

Spreading Out in Expanding Idea Space

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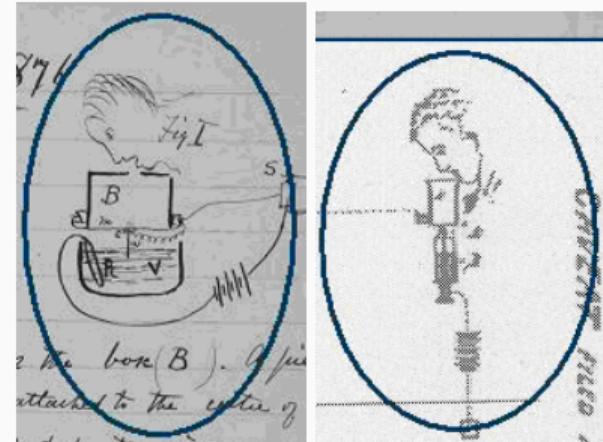
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Introduction

Motivation

Bell and Gray each file a telephone patent — same day (Feb 14, 1876)

- Two independent inventors, **same region of idea space**: simultaneous invention
- A patent dispute, years of litigation: **positioning in idea space has economic consequences**
(cf. choosing a research topic)



Drawings by Bell and Gray, demonstrating their similarity

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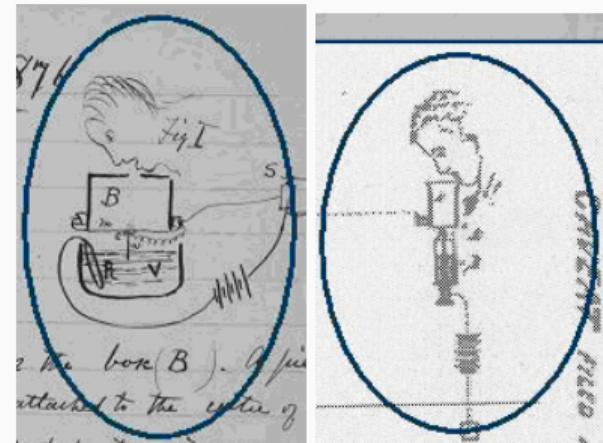
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This paper asks:

Q1: What determines **inventor positioning**?

Q2: What are the **consequences**?

Q3: How do we **measure** positioning?



Drawings by Bell and Gray, demonstrating their similarity

A1: Spatial model of positioning in idea space

- Parsimonious **spatial competition mechanism**, complementing other factors
- Differentiated ideas (adaptation costs → positioning matters) Salop 1979
- Knowledge spillovers vs. competition Bloom et al. 2013, Dasgupta and Maskin 1987
- Sunk and variable costs (burden of knowledge, fishing out) Jones 2009, Kortum 1997

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A2: The model generates surprising implications beyond positioning

- Predicts idea quality, TFP growth, and R&D — without new assumptions

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A2: The model generates surprising implications beyond positioning

- Predicts idea quality, TFP growth, and R&D — without new assumptions

A3: Validated measurement framework

- Systematic comparison using domain-specific tasks
- GTE embeddings outperform TF-IDF; cover 1836–2023

Part I: A Spatial Model of Idea Space

- What determines inventor positioning?

Part II: Predictions and Stylized Facts

- Comparative statics and growth implications
- Benchmarking against eight empirical regularities

Part III: New Empirical Tests

- Measurement challenge and validation
- Evidence from 188 years of U.S. patents

Part I: A Theory of Invention in Idea Space

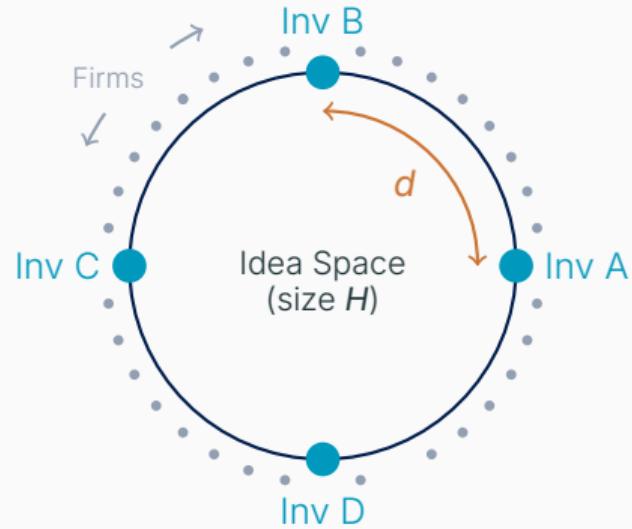
Model Setup: Spatial Competition in Idea Space

Idea space: Circle of circumference H

- $H = \text{size of market for new ideas}$
- "Similar problems have similar solutions"

Idea producers ("inventors" or "inventions"):

Idea consumers ("downstream firms"):



Market for new ideas as Salop (1979) circle

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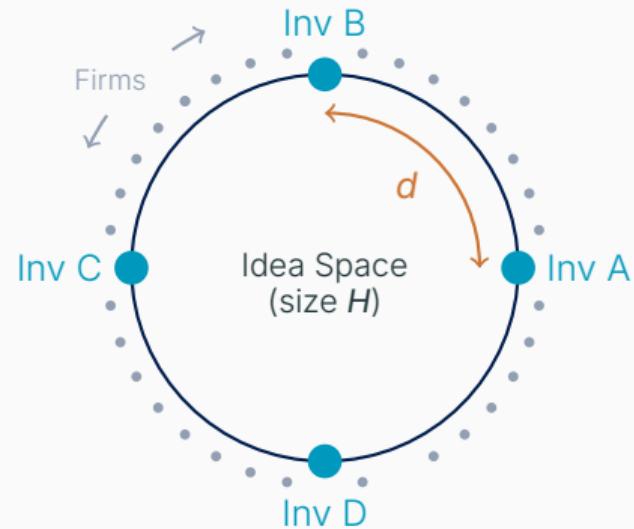
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- Choose: **entry**, location, quality q_i , price p_i
- License non-rival ideas downstream
- "Entry" = undertaking a project (\neq firm)
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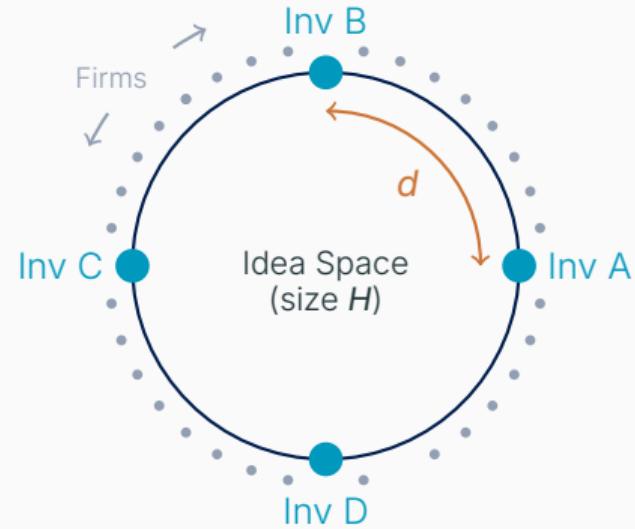
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Idea consumers ("downstream firms"):

- Distributed uniformly on circle
- License ideas to boost their TFP



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Idea Consumers: Downstream Firms

Setup: Mass H of downstream firms uniformly distributed on circle

- Each firm licenses one idea to improve productivity
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TFP from licensing: Firm at distance h from invention i achieves **log TFP**:



$$A_i(h) = Q_i - \tau h$$

- Q_i = realized quality of invention i (including spillovers)
- τh = **adaptation cost** from technological mismatch (Bloom et al. 2013, Arora et al. 2021)

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Net surplus: Firm chooses invention to maximize:

$$\text{Surplus} = \underbrace{Q_i - \tau h}_{\text{TFP gain}} - \underbrace{p_i}_{\text{license fee}}$$

- Adaptation costs create product differentiation among inventions

R&D Technology: Costs and Licensing

R&D investment: Inventor i produces idea of quality q_i at cost:

$$c(q_i) = \frac{1}{2}\gamma q_i^{1+\eta}$$

- $\eta > 0 \Rightarrow$ diminishing returns to R&D effort. Baseline: $\eta = 1$ (quadratic costs).
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- Ideas are **non-rival**—can license to multiple firms at zero marginal cost
- Inventor charges license fee p_i to each downstream firm in territory
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Entry cost: Fixed cost f (sunk costs, setup costs)

Knowledge Spillovers

Realized quality incorporates spillovers from neighbors:

$$Q_i = q_i + \frac{\beta}{2} \left(1 - \frac{d}{\lambda}\right) q_{i-1} + \frac{\beta}{2} \left(1 - \frac{d}{\lambda}\right) q_{i+1}$$

Parameters:

- q_i = own R&D investment
- $\beta \in (0, 1)$ = spillover intensity
- λ = spillover reach (spillovers vanish beyond distance λ)
- d = distance to nearest neighbor

Key property: Spillovers **decay with distance**

- At $d = 0$: maximum spillover βq
- At $d = \lambda$: spillovers vanish

Proximity → spillovers, but also → competition

Equilibrium Analysis

Equilibrium: Pricing and Quality

Symmetric equilibrium: n inventions, equal spacing $d = H/n$, identical (p, q)

