Spreading Out in Expanding Idea Space

Ina Ganguli
UMass–Amherst

Jeffrey Lin
FRB Philadelphia

Vitaly Meursault
FRB Philadelphia

Nicholas Reynolds
U. Essex

February 2026

Introduction

This paper is about idea space:

- The **spatial structure** of **inventions** in **the market for new ideas**

This paper is about idea space:

≡ The **spatial structure** of **inventions** in **the market for new ideas**

To fix ideas, consider "idea production" of publishing in top economics journals

- **Choosing Position**: Which topic/idea to work on? Competition, spillovers
- **Rising Bar**: New data, better methods, richer models, more robustness checks
- **Expanding Frontier**: More papers, more teams, more & new topics

This paper is about idea space:

≡ The **spatial structure** of **inventions** in **the market for new ideas**

To fix ideas, consider "idea production" of publishing in top economics journals

- **Choosing Position**: Which topic/idea to work on? Competition, spillovers
- **Rising Bar**: New data, better methods, richer models, more robustness checks
- **Expanding Frontier**: More papers, more teams, more & new topics

Research Questions:

Q1: What determines **inventor positioning** in idea space?

Q2: What are the **consequences** of inventor positioning?

Q3: How do we **measure** idea space positioning to test predictions?

This Paper

A1: Spatial model of positioning in idea space

- Goal: Baseline **spatial competition mechanism**, complementing other factors
- Differentiated ideas (adaptation costs → positioning matters) Salop 1979
- Knowledge spillovers vs. competition Bloom et al. 2013, Dasgupta and Maskin 1987
- Sunk and variable costs (burden of knowledge, fishing out) Jones 2009, Kortum 1997

This Paper

A1: Spatial model of positioning in idea space

- Goal: Baseline **spatial competition mechanism**, complementing other factors
- Differentiated ideas (adaptation costs → positioning matters) Salop 1979
- Knowledge spillovers vs. competition Bloom et al. 2013, Dasgupta and Maskin 1987
- Sunk and variable costs (burden of knowledge, fishing out) Jones 2009, Kortum 1997

A2: The model has surprising implications beyond just positioning...

This Paper

A1: Spatial model of positioning in idea space

- Goal: Baseline **spatial competition mechanism**, complementing other factors
- Differentiated ideas (adaptation costs → positioning matters) Salop 1979
- Knowledge spillovers vs. competition Bloom et al. 2013, Dasgupta and Maskin 1987
- Sunk and variable costs (burden of knowledge, fishing out) Jones 2009, Kortum 1997

A2: The model has surprising implications beyond just positioning…

A3: Validated measurement framework

- Systematic comparison using domain-specific tasks
- GTE embeddings outperform TF-IDF; cover 1836–2023

Roadmap

Part I: A Spatial Model of Idea Space

- What determines inventor positioning?

Part II: Model Predictions

- Comparative statics and growth implications
- (Spoiler: They match facts beyond just inventor positioning)

Part III: Testing the Predictions

- Measurement challenge and validation
- Evidence from 188 years of U.S. patents

Part I: A Theory of Invention in Idea Space

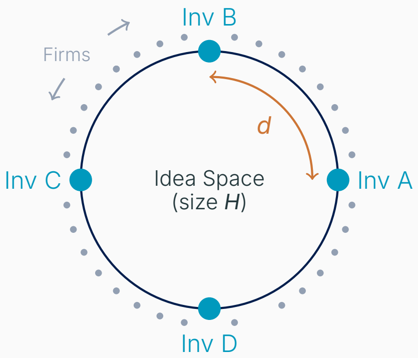
Model Setup: Spatial Competition in Idea Space

Idea space: Circle of circumference H

- H = **size of market for new ideas**
- "Similar problems have similar solutions"

Idea producers ("inventors" or "inventions"):

Idea consumers ("downstream firms"):



Market for new ideas as Salop (1979) circle

Model Setup: Spatial Competition in Idea Space

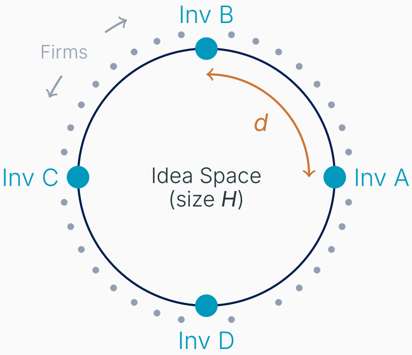
Idea space: Circle of circumference H

- H = **size of market for new ideas**
- "Similar problems have similar solutions"

Idea producers ("inventors" or "inventions"):

- Choose: **entry**, location, quality q_i, price p_i
- License non-rival ideas downstream
- "Entry" = undertaking a project ($\neq$ firm)
- "Inventors" = Individuals, teams, or firms

Idea consumers ("downstream firms"):



Market for new ideas as Salop (1979) circle

Model Setup: Spatial Competition in Idea Space

Idea space: Circle of circumference H

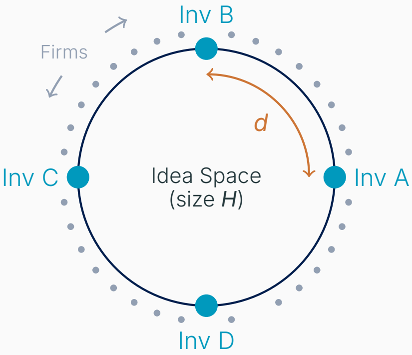
- H = **size of market for new ideas**
- "Similar problems have similar solutions"

Idea producers ("inventors" or "inventions"):

- Choose: **entry**, location, quality q_i, price p_i
- License non-rival ideas downstream
- "Entry" = undertaking a project ($\neq$ firm)
- "Inventors" = Individuals, teams, or firms

Idea consumers ("downstream firms"):

- Distributed uniformly on circle
- License ideas to boost their TFP



Market for new ideas as Salop (1979) circle

Idea Consumers: Downstream Firms

Setup: Mass H of downstream firms uniformly distributed on circle

- Each firm licenses one idea to improve productivity
- Firm location = preferred technological variety

Idea Consumers: Downstream Firms

Setup: Mass H of downstream firms uniformly distributed on circle

- Each firm licenses one idea to improve productivity
- Firm location = preferred technological variety

TFP from licensing: Firm at distance h from invention i achieves **log TFP**:

$$A_i(h) = Q_i - \tau h$$

- Q_i = realized quality of invention i (including spillovers)
- τh = **adaptation cost** from technological mismatch (Bloom et al. 2013, Arora et al. 2021)

Idea Consumers: Downstream Firms

Setup: Mass H of downstream firms uniformly distributed on circle

- Each firm licenses one idea to improve productivity
- Firm location = preferred technological variety

TFP from licensing: Firm at distance h from invention i achieves **log TFP**:

$$A_i(h) = Q_i - \tau h$$

- Q_i = realized quality of invention i (including spillovers)
- τh = **adaptation cost** from technological mismatch (Bloom et al. 2013, Arora et al. 2021)

Net surplus: Firm chooses invention to maximize:

$$\text{Surplus} = \underbrace{Q_i - \tau h}_{\text{TFP gain}} - \underbrace{p_i}_{\text{license fee}}$$

- Adaptation costs create product differentiation among inventions

R&D Technology: Costs and Licensing

R&D investment: Inventor i produces idea of quality q_i at cost:

$$c(q_i) = \frac{1}{2}\gamma q_i^{1+\eta}$$

- $\eta > 0 \Rightarrow$ diminishing returns to R&D effort. Baseline: $\eta = 1$ (quadratic costs).
- Captures "fishing out": harder to improve idea quality Kortum 1997

R&D Technology: Costs and Licensing

R&D investment: Inventor *i* produces idea of quality q_i at cost:

$$c(q_i) = \frac{1}{2}\gamma q_i^{1+\eta}$$

- $\eta > 0 \Rightarrow$ diminishing returns to R&D effort. Baseline: $\eta = 1$ (quadratic costs).
- Captures "fishing out": harder to improve idea quality Kortum 1997

Non-rival licensing:

- Ideas are **non-rival**—can license to multiple firms at zero marginal cost
- Inventor charges license fee p_i to each downstream firm in territory
- Revenue = $p_i \times$ (number of firms served)

R&D Technology: Costs and Licensing

R&D investment: Inventor i produces idea of quality q_i at cost:

$$c(q_i) = \frac{1}{2}\gamma q_i^{1+\eta}$$

- $\eta > 0 \Rightarrow$ diminishing returns to R&D effort. Baseline: $\eta = 1$ (quadratic costs).
- Captures "fishing out": harder to improve idea quality Kortum 1997

Non-rival licensing:

- Ideas are **non-rival**—can license to multiple firms at zero marginal cost
- Inventor charges license fee p_i to each downstream firm in territory
- Revenue = $p_i \times$ (number of firms served)

Entry cost: Fixed cost f (sunk costs, setup costs)

Knowledge Spillovers

Realized quality incorporates spillovers from neighbors:

$$Q_i = q_i + \frac{\beta}{2}\left(1 - \frac{d}{\lambda}\right) q_{i-1} + \frac{\beta}{2}\left(1 - \frac{d}{\lambda}\right) q_{i+1}$$

Parameters:

- q_i = own R&D investment
- $\beta \in (0,1)$ = spillover intensity
- λ = spillover reach (spillovers vanish beyond distance λ)
- d = distance to nearest neighbor

