Research Report

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0.1 Current Research Direction

This week the plan was to complete:

- Wrap up the initial numerical analysis study on hard-walled cylindrical duct acoustic mode convergence study by looking at the first five radial modes; only the first cut-on radial mode for five grids were studied.
- Start the same study on the annular duct case.

0.2 Research Performed This Week

Last week, the convergence study for the cylindrical duct mode was completed for the first cut-on radial mode for a given uniform flow. However, the issue was

Last week, the convergence study for the cylindrical duct mode was completed for the first cut-on radial mode for a given uniform flow. However, the issue was that the analytic computation and importing of numerical results coincided, which can be problematic in convergence studies since the convergence rates can only be computed once all the data from multiple grids is imported. Therefore, importing data was first completed as a relatively quick fix. Then the corresponding analytic computations were carried out, allowing the data to be pulled when needed instead of doing these two steps simultaneously.

Although these compuations are separate, the convergence rate for the next four radial modes need to be computed, the results for the first five grids are below. Note that the number of grids were doubled and not yet subtracted by one.

0.2.1 Discussion

A total of five grids were studied, starting from 33 and doubling until 1056 for the first cut on radial mode. The upstream and downstream mode pairs were used to compute the L2, L_{max}, and convergence rates for a uniform flow in a cylindrical duct. The axial wavenumber's error starts with a magnitude of e-6 and decreases to $e - 12$. A fourth order is expected when computing the convergence rate since a fourth-order central scheme was used. This behavior is shown in the upstream mode, with a rate of convergence of 4.19. However, this behavior is less pronounced in the downstream case. The rate increases to 3.7 but begins to decrease for the last grid pair. A grid spacing choice should have been made such that the jumps are not as large so that the convergence rates can be studied more carefully. For

	L_{2,k_x}	$L_{2,\bar{p}}$	$L_{2,noLBC}$	$L_{2,noRBC}$	$L_{2,noBCS}$	L_{max}	$L_{max,location}$
0	0.00000155279	0.00059786297	0.00019688319	0.00059696793	0.00016546650	0.00324886266	0
1	0.00000028328	0.00010577879	0.00004184232	0.00010658661	0.00004216087	0.00079036919	0
2	0.00000002127	0.00001964476	0.00000996508	0.00001971142	0.00000998706	0.00019476230	0
3	0.00000000162	0.00000384159	0.00000243557	0.00000384595	0.00000243556	0.00004833159	0
4	0.00000000012	0.00000079773	0.00000060218	0.00000079784	0.00000060188	0.00001203774	0
5	0.00000000001	0.00000017589	0.00000014971	0.00000017586	0.00000014965	0.00000300377	0

Table 1: L2 error for downstream radial mode 1

	L_{2,k_x}	$L_{2,\bar{p}}$	$L_{2,noLBC}$	$L_{2,noRBC}$	$L_{2,noBCS}$	L_{max}	$L_{max,location}$
0	0.00000155279	0.00059786297	0.00019688319	0.00059696793	0.00016546650	0.00324886266	0
1	0.00000028331	0.00010577879	0.00004184232	0.00010658661	0.00004216087	0.00079036919	0
2	0.00000002131	0.00001964476	0.00000996508	0.00001971142	0.00000998706	0.00019476231	0
3	0.00000000161	0.00000384159	0.00000243557	0.00000384595	0.00000243556	0.00004833159	0
4	0.00000000011	0.00000079773	0.00000060218	0.00000079784	0.00000060188	0.00001203774	0
5	0.00000000001	0.00000017589	0.00000014971	0.00000017586	0.00000014965	0.00000300377	0

Table 2: L2 error for upstream radial mode 1

	ROC_{k_x}	$ROC_{\bar{p}}$	ROC_{noLBC}	ROC_{noRBC}	ROC_{noBCS}	ROC_{Lmax}
0	2.45454778495	2.49876442834	2.23430536327	2.48562714561	1.97256274680	2.03933614754
1	3.73518793905	2.42883362734	2.07000990043	2.43492294106	2.07777268308	2.02081222716
2	3.71280323136	2.35437009887	2.03262136291	2.35761955726	2.03580376378	2.01067605822
3	3.72874604471	2.26772805114	2.01600070856	2.26917703796	2.01670223610	2.00540238796
4	3.30599575633	2.18125765051	2.00801441568	2.18170434599	2.00792723518	2.00271753800

Table 3: ROC error for downstream radial mode 1

	ROC_{k_x}	$ROC_{\bar{p}}$	ROC_{noLBC}	ROC_{noRBC}	ROC_{noBCS}	ROC_{Lmax}
0	2.45443470920	2.49876442956	2.23430536447	2.48562714682	1.97256274780	2.03933614876
1	3.73304324762	2.42883362073	2.07000989937	2.43492293430	2.07777268140	2.02081221866
2	3.72989837411	2.35437012336	2.03262140864	2.35761958105	2.03580380805	2.01067606674
3	3.92413209222	2.26772808044	2.01600065119	2.26917706705	2.01670217869	2.00540251418
4	4.19379112515	2.18125738030	2.00801435240	2.18170407533	2.00792717211	2.00271680358

Table 4: ROC error for upstream radial mode 1

For the first radial mode, the pressure distribution in the upstream and downstream directions do not reach fourth-order convergence as excpected. Using L'Hopital's rule to obtain a value at the centerline causes the error between the numerical and analytic modes to be highest at the centerline gridpoint. L'Hopital's rule's effect is shown in upstream and downstream directions by identifying the location of the highest error, L_{max}, for pressure radial mode comparisons. When computing the rate of convergence for the L_{max} point, it converges to the second order. The same trend is noticed when both boundaries are removed in the error calculation. When looking at the difference between the excluding of boundaries, it seems that the convergence rate is higher without the wall BC as opposed to being higher without the centerline BC, but when computing the convergence rate of L_{max}, a second-order rate is computed.

For the pressure mode data, neither upstream or downstream reach fourth order convergence. The use of the L'Hopital's rule to obtain a value at the centerline causes the error between the numerical and analytic modes to be highest at the centerline gridpoint. This is shown in both upstream and downstream directions by identifying the location of the highest error L_{max}. When computing the rate of convergence for the L_{max} point, it converges to second order. The same trend is noticed when both boundaries are removed in the error calculation.

When looking at the difference between the excluding of boundaries, it seems that the convergence rate is higher without the wall BC as opposed to being higher without the centerline BC, but when computing the convergence rate of L_{max}, a second order rate is computed.

0.3 Issues and Concerns

The proper grid spacing needs to be used. The distance between grid points isn't halving with the doubling of grid points because of the two points on the boundaries of the domain. As a result the grid should be "doubled" by following $2n - 1$ where n is the number of grid points.

0.4 Planned Research

Now that the post processing routine allows for easier computation of more radial modes, that will be completed first. Then, SWIRL will be revised with the correct method of doubling grid points to preserve a halving of grid spacing Δr .