▶ Existence

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Both price and quality rise as inventions spread out

Free Entry Determines Equilibrium Spacing and Inventions

Zero-profit condition:

$$\underbrace{\tau d^2}_{\text{Revenue}} - \underbrace{\frac{d^2}{2\gamma}}_{\text{R\&D cost}} - \underbrace{f}_{\text{Entry cost}} = 0$$

Solving for equilibrium spacing and number of inventions ($n = H/d$):

$$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}} \quad \Rightarrow \quad n^* = H \sqrt{\frac{\tau - \frac{1}{2\gamma}}{f}}$$

Symmetric equilibrium p^*, q^*, d^*, n^* in terms of costs τ, γ, f , and market size H

Everything Is Connected

Spacing

$$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$$

Pricing

$$p^* = \tau d$$

Quality

$$q^* = \frac{d}{\gamma}$$

Varieties

$$n^* = \frac{H}{d^*}$$

Everything Is Connected

Spacing	Pricing	Quality	Varieties
$d^* = \sqrt{\frac{f}{\tau - \frac{1}{2\gamma}}}$	$p^* = \tau d$	$q^* = \frac{d}{\gamma}$	$n^* = \frac{H}{d^*}$

Notice how both horizontal and vertical features are coupled by spatial forces:

- Spacing **depends on costs** (+fixed f , –variable γ , –adaptation τ)
- Price and quality **depend on spacing** ($p^* = f(d)$, $q^* = f(d)$)
- Number of varieties **depends on idea space size H** and costs

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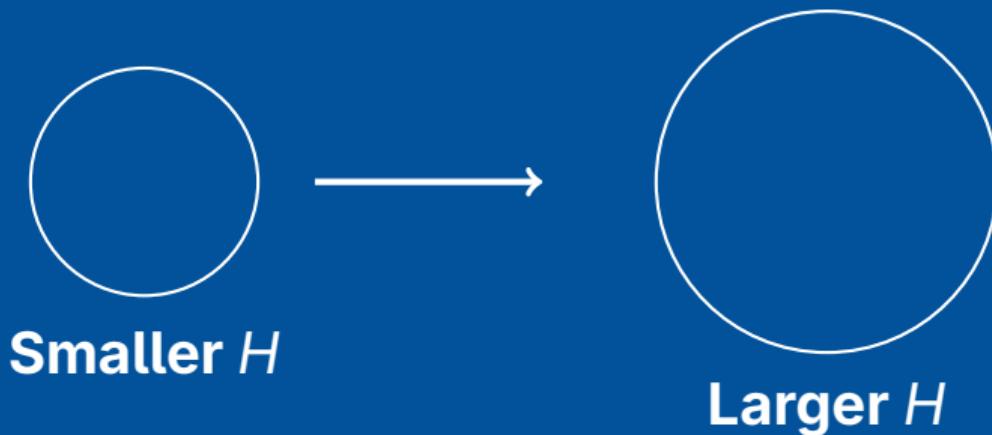
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Positioning is tied to costs (cf. Q1) and quality and pricing too (Q2)

- The size of the market H matters for variety.
- **Key question:** Could spacing, price and quality also depend on H ?

Expanding Idea Space



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Evidence:

- More U.S. patents: 500/year (1840s) → 350,000/year (2020s)
- New technological domains: electricity, chemistry, semiconductors, software...
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- Knowledge accumulation opens new possibilities
- Technology frontiers expand into new domains
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Question: How does equilibrium adjust as H grows?

The Key Structural Relationship

Recall equilibrium spacing:

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Specifying $f(H)$:

- Different relationships $f(H)$ generate different predictions (next slide)
- In principle, $\tau(H)$ or $\gamma(H)$ could also vary—though with less empirical support

Four Scenarios: How Predictions Depend on $f(H)$

As idea space H grows, what happens to spacing d^* and variety n^* ?

Scenario	Spacing d^*	Varieties n^*	q^*, p^*
1. f constant	unchanged	\uparrow (linear in H)	—
2. $f(H)$ decreasing (easier to invent)	\downarrow (clustering!)	$\uparrow\uparrow$ (faster growth)	\downarrow
3. $f(H)$ increasing (harder to invent)	\uparrow (spreading)	\uparrow (grows with H)	\uparrow
4. $f(H)$ increasing rapidly	$\uparrow\uparrow$ rapidly	\downarrow (fewer inventions)	$\uparrow\uparrow$

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- **Key insight:** Growth implications differ dramatically across scenarios

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So which scenario describes reality?

$f(H)$ Increasing: The Burden of Knowledge

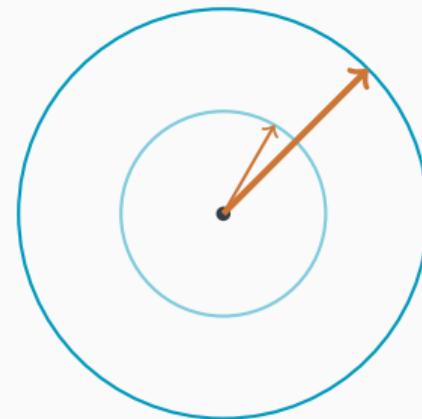
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Empirical evidence: Jones (2009)

- Inventors getting older at first patent
- Larger teams
- More education
- Longer training periods



More effort to reach the frontier

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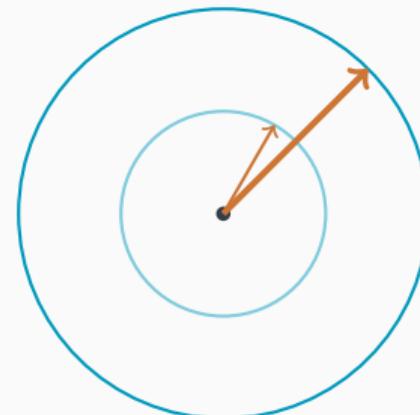
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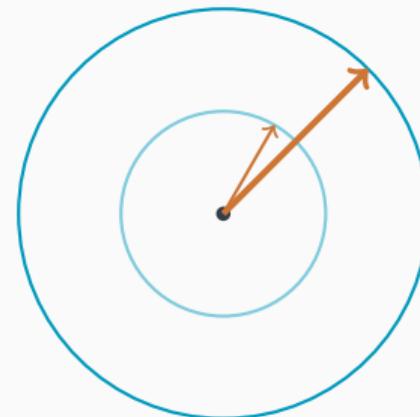
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In idea space: Entry costs rise with market size

Our Model: Entry Costs Rise with Idea Space

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This generates many predictions:

1.

Spreading out:

$$d^* = \sqrt{\frac{\phi H^\alpha}{\tau - \frac{1}{2\gamma}}}$$

— increases with H

Proposition

2. and more...

Part II: Model Predictions

Model Predictions to Evidence

Model Predictions Match Three Categories of Literature Evidence

1. Positioning & Variety

(Horizontal Margins: $dd/dH > 0, dn/dH > 0$)

► Comparative statics

- ✓ Spreading out over time [this paper](#), Kelly+ 2021, [Chiopris 2024](#)
- ✓ More inventions, more firms, expanding idea space [this paper](#), Hirshey+ 2012

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2. Quality & Returns

(Vertical Margins: $dq/dH > 0, dp/dH > 0, d(p \cdot d)/dH > 0$)

- ✓ More R&D investment per firm Hirshey+ 2012
- ✓ Higher gross returns to patents Kogan+ 2017, Bessen+ 2018
- ✓ Higher patent quality Hall+ 2005, Kelly+ 2021
- ✓ R&D spillovers stable ($dq/dH \approx -1 \cdot dd/dH$) Lucking+ 2019

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3. Productivity Decline

(explained next)

- ✓ TFP growth decelerates Bloom+ 2020
- ✓ R&D productivity declines Bloom+ 2020

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Our spatial model unifies many streams of empirical evidence

Declining R&D Productivity

Define aggregate R&D productivity (cf. Bloom et al. 2020)

$$\Pi \equiv \frac{\text{Agg TFP growth}}{\text{Agg R&D}}$$

$$\text{Agg TFP growth} = q \left[1 + \beta \left(1 - \frac{d}{\lambda} \right) \right] - \frac{\tau d}{4}$$

$$\text{Agg R&D} = n \cdot \left[\frac{1}{2} \gamma q^2 + \phi H \right]$$

- **Average** Δ TFP delivered downstream
- Doesn't scale with n
- **Total** R&D across n inventions
- Scales with n

Key insight: As H expands, entry *dilutes* aggregate R&D cf. Howitt 1999, Peretto 1998, 2018

Decomposition Framework

Five forces reduce research productivity

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Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH} \left[1 + \beta \left(1 - \frac{d}{\lambda} \right) \right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda} \frac{dd}{dH}}_{(1) \text{ Spillover attenuation}} - \underbrace{\frac{\tau}{4} \frac{dd}{dH}}_{(2) \text{ Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance

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Forces raising R&D:

$$\frac{d(\text{Agg R&D})}{dH} = \underbrace{\frac{dn}{dH} \cdot [c(q) + f(H)]}_{(5) \text{ Entry expansion}} + \underbrace{n \cdot c'(q)}_{(3) \text{ Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)*} + \underbrace{n \cdot f'(H)}_{(4) \text{ Burden of knowledge}}$$

