0.1 Introduction - Turbomachinery Noise

Turbomachinery noise generation occurs from pressure fluctuations from the series of fans within it's annular duct. While the jet that is produced from this stream of air freely radiates to the observer, the pressure fluctuations produced from the rotor may or may not propagate out of the inlet and exhaust and radiate to the observer. The production of this propagation can be characterized by standing waves referred to as modes, in particular, duct modes because the mode itself is dependent on the geometry of the column of air within the annular duct, as well as the speed of the flow moving through it

0.2 Duct Modes

The pressure field within a duct is governed by the convective wave equation, a second order ODE as a function of radius. Goldstein's *Aeroacoustics* shows that the wave equation can be obtained by taking the divergence of the momentum equation and subtracting the material derivative of the continuity equation, however, the same equation can be obtained by resubstitution of the continuity and momentum equations into the energy equations after the fluctuations have also been substituted for the primative variables. (See Appendix for derivation)

The solution of the convective wave equation are eigenvalues and eigenvectors which may or may not correspond to acoustic disturbances fall into two groups. One group corresponding to the acoustics propagation and the other group corresponding to the convection speed of the flow. Both are modes that are a result from the pressure distribution from within the cylindrical domain.

Modes can be categorized based on the sign of the axial wavenumber and if it is complex in value. For example, for the uniform axial flow case, propagating modes are defined by axial wavenumbers, k_x, that have a real-part only, yielding the assumed fluctuation to resemble Euler's Formula ($e^{ik_x x}$). On the other hand, if the k_x is complex, then the mode will resemble an exponentially decaying function since the imaginary number cancels, leaving a minus sign in front of the axial wavenumber. These two distinctions are referred to as "cut-on" and "cut-off" in the field of ducted sound propagation. Furthermore, the sign of the imaginary part will change the direction of the mode's decay. If k_x is positive, the decay rate occurs in the negative direction. Conversely, if k_x is negative, the decay occurs in the positive direction. The axial wavenumber for uniform axial flow is,

$$k_x = \frac{-M_x k \pm \sqrt{k^2 - (1 - M_x^2)J_{m,n}'^2}}{(1 - M_x^2)}.$$ (1)

where M_x is the axial Mach number, k is the temporal (referred to as reduced) frequency, and $J_{m,n}'$ is the derivative of the Bessel function of the first kind. The $\pm$ accounts for both upstream and downstream modes.

The condition for propagation is such that the axial wavenumber is larger than a "cut-off" value

$$k_{x,real} = \frac{\pm M_x k}{(M_x^2 - 1)}.$$ (2)

Every term that is being raised to the one half i.e. square rooted must be larger than zero to keep axial wavenumber from being imaginary. The mode will propagate or decay based on this condition. Recall thaT the mode is of the form

$$e^{ik_x x} \tag{3}$$

if k_x has a real part, $k_{x,real}$ and an imaginary part $ik_{x,imag}$ then,

$$= e^{ik_x x} \tag{4}$$

$$= e^{i(k_{x,real}+ik_{x,imag})x} \tag{5}$$

$$= \underbrace{e^{ik_{x,real}x}}_{amplitude} \underbrace{e^{-k_{x,imag}x}}_{exponential\ decay} \tag{6}$$

Although the "cut-off" decay to nearly zero rapidly, the rate at which this occured was not much of a concern earlier on in turbomachinery design. As nacelles continue to grow shorter, a mode that is "cut-off" may make it outside the duct.

For this work a desired amplitude was arbitrarily chosen for a mode, $y_{desired}$ and then the axial location at which this occurred, $x_{desired}$ which can be compared against a desired length for a nacelle. Since SWIRL assumes an infinitely long duct, there is nothing limiting the modes propagation with respect to nacelle length. For example, if the desired amplitude is one percent, then $x_{desired}$ is 0.46,

0.3 Analytical Test Case

σ	0.25
k	10
m	2
M_x	0.3

Table 1: Validation test case parameters, Uniform Flow Annular Duct

0.3.1 Axial Wavenumber
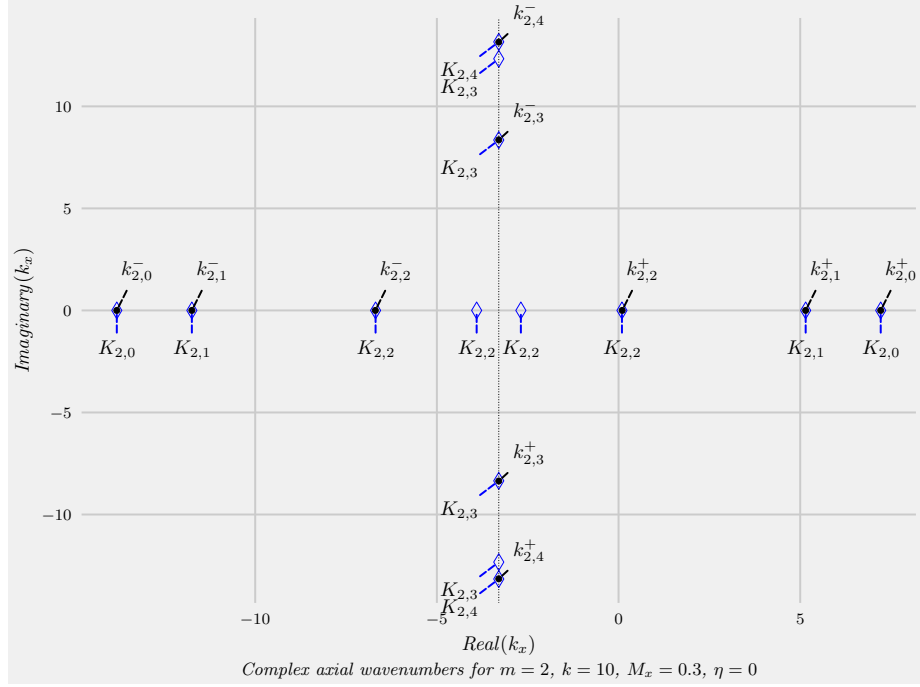
Figure 1: Propagating Axial Wavenumbers for the analytical solution; Uniform Flow in a hard-wall annular duct

0.3.2 Propagating Radial Modes

Second Order, Radial Mode 0
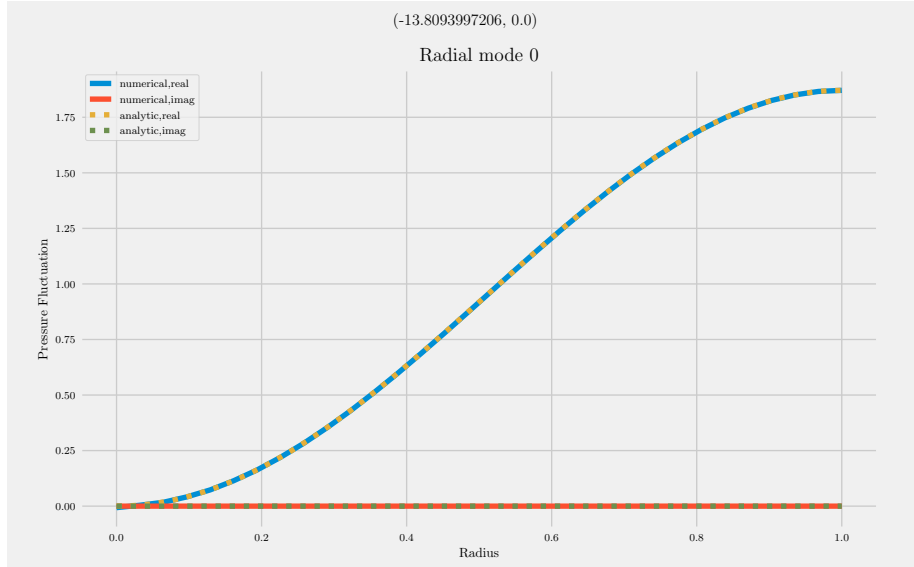
Figure 2: Second Order, Case number 0
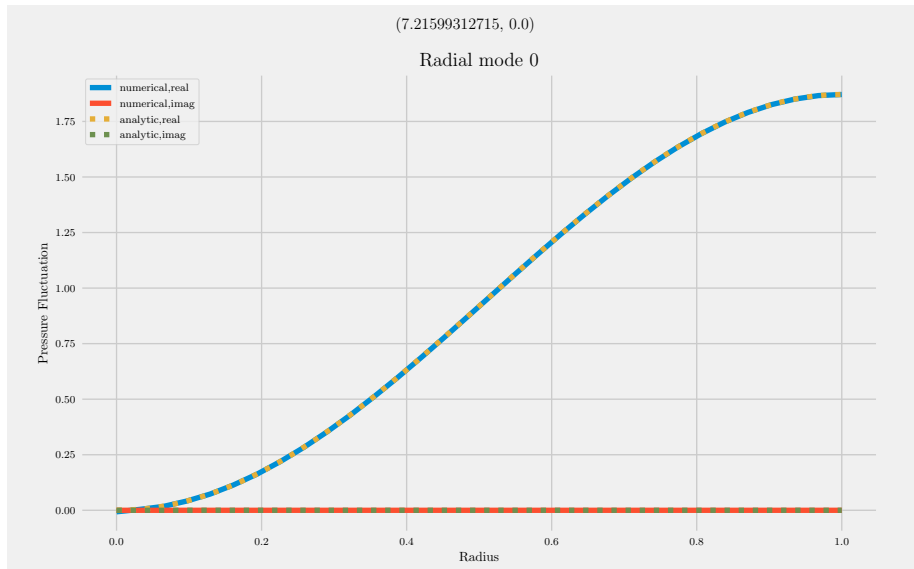
Figure 3: Second Order, Case number 1

