnings Expectations and Major Choice. In a literature studying the determinants of college students' choice of major, a number of papers have found weak relationships between earnings expectations and major choice, though it is important to note that given the lack of a clear theoretical benchmark, such findings could be driven by preference-based explanations such as non-pecuniary components of major choice. Zafar (2013) surveys college students' expectations of the earnings associated with potential majors, in addition to their beliefs about various other pecuniary and non-pecuniary aspects of potential majors. Estimating a discrete choice model of major choice, the paper finds that earnings expectations is not a statistically significant predictor of major choice, despite the fact that students in the sample tended to rank earnings as one of the top considerations of their choice of major. Consistent with these results, Wiswall and Zafar (2015a) study the impact of an information intervention designed to correct students' beliefs about the earnings outcomes associated with various majors in the general population, and leverage the experimental variation in earnings outcomes beliefs generated by this intervention to estimate a model of college major choice. They find that while earnings expectations predict major choice, the magnitude of this relationship is modest when controlling for individual fixed effects in their model: they estimate that a 1% increase in earnings expectations associated with a major increases the subjective likelihood of choosing that major by 0.15%. In contrast to these findings, Arcidiacono et al. (2012) use a similar survey methodology as Zafar (2013) and finds that earnings expectations are a meaningful predictor of major choice.

Policy-Relevant Beliefs and Policy Preferences. A growing literature studying beliefs about policy relevant quantities, such as the extent of social mobility or income inequality, tends to find that while information interventions are often successful in correcting individuals' (often large) misperceptions about these quantities, these changes in beliefs do not translate into changes in attitudes towards policies. For example, in a large, multi-country survey, Alesina et al. (2018) find that while respondents largely over-estimate the degree of social mobility and that providing information has a large corrective effect on these beliefs about social mobility, the intervention has no average impact on respondents' support for policies designed to increase social mobility. Similarly, Kuziemko et al. (2015) find that while providing population statistics has a large effect on respondents' beliefs about income inequality, which on average underestimated the extent of income inequality prior to the intervention, preferences for various policies designed at reducing income inequality were largely unaffected by the intervention. Finally, Haaland and Roth (2021) find that while providing evidence of racial bias in hiring in the U.S. has a large effect on subjects' beliefs about the extent of such bias, this intervention has precisely estimated null effects on respondents' beliefs about pro-black policies, regardless of their political affiliation or whether they initially overestimated/underestimated the extent of racial bias.

Beliefs about Opponents and Play in Games. In the experimental literature on games, a number of studies have found that subjects' behavior in games in inconsistent with their beliefs about their opponents' play in games. In a seminal study, Costa-Gomes and Weizsäcker (2008) elicits subjects' beliefs about their opponents' play across a set of two-player 3×3 games, and finds that subjects fail to best-respond to their beliefs about their opponents' play nearly in nearly half of the games played in the experiment. Importantly, the authors provide evidence that this inconsistency is not purely a product of noise or trembling: in particular, they separately estimate the underlying beliefs implied by subjects' belief reports versus their play, under various assumptions on the structure of error in both sets of elicitations, and reject the null hypothesis that the two sets of elicitations reflect the same underlying beliefs. Similar evidence that subjects fail to best-respond to their beliefs about their opponents' play has been documented for other variants of two-player games (e.g. Ivanov, 2006; Rey-Biel, 2009; Polonio and Coricelli, 2019).

A.2 Derivations of Model Predictions

Main Text Derivations. Here, I derive the model predictions in the main text, re-stated as formal propositions. Recall the maintained assumption in the model: that the DM's prior uncertainty over the decision weight β does not vary across individuals:

Assumption 1. σ_β is constant across individuals.

As in the main text, let $\lambda = \frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\beta^2}$. I begin by noting a basic relationship between counterfactual cognitive uncertainty σ_{cf} and uncertainty over the belief-action map σ_ζ.

Lemma 1. Under Assumption 1, σ_{cf} is increasing in σ_ζ.

Proof. Using standard results on Gaussian information structures, conditional on s_β, the DM's posterior belief over β is given by $N((1-\lambda)s_\beta, \tilde{\sigma}_\beta^2)$, where $\tilde{\sigma}_\beta^2 = \frac{\sigma_\zeta^2\sigma_\beta^2}{\sigma_\zeta^2+\sigma_\beta^2}$. The DM's beliefs over the optimal counterfactual decision a_{cf} is then given by $N\left((1-\lambda)s_\beta\theta_{cf}, \theta_{cf}^2\tilde{\sigma}_\beta^2\right)$, and so we have

$$\begin{aligned}\sigma_{cf} &= |\theta_{cf}|\tilde{\sigma}_\beta \\ &= |\theta^{cf}|\frac{\sigma_\beta\sigma_\zeta}{\sqrt{\sigma_\beta^2+\sigma_\zeta^2}}\end{aligned}$$

which is increasing in σ_ζ, holding fixed σ_β. □

I now derive Predictions 1 and 2 of the main text. As in the main text, let $\hat{\theta}$ and a^* denote the DM's expectation over θ and the DM's choice of action in the first-stage problem, respectively, and let $\Delta\hat{\theta}$ and Δa^* denote the change the information intervention ϕ induces in the DM's expectation over θ and in the DM's choice of action, respectively.

Proposition 1. (Predictions 1 and 2). We have

$$E[a^*|\beta, \hat{\theta}] = (1-\lambda)\beta\hat{\theta}$$
$$E[\Delta a^*|\beta, \Delta\hat{\theta}, \phi] = (1-\lambda)\beta\Delta\hat{\theta}$$

where λ is increasing in σ_ζ, and under Assumption 1, also increasing in σ_{cf}.

Proof. Given $\hat{\theta}$, the DM's action in the first stage problem is given by $a^* = E[\beta|s_\beta]\hat{\theta} = (1-\lambda)s_\beta\hat{\theta}$, where the second equality follows from standard results on Gaussian information structures. This implies that

$$E[a^*|\beta, \hat{\theta}] = (1-\lambda)\beta\hat{\theta}$$

where, λ is increasing in σ_ζ, and increasing in σ_{cf} by Lemma 1. similarly, we have $\Delta a^* = E[\beta|s_\beta]\Delta\hat{\theta} = (1-\lambda)s_\beta\hat{\theta}$, and so

$$E[\Delta a^*|\beta, \Delta\hat{\theta}, \phi] = (1-\lambda)\beta\hat{\Delta}\theta$$

as desired. $\square$

I now derive Prediction 3a of the main text. As in the main text, let τ^* denote the signal precision chosen by the DM in the first-stage problem, and let $\hat{\sigma}_\theta$ denote the DM's posterior standard deviation over θ in the first stage problem.

Proposition 2. (Prediction 3a). Suppose Assumption 1 holds. At an interior solution, we have

$$\tau^* = \frac{(1-\lambda)|s_\beta|}{\sqrt{c}} - \frac{1}{(\sigma_\theta)^2}$$
$$\hat{\sigma}_\theta^2 = \frac{\sqrt{c}}{(1-\lambda)|s_\beta|}$$

and so τ^* is decreasing in σ_{cf} and $\hat{\sigma_\theta}^2$ is increasing in σ_{cf}. Furthermore, $\Delta\hat{\theta}$ is increasing in σ_{cf}.

Proof. For a given signal precision τ and realized signal s_θ, using standard results on Gaussian information structures, the DM's posterior belief over θ is distributed according to $N\left(\hat{\theta}, \frac{\sigma_\theta^2}{1+\tau\sigma_\theta^2}\right)$, for $\hat{\theta} = \tilde{\alpha}\overline{\theta} + (1-\tilde{\alpha})s_\theta$ for $\tilde{\alpha} = \frac{1}{1+\tau\sigma_\theta^2}$.