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3. **Fishing out** Convex R&D costs

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$$\frac{d(\text{Agg R&D})}{dH} = \underbrace{\frac{dn}{dH} \cdot [c(q) + f(H)]}_{(5) \text{ Entry expansion}} + \underbrace{n \cdot c'(q)}_{(3) \text{ Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)*} + \underbrace{n \cdot f'(H)}_{(4) \text{ Burden of knowledge}}$$

3. **Fishing out** Convex R&D costs
4. **Burden of knowledge** Rising fixed costs

Decomposition Framework

Five forces reduce research productivity

- We will use this framework for quantitative decomposition

Forces reducing TFP:

$$\frac{d(\text{Agg TFP growth})}{dH} = \underbrace{\frac{dq}{dH} \left[1 + \beta \left(1 - \frac{d}{\lambda} \right) \right]}_{\text{Quality investment}} - \underbrace{\frac{\beta q}{\lambda} \frac{dd}{dH}}_{(1) \text{ Spillover attenuation}} - \underbrace{\frac{\tau}{4} \frac{dd}{dH}}_{(2) \text{ Adaptation drag}}$$

1. **Spillover attenuation** Knowledge flows weaken with distance
2. **Adaptation drag** Downstream firms farther from inventions

Forces raising R&D:

$$\frac{d(\text{Agg R&D})}{dH} = \underbrace{\frac{dn}{dH} \cdot [c(q) + f(H)]}_{(5) \text{ Entry expansion}} + \underbrace{n \cdot c'(q)}_{(3) \text{ Fishing out}} \cdot \underbrace{\frac{dq}{dH}}_{(5)*} + \underbrace{n \cdot f'(H)}_{(4) \text{ Burden of knowledge}}$$

3. **Fishing out** Convex R&D costs
4. **Burden of knowledge** Rising fixed costs
5. **Entry and territory expansion** More inventions cover larger territories

From Static Model to Growth Rates

If H grows at constant rate g_H ($\dot{H} = g_H \cdot H$) \Rightarrow constant growth in:

Variable	Growth Rate	Baseline ($\alpha = 1, \eta = 1$)
Spacing d	$g_d = \frac{\alpha}{2} g_H$	$\frac{1}{2} g_H$
Quality q	$g_q = g_d$	$\frac{1}{2} g_H$
Entry n	$g_n = (1 - \frac{\alpha}{2}) g_H$	$\frac{1}{2} g_H$
Agg R&D	$g_{R&D} = g_n + \theta g_q + \theta \eta g_q + (1 - \theta) \alpha g_H$	$\frac{3}{2} g_H$

$\theta \equiv$ Variable cost share

► Detailed growth equations

► General model

Static model

→ Comparative statics as H grows exogenously

→ Testable predictions

Part III: Testing Model Predictions

From Growth Rates to Empirical Tests

Growth equations suggest empirical strategy:

Prediction 1: Spreading Out

- Model: $g_d = \frac{1}{2}g_H > 0$
- Empirical: Measure similarity over time → should decline
- Data: 188 years of U.S. patents (1836-2023)

Prediction 2: Declining R&D Productivity

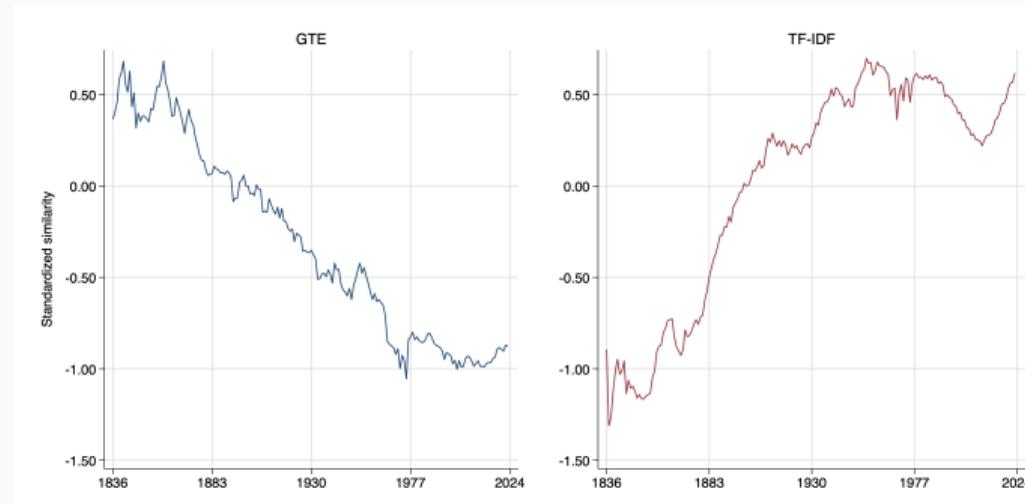
- Model: Five forces decomposition
- Empirical: Regress TFP and R&D growth on $-\Delta \text{Sim}$
- Decompose: Spatial (40-60%) vs non-spatial forces

First challenge: How do we measure similarity?

Measuring Similarity in Idea Space

The Measurement Challenge

Same patent text, opposite conclusions:



- **Left (GTE):** Similarity *declining* — inventions spreading out
- **Right (TF-IDF):** Similarity *increasing* — inventions clustering

Key Question: Which “map” of idea space should we trust?

Data: US Patent Claims, 1836–2023

Patent text corpus:

▶ Details

- **Historical (1836–1975):** ProQuest Patents Core (digitized full text)
- **Modern (1976–2023):** USPTO PatentsView
- Focus on **claims** — defines legal boundaries of invention

Multiple NLP representations tested:

- Traditional: TF-IDF (word frequency)
- Modern neural embeddings: GTE, PaECTER, S-BERT, Doc2vec, USE, OpenAI

Similarity measure:

▶ Computation

- Cosine similarity between patent representations
- Average pairwise similarity by year
- Standardized by cross-sectional standard deviation

▶ Alternatives

Validation Framework: Three Complementary Tasks

Task	Time Period	Granularity	Expertise	
Patent Interferences	2001–2014	Identical	USPTO examiners	
Human Judgments	1850–1975	Continuous	Lay annotators	
Classifications	1850–2023	Categorical	Expert labels	

Why multiple tasks?

- No single ground truth for “similarity”
- Different aspects: legal identity vs. technological relatedness
- Temporal robustness across 175+ years

Models performing well across all tasks are most reliable

Validation Results: Model Performance

Model	Interferences		Human Agreement	Classifications	
	PR AUC	F10		Section	Class
GTE	0.64 (2)	0.90 (1)	0.62 (1)	0.596 (2)	0.656 (3)
PaECTER	0.65 (1)	0.90 (2)	0.51 (3)	0.590 (3)	0.672 (1)
S-BERT	0.52 (3)	0.82 (3)	0.54 (2)	0.600 (1)	0.671 (2)
TF-IDF	0.45 (4)	0.77 (4)	0.35 (4)	0.514 (4)	0.525 (4)

- **GTE and PaECTER** consistently top performers
- **TF-IDF** consistently worst (20–40% lower performance)
- All beat **random chance** — but **magnitudes differ dramatically**

Model Selection: Why We Use GTE

GTE selected for main results because:

1. **Temporal robustness** — best on historical patents (1880–1920)
2. **Near-identical performance on interferences** — our most demanding test
3. **Consistent across all tasks** — ranks 1st or 2nd on 4/5 metrics

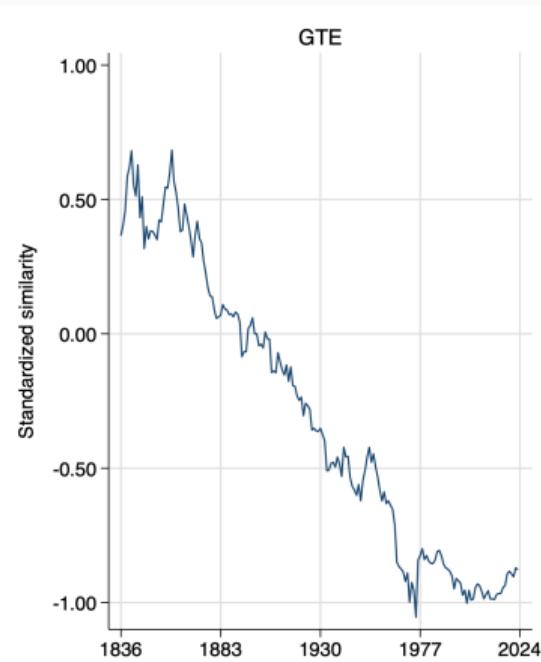
Why TF-IDF fails: ▶ Details

- Overweights period-specific language
- Treats synonyms as unrelated ("velocipede" ≠ "bicycle")
- Would lead to *opposite* conclusions about our theory

Robustness checks with PaECTER, S-BERT, and ensemble measures

Prediction 1: Are Inventions Spreading Out?