Second Order, Radial Mode 1
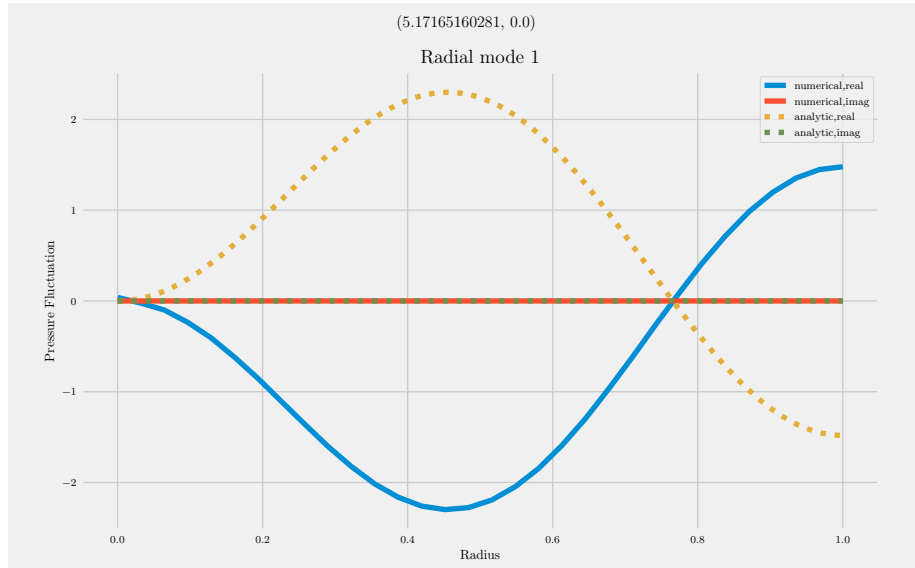
Figure 4: Second Order, Case number 0
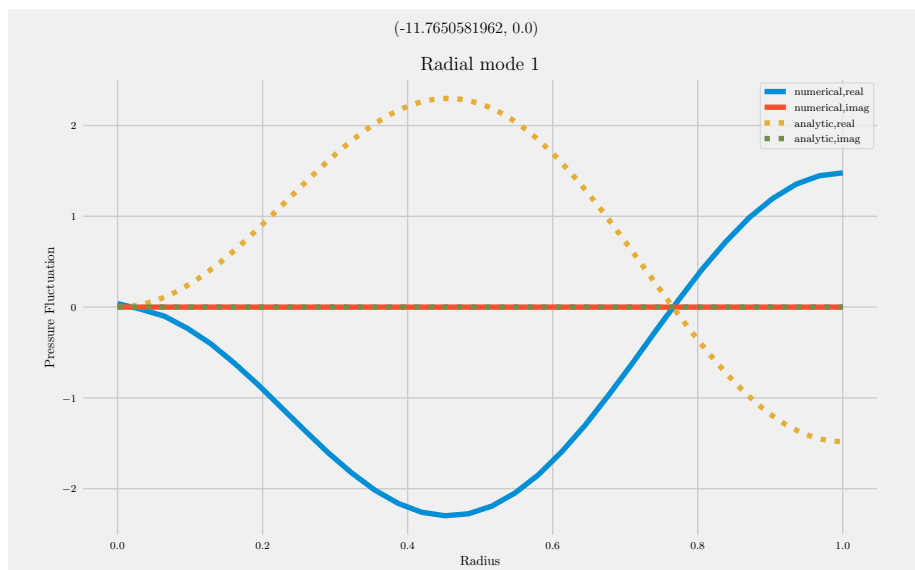
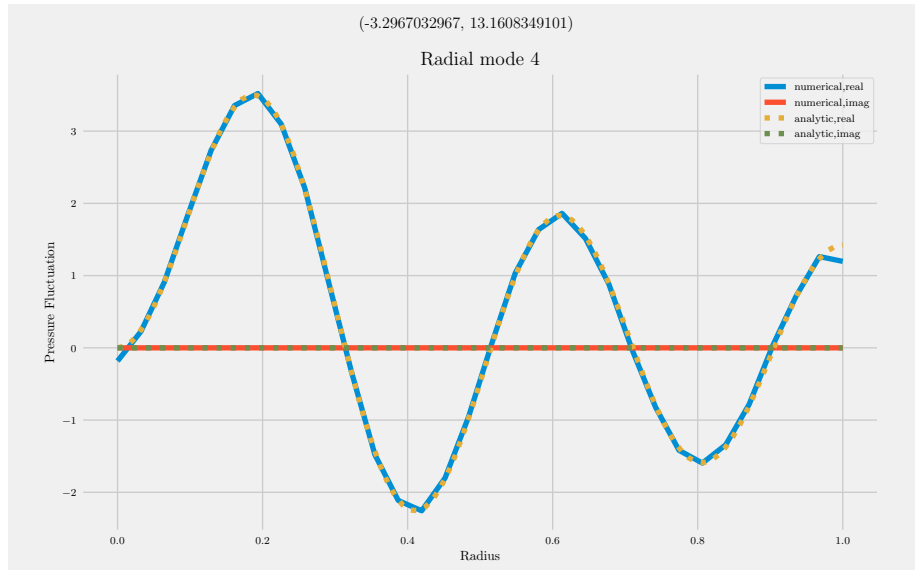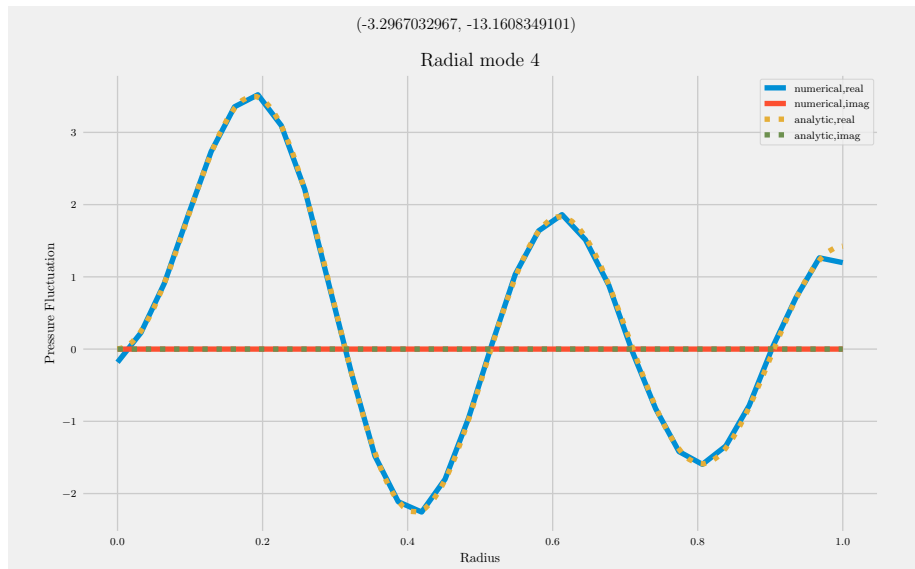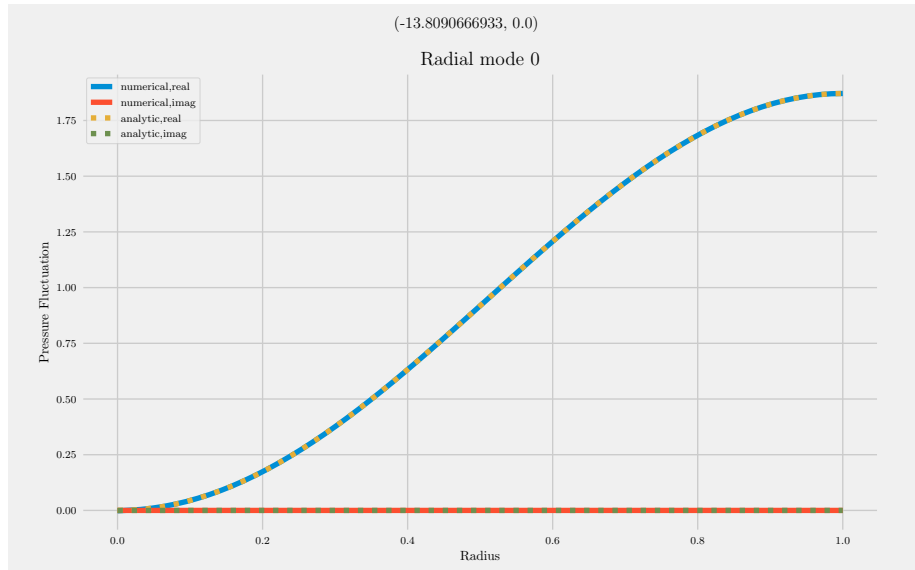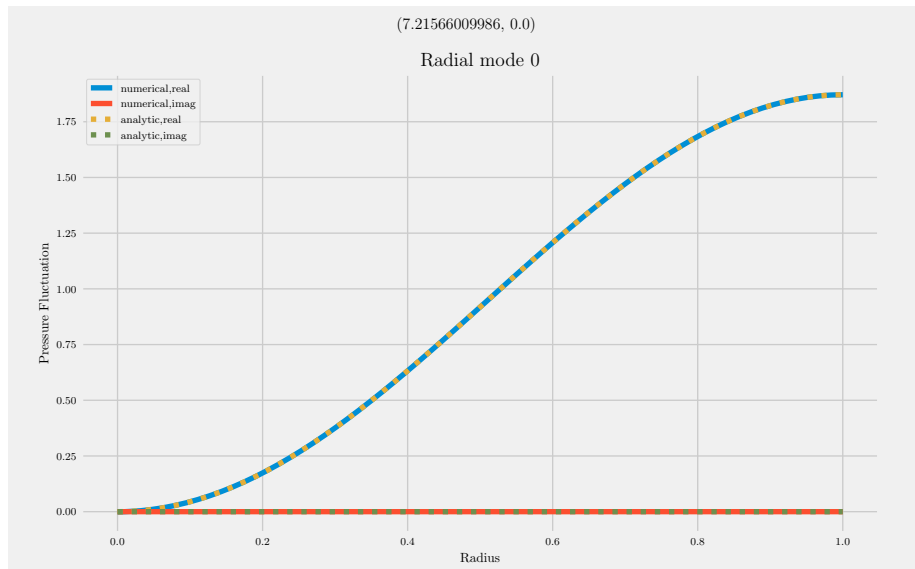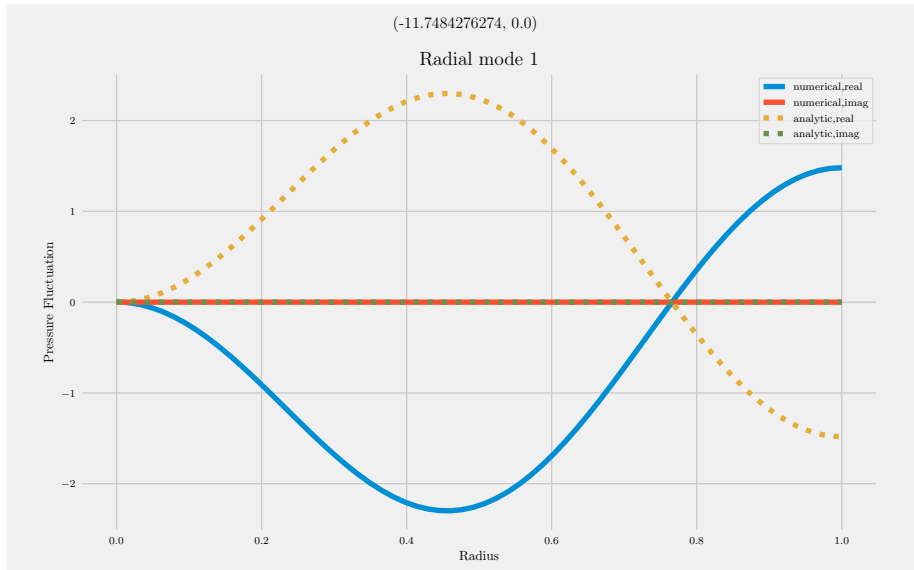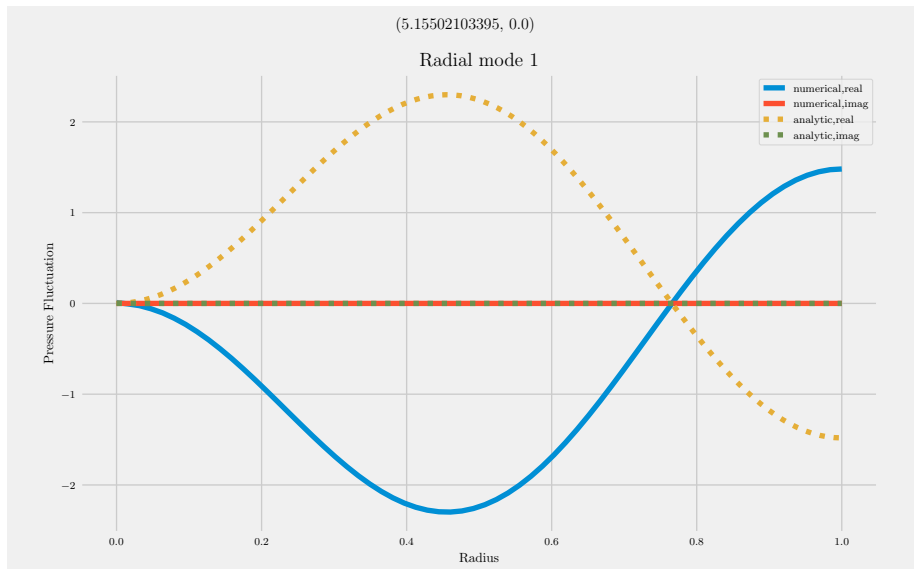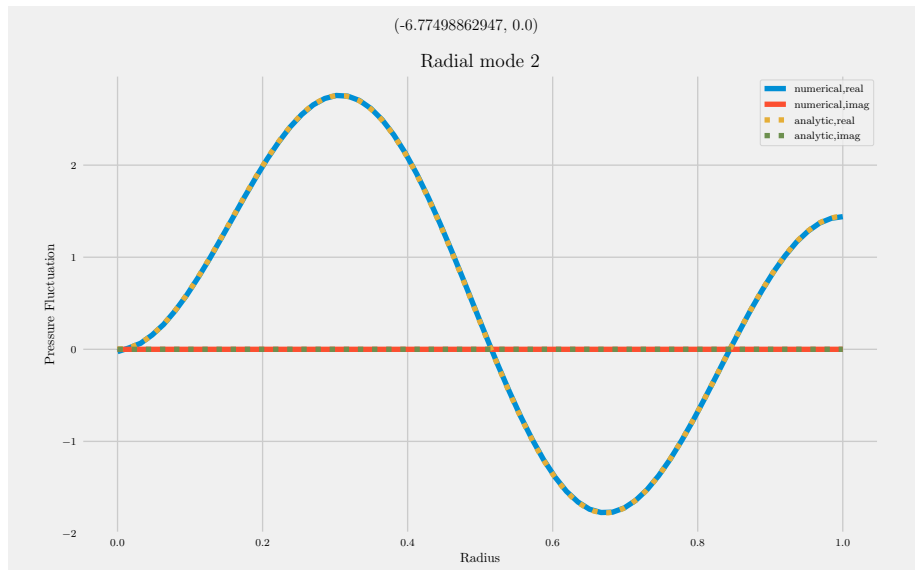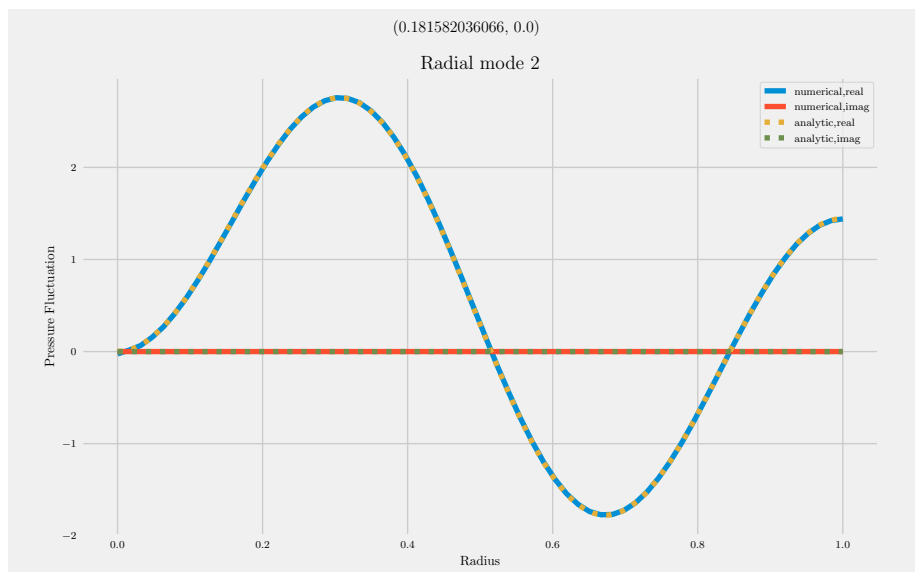Figure 5: Second Order, Case number 1
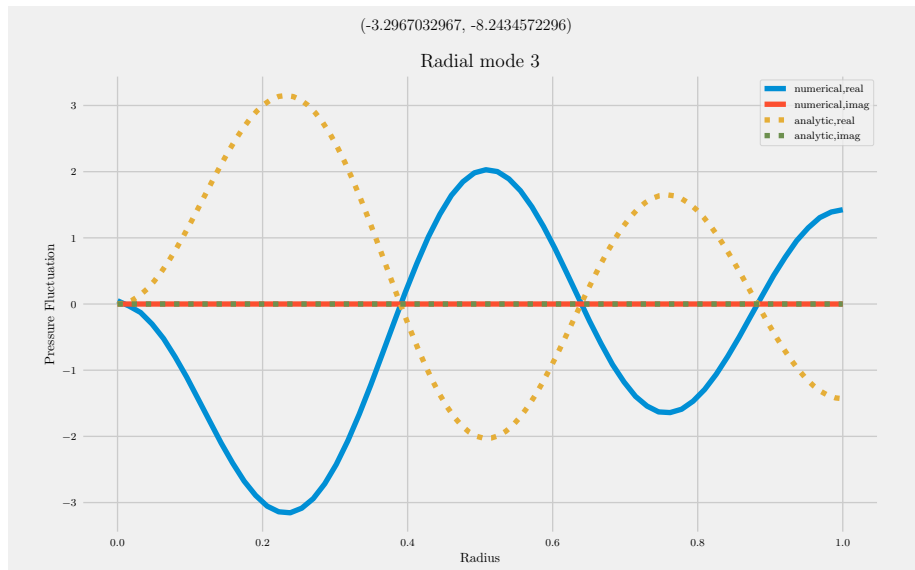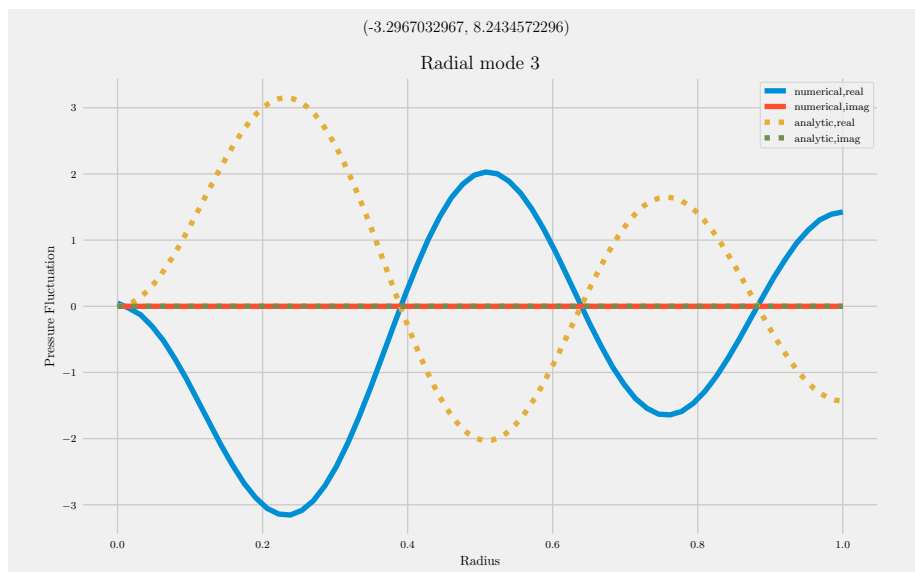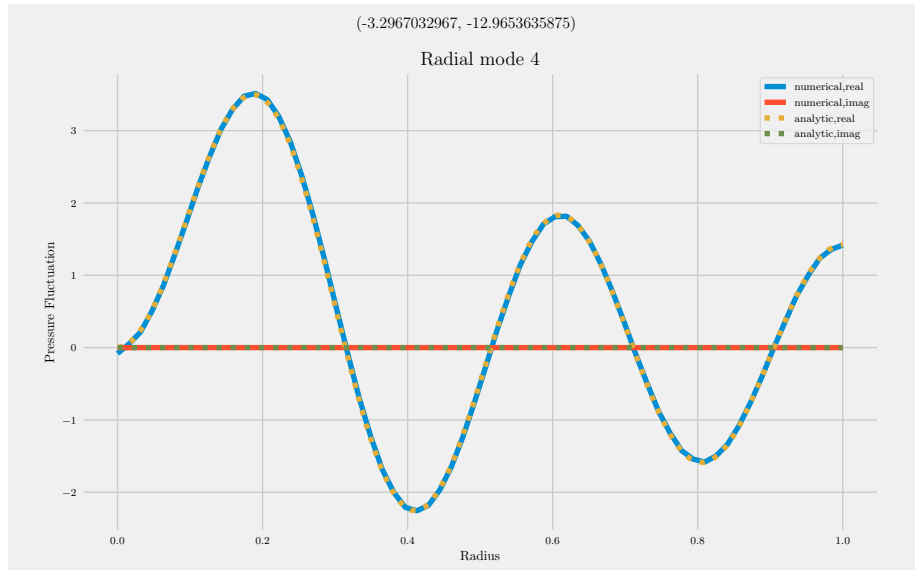
Second Order, Radial Mode 2
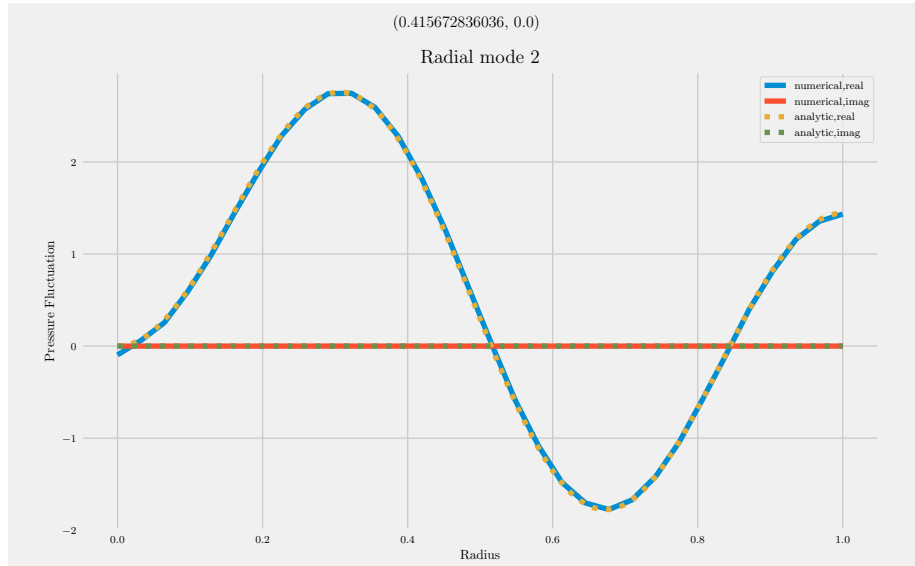
Figure 6: Second Order, Case number 0
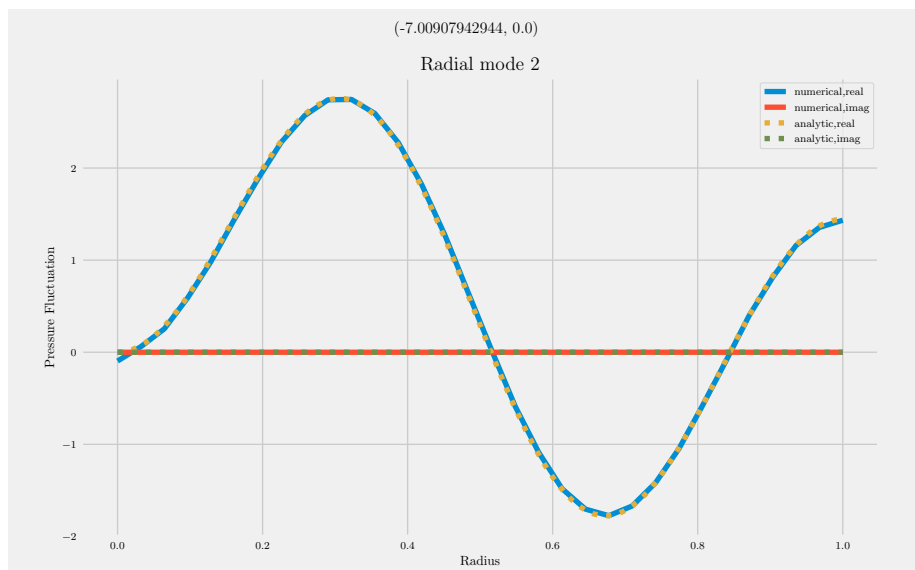
Figure 7: Second Order, Case number 1

Second Order, Radial Mode 3
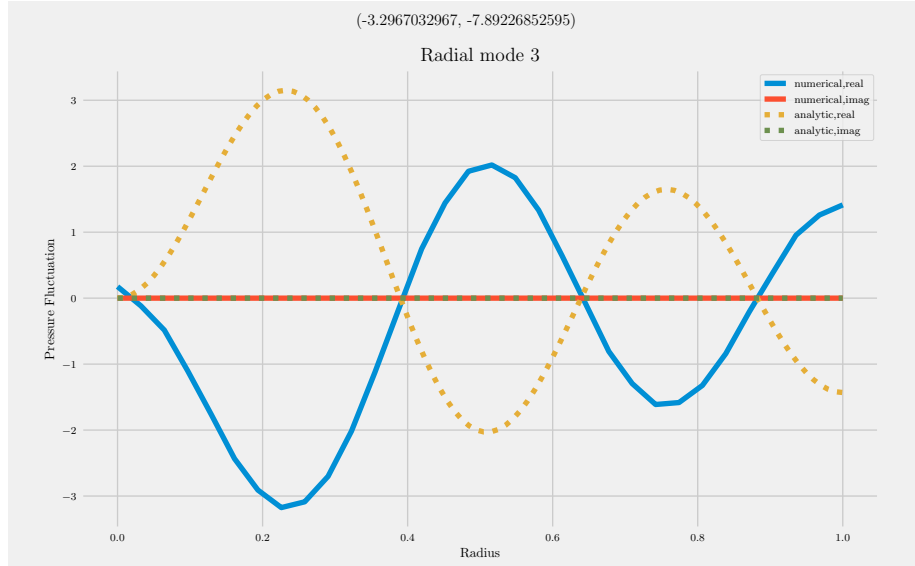
Figure 8: Second Order, Case number 0
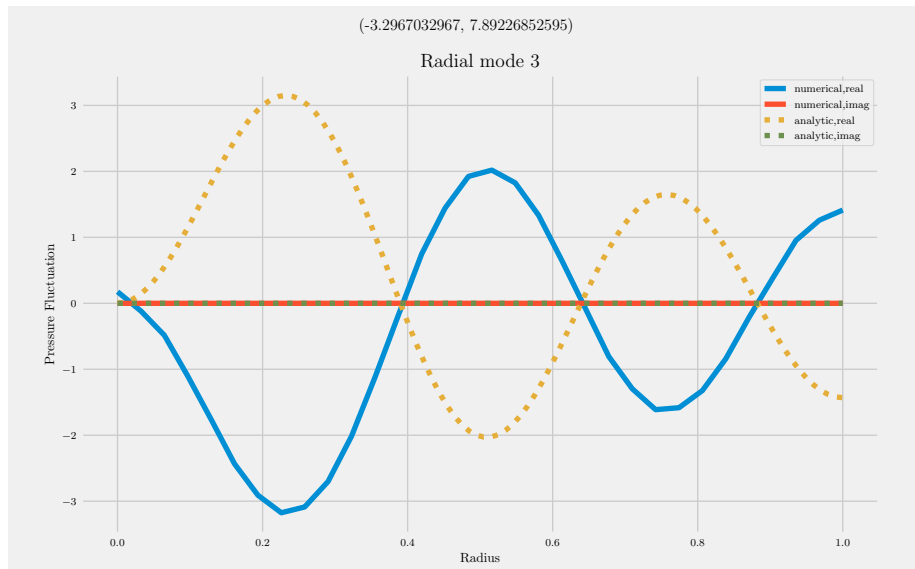
Figure 9: Second Order, Case number 1

