Documentation for the SWIRL code

Ray Hixon
University of Toledo
Toledo, OH 43606
email: dhixon@eng.utoledo.edu

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1 Introduction

This writeup is my notes for the SWIRL code, based on the Kousen technical report and on the code itself.

2 Notes on the Kousen report

From the Summary: there are two types of eigenvalues: first, the acoustic modes (discrete values). Second, the vortical modes, which are convecting with phase velocities that 'correspond to the local mean flow velocities'.

From the Introduction: when the mean flow is nonuniform (swirling or not), there is a question about whether an exponential description of the axial disturbance is correct or not. The SWIRL code assumes an exponential function for the axial variation.

3 Aerodynamic Model

The cylindrical coordinate Euler equations are:

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + v_r \frac{\partial \rho}{\partial r} + \frac{v_\theta}{r}\frac{\partial \rho}{\partial \theta} + v_x \frac{\partial \rho}{\partial x} + \rho\left(\frac{1}{r}\frac{\partial\left(r v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x}\right) &= 0 \\
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_x \frac{\partial v_r}{\partial x} &= -\frac{1}{\rho}\frac{\partial p}{\partial r} \\
\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_x \frac{\partial v_\theta}{\partial x} &= -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} \\
\frac{\partial v_x}{\partial t} + v_r \frac{\partial v_x}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_x}{\partial \theta} + v_x \frac{\partial v_x}{\partial x} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial t} + v_r \frac{\partial p}{\partial r} + \frac{v_\theta}{r}\frac{\partial p}{\partial \theta} + v_x \frac{\partial p}{\partial x} + \gamma p\left(\frac{1}{r}\frac{\partial\left(r v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x}\right) &= 0
\end{aligned}
$$

3.1 Mean Flow

For a steady axisymmetric mean flow with no radial velocity, the Euler equations simplify to:

$$\begin{aligned}\frac{\partial}{\partial x}\left(\overline{\rho}V_x\right) &= 0\\ -\frac{V_\theta^2}{r} &= -\frac{1}{\overline{\rho}}\frac{\partial P}{\partial r}\\ \frac{\partial}{\partial x}\left(V_\theta\right) &= 0\\ \frac{\partial}{\partial x}\left(V_x\right) &= 0\end{aligned}$$

The radial momentum equation can be integrated to obtain the mean pressure distribution:

$$P = \int_{r_{min}}^{r_{max}} \frac{\overline{\rho}V_\theta^2}{r} dr$$

Defining

$$\begin{aligned}\widetilde{r} &= \frac{r}{r_{max}}\\ dr &= \frac{\partial r}{\partial \widetilde{r}} d\widetilde{r}\\ &= r_{max} d\widetilde{r}\end{aligned}$$

and substituting in gives:

$$P\left(\widetilde{r}\right) = P\left(1\right) - \int_{\widetilde{r}}^{1} \frac{\overline{\rho}V_\theta^2}{\widetilde{r}} d\widetilde{r}$$

For reference, the minimum value of $\widetilde{r}$ is

$$\sigma = \frac{r_{min}}{r_{max}}$$

For isentropic flow,

$$\begin{aligned}S &= \frac{P}{\rho^\gamma}\\ \vec{\nabla}S &= 0\\ \vec{\nabla}P &= A^2\vec{\nabla}\overline{\rho}\end{aligned}$$

where A is the local mean speed of sound:

$$A^2 = \frac{\gamma P}{\overline{\rho}}$$

Then,

$$\begin{aligned}
\frac{\partial A^2}{\partial r} &= \frac{\partial}{\partial r}\left(\frac{\gamma P}{\overline{\rho}}\right) \\
&= \frac{\gamma}{\overline{\rho}}\frac{\partial P}{\partial r} - \frac{\gamma P}{\overline{\rho}^2}\frac{\partial \overline{\rho}}{\partial r} \\
&= \frac{\gamma}{\overline{\rho}}\frac{\partial P}{\partial r} - \frac{A^2}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial r} \\
&= \frac{\gamma}{\overline{\rho}}\frac{\partial P}{\partial r} - \frac{1}{\overline{\rho}}\frac{\partial P}{\partial r} \\
&= \frac{\gamma - 1}{\overline{\rho}}\frac{\partial P}{\partial r}
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{\partial P}{\partial r} &= \frac{\overline{\rho} V_\theta^2}{r} \\
\frac{\overline{\rho}}{\gamma - 1}\frac{\partial A^2}{\partial r} &= \frac{\overline{\rho} V_\theta^2}{r} \\
\frac{\partial A^2}{\partial r} &= (\gamma - 1)\frac{V_\theta^2}{r} \\
\frac{1}{A^2}\frac{\partial A^2}{\partial r} &= (\gamma - 1)\frac{M_\theta^2}{r}
\end{aligned}$$

Integrating, and referencing to the conditions at $r = r_{max}$,

$$\begin{aligned}
\ln\left(A^2(r)\right) &= \ln\left(A^2(r_{max})\right) - \int_r^{r_{max}} (\gamma - 1)\frac{M_\theta^2}{r} dr \\
\ln\left(A^2(r)\right) - \ln\left(A^2(r_{max})\right) &= -\int_r^{r_{max}} (\gamma - 1)\frac{M_\theta^2}{r} dr \\
\ln\left(\frac{A^2(r)}{A^2(r_{max})}\right) &= -\int_r^{r_{max}} (\gamma - 1)\frac{M_\theta^2}{r} dr
\end{aligned}$$

Defining

$$\widetilde{A}(r) = \frac{A(r)}{A(r_{max})}$$

gives:

$$\begin{aligned}
\ln\left(\widetilde{A}^2(r)\right) &= (1-\gamma)\int_r^{r_{max}} \frac{M_\theta^2}{r} dr \\
2\ln\left(\widetilde{A}(r)\right) &= (1-\gamma)\int_r^{r_{max}} \frac{M_\theta^2}{r} dr \\
\ln\left(\widetilde{A}(r)\right) &= \frac{1-\gamma}{2}\int_r^{r_{max}} \frac{M_\theta^2}{r} dr \\
\ln\left(\widetilde{A}(\widetilde{r})\right) &= \frac{1-\gamma}{2}\int_{\widetilde{r}}^{1} \frac{M_\theta^2}{\widetilde{r}} d\widetilde{r}
\end{aligned}$$

This gives the general result:

$$\widetilde{A}(\widetilde{r}) = \exp\left(\frac{1-\gamma}{2}\int_{\widetilde{r}}^{1} \frac{M_\theta^2}{\widetilde{r}} d\widetilde{r}\right)$$

which is Eq. (2.6) in the Kousen paper.

4 Linearized Perturbation Equations

The linearized perturbation equations are:

$$\begin{aligned}
\frac{\partial v_r'}{\partial t} + \frac{V_\theta}{r}\frac{\partial v_r'}{\partial \theta} - \frac{2V_\theta v_\theta'}{r} + V_x\frac{\partial v_r'}{\partial x} &= -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial r} + \frac{\rho'}{\bar{\rho}^2}\frac{\partial P}{\partial r} \\
\frac{\partial v_\theta'}{\partial t} + v_r'\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial v_\theta'}{\partial \theta} + \frac{v_r' V_\theta}{r} + V_x\frac{\partial v_\theta'}{\partial x} &= -\frac{1}{\bar{\rho} r}\frac{\partial p'}{\partial \theta} \\
\frac{\partial v_x'}{\partial t} + v_r'\frac{\partial V_x}{\partial r} + \frac{V_\theta}{r}\frac{\partial v_x'}{\partial \theta} + V_x\frac{\partial v_x'}{\partial x} &= -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} \\
\frac{\partial p'}{\partial t} + v_r'\frac{\partial P}{\partial r} + \frac{V_\theta}{r}\frac{\partial p'}{\partial \theta} + V_x\frac{\partial p'}{\partial x} + \gamma P\left(\frac{1}{r}\frac{\partial (r v_r')}{\partial r} + \frac{1}{r}\frac{\partial v_\theta'}{\partial \theta} + \frac{\partial v_x'}{\partial x}\right) &= 0
\end{aligned}$$

Remembering that:

$$\begin{aligned}
\frac{\partial P}{\partial r} &= \frac{\overline{\rho}V_\theta^2}{r} \\
\gamma P &= \overline{\rho}A^2 \\
\rho' &= \frac{1}{A^2}p'
\end{aligned}$$

and substituting in gives Eqs. (2.33-2.36) in the Kousen report:

$$\begin{aligned}
\frac{\partial v_r'}{\partial t} + \frac{V_\theta}{r}\frac{\partial v_r'}{\partial \theta} + V_x\frac{\partial v_r'}{\partial x} - \frac{2V_\theta}{r}v_\theta' &= -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial r} + \frac{V_\theta^2}{\overline{\rho}rA^2}p' \\
\frac{\partial v_\theta'}{\partial t} + \frac{V_\theta}{r}\frac{\partial v_\theta'}{\partial \theta} + V_x\frac{\partial v_\theta'}{\partial x} + \left(\frac{V_\theta}{r} + \frac{\partial V_\theta}{\partial r}\right)v_r' &= -\frac{1}{\overline{\rho}r}\frac{\partial p'}{\partial \theta} \\
\frac{\partial v_x'}{\partial t} + \frac{V_\theta}{r}\frac{\partial v_x'}{\partial \theta} + V_x\frac{\partial v_x'}{\partial x} + \frac{\partial V_x}{\partial r}v_r' &= -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial x} \\
\frac{1}{\overline{\rho}A^2}\left(\frac{\partial p'}{\partial t} + \frac{V_\theta}{r}\frac{\partial p'}{\partial \theta} + V_x\frac{\partial p'}{\partial x}\right) + \frac{V_\theta^2}{A^2 r}v_r' + \frac{\partial v_r'}{\partial r} + \frac{v_r'}{r} + \frac{1}{r}\frac{\partial v_\theta'}{\partial \theta} + \frac{\partial v_x'}{\partial x} &= 0
\end{aligned}$$

Defining the perturbation variables as:

$$\begin{aligned}
v_r' &= v_r(r)\, e^{i(k_x x + m\theta - \omega t)} \\
v_\theta' &= v_\theta(r)\, e^{i(k_x x + m\theta - \omega t)} \\
v_x' &= v_x(r)\, e^{i(k_x x + m\theta - \omega t)} \\
p' &= p(r)\, e^{i(k_x x + m\theta - \omega t)}
\end{aligned}$$

and substituting into the perturbation equations gives:

$$\begin{aligned}
\left(-i\omega + \frac{imV_\theta}{r} + ik_xV_x\right)v_r - \frac{2V_\theta}{r}v_\theta &= -\frac{1}{\overline{\rho}}\frac{\partial p}{\partial r} + \frac{V_\theta^2}{\overline{\rho}rA^2}p \\
\left(-i\omega + \frac{imV_\theta}{r} + ik_xV_x\right)v_\theta + \left(\frac{V_\theta}{r} + \frac{\partial V_\theta}{\partial r}\right)v_r &= -\frac{im}{\overline{\rho}r}p \\
\left(-i\omega + \frac{imV_\theta}{r} + ik_xV_x\right)v_x + \frac{\partial V_x}{\partial r}v_r &= -\frac{ik_x}{\overline{\rho}}p \\
\frac{1}{\overline{\rho}A^2}\left(-i\omega + \frac{imV_\theta}{r} + ik_xV_x\right)p + \frac{V_\theta^2}{A^2 r}v_r + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{im}{r}v_\theta + ik_xv_x &= 0
\end{aligned}$$

Defining

$$
\begin{aligned}
r_T &= r_{max} \\
A_T &= A\left(r_{max}\right) \\
k &= \frac{\omega r_T}{A_T} \\
\overline{\gamma} &= k_x r_T \\
\widetilde{r} &= \frac{r}{r_T} \\
\frac{\partial}{\partial r} &= \frac{\partial \widetilde{r}}{\partial r}\frac{\partial}{\partial \widetilde{r}} \\
&= \frac{1}{r_T}\frac{\partial}{\partial \widetilde{r}} \\
V_\theta &= M_\theta A \\
V_x &= M_x A \\
\widetilde{A} &= \frac{A}{A_T} \\
v_x &= \widetilde{v}_x A \\
v_r &= \widetilde{v}_r A \\
v_\theta &= \widetilde{v}_\theta A \\
p &= \widetilde{p}\overline{\rho}A^2
\end{aligned}
$$

gives:

$$
\begin{aligned}
-i\left(k\frac{A_T}{r_T} - \frac{mM_\theta A}{\widetilde{r}r_T} - \frac{\overline{\gamma}}{r_T}M_x A\right)\widetilde{v}_r A - \frac{2M_\theta A^2}{\widetilde{r}r_T}\widetilde{v}_\theta = -\frac{1}{\overline{\rho}}\frac{1}{r_T}\frac{\partial\left(\widetilde{p}\overline{\rho}A^2\right)}{\partial\widetilde{r}} + \frac{M_\theta^2 A^2}{\overline{\rho}\widetilde{r}r_T A^2}\widetilde{p}\overline{\rho}A^2 \\
-i\left(k\frac{A_T}{r_T} - \frac{mM_\theta A}{\widetilde{r}r_T} - \frac{\overline{\gamma}}{r_T}M_x A\right)\widetilde{v}_\theta A + \left(\frac{M_\theta A}{\widetilde{r}r_T} + \frac{1}{r_T}\frac{\partial\left(M_\theta A\right)}{\partial\widetilde{r}}\right)\widetilde{v}_r A = -\frac{im\overline{\rho}A^2}{\overline{\rho}\widetilde{r}r_T}\widetilde{p} \\
-i\left(k\frac{A_T}{r_T} - \frac{mM_\theta A}{\widetilde{r}r_T} - \frac{\overline{\gamma}}{r_T}M_x A\right)\widetilde{v}_x A + \frac{1}{r_T}\frac{\partial\left(M_x A\right)}{\partial\widetilde{r}}\widetilde{v}_r A = -\frac{i\overline{\gamma\rho}A^2}{\overline{\rho}r_T}\widetilde{p} \\
\frac{-i}{\overline{\rho}A^2}\left(k\frac{A_T}{r_T} - \frac{mM_\theta A}{\widetilde{r}r_T} - \frac{\overline{\gamma}}{r_T}M_x A\right)\overline{\rho}A^2\widetilde{p} + \frac{M_\theta^2 A^2}{A^2\widetilde{r}r_T}\widetilde{v}_r A + \frac{1}{r_T}\frac{\partial\left(\widetilde{v}_r A\right)}{\partial\widetilde{r}} + \frac{\widetilde{v}_r A}{\widetilde{r}r_T} + \frac{im}{\widetilde{r}r_T}\widetilde{v}_\theta A + \frac{i\overline{\gamma}}{r_T}\widetilde{v}_x A = 0
\end{aligned}
$$

which becomes

$$
-i\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}} - \overline{\gamma}M_x\right)\widetilde{v}_r - \frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta = -\frac{1}{\overline{\rho}A^2}\frac{\partial\left(\widetilde{p}\overline{\rho}A^2\right)}{\partial\widetilde{r}} + \frac{M_\theta^2}{\widetilde{r}}\widetilde{p}
$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_\theta+\left(\frac{M_\theta}{\widetilde{r}}+\frac{1}{A}\frac{\partial\left(M_\theta A\right)}{\partial\widetilde{r}}\right)\widetilde{v}_r=-\frac{im}{\widetilde{r}}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_x+\frac{1}{A}\frac{\partial\left(M_x A\right)}{\partial\widetilde{r}}\widetilde{v}_r=-i\overline{\gamma}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{p}+\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_r+\frac{1}{A}\frac{\partial\left(\widetilde{v}_r A\right)}{\partial\widetilde{r}}+\frac{\widetilde{v}_r}{\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_\theta+i\overline{\gamma}\widetilde{v}_x=0$$

Expanding the mean flow derivatives gives:

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta=-\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{\widetilde{p}}{\overline{\rho}A^2}\frac{\partial\left(\overline{\rho}A^2\right)}{\partial\widetilde{r}}+\frac{M_\theta^2}{\widetilde{r}}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_\theta+\left(\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}+\frac{M_\theta}{A}\frac{\partial A}{\partial\widetilde{r}}\right)\widetilde{v}_r=-\frac{im}{\widetilde{r}}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_x+\frac{\partial M_x}{\partial\widetilde{r}}\widetilde{v}_r+\frac{M_x}{A}\frac{\partial A}{\partial\widetilde{r}}\widetilde{v}_r=-i\overline{\gamma}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{p}+\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_r+\frac{\partial\widetilde{v}_r}{\partial\widetilde{r}}+\frac{1}{A}\frac{\partial A}{\partial\widetilde{r}}\widetilde{v}_r+\frac{\widetilde{v}_r}{\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_\theta+i\overline{\gamma}\widetilde{v}_x=0$$

The mean flow derivatives are:

$$\begin{aligned}
\frac{1}{A}\frac{\partial A}{\partial\widetilde{r}} &= \frac{r_T}{A^2}\left(A\frac{\partial A}{\partial r}\right) \\
&= \frac{r_T}{2A^2}\frac{\partial A^2}{\partial r} \\
&= \frac{r_T}{A^2}\left(\frac{\gamma-1}{2}\right)\frac{V_\theta^2}{r} \\
&= \left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{\overline{\rho}A^2}\frac{\partial\left(\overline{\rho}A^2\right)}{\partial\widetilde{r}} &= \frac{\gamma}{\overline{\rho}A^2}\frac{\partial P}{\partial\widetilde{r}} \\
&= \frac{r_T\gamma}{\overline{\rho}A^2}\gamma\frac{\partial P}{\partial r} \\
&= \frac{r_T\gamma}{\overline{\rho}A^2}\frac{\overline{\rho}V_\theta^2}{r}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\gamma}{\overline{\rho} A^2} \frac{\overline{\rho} A^2 M_\theta^2}{\widetilde{r}} \\
&= \frac{\gamma M_\theta^2}{\widetilde{r}}
\end{aligned}
$$

Substituting in,

$$
\begin{aligned}
-i\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}} - \overline{\gamma} M_x\right)\widetilde{v}_r - \frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta &= -\frac{\partial \widetilde{p}}{\partial \widetilde{r}} - \frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} \\
-i\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}} - \overline{\gamma} M_x\right)\widetilde{v}_\theta + \left(\frac{M_\theta}{\widetilde{r}} + \frac{\partial M_\theta}{\partial \widetilde{r}} + \left(\frac{\gamma-1}{2}\right)\frac{M_\theta^3}{\widetilde{r}}\right)\widetilde{v}_r &= -\frac{im}{\widetilde{r}}\widetilde{p} \\
-i\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}} - \overline{\gamma} M_x\right)\widetilde{v}_x + \left(\frac{\partial M_x}{\partial \widetilde{r}} + \left(\frac{\gamma-1}{2}\right)\frac{M_x M_\theta^2}{\widetilde{r}}\right)\widetilde{v}_r &= -i\overline{\gamma}\widetilde{p} \\
-i\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}} - \overline{\gamma} M_x\right)\widetilde{p} + \frac{\partial \widetilde{v}_r}{\partial \widetilde{r}} + \left(\left(\frac{\gamma+1}{2}\right)\frac{M_\theta^2}{\widetilde{r}} + \frac{1}{\widetilde{r}}\right)\widetilde{v}_r + \frac{im}{\widetilde{r}}\widetilde{v}_\theta + i\overline{\gamma}\widetilde{v}_x &= 0
\end{aligned}
$$

which are Eq. (2.38-2.41) in Kousen's report.

