Validation using the Analytical Solution for Duct Mode Propagation in Uniform Flow

Swirl Validation

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This document shows the analytical duct mode solution as well as a numerical comparison.

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0.1 Research Performed

0.2 Results

0.2.1 Discussion

A total of five grids were studied, starting from 33 and doubling until 1056 for the first cut on radial mode. The upstream and downstream mode pair were both used to compute the L2, Lmax, and rate of convergence for a given uniform flow in a cylindrical duct. The axial wavenumber's error starts with a magnitude of $e-6$ and decreases to $e-12$. When computing the convergence rate, a fourth order is expected since a fourth order central scheme was used. This behavior is shown in the upstream mode, with a rate of convergence of 4.19. However, this behaviour is less pronounced in the downstream case. The rate increases to 3.7 but begins to decrease for the last grid pair. A grid spacing choice should have been made such that the jumps are not as large so that the convergence rates can be studied more carefully.

For the pressure mode data, neither upstream or downstream reach fourth order convergence. The use of the L'Hopital's rule to obtain a value at the centerline causes the error between the numerical and analytic modes to be highest at the centerline gridpoint. This is shown in both upstream and downstream directions by identifying the location of the highest error L_{max}. When computing the rate of convergence for the L_{max} point, it converges to second order. The same trend is noticed when both boundaries are removed in the error calculation.

	L_{2,k_x}	$L_{2,\bar{p}}$	$L_{2,noLBC}$	$L_{2,noRBC}$	$L_{2,noBCS}$	L_{max}	$L_{max,location}$
0	0.00000155279	0.00059786297	0.00019688319	0.00059696793	0.00016546650	0.00324886266	0
1	0.00000028328	0.00010577879	0.00004184232	0.00010658661	0.00004216087	0.00079036919	0
2	0.00000002127	0.00001964476	0.00000996508	0.00001971142	0.00000998706	0.00019476230	0
3	0.00000000162	0.00000384159	0.00000243557	0.00000384595	0.00000243556	0.00004833159	0
4	0.00000000012	0.00000079773	0.00000060218	0.00000079784	0.00000060188	0.00001203774	0
5	0.00000000001	0.00000017589	0.00000014971	0.00000017586	0.00000014965	0.00000300377	0

Table 1: L2 error for downstream radial mode 1

	L_{2,k_x}	$L_{2,\bar{p}}$	$L_{2,noLBC}$	$L_{2,noRBC}$	$L_{2,noBCS}$	L_{max}	$L_{max,location}$
0	0.00000155279	0.00059786297	0.00019688319	0.00059696793	0.00016546650	0.00324886266	0
1	0.00000028331	0.00010577879	0.00004184232	0.00010658661	0.00004216087	0.00079036919	0
2	0.00000002131	0.00001964476	0.00000996508	0.00001971142	0.00000998706	0.00019476231	0
3	0.00000000161	0.00000384159	0.00000243557	0.00000384595	0.00000243556	0.00004833159	0
4	0.00000000011	0.00000079773	0.00000060218	0.00000079784	0.00000060188	0.00001203774	0
5	0.00000000001	0.00000017589	0.00000014971	0.00000017586	0.00000014965	0.00000300377	0

Table 2: L2 error for upstream radial mode 1

	ROC_{k_x}	$ROC_{\bar{p}}$	ROC_{noLBC}	ROC_{noRBC}	ROC_{noBCS}	ROC_{Lmax}
0	2.45454778495	2.49876442834	2.23430536327	2.48562714561	1.97256274680	2.03933614754
1	3.73518793905	2.42883362734	2.07000990043	2.43492294106	2.07777268308	2.02081222716
2	3.71280323136	2.35437009887	2.03262136291	2.35761955726	2.03580376378	2.01067605822
3	3.72874604471	2.26772805114	2.01600070856	2.26917703796	2.01670223610	2.00540238796
4	3.30599575633	2.18125765051	2.00801441568	2.18170434599	2.00792723518	2.00271753800

Table 3: ROC error for downstream radial mode 1

	ROC_{k_x}	$ROC_{\bar{p}}$	ROC_{noLBC}	ROC_{noRBC}	ROC_{noBCS}	ROC_{Lmax}
0	2.45443470920	2.49876442956	2.23430536447	2.48562714682	1.97256274780	2.03933614876
1	3.73304324762	2.42883362073	2.07000989937	2.43492293430	2.07777268140	2.02081221866
2	3.72989837411	2.35437012336	2.03262140864	2.35761958105	2.03580380805	2.01067606674
3	3.92413209222	2.26772808044	2.01600065119	2.26917706705	2.01670217869	2.00540251418
4	4.19379112515	2.18125738030	2.00801435240	2.18170407533	2.00792717211	2.00271680358

Table 4: ROC error for upstream radial mode 1

0.3 Issues

When looking at the difference between the excluding of boundaries, it seems that the convergence rate is higher without the wall BC as opposed to being higher without the centerline BC, but when computing the convergence rate of $LMax$, a second order rate is computed. I could have the two collumns swapped.

0.4 Planned Research

The formatting of the latex table doesn't show the Floating point values I am seeing but discussing.

I only looked at the first radial mode at the moment. Once this study can be achieved for multiple radial modes simultaneously, the annular duct mode case will be studied using the same outline.