Second Order, Radial Mode 4
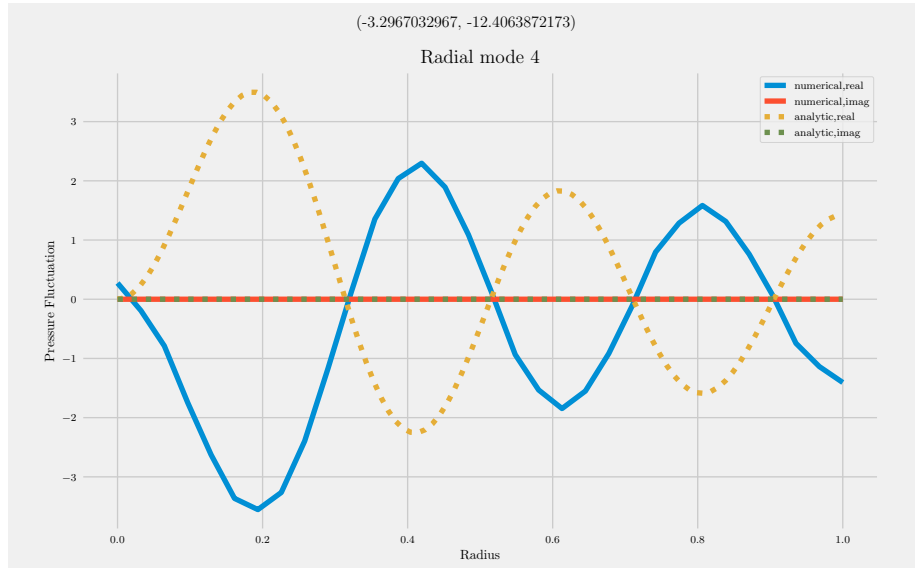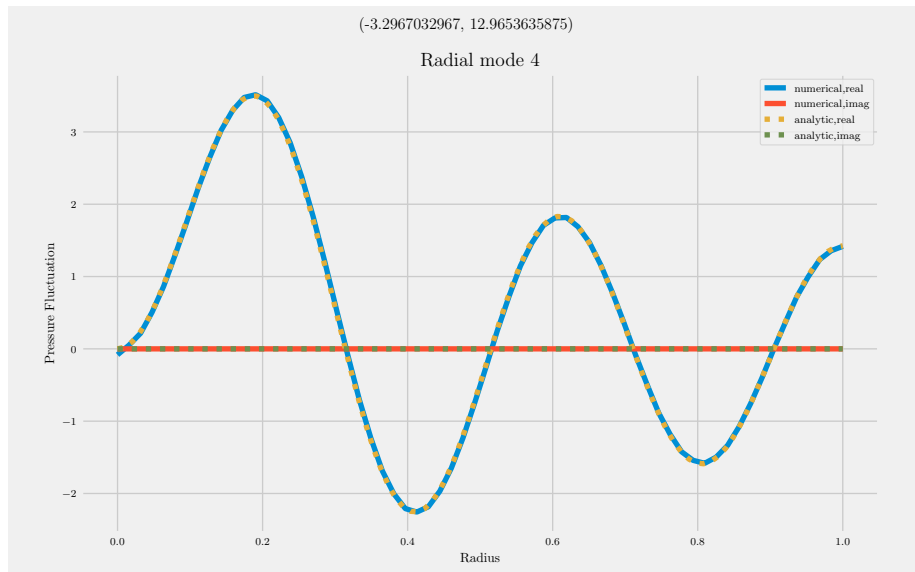

Figure 10: Second Order, Case number 0
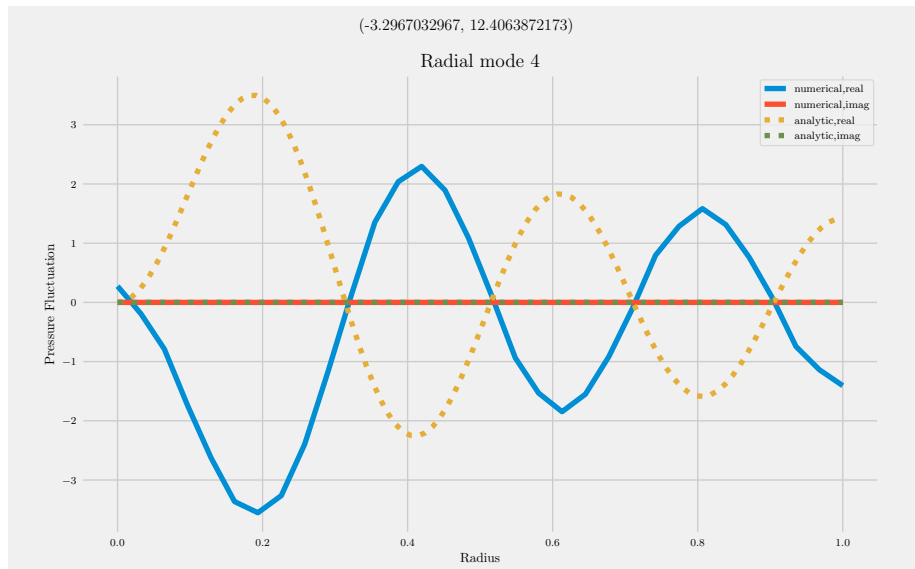
Figure 11: Second Order, Case number 1
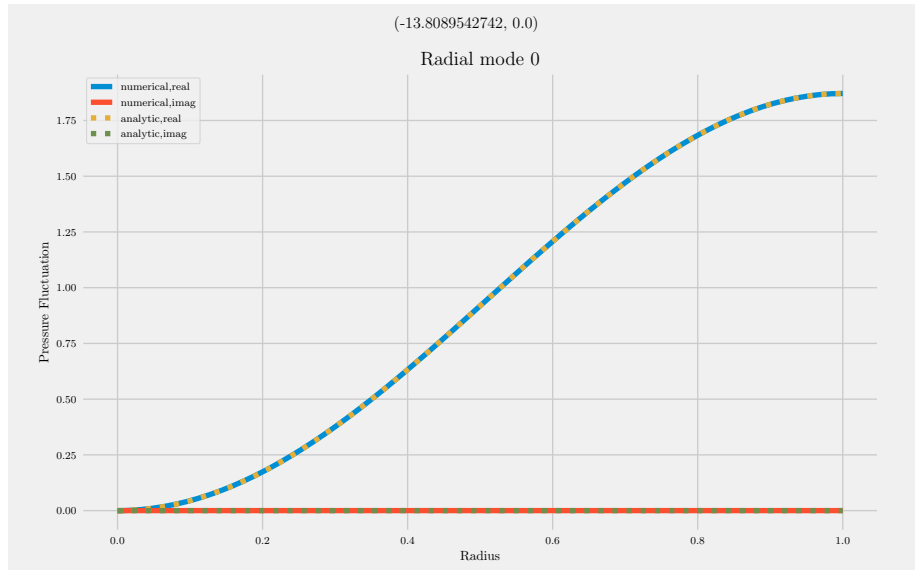
Fourth Order, Radial Mode 0
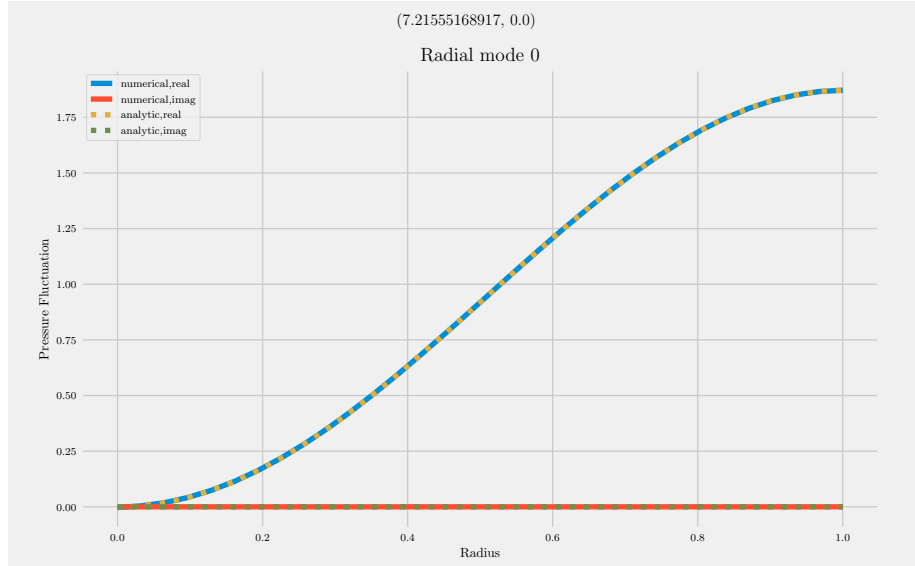
Figure 12: Fourth Order, Case number 0
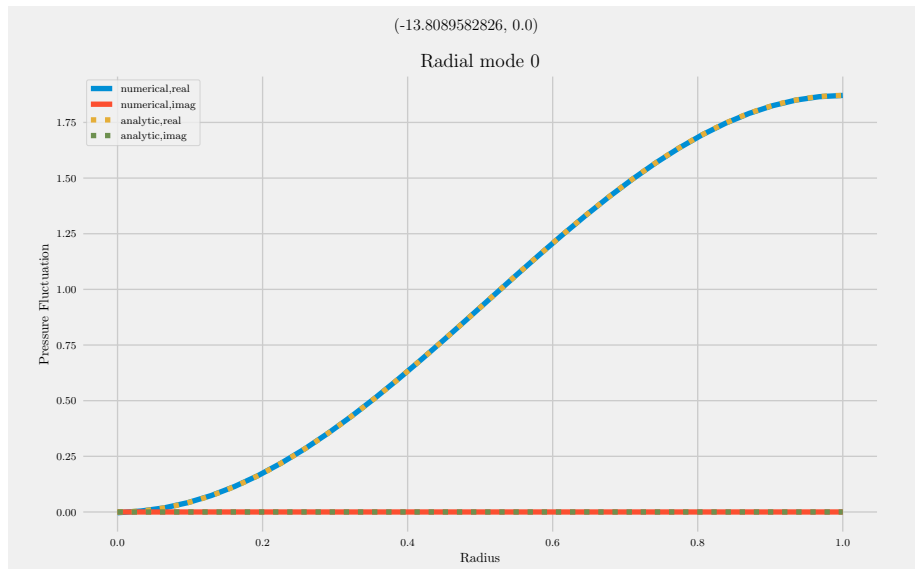
Figure 13: Fourth Order, Case number 1

Fourth Order, Radial Mode 1
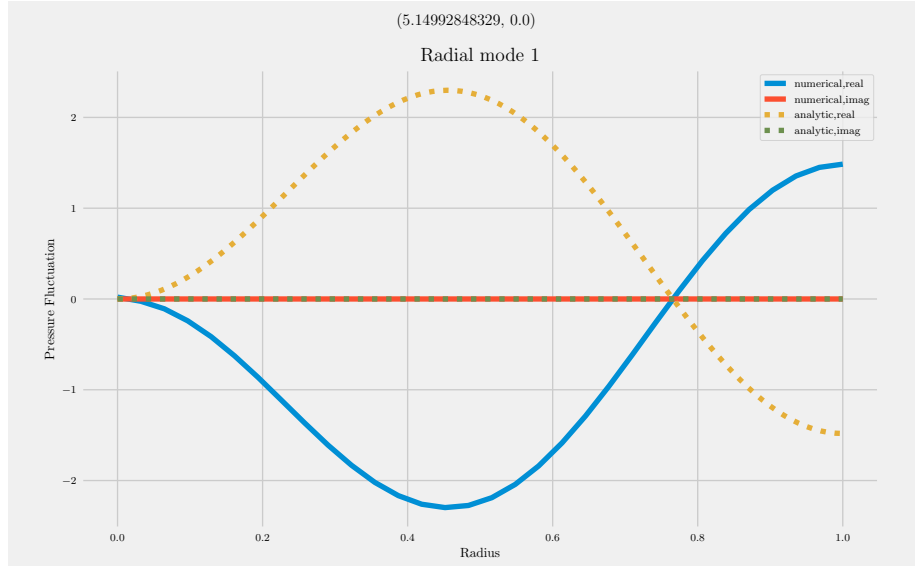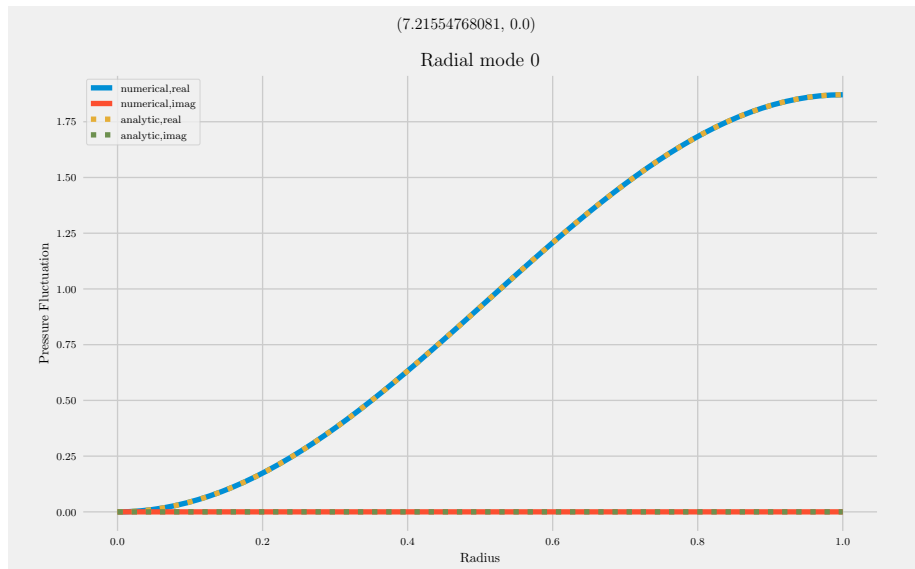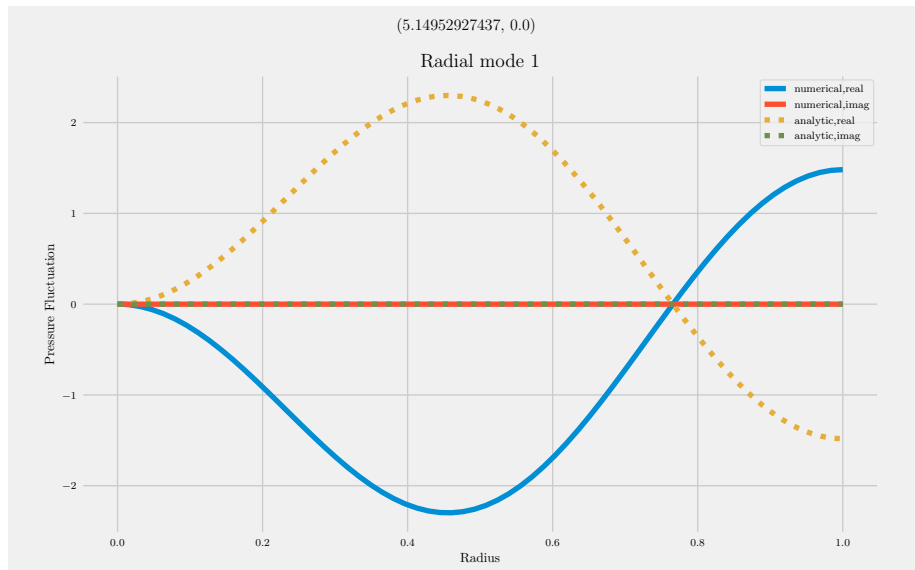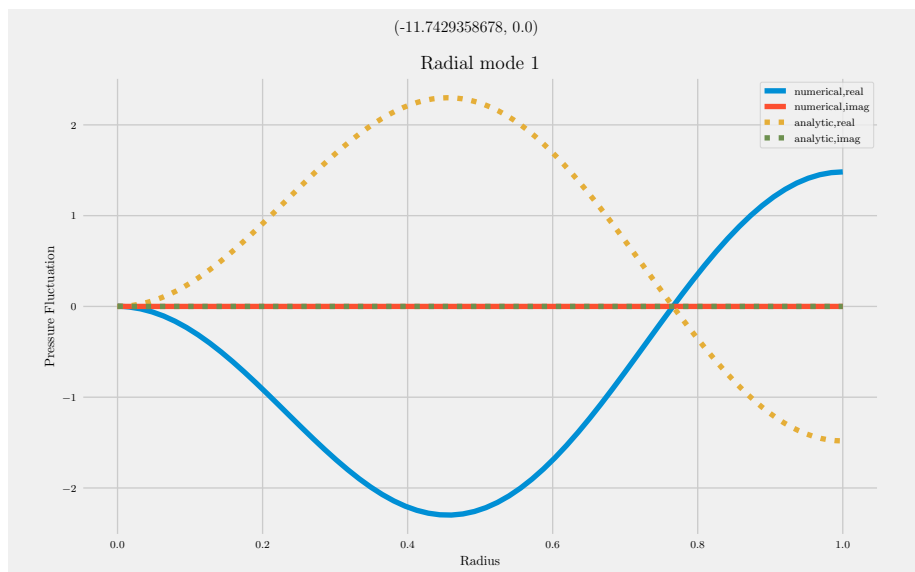

Figure 14: Fourth Order, Case number 0
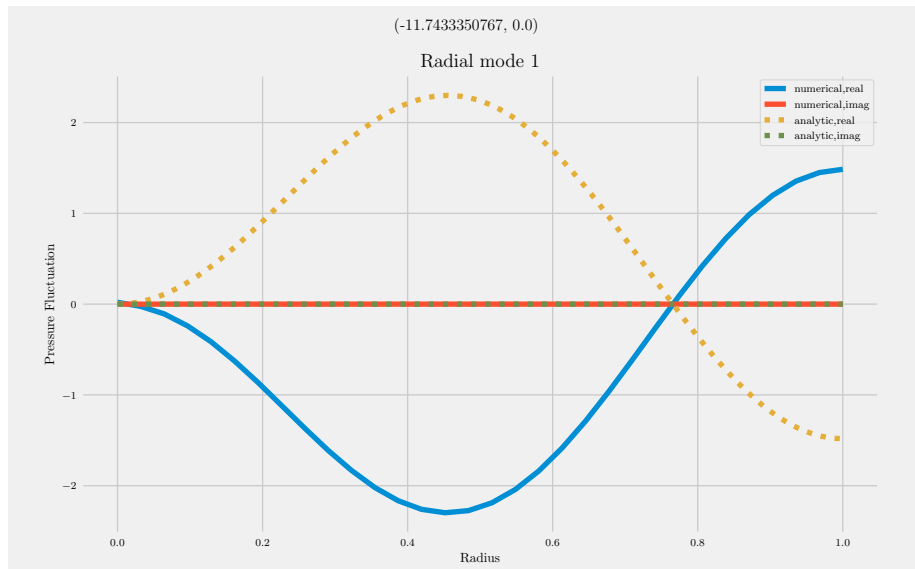
Figure 15: Fourth Order, Case number 1