Main Finding: Secular Decline in Patent Similarity



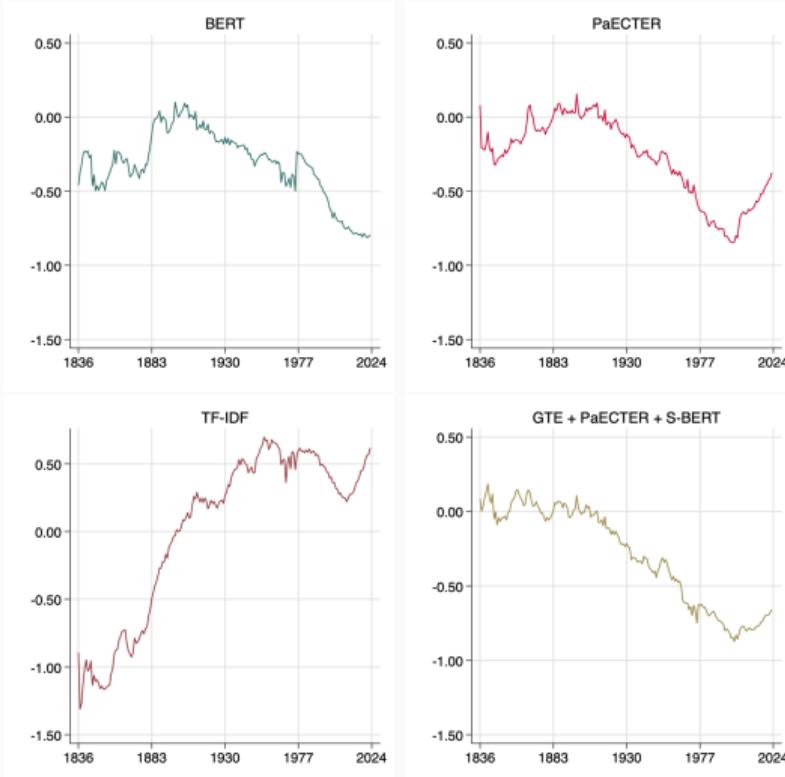
Using validated GTE embeddings:

~ 1.5σ decline in patent similarity, 1836–2023

- Consistent with theory: inventions spreading out
- Spreading out ($d \uparrow$) = Declining similarity (Sim \downarrow)
- Multi-patent entity effect post-2000 (to come)

Confirms Prediction 1: Spreading Out

Why Validation Matters: Comparing Representations



TF-IDF (worst performer):

- $\sim 1.5\sigma$ increase—opposite conclusion!
- Validation correctly discards

PaECTER, S-BERT (cf. GTE):

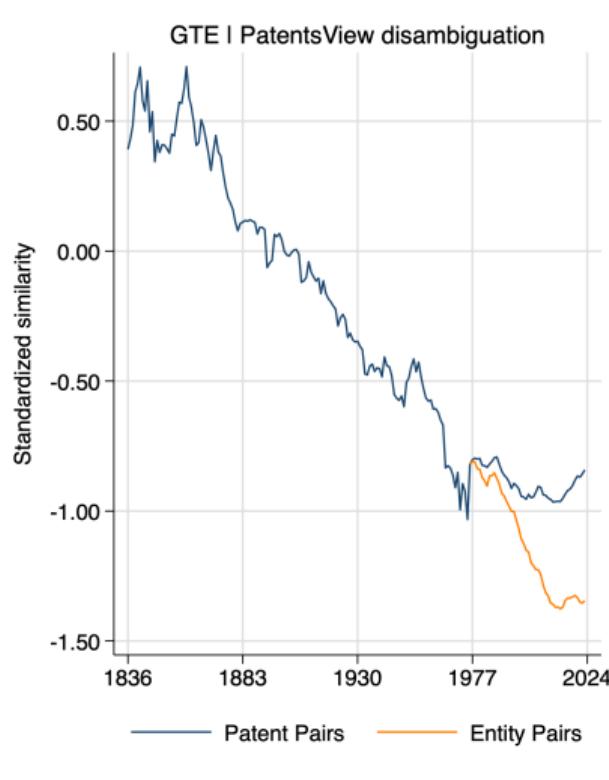
- Similar $\sim 0.8\sigma$ decline, 1880–2000
- Diverge pre-1880 & post-2000

Ensemble (avg of top models):

- $\sim 1.0\sigma$ decline, 1836–2023

**Validated methods agree; unvalidated
TF-IDF misleads**

Robustness: Accounting for Multi-Patent Entities



Concern: Post-2000 dynamics coincide with:
business method patents, non-practicing entities,
increased defensive patenting.

▶ Patents v entities

- **Multiple patents from same entity may be similar but not independent.**

Strategy: Sample 1 patent/entity-year

Result:

- Decline persists after correction
- **Independent inventions** still spreading out

Robustness: Spreading Out Within Technology Classes

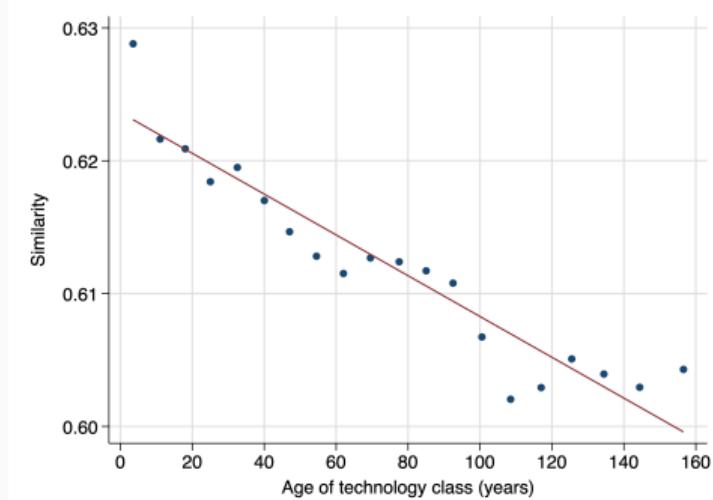
Alternative explanations: Changing patent office practice over time? Shifts across major technology areas?

Test: Within-class similarity by class "age"

- Birth = Class first issued 50 patents
- e.g., Combinatorial Chemistry 2001
- Addresses compositional concerns

▶ Between

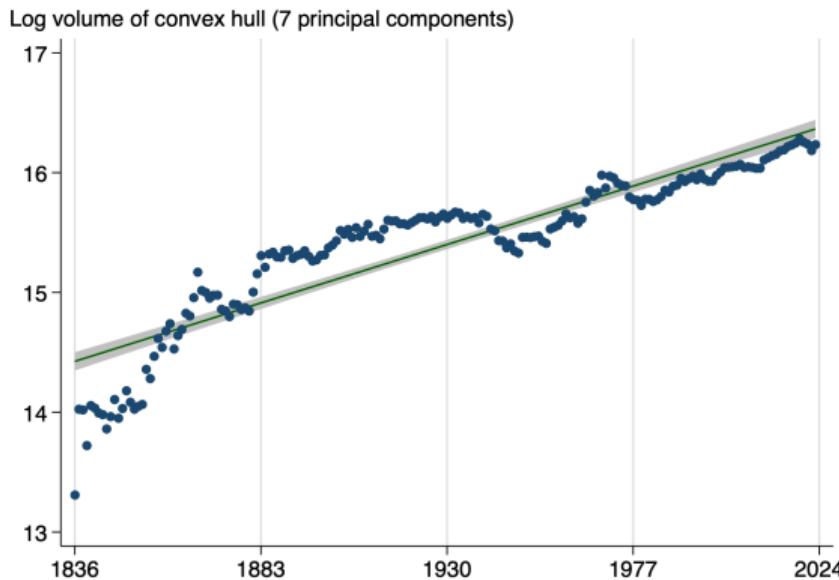
Finding: Within-class similarity declines as classes mature



Spreading out is a dynamic process tied to field evolution

Corroboration: Expanding Convex Hull

◀ R&D regression



Is Idea Space Expanding?

Test:

- 1024 GTE dimensions to 7 PC
- Measure volume of convex hull

Result:

- +0.5%/yr (6 PC: +0.4%/yr)
- Likely under-estimate due to dimensionality reduction

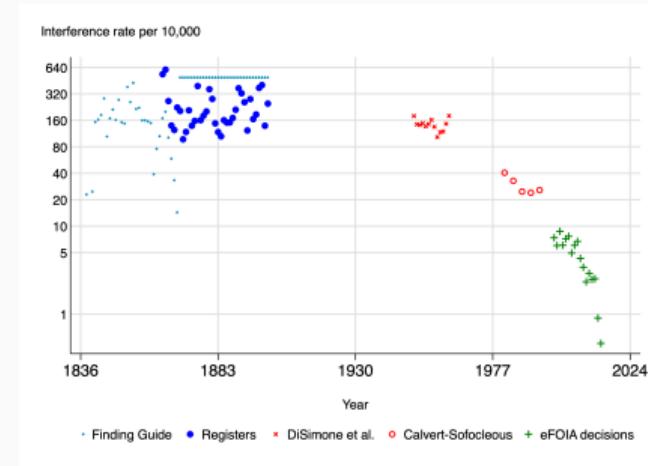
Independent Corroboration: Declining Interference Rates

Patent interferences:

- USPTO determination that two independent inventors made *identical* inventions
- Direct measure of multiple invention ($d = 0$)

Data: Purpose - digitized from 5 sources

- Nat. Archives & Registers (1838–1900)
- Published statistics (1950–1994)
- eFOIA decisions (1998–2014) Ganguli et al. 2020



Finding: Interference rate declined over 150 years

**Same conclusion from
completely different data source**

Summary: Inventions Are Spreading Out

Robust evidence of spreading out:

- ✓ Main finding: 1.5σ decline in similarity, 1836–2023
- ✓ Decline extends after 2000 for independent inventions
- ✓ Robust to spatial scale (local and global)
- ✓ Robust to within vs. between class decomposition
- ✓ Appears within classes as they age
- ✓ Corroborated by interference rates (150 years)
- ✓ Idea space is expanding



Next: What are the consequences for research productivity?