The DM's expected payoff is given by

$$\begin{aligned}
&- E\left[\text{Var}\left(\beta\theta \,\middle|\, s_\beta, s_\theta\right) \,\middle|\, s_\beta\right] - c\tau \\
&= -E\left[\left[E(\beta|s_\beta)^2 + \text{Var}(\beta|s_\beta)\right]\text{Var}(\theta|s_\theta) + \text{Var}(\beta|s_\beta)E(\theta|s_\theta)^2 \,\middle|\, s_\beta\right] - c\tau \\
&= -E\left[\left[(1-\lambda)^2 s_\beta^2 + \text{Var}(\beta|s_\beta)\right]\frac{\sigma_\theta^2}{1+\tau\sigma_\theta^2} + \text{Var}(\beta|s_\beta)\hat{\theta}^2 \,\middle|\, s_\beta\right] - c\tau
\end{aligned}$$

Note that

$$\begin{aligned}
E[\hat{\theta}^2|s_\beta] &= \text{Var}(\hat{\theta}) + E(\hat{\theta})^2 \\
&= \frac{\tau^2\sigma_\theta^4}{(1+\tau\sigma_\theta^2)^2}\left(\sigma_\theta^2 + 1/\tau\right) + \overline{\theta}^2 \\
&= \frac{\tau\sigma_\theta^4}{1+\tau\sigma_\theta^2} + \overline{\theta}^2
\end{aligned}$$

and so we can rewrite the the DM's expected payoff as

$$-(1-\lambda)^2 s_\beta^2 \frac{\sigma_\theta^2}{1+\tau\sigma_\theta^2} - \text{Var}(\beta|s_\beta)\left(\sigma_\theta^2 + \overline{\theta}^2\right) - c\tau$$

And so taking FOCs with respect to τ_k^i yields

$$\begin{aligned}
(1-\lambda)^2 s_\beta^2 \frac{\sigma_\theta^4}{(1+\tau\sigma_\theta^2)^2} &= c \\
\implies \tau^* &= \frac{(1-\lambda)|s_\beta|}{\sqrt{c}} - \frac{1}{\sigma_\theta^2}
\end{aligned}$$

which in turn implies

$$\begin{aligned}
\hat{\sigma}_\theta^2 &= \frac{\sigma_\theta^2}{1+\tau\sigma_\theta^2} \\
&= \frac{\sqrt{c}}{(1-\lambda)|s_\beta|}
\end{aligned}$$

Since λ is increasing in σ_{cf} under Assumption 1, this implies that τ^* and $\hat{\sigma}_\theta^2$ are decreasing and increasing in σ_{cf}, respectively. This in turn implies that $\Delta\hat{\theta} = \frac{\hat{\sigma}_\theta^2}{\hat{\sigma}_\theta^2 + 1/\tau_\phi}(\phi - \hat{\theta})$, which is increasing in $\hat{\sigma}_\theta^2$, is increasing in σ_{cf} under Assumption 1.

□

Finally, I derive Prediction 3b. As in the main text, let $V_0^{\tau^*} \equiv \max_a E(u(a,\theta)|s_\beta, s_\theta)$ denote the DM's expected utility if no additional information is acquired, and let $V_\phi^{\tau^*} \equiv E(\max_a E(u(a,\theta)|s_\beta, s_\theta, \phi)\,|s_\beta, s_\theta)$ denote the DM's expected utility after observing the signal ϕ. Let $WTP_\phi \equiv V_\phi^{\tau^*} - V_0^{\tau^*}$ denote the DM's willingness to pay to acquire the information.

Proposition 3. (Prediction 3b). We have

$$WTP_\phi = (1-\lambda)^2 s_\beta^2 \frac{\tau_\phi \hat{\sigma}_\theta^4}{1+\tau_\phi\hat{\sigma}_\theta^2}$$

In particular, WTP_ϕ is decreasing in σ_ζ, holding fixed $\hat{\sigma}_\theta^2$.

Proof. We have

$$\begin{aligned}
V_0^{\tau^*} &= -\text{Var}\left(\beta\theta \middle| s_\beta, s_\theta\right) \\
&= -\left[E(\beta|s_\beta)^2 + \text{Var}(\beta|s_\beta)\right]\text{Var}(\theta|s_\theta) - \text{Var}(\beta|s_\beta)E(\theta|s_\theta)^2 \\
&= -(1-\lambda)^2 s_\beta^2 \hat{\sigma}_\theta^2 - \text{Var}(\beta|s_\beta)(\hat{\sigma}_\theta^2 + \hat{\theta}^2)
\end{aligned}$$

Given ϕ, the DM's posterior belief over θ is distributed according to $N\left(\alpha\hat{\theta} + (1-\alpha)\phi, \hat{\sigma}_{\theta,\phi}^2\right)$, where $\alpha = \frac{\tau_\phi \hat{\sigma}_\theta^2}{1+\tau_\phi\hat{\sigma}_\theta^2}$ and $\hat{\sigma}_{\theta,\phi}^2 = \frac{\hat{\sigma}_\theta^2}{1+\tau_\phi\hat{\sigma}_\theta^2}$. We have

$$\begin{aligned}
V_\phi^{\tau^*} &= -E\left[\text{Var}\left(\beta\theta \middle| s_\beta, s_\theta, \phi\right) \middle| s_\beta, s_\theta\right] \\
&= -E\left[\left[E(\beta|s_\beta)^2 + \text{Var}(\beta|s_\beta)\right]\text{Var}(\theta|s_\theta,\phi) + \text{Var}(\beta|s_\beta)E(\theta|s_\theta,\phi)^2 \middle| s_\beta, s_\theta\right] \\
&= -E\left[\left[(1-\lambda)^2 s_\beta^2 + \text{Var}(\beta|s_\beta)\right]\frac{\hat{\sigma}_\theta^2}{1+\tau_\phi\hat{\sigma}_\theta^2} + \text{Var}(\beta|s_\beta)(\alpha\hat{\theta} + (1-\alpha)\phi)^2 \middle| s_\beta, s_\theta\right]
\end{aligned}$$

Note that

$$\begin{aligned}
E[(\alpha\hat{\theta} + (1-\alpha)\phi)^2|s_\beta s_\theta] &= \text{Var}((\alpha\hat{\theta} + (1-\alpha)\phi)^2|s_\beta s_\theta) + E[\alpha\hat{\theta} + (1-\alpha)\phi|s_\beta s_\theta]^2 \\
&= \frac{\tau_\phi^2\hat{\sigma}_\theta^2}{(1+\tau_\phi\hat{\sigma}_\phi^2)^2}(\hat{\sigma}_\theta^2 + 1/\tau_\phi) + \hat{\theta}^2 \\
&= \frac{\tau_\phi\hat{\sigma}_\theta^4}{1+\tau_\phi\hat{\sigma}_\theta^2} + \hat{\theta}^2
\end{aligned}$$

Plugging this expression for $V_\phi^{\tau^*}$ yields

$$V_\phi^{\tau^*} = -(1-\lambda)^2 s_\beta^2 \frac{\hat{\sigma}_\theta^2}{1+\tau_\phi\hat{\sigma}_\theta^2} - \text{Var}(\beta|s_\beta)(\hat{\sigma}_\theta^2 + \hat{\theta}^2)$$

and so we have

$$WTP_\phi = (1-\lambda)^2 s_\beta^2 \frac{\tau_\phi\hat{\sigma}_\theta^4}{1+\tau_\phi\hat{\sigma}_\theta^2}$$

which, holding fixed $\hat{\sigma}_\theta^2$, is decreasing in λ and therefore decreasing in σ_ζ. □

Random Choice Model. Here, I show how analogs of the above predictions hold in alternative *random choice* account of the model. Continue to assume that the DM's payoffs are given by $u(a,\theta) = -(a-\beta\theta)^2$ and that the DM holds priors over θ distributed according to $N(\overline{\theta}, \sigma_\theta^2)$, where the the DM can choose the precision τ of a signal $s_\theta \sim N(\theta, 1/\tau)$ at a cost $c\tau$. As before, assume that the DM is uncertain over the decision weight β, with priors $N(0, \sigma_\beta)$. In contrast to the model in the main text, suppose that with probability $(1-\lambda)$, the DM deliberates and generates a cognitive signal of the true decision weight β, and with probability λ, the DM does not generate a cognitive signal. We have the following set of results:

Lemma 2. Under Assumption 1, $E[\sigma_{cf}]$ is increasing in λ.

Proof. Note that if the DM generates the cognitive signal, $\sigma_{cf} = 0$, and $\sigma_{cf} = |\theta^{cf}|\sigma_\beta^2$ otherwise. Therefore, $E[\sigma_{cf}] = \lambda|\theta^{cf}|\sigma_\beta^2$. □

Proposition 4. (Predictions 1–4, Random Choice). We have

$$\begin{aligned} E[a^*|\beta,\hat{\theta}] &= (1-\lambda)\beta\hat{\theta} \\ E[\Delta a^*|\beta,\Delta\hat{\theta},\phi] &= (1-\lambda)\beta\Delta\hat{\theta} \\ E[\tau^*] &= (1-\lambda)\left(\frac{1}{\sqrt{c}} - \frac{1}{\sigma_\theta^2}\right) \\ E[WTP_\phi] &= (1-\lambda)\frac{\tau_\phi\hat{\sigma}_\theta^4}{1+\tau_\phi\hat{\sigma}_\theta^2} \end{aligned}$$

where in particular, under Assumption 1, λ is increasing in $E[\sigma_{cf}]$.

Proof. Follows from Lemma 2 and Propositions 1–3, noting that the random choice account is equivalent to the model in the main text where with probability $1-\lambda$, $\sigma_\zeta = 0$ and with probability λ, $\sigma_\zeta = \infty$. □

A.3 Additional Tables

Table 6: Summary Statistics

	Mean	p10	p25	p50	p75	p90
Age	39.76	25	29	37	49	60
Male	0.51					
College Degree	0.7					
Income ($ thousands)	60.75	5	25	55	87.5	125
Invests in Stock Market	0.67					
Financial Literacy Score	2.66	2	2	3	3	3
Freq. Acquired Stock Info	1.34	0	0	1	3	4
Risk Aversion	0.75	-0.55	0.05	0.65	1.65	2.05
Time Spent on Study (Minutes)	16.52	8.78	10.93	14.2	19.7	27.47

Notes: The financial literacy score corresponds to the three-question measure used in Lusardi and Mitchell (2011). The frequency of stock information acquisition is based on the question "How frequently did you gather information about the performance of the S&P 500 or the stock market in the last 3 months?", with answers ranging from 0 ("Not at all") to 5 ("Daily"). The risk aversion measure is the the difference between expected value of a lottery paying out $6 with 50% chance and the subject's (unincentivized) certainty equivalent for the lottery.