4.1 Second Order Pressure Equations

In the case of axial shear flow, $M_\theta = 0$ and $\widetilde{A} = 1$. This gives:

$$
\begin{aligned}
-i\left(k - \overline{\gamma} M_x\right)\widetilde{v}_r &= -\frac{\partial \widetilde{p}}{\partial \widetilde{r}} \\
-i\left(k - \overline{\gamma} M_x\right)\widetilde{v}_\theta &= -\frac{im}{\widetilde{r}}\widetilde{p} \\
-i\left(k - \overline{\gamma} M_x\right)\widetilde{v}_x + \left(\frac{\partial M_x}{\partial \widetilde{r}}\right)\widetilde{v}_r &= -i\overline{\gamma}\widetilde{p} \\
-i\left(k - \overline{\gamma} M_x\right)\widetilde{p} + \frac{\partial \widetilde{v}_r}{\partial \widetilde{r}} + \left(\frac{1}{\widetilde{r}}\right)\widetilde{v}_r + \frac{im}{\widetilde{r}}\widetilde{v}_\theta + i\overline{\gamma}\widetilde{v}_x &= 0
\end{aligned}
$$

Rearranging the perturbation velocity equations gives:

$$
\begin{aligned}
\widetilde{v}_r &= \frac{-i}{\left(k - \overline{\gamma} M_x\right)}\frac{\partial \widetilde{p}}{\partial \widetilde{r}} \\
\widetilde{v}_\theta &= \frac{m}{\left(k - \overline{\gamma} M_x\right)\widetilde{r}}\widetilde{p} \\
\widetilde{v}_x &= \left(\frac{\overline{\gamma}}{\left(k - \overline{\gamma} M_x\right)}\right)\widetilde{p} + \frac{-\left(\frac{\partial M_x}{\partial \widetilde{r}}\right)}{\left(k - \overline{\gamma} M_x\right)^2}\frac{\partial \widetilde{p}}{\partial \widetilde{r}}
\end{aligned}
$$

Substituting into the pressure equation gives:

$$\begin{pmatrix} \begin{array}{r} -i\left(k-\overline{\gamma}M_x\right)\widetilde{p} \\ +\frac{\partial}{\partial\widetilde{r}}\left(\frac{-i}{(k-\overline{\gamma}M_x)}\frac{\partial\widetilde{p}}{\partial\widetilde{r}}\right) \\ +\left(\frac{1}{\widetilde{r}}\right)\left(\frac{-i}{(k-\overline{\gamma}M_x)}\frac{\partial\widetilde{p}}{\partial\widetilde{r}}\right) \\ +\frac{im}{\widetilde{r}}\frac{m}{(k-\overline{\gamma}M_x)\widetilde{r}}\widetilde{p} \\ +i\overline{\gamma}\left(\left(\frac{\overline{\gamma}}{(k-\overline{\gamma}M_x)}\right)\widetilde{p}+\frac{-\left(\frac{\partial M_x}{\partial\widetilde{r}}\right)}{(k-\overline{\gamma}M_x)^2}\frac{\partial\widetilde{p}}{\partial\widetilde{r}}\right) \end{array} \end{pmatrix} = 0$$

which becomes:

$$\begin{pmatrix} \begin{array}{r} \frac{-i}{(k-\overline{\gamma}M_x)}\frac{\partial^2\widetilde{p}}{\partial\widetilde{r}^2} \\ +\left(\frac{-2i\overline{\gamma}\left(\frac{\partial M_x}{\partial\widetilde{r}}\right)}{(k-\overline{\gamma}M_x)^2}+\frac{-i}{(k-\overline{\gamma}M_x)\widetilde{r}}\right)\frac{\partial\widetilde{p}}{\partial\widetilde{r}} \\ +\left(\frac{im^2}{\widetilde{r}^2}\frac{1}{(k-\overline{\gamma}M_x)}+\frac{i\overline{\gamma}^2}{(k-\overline{\gamma}M_x)}-i\left(k-\overline{\gamma}M_x\right)\right)\widetilde{p} \end{array} \end{pmatrix} = 0$$

and simplifies to:

$$\begin{pmatrix} \begin{array}{r} \frac{\partial^2\widetilde{p}}{\partial\widetilde{r}^2} \\ +\left(\frac{2\overline{\gamma}\left(\frac{\partial M_x}{\partial\widetilde{r}}\right)}{(k-\overline{\gamma}M_x)}+\frac{1}{\widetilde{r}}\right)\frac{\partial\widetilde{p}}{\partial\widetilde{r}} \\ +\left(\frac{-m^2}{\widetilde{r}^2}-\overline{\gamma}^2+\left(k-\overline{\gamma}M_x\right)^2\right)\widetilde{p} \end{array} \end{pmatrix} = 0$$

Gathering and rearranging gives:

$$\frac{\partial^2\widetilde{p}}{\partial\widetilde{r}^2}+\left[\frac{1}{\widetilde{r}}+\frac{2\overline{\gamma}\left(\frac{\partial M_x}{\partial\widetilde{r}}\right)}{(k-\overline{\gamma}M_x)}\right]\frac{\partial\widetilde{p}}{\partial\widetilde{r}}+\left[(k-\overline{\gamma}M_x)^2-\frac{m^2}{\widetilde{r}^2}-\overline{\gamma}^2\right]\widetilde{p}=0$$

and there is a sign error in Kousen's Eq. (2.27).

4.2 Axial Shear and Swirl

In the general case,

$$
\begin{aligned}
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta &= -\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} \\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_\theta+\left(\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^3}{\widetilde{r}}\right)\widetilde{v}_r &= -\frac{im}{\widetilde{r}}\widetilde{p} \\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_x+\left(\frac{\partial M_x}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_xM_\theta^2}{\widetilde{r}}\right)\widetilde{v}_r &= -i\overline{\gamma}\widetilde{p} \\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{p}+\frac{\partial\widetilde{v}_r}{\partial\widetilde{r}}+\left(\left(\frac{\gamma+1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}+\frac{1}{\widetilde{r}}\right)\widetilde{v}_r+\frac{im}{\widetilde{r}}\widetilde{v}_\theta+i\overline{\gamma}\widetilde{v}_x &= 0
\end{aligned}
$$

4.3 Continuing on

Defining:

$$\lambda = -i\overline{\gamma}$$

and defining:

$$\{\vec{x}\} = \left\{\begin{array}{c}\widetilde{v}_r\\ \widetilde{v}_\theta\\ \widetilde{v}_x\\ \widetilde{p}\end{array}\right\}$$

and writing the equation in matrix form gives:

$$
\begin{bmatrix}
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)-\lambda M_x & -\frac{2M_\theta}{\widetilde{r}} & 0 & \frac{\partial}{\partial\widetilde{r}}+\frac{(\gamma-1)M_\theta^2}{\widetilde{r}} \\
\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^3}{\widetilde{r}} & -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)-\lambda M_x & 0 & \frac{im}{\widetilde{r}} \\
\frac{\partial M_x}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_xM_\theta^2}{\widetilde{r}} & 0 & -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)-\lambda M_x & -\lambda \\
\frac{\partial}{\partial\widetilde{r}}+\left(\frac{\gamma+1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}+\frac{1}{\widetilde{r}} & \frac{im}{\widetilde{r}} & -\lambda & -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)-\lambda M_x
\end{bmatrix}
\{\vec{x}\} = 0
$$

Following the Kousen report, this equation can be rewritten as:

$$[A]\{\vec{x}\} = \lambda\,[B]\{\vec{x}\}$$

where

$$[A]=\left[\begin{array}{cccc}
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right) & -\frac{2M_\theta}{\widetilde{r}} & 0 & \frac{\partial}{\partial\widetilde{r}}+\frac{(\gamma-1)M_\theta^2}{\widetilde{r}} \\
\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^3}{\widetilde{r}} & -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right) & 0 & \frac{im}{\widetilde{r}} \\
\frac{\partial M_x}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_x M_\theta^2}{\widetilde{r}} & 0 & -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right) & 0 \\
\frac{\partial}{\partial\widetilde{r}}+\left(\frac{\gamma+1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}+\frac{1}{\widetilde{r}} & \frac{im}{\widetilde{r}} & 0 & -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)
\end{array}\right]$$

and

$$[B]=\left[\begin{array}{cccc}
M_x & 0 & 0 & 0 \\
0 & M_x & 0 & 0 \\
0 & 0 & M_x & 1 \\
0 & 0 & 1 & M_x
\end{array}\right]$$

From the SWIRL code: at $\widetilde{r}=0$,

$$[A]=\left[\begin{array}{cccc}
-i\left(\frac{k}{\widetilde{A}}-m\frac{\partial M_\theta}{\partial\widetilde{r}}\right) & -2\frac{\partial M_\theta}{\partial\widetilde{r}} & 0 & \frac{\partial}{\partial\widetilde{r}}+2\left(\gamma-1\right)M_\theta\frac{\partial M_\theta}{\partial\widetilde{r}} \\
\frac{\partial M_\theta}{\partial\widetilde{r}}+\left(1+3\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\frac{\partial M_\theta}{\partial\widetilde{r}} & -i\left(\frac{k}{\widetilde{A}}-m\frac{\partial M_\theta}{\partial\widetilde{r}}\right) & 0 & 0 \\
\frac{\partial M_x}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)M_\theta\left(M_\theta\frac{\partial M_x}{\partial\widetilde{r}}+2M_x\frac{\partial M_\theta}{\partial\widetilde{r}}\right) & 0 & -i\left(\frac{k}{\widetilde{A}}-m\frac{\partial M_\theta}{\partial\widetilde{r}}\right) & 0 \\
\frac{\partial}{\partial\widetilde{r}}+\left(\gamma+1\right)M_\theta\frac{\partial M_\theta}{\partial\widetilde{r}} & 0 & 0 & -i\left(\frac{k}{\widetilde{A}}-m\frac{\partial M_\theta}{\partial\widetilde{r}}\right)
\end{array}\right]$$

5 Divergence and Vorticity?

The divergence and vorticity of the perturbation velocity in cylindrical coordinates is:

$$\begin{aligned}
\vec{\nabla}\cdot\vec{v'} &= \frac{1}{r}\frac{\partial}{\partial r}\left(rv'_r\right)+\frac{1}{r}\frac{\partial v'_\theta}{\partial\theta}+\frac{\partial v'_x}{\partial x} \\
&= \frac{1}{r_T}\left(\frac{1}{\widetilde{r}}\frac{\partial}{\partial\widetilde{r}}\left(\widetilde{r}v'_r\right)+\frac{1}{\widetilde{r}}\frac{\partial v'_\theta}{\partial\theta}+\frac{\partial v'_x}{\partial\widetilde{x}}\right) \\
\vec{\nabla}\times\vec{v'} &= \left(\begin{array}{r}
\left(\frac{1}{r}\frac{\partial v'_x}{\partial\theta}-\frac{\partial v'_\theta}{\partial x}\right)\vec{e}_r \\
+\left(\frac{\partial v'_r}{\partial x}-\frac{\partial v'_x}{\partial r}\right)\vec{e}_\theta \\
+\frac{1}{r}\left(\frac{\partial}{\partial r}\left(rv'_\theta\right)-\frac{\partial v'_r}{\partial\theta}\right)\vec{e}_x
\end{array}\right)
\end{aligned}$$

$$= \frac{1}{r_T} \left(\begin{array}{r} \left(\frac{1}{\widetilde{r}} \frac{\partial v_x'}{\partial \theta} - \frac{\partial v_\theta'}{\partial \widetilde{x}} \right) \vec{e}_r \\ + \left(\frac{\partial v_r'}{\partial \widetilde{x}} - \frac{\partial v_x'}{\partial \widetilde{r}} \right) \vec{e}_\theta \\ + \frac{1}{\widetilde{r}} \left(\frac{\partial}{\partial \widetilde{r}} \left(\widetilde{r} v_\theta' \right) - \frac{\partial v_r'}{\partial \theta} \right) \vec{e}_x \end{array} \right)$$

Remembering the definitions:

$$\begin{aligned} v_r' &= v_r(r)\, e^{i(k_x x + m\theta - \omega t)} \\ v_\theta' &= v_\theta(r)\, e^{i(k_x x + m\theta - \omega t)} \\ v_x' &= v_x(r)\, e^{i(k_x x + m\theta - \omega t)} \\ p' &= p(r)\, e^{i(k_x x + m\theta - \omega t)} \end{aligned}$$

with the nondimensional counterparts:

$$\begin{aligned} v_r' &= A\widetilde{v}_r(\widetilde{r})\, e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \\ v_\theta' &= A\widetilde{v}_\theta(\widetilde{r})\, e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \\ v_x' &= A\widetilde{v}_x(\widetilde{r})\, e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \\ p' &= \overline{\rho} A^2 \widetilde{p}(\widetilde{r})\, e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \end{aligned}$$

where:

$$\overline{\gamma} = k_x r_T$$

The axial derivatives are:

$$\begin{aligned} \frac{\partial v_r'}{\partial x} &= \frac{1}{r_T} \frac{\partial v_r'}{\partial \widetilde{x}} \\ &= i\overline{\gamma}\widetilde{v}_r \left(\frac{A}{r_T} e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \right) \\ \frac{\partial v_\theta'}{\partial x} &= i\overline{\gamma}\widetilde{v}_\theta \left(\frac{A}{r_T} e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \right) \\ \frac{\partial v_x'}{\partial x} &= i\overline{\gamma}\widetilde{v}_x \left(\frac{A}{r_T} e^{i\left(\overline{\gamma}\widetilde{x} + m\theta - \omega t\right)} \right) \end{aligned}$$

The azimuthal derivatives are:

$$\begin{aligned}
\frac{1}{r}\frac{\partial v_r'}{\partial \theta} &= \frac{im\widetilde{v}_r}{\widetilde{r}}\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)\\
\frac{1}{r}\frac{\partial v_\theta'}{\partial \theta} &= \frac{im\widetilde{v}_\theta}{\widetilde{r}}\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)\\
\frac{1}{r}\frac{\partial v_x'}{\partial \theta} &= \frac{im\widetilde{v}_x}{\widetilde{r}}\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)
\end{aligned}$$

To do the radial derivatives, note that:

$$\frac{\partial A}{\partial r} = \frac{A}{r_T}\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}$$

The radial derivatives (remembering that $A = A\,(r)!$) are:

$$\begin{aligned}
\frac{\partial v_r'}{\partial r} &= \left(\widetilde{v_r'}\frac{\partial A}{\partial r} + \frac{A}{r_T}\frac{\partial v_r'}{\partial \widetilde{r}}\right)e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\\
&= \left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v_r} + \frac{\partial \widetilde{v}_r}{\partial \widetilde{r}}\right)\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)\\
\frac{\partial v_\theta'}{\partial r} &= \left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_\theta + \frac{\partial \widetilde{v}_\theta}{\partial \widetilde{r}}\right)\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)\\
\frac{\partial v_x'}{\partial r} &= \left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_x + \frac{\partial \widetilde{v}_x}{\partial \widetilde{r}}\right)\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)
\end{aligned}$$

gives:

$$\begin{aligned}
\vec{\nabla}\cdot\vec{v'} &= \left(\frac{\widetilde{v}_r}{\widetilde{r}} + \left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_r}{\widetilde{r}} + \frac{\partial \widetilde{v}_r}{\partial \widetilde{r}} + \frac{im}{\widetilde{r}}\widetilde{v}_\theta + i\overline{\gamma}\widetilde{v}_x\right)\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)\\
\vec{\nabla}\times\vec{v'} &= \begin{pmatrix} \left(\frac{im\widetilde{v}_x}{\widetilde{r}} - i\overline{\gamma}\widetilde{v}_\theta\right)\vec{e}_r \\ +\left(i\overline{\gamma}\widetilde{v}_r - \left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_x + \frac{\partial \widetilde{v}_x}{\partial \widetilde{r}}\right)\right)\vec{e}_\theta \\ +\left(\frac{\widetilde{v}_\theta}{\widetilde{r}} + \left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_\theta + \frac{\partial \widetilde{v}_\theta}{\partial \widetilde{r}}\right) - \frac{im\widetilde{v}_r}{\widetilde{r}}\right)\vec{e}_x \end{pmatrix}\left(\frac{A}{r_T}e^{i\left(\overline{\gamma}\widetilde{x}+m\theta-\omega t\right)}\right)
\end{aligned}$$

If all three vorticity components are zero, the perturbations will be irrotational. From the first two velocity components, this gives:

$$
\begin{aligned}
\widetilde{v}_\theta &= \frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_x \\
\widetilde{v}_r &= \frac{-i}{\overline{\gamma}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_x + \frac{\partial\widetilde{v}_x}{\partial\widetilde{r}}\right)
\end{aligned}
$$

As a test, the last vorticity component is:

$$
\begin{aligned}
\frac{\widetilde{v}_\theta}{\widetilde{r}} + \left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_\theta + \frac{\partial\widetilde{v}_\theta}{\partial\widetilde{r}} - \frac{im}{\widetilde{r}}\widetilde{v}_r &= \left(\begin{array}{r}
\frac{m}{\overline{\gamma}\widetilde{r}}\left(\frac{1}{\widetilde{r}} + \left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\right)\widetilde{v}_x \\
+\frac{\partial}{\partial\widetilde{r}}\left(\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_x\right) \\
-\frac{m}{\overline{\gamma}\widetilde{r}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_x + \frac{\partial\widetilde{v}_x}{\partial\widetilde{r}}\right)
\end{array}\right) \\
&= \left(\begin{array}{r}
\frac{m}{\overline{\gamma}\widetilde{r}}\left(\frac{1}{\widetilde{r}} + \left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\right)\widetilde{v}_x \\
-\frac{m}{\overline{\gamma}\widetilde{r}^2}\widetilde{v}_x \\
+\frac{m}{\overline{\gamma}\widetilde{r}}\frac{\partial\widetilde{v}_x}{\partial\widetilde{r}} \\
-\frac{m}{\overline{\gamma}\widetilde{r}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_x + \frac{\partial\widetilde{v}_x}{\partial\widetilde{r}}\right)
\end{array}\right) \\
&= 0
\end{aligned}
$$

showing that these are the irrotational relations between the perturbation velocities.