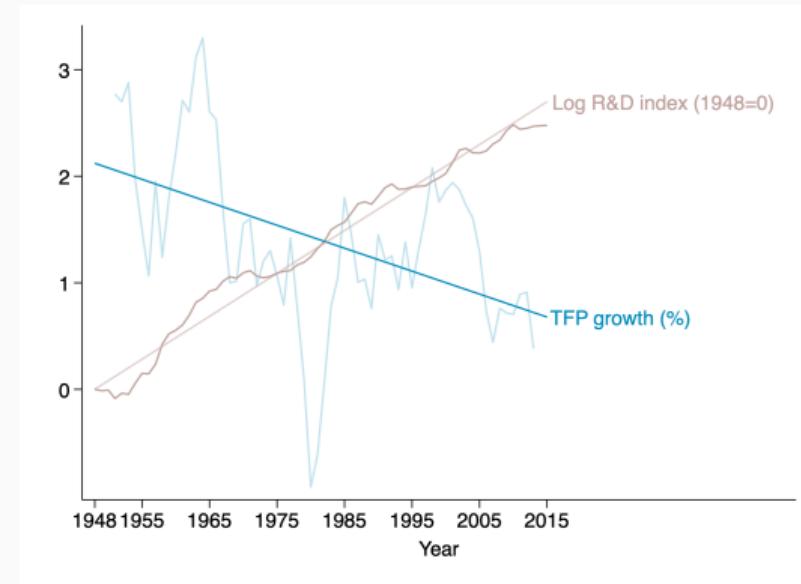
Prediction 2:
Does Spreading Out Reduce R&D Productivity?

The Puzzle: Are Ideas Getting Harder to Find?

The research productivity decline:

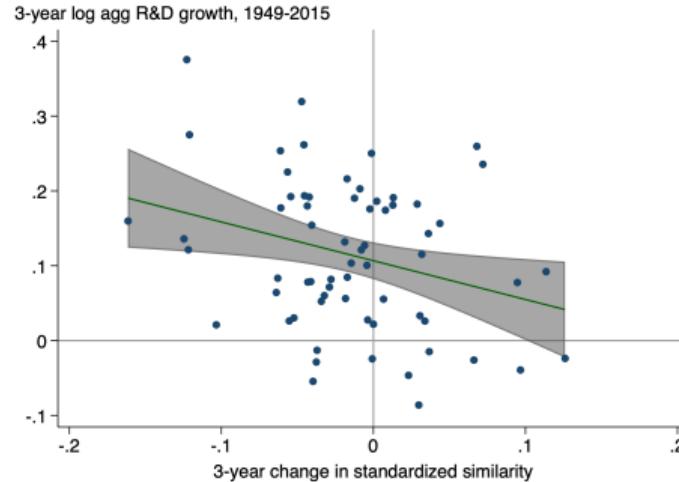
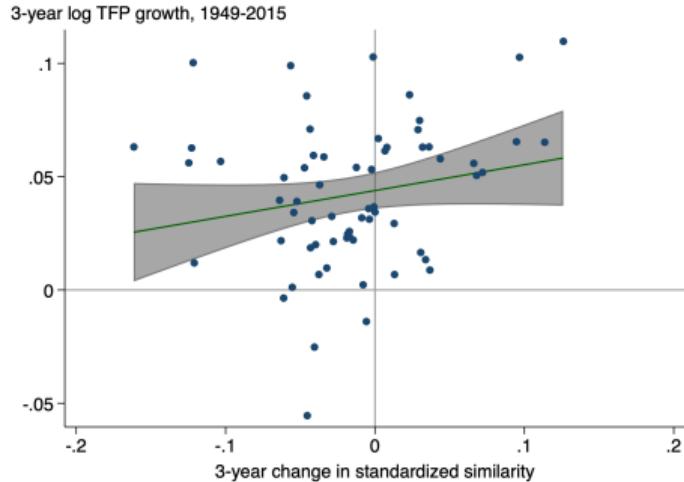
- Real R&D up **>20x** since 1930
- TFP growth slowed by factor of **3x**
- R&D productivity decline >-5%/yr

Key question: Why does it take so much **more** research effort to achieve the same rate of **slower** growth?



Bloom, Jones, Van Reenen and Webb, 2020

Timing: Similarity Predicts TFP and R&D Growth



- **Left:** Declining similarity → **lower TFP growth**
- **Right:** Declining similarity → **higher R&D growth**

Both patterns confirm Prediction 2

TFP and Spreading Out

From Theory to Estimation: TFP

TFP growth equation (from BGP): 

$$g_{TFP} = \underbrace{g_q \left(1 + \beta - \frac{\beta d}{\lambda} \right)}_{\text{Quality (with spillovers)}} - \underbrace{\frac{\beta q}{\lambda} g_d}_{\text{Spillover attenuation}} - \underbrace{\frac{\tau}{4} g_d}_{\text{Adaptation drag}}$$

Substitute equilibrium relationships for unobservables:

- $q^* = d/\gamma$ and $dq^*/dt = (1/\gamma)(dd/dt) \Rightarrow g_q = g_d$

$$g_{TFP} = \underbrace{\left(1 + \beta - \frac{\tau}{4} \right) \cdot g_d}_{b_1} - \underbrace{\beta(1 + 1/\gamma)/\lambda \cdot d \cdot g_d}_{b_2}$$

Suggests the regression:

- Observable proxy: $g_d \approx -\Delta \text{Sim}$ (small annual changes in standardized measure)

$$\Delta \log(\text{TFP})_t = b_0 + b_1 \cdot (-\Delta \text{Sim})_t + b_2 \cdot (-\Delta \text{Sim}) \cdot (-\text{Sim}_{t-1}) + \epsilon_t$$

From Theory to Estimation: TFP Regression

Regression Specification:

$$\Delta \log(\text{TFP})_t = b_0 + b_1 \cdot (-\Delta \text{Sim})_t + b_2 \cdot (-\Delta \text{Sim}) \cdot (-\text{Sim}_{t-1}) + b_3 \cdot t + \epsilon_t$$

Data:

- TFP and Real R&D Inputs, 1948–2015 (Bloom et al., 2020)

Predictions and interpretation:

- $b_1 \leq 0$: Effect on TFP growth from ↑ quality scaling net of ↓ adaptation costs
- $b_2 < 0$: Spillover attenuation and reduced marginal return to R&D
- b_3 : Time trend controls for factors not explicit in the model

TFP Growth and Technological Distance

	Annual		3 - Year	5 - Year
$b_1 : -1 \times \Delta Sim$	-0.169*** (0.057)	-0.171*** (0.083)	-0.278*** (0.095)	-0.269*** (0.098)
$b_2 : (-1 \times \Delta Sim) \times (-1 \times Sim_{t-1})$	—	-0.015 (0.342)	-0.408 (0.320)	-0.571* (0.312)
<i>Implied TFP drag from spreading out ($\bar{\Delta Sim}$, %/yr):</i>				
1948 (Sim = 0.35)	-0.08	-0.08	-0.07	-0.04
1991 (Sim = 0, baseline)	-0.08	-0.09	-0.14	-0.16

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Validation: Implied drag **-0.16%/yr** from ΔSim consistent with quasi-experimental cross-sectional elasticity of **-0.15%/yr** ✓ Bloom et al. 2013, Lucking et al. 2019

► Details

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► Details

Contribution to TFP deceleration: Drag worsened -0.04%/yr (1948) → -0.14%/yr (2015). Change = 0.10 pp = **7% of 1.4 pp total TFP deceleration.**

► Decomposition

R&D and Spreading Out

From Theory to Estimation: R&D

R&D growth equation (from BGP): 

$$g_{R\&D} = \underbrace{g_n}_{\text{Entry}} + \underbrace{\theta(1+\eta)g_q}_{\text{Quality (incl. fishing out)}} + \underbrace{(1-\theta)g_f}_{\text{Rising fixed costs}}$$

Substitute equilibrium relationships:

$$g_{R\&D} = \underbrace{[1 + \alpha(1-\theta)]g_H}_{a_0} + \underbrace{[\theta(1+\eta)-1]g_d}_{a_1}$$

Regression specification:

$$g_{R\&D,t} = a_0 + a_1 \cdot (-\Delta \text{Sim})_t + a_2 \cdot t + \epsilon_t$$

- a_2 captures (unmodeled) acceleration in idea space growth (but: $\hat{a}_2 \approx 0$)

Identification of Structural Parameters

Identification of structural parameters:

$$a_1 = \theta(1 + \eta) - 1$$



$$\theta = \frac{a_1 + 1}{1 + \eta}$$

(variable cost share)

$$a_0 = [1 + \alpha(1 - \theta)]g_H$$



$$g_H = \frac{a_0}{1 + \alpha(1 - \theta)}$$

(idea space growth)

Baseline: $\alpha = 1$, $\eta = 1$. **Later:** Calibration w/ quasi-experimental $\hat{\eta}$ and estimate of α .