Table 7: Beliefs About S&P 500 Returns

	Mean	Std.	p10	p25	p50	p75	p90
Expected 1Y S&P 500 Return	7.11	25.78	-10	3	6.5	12	20
Expected 1Y S&P 500 Return in Range							
Less than $(\hat{\theta} - 30)$%	7.46	11.92	0	1	5	10	15
Between $(\hat{\theta} - 30)$% and $(\hat{\theta} - 15)$%	14.03	10.87	2	5	10	20	28.2
Between $(\hat{\theta} - 15)$% and $(\hat{\theta} + 15)$%	57.41	23.32	25	40	60	75	90
Between $(\hat{\theta} + 30)$% and $(\hat{\theta} + 30)$%	15.03	12.23	2	5	12	20	30
Greater than $(\hat{\theta} + 30)$%	6.08	7.76	0	1	5	10	15
Implied Std. for 1Y S&P 500 Return	16.41	4.32	10.91	13.23	16.4	19.62	21.7

Notes: To construct the implied standard deviation from the distribution question, I first split each bucket into ranges of 5 percentage points. For each of these ranges, I compute the probability that a $N(\hat{\theta}, 15^2)$ distribution assigns to that range, where the standard deviation of this distribution was chosen to match that of historical one-year S&P 500 returns, which is approximately 15%. I then weight these probabilities by the subjective probability of each bucket reported by the respondent. I finally calculate the standard deviation based on the mid-points of the narrower ranges, and their associated subjective probabilities.

Table 8: Correlates of CU over Counterfactual Decision

	Dependent Variable: Counterfactual Cognitive Uncertainty									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age	−0.07									−0.00
	(0.07)									(0.07)
Male		-9.53^{***}								-7.84^{***}
		(1.86)								(1.99)
College Degree			−0.70							1.10
			(2.06)							(2.24)
Income				-0.05^{**}						−0.04
				(0.02)						(0.02)
Invests in Stocks					−2.30					2.24
					(2.03)					(2.31)
Financial Literacy Score						-3.55^{*}				−1.09
						(1.65)				(1.86)
Risk Aversion							2.77^{**}			2.21^{*}
							(0.96)			(0.94)
Return Std. Beliefs								0.77^{***}		0.57^{*}
								(0.23)		(0.23)
Time Spent on Study									−0.08	−0.09
									(0.11)	(0.11)
(Intercept)	41.85^{***}	44.04^{***}	39.50^{***}	42.15^{***}	40.62^{***}	48.49^{***}	36.95^{***}	26.27^{***}	40.35^{***}	36.87^{***}
	(3.04)	(1.38)	(1.73)	(1.50)	(1.68)	(4.53)	(1.15)	(3.93)	(2.04)	(7.14)
R^2	0.00	0.03	0.00	0.01	0.00	0.01	0.01	0.02	0.00	0.06
Num. obs.	777	777	777	777	777	777	777	777	777	777

Standard errors (in parentheses) are robust.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 9: Counterfactual CU vs. Attenuation in Counterfactual Investment Task

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.81***	2.39***	2.30***
	(0.10)	(0.14)	(0.14)
High Cfact. CU		−1.05	−0.03
		(1.97)	(1.97)
Cfact. Returns × High Cfact. CU		−1.24***	−1.04***
		(0.20)	(0.20)
(Intercept)	48.45***	48.47***	38.13***
	(1.00)	(1.43)	(4.82)
Controls	N	N	Y
R^2	0.31	0.35	0.38
Num. obs.	755	755	755

OLS estimates, standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 10: Treatment vs. Attenuation in Baseline Investment Task

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.57***	0.76***	0.87***	0.65***	0.86***	0.95***
	(0.07)	(0.09)	(0.09)	(0.07)	(0.10)	(0.00)
Complex		−12.05***	−11.28***		−11.42***	−10.70***
		(1.65)	(1.64)		(1.72)	(0.00)
Return Beliefs × Complex		−0.55***	−0.55***		−0.60***	−0.60***
		(0.14)	(0.14)		(0.15)	(0.00)
(Intercept)	55.41***	59.70***	47.38***	55.20***	59.23***	46.23***
	(0.83)	(1.03)	(3.39)	(0.85)	(1.05)	(0.00)
Controls	N	N	Y	N	N	Y
R^2	0.06	0.15	0.21	0.07	0.15	0.21
Num. obs.	1167	1167	1167	2176	2176	2176

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 11: Treatment vs. Attenuation in Counterfactual Investment Task

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.43^{***}	1.81^{***}	1.79^{***}
	(0.08)	(0.10)	(0.10)
Complex		-3.26^{*}	-2.68
		(1.61)	(1.64)
Cfact. Returns × Complex		-1.08^{***}	-1.06^{***}
		(0.16)	(0.17)
(Intercept)	47.33^{***}	48.45^{***}	40.08^{***}
	(0.80)	(1.00)	(3.62)
Controls	N	N	Y
R^2	0.21	0.25	0.28
Num. obs.	1167	1167	1167

OLS estimates, standard errors (in parentheses) are robust. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 12: Behavioral Response to Information, Revised Equity Shares

	Dependent Variable: Revised Equity Share					
	(1)	(2)	(3)	(4)	(5)	(6)
Revised Beliefs	1.28***	2.09***	2.12***	0.91***	1.28***	1.30***
	(0.23)	(0.36)	(0.36)	(0.19)	(0.23)	(0.22)
High Cfact. CU		2.25	4.75			
		(3.94)	(3.97)			
Revised Beliefs × High Cfact. CU		−1.33**	−1.37**			
		(0.46)	(0.47)			
Complex					−9.51**	−8.79**
					(2.94)	(2.93)
Revised Beliefs × Complex					−0.90*	−0.87*
					(0.36)	(0.35)
(Intercept)	56.72***	55.05***	38.74***	53.91***	56.72***	42.12***
	(1.96)	(2.79)	(5.21)	(1.54)	(1.96)	(4.00)
Baseline Belief Controls	N	N	Y	N	N	Y
Demographic Controls	N	N	Y	N	N	Y
Num. obs.	723	723	723	1113	1113	1113

IV estimates instrumenting for revised beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 13: Switching Point Analysis

	Dep. Variable: Baseline Equity %		*Dep. Variable*: Revised Equity %		*Dep. Variable*: Cfact. Equity %	
	(1)	(2)	(3)	(4)	(5)	(6)
High Cfact. CU	4.84	-8.54^{***}	11.14^{*}	-9.24^{***}	10.92^{***}	-15.30^{***}
	(3.65)	(1.85)	(5.55)	(1.83)	(3.06)	(2.15)
Baseline Return Beliefs	0.54	0.07				
	(0.27)	(0.14)				
Revised Return Beliefs			0.34	0.92^{***}		
			(0.42)	(0.17)		
Cfact. Returns					0.79	0.87^{***}
					(0.61)	(0.26)
(Intercept)	49.89^{***}	72.74^{***}	44.54^{***}	64.86^{***}	33.30^{***}	65.93^{***}
	(3.40)	(1.84)	(4.57)	(2.10)	(5.15)	(2.97)
$E[r] > r_f$?	N	Y	N	Y	N	Y
R^2	0.04	0.04	0.06	0.08	0.05	0.13
Num. obs.	188	567	100	655	301	454

Standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Models are estimated separately for subjects with expected returns greater than vs. less than or equal to the risk free rate of 2%.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 14: Information Acquisition vs. Measures of Belief Uncertainty

	Dependent Variable: Acquired Expert Estimate			
	(1)	(2)	(3)	(4)
Belief CU	0.20^{**}	0.18^{*}		
	(0.07)	(0.08)		
Freq. Acquired Stock Info			-0.05^{***}	-0.04^{***}
			(0.01)	(0.01)
(Intercept)	0.40^{***}	0.53^{***}	0.58^{***}	0.62^{***}
	(0.05)	(0.11)	(0.02)	(0.09)
Demographic Controls	N	Y	N	Y
R^2	0.01	0.02	0.02	0.03
Num. obs.	755	755	755	755

Standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 15: Measures of Belief Quality vs. CU