5.1 Velocity decomposition

Let's decompose the perturbation velocity field into irrotational and divergence-free components:

$$
\begin{aligned}
\widetilde{v}_x &= \widetilde{v}_{x,\omega} + \widetilde{v}_{x,\phi} \\
\widetilde{v}_r &= \widetilde{v}_{r,\omega} + \widetilde{v}_{r,\phi} \\
\widetilde{v}_\theta &= \widetilde{v}_{\theta,\omega} + \widetilde{v}_{\theta,\phi}
\end{aligned}
$$

with the relations:

$$
\begin{aligned}
\widetilde{v}_{\theta,\phi} &= \frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{x,\phi} \\
\widetilde{v}_{r,\phi} &= \frac{-i}{\overline{\gamma}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_{x,\phi} + \frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right)
\end{aligned}
$$

The velocity divergence gives one relation for the rotational components of the perturbation velocities:

$$
\begin{aligned}
\widetilde{v}_{x,\omega} &= \frac{i}{\overline{\gamma}}\left(\frac{\widetilde{v}_{r,\omega}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\omega}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}\right)-\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{\theta,\omega} \\
&= i\left(\frac{1+\Gamma}{\overline{\gamma}\widetilde{r}}\right)\widetilde{v}_{r,\omega}+\frac{i}{\overline{\gamma}}\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}-\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{\theta,\omega}
\end{aligned}
$$

As a check, the velocity perturbations are put into the velocity divergence equation:

$$
\begin{aligned}
\vec{\nabla}\cdot\vec{v}' &= \left(\frac{\widetilde{v}_r}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_r}{\widetilde{r}}+\frac{\partial\widetilde{v}_r}{\partial\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_\theta+i\overline{\gamma}\widetilde{v}_x\right) \\
&= \left(\begin{array}{r}
\frac{\widetilde{v}_{r,\phi}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\phi}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\phi}}{\partial\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_{\theta,\phi}+i\overline{\gamma}\widetilde{v}_{x,\phi} \\
+\frac{\widetilde{v}_{r,\omega}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\omega}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_{\theta,\omega}+i\overline{\gamma}\widetilde{v}_{x,\omega}
\end{array}\right) \\
&= \left(\begin{array}{r}
\frac{1}{\widetilde{r}}\left(1+\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\frac{-i}{\overline{\gamma}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right) \\
+\frac{\partial}{\partial\widetilde{r}}\left(\frac{-i}{\overline{\gamma}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right)\right) \\
+\frac{im^2}{\overline{\gamma}\widetilde{r}^2}\widetilde{v}_{x,\phi} \\
+i\overline{\gamma}\widetilde{v}_{x,\phi} \\
+\frac{\widetilde{v}_{r,\omega}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\omega}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_{\theta,\omega} \\
+i\overline{\gamma}\left(\frac{i}{\overline{\gamma}}\left(\frac{\widetilde{v}_{r,\omega}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\omega}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}\right)-\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{\theta,\omega}\right)
\end{array}\right) \\
&= \left(\begin{array}{r}
\frac{-i}{\overline{\gamma}\widetilde{r}^2}\left(1+\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\left(\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\widetilde{v}_{x,\phi} \\
-\frac{i}{\overline{\gamma}\widetilde{r}}\left(1+\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\left(\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right) \\
-\frac{i}{\overline{\gamma}}\frac{\partial}{\partial\widetilde{r}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_{x,\phi}\right) \\
-\frac{i}{\overline{\gamma}}\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2} \\
+i\overline{\gamma}\left(1+\frac{m^2}{\overline{\gamma}^2\widetilde{r}^2}\right)\widetilde{v}_{x,\phi} \\
+\frac{\widetilde{v}_{r,\omega}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\omega}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_{\theta,\omega} \\
-\left(\frac{\widetilde{v}_{r,\omega}}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_{r,\omega}}{\widetilde{r}}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}\right)-\frac{im}{\widetilde{r}}\widetilde{v}_{\theta,\omega}
\end{array}\right)
\end{aligned}
$$

$$
= \begin{pmatrix} i\overline{\gamma}\left(\left(1+\frac{m^2}{\overline{\gamma}^2\widetilde{r}^2}\right)-\frac{1}{\overline{\gamma}^2\widetilde{r}^2}\left(1+\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\left(\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\right)\widetilde{v}_{x,\phi} \\ -\frac{i}{\overline{\gamma}\widetilde{r}}\left(1+\left(\frac{\gamma-1}{2}\right)M_\theta^2\right)\left(\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right) \\ +\frac{i}{\overline{\gamma}}\left(\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2}{\widetilde{r}^2}\widetilde{v}_{x,\phi}\right) \\ -\frac{i}{\overline{\gamma}}\frac{\partial M_\theta}{\partial\widetilde{r}}\left(\left(\frac{\gamma-1}{2}\right)\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_{x,\phi}\right) \\ -\frac{i}{\overline{\gamma}\widetilde{r}}\left(\left(\frac{\gamma-1}{2}\right)M_\theta^2\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right) \\ -\frac{i}{\overline{\gamma}}\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2} \end{pmatrix}
$$

$$
= \begin{pmatrix} i\left(\overline{\gamma}+\frac{1}{\overline{\gamma}\widetilde{r}^2}\left(m^2-\Gamma^2-\widetilde{r}\frac{\partial\Gamma}{\partial\widetilde{r}}\right)\right)\widetilde{v}_{x,\phi} \\ -\frac{i}{\overline{\gamma}\widetilde{r}}\left(1+2\Gamma\right)\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}} \\ -\frac{i}{\overline{\gamma}}\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2} \end{pmatrix}
$$

where

$$
\Gamma = \left(\frac{\gamma-1}{2}\right)M_\theta^2
$$

Note that the divergence of velocity can be written solely in terms of $\widetilde{v}_{x,\phi}$:

$$
\left(\frac{\widetilde{v}_r}{\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^2\widetilde{v}_r}{\widetilde{r}}+\frac{\partial\widetilde{v}_r}{\partial\widetilde{r}}+\frac{im}{\widetilde{r}}\widetilde{v}_\theta+i\overline{\gamma}\widetilde{v}_x\right) = \Phi_0\widetilde{v}_{x,\phi}+\Phi_1\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}+\Phi_2\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2}
$$

where

$$
\begin{aligned}
\Phi_0 &= i\left(\overline{\gamma}+\frac{1}{\overline{\gamma}\widetilde{r}^2}\left(m^2-\Gamma^2-\widetilde{r}\frac{\partial\Gamma}{\partial\widetilde{r}}\right)\right) \\
\Phi_1 &= -\frac{i}{\overline{\gamma}\widetilde{r}}\left(1+2\Gamma\right) \\
\Phi_2 &= -\frac{i}{\overline{\gamma}}
\end{aligned}
$$

5.2 Back to business...

The SWIRL code equations are:

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta=-\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_\theta+\left(\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^3}{\widetilde{r}}\right)\widetilde{v}_r=-\frac{im}{\widetilde{r}}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_x+\left(\frac{\partial M_x}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_xM_\theta^2}{\widetilde{r}}\right)\widetilde{v}_r=-i\overline{\gamma}\widetilde{p}$$

$$-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{p}+\frac{\partial\widetilde{v}_r}{\partial\widetilde{r}}+\left(\left(\frac{\gamma+1}{2}\right)\frac{M_\theta^2}{\widetilde{r}}+\frac{1}{\widetilde{r}}\right)\widetilde{v}_r+\frac{im}{\widetilde{r}}\widetilde{v}_\theta+i\overline{\gamma}\widetilde{v}_x=0$$

Defining:

$$\alpha = \frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x$$

and expanding the pressure equation to isolate the divergence of velocity term gives:

$$\begin{aligned}
-i\alpha\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta &= -\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p}\\
-i\alpha\widetilde{v}_\theta+\left(\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_\theta^3}{\widetilde{r}}\right)\widetilde{v}_r &= -\frac{im}{\widetilde{r}}\widetilde{p}\\
-i\alpha\widetilde{v}_x+\left(\frac{\partial M_x}{\partial\widetilde{r}}+\left(\frac{\gamma-1}{2}\right)\frac{M_xM_\theta^2}{\widetilde{r}}\right)\widetilde{v}_r &= -i\overline{\gamma}\widetilde{p}\\
-i\alpha\widetilde{p}+\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_r+\left(\Phi_0\widetilde{v}_{x,\phi}+\Phi_1\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}+\Phi_2\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2}\right) &= 0
\end{aligned}$$

Rewriting this in terms of the rotational and irrotational velocities,

$$\begin{aligned}
-i\alpha\left(\widetilde{v}_{r,\phi}+\widetilde{v}_{r,\omega}\right)-\frac{2M_\theta}{\widetilde{r}}\left(\widetilde{v}_{\theta,\phi}+\widetilde{v}_{\theta,\omega}\right) &= -\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{2\Gamma}{\widetilde{r}}\widetilde{p}\\
-i\alpha\left(\widetilde{v}_{\theta,\phi}+\widetilde{v}_{\theta,\omega}\right)+\left((1+\Gamma)\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}\right)\left(\widetilde{v}_{r,\phi}+\widetilde{v}_{r,\omega}\right) &= -\frac{im}{\widetilde{r}}\widetilde{p}\\
-i\alpha\left(\widetilde{v}_{x,\phi}+\widetilde{v}_{x,\omega}\right)+\left(\frac{\partial M_x}{\partial\widetilde{r}}+\Gamma\frac{M_x}{\widetilde{r}}\right)\left(\widetilde{v}_{r,\phi}+\widetilde{v}_{r,\omega}\right) &= -i\overline{\gamma}\widetilde{p}\\
-i\alpha\widetilde{p}+\frac{M_\theta^2}{\widetilde{r}}\left(\widetilde{v}_{r,\phi}+\widetilde{v}_{r,\omega}\right)+\left(\Phi_0\widetilde{v}_{x,\phi}+\Phi_1\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}+\Phi_2\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2}\right) &= 0
\end{aligned}$$

Substituting in:

$$
\begin{aligned}
\left(\begin{array}{r} -i\alpha\left(\frac{-i}{\overline{\gamma}}\left(\frac{\Gamma}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right)+\widetilde{v}_{r,\omega}\right) \\ -\frac{2M_\theta}{\widetilde{r}}\left(\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{x,\phi}+\widetilde{v}_{\theta,\omega}\right)\end{array}\right) &= -\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{2\Gamma}{\widetilde{r}}\widetilde{p} \\
\left(\begin{array}{r} -i\alpha\left(\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{x,\phi}+\widetilde{v}_{\theta,\omega}\right) \\ +\left((1+\Gamma)\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}}\right)\left(\frac{-i}{\overline{\gamma}}\left(\frac{\Gamma}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right)+\widetilde{v}_{r,\omega}\right)\end{array}\right) &= -\frac{im}{\widetilde{r}}\widetilde{p} \\
\left(\begin{array}{r} -i\alpha\left(\widetilde{v}_{x,\phi}+i\left(\frac{1+\Gamma}{\overline{\gamma}\widetilde{r}}\right)\widetilde{v}_{r,\omega}+\frac{i}{\overline{\gamma}}\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}}-\frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{\theta,\omega}\right) \\ +\left(\frac{\partial M_x}{\partial\widetilde{r}}+\Gamma\frac{M_x}{\widetilde{r}}\right)\left(\frac{-i}{\overline{\gamma}}\left(\frac{\Gamma}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right)+\widetilde{v}_{r,\omega}\right)\end{array}\right) &= -i\overline{\gamma}\widetilde{p} \\
\left(\begin{array}{r} -i\alpha\widetilde{p} \\ +\frac{M_\theta^2}{\widetilde{r}}\left(\frac{-i}{\overline{\gamma}}\left(\frac{\Gamma}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}\right)+\widetilde{v}_{r,\omega}\right) \\ +\left(\Phi_0\widetilde{v}_{x,\phi}+\Phi_1\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}}+\Phi_2\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2}\right)\end{array}\right) &= 0
\end{aligned}
$$

Defining:

$$
\begin{aligned}
\Theta &= (1+\Gamma)\frac{M_\theta}{\widetilde{r}}+\frac{\partial M_\theta}{\partial\widetilde{r}} \\
\tau_\phi &= \frac{\Gamma}{\widetilde{r}}\widetilde{v}_{x,\phi}+\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}} \\
\tau_\omega &= \frac{\Gamma}{\widetilde{r}}\widetilde{v}_{r,\omega}+\frac{\partial\widetilde{v}_{r,\omega}}{\partial\widetilde{r}} \\
S &= \Gamma\frac{M_x}{\widetilde{r}}+\frac{\partial M_x}{\partial\widetilde{r}}
\end{aligned}
$$

and gathering the rotational and irrotational components together:

$$
\begin{aligned}
\left(\begin{array}{r} \frac{\alpha}{\overline{\gamma}}\tau_\phi-\frac{2mM_\theta}{\overline{\gamma}\widetilde{r}^2}\widetilde{v}_{x,\phi} \\ -i\alpha\widetilde{v}_{r,\omega}-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_{\theta,\omega}\end{array}\right) &= -\frac{\partial\widetilde{p}}{\partial\widetilde{r}}-\frac{2\Gamma}{\widetilde{r}}\widetilde{p} \\
\left(\begin{array}{r} -\frac{i\Theta}{\overline{\gamma}}\tau_\phi-\frac{im\alpha}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{x,\phi} \\ +\Theta\widetilde{v}_{r,\omega}-i\alpha\widetilde{v}_{\theta,\omega}\end{array}\right) &= -\frac{im}{\widetilde{r}}\widetilde{p} \\
\left(\begin{array}{r} -i\alpha\widetilde{v}_{x,\phi}+\frac{\alpha}{\overline{\gamma}}\tau_\omega+\frac{\alpha}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{r,\omega}+i\frac{m\alpha}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{\theta,\omega} \\ -i\frac{S}{\overline{\gamma}}\tau_\phi+S\widetilde{v}_{r,\omega}\end{array}\right) &= -i\overline{\gamma}\widetilde{p}
\end{aligned}
$$

$$\left(\begin{array}{r} -i\alpha\widetilde{p} \\ -i\frac{M_\theta^2}{\overline{\gamma}\widetilde{r}}\tau_\phi \\ +\Phi_0\widetilde{v}_{x,\phi} + \Phi_1\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}} + \Phi_2\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2} \\ -i\frac{M_\theta^2}{\widetilde{r}}\widetilde{v}_{r,\omega} \end{array} \right) = 0$$

I can't help but notice these groupings:

$$\begin{aligned} A &= \frac{1}{\overline{\gamma}}\tau_\phi - i\widetilde{v}_{r,\omega} \\ B &= \frac{m}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{x,\phi} + \widetilde{v}_{\theta,\omega} \end{aligned}$$

which gives:

$$\begin{aligned} \alpha A - \frac{2M_\theta}{\widetilde{r}}B &= -\frac{\partial\widetilde{p}}{\partial\widetilde{r}} - \frac{2\Gamma}{\widetilde{r}}\widetilde{p} \\ i\Theta A - i\alpha B &= -\frac{im}{\widetilde{r}}\widetilde{p} \\ \left(\begin{array}{r} -i\alpha\widetilde{v}_{x,\phi} + \frac{\alpha}{\overline{\gamma}}\tau_\omega + \frac{\alpha}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{r,\omega} + i\frac{m\alpha}{\overline{\gamma}\widetilde{r}}\widetilde{v}_{\theta,\omega} \\ -iSA \end{array} \right) &= -i\overline{\gamma}\widetilde{p} \\ \left(\begin{array}{r} -i\alpha\widetilde{p} \\ -i\frac{M_\theta^2}{\widetilde{r}}A \\ +\Phi_0\widetilde{v}_{x,\phi} + \Phi_1\frac{\partial\widetilde{v}_{x,\phi}}{\partial\widetilde{r}} + \Phi_2\frac{\partial^2\widetilde{v}_{x,\phi}}{\partial\widetilde{r}^2} \end{array} \right) &= 0 \end{aligned}$$

I can't help but notice these groupings:

6 The SWIRL Code

In this section, the Fortran2008 version of the SWIRL code is discussed.

The main routine is 'swirl.f90'. The routines called (when Chebyshev polynomials are used) are:

1. input
2. grid
3. derivs

4. smachAndSndspd
5. rmach
6. machout
7. global
8. boundary
9. analysis
10. output

6.1 input

The subroutine 'input' is used to obtain the input data for the code. Input reads a NAMELIST input from the file 'input.data'. The NAMELIST entries are:

1. MM: circumferential mode number (m in report).
2. NPTS: number of radial mesh points.
3. SIG: hub-to-duct radius ratio (σ in report).
4. AKRE: real part of nondimensional frequency ($k = \frac{\omega r_T}{A_T}$ in report).
5. AKIM: imaginary part of nondimensional frequency ($k = \frac{\omega r_T}{A_T}$ in report).
6. IX: this parameter is not actually used in the code.
7. NX: this parameter is not actually used in the code.
8. IR: axial velocity distribution flag (used in rmachModule):
   - IR = 0: uniform axial flow velocity:

$$V_x = RMAX$$

.

- IR = 1: linear shear:

$$V_x = (RMAX - SLOPE\,(\tilde{r} - 1.0))$$

- IR = 2: user axial velocity input from file 'mach.input'. See rmachModule for file format.
- IR = 3: uniform flow plus sine wave boundary layers of thickness $SLOPE$.
- IR = 4: uniform flow plus linear boundary layers of thickness $SLOPE$.
- IR = 5: uniform flow plus $\frac{1}{7}$ power boundary layers.
- IR = 6: hyperbolic secant profile.
- IR = 7: laminar mean flow:

$$V_x = \frac{-4RMAX}{(1-\sigma)^2}\left(\tilde{r}^2 - (1+\sigma)\,\tilde{r} + \sigma\right)$$

- IR = 8: wavy sinusoid for axial Mach number:

$$M_x = \frac{RMAX + SLOPE}{2} + \frac{RMAX - SLOPE}{2}\sin\left(4\pi\left(\frac{2\,(\tilde{r}-\sigma)}{1-\sigma} - 1\right)\right)$$

- IR = 9: Hagen-Poiseuille flow:

$$K = \frac{\sigma^2 - 1}{\ln\sigma}$$

$$V_x = RMAX\left(\frac{1 + K\ln r - \tilde{r}^2}{1 + \frac{K}{2}\ln\frac{K}{2} - \frac{K}{2}}\right)$$

9. RMAX: maximum axial Mach number value.
10. SLOPE: slope of linear axial Mach number distribution.
11. IS: swirl Mach number distribution flag (0-6). See smach for details.

- IS = 0: no swirl.
- IS = 1: solid-body swirl with angular velocity ANGOM.

$$v_\theta = (ANGOM)\,\tilde{r}$$

- IS = 2: free-vortex swirl with strength GAM.

$$v_\theta = \frac{GAM}{\widetilde{r}}$$

- IS = 3: combined solid-body and free-vortex swirl.

$$v_\theta = (ANGOM)\,\widetilde{r} + \frac{GAM}{\widetilde{r}}$$

- IS = 4: Stability test case:

$$v_\theta = \frac{1}{\widetilde{r}^2}$$

- IS = 5: user-input azimuthal velocity profile from file 'swrl.input'. See smachAndSndspdModule for file format.
- IS = 6: constant swirl:

$$v_\theta = ANGOM$$

- IS = 7: trailing line vortex (NOT COMPLETED IN CODE):

$$v_\theta = \frac{GAM}{\widetilde{r}}\left(1 - e^{-\widetilde{r}^2}\right)$$

12. ANGOM: magnitude of solid-body swirl.

13. GAM: magnitude of free-vortex swirl.

14. IREPEAT: this parameter is not actually used in the code.

15. IFD: flag to choose method for derivative calculation:

    - IFD=0: Chebyshev polynomials
    - IFD$\neq$ 0: Finite differencing

16. ITEST: this parameter is not actually used in the code.

17. ETAHR: real part of hub liner admittance (η_H in report).

18. ETAHI: imaginary part of hub liner admittance (η_H in report).

19. ETADR: real part of duct liner admittance (η_D in report).

20. ETADI: imaginary part of duct liner admittance (η_D in report).

21. ED2: second derivative smoothing coefficient.

22. ED4: fourth derivative smoothing coefficient.

23. ICMPR: flag to perform consistency test on selected modes (currently does not work):
    - ICMPR=1: Perform consistency test.
    - ICMPR $\neq$ 1: Do not perform consistency test.

6.2 grid

6.3 derivs

6.4 smachAndSndspd

6.5 rmach

6.6 machout

6.7 globalM

6.8 boundary

6.9 analysis

In this module, the eigendecomposition is actually performed.

The convected wavenumbers are computed as:

$$\lambda_{cvct}\left(\widetilde{r}\right) = \frac{1}{M_x}\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}}\right)$$

(noting that this is a range of convection speeds, due to the different axial and azimuthal mean velocities)

The routine ZGEGV is used to obtain the eigenvalues and eigenvectors. ZGEGV is a LAPACK routine that calculates the generalized eigenvalues (α, β) and the generalized eigenvector $\vec{r}$ that satisfy the equation:

$$
\begin{aligned}
([A] - w\,[B])\,\vec{r} &= 0 \\
w &= \frac{\alpha}{\beta}
\end{aligned}
$$

For SWIRL, the result is:

$$
\begin{aligned}
\lambda &= \frac{\alpha}{\beta} \\
\overline{\gamma} &= \frac{i\alpha}{\beta} \\
k_x &= \frac{1}{r_t}\frac{i\alpha}{\beta}
\end{aligned}
$$

The $\overline{\gamma}$ values are printed to the display, with:

$$
\ldots \tag{1}
$$

7 Boundary conditions

The boundary conditions at the hub and tip are the wall boundary conditions used by Unruh and Eversman [?]:

$$
\begin{aligned}
\widetilde{v}_r - \eta_H \left[1 - \frac{\overline{\gamma}}{k} M_H\right] \widetilde{p} &= 0 \\
\widetilde{v}_r - \eta_T \left[1 - \frac{\overline{\gamma}}{k} M_T\right] \widetilde{p} &= 0
\end{aligned}
$$

where, from before,

$$
\begin{aligned}
\lambda &= -i\overline{\gamma} \\
\overline{\gamma} &= i\lambda
\end{aligned}
$$

which gives:

$$
\begin{aligned}
\widetilde{v}_r - \eta_H \left[1 - \frac{i\lambda}{k} M_H\right] \widetilde{p} &= 0 \\
\widetilde{v}_r - \eta_T \left[1 - \frac{i\lambda}{k} M_T\right] \widetilde{p} &= 0
\end{aligned}
$$

or:

$$
\begin{aligned}
\widetilde{v}_r - \eta_H \left[1 + \frac{\lambda}{ik} M_H\right] \widetilde{p} &= 0 \\
\widetilde{v}_r - \eta_T \left[1 + \frac{\lambda}{ik} M_T\right] \widetilde{p} &= 0
\end{aligned}
$$

Continuing,

$$
\begin{aligned}
ik\widetilde{v}_r - \eta_H \left[ik + \lambda M_H\right] \widetilde{p} &= 0 \\
ik\widetilde{v}_r - \eta_T \left[ik + \lambda M_T\right] \widetilde{p} &= 0
\end{aligned}
$$

which gives:

$$
\begin{aligned}
ik\widetilde{v}_r - ik\eta_H\widetilde{p} &= \lambda M_H \eta_H \widetilde{p} \\
ik\widetilde{v}_r - ik\eta_T\widetilde{p} &= \lambda M_T \eta_T \widetilde{p}
\end{aligned}
$$

which is exactly what is coded in boundaryModule.f90.

