Expanding Idea Space and Declining Research Productivity

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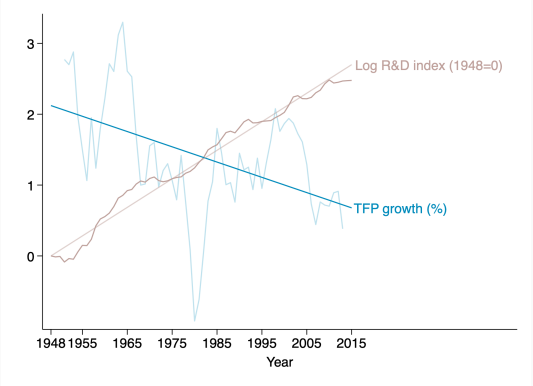
Introduction

The Puzzle: Are Ideas Getting Harder to Find?

The research productivity decline:

- Real R&D up >**20**× since 1930
- TFP growth slowed by factor of **3**×
- R&D productivity decline >-5%/yr

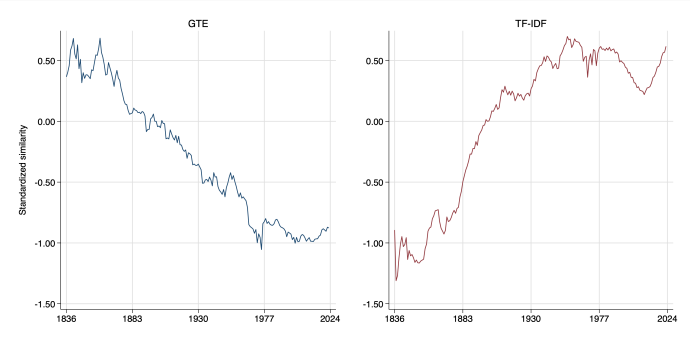
Key question: Why does it take so much **more** research effort to achieve ~~the same rate of~~ **slower** growth?



Bloom, Jones, Van Reenen and Webb, 2020

The Measurement Challenge

Same patent text, opposite conclusions:



- **Left (GTE):** Similarity *declining* — inventions spreading out
- **Right (TF-IDF):** Similarity *increasing* — inventions clustering

Key Question: **Which "map" of idea space should we trust?**

Research Questions

Q1: How do inventors position themselves across idea space?

Q2: What are the consequences for R&D productivity?

Q3: How do we measure positioning in idea space?

This Paper

A1: Spatial model of positioning in idea space

- Rising "burden of knowledge" → inventors spread out to capture larger territories
- Also explains: ↑R&D intensity, ↑returns, ↑quality, ↑breadth (unifies prior findings)

A2: Spreading out over expanding idea space reduces R&D productivity

- Spillover attenuation + adaptation drag + entry & territory expansion
- **Spatial forces can explain 40–60% of R&D productivity decline**

A3: Validated measurement framework

- Systematic comparison using domain-specific tasks
- GTE embeddings outperform TF-IDF; cover 1836–2023

Related Work

Spatial Models of Innovation:

- Dasgupta & Maskin 1987; Salop 1979
- **Connect to growth**

R&D Productivity Decline: Bloom et al. 2020

- Burden of knowledge Jones 2009
- Fishing out Kortum 1997
- **Add new "'spatial" explanations**

Endogenous Growth Theory:

- Howitt 1999; Peretto 1998, 2018
- **Spatially coupled quality/expansion**
- **Connect to ↓R&D productivity**

Patent Similarity Measurement:

- Jaffe et al. 1993; Kelly et al. 2021; Arts et al. 2025
- **Systematic validation**

We bridge these literatures with spatial model + validated measurement

Roadmap

Part I: Theory

- Spatial model of idea space
- Predictions: spreading out + declining productivity

Part II: Measurement

- Validation framework for patent similarity
- Model selection: Why GTE?

Part III: Evidence

- 188 years of spreading out
- Quantifying the spatial contribution to productivity decline

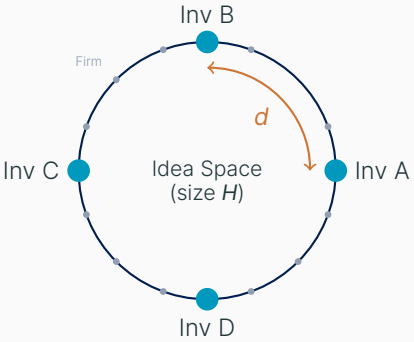
Part I: A Theory of Invention in Idea Space

Model Setup: Spatial Competition in Idea Space

Idea space: Circle of circumference *H*

- *H* = size of market for new ideas
- "Similar problems have similar solutions"
- Expands exogenously w/knowledge, demand

Inventors (idea producers):



Space of new ideas as Salop (1979) circle

Downstream firms (idea consumers):

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- License non-rival ideas downstream
- "Entry" = undertaking a project ($\neq$ firm)

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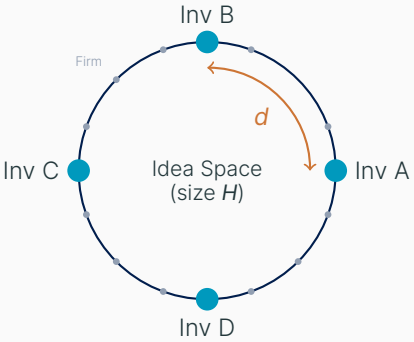

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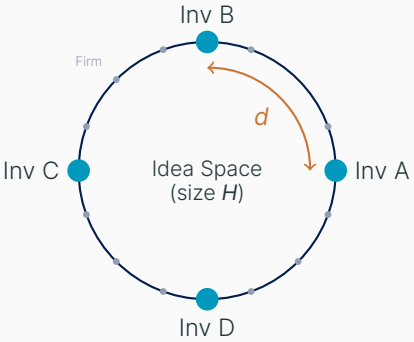
Downstream firms (idea consumers):

- Distributed uniformly on circle
- License ideas to boost their TFP



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Idea Consumers: Downstream Firms

Setup: Mass H of downstream firms uniformly distributed on circle

- Each firm licenses one idea to improve productivity
- Firm location = preferred technological variety

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TFP from licensing: Firm at distance h from invention i achieves **log TFP**:

$$A_i(h) = Q_i - \tau h$$

- Q_i = realized quality of invention i (including spillovers)
- τh = **adaptation cost** from technological mismatch (Bloom et al. 2013, Arora et al. 2021)