Key property: Spillovers **decay with distance**

- At $d = 0$: maximum spillover βq
- At $d = \lambda$: spillovers vanish

Proximity → spillovers, but also → competition

Equilibrium Analysis

Equilibrium: Pricing and Quality

Symmetric equilibrium: n inventions, equal spacing $d = H/n$, identical (p, q) ▸ Existence

Equilibrium: Pricing and Quality

Symmetric equilibrium: n inventions, equal spacing $d = H/n$, identical (p, q) ▸ Existence

Equilibrium pricing (standard differentiated-goods logic): $\boxed{p^* = \tau d}$

- Price proportional to spacing
- Adaptation costs τ create pricing power through differentiation

Equilibrium: Pricing and Quality

Symmetric equilibrium: n inventions, equal spacing $d = H/n$, identical (p, q) ▸ Existence

Equilibrium pricing (standard differentiated-goods logic): $\boxed{p^* = \tau d}$

- Price proportional to spacing
- Adaptation costs τ create pricing power through differentiation

Equilibrium quality (MR = MC for quality investment): $\boxed{q^* = \frac{d}{\gamma}}$

- Quality proportional to spacing
- Larger territories $\Rightarrow$ higher quality investment
- Key insight: adaptation costs make this *necessary*, not just profitable

Equilibrium: Pricing and Quality

Symmetric equilibrium: n inventions, equal spacing $d = H/n$, identical (p, q) ▸ Existence

Equilibrium pricing (standard differentiated-goods logic): $\boxed{p^* = \tau d}$

- Price proportional to spacing
- Adaptation costs τ create pricing power through differentiation

Equilibrium quality (MR = MC for quality investment): $\boxed{q^* = \frac{d}{\gamma}}$

- Quality proportional to spacing
- Larger territories $\Rightarrow$ higher quality investment
- Key insight: adaptation costs make this *necessary*, not just profitable

Both price and quality rise as inventions spread out

Free Entry Determines Equilibrium Spacing and Inventions

Zero-profit condition:

$$\underbrace{\tau d^2}_{\text{Revenue}} - \underbrace{\frac{d^2}{2\gamma}}_{\text{R\&D cost}} - \underbrace{f}_{\text{Entry cost}} = 0$$

Solving for equilibrium spacing and number of inventions ($n = H/d$):

$$\boxed{d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}} \quad \Rightarrow \quad \boxed{n^* = H\sqrt{\frac{\tau - \frac{1}{2\gamma}}{f}}}$$

Symmetric equilibrium p^*, q^*, d^*, n^* in terms of costs τ, γ, f, and market size H

Everything Is Connected

Spacing	Pricing	Quality	Varieties
$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$	$p^* = \tau d$	$q^* = \frac{d}{\gamma}$	$n^* = \frac{H}{d^*}$

Everything Is Connected

Spacing	Pricing	Quality	Varieties
$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$	$p^* = \tau d$	$q^* = \frac{d}{\gamma}$	$n^* = \frac{H}{d^*}$

Notice how both horizontal and vertical features are coupled by spatial forces:

- Spacing **depends on costs** (+fixed f, −variable γ, −adaptation τ)
- Price and quality **depend on spacing** ($p^* = f(d)$, $q^* = f(d)$)
- Number of varieties **depends on idea space size** H and costs

Everything Is Connected

Spacing	Pricing	Quality	Varieties
$d^* = \sqrt{\dfrac{f}{\tau - \frac{1}{2\gamma}}}$	$p^* = \tau d$	$q^* = \dfrac{d}{\gamma}$	$n^* = \dfrac{H}{d^*}$

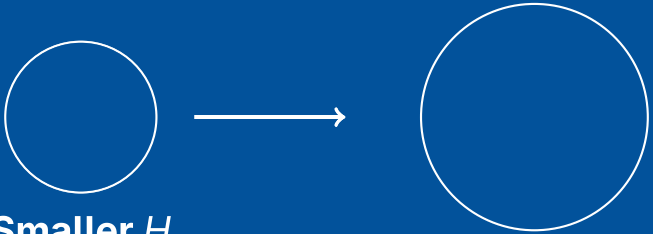
Notice how both horizontal and vertical features are coupled by spatial forces:

- Spacing **depends on costs** ($+$fixed f, $-$variable γ, $-$adaptation τ)
- Price and quality **depend on spacing** ($p^* = f(d)$, $q^* = f(d)$)
- Number of varieties **depends on idea space size** H and costs

Positioning is tied to costs (cf. **Q1**) **and quality and pricing too** (**Q2**)

- The size of the market H matters for variety.
- **Key question:** Could spacing, price and quality also depend on H?

Expanding Idea Space



Smaller *H*

Larger *H*

Expanding Idea Space

Evidence:

- More U.S. patents: 500/year (1840s) → 350,000/year (2020s)
- New technological domains: electricity, chemistry, semiconductors, software…
- More firms doing R&D Hirschey et al. 2012
- Growing scientific knowledge stock

Expanding Idea Space

Evidence:

- More U.S. patents: 500/year (1840s) → 350,000/year (2020s)
- New technological domains: electricity, chemistry, semiconductors, software...
- More firms doing R&D Hirschey et al. 2012
- Growing scientific knowledge stock

Why does H grow?

- Knowledge accumulation opens new possibilities
- Technology frontiers expand into new domains
- Demand for new solutions increases with income, population

Expanding Idea Space

Evidence:

- More U.S. patents: 500/year (1840s) → 350,000/year (2020s)
- New technological domains: electricity, chemistry, semiconductors, software...
- More firms doing R&D Hirschey et al. 2012
- Growing scientific knowledge stock

Why does H grow?

- Knowledge accumulation opens new possibilities
- Technology frontiers expand into new domains
- Demand for new solutions increases with income, population

Our strategy: $\dot{H}$ exogenous—to establish baseline spatial competition mechanism

Expanding Idea Space

Evidence:

- More U.S. patents: 500/year (1840s) → 350,000/year (2020s)
- New technological domains: electricity, chemistry, semiconductors, software...
- More firms doing R&D Hirschey et al. 2012
- Growing scientific knowledge stock

Why does H grow?

- Knowledge accumulation opens new possibilities
- Technology frontiers expand into new domains
- Demand for new solutions increases with income, population

Our strategy: $\dot{H}$ exogenous—to establish baseline spatial competition mechanism

Question: How does equilibrium adjust as H grows?

The Key Structural Relationship

Recall equilibrium spacing:

$$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$$

The Key Structural Relationship

Recall equilibrium spacing:

$$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$$

As idea space H grows, which parameters might change?

- Adaptation cost τ: Mismatch penalty
- R&D cost γ: Production technology
- **Entry cost f: Could respond to idea space size H $\leftarrow$ Our focus**

The Key Structural Relationship

Recall equilibrium spacing:

$$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$$

As idea space H grows, which parameters might change?

- Adaptation cost τ: Mismatch penalty
- R&D cost γ: Production technology
- **Entry cost f: Could respond to idea space size H $\leftarrow$ Our focus**

Specifying $f(H)$:

- Different relationships $f(H)$ generate different predictions (next slide)
- In principle, $\tau(H)$ or $\gamma(H)$ could also vary—though with less empirical support

Four Scenarios: How Predictions Depend on $f(H)$

As idea space H grows, what happens to spacing d^* and variety n^*?

Scenario	Spacing d^*	Varieties n^*
1. f constant	unchanged	$\uparrow$ (linear in H)
2. $f(H)$ decreasing (easier to invent)	$\downarrow$ (clustering!)	$\uparrow\uparrow$ (faster growth)
3. $f(H)$ increasing (harder to invent)	$\uparrow$ (spreading)	$\uparrow$ (grows with H)
4. $f(H)$ increasing rapidly	$\uparrow\uparrow$ rapidly	$\downarrow$ (fewer inventions)

Analysis of $\tau(H)$ or $\gamma(H)$ follows similarly

Four Scenarios: How Predictions Depend on $f(H)$

As idea space H grows, what happens to spacing d^* and variety n^*?

Scenario	**Spacing d^***	**Varieties n^***
1. f constant	unchanged	$\uparrow$ (linear in H)
2. $f(H)$ decreasing (easier to invent)	$\downarrow$ (clustering!)	$\uparrow\uparrow$ (faster growth)
3. $f(H)$ increasing (harder to invent)	$\uparrow$ (spreading)	$\uparrow$ (grows with H)
4. $f(H)$ increasing rapidly	$\uparrow\uparrow$ rapidly	$\downarrow$ (fewer inventions)

Analysis of $\tau(H)$ or $\gamma(H)$ follows similarly

Key insight:

- Quality $q^* = d/\gamma$ and price $p^* = \tau d$ move with spacing
- Growth implications differ dramatically across scenarios

Four Scenarios: How Predictions Depend on $f(H)$

As idea space H grows, what happens to spacing d^* and variety n^*?

Scenario	Spacing d^*	Varieties n^*
1. f constant	unchanged	$\uparrow$ (linear in H)
2. $f(H)$ decreasing (easier to invent)	$\downarrow$ (clustering!)	$\uparrow\uparrow$ (faster growth)
3. $f(H)$ increasing (harder to invent)	$\uparrow$ (spreading)	$\uparrow$ (grows with H)
4. $f(H)$ increasing rapidly	$\uparrow\uparrow$ rapidly	$\downarrow$ (fewer inventions)

Analysis of $\tau(H)$ or $\gamma(H)$ follows similarly

Key insight:

- Quality $q^* = d/\gamma$ and price $p^* = \tau d$ move with spacing
- Growth implications differ dramatically across scenarios

So which scenario describes reality?