	Dep. Variable: Consistent Beliefs				*Dep. Variable*: Rounded Beliefs			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cfact. CU	-0.21^{**}	-0.16^{*}			0.20^{**}	0.17^{*}		
	(0.07)	(0.06)			(0.07)	(0.07)		
Belief CU			-0.20^{**}	-0.14^{*}			0.25^{***}	0.18^{*}
			(0.07)	(0.07)			(0.07)	(0.07)
(Intercept)	0.77^{***}	0.40^{***}	0.80^{***}	0.43^{***}	0.54^{***}	0.82^{***}	0.47^{***}	0.76^{***}
	(0.03)	(0.10)	(0.04)	(0.11)	(0.03)	(0.08)	(0.05)	(0.10)
Controls	N	Y	N	Y	N	Y	N	Y
R^2	0.01	0.05	0.01	0.05	0.01	0.04	0.02	0.04
Num. obs.	755	755	755	755	755	755	755	755

OLS estimates, standard errors (in parentheses) are robust. Dependent variable for columns 1–4 is a dummy for whether baseline return expectations are consistent with return expectations in the repeat elicitation. Dependent variable for coulumns 5–8 is a dummy for whether baseline return expectations are rounded Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 16: Intervention Treatment vs. Behavioral to Information

	Dependent Variable: Change in Equity Share			
	(1)	(2)	(3)	(4)
ΔBeliefs	1.84***	1.16***	1.17***	1.22***
	(0.19)	(0.20)	(0.20)	(0.21)
Intervention		−0.01	−0.35	−0.19
		(1.23)	(1.20)	(1.19)
ΔBeliefs × Intervention		1.63***	1.58***	1.50***
		(0.42)	(0.40)	(0.40)
(Intercept)	1.87**	2.01**	−0.68	11.40**
	(0.61)	(0.75)	(1.31)	(3.69)
Baseline Belief Controls	N	N	Y	Y
Demographic Controls	N	N	N	Y
Num. obs.	781	781	781	781

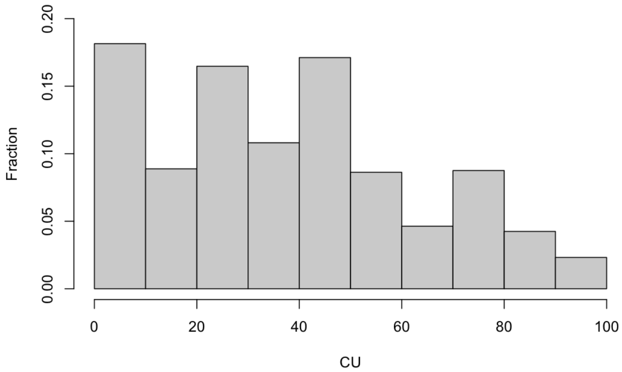
IV estimates instrumenting for the change in beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 17: Treatment vs. Attenuation in Baseline Investment Task, Full Sample

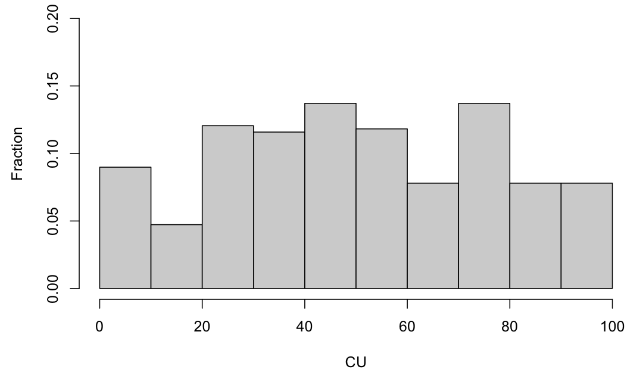
	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.94***	0.62***	0.94***	1.24***	0.82***	1.20***
	(0.12)	(0.17)	(0.16)	(0.15)	(0.23)	(0.20)
Intervention		−4.31	−3.76		−6.48*	−4.88
		(2.43)	(2.27)		(2.83)	(2.71)
Return Beliefs × Intervention		0.64**	0.61**		0.80**	0.66*
		(0.23)	(0.22)		(0.29)	(0.27)
(Intercept)	52.14***	54.18***	19.23***	50.01***	53.35***	16.53**
	(1.26)	(1.81)	(4.40)	(1.45)	(2.18)	(5.06)
Controls	N	N	Y	N	N	Y
R^2	0.08	0.09	0.25	0.08	0.09	0.24
Num. obs.	784	784	784	1430	1430	1430

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

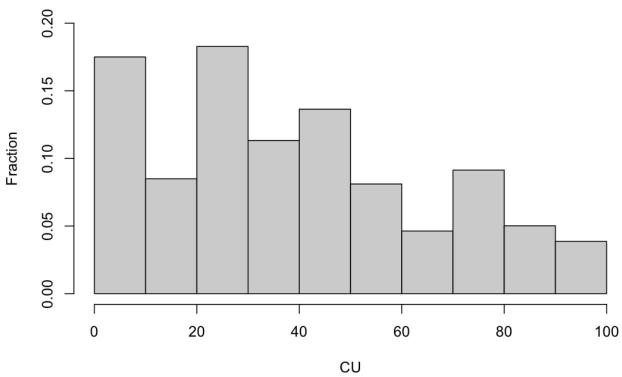
A.4 Additional Figures



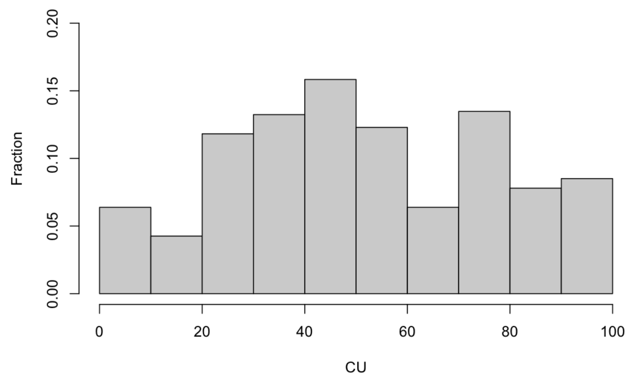
(a) CU over Counterfactual Decision,
Standard Treatment



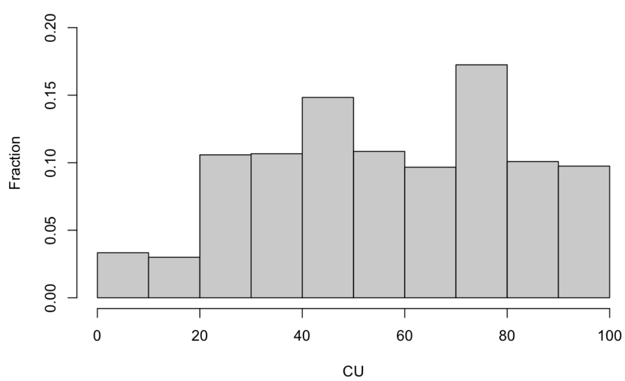
(b) CU over Counterfactual Decision,
Complex Treatment



(c) CU over Baseline Decision,
Standard Treatment



(d) CU over Baseline Decision,
Complex Treatment



(e) CU over Return Expectations

Figure 6: Distribution of cognitive uncertainty measures.

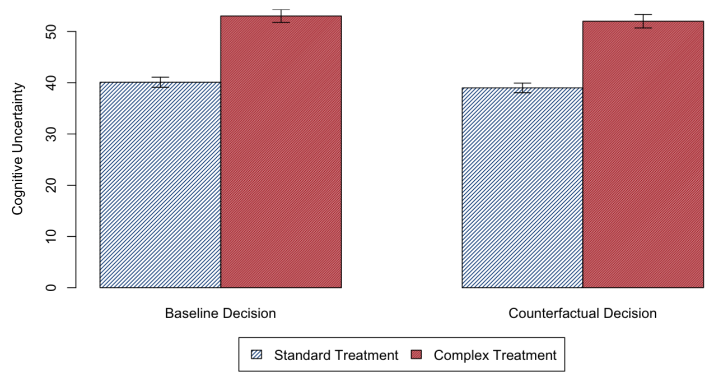


Figure 7: Average cognitive uncertainty over investment decisions by treatment in main study. Whiskers show standard error bars.

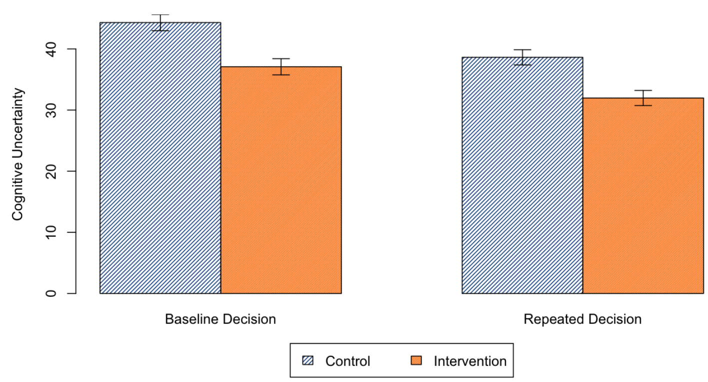


Figure 8: Average cognitive uncertainty over investment decisions by treatment in extension. Whiskers show standard error bars.

