Weekly Research Report

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1   Current Research Direction

Validate annular duct modes. A result has been obtained but does not seem to have the correct boundary conditions.

2   Research Performed This Week

Documentation for the annular duct mode calculations is underway. The procedure was presented in "Aeroacoustic Analysis of Turbofan Noise Generation" by Harold D. Meyer and Edmane Envia. In Chapter 4: Duct Acoustics, Section 4.1 Normal Modes in an Annular Duct , along with the accompanying appendix section, Appendix B: Numerical Computation of the Normal Modes in An Annular Duct, the theoretical background discusses how the boundary conditions are used to obtain the appropriate weighting factors and mode shapes. The FORTRAN 77 library codes have been copied over to SWIRL and a test routine using FORTRAN 90 was written. The test routine contains three main calls, `anrt.f`, `eigen.f` and `rmode.f`. The analytic radial mode shape is of the form,

$$R(k_{r,mn}r) = AJ_m(k_{r,mn}r) + BY_m(k_{r,nmn}r) \tag{1}$$

The key to the numerical procedure is the following "transcendental" equation,

$$\begin{vmatrix} J'_m(k_{r,mn}r_H) & Y'_m(k_{r,mn}r_H) \\ J'_m(k_{r,mn}r_T) & Y'_m(k_{r,mn}r_T) \end{vmatrix} = 0 \tag{2}$$

The non-dimensional roots $k_{r,mn}r_T$ are found using initial guess and then incrementing from there

$$k_{r,mn} = \begin{cases} m & \text{if } n = 1 \\ k_{r,m(n-1)}r_T + \pi, & \text{if } n > 1. \end{cases} \tag{3}$$

The estimate is refined by incrementing the value of $k_{r,mn}$ by $\pi/10$ until the determinant above changes sign. The step size is then halved and also changes sign. This iterative process continues until the absolute value of the determinant is reduced to a preassigned value (error tolerance?). The non dimensional versions of these equations are used in the FORTRAN 77 Code.

The remainder of the procedure is being documented along with directions on how to pass in the correct inputs to the F77 calls

3   Issues and Concerns

Preliminary results appear to not have zero value derivatives at the boundaries since there is an apparent non zero slope. The hypothesis is that the bessel functions are being truncated and not scaled.

4   Planned Research

- Complete investigation and documention on the numerical method.

- writing clear directions and examples will hopefully help me determine the correct annular mode shapes.