$f(H)$ Increasing: The Burden of Knowledge

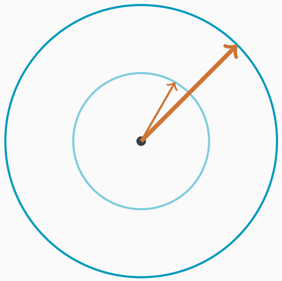
Why might entry costs rise with idea space size? $(f'(H) > 0)$

$f(H)$ Increasing: The Burden of Knowledge

Why might entry costs rise with idea space size? $(f'(H) > 0)$

Empirical evidence: Jones (2009)

- Inventors getting older at first patent
- Larger teams
- More education
- Longer training periods



More effort to reach the frontier

$f(H)$ Increasing: The Burden of Knowledge

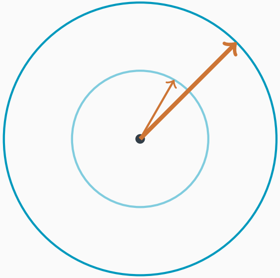
Why might entry costs rise with idea space size? $(f'(H) > 0)$

Empirical evidence: Jones (2009)

- Inventors getting older at first patent
- Larger teams
- More education
- Longer training periods

Mechanism in idea space:

- More prior art to master before contributing
- More labor and managerial costs
- Sophisticated tools/equipment required



More effort to reach the frontier

$f(H)$ Increasing: The Burden of Knowledge

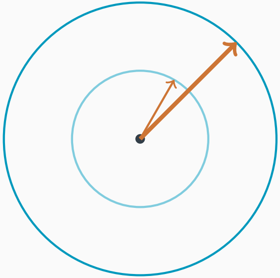
Why might entry costs rise with idea space size? $(f'(H) > 0)$

Empirical evidence: Jones (2009)

- Inventors getting older at first patent
- Larger teams
- More education
- Longer training periods

Mechanism in idea space:

- More prior art to master before contributing
- More labor and managerial costs
- Sophisticated tools/equipment required



More effort to reach the frontier

In idea space: Entry costs rise with market size

Our Model: Entry Costs Rise with Idea Space

Burden of knowledge implies:

$$f(H) = \phi H^{\alpha}, \quad \alpha > 0, \phi > 0$$

Our Model: Entry Costs Rise with Idea Space

Burden of knowledge implies:

$$f(H) = \phi H^{\alpha}, \quad \alpha > 0, \phi > 0$$

Baseline calibration: $\alpha = 1$ (linear)

- Robust to $\alpha \in (0, 2)$

Our Model: Entry Costs Rise with Idea Space

Burden of knowledge implies:

$$f(H) = \phi H^{\alpha}, \quad \alpha > 0, \phi > 0$$

Baseline calibration: $\alpha = 1$ (linear)

- Robust to $\alpha \in (0, 2)$

This generates many predictions:

1. **Spreading out:** $\boxed{d^* = \sqrt{\frac{\phi H^{\alpha}}{\tau - \frac{1}{2\gamma}}}}$ — increases with H Proposition
2. and more...

Part II: Model Predictions

Model Predictions to Evidence

Model Predictions Match Three Categories of Literature Evidence

1. Positioning & Variety (Extensive Margin: $dd/dH > 0, dn/dH > 0$)

Comparative statics

- ✓ Spreading out over time **this paper**, Kelly+ 2021, **Chiopris 2024**
- ✓ More inventions, more firms, expanding idea space this paper, Hirschey+ 2012

Model Predictions Match Three Categories of Literature Evidence

1. Positioning & Variety (Extensive Margin: $dd/dH > 0, dn/dH > 0$)

Comparative statics

- ✓ Spreading out over time **this paper**, Kelly+ 2021, **Chiopris 2024**
- ✓ More inventions, more firms, expanding idea space this paper, Hirschey+ 2012

2. Quality & Returns (Intensive Margin: $dq/dH > 0, dp/dH > 0, d(p \cdot d)/dH > 0$)

- ✓ More R&D investment per firm Hirschey+ 2012
- ✓ Higher gross returns to patents Kogan+ 2017, Bessen+ 2018
- ✓ Higher patent quality Hall+ 2005, **Kelly+ 2021**
- ✓ R&D spillovers stable ($dq/dH \approx -1 \cdot dd/dH$) **Lucking+ 2019**

Replication

Model Predictions Match Three Categories of Literature Evidence

1. Positioning & Variety (Extensive Margin: $dd/dH > 0, dn/dH > 0$)

▸ Comparative statics

- ✓ Spreading out over time **this paper**, Kelly+ 2021, **Chiopris 2024**
- ✓ More inventions, more firms, expanding idea space this paper, Hirschey+ 2012

2. Quality & Returns (Intensive Margin: $dq/dH > 0, dp/dH > 0, d(p \cdot d)/dH > 0$)

- ✓ More R&D investment per firm Hirschey+ 2012
- ✓ Higher gross returns to patents Kogan+ 2017, Bessen+ 2018
- ✓ Higher patent quality Hall+ 2005, **Kelly+ 2021**
- ✓ R&D spillovers stable ($dq/dH \approx -1 \cdot dd/dH$) **Lucking+ 2019**

▸ Replication

3. Productivity Decline (explained next)

- ✓ TFP growth decelerates Bloom+ 2020
- ✓ R&D productivity declines Bloom+ 2020

Model Predictions Match Three Categories of Literature Evidence

1. Positioning & Variety (Extensive Margin: $dd/dH > 0, dn/dH > 0$)

▸ Comparative statics

- ✓ Spreading out over time **this paper**, Kelly+ 2021, **Chiopris 2024**
- ✓ More inventions, more firms, expanding idea space this paper, Hirschey+ 2012

2. Quality & Returns (Intensive Margin: $dq/dH > 0, dp/dH > 0, d(p \cdot d)/dH > 0$)

- ✓ More R&D investment per firm Hirschey+ 2012
- ✓ Higher gross returns to patents Kogan+ 2017, Bessen+ 2018
- ✓ Higher patent quality Hall+ 2005, **Kelly+ 2021**
- ✓ R&D spillovers stable ($dq/dH \approx -1 \cdot dd/dH$) **Lucking+ 2019**

▸ Replication

3. Productivity Decline (explained next)

- ✓ TFP growth decelerates Bloom+ 2020
- ✓ R&D productivity declines Bloom+ 2020

Our spatial model unifies many streams of empirical evidence

Declining R&D Productivity

Define aggregate R&D productivity (cf. Bloom et al. 2020)

$$\Pi \equiv \frac{\text{Agg TFP growth}}{\text{Agg R\&D}}$$

Agg TFP growth $= q\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]-\frac{\tau d}{4}$

- **Average** ΔTFP delivered downstream
- Doesn't scale with n

Agg R&D $= n \cdot \left[\frac{1}{2}\gamma q^2 + \phi H\right]$

- **Total** R&D across n inventions
- Scales with n

Key insight: As H expands, entry *dilutes* aggregate R&D cf. Howitt 1999, Peretto 1998, 2018

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH}\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda}\frac{dd}{dH}}_{\text{(1) Spillover attenuation}} - \underbrace{\frac{\tau}{4}\frac{dd}{dH}}_{\text{(2) Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH}\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda}\frac{dd}{dH}}_{\text{(1) Spillover attenuation}} - \underbrace{\frac{\tau}{4}\frac{dd}{dH}}_{\text{(2) Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance
2. **Adaptation drag** Downstream firms farther from inventions

Forces raising R&D:

$$\frac{d(\text{Agg R\&D})}{dH} = \underbrace{\frac{dn}{dH}\cdot[c(q)+f(H)]}_{\text{(5) Entry expansion}} + \underbrace{n\cdot c'(q)}_{\text{(3) Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)^*} + \underbrace{n\cdot f'(H)}_{\text{(4) Burden of knowledge}}$$

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH}\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda}\frac{dd}{dH}}_{\text{(1) Spillover attenuation}} - \underbrace{\frac{\tau}{4}\frac{dd}{dH}}_{\text{(2) Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance
2. **Adaptation drag** Downstream firms farther from inventions

Forces raising R&D:

$$\frac{d(\text{Agg R\&D})}{dH} = \underbrace{\frac{dn}{dH}\cdot[c(q)+f(H)]}_{\text{(5) Entry expansion}} + \underbrace{n\cdot c'(q)}_{\text{(3) Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)^*} + \underbrace{n\cdot f'(H)}_{\text{(4) Burden of knowledge}}$$

3. **Fishing out** Convex R&D costs

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH}\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda}\frac{dd}{dH}}_{\text{(1) Spillover attenuation}} - \underbrace{\frac{\tau}{4}\frac{dd}{dH}}_{\text{(2) Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance
2. **Adaptation drag** Downstream firms farther from inventions

Forces raising R&D:

$$\frac{d(\text{Agg R\&D})}{dH} = \underbrace{\frac{dn}{dH}\cdot[c(q)+f(H)]}_{\text{(5) Entry expansion}} + \underbrace{n\cdot c'(q)}_{\text{(3) Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)^*} + \underbrace{n\cdot f'(H)}_{\text{(4) Burden of knowledge}}$$

3. **Fishing out** Convex R&D costs
4. **Burden of knowledge** Rising fixed costs

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH}\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda}\frac{dd}{dH}}_{\text{(1) Spillover attenuation}} - \underbrace{\frac{\tau}{4}\frac{dd}{dH}}_{\text{(2) Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance
2. **Adaptation drag** Downstream firms farther from inventions