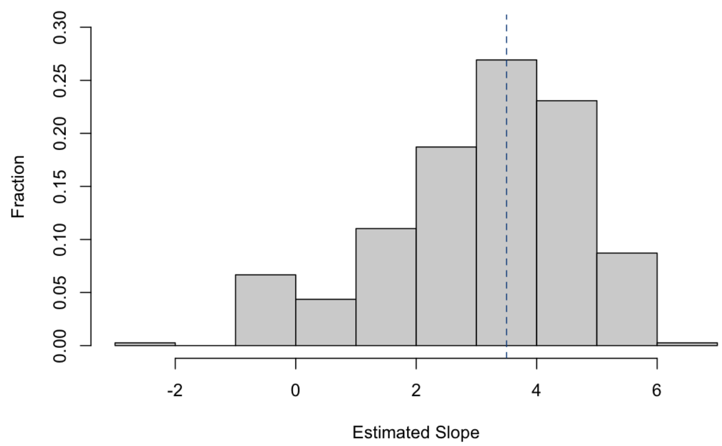


Figure 9: Distribution of slope coefficients for hypothetical decisions. For each subject in the Intervention treatment, slope coefficients are computed by regressing hypothetical portfolio allocations against the corresponding hypothetical expected return using OLS. The dashed line indicates the median slope coefficient.

A.5 Evidence for Attenuation Using Continuous CU Measure

This section reports the corresponding regression analyses in Section 4.1.1 using a continuous CU measure.

Table 18: Counterfactual CU vs. Attenuation in Baseline Investment Task

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.76***	1.30***	1.44***	0.86***	1.34***	1.47***
	(0.09)	(0.16)	(0.15)	(0.10)	(0.18)	(0.17)
Cfact. CU		−0.09*	−0.04		−0.10*	−0.05
		(0.04)	(0.04)		(0.04)	(0.04)
Return Beliefs × Cfact. CU		−0.01***	−0.01***		−0.01***	−0.01**
		(0.00)	(0.00)		(0.00)	(0.00)
(Intercept)	59.70***	63.22***	45.23***	59.23***	62.78***	42.18***
	(1.03)	(1.77)	(4.53)	(1.05)	(1.86)	(5.01)
Controls	N	N	Y	N	N	Y
R^2	0.11	0.16	0.23	0.12	0.16	0.24
Num. obs.	755	755	755	1410	1410	1410

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 19: Counterfactual CU vs. Attenuation in Counterfactual Investment Task

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.81^{***}	2.75^{***}	2.61^{***}
	(0.10)	(0.17)	(0.18)
Cfact. CU		-0.06	-0.03
		(0.04)	(0.04)
Cfact. Returns × Cfact. CU		-0.02^{***}	-0.02^{***}
		(0.00)	(0.00)
(Intercept)	48.45^{***}	50.27^{***}	39.71^{***}
	(1.00)	(1.81)	(5.05)
Controls	N	N	Y
R^2	0.31	0.36	0.38
Num. obs.	755	755	755

OLS estimates, standard errors (in parentheses) are robust. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 20: Counterfactual CU vs. Behavioral Response to Information

	Dependent Variable: Change in Equity Share			
	(1)	(2)	(3)	(4)
ΔBeliefs	1.37^{***}	2.36^{***}	2.38^{***}	2.26^{***}
	(0.21)	(0.39)	(0.39)	(0.38)
Cfact. CU		0.01	0.02	0.01
		(0.03)	(0.03)	(0.03)
ΔBeliefs × Cfact. CU		-0.02^{**}	-0.03^{***}	-0.02^{**}
		(0.01)	(0.01)	(0.01)
(Intercept)	-0.14	-0.19	-2.76	-0.44
	(0.80)	(1.31)	(1.56)	(4.51)
Baseline Belief Controls	N	N	Y	Y
Demographic Controls	N	N	N	Y
Num. obs.	723	723	723	723

IV estimates instrumenting for the change in beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 21: Counterfactual CU vs. Behavioral Response to Information, Revised Equity Shares

	Dependent Variable: Revised Equity Share			
	(1)	(2)	(3)	(4)
Revised Beliefs	1.28^{***}	2.31^{***}	2.30^{***}	2.29^{***}
	(0.23)	(0.46)	(0.45)	(0.45)
Cfact. CU		-0.03	-0.03	0.01
		(0.08)	(0.08)	(0.08)
Revised Beliefs × Cfact. CU		-0.02^{**}	-0.02^{**}	-0.02^{*}
		(0.01)	(0.01)	(0.01)
(Intercept)	56.72^{***}	57.21^{***}	58.29^{***}	41.89^{***}
	(1.96)	(3.73)	(3.33)	(5.50)
Baseline Belief Controls	N	N	Y	Y
Demographic Controls	N	N	N	Y
Num. obs.	723	723	723	723

IV estimates instrumenting for revised beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

A.6 Robustness to Sample Restrictions

Sensitivity to Outlier Restrictions. As pre-registered, the analysis in the main text excludes subjects with return expectations greater than 30% or less than -30%. Here, I show that the results are robust to the inclusion of such outliers. The following tables replicate the analyses in the main text for the sample of subjects with return expectations in $[-100\%, 100\%]$, which includes all but 2 subjects in the sample who reported baseline return expectations of 500% and 600%. In particular, Tables 22–23 provide evidence for Prediction 1a: that higher counterfactual CU predicts greater attenuation between beliefs and behavior, whereas Tables 24–25 provide evidence for Prediction 1b: that cross-sectional attenuation is higher in the complex treatment relative to the standard treatment. Columns 1–3 of Table 26 provide evidence for Prediction 2a: that higher counterfactual CU predicts a weaker behavioral response to information, whereas columns 4–6 provide evidence for Prediction 2b: that the behavioral response to information is weaker in the complex treatment relative to the standard treatment. Tables 27–28 provide evidence for Prediction 3a: in particular, columns 1 and 2 of Table 27 show that subjects with higher counterfactual CU report greater belief CU, columns 3 and 4 of Table 27 show that subjects with higher counterfactual CU report acquiring information about the stock market at a lower frequency, and columns 1–3 of Table 28 show that the return expectations of subjects with higher counterfactual CU are more responsive to receiving information. Table 29 provides evidence for Prediction 3b: that subjects exhibit lower demand for information in complex treatment relative to the standard treatment.

Table 22: Counterfactual CU vs. Attenuation in Baseline Investment Task, Full Sample

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.64***	0.95***	0.99***	0.73***	0.98***	1.07***
	(0.07)	(0.09)	(0.11)	(0.08)	(0.11)	(0.13)
High Cfact. CU		−2.42	−0.55		−2.70	−0.39
		(1.93)	(1.95)		(2.00)	(2.11)
Return Beliefs × High Cfact. CU		−0.65***	−0.59***		−0.54**	−0.54**
		(0.14)	(0.14)		(0.17)	(0.18)
(Intercept)	59.90***	61.29***	45.33***	59.56***	61.05***	43.97***
	(0.97)	(1.25)	(4.27)	(0.99)	(1.32)	(4.76)
Controls	N	N	Y	N	N	Y
R^2	0.11	0.16	0.22	0.04	0.06	0.10
Num. obs.	776	776	776	1528	1528	1528

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 23: Counterfactual CU vs. Attenuation in Counterfactual Investment Task, Full Sample

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.81^{***}	2.41^{***}	2.32^{***}
	(0.10)	(0.13)	(0.13)
High Cfact. CU		-0.55	0.33
		(1.94)	(1.93)
Cfact. Returns × High Cfact. CU		-1.28^{***}	-1.09^{***}
		(0.19)	(0.19)
(Intercept)	48.41^{***}	48.20^{***}	38.97^{***}
	(0.99)	(1.40)	(4.71)
Controls	N	N	Y
R^2	0.31	0.35	0.37
Num. obs.	776	776	776

OLS estimates, standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 24: Treatment vs. Attenuation in Baseline Investment Task, Full Sample

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.47^{***}	0.64^{***}	0.70^{***}	0.50^{***}	0.73^{***}	0.79^{***}
	(0.06)	(0.07)	(0.08)	(0.07)	(0.08)	(0.00)
Complex		-11.82^{***}	-11.56^{***}		-11.75^{***}	-11.67^{***}
		(1.59)	(1.58)		(1.70)	(0.00)
Return Beliefs × Complex		-0.52^{***}	-0.46^{***}		-0.60^{***}	-0.52^{***}
		(0.11)	(0.11)		(0.13)	(0.00)
(Intercept)	55.61^{***}	59.90^{***}	48.04^{***}	55.36^{***}	59.56^{***}	46.78^{***}
	(0.80)	(0.97)	(3.36)	(0.84)	(0.99)	(0.00)
Controls	N	N	Y	N	N	Y
R^2	0.06	0.15	0.21	0.02	0.11	0.15
Num. obs.	1198	1198	1198	2354	2354	2354

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 25: Treatment vs. Attenuation in Counterfactual Investment Task, Full Sample