7.1 Boundary equation work...

The governing equation is:

$$
-i\left(\frac{k}{\widetilde{A}} - \frac{mM_\theta}{\widetilde{r}} - \overline{\gamma} M_x\right)\widetilde{v}_r - \frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta = -\frac{\partial \widetilde{p}}{\partial \widetilde{r}} - \frac{(\gamma - 1)\, M_\theta^2}{\widetilde{r}}\widetilde{p}
$$

which is rearranged as:

$$
\begin{aligned}
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)\widetilde{v}_r+i\overline{\gamma}M_x\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\frac{\partial\widetilde{p}}{\partial\widetilde{r}}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= 0\\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\frac{\partial\widetilde{p}}{\partial\widetilde{r}}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= -i\overline{\gamma}M_x\widetilde{v}_r\\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\frac{\partial\widetilde{p}}{\partial\widetilde{r}}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= \lambda M_x\widetilde{v}_r
\end{aligned}
$$

Is there a way to get the pressure derivative at the wall to give me what I want?

I want both equations to be identical:

$$
\begin{aligned}
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\frac{\partial\widetilde{p}}{\partial\widetilde{r}}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= \lambda M_x\widetilde{v}_r\\
ik\widetilde{v}_r &= ik\eta_H\widetilde{p}+\lambda M_H\eta_H\widetilde{p}
\end{aligned}
$$

7.2 Correction method

We define the 'noBC' boundary equation as:

$$
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial\widetilde{p}}{\partial\widetilde{r}}\right|^{noBC}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} = 0
$$

The desired boundary equation is:

$$
ik\widetilde{v}_r-ik\eta_H\widetilde{p}-\lambda M_H\eta_H\widetilde{p} = 0
$$

and:

$$
\left.\frac{\partial\widetilde{p}}{\partial\widetilde{r}}\right|^{BC} = \left.\frac{\partial\widetilde{p}}{\partial\widetilde{r}}\right|^{noBC}+\Delta\frac{\partial\widetilde{p}}{\partial\widetilde{r}}
$$

Comparing the two equations,

$$
\begin{aligned}
\Delta \frac{\partial \widetilde{p}}{\partial \widetilde{r}} &= \begin{pmatrix} -\left(-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC}+\frac{(\gamma-1)M_\theta^2}{\widetilde{r}}\widetilde{p}\right) \\ +ik\widetilde{v}_r-ik\eta_H\widetilde{p}-\lambda M_H\eta_H\widetilde{p} \end{pmatrix} \\
&= \begin{pmatrix} i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r+\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta-\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC}-\frac{(\gamma-1)M_\theta^2}{\widetilde{r}}\widetilde{p} \\ +ik\widetilde{v}_r-ik\eta_H\widetilde{p}-\lambda M_H\eta_H\widetilde{p} \end{pmatrix} \\
&= \left(i\left(k+\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r+\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta-\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC}-\left(\frac{(\gamma-1)M_\theta^2}{\widetilde{r}}+ik\eta_H+\lambda M_H\eta_H\right)\widetilde{p}\right)
\end{aligned}
$$

As a 'sanity check', we substitute into the 'BC' equation:

$$
\begin{aligned}
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{BC}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= 0 \\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC}+\Delta\frac{\partial \widetilde{p}}{\partial \widetilde{r}}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= 0 \\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{BC}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= 0 \\
-i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC}+\Delta\frac{\partial \widetilde{p}}{\partial \widetilde{r}}+\frac{(\gamma-1)\,M_\theta^2}{\widetilde{r}}\widetilde{p} &= 0 \\
\begin{pmatrix} -i\left(\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r-\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta+\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC}+\frac{(\gamma-1)M_\theta^2}{\widetilde{r}}\widetilde{p} \\ +\begin{pmatrix} i\left(k+\frac{k}{\widetilde{A}}-\frac{mM_\theta}{\widetilde{r}}-\overline{\gamma}M_x\right)\widetilde{v}_r+\frac{2M_\theta}{\widetilde{r}}\widetilde{v}_\theta-\left.\frac{\partial \widetilde{p}}{\partial \widetilde{r}}\right|^{noBC} \\ -\left(\frac{(\gamma-1)M_\theta^2}{\widetilde{r}}+ik\eta_H+\lambda M_H\eta_H\right)\widetilde{p} \end{pmatrix} \end{pmatrix} &= 0 \\
ik\widetilde{v}_r-\left(ik\eta_H+\lambda M_H\eta_H\right)\widetilde{p} &= 0
\end{aligned}
$$

7.3 output

8 SWIRL Verification cases

In the Kousen report, there are a number of verification cases presented. In this section, the inputs for running those cases are determined.

8.1 Cylinder, Uniform Flow with Liner (Table 4.3)

In Table 4.3, a comparison is shown for the case of a cylinder (no centerbody) with uniform flow. For this test case,

$$\begin{aligned}
m &= 2 \\
k &= \frac{\omega r_T}{A_T} = -1 \\
M_x &= 0.5 \\
\eta_T &= 0.72 + 0.42i
\end{aligned}$$

Recall that in the SWIRL code, A_T is the speed of sound at the outer radius, r_T. Also, η_T is the admittance of the liner on the outer wall. The azimuthal mode number is m.

The input deck for this test case is:

```
&INPUTS
 MM      =    2,
 NPTS    =   32,
 SIG     = 0.00,
 AKRE    =  -1.,
 AKIM    =   0.,
 IX      =    0,
 NX      =    0,
 IR      =    0,
 RMAX    = 0.50,
 SLOPE   = 0.00,
 IS      =    0,
 ANGOM   = 0.00,
 GAM     = 0.00,
 IREPEAT =    0,
 IFD     =    0,
 ITEST   =    0,
 ETAHR   = 0.00,
 ETAHI   = 0.00,
 ETADR   = 0.72,
 ETADI   = 0.42,
 ED2     =  0.0,
 ED4     =  0.0,
&END
```

The resulting γ values are given in 'gam.acc'. The data from left to right are:

1. Eigenvalue ID number, j.
2. Real part of the nondimensional axial wavenumber $\overline{\gamma}_j$
3. Imaginary part of the nondimensional axial wavenumber $\overline{\gamma}_j$
4. Real part of $\frac{k}{\overline{\gamma}_j}$

5. Imaginary part of $\frac{k}{\overline{\gamma}_j}$

6. Real part of $\frac{\overline{\gamma}_j}{k}$

7. Imaginary part of $\frac{\overline{\gamma}_j}{k}$

(I have no idea why the last two columns are needed.)
The output in 'gam.acc' for this test case is:

```
#   j         Re{gam}            Im{gam}           Re{gam/ak}         Im{gam/ak}     nz
    1  0.666661832386E+00 -0.863181766372E+03 -0.666661832386E+00  0.863181766372E+03 -0.894748166120E-06 -0.115850385453E-02
    2  0.666671535394E+00  0.863181766432E+03 -0.666671535394E+00 -0.863181766432E+03 -0.894761188697E-06  0.115850385443E-02
    3  0.486904334131E+03  0.711226578236E-05 -0.486904334131E+03 -0.711226578236E-05 -0.205379153542E-02  0.299999614658E-10
    4 -0.485571000887E+03 -0.727158271276E-05  0.485571000887E+03  0.727158271276E-05  0.205943105781E-02 -0.308406458597E-10
    5  0.666186196644E+00 -0.270964395496E+03 -0.666186196644E+00  0.270964395496E+03 -0.907336845842E-05 -0.369049946071E-02
    6  0.667159178385E+00  0.270964414114E+03 -0.667159178385E+00 -0.270964414114E+03 -0.908661893262E-05  0.369049914192E-02
    7 -0.152502349084E+00 -0.235471154884E+03  0.152502349084E+00  0.235471154884E+03  0.275043207226E-05 -0.424680288779E-02
    8  0.150782379084E+01  0.235508741734E+03 -0.150782379084E+01 -0.235508741734E+03 -0.271843357482E-04  0.424595284000E-02
    9  0.654577649002E+00 -0.154440881062E+03 -0.654577649002E+00  0.154440881062E+03 -0.274428258331E-04 -0.647485322322E-02
   10  0.679360666482E+00  0.154441666296E+03 -0.679360666482E+00 -0.154441666296E+03 -0.284815117974E-04  0.647481133017E-02
   11  0.203817254085E+00 -0.147580882502E+03 -0.203817254085E+00  0.147580882502E+03 -0.935793257979E-05 -0.677593246321E-02
   12  0.114908693497E+01  0.147615287403E+03 -0.114908693497E+01 -0.147615287403E+03 -0.527307439101E-04  0.677395563416E-02
   13  0.659171278977E+00 -0.109556505279E+03 -0.659171278977E+00  0.109556505279E+03 -0.549169253183E-04 -0.912737949970E-02
   14  0.674649119395E+00  0.109557229559E+03 -0.674649119395E+00 -0.109557229559E+03 -0.562055764807E-04  0.912730346485E-02
   15  0.398023285909E+00 -0.101971304048E+03 -0.398023285909E+00  0.101971304048E+03 -0.382777073285E-04 -0.980653110120E-02
   16  0.952415052147E+00  0.102000245487E+03 -0.952415052147E+00 -0.102000245487E+03 -0.915347402439E-04  0.980304327863E-02
   17  0.631981427627E+00 -0.878632034233E+02 -0.631981427627E+00  0.878632034233E+02 -0.818592537062E-04 -0.113807399174E-01
   18  0.704204711787E+00  0.878674124623E+02 -0.704204711787E+00 -0.878674124623E+02 -0.912043090777E-04  0.113800525755E-01
   19  0.512907574867E+00 -0.842237595480E+02 -0.512907574867E+00  0.842237595480E+02 -0.723025742788E-04 -0.118726938910E-01
   20  0.831861539314E+00  0.842442317160E+02 -0.831861539314E+00 -0.842442317160E+02 -0.117200200343E-03  0.118690916315E-01
   21  0.631266364148E+00 -0.757541009147E+02 -0.631266364148E+00  0.757541009147E+02 -0.109994304117E-03 -0.131996888911E-01
   22  0.705249647838E+00  0.757592371570E+02 -0.705249647838E+00 -0.757592371570E+02 -0.122866661541E-03  0.131985667472E-01
   23  0.584693633914E+00 -0.720556587549E+02 -0.584693633914E+00  0.720556587549E+02 -0.112606532211E-03 -0.138772467972E-01
   24  0.756173116912E+00  0.720683291681E+02 -0.756173116912E+00 -0.720683291681E+02 -0.145574235158E-03  0.138741931750E-01
   25  0.608943525213E+00 -0.679734189871E+02 -0.608943525213E+00  0.679734189871E+02 -0.131784378831E-03 -0.147104524924E-01
   26  0.729943721425E+00  0.679829463997E+02 -0.729943721425E+00 -0.679829463997E+02 -0.157920803754E-03  0.147078757196E-01
   27  0.603587261705E+00 -0.643648950046E+02 -0.603587261705E+00  0.643648950046E+02 -0.145681278991E-03 -0.155350532082E-01
   28  0.736150415165E+00  0.643759364750E+02 -0.736150415165E+00 -0.643759364750E+02 -0.177608053949E-03  0.155317236301E-01
   29  0.602714553949E+00 -0.606583474684E+02 -0.602714553949E+00  0.606583474684E+02 -0.163790102422E-03 -0.164841497181E-01
   30  0.598330594187E+00 -0.570207468632E+02 -0.598330594187E+00  0.570207468632E+02 -0.184004138383E-03 -0.175355455637E-01
   31  0.593992620827E+00 -0.533940329180E+02 -0.593992620827E+00  0.533940329180E+02 -0.208325223037E-03 -0.187263670060E-01
   32  0.668203564553E+00 -0.503674749089E+02 -0.668203564553E+00  0.503674749089E+02 -0.263349198330E-03 -0.198505887169E-01
   33  0.587454135440E+00 -0.497669424711E+02 -0.587454135440E+00  0.497669424711E+02 -0.237154591154E-03 -0.200908601756E-01
   34  0.737556431878E+00  0.606702564914E+02 -0.737556431878E+00 -0.606702564914E+02 -0.200345417813E-03  0.164801056031E-01
   35  0.583528302269E+00 -0.461144238809E+02 -0.583528302269E+00  0.461144238809E+02 -0.274358750369E-03 -0.216817173405E-01
   36  0.742927474892E+00  0.570343361984E+02 -0.742927474892E+00 -0.570343361984E+02 -0.228349441645E-03  0.175303233029E-01
   37  0.748373723996E+00  0.534095424830E+02 -0.748373723996E+00 -0.534095424830E+02 -0.262298348430E-03  0.187195706296E-01
   38  0.664837047477E+00  0.503671839626E+02 -0.664837047477E+00 -0.503671839626E+02 -0.262025892761E-03  0.198507384535E-01
   39  0.756588821850E+00  0.497851192860E+02 -0.756588821850E+00 -0.497851192860E+02 -0.305183132961E-03  0.200816853749E-01
   40  0.761911557798E+00  0.461352109598E+02 -0.761911557798E+00 -0.461352109598E+02 -0.357866551181E-03  0.216695083113E-01
   41  0.577042153605E+00 -0.424768910577E+02 -0.577042153605E+00  0.424768910577E+02 -0.319758393967E-03 -0.235378687336E-01
   42  0.569568800457E+00 -0.388329943784E+02 -0.569568800457E+00  0.388329943784E+02 -0.377616545059E-03 -0.257457591773E-01
   43  0.770559879483E+00  0.425014080621E+02 -0.770559879483E+00 -0.425014080621E+02 -0.426439106763E-03  0.235209008057E-01
   44  0.780827155751E+00  0.388623195128E+02 -0.780827155751E+00 -0.388623195128E+02 -0.516799620431E-03  0.257214824374E-01
   45  0.560740641222E+00 -0.351878517665E+02 -0.560740641222E+00  0.351878517665E+02 -0.452758126067E-03 -0.284116838604E-01
   46  0.793358476458E+00  0.352235698890E+02 -0.793358476458E+00 -0.352235698890E+02 -0.639120079320E-03  0.283756857076E-01
   47  0.550240023912E+00 -0.315368072415E+02 -0.550240023912E+00  0.315368072415E+02 -0.553075656592E-03 -0.316993304992E-01
   48  0.537545130773E+00 -0.278800642562E+02 -0.537545130773E+00  0.278800642562E+02 -0.691299075016E-03 -0.358545943927E-01
   49  0.808917325270E+00  0.315812623997E+02 -0.808917325270E+00 -0.315812623997E+02 -0.810513646490E-03  0.316435850101E-01
   50  0.522026084592E+00 -0.242129844673E+02 -0.522026084592E+00  0.242129844673E+02 -0.890007639072E-03 -0.412809661752E-01
   51  0.502872959887E+00 -0.205307641185E+02 -0.502872959887E+00  0.205307641185E+02 -0.119230560451E-02 -0.486781892764E-01
   52  0.828772145700E+00  0.279369289497E+02 -0.828772145700E+00 -0.279369289497E+02 -0.106095211305E-02  0.357634410798E-01
   53  0.854906548852E+00  0.242883150612E+02 -0.854906548852E+00 -0.242883150612E+02 -0.144739192767E-02  0.411211157565E-01
   54  0.890752800052E+00  0.206353530687E+02 -0.890752800052E+00 -0.206353530687E+02 -0.208797313144E-02  0.483703927316E-01
   55  0.479527868207E+00 -0.168216752549E+02 -0.479527868207E+00  0.168216752549E+02 -0.169325586289E-02 -0.593988423559E-01
   56  0.942504798216E+00  0.169767389380E+02 -0.942504798216E+00 -0.169767389380E+02 -0.326015697984E-02  0.587231322838E-01
   57  0.453892583503E+00 -0.130615530535E+02 -0.453892583503E+00  0.130615530535E+02 -0.265729224010E-02 -0.764682324278E-01
   58 -0.581953517139E+01 -0.389677944159E+01  0.581953517139E+01  0.389677944159E+01  0.118640411246E+00 -0.794420004123E-01
   59  0.445334863371E+00 -0.918684055908E+01 -0.445334863371E+00  0.918684055908E+01 -0.526423181437E-02 -0.108596165094E+00
```

60	0.619541702280E+00	-0.501386488846E+01	-0.619541702280E+00	0.501386488846E+01	-0.242741699232E-01	-0.196447483400E+00
61	0.102183778539E+01	0.133150046366E+02	-0.102183778539E+01	-0.133150046366E+02	-0.572992612188E-02	0.746635072323E-01
62	0.114567394058E+01	0.966787961563E+01	-0.114567394058E+01	-0.966787961563E+01	-0.120876589928E-01	0.102002871705E+00
63	0.125949805048E+01	0.608517533075E+01	-0.125949805048E+01	-0.608517533075E+01	-0.326162219368E-01	0.157582958573E+00
64	0.410097973544E+00	0.129037864478E+01	-0.410097973544E+00	-0.129037864478E+01	-0.223699069595E+00	0.703872052248E+00
65	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
66	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
67	0.000000000000E+00	0.000000000000E+00	-0.000000000000E+00	-0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
68	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
69	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
70	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
71	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
72	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
73	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
74	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
75	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
76	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
77	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
78	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
79	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
80	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
81	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
82	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
83	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
84	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
85	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
86	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
87	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
88	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
89	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
90	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
91	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
92	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
93	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
94	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
95	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
96	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
97	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
98	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
99	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
100	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
101	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
102	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
103	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
104	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
105	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
106	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
107	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
108	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
109	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
110	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
111	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
112	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
113	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
114	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
115	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
116	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
117	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
118	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
119	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
120	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
121	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
122	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
123	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
124	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
125	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
126	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
127	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00
128	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00

Comparing to the data in Table 4.3 of the paper, the first eigenvalues are well predicted. However, there are a number of eigenvalues that lie between the tabulated results for γ_8^+ and γ_9^+, and between γ_8^- and γ_9^-.