Forces raising R&D:

$$\frac{d(\text{Agg R\&D})}{dH} = \underbrace{\frac{dn}{dH}\cdot[c(q)+f(H)]}_{\text{(5) Entry expansion}} + \underbrace{n\cdot c'(q)}_{\text{(3) Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)^*} + \underbrace{n\cdot f'(H)}_{\text{(4) Burden of knowledge}}$$

3. **Fishing out** Convex R&D costs
4. **Burden of knowledge** Rising fixed costs
5. **Entry and territory expansion** More inventions cover larger territories

From Static Model to Growth Rates

If H grows at constant rate g_H ($\dot{H} = g_H \cdot H$) $\Rightarrow$ constant growth in:

Variable	**Growth Rate**	**Baseline ($\alpha = 1, \eta = 1$)**
Spacing d	$g_d = \frac{\alpha}{2} g_H$	$\frac{1}{2} g_H$
Quality q	$g_q = g_d$	$\frac{1}{2} g_H$
Entry n	$g_n = (1 - \frac{\alpha}{2}) g_H$	$\frac{1}{2} g_H$
Agg R&D	$g_{R\&D} = g_n + \theta g_q + \theta \eta g_q + (1-\theta)\alpha g_H$	$\frac{3}{2} g_H$

$\theta \equiv$ Variable cost share ▸ Detailed growth equations ▸ General model

Static model

$\longrightarrow$ Comparative statics as H grows exogenously

$\longrightarrow$ Testable predictions

Part III: Testing Model Predictions

From Growth Rates to Empirical Tests

Growth equations suggest empirical strategy:

Prediction 1: Spreading Out

- Model: $g_d = \frac{1}{2} g_H > 0$
- Empirical: Measure similarity over time $\rightarrow$ should decline
- Data: 188 years of U.S. patents (1836-2023)

Prediction 2: Declining R&D Productivity

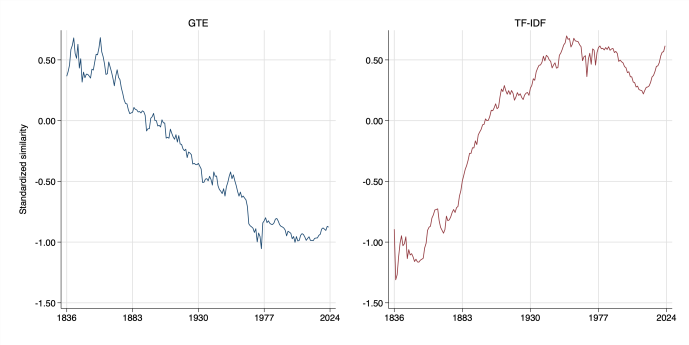
- Model: Five forces decomposition
- Empirical: Regress TFP and R&D growth on $-\Delta$Sim
- Decompose: Spatial (40-60%) vs non-spatial forces

First challenge: How do we measure similarity?

Measuring Similarity in Idea Space

The Measurement Challenge

Same patent text, opposite conclusions:



- **Left (GTE):** Similarity *declining* — inventions spreading out
- **Right (TF-IDF):** Similarity *increasing* — inventions clustering

Key Question: **Which "map" of idea space should we trust?**

Data: US Patent Claims, 1836–2023

Patent text corpus: ▸ Details

- **Historical (1836–1975):** ProQuest Patents Core (digitized full text)
- **Modern (1976–2023):** USPTO PatentsView
- Focus on **claims** — defines legal boundaries of invention

Multiple NLP representations tested:

- Traditional: TF-IDF (word frequency)
- Modern neural embeddings: GTE, PaECTER, S-BERT, Doc2vec, USE, OpenAI

Similarity measure: ▸ Computation

- Cosine similarity between patent representations
- Average pairwise similarity by year
- Standardized by cross-sectional standard deviation

▸ Alternatives

Validation Framework: Three Complementary Tasks

Task	Time Period	Granularity	Expertise	
Patent Interferences	2001–2014	Identical	USPTO examiners	
Human Judgments	1850–1975	Continuous	Lay annotators	
Classifications	1850–2023	Categorical	Expert labels	

Why multiple tasks?

- No single ground truth for “similarity”
- Different aspects: legal identity vs. technological relatedness
- Temporal robustness across 175+ years

Models performing well across all tasks are most reliable

Validation Results: Model Performance

Model	Interferences		Human	Classifications	
	PR AUC	F10	Agreement	Section	Class
GTE	0.64 (2)	**0.90** (1)	**0.62** (1)	0.596 (2)	0.656 (3)
PaECTER	**0.65** (1)	0.90 (2)	0.51 (3)	0.590 (3)	**0.672** (1)
S-BERT	0.52 (3)	0.82 (3)	0.54 (2)	**0.600** (1)	0.671 (2)
TF-IDF	0.45 (4)	0.77 (4)	0.35 (4)	0.514 (4)	0.525 (4)

- **GTE and PaECTER** consistently top performers
- **TF-IDF** consistently worst (20–40% lower performance)
- All beat **random chance** — but **magnitudes differ dramatically**

Model Selection: Why We Use GTE

GTE selected for main results because:

1. **Temporal robustness** — best on historical patents (1880–1920)
2. **Near-identical performance on interferences** — our most demanding test
3. **Consistent across all tasks** — ranks 1st or 2nd on 4/5 metrics

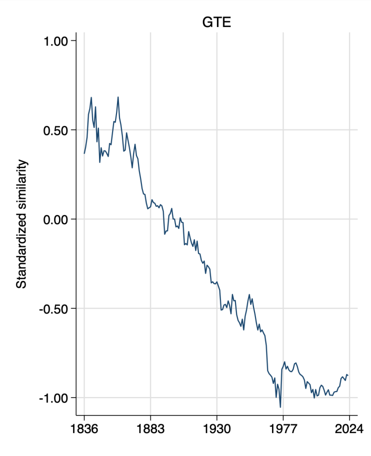
Why TF-IDF fails: ▸ Details

- Overweights period-specific language
- Treats synonyms as unrelated (“velocipede” $\neq$ “bicycle”)
- Would lead to *opposite* conclusions about our theory

Robustness checks with PaECTER, S-BERT, and ensemble measures

Prediction 1: Are Inventions Spreading Out?

Main Finding: Secular Decline in Patent Similarity



Average annual pairwise cosine similarity, standardized by cross-sectional SD. Indexed to 0 in 1900.

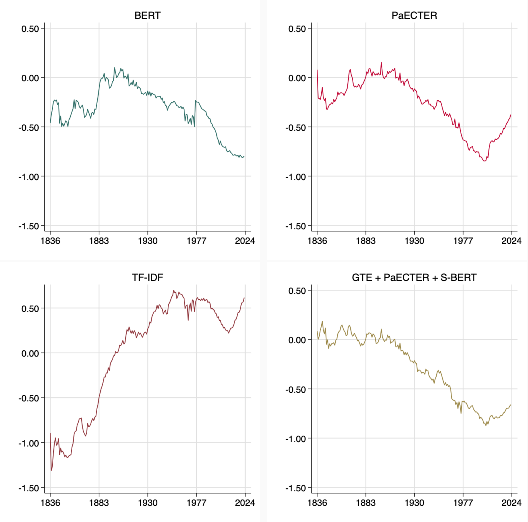
Using validated GTE embeddings:

$\sim$1.5σ decline in patent similarity, 1836–2023

- Consistent with theory: inventions spreading out
- Spreading out ($d\uparrow$) = Declining similarity (Sim $\downarrow$)
- Multi-patent entity effect post-2000 (to come)

Confirms Prediction 1: Spreading Out

Why Validation Matters: Comparing Representations



TF-IDF (worst performer):

- ~1.5σ increase—opposite conclusion!
- Validation correctly discards

PaECTER, S-BERT (cf. GTE):

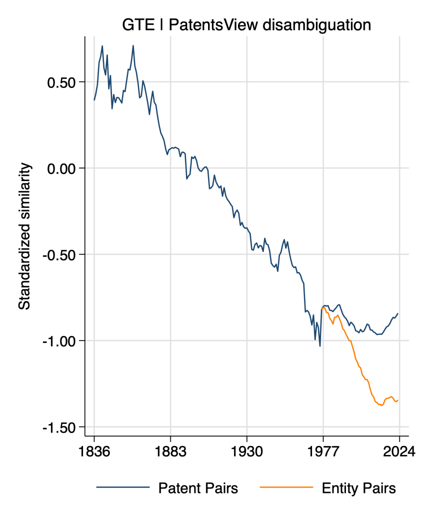
- Similar ~0.8σ decline, 1880–2000
- Diverge pre-1880 & post-2000

Ensemble (avg of top models):

- ~1.0σ decline, 1836–2023

Validated methods agree; unvalidated TF-IDF misleads

Robustness: Accounting for Multi-Patent Entities



Concern: Post-2000 dynamics coincide with: business method patents, non-practicing entities, increased defensive patenting.

▸ Patents v entities

- **Multiple patents from same entity may be similar but not independent.**

Strategy: Sample 1 patent/entity–year

Result:

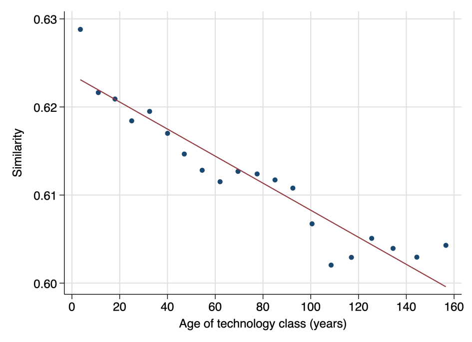
- Decline persists after correction
- **Independent inventions** still spreading out

Robustness: Spreading Out Within Technology Classes

Alternative explanations: Changing patent office practice over time? Shifts across major technology areas?