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.43^{***}	1.81^{***}	1.79^{***}
	(0.08)	(0.10)	(0.10)
Complex		-3.31^{*}	-2.69
		(1.58)	(1.61)
Cfact. Returns × Complex		-1.07^{***}	-1.06^{***}
		(0.16)	(0.16)
(Intercept)	47.29^{***}	48.41^{***}	40.42^{***}
	(0.79)	(0.99)	(3.53)
Controls	N	N	Y
R^2	0.21	0.25	0.28
Num. obs.	1198	1198	1198

OLS estimates, standard errors (in parentheses) are robust. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 26: Behavioral Response to Information, Full Sample

	Dependent Variable: Change in Equity Share					
	(1)	(2)	(3)	(4)	(5)	(6)
ΔBeliefs	1.42***	2.32***	2.00***	1.07***	1.42***	1.26***
	(0.24)	(0.44)	(0.36)	(0.17)	(0.24)	(0.21)
High Cfact. CU		0.48	−0.10			
		(1.65)	(1.53)			
ΔBeliefs × High Cfact. CU		−1.48**	−1.22**			
		(0.52)	(0.46)			
Complex					−0.10	−0.36
					(1.16)	(1.17)
ΔBeliefs × Complex					−0.90**	−0.75*
					(0.31)	(0.33)
(Intercept)	0.55	0.51	−0.22	0.62	0.55	−1.65
	(0.83)	(1.17)	(4.35)	(0.60)	(0.83)	(3.16)
Baseline Belief Controls	N	N	Y	N	N	Y
Demographic Controls	N	N	Y	N	N	Y
Num. obs.	742	742	742	1143	1143	1143

IV estimates instrumenting for the change in beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 27: Counterfactual CU vs. Information Acquisition, Full Sample

	Dep. Variable: Belief CU		*Dep. Variable*: Freq. Acquired Stock Info		*Dep. Variable*: Acquired Expert Estimate		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cfact. CU	0.35***	0.33***	−1.25***	−0.84***	−0.08	−0.17*	−0.20**
	(0.03)	(0.03)	(0.22)	(0.19)	(0.07)	(0.07)	(0.07)
Belief CU						0.27***	0.26***
						(0.08)	(0.08)
(Intercept)	0.44***	0.63***	1.86***	−0.85***	0.54***	0.42***	0.53***
	(0.02)	(0.04)	(0.11)	(0.20)	(0.03)	(0.05)	(0.11)
Controls	N	Y	N	Y	N	N	Y
R^2	0.14	0.19	0.04	0.30	0.00	0.02	0.03
Num. obs.	776	776	776	776	776	776	776

OLS estimates, standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 28: Implied Information Weight vs. CU Measures, Full Sample

	Dependent Variable: Implied Information Weight					
	(1)	(2)	(3)	(4)	(5)	(6)
Cfact. CU	0.36***	0.36***	0.29***			
	(0.05)	(0.05)	(0.05)			
Belief CU				0.45***	0.47***	0.36***
				(0.06)	(0.06)	(0.06)
(Intercept)	45.01***	42.69***	90.23***	32.99***	29.71***	74.70***
	(2.89)	(2.96)	(8.04)	(3.82)	(3.83)	(8.84)
Baseline Belief Controls	N	Y	Y	N	Y	Y
Demographic Controls	N	N	Y	N	N	Y
R^2	0.05	0.07	0.15	0.08	0.09	0.16
Num. obs.	653	653	653	653	653	653

OLS estimates, standard errors (in parentheses) are robust. Demographic controls include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 29: Information Acquisition vs. Treatment, Full Sample

	Dependent Variable: Acquired Expert Estimate		
	(1)	(2)	(3)
Complex	−0.10***	−0.11***	−0.10***
	(0.03)	(0.03)	(0.03)
Belief CU		0.15**	0.16**
		(0.06)	(0.06)
Freq. Acquired Stock Info		−0.03***	−0.04***
		(0.01)	(0.01)
(Intercept)	0.51***	0.47***	0.33***
	(0.02)	(0.04)	(0.08)
Controls	N	N	Y
R^2	0.01	0.03	0.04
Num. obs.	1198	1198	1198

OLS estimates, standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Restricting to Sample of Investors. The following tables demonstrate that the main predictions hold for the subsample of subjects who report that they invest in stocks. In particular, Tables 30–31 provide evidence for Prediction 1a: that higher counterfactual CU predicts greater attenuation between beliefs and behavior, whereas Tables 32–33 provide evidence for Prediction 1b: that cross-sectional attenuation is higher in the complex treatment relative to the standard treatment. Columns 1–3 of Table 34 provide evidence for Prediction 2a: that higher counterfactual CU predicts a weaker behavioral response to information, whereas columns 4–6 provide evidence for Prediction 2b: that the behavioral response to information is weaker in the complex treatment relative to the standard treatment. Tables 35–36 provide evidence for Prediction 3a: in particular, columns 1 and 2 of Table 35 show that subjects with higher counterfactual CU report greater belief CU, columns 3 and 4 of Table 35 show that subjects with higher counterfactual CU report acquiring information about the stock market at a lower frequency, and columns 1–3 of Table 36 show that the return expectations of subjects with higher counterfactual CU are more responsive to receiving information. Table 37 provides evidence for Prediction 3b: that subjects exhibit lower demand for information in complex treatment relative to the standard treatment.

Table 30: Counterfactual CU vs. Attenuation in Baseline Investment Task, Investor Sample

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	1.01***	1.38***	1.43***	1.09***	1.46***	1.49***
	(0.12)	(0.15)	(0.14)	(0.12)	(0.16)	(0.16)
High Cfact. CU		−0.32	1.56		−0.58	1.60
		(2.34)	(2.29)		(2.38)	(2.40)
Return Beliefs × High Cfact. CU		−0.75***	−0.75***		−0.72**	−0.74**
		(0.22)	(0.21)		(0.23)	(0.22)
(Intercept)	61.03***	61.14***	43.46***	60.64***	60.82***	43.40***
	(1.19)	(1.51)	(7.04)	(1.20)	(1.60)	(8.07)
Controls	N	N	Y	N	N	Y
R^2	0.17	0.20	0.26	0.18	0.21	0.26
Num. obs.	534	534	534	1018	1018	1018

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. “High Cfact. CU” is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 31: Counterfactual CU vs. Attenuation in Counterfactual Investment Task, Investor Sample

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.93***	2.38***	2.29***
	(0.12)	(0.16)	(0.16)
High Cfact. CU		−2.54	−1.23
		(2.32)	(2.33)
Cfact. Returns × High Cfact. CU		−1.05***	−0.89***
		(0.23)	(0.23)
(Intercept)	49.67***	50.37***	39.49***
	(1.16)	(1.69)	(5.78)
Controls	N	N	Y
R^2	0.34	0.37	0.40
Num. obs.	534	534	534

OLS estimates, standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 32: Treatment vs. Attenuation in Baseline Investment Task, Investor Sample

	Dependent Variable: Equity Share, Baseline Task					
	OLS			ORIV		
	(1)	(2)	(3)	(4)	(5)	(6)
Return Beliefs	0.78***	1.01***	1.05***	0.83***	1.09***	1.11***
	(0.09)	(0.12)	(0.11)	(0.09)	(0.12)	(0.00)
Complex		−14.46***	−13.78***		−14.12***	−13.48***
		(1.98)	(1.96)		(2.04)	(0.00)
Return Beliefs × Complex		−0.81***	−0.81***		−0.84***	−0.83***
		(0.18)	(0.18)		(0.19)	(0.00)
(Intercept)	56.28***	61.03***	49.91***	55.97***	60.64***	49.76***
	(0.99)	(1.19)	(5.44)	(1.01)	(1.20)	(0.00)
Controls	N	N	Y	N	N	Y
R^2	0.09	0.22	0.26	0.10	0.22	0.26
Num. obs.	793	793	793	1518	1518	1518

Standard errors (in parentheses) are robust, and clustered at the subject level for ORIV estimates. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 33: Treatment vs. Attenuation in Counterfactual Investment Task, Investor Sample

	Dependent Variable: Equity Share, Counterfactual Task		
	(1)	(2)	(3)
Cfact. Returns	1.54^{***}	1.93^{***}	1.89^{***}
	(0.10)	(0.12)	(0.12)
Complex		-4.75^{*}	-4.27^{*}
		(2.12)	(2.13)
Cfact. Returns × Complex		-1.15^{***}	-1.14^{***}
		(0.21)	(0.21)
(Intercept)	47.97^{***}	49.67^{***}	49.14^{***}
	(0.99)	(1.16)	(3.44)
Controls	N	N	Y
R^2	0.23	0.28	0.30
Num. obs.	793	793	793

OLS estimates, standard errors (in parentheses) are robust. Control variables include (1) risk aversion (de-meaned) and its interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 34: Behavioral Response to Information, Investor Sample