Regression coefficients \rightarrow structural parameters (θ, g_H)

R&D Growth and Technological Distance

	Annual	3 - Year	5 - Year
$a_1: -1 \times \Delta \text{Sim}$	0.165 (0.177)	0.448** (0.219)	0.438* (0.244)
$a_0: \text{Constant}$	0.034*** (0.006)	0.102*** (0.013)	0.173*** (0.018)
Implied θ (variable cost share)	0.58	0.72	0.72
Implied g_H (idea space growth)	2.4%/yr	2.7%/yr	2.7%/yr

Validation:

- $\theta = 72\%$ aligns with NSF survey data (labor = 69% of R&D) ✓
- $g_H = 2.7\%/\text{yr}$ consistent with patent embedding volume growth ✓ 
- BGP consistency: Model predicts $g_d/g_{R&D} = 1/3$; In data, $-\Delta \text{Sim}/g_{R&D} = 0.31$ ✓

Growth Accounting

The Research Productivity Decline

Research productivity:

$$\Pi \equiv g_{TFP}/\text{Agg R&D} \text{ (TFP growth per unit R&D)}$$

The decline (1948–2015):

- TFP growth fell: $2.1\%/\text{yr} \rightarrow 0.7\%/\text{yr}$ ($g_{g_{TFP}} = -1.6\%/\text{yr}$)
- R&D spending grew: $4.0\%/\text{yr}$

$$g_{\Pi} = g_{g_{TFP}} - g_{\text{R&D}} = -1.6\% - 4.0\% = \boxed{-5.6\%/\text{yr}}$$

Goal: Decompose this decline into spatial and non-spatial components

From Regressions to Parameters

What we estimated from R&D regression:

Parameter	Value	Source
θ (variable cost share)	0.72	R&D regression coefficient a_2 ($\eta = 1$)
g_H (idea space growth)	2.7%/yr	R&D regression constant a_0 ($\alpha = 1$)

What we assume (baseline):

Parameter	Value	Interpretation
α (entry cost curvature)	1.0	Entry costs scale linearly with H
η (R&D cost curvature)	1.0	Quadratic R&D costs

Decomposing the R&D Productivity Decline

Model implies: $g_d = \frac{\alpha}{2} g_H = 1.35\%/\text{yr}$ (spreading rate if $\alpha = 1$)

Component	Contribution	Classification	Comment
TFP deceleration	-1.6%/yr		
Spatial drag worsened	-0.11%/yr	Spatial	7% of deceleration
Unmodeled factors	-1.49%/yr	Non-spatial	

Decomposing the R&D Productivity Decline

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Spatial drag worsened	-0.11%/yr	Spatial	7% of deceleration  TFP regression
Unmodeled factors	-1.49%/yr	Non-spatial	
<i>R&D growth</i>	+4.0%/yr		
Entry expansion $(1 - \frac{\alpha}{2})g_H$	+1.35%/yr	Spatial	(new inventions)
Quality scaling $(\theta \frac{\alpha}{2} g_H)$	+0.97%/yr	Spatial	(larger territories; TFP units)
Fishing out $(\theta \eta \frac{\alpha}{2} g_H)$	+0.97%/yr	Non-spatial	(convex costs)
Burden of knowledge $(1 - \theta)(\alpha g_H)$	+0.76%/yr	Non-spatial	(rising fixed costs)
Unmodeled factors	-0.05%/yr	Non-spatial	

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Model implies: $g_d = \frac{\alpha}{2} g_H = 1.35\%/\text{yr}$ (spreading rate if $\alpha = 1$)

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Fishing out $(\theta \eta \frac{\alpha}{2} g_H)$	+0.97%/yr	Non-spatial	(convex costs)
Burden of knowledge $(1 - \theta)(\alpha g_H)$	+0.76%/yr	Non-spatial	(rising fixed costs)
Unmodeled factors	-0.05%/yr	Non-spatial	
Total decline	-5.6%/yr		
Spatial contribution	-2.43%/yr		43%
Non-spatial contribution	-3.17%/yr		57%

Robustness: Spatial Share Increases with Better Calibration

Baseline assumptions: $\alpha = 1$, $\eta = 1$

Alternative calibration:

- $\eta = 0.625$: Guceri-Liu (2019)
- $\theta = 0.89$: From R&D regression a_2
- $\alpha = 0.76$: Constrain sum to 4.0%
- $g_H = 3.2\%/\text{yr}$: From R&D regression a_0
- $g_d = \frac{\alpha}{2} g_H = 1.2\%/\text{yr}$

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Alternative decomposition:

	Baseline	Alternative
Entry expansion	1.35%/ yr	1.98%/ yr
Quality scaling	0.97%/ yr	1.08%/ yr
Fishing out	0.97%/ yr	0.67%/ yr
Burden of knowledge	0.76%/ yr	0.27%/ yr
Sum	4.05%/ yr	4.00%/ yr
Spatial share	43%	57%

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Sum	4.05%/ yr	4.00%/ yr
Spatial share	43%	57%

Conservative baseline; higher spatial share with alternative calibration

- $\eta = 0.625 < 1$: R&D costs grow sub-quadratically with q
- $\hat{\alpha} = 0.76 < 1$: Entry costs grow sub-linearly with H
- Entry expansion (1.98%) < patent growth (3.9%) $\Rightarrow \downarrow$ **ideas per patent** (-1.9%/ yr)

Conclusion

Summary

1. Inventions are spreading out across expanding idea space

- Documented over 188 years using validated NLP measures
- Independently corroborated by patent interference trend

2. Spatial model explains spreading out as response to burden of knowledge

- Unifies positioning, quality, and R&D productivity with a single mechanism

3. Spatial forces account for 40–60% of the long-run R&D productivity decline

- Spreading out reduces TFP growth and raises R&D — simultaneously
- Complements fishing-out and burden of knowledge

Spatial forces are a first-order driver of declining research productivity

Backup Slides

Backup: Comparative Statics Derivations

[◀ Return](#)

Spreading out: From zero-profit condition $d^2(\tau - \frac{1}{2\gamma}) = \phi H$:

$$\frac{dd}{dH} = \frac{\phi}{2d(\tau - \frac{1}{2\gamma})} = \frac{\phi}{dR/dd - dc/dd} > 0$$

Rising quality and prices:

$$\frac{dq}{dH} = \frac{1}{\gamma} \frac{dd}{dH} > 0, \quad \frac{dp}{dH} = \tau \frac{dd}{dH} > 0$$

Rising entry:

$$\frac{dn}{dH} = \frac{1}{d} - \frac{H}{d^2} \frac{dd}{dH} > 0 \text{ under spreading-out condition}$$

Declining productivity:

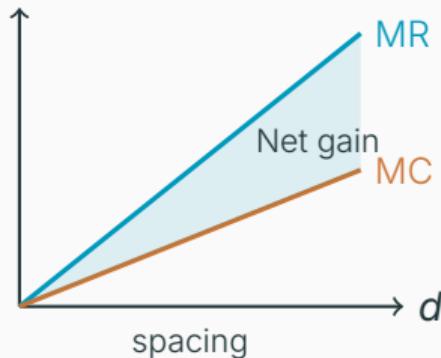
$$\frac{d\rho}{dH} < 0, \quad \frac{d\Pi}{dH} < 0$$

Proposition (Spreading Out)

For $\tau\gamma > \frac{1}{2}$, equilibrium spacing increases with opportunity space: $\frac{dd^*}{dH} > 0$.

Inventions become **less similar** over time.

Why is spreading out profitable?



Marginal revenue of expanding territory:

- Revenue $R = \tau d^2 \Rightarrow MR = 2\tau d$

Marginal cost of expanding territory:

- Need higher quality: $q = d/\gamma$
- $MC = d/\gamma$

Spreading profitable when:

$$MR > MC \Rightarrow \boxed{\tau\gamma > \frac{1}{2}}$$

Adaptation costs must create sufficient pricing power

Backup: Model Equations

Fixed cost (burden of knowledge): $f(H) = \phi H^\alpha$

R&D cost: $c(q_i) = \frac{1}{2} \gamma q_i^{1+\eta}$

Realized quality (with spillovers): $Q_i = q_i + \frac{1}{2} \beta \left(1 - \frac{d}{\lambda}\right) (q_{i-1} + q_{i+1})$

Baseline model: $\alpha = 1, \eta = 1$

Equilibrium pricing and quality: $p^* = \tau d, \quad q^* = \frac{d}{\gamma}$

Equilibrium spacing: $d^*(H) = \sqrt{\frac{\phi H}{\tau - \frac{1}{2\gamma}}}$

Equilibrium entry: $n^* = \frac{H}{d^*}$

Equilibrium revenue: $R^* = p^* \cdot d^* = \tau d^2$

Backup: Robustness to Entry Cost Curvature

◀ Return

With $f(H) = \phi H^\alpha$, equilibrium spacing satisfies $d \propto H^{\alpha/2}$:

Condition	Entry Growth	Prediction
$0 < \alpha < 2$	$g_n = (1 - \frac{\alpha}{2})g_H > 0$	Entry grows ✓
$\alpha = 2$	$g_n = 0$	Entry stagnates
$\alpha > 2$	$g_n < 0$	Entry declines ✗