Fourth Order, Radial Mode 2
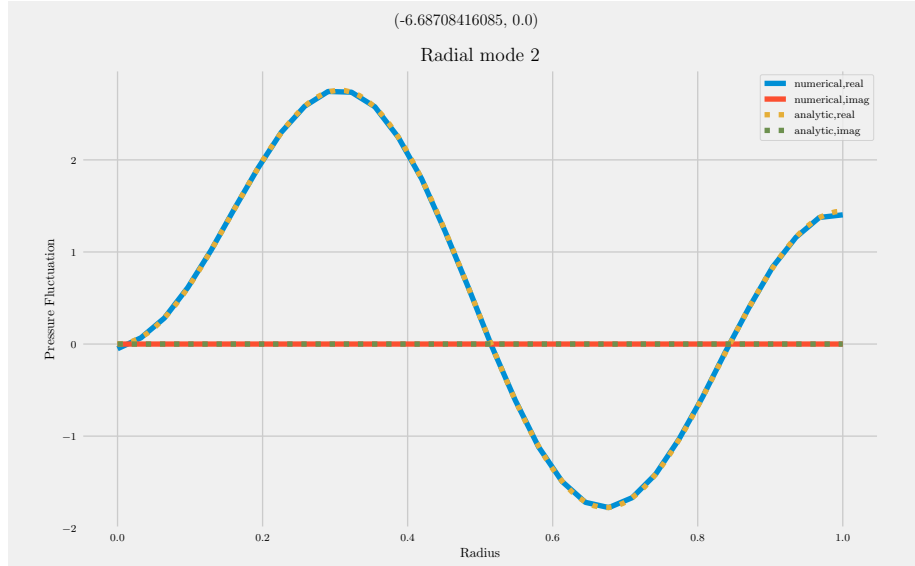
Figure 16: Fourth Order, Case number 0
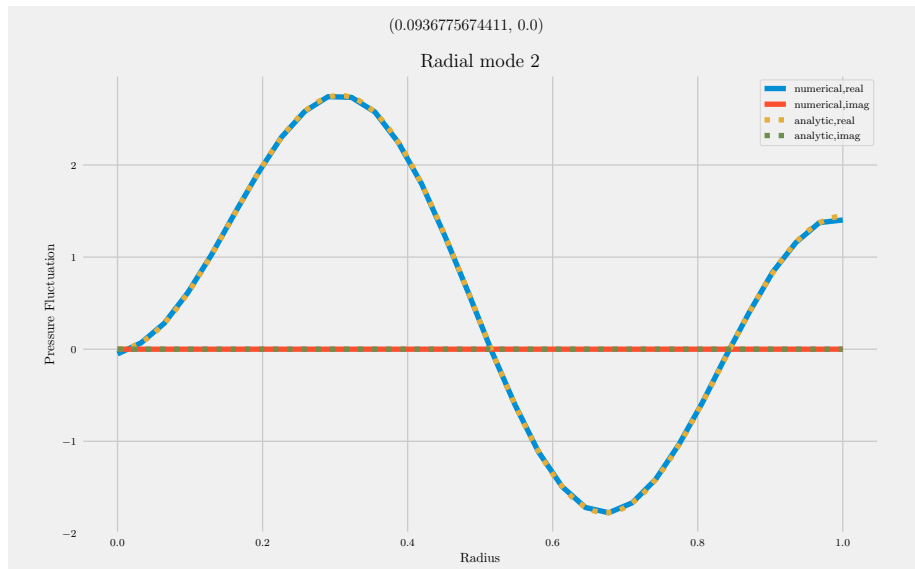
Figure 17: Fourth Order, Case number 1
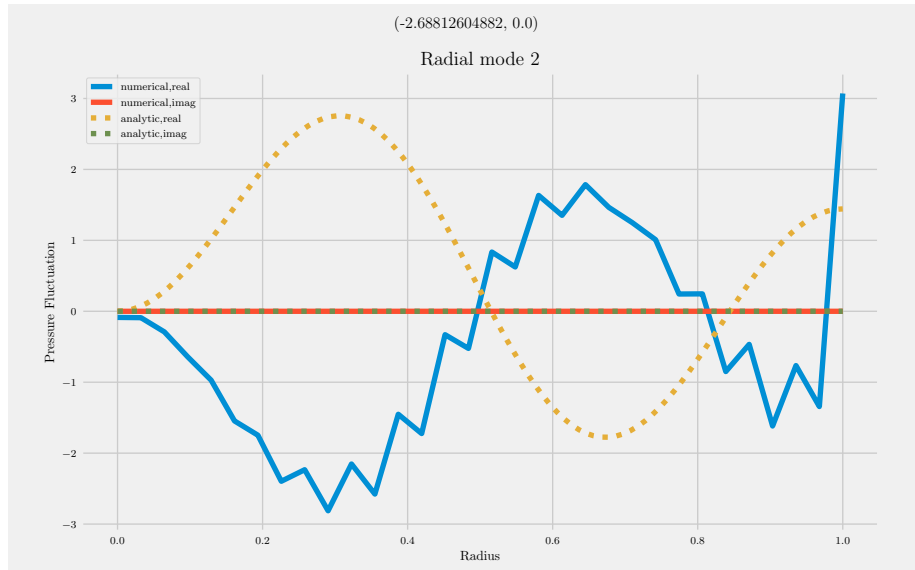
Figure 18: Fourth Order, Case number 2
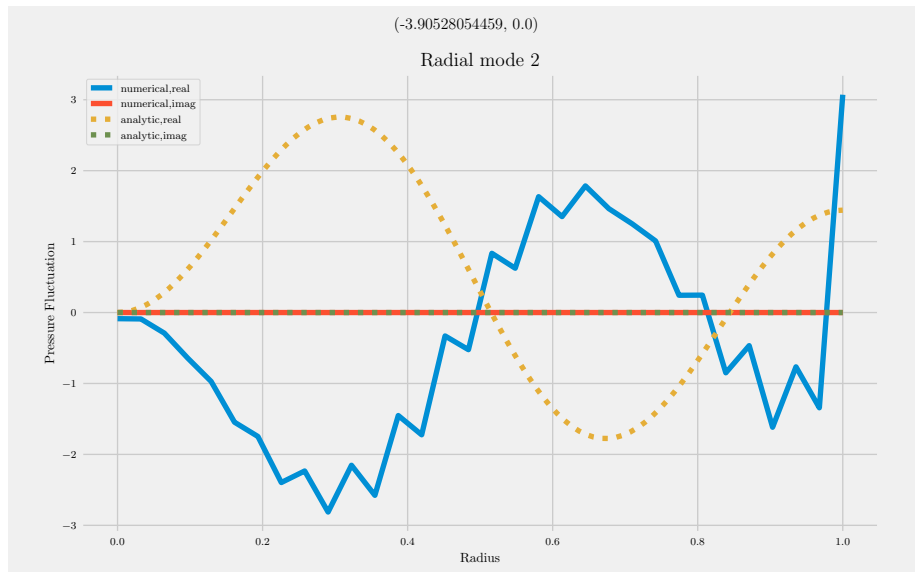
Figure 19: Fourth Order, Case number 3

Fourth Order, Radial Mode 3
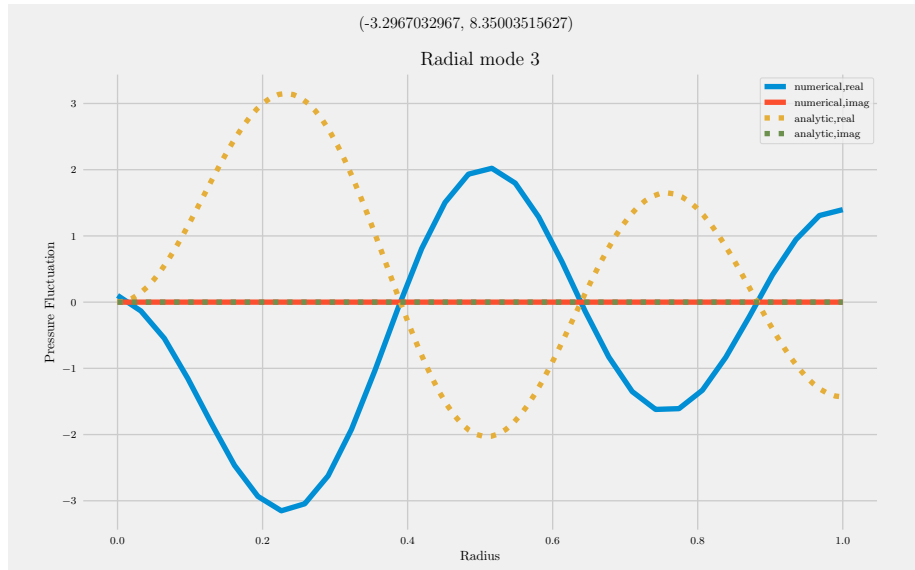
Figure 20: Fourth Order, Case number 0
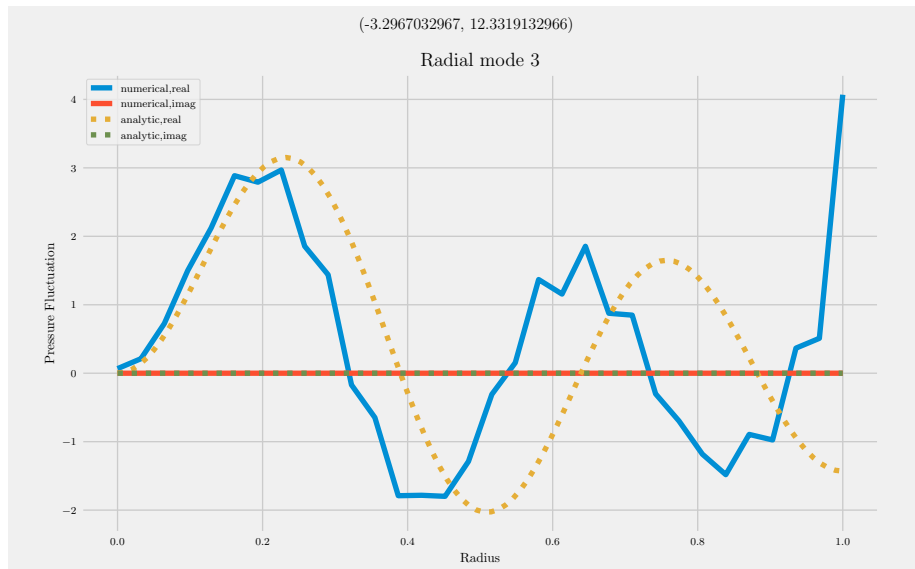
Figure 21: Fourth Order, Case number 1
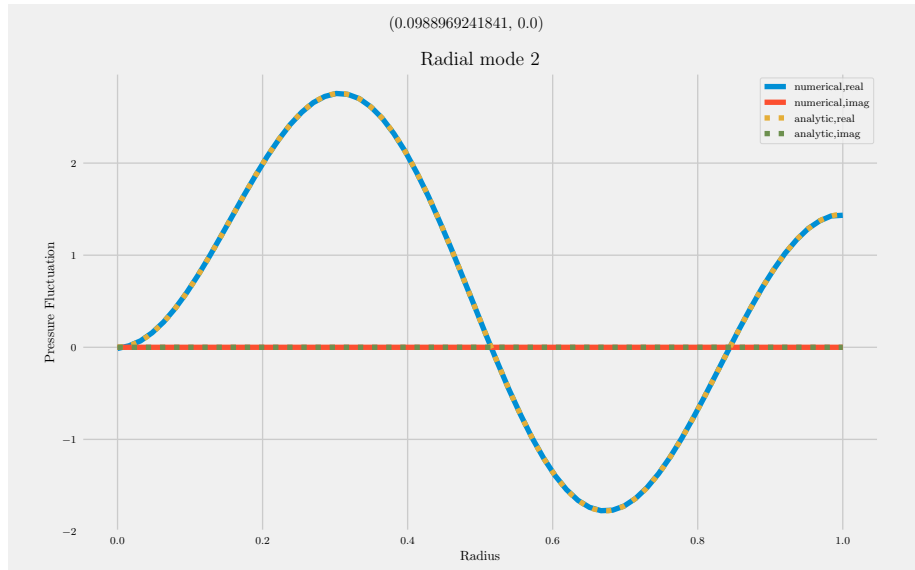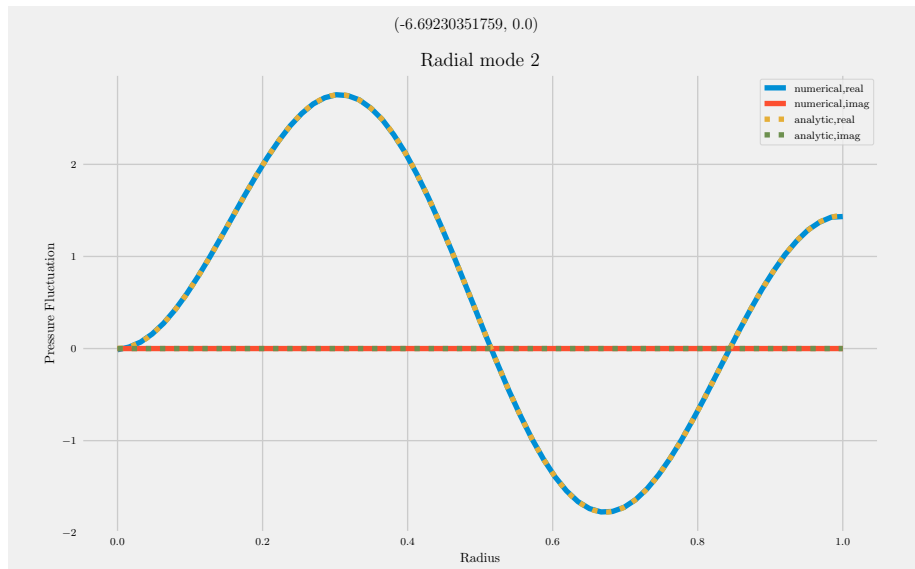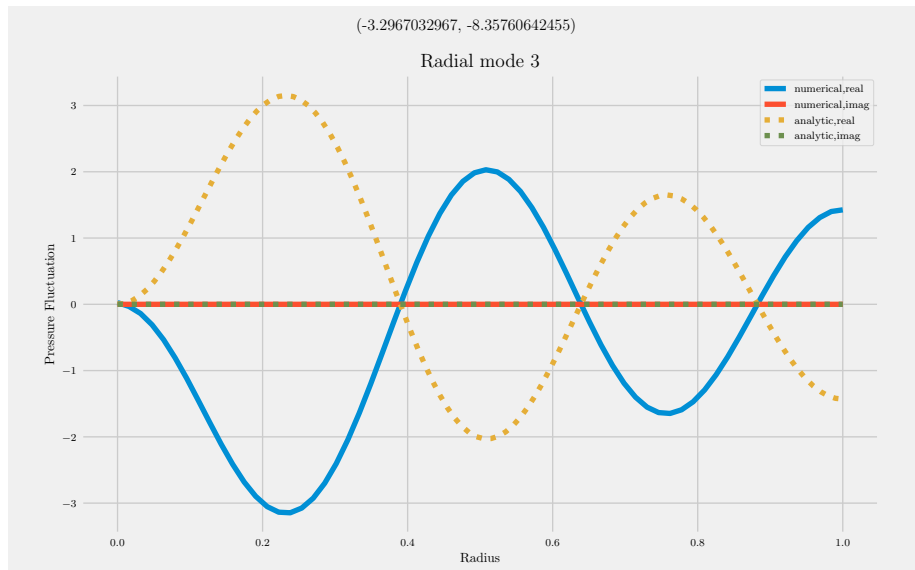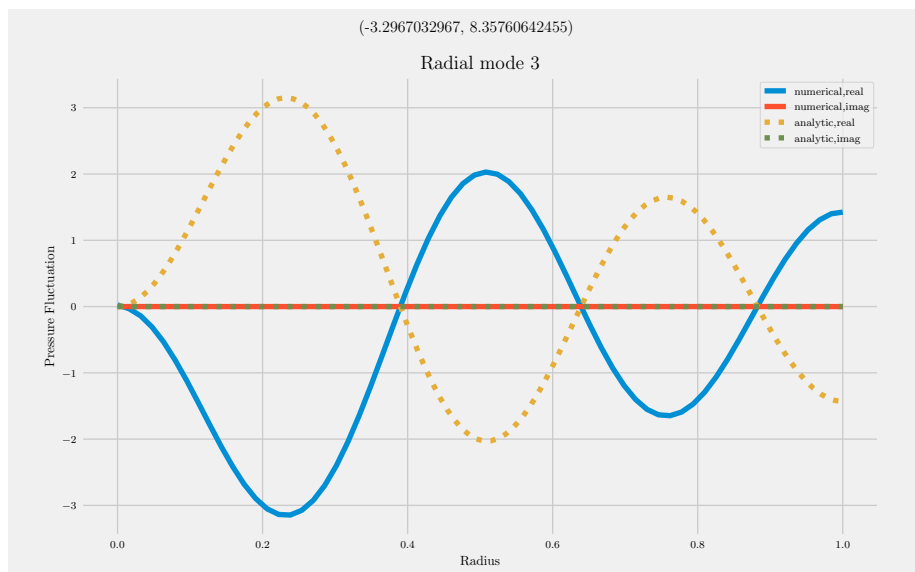
Fourth Order, Radial Mode 4
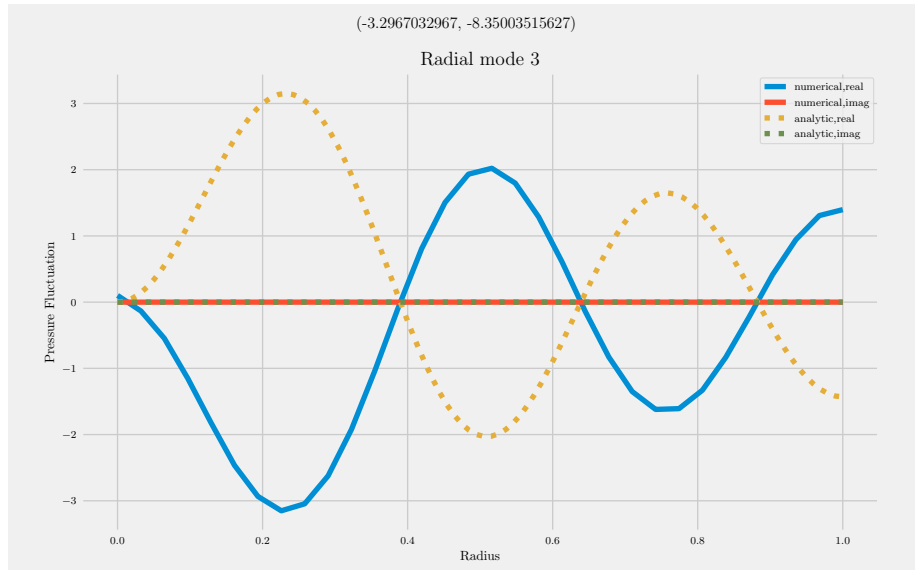
Figure 22: Fourth Order, Case number 2
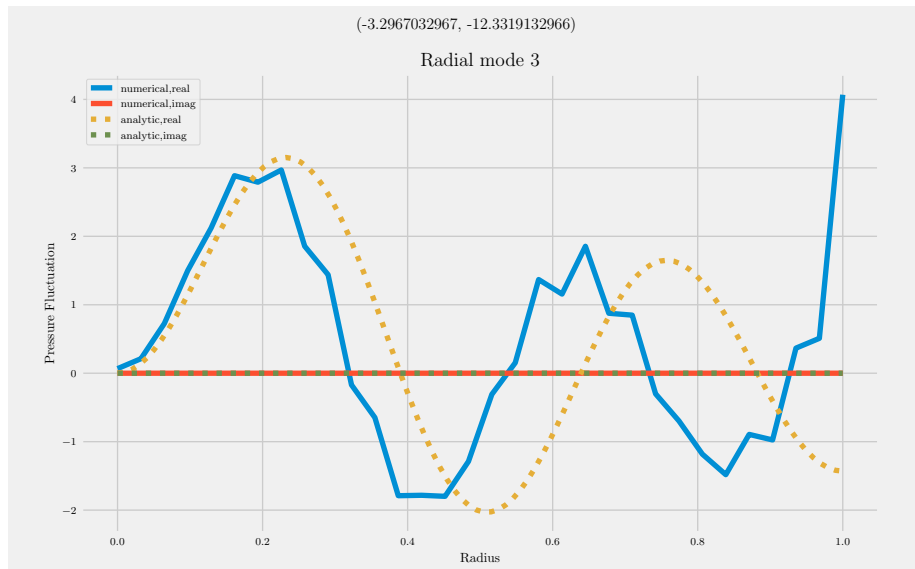
Figure 23: Fourth Order, Case number 3