	Dependent Variable: Change in Equity Share					
	(1)	(2)	(3)	(4)	(5)	(6)
ΔBeliefs	1.46***	2.58***	2.36***	1.16***	1.46***	1.33***
	(0.27)	(0.51)	(0.48)	(0.21)	(0.27)	(0.25)
High Cfact. CU		1.63	0.99			
		(1.82)	(1.79)			
ΔBeliefs × High Cfact. CU		−1.85**	−1.69**			
		(0.58)	(0.57)			
Complex					1.61	1.46
					(1.79)	(1.97)
ΔBeliefs × Complex					−0.91*	−0.83
					(0.43)	(0.49)
(Intercept)	−0.96	−1.39	0.75	−0.45	−0.96	−0.95
	(0.95)	(1.30)	(6.75)	(0.80)	(0.95)	(5.39)
Baseline Belief Controls	N	N	Y	N	N	Y
Demographic Controls	N	N	Y	N	N	Y
Num. obs.	509	509	509	752	752	752

IV estimates instrumenting for the change in beliefs and its interactions using the expert estimate and its corresponding interactions. Standard errors (in parentheses) are robust. "High Cfact. CU" is a dummy for whether CU is above the median CU of subjects in the standard treatment. Control variables include (1) beliefs about the standard deviation of S&P 500 returns (de-meaned) and risk aversion (de-meaned), as well as their interactions with baseline beliefs, and (2) demographic controls, which include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 35: Counterfactual CU vs. Information Acquisition, Investor Sample

	Dep. Variable: Belief CU		*Dep. Variable*: Freq. Acquired Stock Info		*Dep. Variable*: Acquired Expert Estimate		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cfact. CU	0.38***	0.36***	−1.42***	−1.01***	−0.05	−0.13	−0.16
	(0.04)	(0.04)	(0.26)	(0.25)	(0.08)	(0.09)	(0.09)
Belief CU						0.20*	0.21*
						(0.10)	(0.10)
(Intercept)	0.41***	0.57***	2.37***	−0.58	0.52***	0.43***	0.61***
	(0.02)	(0.07)	(0.13)	(0.38)	(0.04)	(0.05)	(0.16)
Controls	N	Y	N	Y	N	N	Y
R^2	0.16	0.20	0.05	0.17	0.00	0.01	0.02
Num. obs.	534	534	534	534	534	534	534

OLS estimates, standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 36: Implied Information Weight vs. CU Measures, Investor Sample

	Dependent Variable: Implied Information Weight					
	(1)	(2)	(3)	(4)	(5)	(6)
Cfact. CU	0.36^{***}	0.36^{***}	0.29^{***}			
	(0.06)	(0.06)	(0.06)			
Belief CU				0.43^{***}	0.44^{***}	0.34^{***}
				(0.07)	(0.07)	(0.07)
(Intercept)	41.11^{***}	39.98^{***}	79.83^{***}	31.49^{***}	29.70^{***}	67.15^{***}
	(3.32)	(3.52)	(13.88)	(4.41)	(4.51)	(14.10)
Baseline Belief Controls	N	Y	Y	N	Y	Y
Demographic Controls	N	N	Y	N	N	Y
R^2	0.05	0.06	0.13	0.07	0.07	0.13
Num. obs.	458	458	458	458	458	458

OLS estimates, standard errors (in parentheses) are robust. Demographic controls include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

Table 37: Information Acquisition vs. Treatment, Investor Sample

	Dependent Variable: Acquired Expert Estimate		
	(1)	(2)	(3)
Complex	-0.10^{**}	-0.10^{**}	-0.10^{**}
	(0.04)	(0.04)	(0.04)
Belief CU		0.16^{*}	0.16^{*}
		(0.07)	(0.07)
Freq. Acquired Stock Info		-0.04^{***}	-0.04^{***}
		(0.01)	(0.01)
(Intercept)	0.49^{***}	0.48^{***}	0.35^{**}
	(0.02)	(0.05)	(0.12)
Controls	N	N	Y
R^2	0.01	0.04	0.05
Num. obs.	793	793	793

OLS estimates, standard errors (in parentheses) are robust. Control variables include age, gender, college education, income, stock market participation, and financial literacy.
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$.

A.7 Experimental Instructions

Instructions.

This study will consist of two parts.

Part 1
In Part 1 of the study, you will complete three **estimation tasks** in which we will ask you to give estimates on the performance of the stock market. For example, you may be asked to estimate the change in value of a particular investment over the next 12 months.

Part 2
In Part 2 of the study, you will complete three **investment tasks.** In each investment task, you will decide how to split $1000 between two investment accounts. Over the next 12 months, the values of these investment accounts may increase or decrease, based on a procedure described in the task. **Your bonus payment will depend on the performance of your investments over the next 12 months.**

Below, we give an example of investment task and describe the procedure for determining your bonus payment. This procedure may seem complicated, but all it means is that **you should invest the money as well as possible in each investment task, since your investments will affect your bonus payment.**

Your Bonus Payment
12 months after the conclusion of the study, we will compute your final wealth for each task, which will be equal to the total value of your investments for that task.

For example, suppose that the following investment task, you invested $500 in Account A and $500 in Account B, as shown below:

> Suppose you are given $1000. You can allocate this money between two investments:
>
> - **Account A**: The value of this investment will increase by 20% over the next 12 months.
> - **Account B**: The value of this investment will increase by 10% over the next 12 months.
>
> What proportion of the $1000 would you invest in Account A?
>
> Investment in **Account A**: $500
> Investment in **Account B**: $500
>
> 0% 25% 50% 75% 100%

Your final wealth for this task would be

($500 in Account A) × 1.20 + ($500 in Account B) × 1.10 = $1150.

At the end of the study, there is a 10% chance that one of the three investment tasks will be randomly selected for payment. If an investment task is selected for payment, your bonus payment (which you will receive 12 months after the study) will be equal to your final wealth for that task divided by 100. For example, if this investment task above were to be selected for payment, you would earn a bonus of $11.50.

Certainty Questions

For the estimation and investment tasks, you may be uncertain over the quality of your estimate, or uncertain over whether you actually made the best investment. After some of these tasks, we will ask you a **certainty question** in which you will indicate how certain you are in your estimate or your investment decision.

For example, suppose in the investment task described above, you invested $500 in Account A and $500 in Account B. The certainty question for this task would look like this:

You indicated that you would invest **$500** of the $1000 in Account A.

How certain are you that you would be best off investing between $480 and $520 in Account A, given your own preferences and the available information?

Very uncertain **Completely certain**

0% 25% 50% 75% 100% %

For this question, you would move the slider to indicate your level of certainty.

Comprehension Check Questions.

To proceed with the study, you must correctly answer the comprehension checks below. You will have two chances to answer the comprehension checks correctly.

Click here to review the instructions.

Which of the following statements correctly describes how values of the investment accounts in each of the investment tasks will change over the next 12 months?

- The value of the investment accounts will always increase over the next 12 months.
- The value of the investment accounts will always decrease over the next 12 months.
- The value of some investment accounts will always increase over the next 12 months, while the value of other investment accounts will always decrease over the next 12 months.
- The value of each investment account may either increase or decrease over the next 12 months, depending on the procedure described in the task.

Suppose that in an investment task that involved splitting $1000 between investment accounts A and B, you invested $500 in Account A and $500 in Account B. If the value of Account A increased by 20% over the next 12 months and the value of Account B did not change over the next 12 months, which of the following statements would be true?

- My final wealth would have be higher if I had invested more money in Account A.
- My final wealth would have be higher if I had invested more money in Account B.
- My final wealth would not depend on how I invested the $1000.

In the investment task above, continue to suppose that you invested $500 in Account A and $500 in Account B. Which of the following statements about your final wealth for this task is correct?

- Based on my investment, my final wealth for this task will depend only on the value of Account A.
- Based on my investment, my final wealth for this task will depend only on the value of Account B.
- Based on my investment, my final wealth for this task will depend on the values of both Account A and Account B.

In the investment task above, suppose that you are uncertain whether investing $500 in Account A is actually the right investment. In particular, suppose that you are only 30% certain that you would be best off investing between $480 and $520 in Account A, given your own preferences and the available information.

Please move the slider to accurately reflect this level of certainty.

Very uncertain **Completely certain**

0% 25% 50% 75% 100%

%

Baseline Belief Elicitation.

The S&P 500 is an American stock market index that includes 500 of the largest companies based in the United States.

Consider the value of an investment in the S&P 500. Do you expect the value of the investment to increase or decrease over the next 12 months?

Increase

In percentage terms, by how much do you expect the value of such an investment to increase over the next 12 months?

12 % increase over the next 12 months

According to your estimate, if you invested $100 in the S&P 500 today, you would expect the investment to be worth **$ 112** in 12 months, a return of **12 %**.

Belief CU Elicitation.

Each year, professional forecasters provide their estimates of the return of the S&P 500. To arrive at an **expert estimate** of the S&P 500 returns over this year, we averaged the estimates of S&P 500 returns made by a sample of these professional forecasters.

How certain are you that the expert estimate of the return of the S&P 500 is within 3 percentage points of your own estimate (between 9% and 15%)?

Very uncertain **Completely certain**

0% 25% 50% 75% 100%

20 %

Subjective Return Variance Elicitation.