$\gamma_n^{\pm}$	Kousen Ref. [15]	Kousen report	current	index
γ_0^+	$0.620 - 5.014i$	$0.6195 - 5.0139i$	$0.61954 - 5.01386i$	60
γ_1^+	$-5.820 - 3.897i$	$-5.8195 - 3.8968i$	$-5.81953 - 3.89677i$	58
γ_2^+	$0.445 - 9.187i$	$0.4453 - 9.1868i$	$0.44533 - 9.18684i$	59
γ_3^+	$0.453 - 13.062i$	$0.4539 - 13.062i$	$0.45389 - 13.0615i$	57
γ_4^+	$0.480 - 16.822i$	$0.4795 - 16.822i$	$0.47952 - 16.8216i$	55
γ_5^+	$0.503 - 20.531i$	$0.5029 - 20.531i$	$0.50287 - 20.5307i$	51
γ_6^+	$0.522 - 24.213i$	$0.5220 - 24.213i$	$0.52202 - 24.2129i$	50
γ_7^+	$0.538 - 27.880i$	$0.5376 - 27.880i$	$0.53754 - 27.8800i$	48
γ_8^+	$0.550 - 31.537i$	$0.5502 - 31.537i$	$0.55024 - 31.5368i$	47
γ_9^+	$0.589 - 49.75i$	$0.5891 - 49.754i$	$0.58745 - 49.7669i$	33
γ_0^-	$0.410 + 1.290i$	$0.4101 + 1.2904i$	$0.41009 + 1.29037i$	64
γ_1^-	$1.259 + 6.085i$	$1.2595 + 6.0852i$	$1.25949 + 6.08517i$	63
γ_2^-	$1.146 + 9.668i$	$1.1457 + 9.6679i$	$1.14567 + 9.66787i$	62
γ_3^-	$1.022 + 13.315i$	$1.0218 + 13.315i$	$1.02183 + 13.3150i$	61
γ_4^-	$0.943 + 16.977i$	$0.9425 + 16.977i$	$0.94250 + 16.9767i$	56
γ_5^-	$0.891 + 20.635i$	$0.8908 + 20.635i$	$0.89075 + 20.6353i$	54
γ_6^-	$0.855 + 24.288i$	$0.8549 + 24.288i$	$0.85490 + 24.2883i$	53
γ_7^-	$0.829 + 27.937i$	$0.8288 + 27.937i$	$0.82877 + 27.9369i$	52
γ_8^-	$0.809 + 31.581i$	$0.8089 + 31.581i$	$0.80891 + 31.5812i$	49
γ_9^-	$0.755 + 49.77i$	$0.7547 + 49.772i$	$0.75658 + 49.7851i$	39

Table 1: Table 4.3 data

8.2 Cylinder, Shear Flow (Table 4.4)

This test case is given in Ref. 10 by Shankar. The data corresponds to the results given in Table 1 of Ref. 10. This is a hard-wall duct ($\eta = 0$), with an axial Mach number profile given by:

$$
\begin{aligned}
b &= r_T - r_H \\
\widetilde{r} &= \frac{r}{b} \\
M(\widetilde{r}) &= M_0 (1 - \widetilde{r})^{\frac{1}{7}} \\
M_0 &= 0.3 \\
m &= 0 \\
kb &= 20
\end{aligned}
$$

Next, we translate Shankar's notation into the SWIRL notation. Comparing Shankar and SWIRL,

$$
\begin{aligned}
p_n(r, x, t) &= a_n \phi_n(r) e^{ik(\beta_n x - ct)} \\
\widetilde{p}_n(r, x, \theta, t) &= p_n(r) e^{i(k_x x + m\theta - \omega t)}
\end{aligned}
$$

Translating this into SWIRL inputs:

$$
\begin{aligned}
(kb)_{Shankar} &= \frac{\omega}{A_T} r_T \\
&= k_{SWIRL}
\end{aligned}
$$

The input deck for this test case is:

```
&INPUTS
 MM      =    0,
 NPTS    =   32,
 SIG     = 0.00,
 AKRE    =  20.,
 AKIM    =   0.,
 IX      =    0,
 NX      =    0,
 IR      =    5,
 RMAX    = 0.30,
 SLOPE   = 0.00,
 IS      =    0,
 ANGOM   = 0.00,
 GAM     = 0.00,
 IREPEAT =    0,
 IFD     =    0,
 ITEST   =    0,
 ETAHR   = 0.00,
```

$\gamma_n^{\pm}$	Shankar β_n	$\frac{\overline{\gamma}_n}{k}$ (16 points)	index	$\frac{\overline{\gamma}_n}{k}$ (32 points)	index
γ_0^-	0.81500	0.816009	45	0.815493	69
γ_1^-	0.76944	0.769595	47	0.769520	71
γ_2^-	0.72751	0.727753	46	0.727642	70
γ_3^-	0.65329	0.653542	44	0.653472	68
γ_4^-	0.54028	0.540499	43	0.540524	67
γ_5^-	0.36933	0.369465	42	0.369746	66
γ_6^-	0.06361	0.064613	41	0.064754	65
γ_7^-	$-0.28313 + 0.48807i$	$-0.278964 + 0.487173i$	39	$-0.282743 + 0.486526i$	63
γ_8^-	$-0.28357 + 0.80635i$	$-0.289882 + 0.854887i$	29	$-0.283251 + 0.804397i$	62
γ_9^-	$-0.28410 + 1.05658i$	$-0.263260 + 1.090093i$	27	$-0.283670 + 1.053847i$	57
γ_{10}^-	$-0.28410 + 1.27947i$	$-0.315161 + 1.655351i$	25	$-0.284027 + 1.275635i$	50

Table 2: Table 4.4 data

```
 ETAHI   = 0.00,
 ETADR   = 0.00,
 ETADI   = 0.00,
 ED2     =  0.0,
 ED4     =  0.0,
&END
```

For the cylinder case of Shankar, the results in Table 4.4 are for $\frac{\overline{\gamma}_n}{k}$ in order to compare with the β_n results:

The output in 'gam.acc' for this test case (using 16 points) is:

```
#    j          Re{gam}              Im{gam}            Re{gam/ak}           Im{gam/ak}       nz
     1  0.163319288641E+03  0.000000000000E+00  0.816596443205E+01  0.000000000000E+00  0.122459509629E+00  0.000000000000E+00
     2 -0.117303925650E+03  0.000000000000E+00 -0.586519628252E+01  0.000000000000E+00 -0.170497277811E+00 -0.000000000000E+00
     3 -0.190345763833E+02 -0.825374426224E+02 -0.951728819167E+00 -0.412687213112E+01 -0.530599418459E-01  0.230077718435E+00
     4 -0.190345763833E+02  0.825374426224E+02 -0.951728819167E+00  0.412687213112E+01 -0.530599418459E-01 -0.230077718435E+00
     5 -0.662048021457E+01 -0.743548673755E+02 -0.331024010729E+00 -0.371774336877E+01 -0.237613547529E-01  0.266864687160E+00
     6 -0.662048021457E+01  0.743548673755E+02 -0.331024010729E+00  0.371774336877E+01 -0.237613547529E-01 -0.266864687160E+00
     7  0.105980545858E+03  0.000000000000E+00  0.529902729289E+01  0.000000000000E+00  0.188713879874E+00  0.000000000000E+00
     8  0.104346612102E+03  0.000000000000E+00  0.521733060512E+01  0.000000000000E+00  0.191668896546E+00  0.000000000000E+00
     9  0.939100552891E+02  0.000000000000E+00  0.469550276446E+01  0.000000000000E+00  0.212969739379E+00  0.000000000000E+00
    10  0.865107830636E+02  0.000000000000E+00  0.432553915318E+01  0.000000000000E+00  0.231185053374E+00  0.000000000000E+00
    11  0.814508584375E+02  0.000000000000E+00  0.407254292187E+01  0.000000000000E+00  0.245546828894E+00  0.000000000000E+00
    12  0.777325529553E+02  0.000000000000E+00  0.388662764776E+01  0.000000000000E+00  0.257292462934E+00  0.000000000000E+00
    13  0.748921084828E+02  0.000000000000E+00  0.374460542414E+01  0.000000000000E+00  0.267050833595E+00  0.000000000000E+00
    14  0.726768738467E+02  0.000000000000E+00  0.363384369234E+01  0.000000000000E+00  0.275190702921E+00  0.000000000000E+00
    15  0.709361335325E+02  0.000000000000E+00  0.354680667662E+01  0.000000000000E+00  0.281943757068E+00  0.000000000000E+00
    16  0.695750820389E+02  0.000000000000E+00  0.347875410194E+01  0.000000000000E+00  0.287459237042E+00  0.000000000000E+00
    17  0.685320076963E+02  0.000000000000E+00  0.342660038481E+01  0.000000000000E+00  0.291834438714E+00  0.000000000000E+00
    18  0.677612959441E+02  0.000000000000E+00  0.338806479721E+01  0.000000000000E+00  0.295153741105E+00  0.000000000000E+00
    19  0.672124795891E+02  0.000000000000E+00  0.336062397946E+01  0.000000000000E+00  0.297563787592E+00  0.000000000000E+00
    20  0.665514226573E+02  0.000000000000E+00  0.332757113286E+01  0.000000000000E+00  0.300519496074E+00  0.000000000000E+00
    21  0.668152109662E+02  0.000000000000E+00  0.334076054831E+01  0.000000000000E+00  0.299333036756E+00  0.000000000000E+00
    22 -0.471272186164E+01 -0.379656309224E+02 -0.235636093082E+00 -0.189828154612E+01 -0.643990909023E-01  0.518798305675E+00
    23 -0.630323705080E+01 -0.331070325502E+02 -0.315161852540E+00 -0.165535162751E+01 -0.110991398568E+00  0.582969641721E+00
    24 -0.471272186164E+01  0.379656309224E+02 -0.235636093082E+00  0.189828154612E+01 -0.643990909023E-01 -0.518798305675E+00
    25 -0.630323705080E+01  0.331070325502E+02 -0.315161852540E+00  0.165535162751E+01 -0.110991398568E+00 -0.582969641721E+00
    26 -0.526520450645E+01 -0.218018701463E+02 -0.263260225323E+00 -0.109009350732E+01 -0.209333789485E+00  0.866797878412E+00
    27 -0.526520450645E+01  0.218018701463E+02 -0.263260225323E+00  0.109009350732E+01 -0.209333789485E+00 -0.866797878412E+00
    28 -0.579764528750E+01 -0.170977580948E+02 -0.289882264375E+00 -0.854887904738E+00 -0.355742661846E+00  0.104911592114E+01
    29 -0.579764528750E+01  0.170977580948E+02 -0.289882264375E+00  0.854887904738E+00 -0.355742661846E+00 -0.104911592114E+01
    30 -0.277384694568E+02  0.000000000000E+00 -0.138692347284E+01  0.000000000000E+00 -0.721020315527E+00 -0.000000000000E+00
```

```
 31 -0.265814970586E+02  0.000000000000E+00 -0.132907485293E+01  0.000000000000E+00 -0.752403070299E+00 -0.000000000000E+00
 32 -0.254247235137E+02  0.000000000000E+00 -0.127123617569E+01  0.000000000000E+00 -0.786635889638E+00 -0.000000000000E+00
 33 -0.240697952822E+02  0.000000000000E+00 -0.120348976411E+01  0.000000000000E+00 -0.830916913316E+00 -0.000000000000E+00
 34 -0.219589611241E+02  0.000000000000E+00 -0.109794805621E+01  0.000000000000E+00 -0.910789899711E+00 -0.000000000000E+00
 35 -0.186181534184E+02  0.000000000000E+00 -0.930907670922E+00  0.000000000000E+00 -0.107422038859E+01 -0.000000000000E+00
 36 -0.125335327984E+02  0.000000000000E+00 -0.626676639920E+00  0.000000000000E+00 -0.159571928535E+01 -0.000000000000E+00
 37 -0.557929881970E+01 -0.974346480870E+01 -0.278964940985E+00 -0.487173240435E+00 -0.885155619784E+00  0.154580045097E+01
 38 -0.543485433791E+01 -0.550803802019E+01 -0.271742716896E+00 -0.275401901009E+00 -0.181536610218E+01  0.183981113194E+01
 39 -0.557929881970E+01  0.974346480870E+01 -0.278964940985E+00  0.487173240435E+00 -0.885155619784E+00 -0.154580045097E+01
 40 -0.543485433791E+01  0.550803802019E+01 -0.271742716896E+00  0.275401901009E+00 -0.181536610218E+01 -0.183981113194E+01
 41  0.129227898584E+01  0.000000000000E+00  0.646139492918E-01  0.000000000000E+00  0.154765342617E+02  0.000000000000E+00
 42  0.738930776482E+01  0.000000000000E+00  0.369465388241E+00  0.000000000000E+00  0.270661347944E+01  0.000000000000E+00
 43  0.108099837563E+02  0.000000000000E+00  0.540499187815E+00  0.000000000000E+00  0.185014154053E+01  0.000000000000E+00
 44  0.130708424342E+02  0.000000000000E+00  0.653542121712E+00  0.000000000000E+00  0.153012325721E+01  0.000000000000E+00
 45  0.163201998955E+02  0.000000000000E+00  0.816009994775E+00  0.000000000000E+00  0.122547518585E+01  0.000000000000E+00
 46  0.145550786146E+02  0.000000000000E+00  0.727753930731E+00  0.000000000000E+00  0.137409082627E+01  0.000000000000E+00
 47  0.153919047635E+02  0.000000000000E+00  0.769595238174E+00  0.000000000000E+00  0.129938433919E+01  0.000000000000E+00
 48  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 49  0.666666666667E+02  0.000000000000E+00  0.333333333333E+01  0.000000000000E+00  0.300000000000E+00  0.000000000000E+00
 50  0.667724010447E+02  0.000000000000E+00  0.333862005224E+01  0.000000000000E+00  0.299524948737E+00  0.000000000000E+00
 51  0.670926323634E+02  0.000000000000E+00  0.335463161817E+01  0.000000000000E+00  0.298095324263E+00  0.000000000000E+00
 52  0.676368394188E+02  0.000000000000E+00  0.338184197094E+01  0.000000000000E+00  0.295696844676E+00  0.000000000000E+00
 53  0.684221959778E+02  0.000000000000E+00  0.342110979889E+01  0.000000000000E+00  0.292302807798E+00  0.000000000000E+00
 54  0.694758827877E+02  0.000000000000E+00  0.347379413939E+01  0.000000000000E+00  0.287869677901E+00  0.000000000000E+00
 55  0.708389526804E+02  0.000000000000E+00  0.354194763402E+01  0.000000000000E+00  0.282330543341E+00  0.000000000000E+00
 56  0.725727534440E+02  0.000000000000E+00  0.362863767220E+01  0.000000000000E+00  0.275585520059E+00  0.000000000000E+00
 57  0.747700003610E+02  0.000000000000E+00  0.373850001805E+01  0.000000000000E+00  0.267486958719E+00  0.000000000000E+00
 58  0.775750782186E+02  0.000000000000E+00  0.387875391093E+01  0.000000000000E+00  0.257814757771E+00  0.000000000000E+00
 59  0.812245018074E+02  0.000000000000E+00  0.406122509037E+01  0.000000000000E+00  0.246231119366E+00  0.000000000000E+00
 60  0.861370575300E+02  0.000000000000E+00  0.430685287650E+01  0.000000000000E+00  0.232188103164E+00  0.000000000000E+00
 61  0.931487672395E+02  0.000000000000E+00  0.465743836198E+01  0.000000000000E+00  0.214710302591E+00  0.000000000000E+00
 62  0.104294381450E+03  0.000000000000E+00  0.521471907251E+01  0.000000000000E+00  0.191764884377E+00  0.000000000000E+00
 63  0.126919835334E+03  0.000000000000E+00  0.634599176670E+01  0.000000000000E+00  0.157579782131E+00  0.000000000000E+00
 64  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
```

The output in 'gam.acc' for this test case (using 32 points) is:

```
#    j         Re{gam}             Im{gam}            Re{gam/ak}           Im{gam/ak}       nz
     1 -0.419148876321E+03  0.000000000000E+00 -0.209574438160E+02  0.000000000000E+00 -0.477157428539E-01 -0.000000000000E+00
     2  0.405930803348E+03  0.000000000000E+00  0.202965401674E+02  0.000000000000E+00  0.492694809930E-01  0.000000000000E+00
     3 -0.658653414210E+01 -0.290862176480E+03 -0.329326707105E+00 -0.145431088240E+02 -0.155628739242E-02  0.687258470725E-01
     4 -0.658653414210E+01  0.290862176480E+03 -0.329326707105E+00  0.145431088240E+02 -0.155628739242E-02 -0.687258470725E-01
     5 -0.784863705464E+02 -0.247706992620E+03 -0.392431852732E+01 -0.123853496310E+02 -0.232487234685E-01  0.733741582460E-01
     6 -0.784863705464E+02  0.247706992620E+03 -0.392431852732E+01  0.123853496310E+02 -0.232487234685E-01 -0.733741582460E-01
     7  0.309500174533E+03  0.000000000000E+00  0.154750087267E+02  0.000000000000E+00  0.646203189712E-01  0.000000000000E+00
     8 -0.466033764842E+01 -0.153375838349E+03 -0.233016882421E+00 -0.766879191744E+01 -0.395851869904E-02  0.130278355322E+00
     9 -0.653017072648E+01 -0.145048816442E+03 -0.326508536324E+00 -0.725244082209E+01 -0.619507823807E-02  0.137605708003E+00
    10 -0.466033764843E+01  0.153375838349E+03 -0.233016882421E+00  0.766879191744E+01 -0.395851869904E-02 -0.130278355322E+00
    11 -0.653017072648E+01  0.145048816442E+03 -0.326508536324E+00  0.725244082209E+01 -0.619507823807E-02 -0.137605708003E+00
    12  0.134486754616E+03  0.000000000000E+00  0.672433773078E+01  0.000000000000E+00  0.148713529873E+00  0.000000000000E+00
    13 -0.742683514153E+01  0.101949221883E+03 -0.371341757077E+00  0.509746109414E+01 -0.142156686262E-01 -0.195140504315E+00
    14 -0.657782693991E+01  0.982123578086E+02 -0.328891346996E+00  0.491061789043E+01 -0.135780184656E-01 -0.202730965722E+00
    15 -0.742683514153E+01 -0.101949221883E+03 -0.371341757077E+00 -0.509746109414E+01 -0.142156686262E-01  0.195140504315E+00
    16 -0.657782693991E+01 -0.982123578086E+02 -0.328891346996E+00 -0.491061789043E+01 -0.135780184656E-01  0.202730965722E+00
    17  0.115944495642E+03  0.000000000000E+00  0.579722478208E+01  0.000000000000E+00  0.172496330156E+00  0.000000000000E+00
    18 -0.518443551179E+01 -0.789084126835E+02 -0.259221775589E+00 -0.394542063418E+01 -0.165811311507E-01  0.252368987255E+00
    19 -0.610033946865E+01 -0.762435857044E+02 -0.305016973433E+00 -0.381217928522E+01 -0.208547972923E-01  0.260648531589E+00
    20  0.105446208485E+03  0.000000000000E+00  0.527231042423E+01  0.000000000000E+00  0.189670167258E+00  0.000000000000E+00
    21 -0.518443551179E+01  0.789084126835E+02 -0.259221775589E+00  0.394542063418E+01 -0.165811311507E-01 -0.252368987255E+00
    22 -0.610033946865E+01  0.762435857044E+02 -0.305016973433E+00  0.381217928522E+01 -0.208547972923E-01 -0.260648531589E+00
    23  0.986084901449E+02  0.000000000000E+00  0.493042450725E+01  0.000000000000E+00  0.202822292184E+00  0.000000000000E+00
    24 -0.570634712692E+01 -0.665650030025E+02 -0.285317356346E+00 -0.332825015012E+01 -0.255691528167E-01  0.298266245666E+00
    25 -0.570634712692E+01  0.665650030025E+02 -0.285317356346E+00  0.332825015012E+01 -0.255691528167E-01 -0.298266245666E+00
    26 -0.587913448638E+01  0.636369771958E+02 -0.293956724319E+00  0.318184885979E+01 -0.287894452460E-01 -0.311622956549E+00
    27 -0.587913448637E+01 -0.636369771958E+02 -0.293956724319E+00 -0.318184885979E+01 -0.287894452460E-01  0.311622956549E+00
    28  0.935764252574E+02  0.000000000000E+00  0.467882126287E+01  0.000000000000E+00  0.213729044949E+00  0.000000000000E+00
    29 -0.569095796687E+01  0.588308495638E+02 -0.284547898344E+00  0.294154247819E+01 -0.325806776446E-01 -0.336806027448E+00
    30 -0.569095796687E+01 -0.588308495638E+02 -0.284547898344E+00 -0.294154247819E+01 -0.325806776446E-01  0.336806027448E+00
    31 -0.571582832349E+01 -0.554812940612E+02 -0.285791416174E+00 -0.277406470306E+01 -0.367477576278E-01  0.356696008285E+00
    32 -0.571582832349E+01  0.554812940612E+02 -0.285791416174E+00  0.277406470306E+01 -0.367477576278E-01 -0.356696008285E+00
    33 -0.571740595343E+01 -0.518791633984E+02 -0.285870297671E+00 -0.259395816992E+01 -0.419759165456E-01  0.380885221550E+00
    34 -0.571740595343E+01  0.518791633984E+02 -0.285870297671E+00  0.259395816992E+01 -0.419759165456E-01 -0.380885221550E+00
    35 -0.571464946639E+01 -0.483366390993E+02 -0.285732473319E+00 -0.241683195497E+01 -0.482434566599E-01  0.408061171064E+00
    36 -0.571464946639E+01  0.483366390993E+02 -0.285732473319E+00  0.241683195497E+01 -0.482434566599E-01 -0.408061171064E+00
    37 -0.570863182790E+01 -0.447486900786E+02 -0.285431591395E+00 -0.223743450393E+01 -0.561035594901E-01  0.439783274104E+00
```