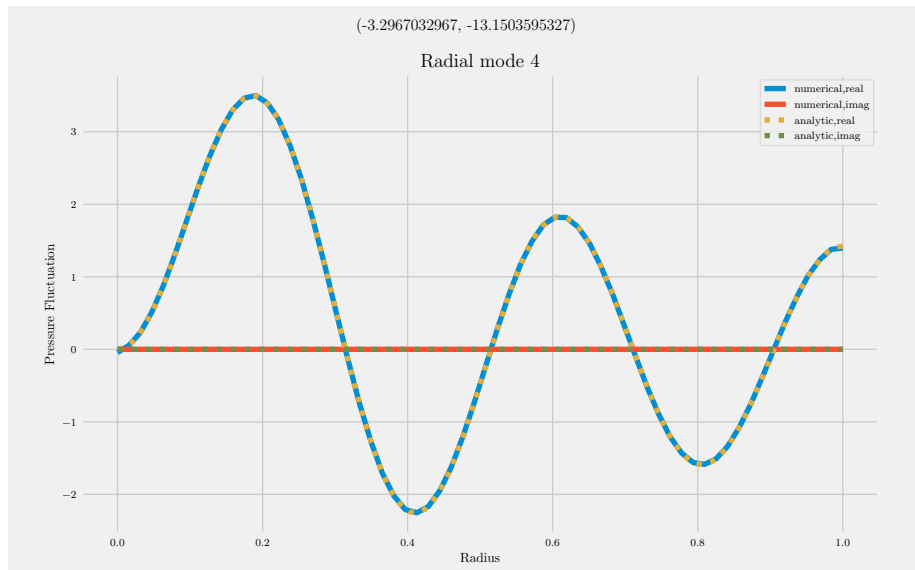### 0.3.3 64 points

Second Order, Radial Mode 0

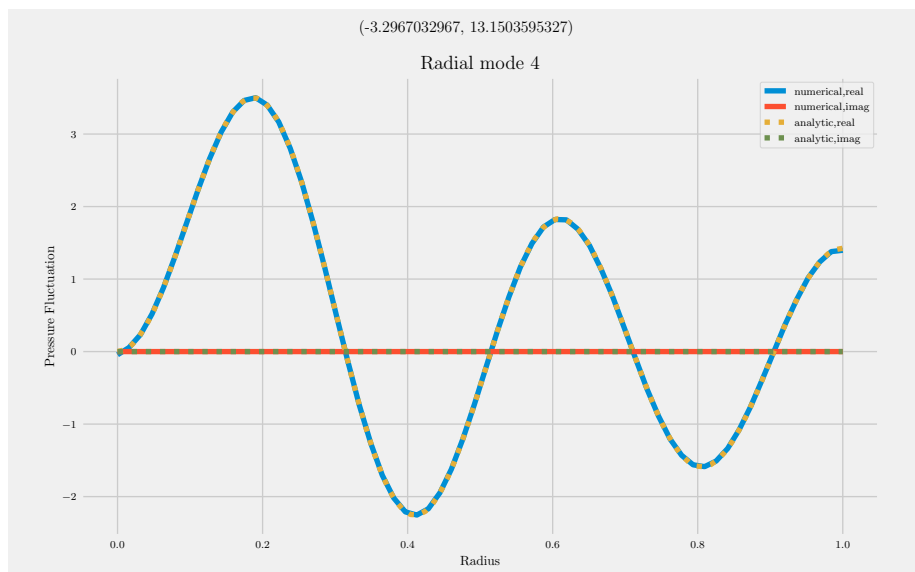Figure 24: Fourth Order, Case number 0

Figure 25: Fourth Order, Case number 1
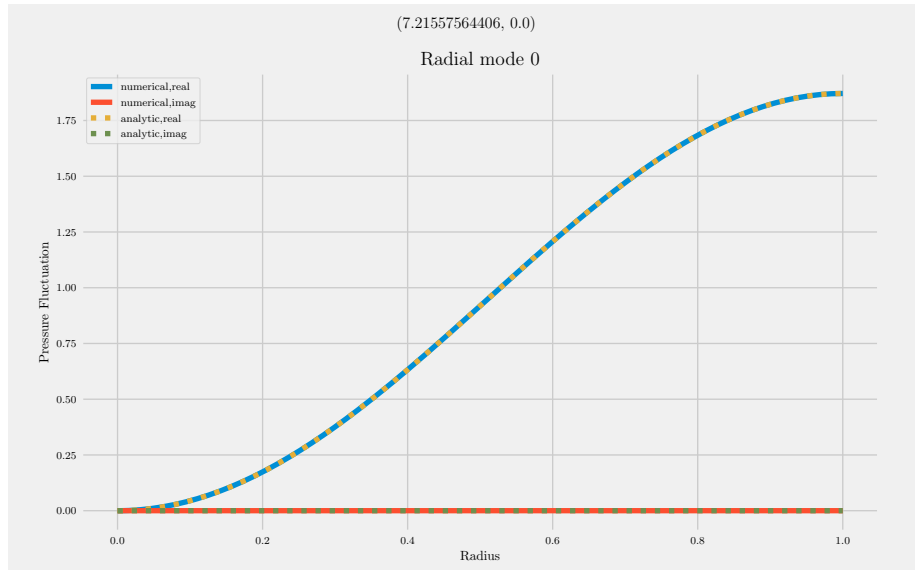
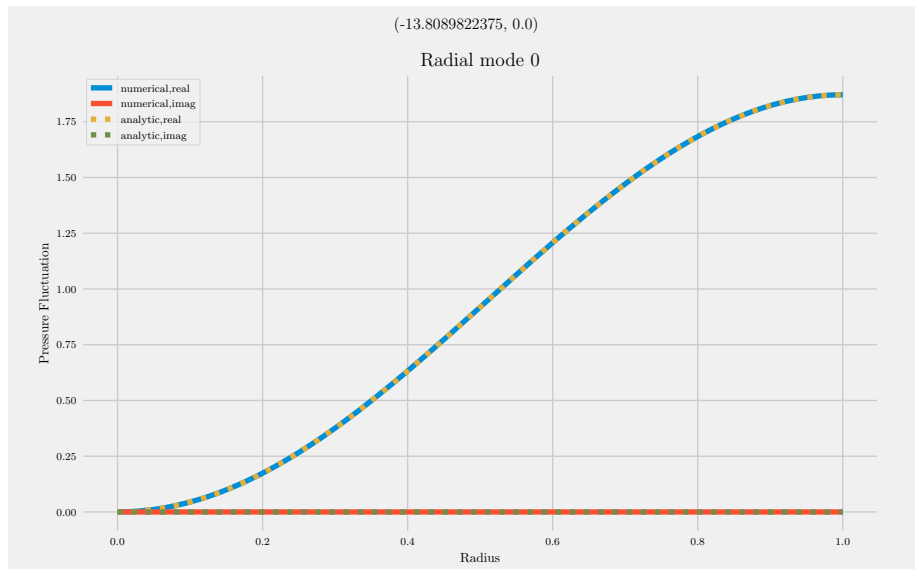**Second Order, Radial Mode 1**

Figure 26: Second Order, Case number 0

Figure 27: Second Order, Case number 1
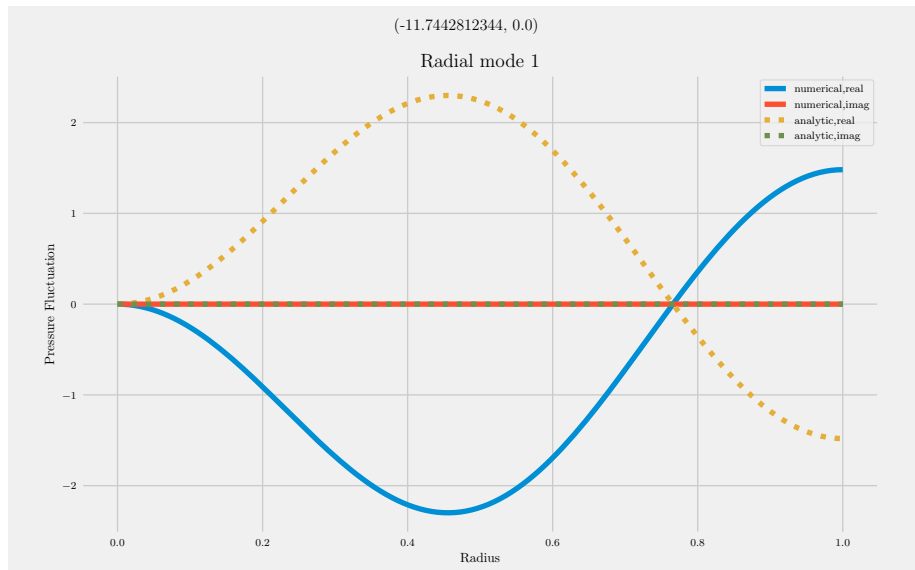
Second Order, Radial Mode 2
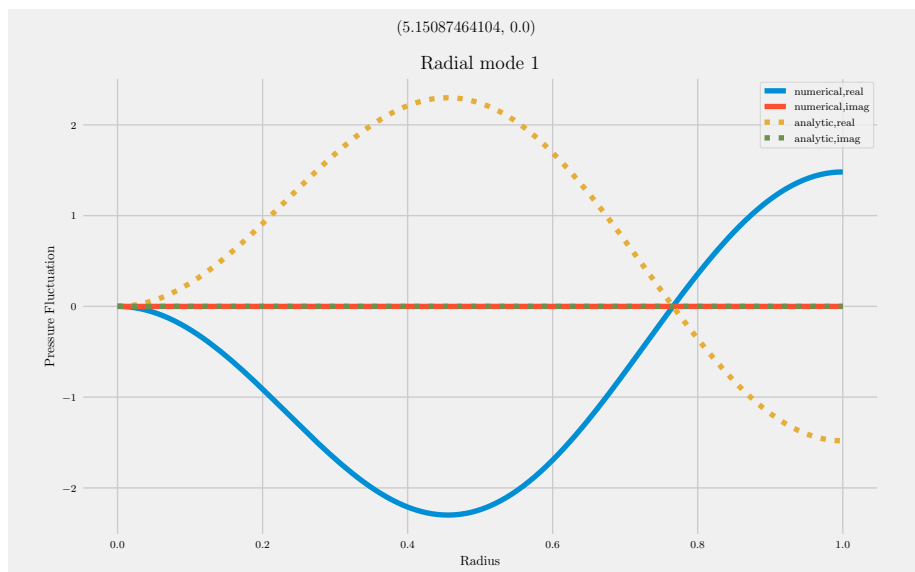
Figure 28: Second Order, Case number 0

Figure 29: Second Order, Case number 1
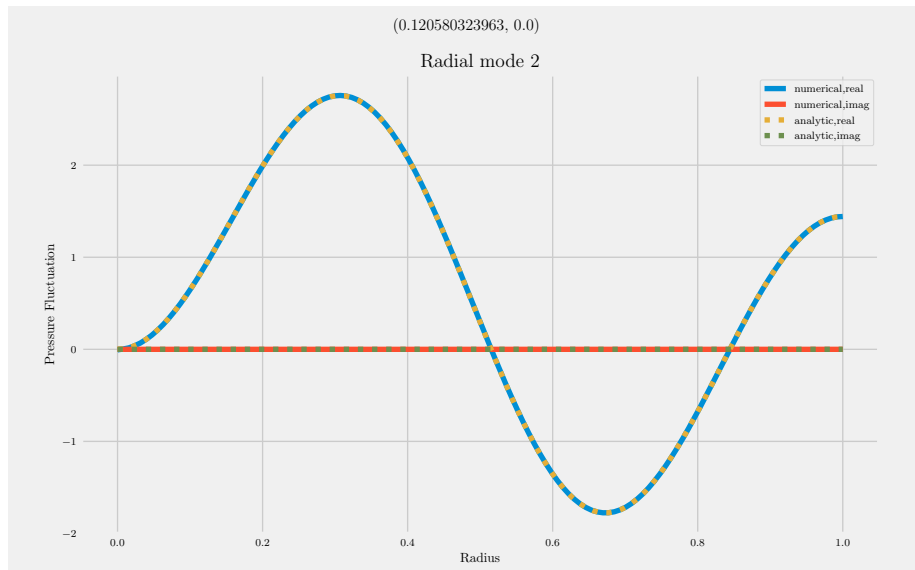
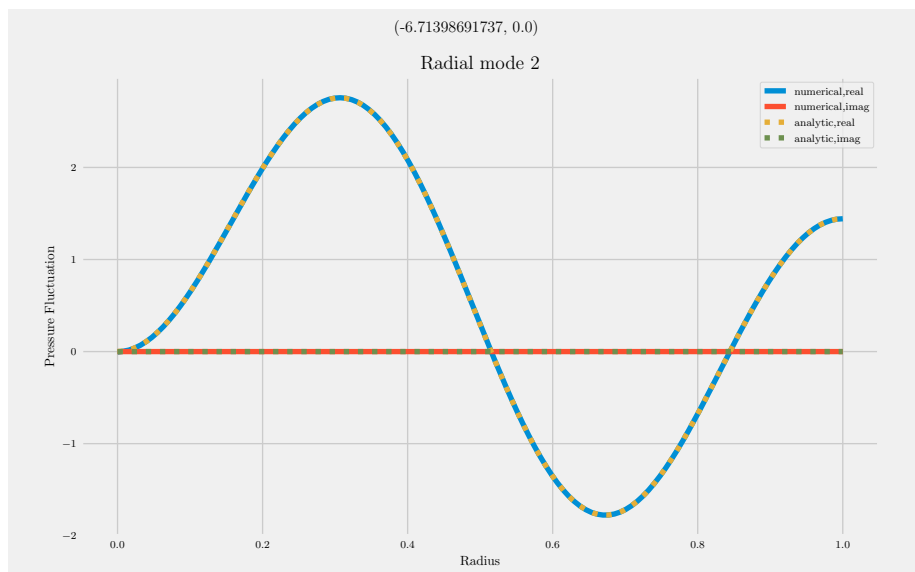**Second Order, Radial Mode 3**

Figure 30: Second Order, Case number 0

Figure 31: Second Order, Case number 1
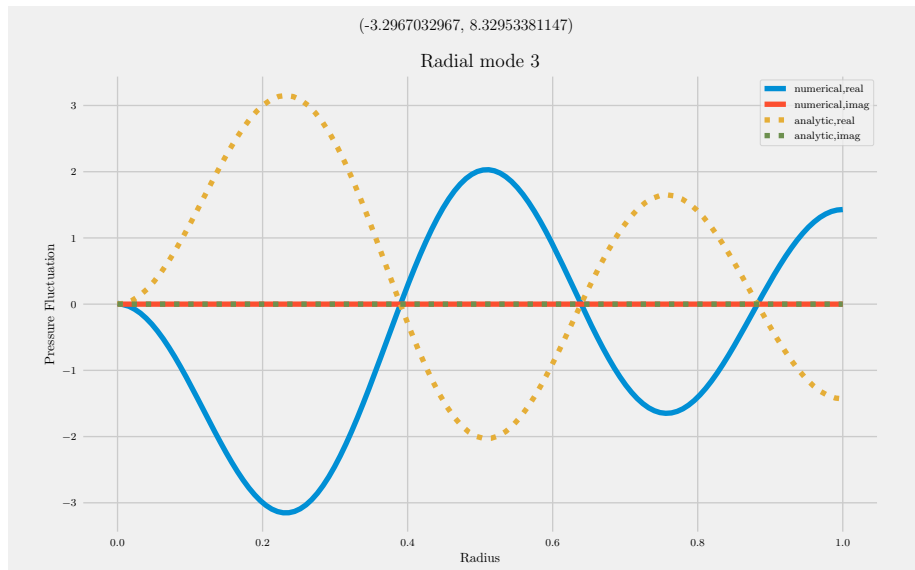
Second Order, Radial Mode 4