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Net surplus: Firm chooses invention to maximize:

$$\text{Surplus} = \underbrace{Q_i - \tau h}_{\text{TFP gain}} - \underbrace{p_i}_{\text{license fee}}$$

- Adaptation costs create product differentiation among inventions

R&D Technology: Costs and Licensing

R&D investment: Inventor *i* produces idea of quality q_i at cost:

$$c(q_i) = \frac{1}{2}\gamma q_i^{1+\eta}$$

- $\eta > 0 \Rightarrow$ diminishing returns to R&D effort. Baseline: $\eta = 1$ (quadratic costs).
- Captures "fishing out" (Kortum 1997): harder to find new ideas

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- Ideas are **non-rival**—can license to multiple firms at zero marginal cost
- Inventor charges license fee p_i to each downstream firm in territory
- Revenue = $p_i \times$ (number of firms served)

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Entry cost: Fixed cost $f(H) = \phi H^{\alpha}$, $\alpha > 0$ captures burden of knowledge (Jones 2009)

- More education, larger teams, sophisticated equipment
- Baseline: $\alpha = 1$ (linear), but $\alpha < 2$ preserves main results.

Knowledge Spillovers

Realized quality incorporates spillovers from neighbors:

$$Q_i = q_i + \frac{\beta}{2}\left(1 - \frac{d}{\lambda}\right) q_{i-1} + \frac{\beta}{2}\left(1 - \frac{d}{\lambda}\right) q_{i+1}$$

Parameters:

- q_i = own R&D investment
- $\beta \in (0,1)$ = spillover intensity
- λ = spillover reach (spillovers vanish beyond distance λ)
- d = distance to nearest neighbor

Key property: Spillovers **decay with distance**

- At $d = 0$: maximum spillover βq
- At $d = \lambda$: spillovers vanish

Proximity → spillovers, but also → competition

Equilibrium Analysis

Key Mechanism: Rising Entry Costs Drive Spreading Out

As idea space expands (*H* grows):

1. **Burden of knowledge** rises

2. **Inventors respond by spreading out**

   - Invest more in R&D quality
   - Charge higher prices (adaptation costs create differentiation)
   - Capture larger territories (serve more downstream firms)
   - Earn sufficient revenue to cover rising entry costs

Spreading out restores zero profits as entry costs rise

Equilibrium: Pricing and Quality

Symmetric equilibrium: n inventions, equal spacing $d = H/n$, identical (p, q) ▸ Existence

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- Larger territories ⇒ higher quality investment
- Key insight: adaptation costs make this *necessary*, not just profitable

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Both price and quality rise as inventions spread out

- Validated by empirical trends Hirschey et al. 2019, Bessen et al. 2018, KPSS 2017, Kelly et al. 2021

▸ Kelly replication

Free Entry Determines Equilibrium Spacing and Inventions

Zero-profit condition:

$$\underbrace{\tau d^2}_{\text{Revenue}} - \underbrace{\frac{d^2}{2\gamma}}_{\text{R\&D cost}} - \underbrace{\phi H}_{\text{Entry cost}} = 0$$

Solving for equilibrium spacing and number of inventions ($n = H/d$):

$$\boxed{d^*(H) = \sqrt{\frac{\phi H}{\tau - \frac{1}{2\gamma}}}} \quad \Rightarrow \quad \boxed{n^* = \sqrt{\frac{1}{\phi} H (\tau - \frac{1}{2\gamma})}}$$

Key observations:

- Spacing d^* **increases** with idea space H (spreading out!)
- Requires $\tau\gamma > \frac{1}{2}$ for real solution (economic interpretation next slide)
- Number of inventions n^* **also increases** with idea space H

↑ entry costs and ↑ idea space ⇒ Spreading out *and* more new ideas

Prediction 1: Spreading Out

Spreading Out

Proposition (Spreading Out)

Derivations

For $\tau\gamma > \frac{1}{2}$, equilibrium spacing increases with opportunity space: $\frac{dd^*}{dH} > 0$.

Inventions become **less similar** over time.

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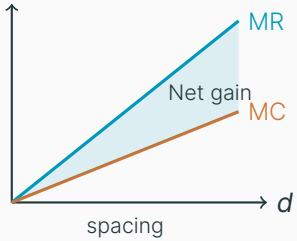
Why is spreading out profitable?



Marginal revenue of expanding territory:

- Revenue $R = \tau d^2 \quad \Rightarrow \quad \text{MR} = 2\tau d$

Marginal cost of expanding territory:

- Need higher quality: $q = d/\gamma$
- $\text{MC} = d/\gamma$

Spreading profitable when:

$$\text{MR} > \text{MC} \quad \Rightarrow \quad \boxed{\tau\gamma > \frac{1}{2}}$$

Adaptation costs must create sufficient pricing power

Prediction 2: Expanding Idea Space and Declining R&D Productivity

Declining R&D Productivity

Define aggregate R&D productivity (cf. Bloom et al. 2020)

$$\Pi \equiv \frac{\text{Agg TFP growth}}{\text{Agg R\&D}}$$

Key insight: As idea space *H* expands, entry *dilutes* aggregate productivity

Agg TFP growth $= q\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]-\frac{\tau d}{4}$

- **Average** ΔTFP delivered downstream
- Doesn't scale with *n*

Agg R&D $= n\cdot\left[\frac{1}{2}\gamma q^2+\phi H\right]$

- **Total** R&D across *n* inventions
- Scales with *n* ▸ Literature

Five Forces Reduce R&D Productivity

As idea space expands, five forces reduce R&D productivity:

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Forces reducing TFP:

Forces raising R&D:

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3. **Fishing out** Convex R&D costs
4. **Burden of knowledge** Rising fixed costs
5. **Entry and territory expansion**
   More inventions cover larger territories

Productivity Decline: The Decomposition

Aggregate TFP response:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH}\left[1+\beta\left(1-\frac{d}{\lambda}\right)\right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda}\frac{dd}{dH}}_{\text{(1) Spillover attenuation}} - \underbrace{\frac{\tau}{4}\frac{dd}{dH}}_{\text{(2) Adaptation drag}}$$

Aggregate R&D response:

$$\frac{d(\text{Agg R\&D})}{dH} = \underbrace{\frac{dn}{dH}\cdot[c(q)+f(H)]}_{\text{(5) Entry expansion}} + \underbrace{n\cdot c'(q)}_{\text{(3) Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{\text{(5)*}} + \underbrace{n\cdot f'(H)}_{\text{(4) Burden of knowledge}}$$

- (5)* = territory expansion per invention
- Terms (1)–(5) correspond to five forces on previous slide

Growth Implications

Constant Growth in Fundamentals

Key question: If idea space H grows over time, what are the testable predictions?