Main results robust for $\alpha < 2$:

- Spreading out: $g_d = \frac{\alpha}{2}g_H > 0$ for any $\alpha > 0$
- Declining R&D productivity: Holds throughout range
- Higher $\alpha \rightarrow$ faster spreading, but lower spatial share of productivity decline

Counterfactual boundary: Patent counts grow over time, ruling out $\alpha \geq 2$

Backup: TFP and R&D Growth Equations

[◀ Growth rates](#) [◀ TFP regression](#) [◀ R&D regression](#)

TFP growth:

$$g_{TFP} = \underbrace{g_q \left(1 + \beta - \frac{\beta d}{\lambda}\right)}_{\text{Quality (with spillovers)}} - \underbrace{\frac{\beta q}{\lambda} g_d}_{\text{Spillover attenuation}} - \underbrace{\frac{\tau}{4} g_d}_{\text{Adaptation drag}}$$

R&D growth:

$$g_{R&D} = \underbrace{g_n}_{\text{Entry}} + \underbrace{\theta \cdot g_q}_{\text{Quality scaling}} + \underbrace{\theta \cdot g_q}_{\text{Fishing out}} + \underbrace{(1 - \theta) g_f}_{\text{Burden of knowledge}}$$

where θ = variable cost share; $\alpha = 1$, $\eta = 1$

The asymmetry:

- TFP: grows with g_q minus spatial drags
- R&D: grows with g_q plus entry (g_n) plus fixed costs (g_f)

$g_{R&D} > g_{TFP} \Rightarrow \text{Research productivity declines}$

Backup: Growth Equations with General Cost Curvatures

◀ Growth Rates

◀ Decomposition

Component	General	Baseline ($\alpha = 1, \eta = 1$)
Entry cost	$f(H) = \phi H^\alpha$	ϕH
R&D cost	$c(q) = \frac{1}{2} \gamma q^{1+\eta}$	$\frac{1}{2} \gamma q^2$
Spacing growth	$g_d = \frac{\alpha}{2} g_H$	$\frac{1}{2} g_H$
Quality growth	$g_q = g_d$	$\frac{1}{2} g_H$
Entry growth	$g_n = (1 - \frac{\alpha}{2}) g_H$	$\frac{1}{2} g_H$
Fixed cost growth	$g_f = \alpha g_H$	g_H

R&D growth equation:

$$g_{R\&D} = \underbrace{(1 - \frac{\alpha}{2}) g_H}_{\text{Entry}} + \underbrace{\frac{\theta \alpha}{2} g_H}_{\text{Quality scaling}} + \underbrace{\frac{\theta \eta \alpha}{2} g_H}_{\text{Fishing out}} + \underbrace{(1 - \theta) \alpha g_H}_{\text{Burden of knowledge}}$$

Why linear in log TFP? $A_i(h) = Q_i - \tau h$

Standard in spatial competition (Salop 1979):

- Idea consumers have preferences linear in quality net of distance costs
- $A_i(h)$ interpreted as log TFP \Rightarrow firms care about *proportional* productivity gains

Microfoundation: Each downstream firm has one unit of fixed input ℓ and produces:

$$y = e^A \cdot \ell$$

With output price = 1 and $\ell = 1$, profit is $\pi = e^A$. Willingness to pay for technology delivering incremental log TFP A (relative to baseline $e^0 = 1$):

$$WTP = e^A - 1 \approx A \quad (\text{first-order Taylor approximation})$$

Accuracy: For annual TFP increments ($A \approx 0.015/\text{year}$), approximation error < 0.01%

Advantage: Predictions directly comparable to empirical TFP elasticities (Bloom et al. 2013) and growth accounting (Bloom et al. 2020)

Spreading-out condition:

$$\tau\gamma > \frac{1}{2}$$

Marginal revenue of expanding territory exceeds marginal cost.

Second-order conditions:

- Pricing: $\partial^2 R / \partial p^2 < 0$ (satisfied)
- Quality: $\partial^2 \pi / \partial q^2 = -\gamma < 0$ (satisfied)
- No spatial deviation (verified in paper)

Additional conditions:

- Spillover reach: $d < \lambda$ (spillovers active)
- Full coverage: All downstream firms adopt some technology

Patent interferences (2001–2014):

- **First to invent:** USPTO proceeding for multiple applicants w/ identical claims
- Provides ground truth for “identical” similarity
- 322 true interfering pairs among 96,580 application pairs

Economic intuition: Examiner ranks pairs by similarity, investigates above threshold

- Higher threshold → fewer false positives but miss true interferences
- Lower threshold → catch more but burden staff with unnecessary investigations

Metrics:

- F10: Weights recall 10× more than precision (missing interferences is costly)
- PR AUC: Precision - Recall area under curve across all thresholds

Key result: GTE, PaECTER, OpenAI retrieve ~90% of true interferences with 2–5× fewer false positives than TF-IDF/S-BERT

Historical patents (1850–1975):

- Sampled patent pairs that each model ranked at least 50 percentiles apart
- Annotators rank **relative** similarity of 2 patent pairs
- Tests temporal robustness (historical language): Oversample 1880–1920

Task: Do model rankings agree with human rankings?

- For each patent, rank others by similarity
- Compare model ranking to human ranking

Metric: Agreement coefficient from regression

$$\text{Human Rank} = \alpha + \beta \cdot \text{Model Rank} + \epsilon$$

Higher β = better agreement

Backup: Classification Validation Task

[◀ Return](#)

USPTO Classifications (1850–2023):

- CPC technology codes assigned by examiners
- Section level (8 categories) and Class level (120+ categories)
- Captures expert judgment of technological relatedness

Task: Predict whether patent pair shares classification

- Same Section (coarse): 8 top-level categories
- Same Class (fine): 3-digit classification

Metric: ROC AUC

- Area under Receiver Operating Characteristic curve
- 0.5 = random, 1.0 = perfect

TF-IDF overweights period-specific language:

- Treats “velocipede” (1880s) and “bicycle” (modern) as unrelated
- Period-specific terminology dominates similarity scores
- Creates spurious correlation with time

Example: 1880 velocipede patent

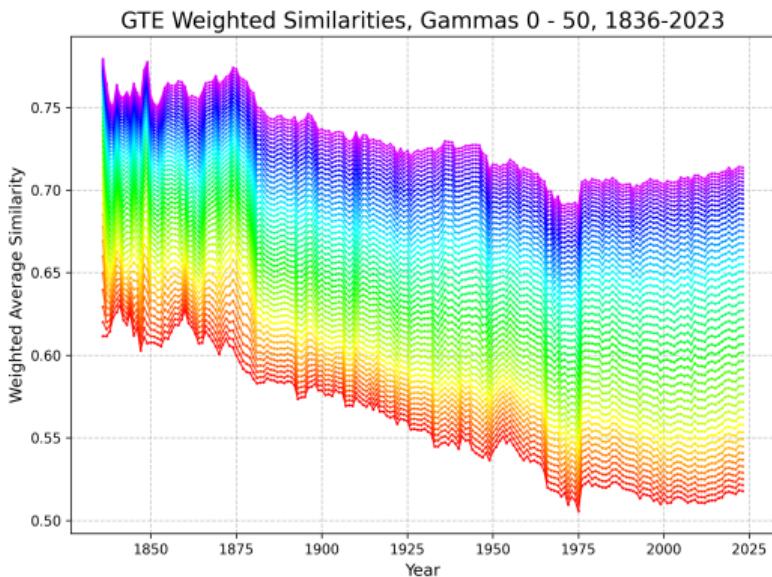
- TF-IDF: High similarity to other 1880s patents (shared vocabulary)
- GTE: High similarity to modern bicycle patents (shared concepts)

Evidence:

- TF-IDF similarity correlates with word overlap
- GTE similarity correlates with conceptual similarity
- Google Ngrams shows vocabulary shifts over time

Backup: Similarity at Different Spatial Scales

[◀ Return](#)

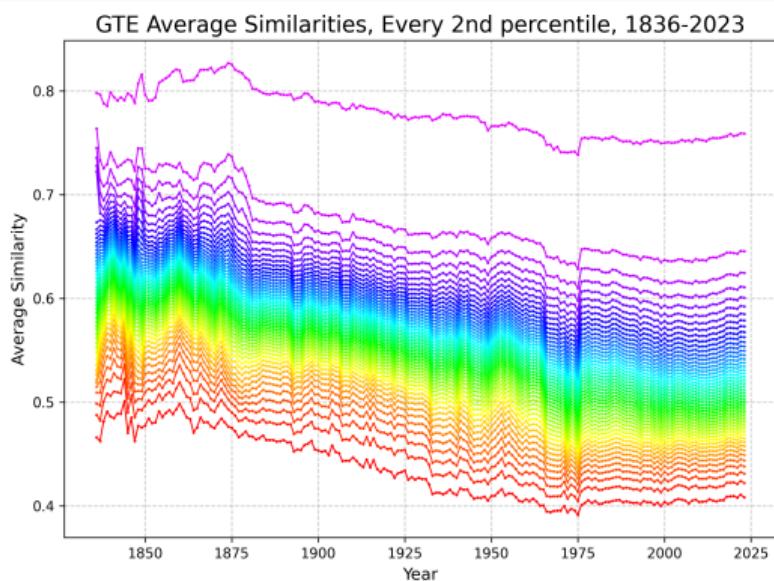


Weighted average: $\equiv \frac{1}{n} \sum_{i=1}^n \frac{\sum_{j \neq i} (1-d_{ij}) e^{-\gamma d_{ij}}}{\sum_{j \neq i} e^{-\gamma d_{ij}}}$
where γ from 0 (global) to 50 (local)