Figure 32: Second Order, Case number 0
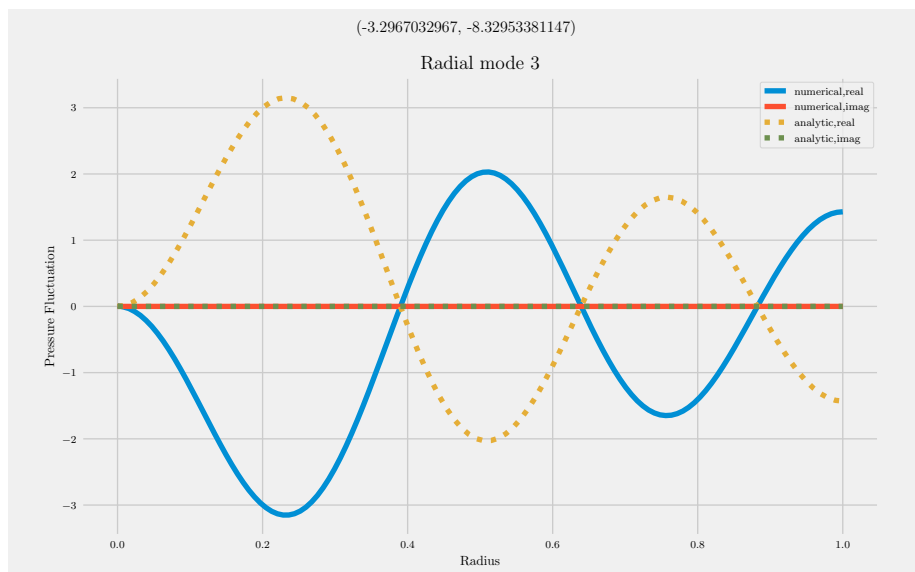

Figure 33: Second Order, Case number 1
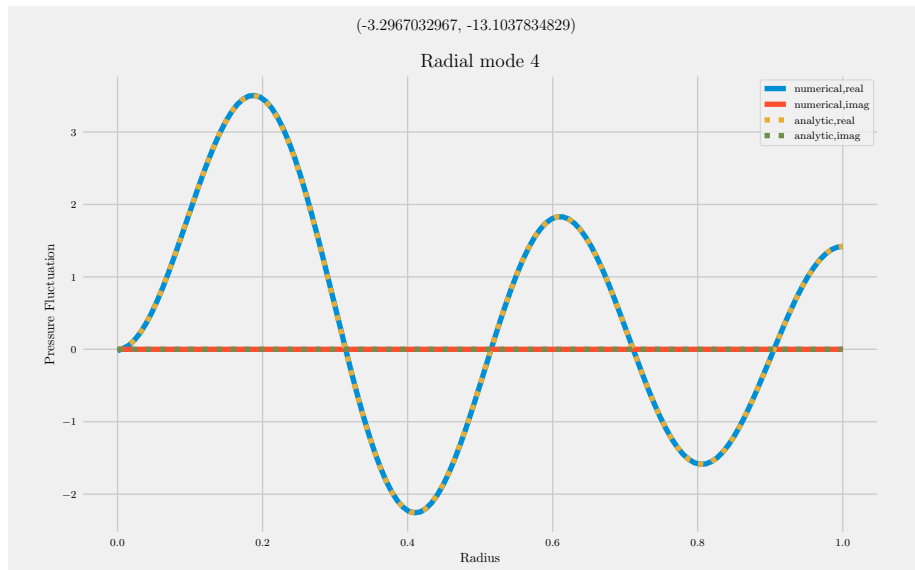
Fourth Order, Radial Mode 0

Figure 34: Second Order, Case number 0

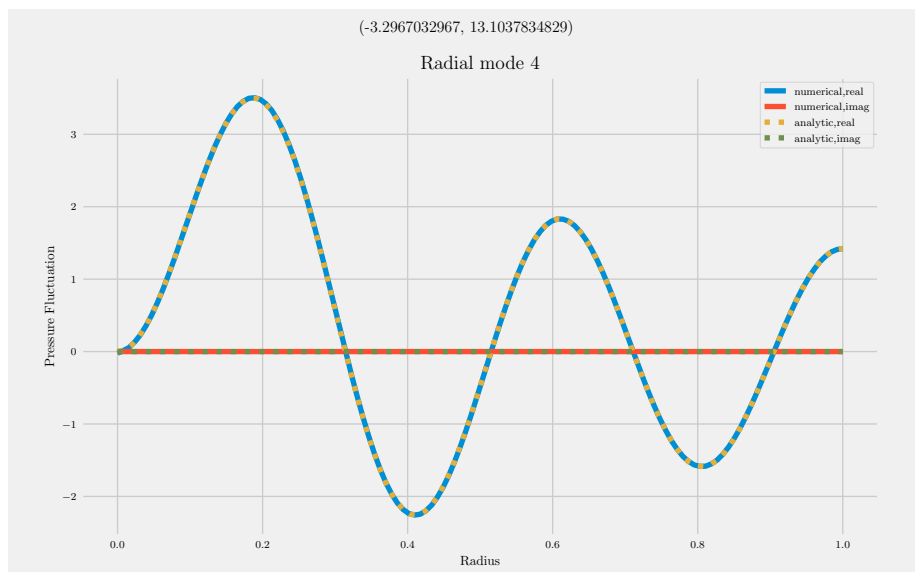
Figure 35: Second Order, Case number 1
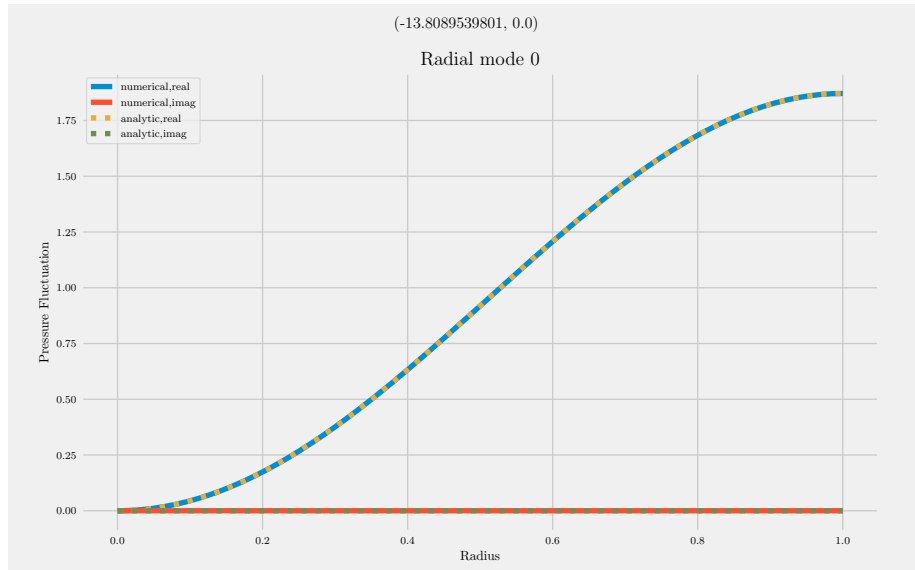
Fourth Order, Radial Mode 1

Figure 36: Fourth Order, Case number 0

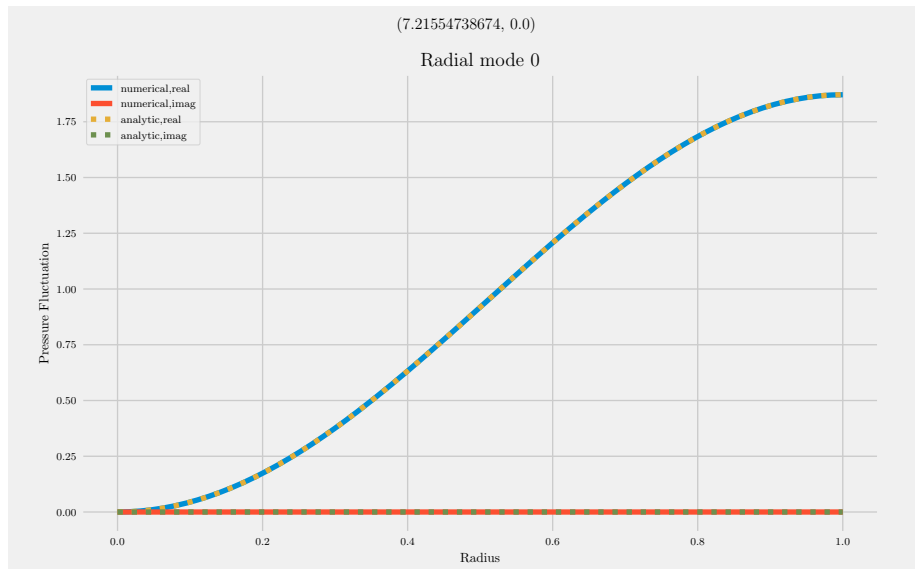
Figure 37: Fourth Order, Case number 1

Fourth Order, Radial Mode 2

Figure 38: Fourth Order, Case number 0

Figure 39: Fourth Order, Case number 1
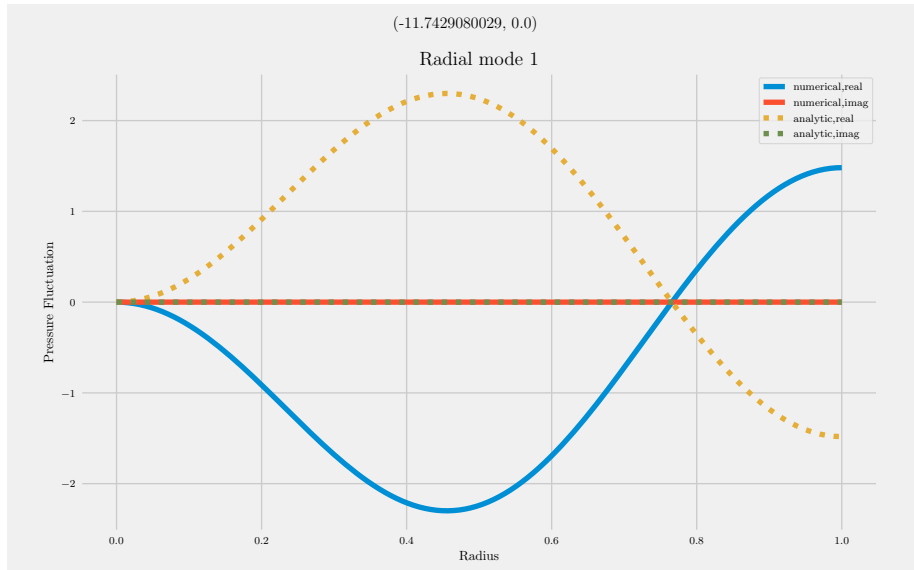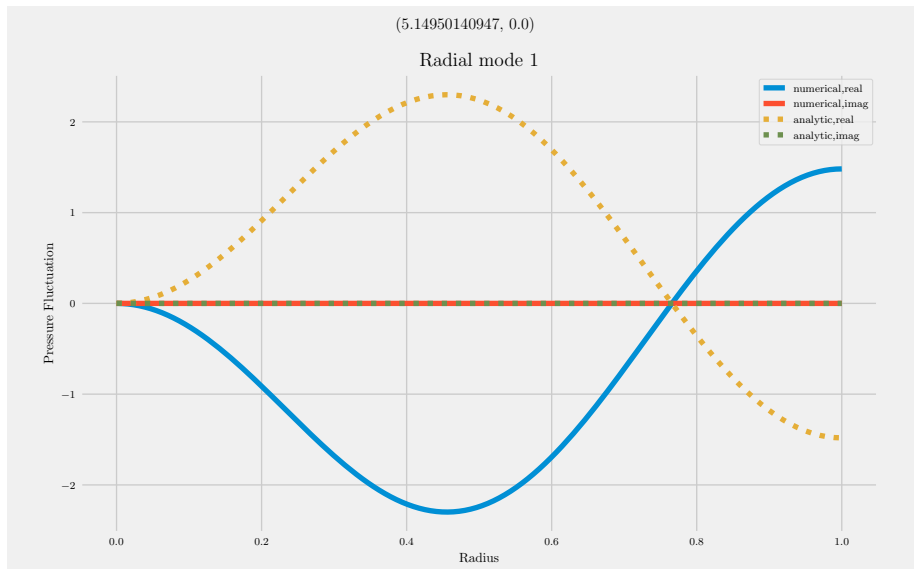
Fourth Order, Radial Mode 3

Figure 40: Fourth Order, Case number 0

Figure 41: Fourth Order, Case number 1

Fourth Order, Radial Mode 4

0.3.4 128 points
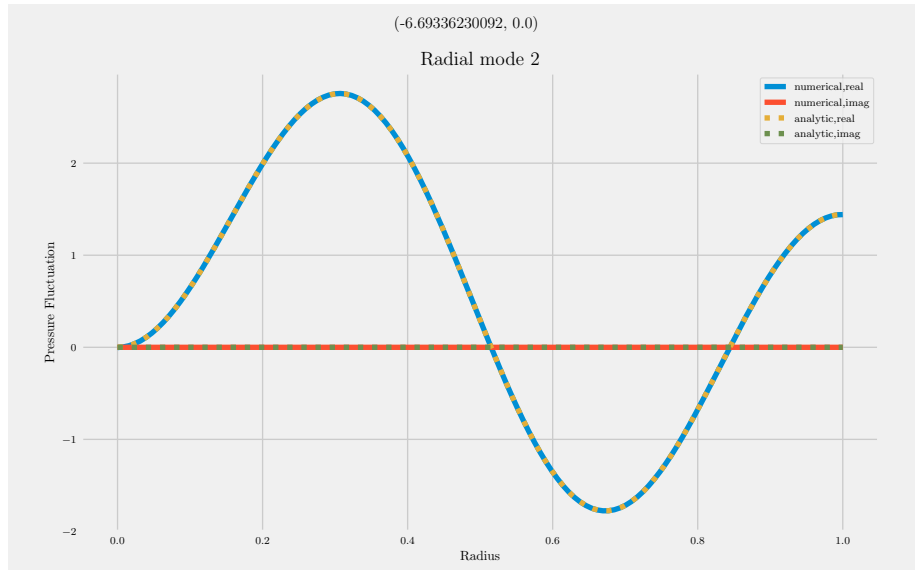
Second Order, Radial Mode 0

Figure 42: Fourth Order, Case number 0

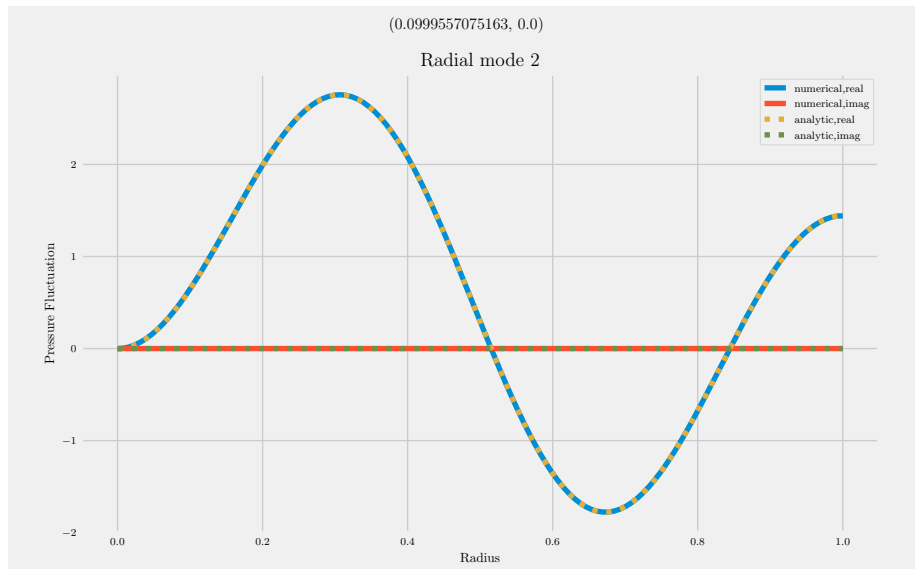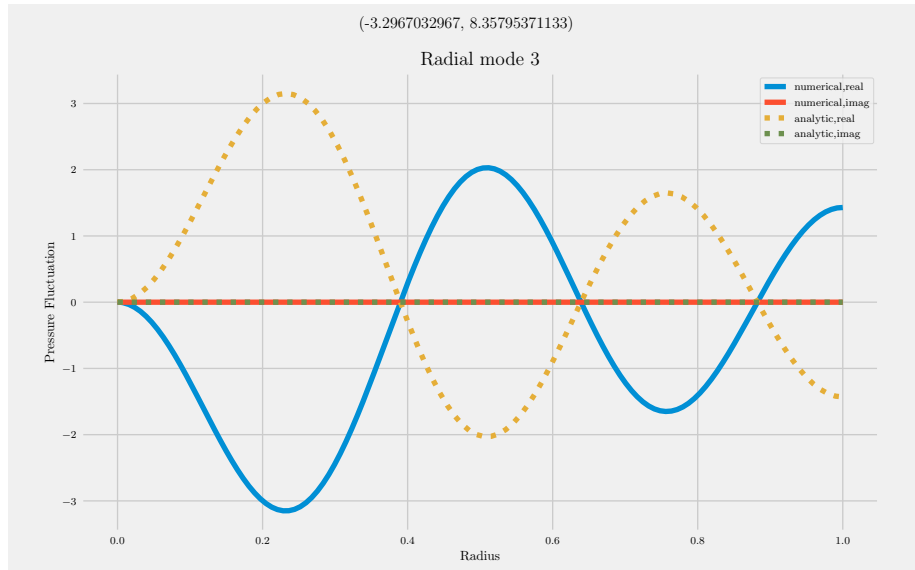
Figure 43: Fourth Order, Case number 1