Setup: Size of idea space expands at constant rate g_H: $\dot{H} = g_H \cdot H$ cf. Jones 1995

Fundamental variables grow at constant rates: ▸ With general cost curves

- Spacing: $d^* = C \cdot \sqrt{H} \Rightarrow g_d = \frac{1}{2} g_H$
- Quality: $q^* = d/\gamma \Rightarrow g_q = g_d = \frac{1}{2} g_H$
- Entry: $n^* = H/d \Rightarrow g_n = g_H - g_d = \frac{1}{2} g_H$
- R&D: $g_{R\&D} = \frac{3}{2} g_H$ (next slide)
- Note: TFP growth rate g_{TFP} declines as d and q grow (next slide)

No knife-edge: Balanced growth holds for *any* admissible (τ, γ, ϕ) (Parallels Peretto 2018)

TFP and R&D Growth Equations

TFP R&D

TFP growth:

$$g_{TFP} = \underbrace{g_q\left(1+\beta-\frac{\beta d}{\lambda}\right)}_{\text{Quality (with spillovers)}} - \underbrace{\frac{\beta q}{\lambda} g_d}_{\text{Spillover attenuation}} - \underbrace{\frac{\tau}{4} g_d}_{\text{Adaptation drag}}$$

R&D growth:

$$g_{R\&D} = \underbrace{g_n}_{\text{Entry}} + \underbrace{\theta \cdot g_q}_{\text{Quality scaling}} + \underbrace{\theta \cdot g_q}_{\text{Fishing out}} + \underbrace{(1-\theta) g_f}_{\text{Burden of knowledge}}$$

where θ = variable cost share; α = 1, η = 1

We can use these growth equations to decompose R&D productivity decline

From Theory to Empirics

Key predictions ⇒ testable implications:

- Spreading out: ↑ d over time ⇒ ↓ **similarity over time**
- R&D productivity: ↑ d → ↓ Π ⇒ ↓ **similarity** → ↑ **R&D,** ↓ **TFP growth**

Decompose R&D productivity decline into spatial and non-spatial components:

- Spatial: spillover attenuation, adaptation drag, entry & territory expansion
- Non-spatial: fishing out, burden of knowledge

Part II: Measuring Similarity in Idea Space

Data: US Patent Claims, 1836–2023

Patent text corpus: Details

- **Historical (1836–1975):** ProQuest Patents Core (digitized full text)
- **Modern (1976–2023):** USPTO PatentsView
- Focus on **claims** — defines legal boundaries of invention

Multiple NLP representations tested:

- Traditional: TF-IDF (word frequency)
- Modern neural embeddings: GTE, PaECTER, S-BERT, Doc2vec, USE, OpenAI

Similarity measure: Computation

- Cosine similarity between patent representations
- Average pairwise similarity by year
- Standardized by cross-sectional standard deviation

Validation Framework: Three Complementary Tasks

Task	Time Period	Granularity	Expertise	
Patent Interferences	2001–2014	Identical	USPTO examiners	
Human Judgments	1850–1975	Continuous	Lay annotators	
Classifications	1850–2023	Categorical	Expert labels	

Why multiple tasks?

- No single ground truth for “similarity”
- Different aspects: legal identity vs. technological relatedness
- Temporal robustness across 175+ years

Models performing well across all tasks are most reliable

Validation Results: Model Performance

Model	Interferences		Human	Classifications	
	PR AUC	F10	Agreement	Section	Class
GTE	0.64 (2)	**0.90** (1)	**0.62** (1)	0.596 (2)	0.656 (3)
PaECTER	**0.65** (1)	0.90 (2)	0.51 (3)	0.590 (3)	**0.672** (1)
S-BERT	0.52 (3)	0.82 (3)	0.54 (2)	**0.600** (1)	0.671 (2)
TF-IDF	0.45 (4)	0.77 (4)	0.35 (4)	0.514 (4)	0.525 (4)

- **GTE and PaECTER** consistently top performers
- **TF-IDF** consistently worst (20–40% lower performance)
- All beat **random chance** — but **magnitudes differ dramatically**

Model Selection: Why We Use GTE

GTE selected for main results because:

1. **Temporal robustness** — best on historical patents (1880–1920)
2. **Near-identical performance on interferences** — our most demanding test
3. **Consistent across all tasks** — ranks 1st or 2nd on 4/5 metrics

Why TF-IDF fails:

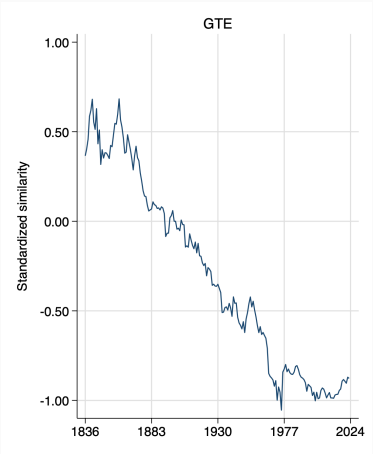
- Overweights period-specific language
- Treats synonyms as unrelated ("velocipede" ≠ "bicycle")
- Would lead to *opposite* conclusions about our theory

Robustness checks with PaECTER, S-BERT, and ensemble measures

Part III: Evidence

Prediction 1: Are Inventions Spreading Out?

Main Finding: Secular Decline in Patent Similarity



Average annual pairwise cosine similarity, standardized by cross-sectional SD. Indexed to 0 in 1900.