Test: Within-class similarity by class "age"

- Birth = Class first issued 50 patents
- e.g., Combinatorial Chemistry 2001
- Addresses compositional concerns ▸ Between

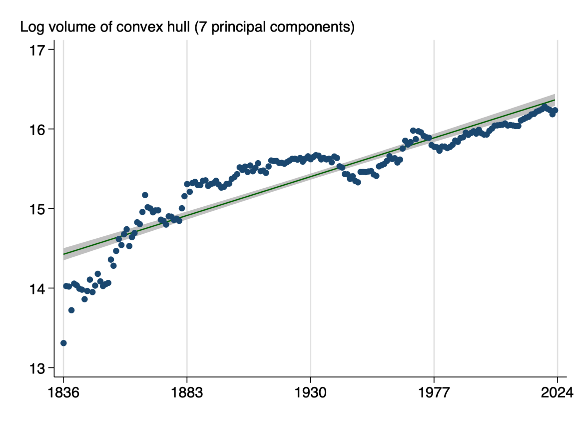


Finding: Within-class similarity declines as classes mature

Spreading out is a dynamic process tied to field evolution

Corroboration: Expanding Convex Hull

R&D regression



Is Idea Space Expanding?

Test:

- 1024 GTE dimensions to 7 PC
- Measure volume of convex hull

Result:

- +0.5%/yr (6 PC: +0.4%/yr)
- Likely under-estimate due to dimensionality reduction

Independent Corroboration: Declining Interference Rates

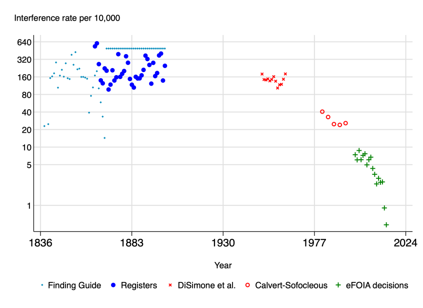
Patent interferences:

- USPTO determination that two independent inventors made *identical* inventions
- Direct measure of multiple invention ($d = 0$)

Data: Purpose-digitized from 5 sources

- Nat. Archives & Registers (1838–1900)
- Published statistics (1950–1994)
- eFOIA decisions (1998–2014) Ganguli et al. 2020

Finding: Interference rate declined over 150 years



Same conclusion from completely different data source

Summary: Inventions Are Spreading Out

Robust evidence of spreading out:

- ✓ Main finding: 1.5σ decline in similarity, 1836–2023
- ✓ Decline extends after 2000 for independent inventions
- ✓ Robust to spatial scale (local and global)
- ✓ Robust to within vs. between class decomposition
- ✓ Appears within classes as they age
- ✓ Corroborated by interference rates (150 years)
- ✓ Idea space is expanding

Next: What are the consequences for research productivity?

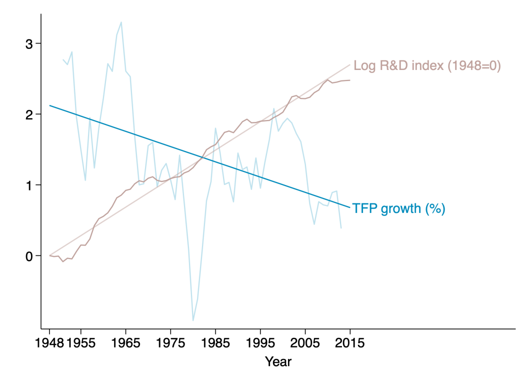
Prediction 2:
Does Spreading Out Reduce R&D Productivity?

The Puzzle: Are Ideas Getting Harder to Find?

The research productivity decline:

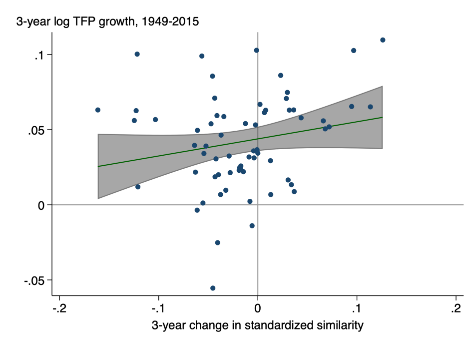
- Real R&D up **>20×** since 1930
- TFP growth slowed by factor of **3×**
- R&D productivity decline >-5%/yr

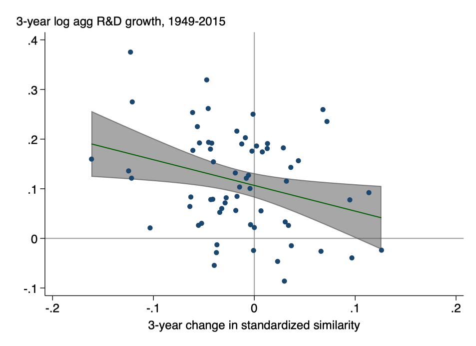
Key question: Why does it take so much **more** research effort to achieve ~~the same rate of~~ **slower** growth?



Bloom, Jones, Van Reenen and Webb, 2020

Timing: Similarity Predicts TFP and R&D Growth





- **Left:** Declining similarity → **lower TFP growth**
- **Right:** Declining similarity → **higher R&D growth**

Both patterns confirm Prediction 2

TFP and Spreading Out

From Theory to Estimation: TFP

TFP growth equation (from BGP):

$$g_{TFP} = \underbrace{g_q\left(1+\beta-\frac{\beta d}{\lambda}\right)}_{\text{Quality (with spillovers)}} - \underbrace{\frac{\beta q}{\lambda} g_d}_{\text{Spillover attenuation}} - \underbrace{\frac{\tau}{4} g_d}_{\text{Adaptation drag}}$$

Substitute equilibrium relationships for unobservables:

- $q^* = d/\gamma$ and $dq^*/dt = (1/\gamma)(dd/dt) \Rightarrow g_q = g_d$

$$g_{TFP} = \underbrace{(1+\beta-\frac{\tau}{4})}_{b_1} \cdot g_d - \underbrace{\beta(1+1/\gamma)/\lambda}_{b_2} \cdot d \cdot g_d$$

Suggests the regression:

- Observable proxy: $g_d \approx -\Delta\text{Sim}$ (small annual changes in standardized measure)

$$\Delta\log(\text{TFP})_t = b_0 + b_1 \cdot (-\Delta\text{Sim})_t + b_2 \cdot (-\Delta\text{Sim}) \cdot (-\text{Sim}_{t-1}) + \epsilon_t$$

From Theory to Estimation: TFP

Regression Specification:

$$\Delta \log(\mathrm{TFP})_t = b_0 + b_1 \cdot (-\Delta \mathrm{Sim})_t + b_2 \cdot (-\Delta \mathrm{Sim}) \cdot (-\mathrm{Sim}_{t-1}) + b_3 \cdot t + \epsilon_t$$

Data:

- TFP and Real R&D Inputs, 1948–2015 (Bloom et al., 2020)

Predictions and interpretation:

- $b_1 \lesseqgtr 0$: Effect on TFP growth from $\uparrow$ quality scaling net of $\downarrow$ adaptation costs
- $b_2 < 0$: Spillover attenuation and reduced marginal return to R&D
- b_3: Time trend controls for factors not explicit in the model

TFP Growth and Technological Distance

	Annual		3-Year	5-Year
$b_1 : -1 \times \Delta\text{Sim}$	-0.169^{***}	-0.171^{***}	-0.278^{***}	-0.269^{***}
	(0.057)	(0.083)	(0.095)	(0.098)
$b_2 : (-1 \times \Delta\text{Sim}) \times (-1 \times \text{Sim}_{t-1})$	—	-0.015	-0.408	-0.571^{*}
		(0.342)	(0.320)	(0.312)
Implied TFP drag from spreading out ($\overline{\Delta\text{Sim}}$, %/yr):				
1948 (Sim $=$ 0.35)	-0.08	-0.08	-0.07	-0.04
1991 (Sim $=$ 0, baseline)	-0.08	-0.09	-0.14	-0.16

TFP Growth and Technological Distance

	Annual		3-Year	5-Year
$b_1 : -1 \times \Delta\text{Sim}$	-0.169^{***}	-0.171^{***}	-0.278^{***}	-0.269^{***}
	(0.057)	(0.083)	(0.095)	(0.098)
$b_2 : (-1 \times \Delta\text{Sim}) \times (-1 \times \text{Sim}_{t-1})$	—	-0.015	-0.408	-0.571^{*}
		(0.342)	(0.320)	(0.312)
Implied TFP drag from spreading out ($\overline{\Delta\text{Sim}}$, %/yr):				
1948 ($\text{Sim} = 0.35$)	-0.08	-0.08	-0.07	-0.04
1991 ($\text{Sim} = 0$, baseline)	-0.08	-0.09	-0.14	-0.16

Validation: Implied drag (–0.16%/yr) from ΔSim consistent with quasi-experimental cross-sectional elasticity ✓ Bloom et al. (2013) (–0.14 to –0.16%/yr, 1981–2001)

TFP Growth and Technological Distance

	Annual		3-Year	5-Year
$b_1 : -1 \times \Delta\text{Sim}$	-0.169^{***}	-0.171^{***}	-0.278^{***}	-0.269^{***}
	(0.057)	(0.083)	(0.095)	(0.098)
$b_2 : (-1 \times \Delta\text{Sim}) \times (-1 \times \text{Sim}_{t-1})$	—	-0.015	-0.408	-0.571^{*}
		(0.342)	(0.320)	(0.312)
Implied TFP drag from spreading out ($\overline{\Delta \text{Sim}}$, %/yr):				
1948 (Sim $=$ 0.35)	-0.08	-0.08	-0.07	-0.04
1991 (Sim $=$ 0, baseline)	-0.08	-0.09	-0.14	-0.16

Validation: Implied drag (–0.16%/yr) from ΔSim consistent with quasi-experimental cross-sectional elasticity ✓ Bloom et al. (2013) (–0.14 to –0.16%/yr, 1981–2001)

Contribution to TFP deceleration: Drag worsened -0.04%/yr (1948) → -0.14%/yr (2015). Change = 0.10 pp = **7% of 1.4 pp total TFP deceleration**.