Second Order, Radial Mode 1

Figure 44: Fourth Order, Case number 0

Figure 45: Fourth Order, Case number 1

Second Order, Radial Mode 2

Figure 46: Second Order, Case number 0

Figure 47: Second Order, Case number 1

Second Order, Radial Mode 3

Figure 48: Second Order, Case number 0

Figure 49: Second Order, Case number 1

Second Order, Radial Mode 4

Figure 50: Second Order, Case number 0

Figure 51: Second Order, Case number 1

Fourth Order, Radial Mode 0

Figure 52: Second Order, Case number 0

Figure 53: Second Order, Case number 1

Fourth Order, Radial Mode 1

Figure 54: Second Order, Case number 0

Figure 55: Second Order, Case number 1

Fourth Order, Radial Mode 2

Figure 56: Fourth Order, Case number 0

Figure 57: Fourth Order, Case number 1

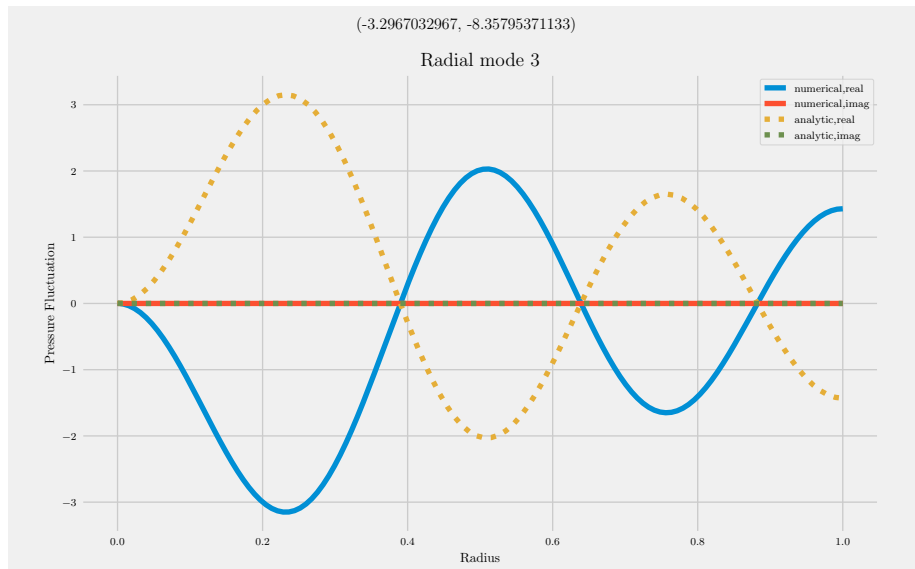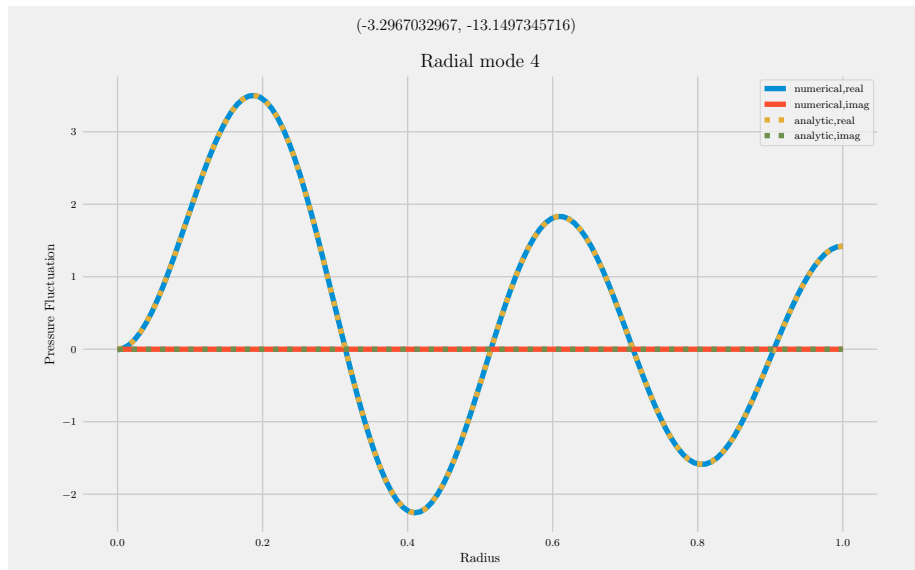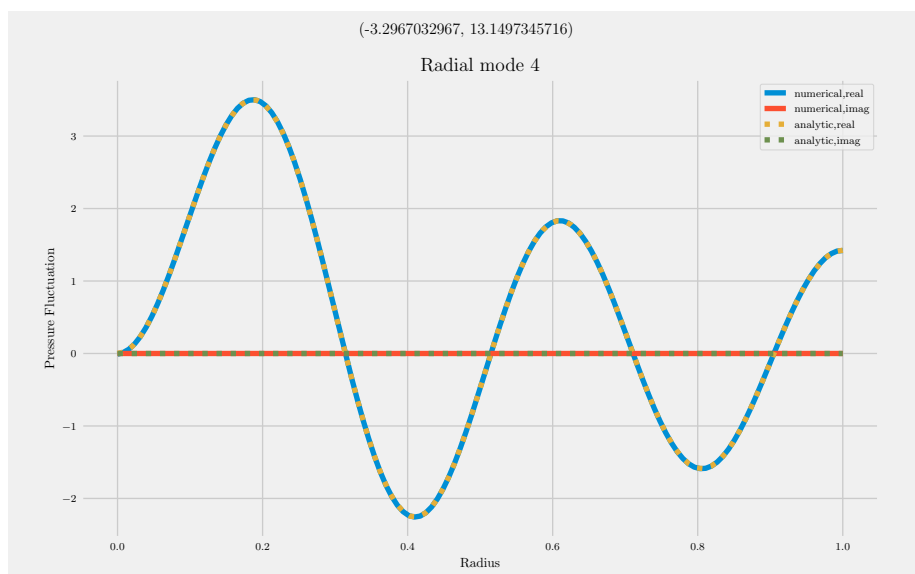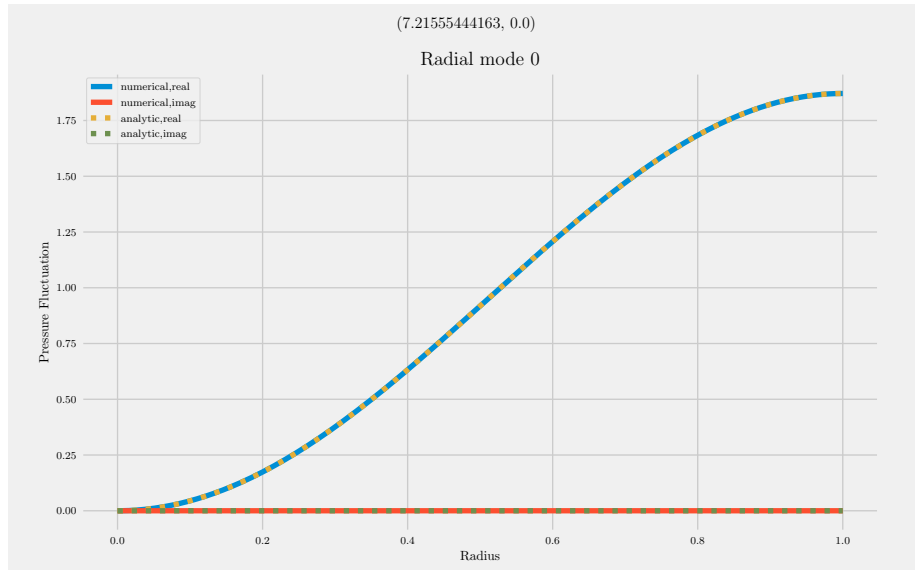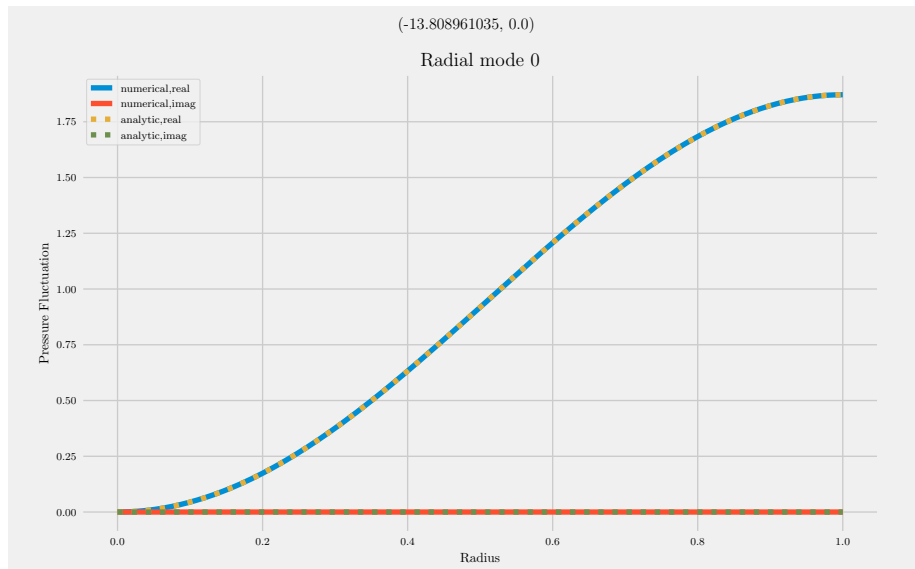**Fourth Order, Radial Mode 3**

Figure 58: Fourth Order, Case number 0

Figure 59: Fourth Order, Case number 1

Fourth Order, Radial Mode 4

0.3.5   256 points

Second Order, Radial Mode 0

Figure 60: Fourth Order, Case number 0

Figure 61: Fourth Order, Case number 1

Second Order, Radial Mode 1

Figure 62: Fourth Order, Case number 0

Figure 63: Fourth Order, Case number 1

Second Order, Radial Mode 2

Figure 64: Fourth Order, Case number 0

Figure 65: Fourth Order, Case number 1

Second Order, Radial Mode 3

Figure 66: Second Order, Case number 0

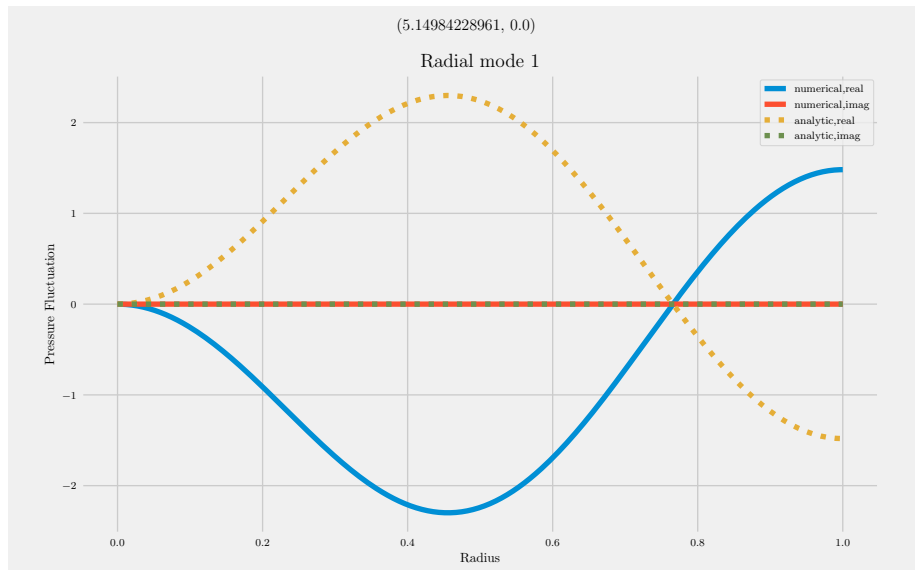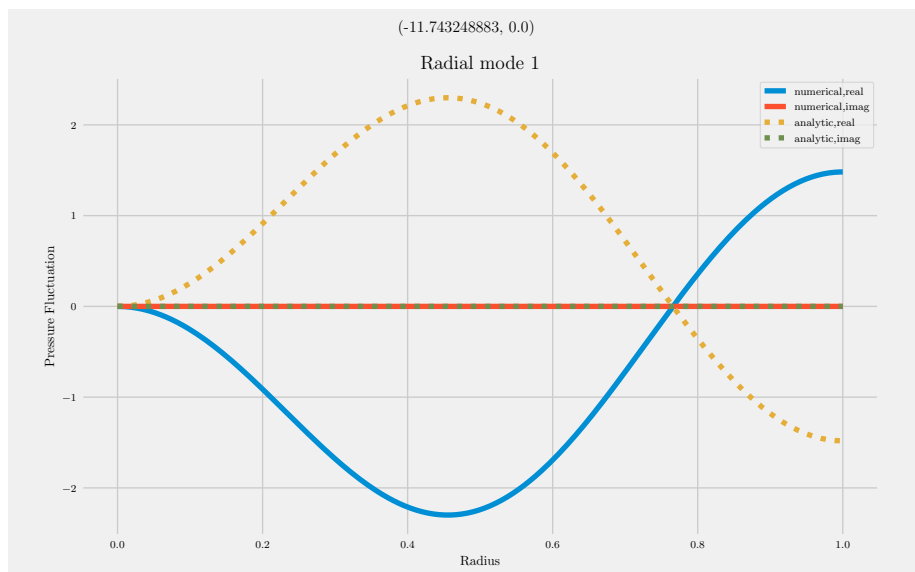Figure 67: Second Order, Case number 1
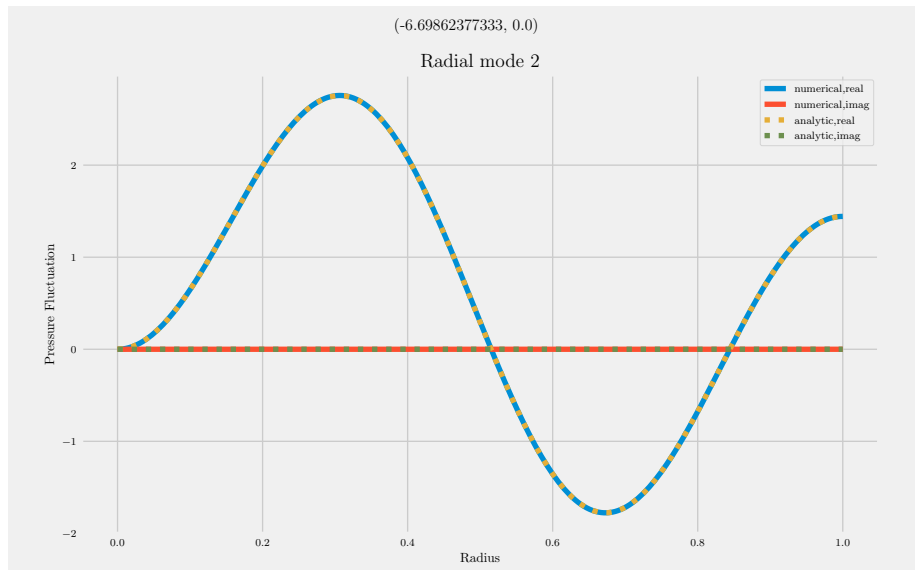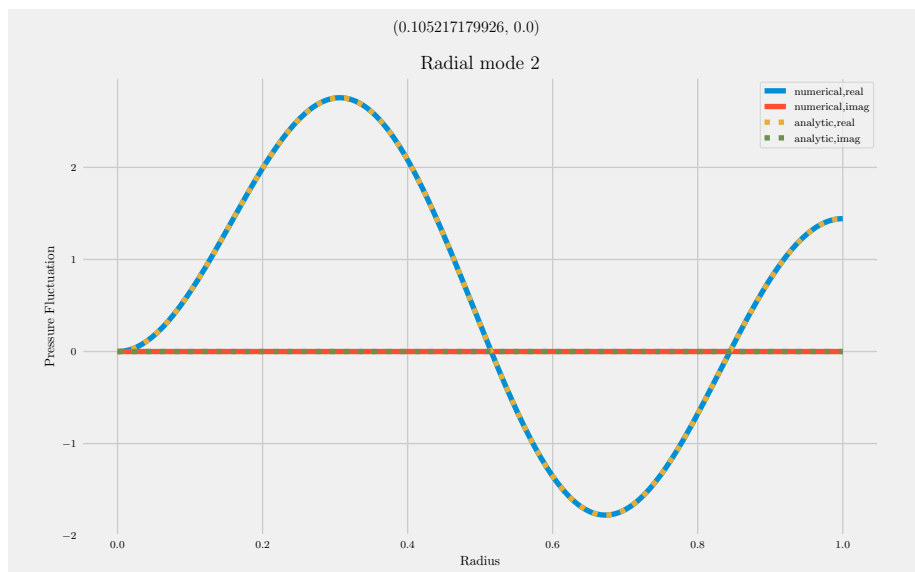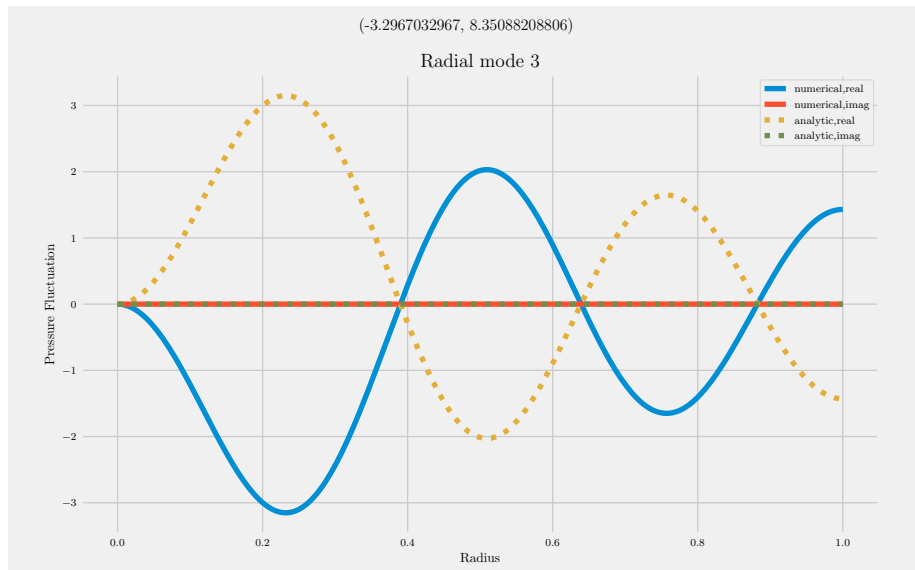
Second Order, Radial Mode 4

(5.14984228961, 0.0)

Figure 68: Second Order, Case number 0

(-11.743248883, 0.0)