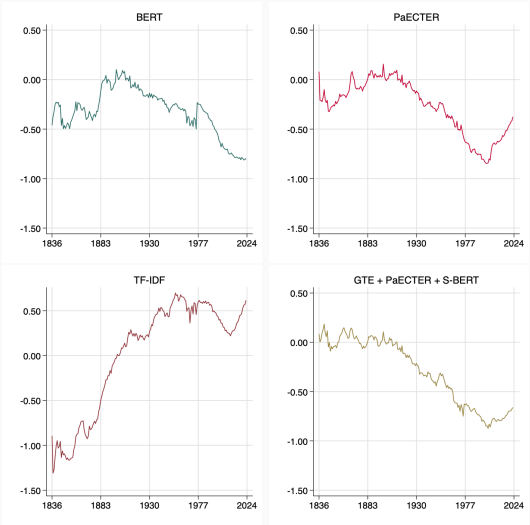
Using validated GTE embeddings:

~1.5σ decline in patent similarity, 1836–2023

- Consistent with theory: inventions spreading out
- Spreading out ($d\uparrow$) = Declining similarity (Sim $\downarrow$)
- Multi-patent entity effect post-2000 (to come)

Confirms Prediction 1: Spreading Out

Why Validation Matters: Comparing Representations



TF-IDF (worst performer):

- $\sim 1.5\sigma$ increase—opposite conclusion!
- Validation correctly discards

PaECTER, S-BERT (cf. GTE):

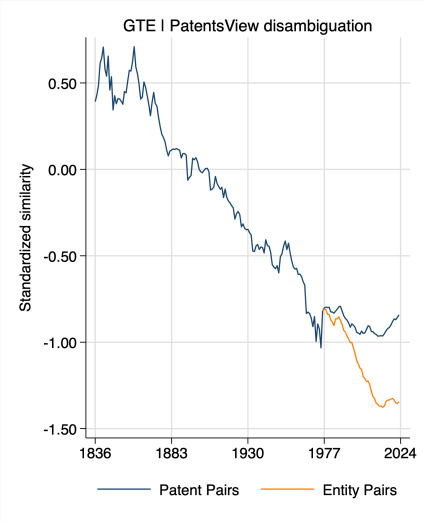
- Similar $\sim 0.8\sigma$ decline, 1880–2000
- Diverge pre-1880 & post-2000

Ensemble (avg of top models):

- $\sim 1.0\sigma$ decline, 1836–2023

Validated methods agree; unvalidated TF-IDF misleads

Robustness: Accounting for Multi-Patent Entities



Concern: Post-2000 dynamics coincide with: business method patents, non-practicing entities, increased defensive patenting.

- **Multiple patents from same entity may be similar but not independent.**

Strategy: Sample 1 patent/entity–year

Result:

- Decline persists after correction
- **Independent inventions** still spreading out

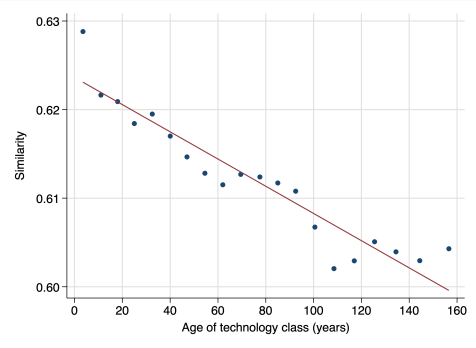
Robustness: Spreading Out Within Technology Classes

Alternative explanations: Changing patent office practice over time? Shifts across major technology areas?

Test: Within-class similarity by class "age"

- Birth = Class first issued 50 patents
- e.g., Combinatorial Chemistry 2001
- Addresses compositional concerns ▸ Between

Finding: Within-class similarity declines as classes mature



Spreading out is a dynamic process tied to field evolution

Independent Corroboration: Declining Interference Rates

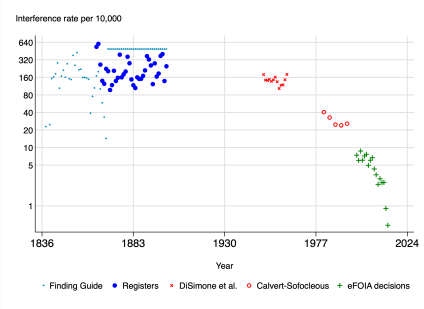
Patent interferences:

- USPTO determination that two independent inventors made *identical* inventions
- Direct measure of multiple invention ($d = 0$)

Data: Purpose-digitized from 5 sources

- Nat'l Archives files & Registers (1838–1900)
- Published statistics (1950–1994)
- eFOIA decisions (1998–2014) Ganguli et al. 2020

Finding: Interference rate declined over 150 years



Same conclusion from completely different data source

Summary: Inventions Are Spreading Out

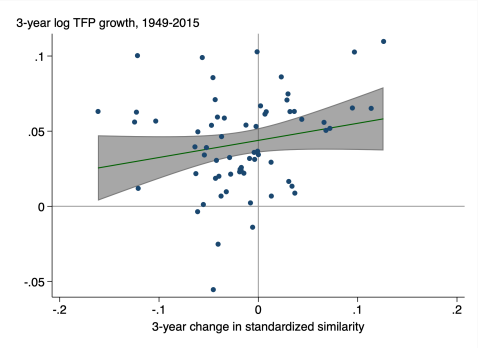
Robust evidence of spreading out:

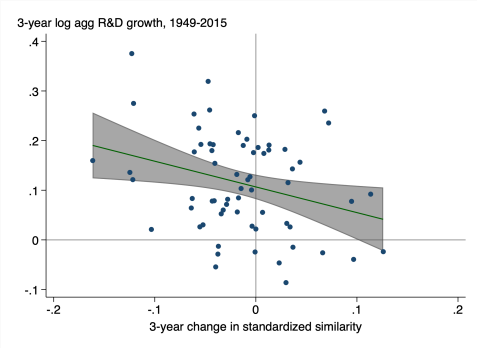
- ✓ Main finding: 1.5σ decline in similarity, 1836–2023
- ✓ Decline extends after 2000 for independent inventions
- ✓ Robust to spatial scale (local and global)
- ✓ Robust to within vs. between class decomposition
- ✓ Appears within classes as they age
- ✓ Corroborated by interference rates (150 years)

Next: What are the consequences for research productivity?

Prediction 2:
Does Spreading Out Reduce R&D Productivity?

Timing: Similarity Predicts TFP and R&D Growth





- **Left:** Declining similarity → **lower TFP growth**
- **Right:** Declining similarity → **higher R&D growth**

Both patterns confirm Prediction 2

TFP and Spreading Out

From Theory to Estimation: TFP

TFP growth equation (from BGP):

$$g_{TFP} = \underbrace{g_q\left(1+\beta-\frac{\beta d}{\lambda}\right)}_{\text{Quality (with spillovers)}} - \underbrace{\frac{\beta q}{\lambda} g_d}_{\text{Spillover attenuation}} - \underbrace{\frac{\tau}{4} g_d}_{\text{Adaptation drag}}$$

Substitute equilibrium relationships for unobservables:

- $q^* = d/\gamma$ and $dq^*/dt = (1/\gamma)(dd/dt) \Rightarrow g_q = g_d$

$$g_{TFP} = \underbrace{(1+\beta-\frac{\tau}{4})}_{b_1} \cdot g_d - \underbrace{\frac{2\beta}{\lambda}}_{b_2} \cdot d \cdot g_d$$