▸ Decomposition

R&D and Spreading Out

From Theory to Estimation: R&D

R&D growth equation (from BGP):

$$g_{R\&D} = \underbrace{g_n}_{\text{Entry}} + \underbrace{\theta(1+\eta)g_q}_{\text{Quality (incl. fishing out)}} + \underbrace{(1-\theta)g_f}_{\text{Rising fixed costs}}$$

Substitute equilibrium relationships:

$$g_{R\&D} = \underbrace{[1+\alpha(1-\theta)]g_H}_{a_0} + \underbrace{[\theta(1+\eta)-1]}_{a_1} g_d$$

Regression specification:

$$g_{R\&D,t} = a_0 + a_1 \cdot (-\Delta \text{Sim})_t + a_2 \cdot t + \epsilon_t$$

- a_2 captures (unmodeled) acceleration in idea space growth (but: $\hat{a}_2 \approx 0$)

Identification of Structural Parameters

Identification of structural parameters:

$$a_1 = \theta(1+\eta) - 1 \quad \Downarrow \quad \boxed{\theta = \frac{a_1 + 1}{1+\eta}}$$

(variable cost share)

$$a_0 = [1+\alpha(1-\theta)]g_H \quad \Downarrow \quad \boxed{g_H = \frac{a_0}{1+\alpha(1-\theta)}}$$

(idea space growth)

Baseline: $\alpha = 1$, $\eta = 1$. **Later:** Calibration w/ quasi-experimental $\hat{\eta}$ and estimate of α.

Regression coefficients $\rightarrow$ structural parameters (θ, g_H)

R&D Growth and Technological Distance

	Annual	3-Year	5-Year
a_1: $-1 \times \Delta\mathrm{Sim}$	0.165	**0.448****	**0.438***
	(0.177)	(0.219)	(0.244)
a_0: Constant	**0.034*****	**0.102*****	**0.173*****
	(0.006)	(0.013)	(0.018)
Implied θ (variable cost share)	0.58	0.72	**0.72**
Implied g_H (idea space growth)	2.4%/yr	2.7%/yr	**2.7%/yr**

Validation:

- $\theta = 72\%$ aligns with NSF survey data (labor = 69% of R&D) ✓
- $g_H = 2.7\%$/yr consistent with patent embedding volume growth ✓
- BGP consistency: Model predicts $g_d/g_{R\&D} = 1/3$; In data, $-\Delta\mathrm{Sim}/g_{R\&D} = 0.31$ ✓

Growth Accounting

The Research Productivity Decline

Research productivity:

$$\Pi \equiv g_{TFP}/\text{Agg R\&D (TFP growth per unit R\&D)}$$

The decline (1948–2015):

- TFP growth fell: 2.1%/yr → 0.7%/yr ($g_{g_{TFP}} = -1.6\%/\text{yr}$)
- R&D spending grew: 4.0%/yr

$$g_\Pi = g_{g_{\text{TFP}}} - g_{\text{R\&D}} = -1.6\% - 4.0\% = \boxed{-5.6\%/\text{yr}}$$

Goal: Decompose this decline into spatial and non-spatial components

From Regressions to Parameters

What we estimated from R&D regression:

Parameter	Value	Source
θ (variable cost share)	0.72	R&D regression coefficient a_2 $(\eta = 1)$
g_H (idea space growth)	2.7%/yr	R&D regression constant a_0 $(\alpha = 1)$

What we assume (baseline):

Parameter	Value	Interpretation
α (entry cost curvature)	1.0	Entry costs scale linearly with H
η (R&D cost curvature)	1.0	Quadratic R&D costs

Decomposing the R&D Productivity Decline

Model implies: $g_d = \frac{\alpha}{2} g_H = 1.35\%/\text{yr}$ (spreading rate if $\alpha = 1$)

Component	Contribution	Classification	Comment
TFP deceleration	–1.6%/yr		
Spatial drag worsened	–0.11%/yr	**Spatial**	7% of deceleration ▸ TFP regression
Unmodeled factors	–1.49%/yr	Non-spatial	

Decomposing the R&D Productivity Decline

Model implies: $g_d = \frac{\alpha}{2} g_H = 1.35\%/\text{yr}$ (spreading rate if $\alpha = 1$)

Component	Contribution	Classification	Comment
TFP deceleration	$-1.6\%/\text{yr}$		
Spatial drag worsened	$-0.11\%/\text{yr}$	**Spatial**	7% of deceleration ▸ TFP regression
Unmodeled factors	$-1.49\%/\text{yr}$	Non-spatial	
R&D growth	$+4.0\%/\text{yr}$		
Entry expansion $(1-\frac{\alpha}{2})g_H$	$+1.35\%/\text{yr}$	**Spatial**	(new inventions)
Quality scaling $(\theta \frac{\alpha}{2} g_H)$	$+0.97\%/\text{yr}$	**Spatial**	(larger territories; TFP units)
Fishing out $(\theta \eta \frac{\alpha}{2} g_H)$	$+0.97\%/\text{yr}$	Non-spatial	(convex costs)
Burden of knowledge $(1-\theta)(\alpha g_H)$	$+0.76\%/\text{yr}$	Non-spatial	(rising fixed costs)
Unmodeled factors	$-0.05\%/\text{yr}$	Non-spatial	

Decomposing the R&D Productivity Decline

Model implies: $g_d = \frac{\alpha}{2} g_H = 1.35\%/\text{yr}$ (spreading rate if $\alpha = 1$)

Component	Contribution	Classification	Comment
TFP deceleration	–1.6%/yr		
Spatial drag worsened	–0.11%/yr	**Spatial**	7% of deceleration ▸ TFP regression
Unmodeled factors	–1.49%/yr	Non-spatial	
R&D growth	+4.0%/yr		
Entry expansion $(1-\frac{\alpha}{2})g_H$	+1.35%/yr	**Spatial**	(new inventions)
Quality scaling $(\theta \frac{\alpha}{2} g_H)$	+0.97%/yr	**Spatial**	(larger territories; TFP units)
Fishing out $(\theta \eta \frac{\alpha}{2} g_H)$	+0.97%/yr	Non-spatial	(convex costs)
Burden of knowledge $(1-\theta)(\alpha g_H)$	+0.76%/yr	Non-spatial	(rising fixed costs)
Unmodeled factors	–0.05%/yr	Non-spatial	
Total decline	–5.6%/yr		
Spatial contribution	–2.43%/yr		**43%**
Non-spatial contribution	–3.17%/yr		57%

Robustness: Spatial Share Increases with Better Calibration

Baseline assumptions: $\alpha = 1$, $\eta = 1$

Alternative calibration:

- $\eta = 0.625$: Guceri-Liu (2019)
- $\theta = 0.89$: From R&D regression a_2
- $\alpha = 0.76$: Constrain sum to 4.0%
- $g_H = 3.2\%/\text{yr}$: From R&D regression a_0
- $g_d = \frac{\alpha}{2} g_H = 1.2\%/\text{yr}$

Robustness: Spatial Share Increases with Better Calibration

Baseline assumptions: $\alpha = 1$, $\eta = 1$

Alternative calibration:

- $\eta = 0.625$: Guceri-Liu (2019)
- $\theta = 0.89$: From R&D regression a_2
- $\alpha = 0.76$: Constrain sum to 4.0%
- $g_H = 3.2\%/\text{yr}$: From R&D regression a_0
- $g_d = \frac{\alpha}{2} g_H = 1.2\%/\text{yr}$

Alternative decomposition:

	Baseline	Alternative
Entry expansion	1.35%/yr	1.98%/yr
Quality scaling	0.97%/yr	1.08%/yr
Fishing out	0.97%/yr	0.67%/yr
Burden of knowledge	0.76%/yr	0.27%/yr
Sum	4.05%/yr	4.00%/yr
Spatial share	**43%**	**57%**

Robustness: Spatial Share Increases with Better Calibration

Baseline assumptions: $\alpha = 1$, $\eta = 1$

Alternative calibration:

- $\eta = 0.625$: Guceri-Liu (2019)
- $\theta = 0.89$: From R&D regression a_2
- $\alpha = 0.76$: Constrain sum to 4.0%
- $g_H = 3.2\%$/yr: From R&D regression a_0
- $g_d = \frac{\alpha}{2} g_H = 1.2\%$/yr

Alternative decomposition:

	Baseline	**Alternative**
Entry expansion	1.35%/yr	1.98%/yr
Quality scaling	0.97%/yr	1.08%/yr
Fishing out	0.97%/yr	0.67%/yr
Burden of knowledge	0.76%/yr	0.27%/yr
Sum	4.05%/yr	4.00%/yr
Spatial share	**43%**	**57%**

Conservative baseline; higher spatial share with alternative calibration

- $\eta = 0.625 < 1$: R&D costs grow sub-quadratically with q
- $\hat{\alpha} = 0.76 < 1$: Entry costs grow sub-linearly with H
- Entry expansion (1.98%) < patent growth (3.9%) $\Rightarrow \downarrow$ **ideas per patent** (-1.9%/yr)

Conclusion

Summary

1. **Theory:**
   - Spatial model predicts as idea space expands, inventions spread out
   - Space unifies new & old evidence: horizontal, vertical, R&D productivity

2. **Measurement:** Validated NLP methods using domain-specific tasks
   - Representation choice fundamentally affects conclusions
   - GTE outperforms traditional workhorse TF-IDF

3. **Empirics:** Nearly 2 centuries of spreading out in expanding idea space
   - Robust across multiple tests and data sources
   - **Spatial forces can explain 40–60% of R&D productivity decline**

Backup Slides

Backup: Comparative Statics Derivations

Return

Spreading out: From zero-profit condition $d^2(\tau - \frac{1}{2\gamma}) = \phi H$:

$$\frac{dd}{dH} = \frac{\phi}{2d(\tau - \frac{1}{2\gamma})} = \frac{\phi}{dR/dd - dc/dd} > 0$$

Rising quality and prices:

$$\frac{dq}{dH} = \frac{1}{\gamma}\frac{dd}{dH} > 0, \qquad \frac{dp}{dH} = \tau\frac{dd}{dH} > 0$$

Rising entry:

$$\frac{dn}{dH} = \frac{1}{d} - \frac{H}{d^2}\frac{dd}{dH} > 0 \text{ under spreading-out condition}$$

Declining productivity:

$$\frac{d\rho}{dH} < 0, \qquad \frac{d\Pi}{dH} < 0$$

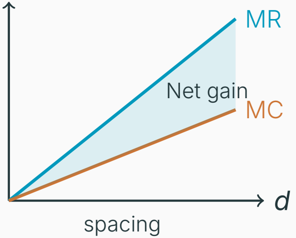
Backup: Spreading Out

Return

Proposition (Spreading Out)

For $\tau\gamma > \frac{1}{2}$, equilibrium spacing increases with opportunity space: $\dfrac{dd^*}{dH} > 0$.
Inventions become **less similar** over time.

Why is spreading out profitable?



Marginal revenue of expanding territory:

- Revenue $R = \tau d^2 \quad \Rightarrow \quad \text{MR} = 2\tau d$

Marginal cost of expanding territory:

- Need higher quality: $q = d/\gamma$
- $\text{MC} = d/\gamma$

Spreading profitable when:

$$\text{MR} > \text{MC} \quad \Rightarrow \quad \boxed{\tau\gamma > \frac{1}{2}}$$

Adaptation costs must create sufficient pricing power

Backup: Model Equations

Fixed cost (burden of knowledge):	$f(H) = \phi H^{\alpha}$
R&D cost:	$c(q_i) = \frac{1}{2}\gamma q_i^{1+\eta}$
Realized quality (with spillovers):	$Q_i = q_i + \frac{1}{2}\beta\left(1 - \frac{d}{\lambda}\right)(q_{i-1} + q_{i+1})$
Baseline model:	$\alpha = 1, \eta = 1$
Equilibrium pricing and quality:	$p^* = \tau d, \qquad q^* = \frac{d}{\gamma}$
Equilibrium spacing:	$d^*(H) = \sqrt{\frac{\phi H}{\tau - \frac{1}{2\gamma}}}$
Equilibrium entry:	$n^* = \frac{H}{d^*}$
Equilibrium revenue:	$R^* = p^* \cdot d^* = \tau d^2$

Backup: Robustness to Entry Cost Curvature

Return

With $f(H) = \phi H^{\alpha}$, equilibrium spacing satisfies $d \propto H^{\alpha/2}$:

Condition	Entry Growth	Prediction
$0 < \alpha < 2$	$g_n = (1 - \frac{\alpha}{2}) g_H > 0$	Entry grows ✓
$\alpha = 2$	$g_n = 0$	Entry stagnates
$\alpha > 2$	$g_n < 0$	Entry declines ×

Main results robust for $\alpha < 2$:

- Spreading out: $g_d = \frac{\alpha}{2} g_H > 0$ for any $\alpha > 0$
- Declining R&D productivity: Holds throughout range
- Higher $\alpha \longrightarrow$ faster spreading, but lower spatial share of productivity decline

Counterfactual boundary: Patent counts grow over time, ruling out $\alpha \geq 2$

Backup: TFP and R&D Growth Equations

Growth rates | TFP regression | R&D regression

TFP growth:

$$g_{TFP} = \underbrace{g_q\left(1+\beta-\frac{\beta d}{\lambda}\right)}_{\text{Quality (with spillovers)}} - \underbrace{\frac{\beta q}{\lambda} g_d}_{\text{Spillover attenuation}} - \underbrace{\frac{\tau}{4} g_d}_{\text{Adaptation drag}}$$

R&D growth:

$$g_{R\&D} = \underbrace{g_n}_{\text{Entry}} + \underbrace{\theta \cdot g_q}_{\text{Quality scaling}} + \underbrace{\theta \cdot g_q}_{\text{Fishing out}} + \underbrace{(1-\theta) g_f}_{\text{Burden of knowledge}}$$

where θ = variable cost share; α = 1, η = 1

The asymmetry:

- TFP: grows with g_q *minus* spatial drags
- R&D: grows with g_q *plus* entry (g_n) *plus* fixed costs (g_f)

$g_{R\&D} > g_{TFP} \Rightarrow$ **Research productivity declines**

Backup: Growth Equations with General Cost Curvatures

Growth Rates Decomposition

Component	General	Baseline ($\alpha = 1, \eta = 1$)
Entry cost	$f(H) = \phi H^{\alpha}$	ϕH
R&D cost	$c(q) = \frac{1}{2}\gamma q^{1+\eta}$	$\frac{1}{2}\gamma q^2$
Spacing growth	$g_d = \frac{\alpha}{2} g_H$	$\frac{1}{2} g_H$
Quality growth	$g_q = g_d$	$\frac{1}{2} g_H$
Entry growth	$g_n = (1 - \frac{\alpha}{2}) g_H$	$\frac{1}{2} g_H$
Fixed cost growth	$g_f = \alpha g_H$	g_H

R&D growth equation:

$$g_{R\&D} = \underbrace{(1 - \tfrac{\alpha}{2}) g_H}_{\text{Entry}} + \underbrace{\frac{\theta\alpha}{2} g_H}_{\text{Quality scaling}} + \underbrace{\frac{\theta\eta\alpha}{2} g_H}_{\text{Fishing out}} + \underbrace{(1-\theta)\alpha g_H}_{\text{Burden of knowledge}}$$

Log TFP Specification

◂ Return

Why linear in log TFP? $A_i(h) = Q_i - \tau h$

Standard in spatial competition (Salop 1979):

- Idea consumers have preferences linear in quality net of distance costs
- $A_i(h)$ interpreted as log TFP $\Rightarrow$ firms care about *proportional* productivity gains

Microfoundation: Each downstream firm has one unit of fixed input ℓ and produces:

$$y = e^A \cdot \ell$$

With output price = 1 and $\ell = 1$, profit is $\pi = e^A$. Willingness to pay for technology delivering incremental log TFP A (relative to baseline $e^0 = 1$):

$$WTP = e^A - 1 \approx A \quad \text{(first-order Taylor approximation)}$$

Accuracy: For annual TFP increments ($A \approx 0.015$/year), approximation error $< 0.01\%$

Advantage: Predictions directly comparable to empirical TFP elasticities (Bloom et al. 2013) and growth accounting (Bloom et al. 2020)

Backup: Equilibrium Existence Conditions

◂ Return

Spreading-out condition:

$$\tau\gamma > \frac{1}{2}$$

Marginal revenue of expanding territory exceeds marginal cost.

Second-order conditions:

- Pricing: $\partial^2 R/\partial p^2 < 0$ (satisfied)
- Quality: $\partial^2 \pi/\partial q^2 = -\gamma < 0$ (satisfied)
- No spatial deviation (verified in paper)

Additional conditions:

- Spillover reach: $d < \lambda$ (spillovers active)
- Full coverage: All downstream firms adopt some technology

Backup: Interference Validation Task

◂ Return

Patent interferences (2001–2014):

- **First to invent**: USPTO proceeding for multiple applicants w/ identical claims
- Provides ground truth for "identical" similarity
- 322 true interfering pairs among 96,580 application pairs

Economic intuition: Examiner ranks pairs by similarity, investigates above threshold

- Higher threshold → fewer false positives but miss true interferences
- Lower threshold → catch more but burden staff with unnecessary investigations

Metrics:

- F10: Weights recall 10× more than precision (missing interferences is costly)
- PR AUC: Precision-Recall area under curve across all thresholds

Key result: GTE, PaECTER, OpenAI retrieve ~90% of true interferences with 2–5× fewer false positives than TF-IDF/S-BERT

Backup: Human Annotation Task

Return

Historical patents (1850–1975):

- Sampled patent pairs that each model ranked at least 50 percentiles apart
- Annotators rank **relative** similarity of 2 patent pairs
- Tests temporal robustness (historical language): Oversample 1880–1920

Task: Do model rankings agree with human rankings?