Figure 69: Second Order, Case number 1

Fourth Order, Radial Mode 0

Figure 70: Second Order, Case number 0

Figure 71: Second Order, Case number 1

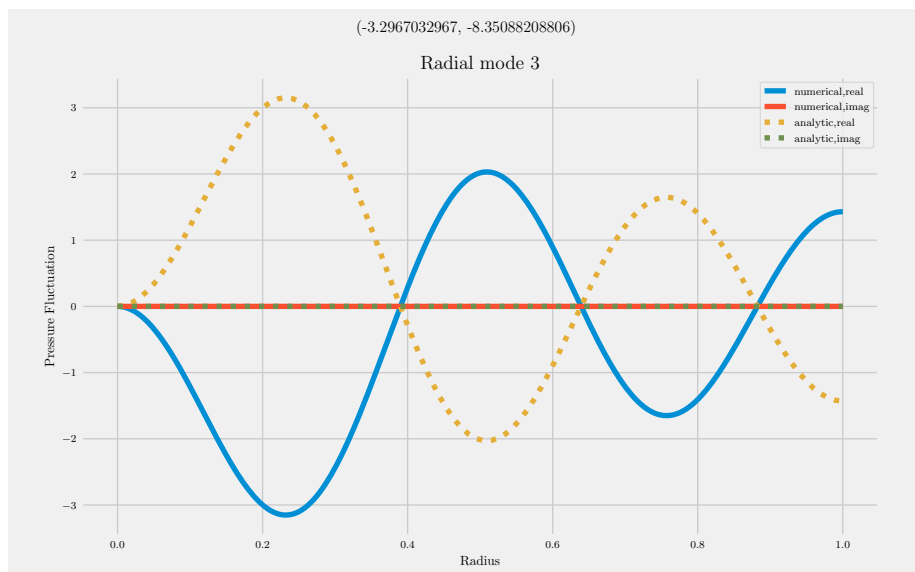**Fourth Order, Radial Mode 1**

Figure 72: Second Order, Case number 0

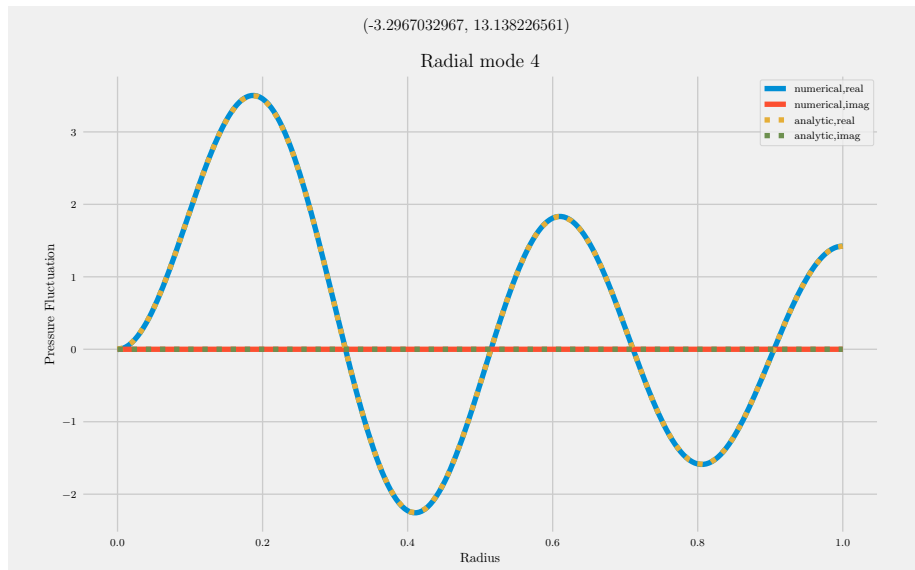Figure 73: Second Order, Case number 1

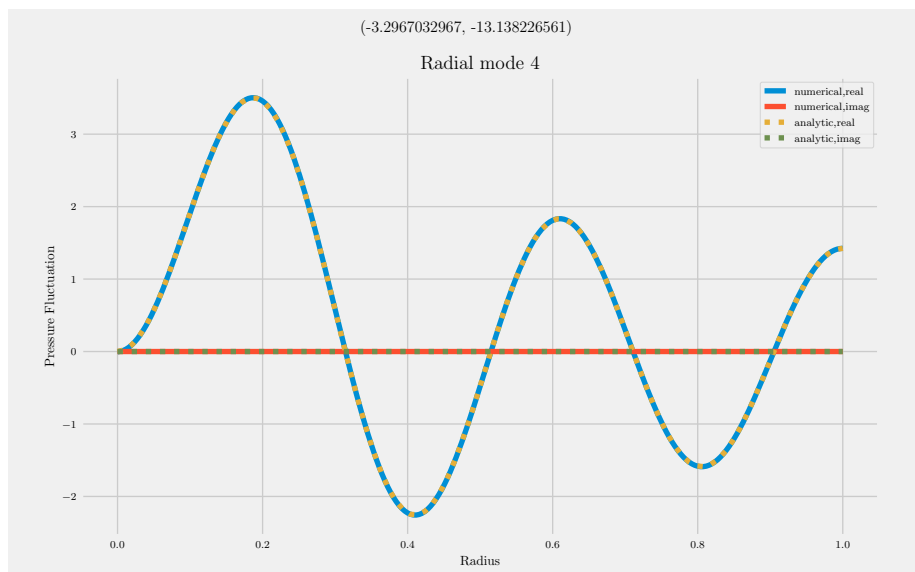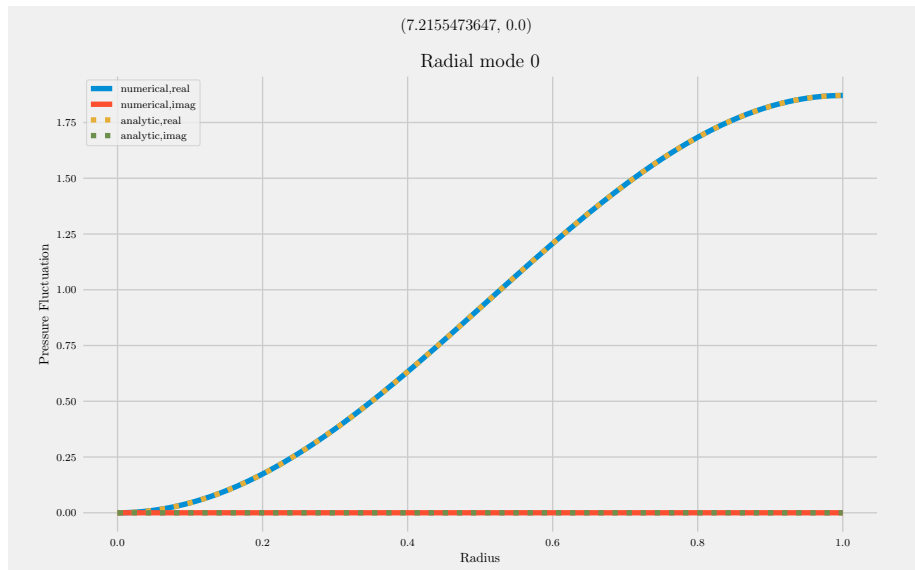**Fourth Order, Radial Mode 2**

Figure 74: Second Order, Case number 0

Figure 75: Second Order, Case number 1

Fourth Order, Radial Mode 3

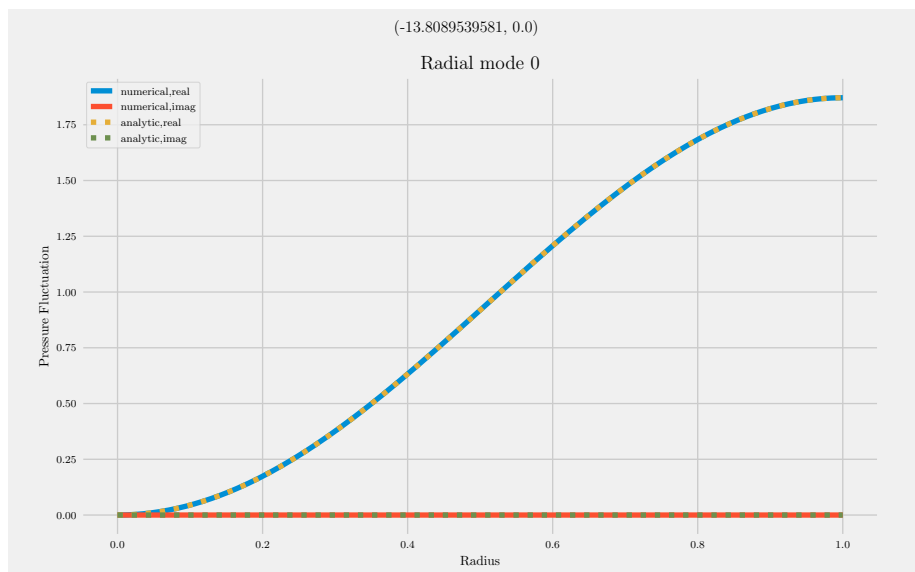Figure 76: Fourth Order, Case number 0

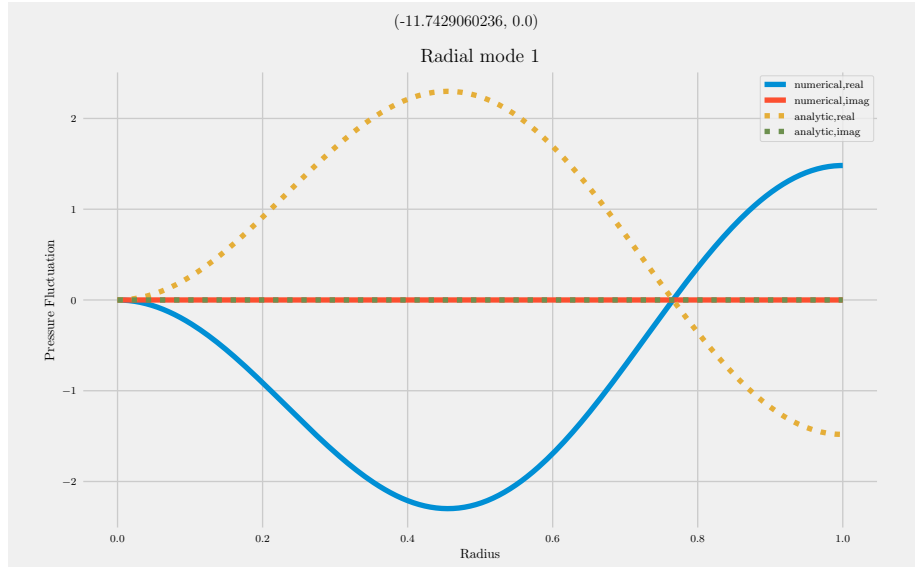Figure 77: Fourth Order, Case number 1

Fourth Order, Radial Mode 4

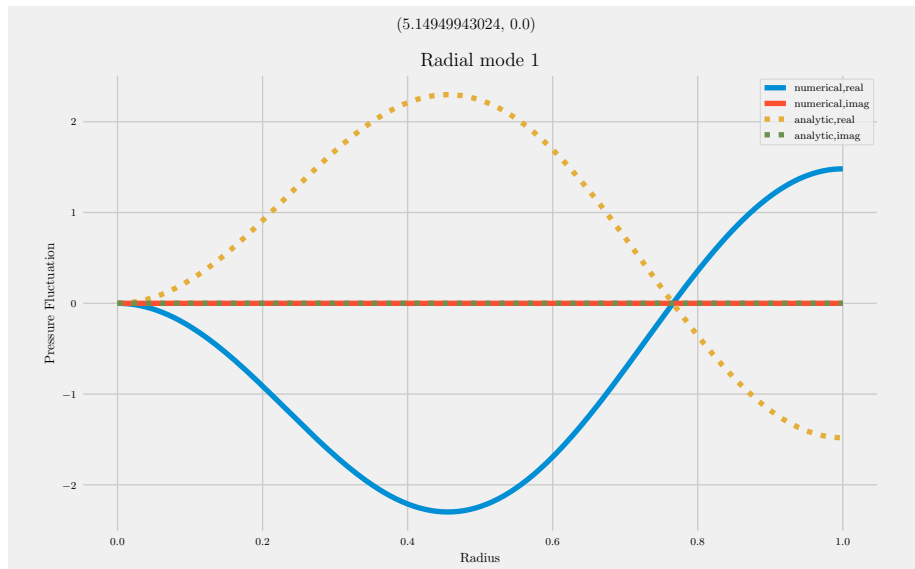Figure 78: Fourth Order, Case number 0
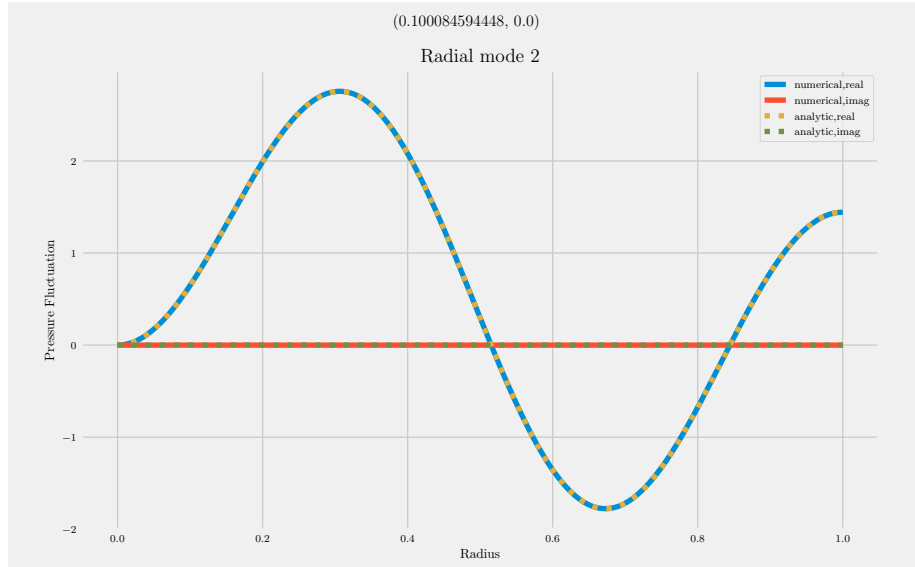

Figure 79: Fourth Order, Case number 1

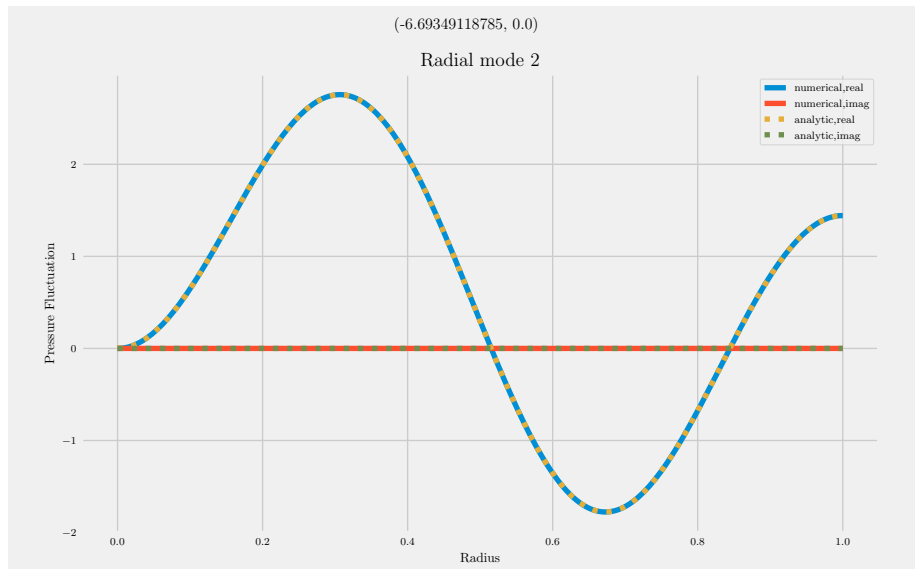Figure 80: Fourth Order, Case number 0
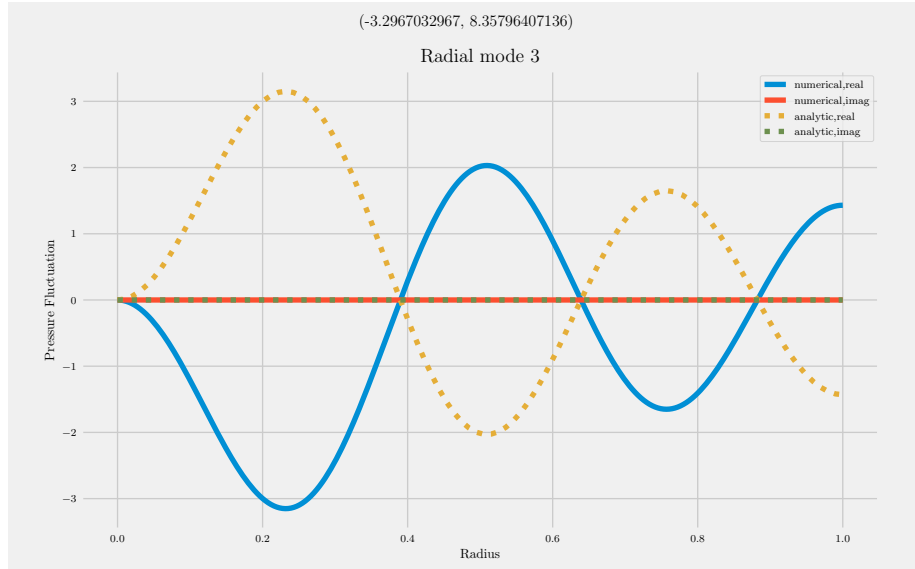

Figure 81: Fourth Order, Case number 1

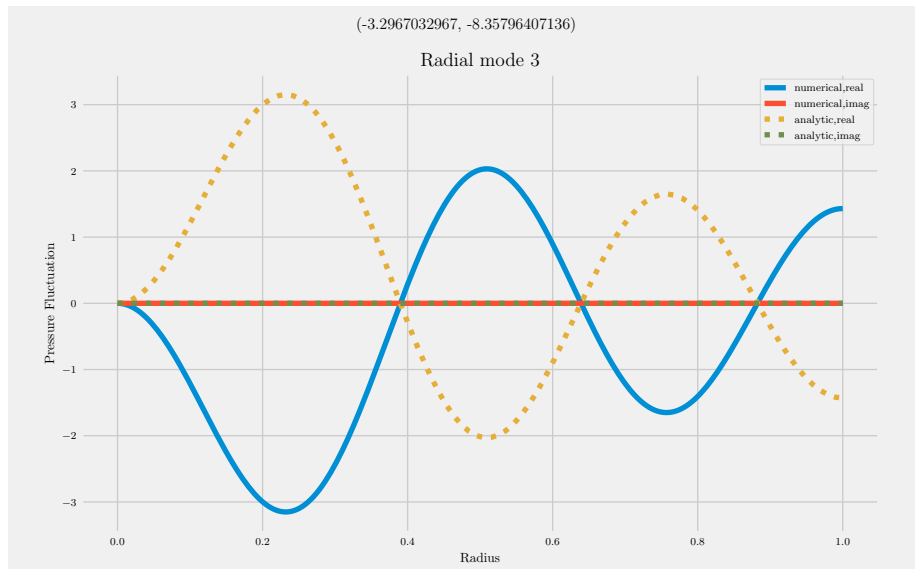Figure 82: Fourth Order, Case number 0
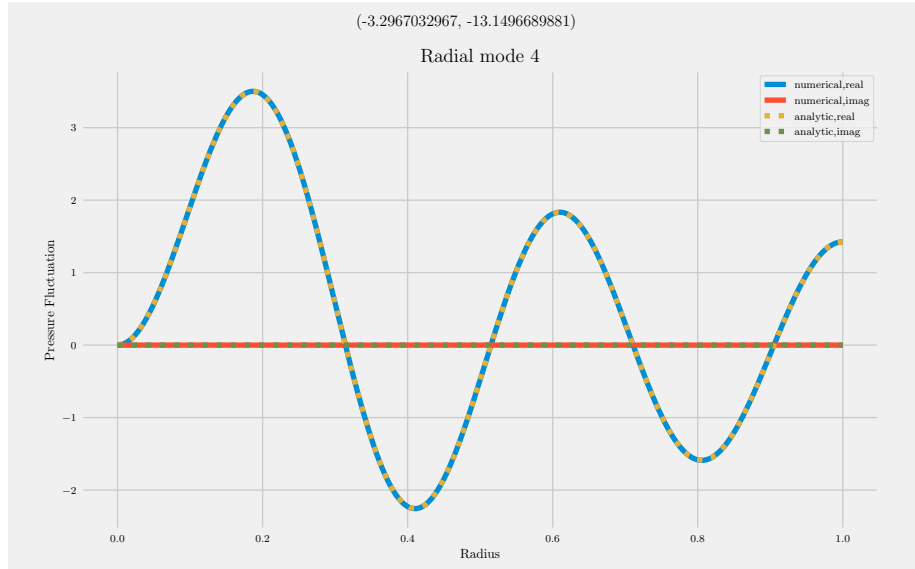

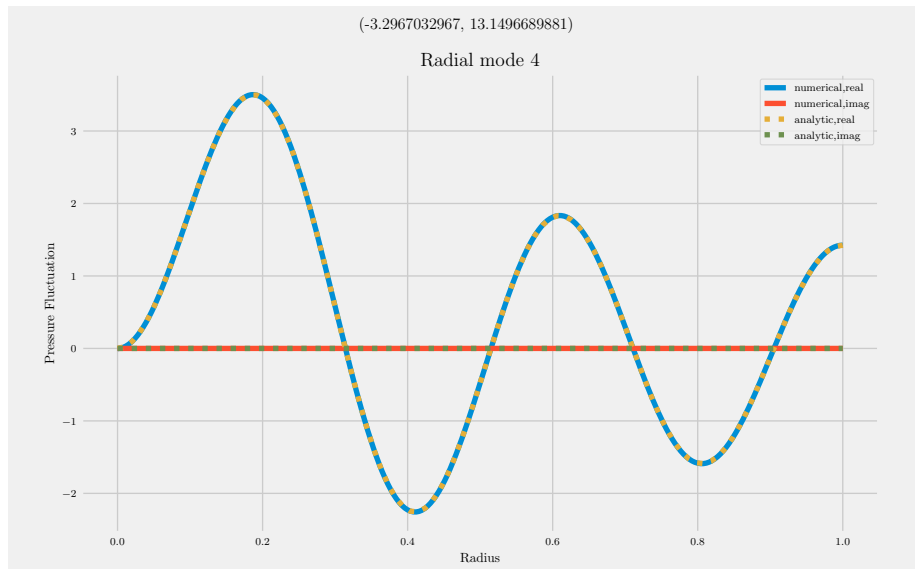Figure 83: Fourth Order, Case number 1

Figure 84: Fourth Order, Case number 0

Figure 85: Fourth Order, Case number 1