Suggests the regression:

- Observable proxy: $g_d \approx -\Delta\text{Sim}$ (small annual changes in standardized measure)

$$\Delta \log(\text{TFP})_t = b_0 + b_1 \cdot (-\Delta\text{Sim})_t + b_2 \cdot (-\Delta\text{Sim}) \cdot (-\text{Sim}_{t-1}) + \epsilon_t$$

From Theory to Estimation: TFP

Regression Specification:

$$\Delta \log(\text{TFP})_t = b_0 + b_1 \cdot (-\Delta \text{Sim})_t + b_2 \cdot (-\Delta \text{Sim}) \cdot (-\text{Sim}_{t-1}) + b_3 \cdot t + \epsilon_t$$

Data:

- TFP and Real R&D Inputs, 1948–2015 (Bloom et al., 2020)

Predictions and interpretation:

- $b_1 \lessgtr 0$: Effect on TFP growth from ↑ quality scaling net of ↓ adaptation costs
- $b_2 < 0$: Spillover attenuation and reduced marginal return to R&D
- b_3: Time trend controls for factors not explicit in the model

TFP Growth and Technological Distance

	Annual		3-Year	5-Year
$b_1 : -1 \times \Delta\text{Sim}$	−0.169***	−0.171***	−0.278***	−0.269***
	(0.057)	(0.083)	(0.095)	(0.098)
$b_2 : (-1 \times \Delta\text{Sim}) \times (-1 \times \text{Sim}_{t-1})$	—	−0.015	−0.408	−0.571*
		(0.342)	(0.320)	(0.312)
Implied TFP drag from spreading out ($\overline{\Delta Sim}$, %/yr):				
1948 (Sim = 0.35)	−0.08	−0.08	−0.07	−0.04
1991 (Sim = 0, baseline)	−0.08	−0.09	−0.14	−0.16

Validation: Implied drag (−0.16%/yr) from ΔSim consistent with quasi-experimental cross-sectional elasticity ✓ Bloom et al. (2013) (−0.14 to −0.16%/yr, 1981–2001)

Contribution to TFP deceleration: Drag worsened -0.04%/yr (1948) → -0.14%/yr (2015). Change = 0.10 pp = **7% of 1.4 pp total TFP deceleration**.

▸ Decomposition

R&D and Spreading Out

From Theory to Estimation: R&D

R&D growth equation (from BGP):

$$g_{R\&D} = \underbrace{g_n}_{\text{Entry}} + \underbrace{\theta(1+\eta)g_q}_{\text{Quality (incl. fishing out)}} + \underbrace{(1-\theta)g_f}_{\text{Rising fixed costs}}$$

Substitute equilibrium relationships:

$$g_{R\&D} = \underbrace{[1+\alpha(1-\theta)]g_H}_{a_0} + \underbrace{[\theta(1+\eta)-1]}_{a_1} g_d$$

Regression specification:

$$g_{R\&D,t} = a_0 + a_1 \cdot (-\Delta \text{Sim})_t + a_2 \cdot t + \epsilon_t$$

- a_2 captures (unmodeled) acceleration in idea space growth (but: $\hat{a}_2 \approx 0$)

Identification of Structural Parameters

Identification of structural parameters:

$$a_1 = \theta(1+\eta) - 1$$

$$\Downarrow$$

$$\boxed{\theta = \frac{a_1 + 1}{1+\eta}}$$

(variable cost share)

$$a_0 = [1 + \alpha(1-\theta)]g_H$$

$$\Downarrow$$

$$\boxed{g_H = \frac{a_0}{1+\alpha(1-\theta)}}$$

(idea space growth)

Baseline: $\alpha = 1$, $\eta = 1$. **Later:** Calibration w/ quasi-experimental $\hat{\eta}$ and estimate of α.

Regression coefficients → structural parameters (θ, g_H)

R&D Growth and Technological Distance

	Annual	3-Year	5-Year
a_1: $-1 \times \Delta$Sim	0.165	0.448**	0.438*
	(0.177)	(0.219)	(0.244)
a_0: Constant	0.034***	0.102***	0.173***
	(0.006)	(0.013)	(0.018)
Implied θ (variable cost share)	0.58	0.72	**0.72**
Implied g_H (idea space growth)	2.4%/yr	2.7%/yr	**2.7%/yr**

Validation:

- $\theta = 72\%$ aligns with NSF survey data (labor = 69% of R&D) ✓
- $g_H = 2.7\%$/yr consistent with patent embedding volume growth ✓
- BGP consistency: Model predicts $g_d/g_{R\&D} = 1/3$; In data, $-\Delta\text{Sim}/g_{R\&D} = 0.31$ ✓

Growth Accounting

The Research Productivity Decline

Research productivity:

$$\Pi \equiv g_{TFP}/\text{Agg R\&D (TFP growth per unit R\&D)}$$

The decline (1948–2015):

- TFP growth fell: 2.1%/yr → 0.7%/yr $\quad (g_{g_{TFP}} = -1.6\%/\text{yr})$
- R&D spending grew: 4.0%/yr

$$g_{\Pi} = g_{g_{\text{TFP}}} - g_{\text{R\&D}} = -1.6\% - 4.0\% = \boxed{-5.6\%/\text{yr}}$$

Goal: Decompose this decline into spatial and non-spatial components

From Regressions to Parameters

What we estimated from R&D regression:

Parameter	Value	Source
θ (variable cost share)	0.72	R&D regression coefficient a_2 ($\eta = 1$)
g_H (idea space growth)	2.7%/yr	R&D regression constant a_0 ($\alpha = 1$)

What we assume (baseline):

Parameter	Value	Interpretation
α (entry cost curvature)	1.0	Entry costs scale linearly with H
η (R&D cost curvature)	1.0	Quadratic R&D costs

Decomposing the R&D Productivity Decline

Model implies: $g_d = \frac{\alpha}{2} g_H = 1.35\%/\text{yr}$ (spreading rate if $\alpha = 1$)

Component	Contribution	Classification	Comment
TFP deceleration	−1.6%/yr		
Spatial drag worsened	−0.11%/yr	**Spatial**	7% of deceleration ▸ TFP regression
Unmodeled factors	−1.49%/yr	Non-spatial	