Consider five possible scenarios for S&P 500 returns over the next 12 months: the S&P 500 returns can be

- greater than 40%
- between 25% and 40%
- between -5% and 25%
- between -20% and -5%
- less than -20%

Enter the probability, in percent, that you attach to each scenario. The probabilities of the five scenarios have to sum to 100%.

greater than 40%	10 %
between 25% and 40%	30 %
between -5% and 25%	40 %
between -20% and -5%	20 %
less than -20%	0 %
Total:	**100** %

Note: The ranges of returns used for the subjective return variance elicitation are given by $\{(-\infty, \hat{\theta}^{rd} - 30), (\hat{\theta}^{rd} - 30, \hat{\theta}^{rd} - 15), (\hat{\theta}^{rd} - 15, \hat{\theta}^{rd} + 15), (\hat{\theta}^{rd} + 15, \hat{\theta}^{rd} + 30), (\hat{\theta}^{rd} + 30, \infty)\}$, where $\hat{\theta}^{rd}$ is the subjects baseline return expectation, rounded to the nearest 5%.

Repeated Belief Elicitation.

If you were to invest $100 in the S&P 500 today, how much would you expect your investment to be worth <u>after 12 months</u>?

$ 112

Baseline Investment Task.

Suppose you are given $1000. You can allocate this money between two investments:

- **Stock Account**: The value of this investment tracks the value of the S&P 500 over the next 12 months. This means that the return on this investment will be equal to the return of the S&P 500 over the next 12 months.
- **Bank Account**: The value of this investment will increase by a guaranteed 2% over the next 12 months.

Your final wealth will be the value of your investment in the Stock Account after 12 months plus the value of your investment in the Bank Account after 12 months.

What proportion of the $1000 would you invest in the Stock Account?

Investment in **Stock Account**: $200
Investment in **Bank Account**: $800

0% 25% 50% 75% 100%

Baseline Investment Task: Complex Treatment.

Suppose you are given $1000. You can allocate this money between two investments:

- **Account A**: The value of this investment will track the value of the portfolio below:

Name	Portfolio Weight	Description (hover for details)
SDS	15%	Inverse leveraged S&P 500 ETF
UPRO	35%	Leveraged S&P 500 ETF
SH	25%	Inverse S&P 500 ETF
SPUU	25%	Leveraged S&P 500 ETF

Designed to produce daily returns that are 2x the daily returns of the S&P 500

- **Account B**: The value of this investment ... :

Name	Portfolio Weight	Description (hover for details)
IVV	25%	S&P 500 ETF
SPXU	15%	Inverse leveraged S&P 500 ETF
SPDN	10%	Leveraged S&P 500 ETF
Interest Account	50%	4% APY interest account

Click here for more details on the portfolios.

Your final wealth will be the value of your investment in Account A after 12 months plus the value of your investment in Account B after 12 months.

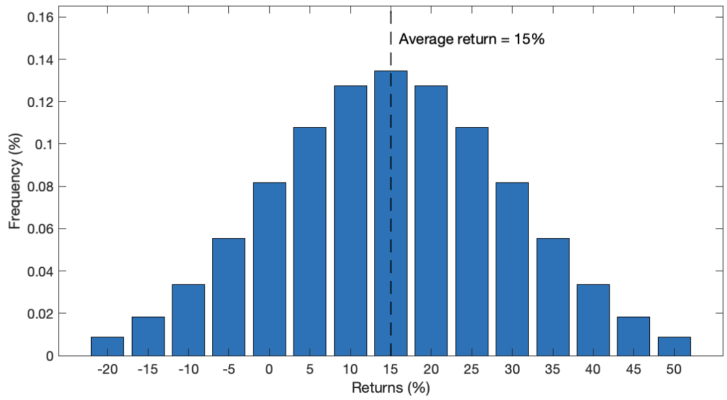
What proportion of the $1000 would you invest in Account A?

Investment in **Account A**: $500
Investment in **Account B**: $500

0% 25% 50% 75% 100%

Counterfactual Investment Task.

Suppose you knew that the 12-month S&P 500 return will be a random draw from a range of returns between -20% and 50%. The figure below gives a visual representation of the likelihood that each value of returns is drawn.



According to this procedure, **the 12-month S&P 500 return will be 15% on average**. This means that on average, a $100 investment in the S&P 500 will be worth **$115** after 12 months.

Note: Since returns are random, the actual return may be higher or lower than the average return, as shown in the figure.

Now, suppose you are given $1000. You can allocate this money between two investments:

- **Stock Account**: The value of this investment tracks the value of the S&P 500 over the next 12 months. This means that the return on this investment will be equal to the return of the S&P 500 over the next 12 months.
- **Bank Account**: The value of this investment will increase by a guaranteed 2% over the next 12 months.

Your final wealth will be the value of your investment in the Stock Account after 12 months plus the value of your investment in the Bank Account after 12 months.

Knowing that the S&P 500 return will be determined by the procedure described above, what proportion of the $1000 would you invest in the Stock Account?

Investment in **Stock Account**: $500
Investment in **Bank Account**: $500

0% 25% 50% 75% 100%

Counterfactual CU Elicitation.

On the previous screen, you indicated that you would invest **$500** of the $1000 in the Stock Account if you knew the S&P 500 return was determined by the procedure we described. In this next question, we are interested in **how certain** you are in your decision.

How certain are you that you would actually be best off investing between $480 and $520 in the stock account, given your own preferences and the available information?

Very uncertain **Completely certain**

0% 25% 50% 75% 100%

15 %

Repeated Investment Task: Information Acquisition.

Suppose you are given $1000. You can allocate this money between two investments:

- **Stock Account**: The value of this investment tracks the S&P 500. This means that the return on this investment will be equal to the return of the S&P 500 over the next 12 months.
- **Bank Account**: The value of this investment will increase by a guaranteed 2% over the next 12 months.

Before you decide your investment, we would like to give you a chance to obtain an **expert estimate** of the return of the S&P 500 over the next 12 months. This estimate is the average forecast made by a sample of professional forecasters.

How useful do you think this information will be in helping you decide your investment above?

Not very useful

Taking this into account: would you rather obtain the estimate, or instead receive $0.20 in additional bonus payment?

Obtain the expert estimate | Receive $0.20 in bonus payment

Repeated Investment Task: Information Intervention.

Your choice was not selected to count. Therefore, you will receive the expert estimate.

Note: this estimate will be shown to you only once and you will not be able to go back to the estimate later in the study.

Expert Estimate

Below, we report the consensus expert estimate of the 12-month S&P 500 return, which we compute by averaging the estimates made by a sample of professional forecasters.

According to this consensus estimate, the expected return of the S&P 500 over this year will be **3.2%**. This means that the forecasters expect a $100 investment in the S&P 500 to be worth **$103.2** on average after one year.

To check your understanding of the estimate, please answer the question below.

What was the consensus expert estimate of the return of the S&P 500 over this year?

3.2 %

Self-Reported Information Gathering.

How frequently did you gather information about the performance of the S&P 500 or the stock market in the last 3 months?

Not at all	Once a month	Twice a month	Weekly	Several times a week	Daily

Extension: Hypothetical Decisions, Intervention Treatment.

What proportion of the $1000 would you invest in the **Stock Account** if you expect the S&P 500 return to be:

0% 25% 50% 75% 100%

0% on average? (range: -15% to 15%)	0 % in stocks
5% on average? (range: -10% to 20%)	23 % in stocks
10% on average? (range: -5% to 25%)	43 % in stocks
15% on average? (range: 0% to 30%)	60 % in stocks
20% on average? (range: 5% to 35%)	% in stocks

Extension: Investment Task, Intervention Treatment.

Your hypothetical investment decisions are summarized below.

	0% 25% 50% 75% 100%	
S&P 500 return: **0%**		0 % in stocks
S&P 500 return: **5%**		23 % in stocks
S&P 500 return: **10%**		43 % in stocks
S&P 500 return: **15%**		60 % in stocks
S&P 500 return: **20%**		70 % in stocks

Now, please make your actual investment decision.

What proportion of the $1000 would you invest in the **Stock Account**?
Remember: you estimated that the S&P 500 would have a 12-month return of **15%**.

Investment in **Stock Account**: $610
Investment in **Bank Account**: $390

0% 25% 50% 75% 100%

61 % in stocks

Extension: Investment Task, Control Treatment.

Suppose you are given $1000. You can allocate this money between two investments:

- **Stock Account**: The value of this investment tracks the S&P 500. This means that the return on this investment will be equal to the return of the S&P 500 over the next 12 months.
- **Bank Account**: The value of this investment will increase by a guaranteed 2% over the next 12 months.

Your final wealth will be the value of your investment in the Stock Account after 12 months plus the value of your investment in the Bank Account after 12 months.

What proportion of the $1000 would you invest in the Stock Account?
Remember: you estimated that the S&P 500 would have a 12-month return of **15%**.

Investment in **Stock Account**: $590
Investment in **Bank Account**: $410

0% 25% 50% 75% 100%