- For each patent, rank others by similarity
- Compare model ranking to human ranking

Metric: Agreement coefficient from regression

$$\text{Human Rank} = \alpha + \beta \cdot \text{Model Rank} + \epsilon$$

Higher β = better agreement

Backup: Classification Validation Task

◂ Return

USPTO Classifications (1850–2023):

- CPC technology codes assigned by examiners
- Section level (8 categories) and Class level (120+ categories)
- Captures expert judgment of technological relatedness

Task: Predict whether patent pair shares classification

- Same Section (coarse): 8 top-level categories
- Same Class (fine): 3-digit classification

Metric: ROC AUC

- Area under Receiver Operating Characteristic curve
- 0.5 = random, 1.0 = perfect

Backup: Why TF-IDF Fails

◂ Return

TF-IDF overweights period-specific language:

- Treats “velocipede” (1880s) and “bicycle” (modern) as unrelated
- Period-specific terminology dominates similarity scores
- Creates spurious correlation with time

Example: 1880 velocipede patent

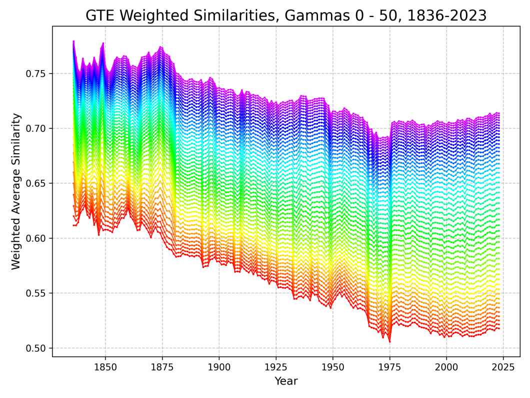
- TF-IDF: High similarity to other 1880s patents (shared vocabulary)
- GTE: High similarity to modern bicycle patents (shared concepts)

Evidence:

- TF-IDF similarity correlates with word overlap
- GTE similarity correlates with conceptual similarity
- Google Ngrams shows vocabulary shifts over time

Backup: Similarity at Different Spatial Scales

◀ Return



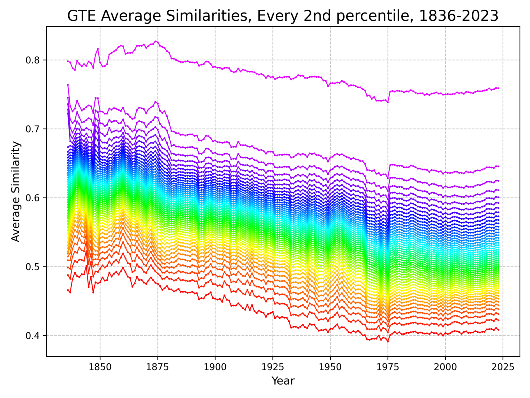
Weighted average: $\equiv \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j\neq i}(1-d_{ij})e^{-\gamma d_{ij}}}{\sum_{j\neq i} e^{-\gamma d_{ij}}}$

where γ from 0 (global) to 50 (local)

- **Key finding:** Similar declining trends across all spatial scales
- Model predictions concern averages — important to verify pattern holds across distribution
- Post-2000 arrest slightly stronger at local scales (consistent with entity correction)

Backup: Similarity at Different Quantiles

Return

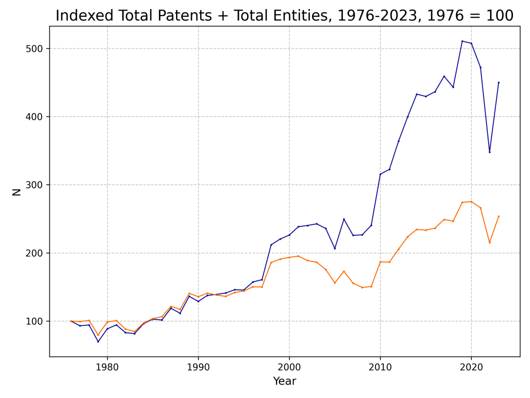


Similarity at Different Quantiles:

- 50 quantiles of pairwise similarity in each year
- Secular decline is robust across all quantiles
- Post-2000 increase in similarity is slightly faster for higher quantiles

Backup: Growth in Patents vs. Patenting Entities

Return

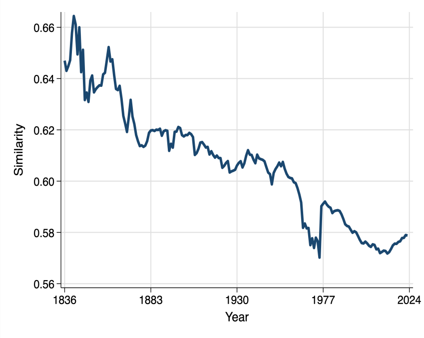


- Number of issued utility patents and unique patenting entities per year
- Divergence after 1999: substantial growth in patents per entity
- Driven by business method patents and non-practicing entities
- Motivates sampling 1 patent per entity per year for robustness

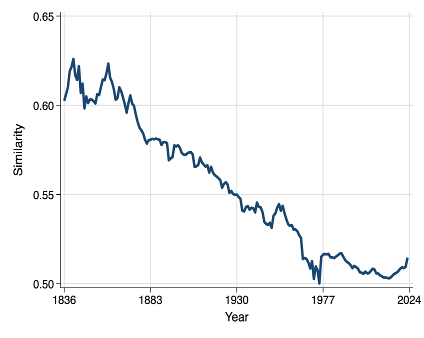
Backup: Within vs. Between Technology Classes

◂ Return

Within-class similarity



Between-class similarity



- **Addresses compositional concern:** Decline not driven by shifts across technology fields — spreading out occurs *within* established classes

Backup: Alternative Normalizations

Return

Main specification: Standardize by annual cross-sectional SD

Robustness checks:

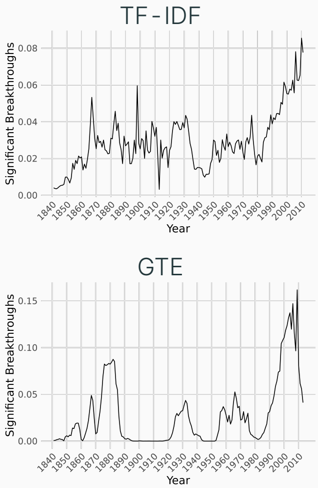
1. Time-invariant global SD → Nearly identical results
2. Raw similarity (no standardization) → Same qualitative pattern
3. Different sample sizes per year → Robust

Why standardize?

- Different representations have different scales
- No intrinsic economic interpretation of raw similarity
- SD provides meaningful units for comparison

Backup: Kelly et al. Breakthrough Replication

Return



Kelly et al. (2021): Identify "breakthrough" patents using similarity to future patents

Our replication with GTE:

- Qualitative conclusions align (more breakthroughs today)
- Quantitative results more robust (less sensitivity to methodological choices)
- TF-IDF produces noisier breakthrough classification

Implication: Validated similarity measures improve downstream analyses

Backup: Patent Corpus Details

Return

Historical (1836–1975): ProQuest Patents Core

- OCR-digitized patent images
- Full text of claims extracted
- Quality varies with original document condition

Modern (1976–2023): USPTO PatentsView

- Machine-readable full text
- Structured data with claim parsing
- Consistent quality

Potential discontinuity at 1976:

- Some evidence of break in levels
- Trends consistent across periods
- Results robust to excluding transition years

Backup: Computing Similarity Efficiently

◂ Return

Challenge: $O(N^2)$ pairwise comparisons infeasible for millions of patents

Solution: For unit-normalized vectors, average cosine similarity reduces to:

$$\bar{S} = \frac{1}{N(N-1)} \sum_{i \neq j} \cos(v_i, v_j) = \frac{||\sum_i v_i||^2 - N}{N(N-1)}$$

Complexity: $O(N \cdot d)$ where d = embedding dimension

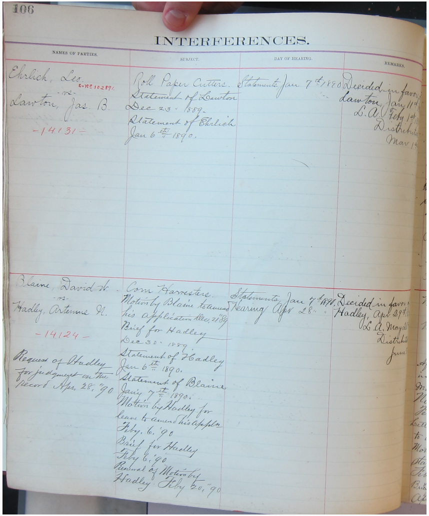
Implementation:

1. Normalize all vectors to unit length
2. Sum vectors: $S = \sum_i v_i$
3. Compute $||S||^2$
4. Apply formula

Cross-sectional SD: Subsample up to 10,000 patents/year

Example: Register of Interferences (1890)

Return



Purpose - digitized from National Archives:

- USPTO *Registers of Interferences*, 1864–1900
- 19,388 interference cases documented
- Average 504 annual terminations

Example cases (Jan 7, 1890):

- Ehrlich v. Lawton: Roll paper cutters
- Blaine v. Hadley: Corn harvesters