Decomposing the R&D Productivity Decline

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Unmodeled factors	−1.49%/yr	Non-spatial	
R&D growth	+4.0%/yr		
Entry expansion $(1-\frac{\alpha}{2})g_H$	+1.35%/yr	**Spatial**	(new inventions)
Quality scaling $(\theta \frac{\alpha}{2} g_H)$	+0.97%/yr	**Spatial**	(larger territories; TFP units)
Fishing out $(\theta\eta\frac{\alpha}{2} g_H)$	+0.97%/yr	Non-spatial	(convex costs)
Burden of knowledge $(1-\theta)(\alpha g_H)$	+0.76%/yr	Non-spatial	(rising fixed costs)
Unmodeled factors	−0.05%/yr	Non-spatial	

Decomposing the R&D Productivity Decline

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Unmodeled factors	−0.05%/yr	Non-spatial	
Total decline	−5.6%/yr		
Spatial contribution	−2.43%/yr		**43%**
Non-spatial contribution	−3.17%/yr		57%

Robustness: Spatial Share Increases with Better Calibration

Baseline assumptions: $\alpha = 1$, $\eta = 1$

Alternative calibration:

- $\eta = 0.625$: Guceri-Liu (2019)
- $\theta = 0.89$: From R&D regression a_2
- $\alpha = 0.76$: Constrain sum to 4.0%
- $g_H = 3.2\%$/yr: From R&D regression a_0
- $g_d = \frac{\alpha}{2} g_H = 1.2\%$/yr

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- $g_H = 3.2\%$/yr: From R&D regression a_0
- $g_d = \frac{\alpha}{2} g_H = 1.2\%$/yr

Alternative decomposition:

	Baseline	Alternative
Entry expansion	1.35%/yr	1.98%/yr
Quality scaling	0.97%/yr	1.08%/yr
Fishing out	0.97%/yr	0.67%/yr
Burden of knowledge	0.76%/yr	0.27%/yr
Sum	4.05%/yr	4.00%/yr
Spatial share	**43%**	**57%**

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Entry expansion	1.35%/yr	1.98%/yr
Quality scaling	0.97%/yr	1.08%/yr
Fishing out	0.97%/yr	0.67%/yr
Burden of knowledge	0.76%/yr	0.27%/yr
Sum	4.05%/yr	4.00%/yr
Spatial share	**43%**	**57%**

Conservative baseline; higher spatial share with alternative calibration

- $\eta = 0.625 < 1$: R&D costs grow sub-quadratically with q
- $\hat{\alpha} = 0.76 < 1$: Entry costs grow sub-linearly with H
- Entry expansion (1.98%) < patent growth (3.9%) $\Rightarrow \downarrow$ ideas per patent (-1.9%/yr)

Conclusion

Summary

1. **Theory:** Spatial model predicts:
   - As idea space expands, inventions spread out
   - R&D productivity declines through spillover attenuation, adaptation drag, and entry & territory expansion

2. **Measurement:** Validated NLP methods using domain-specific tasks
   - Representation choice fundamentally affects conclusions
   - GTE outperforms traditional workhorse TF-IDF

3. **Empirics:** Nearly 2 centuries of spreading out
   - Robust across multiple tests and data sources
   - **Spatial forces can explain 40–60% of R&D productivity decline**

Backup Slides

Backup: Model Equations

Fixed cost (burden of knowledge):	$f(H) = \phi H, \quad \phi > 0$
R&D cost:	$c(q_i) = \frac{1}{2}\gamma q_i^2$
Realized quality (with spillovers):	$Q_i = q_i + \frac{1}{2}\beta\left(1 - \frac{d}{\lambda}\right)(q_{i-1} + q_{i+1})$
Equilibrium pricing and quality:	$p^* = \tau d, \qquad q^* = \frac{d}{\gamma}$
Equilibrium spacing:	$d^*(H) = \sqrt{\frac{\phi H}{\tau - \frac{1}{2\gamma}}}$
Equilibrium entry:	$n^* = \frac{H}{d^*}$
Equilibrium revenue:	$R^* = p^* \cdot d^* = \tau d^2$

Backup: Robustness to Entry Cost Curvature

Return

With $f(H) = \phi H^{\alpha}$, equilibrium spacing satisfies $d \propto H^{\alpha/2}$:

Condition	Entry Growth	Prediction
$0 < \alpha < 2$	$g_n = (1 - \frac{\alpha}{2}) g_H > 0$	Entry grows ✓
$\alpha = 2$	$g_n = 0$	Entry stagnates
$\alpha > 2$	$g_n < 0$	Entry declines ×

Main results robust for $\alpha < 2$:

- Spreading out: $g_d = \frac{\alpha}{2} g_H > 0$ for any $\alpha > 0$
- Declining R&D productivity: Holds throughout range
- Higher $\alpha \rightarrow$ faster spreading, but lower spatial share of productivity decline

Counterfactual boundary: Patent counts grow over time, ruling out $\alpha \geq 2$

Backup: Model with General Cost Curvatures

Growth equations | Decomposition

Component	General	Baseline ($\alpha = 1, \eta = 1$)
Entry cost	$f(H) = \phi H^{\alpha}$	ϕH
R&D cost	$c(q) = \frac{1}{2}\gamma q^{1+\eta}$	$\frac{1}{2}\gamma q^2$
Spacing growth	$g_d = \frac{\alpha}{2} g_H$	$\frac{1}{2} g_H$
Quality growth	$g_q = g_d$	$\frac{1}{2} g_H$
Entry growth	$g_n = (1 - \frac{\alpha}{2}) g_H$	$\frac{1}{2} g_H$
Fixed cost growth	$g_f = \alpha g_H$	g_H

R&D growth equation:

$$g_{R\&D} = \underbrace{(1 - \tfrac{\alpha}{2}) g_H}_{\text{Entry}} + \underbrace{\tfrac{\theta\alpha}{2} g_H}_{\text{Quality scaling}} + \underbrace{\tfrac{\theta\eta\alpha}{2} g_H}_{\text{Fishing out}} + \underbrace{(1-\theta)\alpha g_H}_{\text{Burden of knowledge}}$$

Log TFP Specification

Return

Why linear in log TFP? $A_i(h) = Q_i - \tau h$

Standard in spatial competition (Salop 1979):

- Idea consumers have preferences linear in quality net of distance costs
- $A_i(h)$ interpreted as log TFP ⇒ firms care about *proportional* productivity gains

Microfoundation: Each downstream firm has one unit of fixed input ℓ and produces:

$$y = e^A \cdot \ell$$

With output price = 1 and $\ell = 1$, profit is $\pi = e^A$. Willingness to pay for technology delivering incremental log TFP A (relative to baseline $e^0 = 1$):

$$WTP = e^A - 1 \approx A \quad \text{(first-order Taylor approximation)}$$

Accuracy: For annual TFP increments ($A \approx 0.015$/year), approximation error $< 0.01\%$

Advantage: Predictions directly comparable to empirical TFP elasticities (Bloom et al. 2013) and growth accounting (Bloom et al. 2020)

Backup: Equilibrium Existence Conditions

Return

Spreading-out condition:

$$\tau\gamma > \frac{1}{2}$$

Marginal revenue of expanding territory exceeds marginal cost.