38	-0.570863182790E+01	0.447486900786E+02	-0.285431591395E+00	0.223743450393E+01	-0.561035594901E-01	-0.439783274104E+00
39	-0.570360025930E+01	-0.411061047912E+02	-0.285180012965E+00	-0.205530523956E+01	-0.662345623378E-01	0.477355483638E+00
40	-0.687998218961E+01	-0.380630218933E+02	-0.343999109480E+00	-0.190315109466E+01	-0.919705152587E-01	0.508820464843E+00
41	-0.569664019370E+01	-0.373916704144E+02	-0.284832009685E+00	-0.186958352072E+01	-0.796405017132E-01	0.522745213045E+00
42	-0.570360025930E+01	0.411061047912E+02	-0.285180012965E+00	0.205530523956E+01	-0.662345623378E-01	-0.477355483638E+00
43	-0.687998218961E+01	0.380630218933E+02	-0.343999109480E+00	0.190315109466E+01	-0.919705152587E-01	-0.508820464843E+00
44	-0.569664019370E+01	0.373916704144E+02	-0.284832009685E+00	0.186958352072E+01	-0.796405017132E-01	-0.522745213045E+00
45	-0.569255861892E+01	-0.335842778282E+02	-0.284627930946E+00	-0.167921389141E+01	-0.981214298780E-01	0.578885099394E+00
46	-0.569255861892E+01	0.335842778282E+02	-0.284627930946E+00	0.167921389141E+01	-0.981214298780E-01	-0.578885099394E+00
47	-0.568686921779E+01	-0.296444024663E+02	-0.284343460889E+00	-0.148222012331E+01	-0.124830978733E+00	0.650716524698E+00
48	-0.568686921779E+01	0.296444024663E+02	-0.284343460890E+00	0.148222012331E+01	-0.124830978733E+00	-0.650716524698E+00
49	-0.568055017101E+01	-0.255127184518E+02	-0.284027508550E+00	-0.127563592259E+01	-0.166300374694E+00	0.746895020775E+00
50	-0.568055017101E+01	0.255127184518E+02	-0.284027508550E+00	0.127563592259E+01	-0.166300374694E+00	-0.746895020775E+00
51	-0.277387784651E+02	0.000000000000E+00	-0.138693892325E+01	0.000000000000E+00	-0.721012283406E+00	-0.000000000000E+00
52	-0.265826768992E+02	0.000000000000E+00	-0.132913384496E+01	0.000000000000E+00	-0.752369675780E+00	-0.000000000000E+00
53	-0.254341931169E+02	0.000000000000E+00	-0.127170965584E+01	0.000000000000E+00	-0.786343011084E+00	-0.000000000000E+00
54	-0.240896983111E+02	0.000000000000E+00	-0.120448491555E+01	0.000000000000E+00	-0.830230405617E+00	-0.000000000000E+00
55	-0.219801383439E+02	0.000000000000E+00	-0.109900691719E+01	0.000000000000E+00	-0.909912380309E+00	-0.000000000000E+00
56	-0.186373158152E+02	0.000000000000E+00	-0.931865790762E+00	0.000000000000E+00	-0.107311590351E+01	-0.000000000000E+00
57	-0.567340162149E+01	0.210769456156E+02	-0.283670081075E+00	0.105384728078E+01	-0.238165510351E+00	-0.884795726456E+00
58	-0.567340162149E+01	-0.210769456156E+02	-0.283670081075E+00	-0.105384728078E+01	-0.238165510351E+00	0.884795726456E+00
59	-0.125777769077E+02	0.000000000000E+00	-0.628888845383E+00	0.000000000000E+00	-0.159010611707E+01	-0.000000000000E+00
60	-0.566503698651E+01	-0.160879564984E+02	-0.283251849325E+00	-0.804397824922E+00	-0.389463397438E+00	0.110602458742E+01
61	-0.565486924979E+01	-0.973053693345E+01	-0.282743462489E+00	-0.486526846673E+00	-0.892914776707E+00	0.153647057596E+01
62	-0.566503698651E+01	0.160879564984E+02	-0.283251849325E+00	0.804397824922E+00	-0.389463397438E+00	-0.110602458742E+01
63	-0.565486924978E+01	0.973053693345E+01	-0.282743462489E+00	0.486526846673E+00	-0.892914776707E+00	-0.153647057596E+01
64	0.896639357693E+02	0.000000000000E+00	0.448319678847E+01	0.000000000000E+00	0.223055120528E+00	0.000000000000E+00
65	0.129509349093E+01	0.000000000000E+00	0.647546745465E-01	0.000000000000E+00	0.154429005628E+02	0.000000000000E+00
66	0.739493575533E+01	0.000000000000E+00	0.369746787766E+00	0.000000000000E+00	0.270455358393E+01	0.000000000000E+00
67	0.108104868030E+02	0.000000000000E+00	0.540524340149E+00	0.000000000000E+00	0.185005544750E+01	0.000000000000E+00
68	0.130694531452E+02	0.000000000000E+00	0.653472657260E+00	0.000000000000E+00	0.153028591004E+01	0.000000000000E+00
69	0.163098658470E+02	0.000000000000E+00	0.815493292350E+00	0.000000000000E+00	0.122625165575E+01	0.000000000000E+00
70	0.145528553481E+02	0.000000000000E+00	0.727642767404E+00	0.000000000000E+00	0.137430074866E+01	0.000000000000E+00
71	0.153904181199E+02	0.000000000000E+00	0.769520905996E+00	0.000000000000E+00	0.129950985374E+01	0.000000000000E+00
72	0.865054770642E+02	0.000000000000E+00	0.432527385321E+01	0.000000000000E+00	0.231199233607E+00	0.000000000000E+00
73	0.838882082668E+02	0.000000000000E+00	0.419441041334E+01	0.000000000000E+00	0.238412530357E+00	0.000000000000E+00
74	0.816766572258E+02	0.000000000000E+00	0.408383286129E+01	0.000000000000E+00	0.244868003654E+00	0.000000000000E+00
75	0.797794690015E+02	0.000000000000E+00	0.398897345007E+01	0.000000000000E+00	0.250691064384E+00	0.000000000000E+00
76	0.781323134320E+02	0.000000000000E+00	0.390661567160E+01	0.000000000000E+00	0.255976037589E+00	0.000000000000E+00
77	0.766883760716E+02	0.000000000000E+00	0.383441880358E+01	0.000000000000E+00	0.260795716698E+00	0.000000000000E+00
78	0.754127097705E+02	0.000000000000E+00	0.377063548852E+01	0.000000000000E+00	0.265207284831E+00	0.000000000000E+00
79	0.742787034150E+02	0.000000000000E+00	0.371393517075E+01	0.000000000000E+00	0.269256180850E+00	0.000000000000E+00
80	0.732657804025E+02	0.000000000000E+00	0.366328902012E+01	0.000000000000E+00	0.272978734276E+00	0.000000000000E+00
81	0.723578447929E+02	0.000000000000E+00	0.361789223964E+01	0.000000000000E+00	0.276404031342E+00	0.000000000000E+00
82	0.715421978123E+02	0.000000000000E+00	0.357710989062E+01	0.000000000000E+00	0.279555292003E+00	0.000000000000E+00
83	0.708087562810E+02	0.000000000000E+00	0.354043781405E+01	0.000000000000E+00	0.282450943223E+00	0.000000000000E+00
84	0.701494654443E+02	0.000000000000E+00	0.350747327222E+01	0.000000000000E+00	0.285105522520E+00	0.000000000000E+00
85	0.695578365653E+02	0.000000000000E+00	0.347789182827E+01	0.000000000000E+00	0.287530506807E+00	0.000000000000E+00
86	0.690285708194E+02	0.000000000000E+00	0.345142854097E+01	0.000000000000E+00	0.289735101895E+00	0.000000000000E+00
87	0.685572669257E+02	0.000000000000E+00	0.342786334628E+01	0.000000000000E+00	0.291726915276E+00	0.000000000000E+00
88	0.681402464491E+02	0.000000000000E+00	0.340701232245E+01	0.000000000000E+00	0.293512293281E+00	0.000000000000E+00
89	0.677745443370E+02	0.000000000000E+00	0.338872721685E+01	0.000000000000E+00	0.295096045214E+00	0.000000000000E+00
90	0.674580039506E+02	0.000000000000E+00	0.337290019753E+01	0.000000000000E+00	0.296480755859E+00	0.000000000000E+00
91	0.671895173396E+02	0.000000000000E+00	0.335947586698E+01	0.000000000000E+00	0.297665481044E+00	0.000000000000E+00
92	0.666544306987E+02	0.000000000000E+00	0.333272153493E+01	0.000000000000E+00	0.300055071964E+00	0.000000000000E+00
93	0.666943560726E+02	0.000000000000E+00	0.333471780363E+01	0.000000000000E+00	0.299875449404E+00	0.000000000000E+00
94	0.668000530852E+02	0.000000000000E+00	0.334000265426E+01	0.000000000000E+00	0.299400959674E+00	0.000000000000E+00
95	0.669693098976E+02	0.000000000000E+00	0.334846549488E+01	0.000000000000E+00	0.298644260044E+00	0.000000000000E+00
96	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
97	0.666666666667E+02	0.000000000000E+00	0.333333333333E+01	0.000000000000E+00	0.300000000000E+00	0.000000000000E+00
98	0.666944512249E+02	0.000000000000E+00	0.333472256125E+01	0.000000000000E+00	0.299875021575E+00	0.000000000000E+00
99	0.667776922150E+02	0.000000000000E+00	0.333888461075E+01	0.000000000000E+00	0.299501215700E+00	0.000000000000E+00
100	0.669160870679E+02	0.000000000000E+00	0.334580435339E+01	0.000000000000E+00	0.298881791754E+00	0.000000000000E+00
101	0.671092425331E+02	0.000000000000E+00	0.335546212666E+01	0.000000000000E+00	0.298021542862E+00	0.000000000000E+00
102	0.673568188899E+02	0.000000000000E+00	0.336784094449E+01	0.000000000000E+00	0.296926136502E+00	0.000000000000E+00
103	0.676586963257E+02	0.000000000000E+00	0.338293481629E+01	0.000000000000E+00	0.295601320837E+00	0.000000000000E+00
104	0.680151441632E+02	0.000000000000E+00	0.340075720816E+01	0.000000000000E+00	0.294052159208E+00	0.000000000000E+00
105	0.684269818625E+02	0.000000000000E+00	0.342134909312E+01	0.000000000000E+00	0.292282363706E+00	0.000000000000E+00
106	0.688957299739E+02	0.000000000000E+00	0.344478649869E+01	0.000000000000E+00	0.290293752713E+00	0.000000000000E+00
107	0.694237569101E+02	0.000000000000E+00	0.347118784551E+01	0.000000000000E+00	0.288085820908E+00	0.000000000000E+00
108	0.700144328307E+02	0.000000000000E+00	0.350072164153E+01	0.000000000000E+00	0.285655388345E+00	0.000000000000E+00
109	0.706723056747E+02	0.000000000000E+00	0.353361528373E+01	0.000000000000E+00	0.282996285590E+00	0.000000000000E+00
110	0.714033176781E+02	0.000000000000E+00	0.357016588390E+01	0.000000000000E+00	0.280099029714E+00	0.000000000000E+00
111	0.722150849613E+02	0.000000000000E+00	0.361075424807E+01	0.000000000000E+00	0.276950446167E+00	0.000000000000E+00
112	0.731172694815E+02	0.000000000000E+00	0.365586347408E+01	0.000000000000E+00	0.273533190474E+00	0.000000000000E+00
113	0.741220836745E+02	0.000000000000E+00	0.370610418372E+01	0.000000000000E+00	0.269825118352E+00	0.000000000000E+00
114	0.752449862794E+02	0.000000000000E+00	0.376224931397E+01	0.000000000000E+00	0.265798440387E+00	0.000000000000E+00

```
115  0.765056579357E+02  0.000000000000E+00  0.382528289678E+01  0.000000000000E+00  0.261418573994E+00  0.000000000000E+00
116  0.779293959244E+02  0.000000000000E+00  0.389646979622E+01  0.000000000000E+00  0.256642564244E+00  0.000000000000E+00
117  0.795491556641E+02  0.000000000000E+00  0.397745778321E+01  0.000000000000E+00  0.251416873417E+00  0.000000000000E+00
118  0.814086258893E+02  0.000000000000E+00  0.407043129447E+01  0.000000000000E+00  0.245674211811E+00  0.000000000000E+00
119  0.835670258134E+02  0.000000000000E+00  0.417835129067E+01  0.000000000000E+00  0.239328847776E+00  0.000000000000E+00
120  0.861069152010E+02  0.000000000000E+00  0.430534576005E+01  0.000000000000E+00  0.232269382236E+00  0.000000000000E+00
121  0.891475960278E+02  0.000000000000E+00  0.445737980139E+01  0.000000000000E+00  0.224347047942E+00  0.000000000000E+00
122  0.928696688792E+02  0.000000000000E+00  0.464348344396E+01  0.000000000000E+00  0.215355564862E+00  0.000000000000E+00
123  0.975639677421E+02  0.000000000000E+00  0.487819838710E+01  0.000000000000E+00  0.204993712975E+00  0.000000000000E+00
124  0.103740573661E+03  0.000000000000E+00  0.518702868305E+01  0.000000000000E+00  0.192788600392E+00  0.000000000000E+00
125  0.112412971203E+03  0.000000000000E+00  0.562064856014E+01  0.000000000000E+00  0.177915411238E+00  0.000000000000E+00
126  0.126043252760E+03  0.000000000000E+00  0.630216263801E+01  0.000000000000E+00  0.158675689194E+00  0.000000000000E+00
127  0.153516836704E+03  0.000000000000E+00  0.767584183522E+01  0.000000000000E+00  0.130278869923E+00  0.000000000000E+00
128  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
```

8.3 Annulus, Shear Flow (Table 4.5)

This test case is given in Ref. 10 by Shankar. In order to get the correct input data,

$$
\begin{aligned}
\widetilde{r}_i &= 6.0 \\
\frac{r_i}{b} &= 6.0 \\
\frac{r_i}{r_o - r_i} &= 6.0 \\
r_i &= 6r_o - 6r_i \\
7r_i &= 6r_o \\
\frac{r_i}{r_o} &= \frac{6}{7} \\
\sigma &= \frac{6}{7} \\
b &= \frac{1}{7} \\
kb &= 10 \\
k &= 70
\end{aligned}
$$

The axial Mach number profile is given by:

$$
\begin{aligned}
M\left(\frac{r}{b}\right) &= M_0\left(1 - 2\left|\frac{r_i - r}{b} + 0.5\right|\right)^{\frac{1}{7}} \\
M_0 &= 0.3
\end{aligned}
$$

The input data for SWIRL is then:

```
&INPUTS
 MM      =    0,
 NPTS    =   64,
 SIG     = 0.857142857143,
 AKRE    =  70.,
 AKIM    =   0.,
 IX      =    0,
 NX      =    0,
 IR      =    5,
 RMAX    = 0.30,
 SLOPE   = 0.00,
 IS      =    0,
 ANGOM   = 0.00,
 GAM     = 0.00,
 IREPEAT =    0,
 IFD     =    0,
 ITEST   =    0,
 ETAHR   = 0.00,
 ETAHI   = 0.00,
 ETADR   = 0.00,
 ETADI   = 0.00,
 ED2     =  0.0,
 ED4     =  0.0,
&END
```