- **Key finding:** Similar declining trends across all spatial scales
- Model predictions concern averages — important to verify pattern holds across distribution
- Post-2000 arrest slightly stronger at local scales (consistent with entity correction)

Backup: Similarity at Different Quantiles

[◀ Return](#)

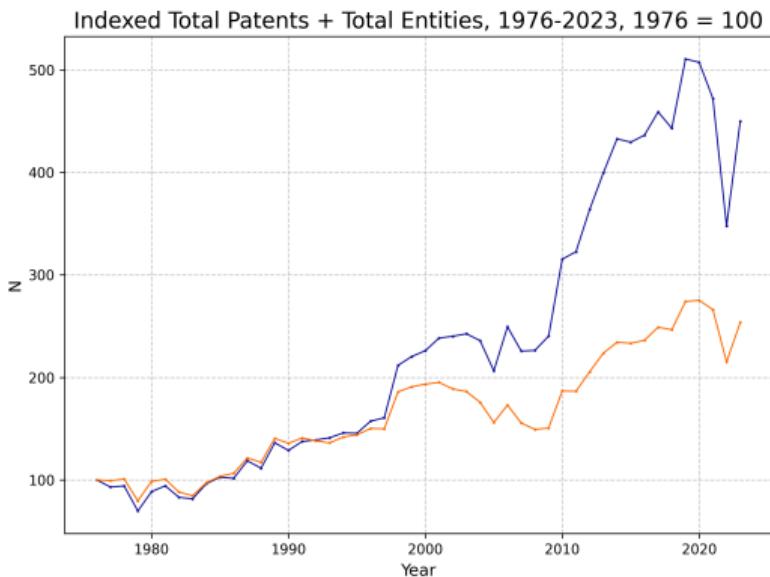


Similarity at Different Quantiles:

- 50 quantiles of pairwise similarity in each year
- Secular decline is robust across all quantiles
- Post-2000 increase in similarity is slightly faster for higher quantiles

Backup: Growth in Patents vs. Patenting Entities

[◀ Return](#)



- Number of issued utility patents and unique patenting entities per year
- Divergence after 1999: substantial growth in patents per entity
- Driven by business method patents and non-practicing entities
- Motivates sampling 1 patent per entity per year for robustness

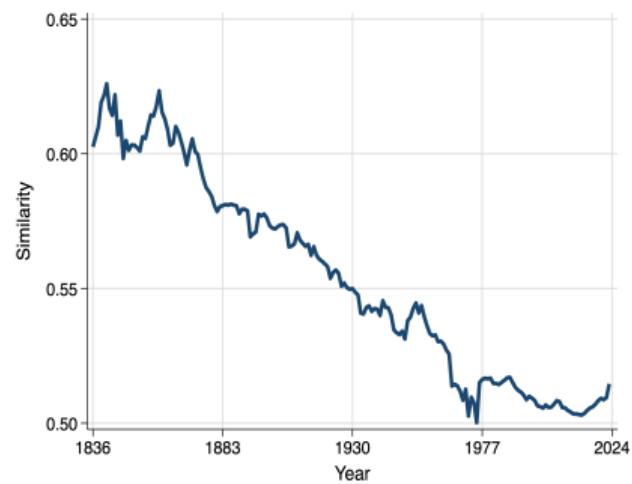
Backup: Within vs. Between Technology Classes

[◀ Return](#)

Within-class similarity



Between-class similarity



- **Addresses compositional concern:** Decline not driven by shifts across technology fields — spreading out occurs *within* established classes

Main specification: Standardize by annual cross-sectional SD

Robustness checks:

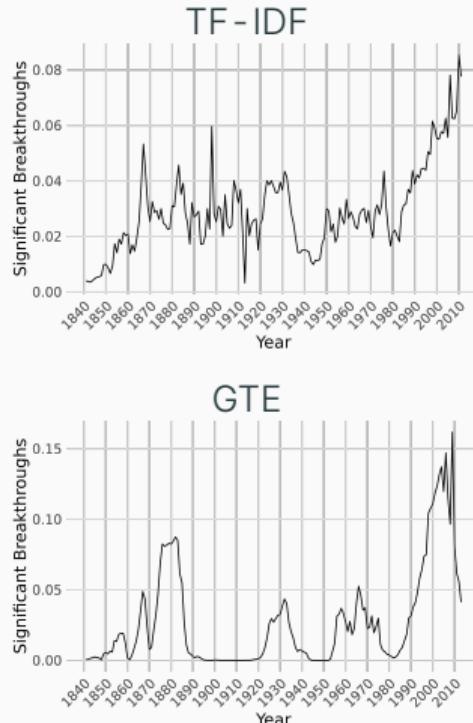
1. Time-invariant global SD → Nearly identical results
2. Raw similarity (no standardization) → Same qualitative pattern
3. Different sample sizes per year → Robust

Why standardize?

- Different representations have different scales
- No intrinsic economic interpretation of raw similarity
- SD provides meaningful units for comparison

Backup: Kelly et al. Breakthrough Replication

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Kelly et al. (2021): Identify
“breakthrough” patents using similarity to
future patents

Our replication with GTE:

- Qualitative conclusions align (more breakthroughs today)
- Quantitative results more robust (less sensitivity to methodological choices)
- TF-IDF produces noisier breakthrough classification

Implication: Validated similarity measures
improve downstream analyses

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Historical (1836–1975): ProQuest Patents Core

- OCR-digitized patent images
- Full text of claims extracted
- Quality varies with original document condition

Modern (1976–2023): USPTO PatentsView

- Machine-readable full text
- Structured data with claim parsing
- Consistent quality

Potential discontinuity at 1976:

- Some evidence of break in levels
- Trends consistent across periods
- Results robust to excluding transition years

Backup: Computing Similarity Efficiently

◀ Return

Challenge: $O(N^2)$ pairwise comparisons infeasible for millions of patents

Solution: For unit-normalized vectors, average cosine similarity reduces to:

$$\bar{S} = \frac{1}{N(N-1)} \sum_{i \neq j} \cos(v_i, v_j) = \frac{\|\sum_i v_i\|^2 - N}{N(N-1)}$$

Complexity: $O(N \cdot d)$ where d = embedding dimension

Implementation:

1. Normalize all vectors to unit length
2. Sum vectors: $S = \sum_i v_i$
3. Compute $\|S\|^2$
4. Apply formula

Cross-sectional SD: Subsample up to 10,000 patents/year

Example: Register of Interferences (1890)

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INTERFERENCES.			
NAME OF PARTIES	SEARCH	DAY OF HEARING	DISPOSITION
Ehrlich, Leo -14131-	Roll Paper Cutters, Statement Jan 7, 1890	Decided in favor of Ehrlich	Lawton, Jan 11, 1890
Lawton, Jas. B.	Statement of Lawton Dec 25, 1889		L. A. Ehrlich, Jan 11, 1890
	Statement of Ehrlich Jan 6, 1890		Decision by Hadley, May 14, 1890
Blaine, David W. -14124-	Corn Harvesters, Statement Jan 7, 1890	Decided in favor of Blaine	Hadley, Apr 27, 1890
Hadley, Artemus L.	Motion by Blaine to amend his application, Mar 28, 1890	Brief for Hadley Dec 31, 1890	L. A. May, Dec 31, 1890
	Statement of Hadley Jan 6, 1890		Decision by Hadley, Jan 6, 1890
Request of Hadley for judgment in the case Dec 28, 1890	Statement of Blaine Jan 7, 1890		Decision by Hadley for leave to amend his application Feb 6, 1890
	Motion by Hadley for leave to amend his application Feb 6, 1890	Brief for Hadley Feb 6, 1890	Decision by Hadley by Hadley, Feb 6, 1890

Purpose - digitized from National Archives:

- USPTO Registers of Interferences, 1864–1900
- 19,388 interference cases documented
- Average 504 annual terminations

Example cases (Jan 7, 1890):

- Ehrlich v. Lawton: Roll paper cutters
- Blaine v. Hadley: Corn harvesters

Quasi-experimental estimates of R&D spillovers

Bloom et al. 2013, Lucking et al. 2019

- Firm-level TFP elasticity to shocks to spillover pool
- SPILLTECH \equiv sum of neighbors' R&D, weighted by idea distance
- IV: State R&D tax credit shocks

Ingredients:

- Bloom et al. 2013 (1981-2001): $\hat{\beta} = \mathbf{0.206}$
- Standard deviation $\log(\text{SPILLTECH}) = \mathbf{1.04}$
- Avg change in similarity (Bloom sample period): $\mathbf{-0.007\sigma/\text{yr}}$

Implied TFP drag from average rate of spreading out:

- $-0.007 \times 1.04 \times 0.206 = \mathbf{-0.15\%/\text{yr}} \checkmark$