Second-order conditions:

- Pricing: $\partial^2 R/\partial p^2 < 0$ (satisfied)
- Quality: $\partial^2 \pi/\partial q^2 = -\gamma < 0$ (satisfied)
- No spatial deviation (verified in paper)

Additional conditions:

- Spillover reach: $d < \lambda$ (spillovers active)
- Full coverage: All downstream firms adopt some technology

Backup: Comparative Statics Derivations

Return

Spreading out: From zero-profit condition $d^2(\tau - \frac{1}{2\gamma}) = \phi H$:

$$\frac{dd}{dH} = \frac{\phi}{2d(\tau - \frac{1}{2\gamma})} = \frac{\phi}{dR/dd - dc/dd} > 0$$

Rising quality and prices:

$$\frac{dq}{dH} = \frac{1}{\gamma}\frac{dd}{dH} > 0, \qquad \frac{dp}{dH} = \tau\frac{dd}{dH} > 0$$

Rising entry:

$$\frac{dn}{dH} = \frac{1}{d} - \frac{H}{d^2}\frac{dd}{dH} > 0 \text{ under spreading-out condition}$$

Declining productivity:

$$\frac{d\rho}{dH} < 0, \qquad \frac{d\Pi}{dH} < 0$$

MR > MC Derivation

Return

Question: When is spreading out profitable?

Marginal revenue of expanding spacing:

$$R = p \cdot d = \tau d \cdot d = \tau d^2 \quad \Rightarrow \quad \frac{dR}{dd} = 2\tau d$$

Marginal cost of expanding spacing (need higher quality to serve larger territory):

$$c(q) = \frac{1}{2}\gamma q^2, \quad q = \frac{d}{\gamma} \quad \Rightarrow \quad c = \frac{d^2}{2\gamma} \quad \Rightarrow \quad \frac{dc}{dd} = \frac{d}{\gamma}$$

Spreading out profitable when MR > MC:

$$2\tau d > \frac{d}{\gamma} \quad \Rightarrow \quad 2\tau > \frac{1}{\gamma} \quad \Rightarrow \quad \boxed{\tau\gamma > \frac{1}{2}}$$

Interpretation: Adaptation cost intensity (τ) × R&D cost parameter (γ) must exceed $\frac{1}{2}$. Pricing power from differentiation must outweigh quality investment costs.

Backup: Connection to Endogenous Growth Literature

◂ Related Work ◂ Productivity

Scale effects debate (1990s):

- Early models: bigger economy → faster growth (Romer 1990)
- Empirically rejected: R&D workforce grew, growth rates didn't (Jones 1995)
- Resolution: hybrid models with horizontal + vertical innovation

Howitt (1999), Peretto (1998):

- Horizontal R&D expands varieties but **doesn't improve average quality**
- New entrants match (not exceed) existing productivity levels
- Entry dissipates scale effects — addressed 1990s debate
- **Our contribution:** Connect this to *declining R&D productivity* (Bloom et al. 2020)

No knife-edge assumptions:

- Endogenous spacing $d(H)$ adjusts for any admissible parameters
- Balanced growth emerges as equilibrium outcome, not assumption
- Parallels Peretto (2018) where market mechanisms achieve steady-state

Backup: Interference Validation Task

Return

Patent interferences (2001–2014):

- **First to invent**: USPTO proceeding for multiple applicants w/ identical claims
- Provides ground truth for "identical" similarity
- 322 true interfering pairs among 96,580 application pairs

Economic intuition: Examiner ranks pairs by similarity, investigates above threshold

- Higher threshold → fewer false positives but miss true interferences
- Lower threshold → catch more but burden staff with unnecessary investigations

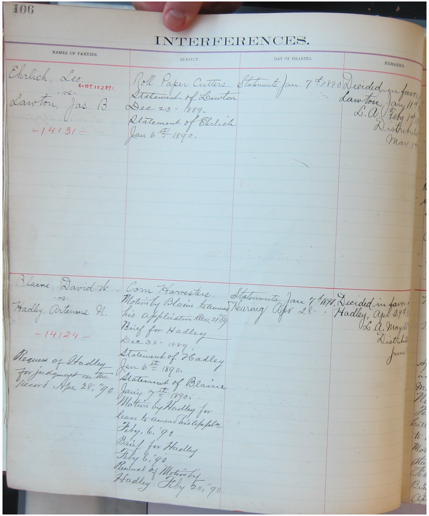
Metrics:

- F10: Weights recall 10× more than precision (missing interferences is costly)
- PR AUC: Precision-Recall area under curve across all thresholds

Key result: GTE, PaECTER, OpenAI retrieve ~90% of true interferences with 2–5× fewer false positives than TF-IDF/S-BERT

Example: Register of Interferences (1890)

Return



Purpose-digitized from National Archives:

- USPTO *Registers of Interferences*, 1864–1900
- 19,388 interference cases documented
- Average 504 annual terminations

Example cases (Jan 7, 1890):

- Ehrlich v. Lawton: Roll paper cutters
- Blaine v. Hadley: Corn harvesters

Backup: Human Annotation Task

Return

Historical patents (1850–1975):

- Sampled patent pairs that each model ranked at least 50 percentiles apart
- Annotators rank **relative** similarity of 2 patent pairs
- Tests temporal robustness (historical language): Oversample 1880–1920

Task: Do model rankings agree with human rankings?

- For each patent, rank others by similarity
- Compare model ranking to human ranking

Metric: Agreement coefficient from regression

$$\text{Human Rank} = \alpha + \beta \cdot \text{Model Rank} + \epsilon$$

Higher β = better agreement

Backup: Classification Validation Task

Return

USPTO Classifications (1850–2023):

- CPC technology codes assigned by examiners
- Section level (8 categories) and Class level (120+ categories)
- Captures expert judgment of technological relatedness

Task: Predict whether patent pair shares classification

- Same Section (coarse): 8 top-level categories
- Same Class (fine): 3-digit classification

Metric: ROC AUC

- Area under Receiver Operating Characteristic curve
- 0.5 = random, 1.0 = perfect

Backup: Why TF-IDF Fails

Return

TF-IDF overweights period-specific language:

- Treats “velocipede” (1880s) and “bicycle” (modern) as unrelated
- Period-specific terminology dominates similarity scores
- Creates spurious correlation with time

Example: 1880 velocipede patent

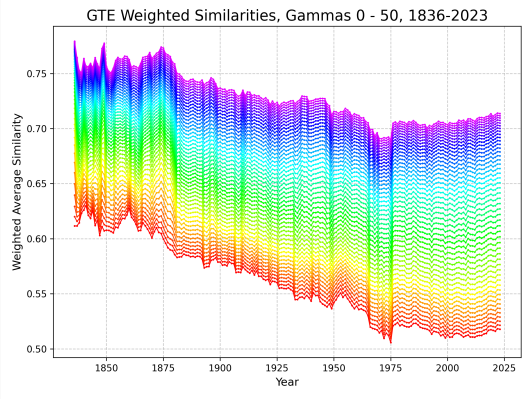
- TF-IDF: High similarity to other 1880s patents (shared vocabulary)
- GTE: High similarity to modern bicycle patents (shared concepts)

Evidence:

- TF-IDF similarity correlates with word overlap
- GTE similarity correlates with conceptual similarity
- Google Ngrams shows vocabulary shifts over time

Backup: Similarity at Different Spatial Scales

Return



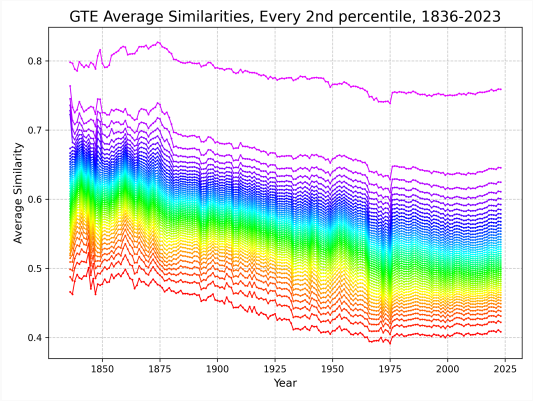
Weighted average: $\equiv \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j\neq i}(1-d_{ij})e^{-\gamma d_{ij}}}{\sum_{j\neq i} e^{-\gamma d_{ij}}}$

where γ from 0 (global) to 50 (local)

- **Key finding:** Similar declining trends across all spatial scales
- Model predictions concern averages — important to verify pattern holds across distribution
- Post-2000 arrest slightly stronger at local scales (consistent with entity correction)

Backup: Similarity at Different Quantiles

Return

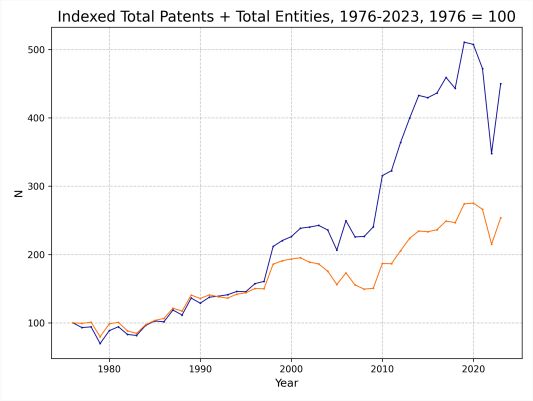


Similarity at Different Quantiles:

- 50 quantiles of pairwise similarity in each year
- Secular decline is robust across all quantiles
- Post-2000 increase in similarity is slightly faster for higher quantiles

Backup: Growth in Patents vs. Patenting Entities

Return



- Number of issued utility patents and unique patenting entities per year
- Divergence after 1999: substantial growth in patents per entity
- Driven by business method patents and non-practicing entities
- Motivates sampling 1 patent per entity per year for robustness

Backup: Within vs. Between Technology Classes

Return

Within-class similarity

Between-class similarity

- **Addresses compositional concern:** Decline not driven by shifts across technology fields — spreading out occurs *within* established classes

Backup: Alternative Normalizations

Return

Main specification: Standardize by annual cross-sectional SD

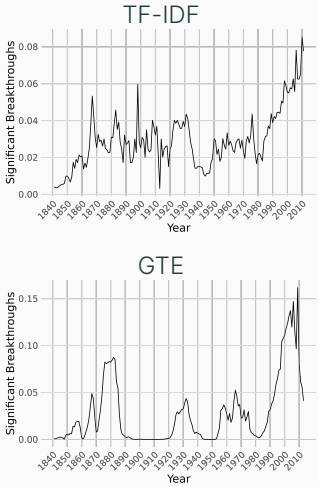
Robustness checks:

1. Time-invariant global SD → Nearly identical results
2. Raw similarity (no standardization) → Same qualitative pattern
3. Different sample sizes per year → Robust

Why standardize?

- Different representations have different scales
- No intrinsic economic interpretation of raw similarity
- SD provides meaningful units for comparison

Backup: Kelly et al. Breakthrough Replication



Kelly et al. (2021): Identify "breakthrough" patents using similarity to future patents

Our replication with GTE:

- Qualitative conclusions align (more breakthroughs today)
- Quantitative results more robust (less sensitivity to methodological choices)
- TF-IDF produces noisier breakthrough classification

Implication: Validated similarity measures improve downstream analyses

Backup: Patent Corpus Details

Return

Historical (1836–1975): ProQuest Patents Core

- OCR-digitized patent images
- Full text of claims extracted
- Quality varies with original document condition

Modern (1976–2023): USPTO PatentsView

- Machine-readable full text
- Structured data with claim parsing
- Consistent quality

Potential discontinuity at 1976:

- Some evidence of break in levels
- Trends consistent across periods
- Results robust to excluding transition years

Backup: Computing Similarity Efficiently

Return

Challenge: $O(N^2)$ pairwise comparisons infeasible for millions of patents

Solution: For unit-normalized vectors, average cosine similarity reduces to:

$$\bar{S} = \frac{1}{N(N-1)} \sum_{i \neq j} \cos(v_i, v_j) = \frac{\|\sum_i v_i\|^2 - N}{N(N-1)}$$

Complexity: $O(N \cdot d)$ where d = embedding dimension

Implementation:

1. Normalize all vectors to unit length
2. Sum vectors: $S = \sum_i v_i$
3. Compute $\|S\|^2$
4. Apply formula

Cross-sectional SD: Subsample up to 10,000 patents/year

Backup: Convex Hull

Return