The output in 'gam.acc' for this test case is:

```
#    j          Re{gam}             Im{gam}            Re{gam/ak}          Im{gam/ak}      nz
     1 -0.120741787661E+04 -0.349111287122E+04 -0.172488268087E+02 -0.498730410175E+02 -0.619382545985E-02  0.179087490784E-01
     2 -0.120705227993E+04 -0.347634030347E+04 -0.172436039989E+02 -0.496620043353E+02 -0.623941639104E-02  0.179696729222E-01
     3 -0.120741787661E+04  0.349111287122E+04 -0.172488268087E+02  0.498730410175E+02 -0.619382545984E-02 -0.179087490784E-01
     4 -0.120705227993E+04  0.347634030347E+04 -0.172436039989E+02  0.496620043353E+02 -0.623941639104E-02 -0.179696729222E-01
     5  0.363090674699E+03 -0.311800080711E+04  0.518700963856E+01 -0.445428686729E+02  0.257935420021E-02  0.221499174682E-01
     6  0.364635350207E+03 -0.310984927648E+04  0.520907643153E+01 -0.444264182354E+02  0.260344331880E-02  0.222038711187E-01
     7  0.364635350207E+03  0.310984927648E+04  0.520907643153E+01  0.444264182354E+02  0.260344331880E-02 -0.222038711187E-01
     8  0.363090674699E+03  0.311800080711E+04  0.518700963856E+01  0.445428686729E+02  0.257935420021E-02 -0.221499174682E-01
     9  0.276837698579E+04  0.000000000000E+00  0.395482426541E+02  0.000000000000E+00  0.252855735904E-01  0.000000000000E+00
    10  0.271834068634E+04  0.000000000000E+00  0.388334383762E+02  0.000000000000E+00  0.257510033058E-01  0.000000000000E+00
    11 -0.283785246651E+03 -0.198121755313E+04 -0.405407495216E+01 -0.283031079018E+02 -0.495910435226E-02  0.346214777070E-01
    12 -0.288389122393E+03 -0.196261386206E+04 -0.411984460562E+01 -0.280373408866E+02 -0.513014701083E-02  0.349128897591E-01
    13 -0.283785246651E+03  0.198121755313E+04 -0.405407495216E+01  0.283031079018E+02 -0.495910435226E-02 -0.346214777070E-01
    14 -0.288389122393E+03  0.196261386206E+04 -0.411984460562E+01  0.280373408866E+02 -0.513014701082E-02 -0.349128897591E-01
    15  0.359086850807E+02 -0.163143883751E+04  0.512981215439E+00 -0.233062691073E+02  0.943942633522E-03  0.428861337933E-01
    16  0.368238415522E+02 -0.161980711915E+04  0.526054879317E+00 -0.231401017021E+02  0.981920068708E-03  0.431926993677E-01
    17  0.359086850806E+02  0.163143883751E+04  0.512981215438E+00  0.233062691073E+02  0.943942633520E-03 -0.428861337933E-01
    18  0.368238415522E+02  0.161980711915E+04  0.526054879318E+00  0.231401017021E+02  0.981920068709E-03 -0.431926993677E-01
    19 -0.742193248141E+02 -0.136832783077E+04 -0.106027606877E+01 -0.195475404395E+02 -0.276667959964E-02  0.510072640041E-01
    20 -0.771377775956E+02 -0.135027287940E+04 -0.110196825137E+01 -0.192896125629E+02 -0.295193666861E-02  0.516727361039E-01
    21 -0.742193248141E+02  0.136832783077E+04 -0.106027606877E+01  0.195475404395E+02 -0.276667959963E-02 -0.510072640041E-01
    22 -0.771377775956E+02  0.135027287940E+04 -0.110196825137E+01  0.192896125629E+02 -0.295193666861E-02 -0.516727361039E-01
    23 -0.291264334872E+01 -0.122047595092E+04 -0.416091906960E-01 -0.174353707274E+02 -0.136875091649E-03  0.573543471126E-01
    24 -0.299392234434E+01 -0.120409350298E+04 -0.427703192049E-01 -0.172013357568E+02 -0.144549122790E-03  0.581346606872E-01
    25 -0.291264334873E+01  0.122047595092E+04 -0.416091906962E-01  0.174353707274E+02 -0.136875091650E-03 -0.573543471126E-01
    26 -0.299392234443E+01  0.120409350298E+04 -0.427703192061E-01  0.172013357568E+02 -0.144549122794E-03 -0.581346606872E-01
    27 -0.334118131995E+02 -0.112830252029E+04 -0.477311617136E+00 -0.161186074327E+02 -0.183555035562E-02  0.619857437847E-01
    28 -0.351299647894E+02 -0.110503918080E+04 -0.501856639849E+00 -0.157862740114E+02 -0.201178563818E-02  0.632822141121E-01
    29 -0.334118131996E+02  0.112830252029E+04 -0.477311617136E+00  0.161186074327E+02 -0.183555035562E-02 -0.619857437847E-01
    30 -0.351299647894E+02  0.110503918080E+04 -0.501856639848E+00  0.157862740114E+02 -0.201178563818E-02 -0.632822141121E-01
    31 -0.171140520383E+02 -0.107070602389E+04 -0.244486457690E+00 -0.152958003413E+02 -0.104471890720E-02  0.653607237323E-01
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    33 -0.167024565682E+02 -0.104557021514E+04 -0.238606522403E+00 -0.149367173591E+02 -0.106920547464E-02  0.669320344334E-01
    34 -0.167024565682E+02  0.104557021514E+04 -0.238606522403E+00  0.149367173591E+02 -0.106920547464E-02 -0.669320344334E-01
    35 -0.212710709991E+02 -0.102328071558E+04 -0.303872442845E+00 -0.146182959368E+02 -0.142137999173E-02  0.683778797551E-01
    36 -0.212710709991E+02  0.102328071558E+04 -0.303872442845E+00  0.146182959368E+02 -0.142137999173E-02 -0.683778797551E-01
    37 -0.228054873142E+02 -0.998324712386E+03 -0.325792675918E+00 -0.142617816055E+02 -0.160091097336E-02  0.700808961023E-01
    38 -0.212774834152E+02 -0.976894964284E+03 -0.303964048789E+00 -0.139556423469E+02 -0.155997119231E-02  0.716216280121E-01
    39 -0.228054873142E+02  0.998324712386E+03 -0.325792675918E+00  0.142617816055E+02 -0.160091097336E-02 -0.700808961023E-01
    40 -0.208993998168E+02 -0.953074686225E+03 -0.298562854526E+00 -0.136153526604E+02 -0.160978991568E-02  0.734111999492E-01
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    46 -0.211032442940E+02 -0.885049284498E+03 -0.301474918485E+00 -0.126435612071E+02 -0.188480103133E-02  0.790466992164E-01
    47 -0.209953926758E+02  0.930873564586E+03 -0.299934181083E+00  0.132981937798E+02 -0.169519543977E-02 -0.751599479970E-01
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48	-0.214307849008E+02	0.907211471256E+03	-0.306154070012E+00	0.129601638751E+02	-0.182169962394E-02	-0.771164847049E-01
49	-0.750743340143E+02	0.867636273988E+03	-0.107249048592E+01	0.123948039141E+02	-0.692906567927E-02	-0.800794147177E-01
50	-0.211032442940E+02	0.885049284498E+03	-0.301474918485E+00	0.126435612071E+02	-0.188480103133E-02	-0.790466992164E-01
51	-0.208598209490E+02	-0.861876916702E+03	-0.297997442128E+00	-0.123125273815E+02	-0.196455319739E-02	0.811705458359E-01
52	-0.208598209490E+02	0.861876916702E+03	-0.297997442128E+00	0.123125273815E+02	-0.196455319739E-02	-0.811705458359E-01
53	-0.210016771115E+02	-0.839223423030E+03	-0.300023958735E+00	-0.119889060433E+02	-0.208605103367E-02	0.833582422869E-01
54	-0.210016771115E+02	0.839223423030E+03	-0.300023958735E+00	0.119889060433E+02	-0.208605103367E-02	-0.833582422869E-01
55	-0.208255231249E+02	-0.816057187878E+03	-0.297507473213E+00	-0.116579598268E+02	-0.218761063733E-02	0.857224749726E-01
56	-0.208255231249E+02	0.816057187878E+03	-0.297507473213E+00	0.116579598268E+02	-0.218761063733E-02	-0.857224749726E-01
57	-0.210021156194E+02	-0.793422863730E+03	-0.300030223134E+00	-0.113346123390E+02	-0.233371313108E-02	0.881635635733E-01
58	-0.210021156194E+02	0.793422863730E+03	-0.300030223134E+00	0.113346123390E+02	-0.233371313107E-02	-0.881635635733E-01
59	-0.208952736248E+02	-0.770454251176E+03	-0.298503908926E+00	-0.110064893025E+02	-0.246225523958E-02	0.907887137961E-01
60	-0.208952736248E+02	0.770454251176E+03	-0.298503908926E+00	0.110064893025E+02	-0.246225523958E-02	-0.907887137961E-01
61	-0.209430891283E+02	-0.747600810660E+03	-0.299186987547E+00	-0.106800115809E+02	-0.262094897695E-02	0.935594347072E-01
62	-0.209430891283E+02	0.747600810660E+03	-0.299186987547E+00	0.106800115809E+02	-0.262094897695E-02	-0.935594347072E-01
63	-0.208625296440E+02	-0.724630850142E+03	-0.298036137771E+00	-0.103518692877E+02	-0.277889112402E-02	0.965209047997E-01
64	-0.208625296440E+02	0.724630850142E+03	-0.298036137771E+00	0.103518692877E+02	-0.277889112402E-02	-0.965209047997E-01
65	-0.209214362523E+02	-0.701768298438E+03	-0.298877660747E+00	-0.100252614063E+02	-0.297109283760E-02	0.996594468947E-01
66	-0.209214362523E+02	0.701768298438E+03	-0.298877660747E+00	0.100252614063E+02	-0.297109283760E-02	-0.996594468947E-01
67	-0.208630001646E+02	-0.678820453272E+03	-0.298042859494E+00	-0.969743504674E+01	-0.316632088679E-02	0.103022737028E+00
68	-0.208630001646E+02	0.678820453272E+03	-0.298042859494E+00	0.969743504674E+01	-0.316632088679E-02	-0.103022737028E+00
69	-0.208784000846E+02	0.655904071130E+03	-0.298262858352E+00	0.937005815900E+01	-0.339371039949E-02	-0.106614896651E+00
70	-0.208784000846E+02	-0.655904071130E+03	-0.298262858352E+00	-0.937005815900E+01	-0.339371039949E-02	0.106614896651E+00
71	-0.208320767358E+02	0.632948449270E+03	-0.297601096225E+00	0.904212070386E+01	-0.363599880390E-02	-0.110473854031E+00
72	-0.208320767358E+02	-0.632948449270E+03	-0.297601096225E+00	-0.904212070386E+01	-0.363599880390E-02	0.110473854031E+00
73	-0.208512627129E+02	0.610008500017E+03	-0.297875181613E+00	0.871440714310E+01	-0.391788326466E-02	-0.114618578569E+00
74	-0.208512627129E+02	-0.610008500017E+03	-0.297875181613E+00	-0.871440714310E+01	-0.391788326466E-02	0.114618578569E+00
75	-0.208108332464E+02	0.587043961552E+03	-0.297297617806E+00	0.838634230788E+01	-0.422183072420E-02	-0.119091830874E+00
76	-0.208108332464E+02	-0.587043961552E+03	-0.297297617806E+00	-0.838634230788E+01	-0.422183072420E-02	0.119091830874E+00
77	-0.208154256827E+02	0.564065047174E+03	-0.297363224039E+00	0.805807210248E+01	-0.457334474415E-02	-0.123930394611E+00
78	-0.208154256827E+02	-0.564065047174E+03	-0.297363224039E+00	-0.805807210248E+01	-0.457334474415E-02	0.123930394611E+00
79	-0.207805939168E+02	0.541079945049E+03	-0.296865627382E+00	0.772971350070E+01	-0.496127165254E-02	-0.129180359516E+00
80	-0.207805939168E+02	-0.541079945049E+03	-0.296865627382E+00	-0.772971350070E+01	-0.496127165254E-02	0.129180359516E+00
81	-0.207858979626E+02	0.518064260320E+03	-0.296941399466E+00	0.740091800457E+01	-0.541253855727E-02	-0.134901209905E+00
82	-0.207858979626E+02	-0.518064260320E+03	-0.296941399466E+00	-0.740091800457E+01	-0.541253855727E-02	0.134901209905E+00
83	-0.207517019845E+02	0.495049879093E+03	-0.296452885493E+00	0.707214112990E+01	-0.591686134575E-02	-0.141151867736E+00
84	-0.207517019845E+02	-0.495049879093E+03	-0.296452885493E+00	-0.707214112990E+01	-0.591686134575E-02	0.141151867736E+00
85	-0.207513244313E+02	0.471984353313E+03	-0.296447491876E+00	0.674263361876E+01	-0.650803630990E-02	-0.148023868030E+00
86	-0.207513244313E+02	-0.471984353313E+03	-0.296447491876E+00	-0.674263361876E+01	-0.650803630990E-02	0.148023868030E+00
87	-0.207178790272E+02	0.448927212234E+03	-0.295969700389E+00	0.641324588906E+01	-0.718071174806E-02	-0.155595893898E+00
88	-0.207178790272E+02	-0.448927212234E+03	-0.295969700389E+00	-0.641324588906E+01	-0.718071174806E-02	0.155595893898E+00
89	0.541183502189E+03	0.000000000000E+00	0.773119288841E+01	0.000000000000E+00	0.129346145470E+00	0.000000000000E+00
90	0.527777955973E+03	0.000000000000E+00	0.753968508533E+01	0.000000000000E+00	0.132631534167E+00	0.000000000000E+00
91	-0.207168290590E+02	-0.425800168716E+03	-0.295954700842E+00	-0.608285955309E+01	-0.797963115552E-02	0.164008125116E+00
92	-0.206803371920E+02	-0.402681502135E+03	-0.295433388458E+00	-0.575259288764E+01	-0.890406565041E-02	0.173377372812E+00
93	-0.207168290590E+02	0.425800168716E+03	-0.295954700842E+00	0.608285955309E+01	-0.797963115552E-02	-0.164008125116E+00
94	-0.206803371920E+02	0.402681502135E+03	-0.295433388458E+00	0.575259288764E+01	-0.890406565041E-02	-0.173377372812E+00
95	-0.206760924777E+02	-0.379469735625E+03	-0.295372749681E+00	-0.542099622322E+01	-0.100213167218E-01	0.183921909381E+00
96	-0.206760924777E+02	0.379469735625E+03	-0.295372749681E+00	0.542099622322E+01	-0.100213167218E-01	-0.183921909381E+00
97	-0.206354282100E+02	-0.356261090256E+03	-0.294791831572E+00	-0.508944414652E+01	-0.113427961938E-01	0.195828111606E+00
98	-0.206354282100E+02	0.356261090256E+03	-0.294791831572E+00	0.508944414652E+01	-0.113427961938E-01	-0.195828111606E+00
99	-0.206289181188E+02	-0.332930086489E+03	-0.294698830268E+00	-0.475614409270E+01	-0.129778942411E-01	0.209450220669E+00
100	-0.206289181188E+02	0.332930086489E+03	-0.294698830268E+00	0.475614409270E+01	-0.129778942411E-01	-0.209450220669E+00
101	-0.205805304209E+02	-0.309584898047E+03	-0.294007577442E+00	-0.442264140067E+01	-0.149651136674E-01	0.225114372382E+00
102	-0.205805304209E+02	0.309584898047E+03	-0.294007577442E+00	0.442264140067E+01	-0.149651136674E-01	-0.225114372382E+00
103	-0.205698407053E+02	-0.286074515835E+03	-0.293854867219E+00	-0.408677879764E+01	-0.175037475210E-01	0.243432905928E+00
104	-0.205698407053E+02	0.286074515835E+03	-0.293854867219E+00	0.408677879764E+01	-0.175037475210E-01	-0.243432905928E+00
105	-0.205109474144E+02	-0.262511753301E+03	-0.293013534491E+00	-0.375016790430E+01	-0.207082319042E-01	0.265036721859E+00
106	-0.205109474144E+02	0.262511753301E+03	-0.293013534491E+00	0.375016790430E+01	-0.207082319042E-01	-0.265036721859E+00
107	-0.204942612897E+02	-0.238712301396E+03	-0.292775161281E+00	-0.341017573423E+01	-0.249914439251E-01	0.291094419567E+00
108	-0.204942612897E+02	0.238712301396E+03	-0.292775161281E+00	0.341017573423E+01	-0.249914439251E-01	-0.291094419567E+00
109	-0.204204262324E+02	-0.214775102881E+03	-0.291720374749E+00	-0.306821575544E+01	-0.307104831225E-01	0.323002423998E+00
110	-0.204204262324E+02	0.214775102881E+03	-0.291720374749E+00	0.306821575544E+01	-0.307104831225E-01	-0.323002423998E+00
111	-0.203933329250E+02	-0.190459220694E+03	-0.291333327500E+00	-0.272084600992E+01	-0.389073276745E-01	0.363366759883E+00
112	-0.203933329250E+02	0.190459220694E+03	-0.291333327500E+00	0.272084600992E+01	-0.389073276745E-01	-0.363366759883E+00
113	-0.202992137931E+02	-0.165795983452E+03	-0.289988768472E+00	-0.236851404932E+01	-0.509292585927E-01	0.415970125787E+00
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115	-0.202524134771E+02	-0.140393405205E+03	-0.289320192530E+00	-0.200562007435E+01	-0.704590424940E-01	0.488434867992E+00
116	-0.958004151274E+02	0.000000000000E+00	-0.136857735896E+01	0.000000000000E+00	-0.730685769022E+00	-0.000000000000E+00
117	-0.909130705564E+02	0.000000000000E+00	-0.129875815081E+01	0.000000000000E+00	-0.769966293863E+00	-0.000000000000E+00
118	-0.892966716966E+02	0.000000000000E+00	-0.127566673852E+01	0.000000000000E+00	-0.783903796973E+00	-0.000000000000E+00
119	-0.784690055423E+02	0.000000000000E+00	-0.112098579346E+01	0.000000000000E+00	-0.892071965437E+00	-0.000000000000E+00
120	-0.202524134771E+02	0.140393405205E+03	-0.289320192530E+00	0.200562007435E+01	-0.704590424941E-01	-0.488434867992E+00
121	-0.201287621222E+02	-0.113979762453E+03	-0.287553744604E+00	-0.162828232076E+01	-0.105177303116E+00	0.595569859281E+00
122	-0.201287621223E+02	0.113979762453E+03	-0.287553744604E+00	0.162828232076E+01	-0.105177303116E+00	-0.595569859281E+00
123	-0.506983875785E+02	0.000000000000E+00	-0.724262679693E+00	0.000000000000E+00	-0.138071452256E+01	-0.000000000000E+00
124	-0.200357564113E+02	-0.853889156538E+02	-0.286225091590E+00	-0.121984165220E+01	-0.182316121921E+00	0.776999661880E+00

125	-0.200357564113E+02	0.853889156538E+02	-0.286225091589E+00	0.121984165220E+01	-0.182316121921E+00	-0.776999661881E+00
126	-0.198546281750E+02	-0.514027770570E+02	-0.283637545357E+00	-0.734325386528E+00	-0.457713175468E+00	0.118499969414E+01
127	-0.198546281750E+02	0.514027770569E+02	-0.283637545358E+00	0.734325386528E+00	-0.457713175468E+00	-0.118499969414E+01
128	0.448963357643E+03	0.000000000000E+00	0.641376225205E+01	0.000000000000E+00	0.155914728470E+00	0.000000000000E+00
129	0.440733634069E+03	0.000000000000E+00	0.629619477242E+01	0.000000000000E+00	0.158826090384E+00	0.000000000000E+00
130	0.115005890022E+02	0.000000000000E+00	0.164294128603E+00	0.000000000000E+00	0.608664477849E+01	0.000000000000E+00
131	0.401187152504E+02	0.000000000000E+00	0.573124503578E+00	0.000000000000E+00	0.174482157674E+01	0.000000000000E+00
132	0.526993221422E+02	0.000000000000E+00	0.752847459174E+00	0.000000000000E+00	0.132829033002E+01	0.000000000000E+00
133	0.567458536332E+02	0.000000000000E+00	0.810655051903E+00	0.000000000000E+00	0.123357030546E+01	0.000000000000E+00
134	0.555434885843E+02	0.000000000000E+00	0.793478408347E+00	0.000000000000E+00	0.126027373837E+01	0.000000000000E+00
135	0.402641558115E+03	0.000000000000E+00	0.575202225878E+01	0.000000000000E+00	0.173851900255E+00	0.000000000000E+00
136	0.396121698821E+03	0.000000000000E+00	0.565888141173E+01	0.000000000000E+00	0.176713369170E+00	0.000000000000E+00
137	0.373309777565E+03	0.000000000000E+00	0.533299682236E+01	0.000000000000E+00	0.187511831210E+00	0.000000000000E+00
138	0.368150631881E+03	0.000000000000E+00	0.525929474116E+01	0.000000000000E+00	0.190139562283E+00	0.000000000000E+00
139	0.352330933134E+03	0.000000000000E+00	0.503329904478E+01	0.000000000000E+00	0.198676850134E+00	0.000000000000E+00
140	0.347895533404E+03	0.000000000000E+00	0.496993619148E+01	0.000000000000E+00	0.201209826741E+00	0.000000000000E+00
141	0.336356851415E+03	0.000000000000E+00	0.480509787735E+01	0.000000000000E+00	0.208112306039E+00	0.000000000000E+00
142	0.332545345539E+03	0.000000000000E+00	0.475064779342E+01	0.000000000000E+00	0.210497608639E+00	0.000000000000E+00
143	0.323605489636E+03	0.000000000000E+00	0.462293556623E+01	0.000000000000E+00	0.216312770462E+00	0.000000000000E+00
144	0.320212435720E+03	0.000000000000E+00	0.457446336742E+01	0.000000000000E+00	0.218604876612E+00	0.000000000000E+00
145	0.313118185803E+03	0.000000000000E+00	0.447311694005E+01	0.000000000000E+00	0.223557759254E+00	0.000000000000E+00
146	0.310085459170E+03	0.000000000000E+00	0.442979227386E+01	0.000000000000E+00	0.225744219633E+00	0.000000000000E+00
147	0.304281414191E+03	0.000000000000E+00	0.434687734559E+01	0.000000000000E+00	0.230050199372E+00	0.000000000000E+00
148	0.301526331610E+03	0.000000000000E+00	0.430751902300E+01	0.000000000000E+00	0.232152195883E+00	0.000000000000E+00
149	0.296704614015E+03	0.000000000000E+00	0.423863734307E+01	0.000000000000E+00	0.235924878460E+00	0.000000000000E+00
150	0.294188782745E+03	0.000000000000E+00	0.420269689635E+01	0.000000000000E+00	0.237942450922E+00	0.000000000000E+00
151	0.290112734356E+03	0.000000000000E+00	0.414446763365E+01	0.000000000000E+00	0.241285513218E+00	0.000000000000E+00
152	0.287795224939E+03	0.000000000000E+00	0.411136035628E+01	0.000000000000E+00	0.243228496980E+00	0.000000000000E+00
153	0.284310943755E+03	0.000000000000E+00	0.406158491078E+01	0.000000000000E+00	0.246209305472E+00	0.000000000000E+00
154	0.282165506001E+03	0.000000000000E+00	0.403093580002E+01	0.000000000000E+00	0.248081351233E+00	0.000000000000E+00
155	0.279153483577E+03	0.000000000000E+00	0.398790690824E+01	0.000000000000E+00	0.250758110209E+00	0.000000000000E+00
156	0.277155900145E+03	0.000000000000E+00	0.395937000208E+01	0.000000000000E+00	0.252565433257E+00	0.000000000000E+00
157	0.274530082035E+03	0.000000000000E+00	0.392185831479E+01	0.000000000000E+00	0.254981164472E+00	0.000000000000E+00
158	0.272661978839E+03	0.000000000000E+00	0.389517112628E+01	0.000000000000E+00	0.256728130185E+00	0.000000000000E+00
159	0.270354802085E+03	0.000000000000E+00	0.386221145835E+01	0.000000000000E+00	0.258919018491E+00	0.000000000000E+00
160	0.268600082685E+03	0.000000000000E+00	0.383714403836E+01	0.000000000000E+00	0.260610493117E+00	0.000000000000E+00
161	0.266559862753E+03	0.000000000000E+00	0.380799803933E+01	0.000000000000E+00	0.262605177227E+00	0.000000000000E+00
162	0.264905307952E+03	0.000000000000E+00	0.378436154217E+01	0.000000000000E+00	0.264245365792E+00	0.000000000000E+00
163	0.263090838979E+03	0.000000000000E+00	0.375844055684E+01	0.000000000000E+00	0.266067797236E+00	0.000000000000E+00
164	0.261524965584E+03	0.000000000000E+00	0.373607093692E+01	0.000000000000E+00	0.267660870707E+00	0.000000000000E+00
165	0.259903550422E+03	0.000000000000E+00	0.371290786317E+01	0.000000000000E+00	0.269330680117E+00	0.000000000000E+00
166	0.258416541210E+03	0.000000000000E+00	0.369166487443E+01	0.000000000000E+00	0.270880492681E+00	0.000000000000E+00
167	0.256961692367E+03	0.000000000000E+00	0.367088131953E+01	0.000000000000E+00	0.272414146075E+00	0.000000000000E+00
168	0.255544939220E+03	0.000000000000E+00	0.365064198885E+01	0.000000000000E+00	0.273924422897E+00	0.000000000000E+00
169	0.234425495389E+03	0.000000000000E+00	0.334893564842E+01	0.000000000000E+00	0.298602333692E+00	0.000000000000E+00
170	0.234637004147E+03	0.000000000000E+00	0.335195720211E+01	0.000000000000E+00	0.298333164687E+00	0.000000000000E+00
171	0.235363438154E+03	0.000000000000E+00	0.336233483077E+01	0.000000000000E+00	0.297412378698E+00	0.000000000000E+00
172	0.254235150367E+03	0.000000000000E+00	0.363193071953E+01	0.000000000000E+00	0.275335648508E+00	0.000000000000E+00
173	0.252881099538E+03	0.000000000000E+00	0.361258713626E+01	0.000000000000E+00	0.276809932130E+00	0.000000000000E+00
174	0.236219246406E+03	0.000000000000E+00	0.337456066294E+01	0.000000000000E+00	0.296334871375E+00	0.000000000000E+00
175	0.236798796630E+03	0.000000000000E+00	0.338283995185E+01	0.000000000000E+00	0.295609610337E+00	0.000000000000E+00
176	0.251698717418E+03	0.000000000000E+00	0.359569596311E+01	0.000000000000E+00	0.278110276914E+00	0.000000000000E+00
177	0.237809858589E+03	0.000000000000E+00	0.339728369413E+01	0.000000000000E+00	0.294352809489E+00	0.000000000000E+00
178	0.238401550533E+03	0.000000000000E+00	0.340573643618E+01	0.000000000000E+00	0.293622251380E+00	0.000000000000E+00
179	0.250400611875E+03	0.000000000000E+00	0.357715159822E+01	0.000000000000E+00	0.279552032544E+00	0.000000000000E+00
180	0.239473359952E+03	0.000000000000E+00	0.342104799931E+01	0.000000000000E+00	0.292308088107E+00	0.000000000000E+00
181	0.240124014968E+03	0.000000000000E+00	0.343034307097E+01	0.000000000000E+00	0.291516031870E+00	0.000000000000E+00
182	0.249331116082E+03	0.000000000000E+00	0.356187308689E+01	0.000000000000E+00	0.280751159742E+00	0.000000000000E+00
183	0.248082903115E+03	0.000000000000E+00	0.354404147308E+01	0.000000000000E+00	0.282163740915E+00	0.000000000000E+00
184	0.241942301496E+03	0.000000000000E+00	0.345631859279E+01	0.000000000000E+00	0.289325180290E+00	0.000000000000E+00
185	0.241223340883E+03	0.000000000000E+00	0.344604772690E+01	0.000000000000E+00	0.290187507327E+00	0.000000000000E+00
186	0.247114276078E+03	0.000000000000E+00	0.353020394397E+01	0.000000000000E+00	0.283269753213E+00	0.000000000000E+00
187	0.245910310431E+03	0.000000000000E+00	0.351300443474E+01	0.000000000000E+00	0.284656628985E+00	0.000000000000E+00
188	0.245032674911E+03	0.000000000000E+00	0.350046678444E+01	0.000000000000E+00	0.285676185943E+00	0.000000000000E+00
189	0.243073018772E+03	0.000000000000E+00	0.347247169675E+01	0.000000000000E+00	0.287979309072E+00	0.000000000000E+00
190	0.243867966407E+03	0.000000000000E+00	0.348382809153E+01	0.000000000000E+00	0.287040569663E+00	0.000000000000E+00
191	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
192	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
193	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
194	0.567911472111E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
195	0.466240772809E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
196	0.415767284045E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
197	0.383621352506E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
198	0.360695293338E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
199	0.343240039104E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
200	0.329368527703E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
201	0.318004530586E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00

202	0.308479382536E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
203	0.300350954098E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
204	0.293312800074E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
205	0.287144565956E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
206	0.281683072401E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
207	0.276804538102E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
208	0.272413165137E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
209	0.268433540210E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
210	0.264805420345E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
211	0.261480061434E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
212	0.258417575385E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
213	0.255584990968E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
214	0.252954806970E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
215	0.250503896645E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
216	0.248212667187E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
217	0.246064407246E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
218	0.244044775008E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
219	0.242141392684E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
220	0.240343522485E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
221	0.238641805624E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
222	0.237028050595E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
223	0.235495060273E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
224	0.234036489934E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
225	0.234036489934E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
226	0.235495060273E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
227	0.237028050595E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
228	0.238641805624E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
229	0.240343522485E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
230	0.242141392684E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
231	0.244044775008E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
232	0.246064407246E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
233	0.248212667187E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
234	0.250503896645E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
235	0.252954806970E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
236	0.255584990968E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
237	0.258417575385E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
238	0.261480061434E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
239	0.264805420345E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
240	0.268433540210E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
241	0.272413165137E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
242	0.276804538102E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
243	0.281683072401E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
244	0.287144565956E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
245	0.293312800074E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
246	0.300350954098E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
247	0.308479382536E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
248	0.318004530586E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00
249	0.329368527703E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
250	0.343240039104E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
251	0.360695293338E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
252	0.383621352506E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
253	0.415767284045E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
254	0.466240772809E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
255	0.567911472111E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
256	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00

The comparison with the original data is given in the table. Note that, in order to make sure of a valid comparison, 64 radial grid points were run.

In the data, note the propagating mode at index 133. This needs to be investigated further.

8.4 Lined Annulus, Shear Flow (Table 4.6)

This test case is given in Ref. 10 by Shankar. In order to get the correct input data,

$\gamma_n^{\pm}$	Shankar β_n	Kousen report	$\frac{\overline{\gamma}_n}{k}$ (current)	index
γ_0^-	0.79293	0.79353	0.793478	134
γ_1^-	0.75075	0.75292	0.752847	132
γ_2^-	0.57143	0.57320	0.573124	131
γ_3^-	-0.00969	0.16437	0.164294	130
γ_4^-	$-0.28733 + 0.73219i$	$-0.28357 + 0.73425i$	$-0.283637 + 0.734325i$	127
γ_5^-	$-0.29118 + 1.21721i$	$-0.28622 + 1.2198i$	$-0.286225 + 1.219841i$	125
γ_6^-	$-0.29248 + 1.62569i$	$-0.28766 + 1.6281i$	$-0.287553 + 1.628282i$	122
γ_7^-	$-0.29519 + 2.00221i$	$-0.28947 + 2.0055i$	$-0.289320 + 2.005620i$	120
γ_8^-	$-0.29567 + 2.36511i$	$-0.29035 + 2.3683i$	$-0.289988 + 2.368514i$	114
γ_9^-	$-0.29768 + 2.71665i$	$-0.29167 + 2.7209i$	$-0.291333 + 2.720846i$	112
γ_{10}^-	$-0.29776 + 3.06414i$	$-0.29243 + 3.0679i$	$-0.291720 + 3.068215i$	110

Table 3: Table 4.5 data

$$
\begin{aligned}
\widetilde{r}_i &= 2.0 \\
\frac{r_i}{b} &= 2.0 \\
\frac{r_i}{r_o - r_i} &= 2.0 \\
r_i &= 2r_o - 2r_i \\
3r_i &= 2r_o \\
\frac{r_i}{r_o} &= \frac{2}{3} \\
\sigma &= \frac{2}{3} \\
b &= \frac{1}{3} \\
kb &= 10 \\
k &= 30
\end{aligned}
$$

The axial Mach number profile is again given by:

$$
\begin{aligned}
M\left(\frac{r}{b}\right) &= M_0 \left(1 - 2\left|\frac{r_i - r}{b} + 0.5\right|\right)^{\frac{1}{7}} \\
M_0 &= 0.3
\end{aligned}
$$

The admittance is given as:

$$\eta \;=\; 0.3 + 0.1i$$

It is not clear from Shankar whether both walls are lined or only one. Running the code for each case, it appears that the outer wall is lined and the inner wall is hard.

The input data for SWIRL is then:

```
&INPUTS
 MM      =    0,
 NPTS    =   64,
 SIG     = 0.666666666667,
 AKRE    =  30.,
 AKIM    =   0.,
 IX      =    0,
 NX      =    0,
 IR      =    5,
 RMAX    = 0.30,
 SLOPE   = 0.00,
 IS      =    0,
 ANGOM   = 0.00,
 GAM     = 0.00,
 IREPEAT =    0,
 IFD     =    0,
 ITEST   =    0,
 ETAHR   = 0.30,
 ETAHI   = 0.10,
 ETADR   = 0.00,
 ETADI   = 0.00,
 ED2     =  0.0,
 ED4     =  0.0,
&END
```

The output in 'gam.acc' for this test case is:

```
#    j          Re{gam}            Im{gam}           Re{gam/ak}          Im{gam/ak}      nz
     1  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
     2 -0.494841896977E+03  0.150986662444E+04 -0.164947298992E+02  0.503288874813E+02 -0.588032102032E-02 -0.179420952506E-01
     3 -0.497294064275E+03  0.149993467498E+04 -0.165764688092E+02  0.499978224995E+02 -0.597444618683E-02 -0.180200803573E-01
     4 -0.505958390575E+03 -0.149630330504E+04 -0.168652796858E+02 -0.498767768346E+02 -0.608386925721E-02  0.179922180293E-01
     5 -0.498555082515E+03 -0.150141804284E+04 -0.166185027505E+02 -0.500472680945E+02 -0.597593612666E-02  0.179967643257E-01
     6  0.142037265777E+03  0.133362305854E+04  0.473457552591E+01  0.444541019515E+02  0.236896571828E-02 -0.222428057138E-01
     7  0.139416488582E+03  0.133894446939E+04  0.464721628607E+01  0.446314823132E+02  0.230795348761E-02 -0.221653951357E-01
     8  0.141036909142E+03 -0.134302692906E+04  0.470123030475E+01 -0.447675643021E+02  0.232017860864E-02  0.220939495242E-01
     9  0.139746687507E+03 -0.133783150023E+04  0.465822291691E+01 -0.445943833410E+02  0.231711106343E-02  0.221823087580E-01
    10  0.116993225396E+04  0.533196962418E+01  0.389977417986E+02  0.177732320806E+00  0.256419778038E-01 -0.116863387851E-03
    11  0.115495640560E+04  0.118684119891E+02  0.384985468533E+02  0.395613732970E+00  0.259722637678E-01 -0.266892780707E-03
    12 -0.115674962011E+03  0.847926389266E+03 -0.385583206704E+01  0.282642129755E+02 -0.473844766381E-02 -0.347340059460E-01
    13 -0.117275874013E+03  0.841217168836E+03 -0.390919580044E+01  0.280405722945E+02 -0.487701197132E-02 -0.349826956090E-01
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    15 -0.120697214617E+03 -0.843119145332E+03 -0.402324048723E+01 -0.281039715111E+02 -0.499149163909E-02  0.348675997042E-01
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31	-0.144935585303E+02	0.473271385981E+03	-0.483118617676E+00	0.157757128660E+02	-0.193940575147E-02	-0.633291848970E-01
32	-0.752876863706E+01	-0.459004952518E+03	-0.250958954569E+00	-0.153001650839E+02	-0.107175035780E-02	0.653411926712E-01
33	-0.743135075327E+01	0.458704726677E+03	-0.247711691776E+00	0.152901575559E+02	-0.105927473193E-02	-0.653843887226E-01
34	-0.744149683653E+01	-0.448246417354E+03	-0.248049894551E+00	-0.149415472451E+02	-0.111078044309E-02	0.669090325537E-01
35	-0.920576576135E+01	-0.438615627382E+03	-0.306858858712E+00	-0.146205209127E+02	-0.143490023161E-02	0.683668997924E-01
36	-0.729169063155E+01	0.447958336767E+03	-0.243056354385E+00	0.149319445589E+02	-0.108983107347E-02	-0.669527740134E-01
37	-0.905199945737E+01	0.438343085580E+03	-0.301733315246E+00	0.146114361860E+02	-0.141270763053E-02	-0.684103688589E-01
38	-0.982781908426E+01	-0.427924858523E+03	-0.327593969475E+00	-0.142641619508E+02	-0.160921610079E-02	0.700688083856E-01
39	-0.964650345945E+01	0.427678776294E+03	-0.321550115315E+00	0.142559592098E+02	-0.158137507886E-02	-0.701104354994E-01
40	-0.920768274853E+01	-0.418744951766E+03	-0.306922758284E+00	-0.139581650589E+02	-0.157457191233E-02	0.716080318458E-01
41	-0.905697339145E+01	0.418467949883E+03	-0.301899113048E+00	0.139489316628E+02	-0.155087414673E-02	-0.716565122430E-01
42	-0.909752712317E+01	-0.408529278134E+03	-0.303250904106E+00	-0.136176426045E+02	-0.163449247002E-02	0.733977508232E-01
43	-0.891499448382E+01	0.408296879301E+03	-0.297166482794E+00	0.136098959767E+02	-0.160355260706E-02	-0.734409344218E-01
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45	-0.929621464180E+01	-0.388858401426E+03	-0.309873821393E+00	-0.129619467142E+02	-0.184330112669E-02	0.771048385920E-01
46	-0.319851132466E+02	-0.372057084071E+03	-0.106617044155E+01	-0.124019028024E+02	-0.688100890280E-02	0.800412394387E-01
47	-0.914406205284E+01	-0.379388111387E+03	-0.304802068428E+00	-0.126462703796E+02	-0.190476222746E-02	0.790287882938E-01
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50	-0.318473298411E+02	0.371657633048E+03	-0.106157766137E+01	0.123885877683E+02	-0.686642836164E-02	-0.801310667209E-01
51	-0.898186122908E+01	0.379117826555E+03	-0.299395374303E+00	0.126372608852E+02	-0.187368029162E-02	-0.790866816686E-01
52	-0.891763543528E+01	0.369209586090E+03	-0.297254514509E+00	0.123069862030E+02	-0.196142517728E-02	-0.812072867415E-01
53	-0.903194288961E+01	-0.369394909142E+03	-0.301064762987E+00	-0.123131636381E+02	-0.198454553156E-02	0.811653733066E-01
54	-0.893940291858E+01	0.359477445153E+03	-0.297980097286E+00	0.119825815051E+02	-0.207404410167E-02	-0.834028940851E-01
55	-0.910218986587E+01	-0.359753714580E+03	-0.303406328862E+00	-0.119917904860E+02	-0.210852447751E-02	0.833370347403E-01
56	-0.889225304768E+01	0.349558391338E+03	-0.296408434923E+00	0.116519463779E+02	-0.218178852980E-02	-0.857670699008E-01
57	-0.899450067985E+01	-0.349811161187E+03	-0.299816689328E+00	-0.116603720396E+02	-0.220365680681E-02	0.857038955120E-01
58	-0.893126655765E+01	0.339847097800E+03	-0.297708885255E+00	0.113282365933E+02	-0.231828770710E-02	-0.882140672926E-01
59	-0.910464716525E+01	-0.340131445635E+03	-0.303488238842E+00	-0.113377148545E+02	-0.235928133903E-02	0.881380418087E-01
60	-0.890093534012E+01	0.329998891847E+03	-0.296697844671E+00	0.109999630616E+02	-0.245028213633E-02	-0.908433056529E-01
61	-0.903322163435E+01	-0.330282612282E+03	-0.301107387812E+00	-0.110094204094E+02	-0.248237711543E-02	0.907634099483E-01
62	-0.889958958471E+01	0.320210999808E+03	-0.296652986157E+00	0.106736999936E+02	-0.260185686715E-02	-0.936159112574E-01
63	-0.908471779200E+01	-0.320499735910E+03	-0.302823926400E+00	-0.106833245303E+02	-0.265111485297E-02	0.935286741645E-01
64	-0.887526605486E+01	0.310364676562E+03	-0.295842201829E+00	0.103454892187E+02	-0.276186892691E-02	-0.965815053778E-01
65	-0.903311156364E+01	-0.310658567968E+03	-0.301103718788E+00	-0.103552855989E+02	-0.280559462379E-02	0.964874619322E-01
66	-0.888274470653E+01	0.300570433265E+03	-0.296091490218E+00	0.100190144422E+02	-0.294711296615E-02	-0.997231205422E-01
67	-0.908038034459E+01	-0.300863070781E+03	-0.302679344820E+00	-0.100287690260E+02	-0.300671392218E-02	0.996223890695E-01
68	-0.886297830793E+01	0.290732733321E+03	-0.295432610264E+00	0.969109111071E+01	-0.314274874427E-02	-0.103091748712E+00
69	-0.904203678693E+01	-0.291032569029E+03	-0.301401226231E+00	-0.970108563428E+01	-0.319952367891E-02	0.102981841136E+00
70	-0.885701100393E+01	0.280917084543E+03	-0.295233700131E+00	0.936390281810E+01	-0.336372612447E-02	-0.106687022932E+00
71	-0.906811637152E+01	-0.281213652143E+03	-0.302270545717E+00	-0.937378840476E+01	-0.343648285443E-02	0.106569639650E+00
72	-0.884016079083E+01	0.271077478227E+03	-0.294672026361E+00	0.903591594091E+01	-0.360522955685E-02	-0.110551896037E+00
73	-0.903898154434E+01	-0.271381807093E+03	-0.301299384811E+00	-0.904606023642E+01	-0.367788179221E-02	0.110422861486E+00
74	-0.883679750612E+01	0.261250410280E+03	-0.294559916871E+00	0.870834700935E+01	-0.387976650971E-02	-0.114701122409E+00
75	-0.906314714190E+01	-0.261551324261E+03	-0.302104904730E+00	-0.871837747538E+01	-0.396976942885E-02	0.114562682793E+00
76	-0.881985480292E+01	0.251407374339E+03	-0.293995160097E+00	0.838024581130E+01	-0.418111863130E-02	-0.119181560284E+00
77	-0.881191261802E+01	0.241563896357E+03	-0.293730420601E+00	0.805212987858E+01	-0.452428399316E-02	-0.124025704406E+00
78	-0.903881993856E+01	-0.251716344723E+03	-0.301293997952E+00	-0.839054482409E+01	-0.427415749664E-02	0.119028292314E+00
79	-0.879552096851E+01	0.231713064885E+03	-0.293184032284E+00	0.772376882951E+01	-0.490745665081E-02	-0.129284191968E+00
80	-0.905578809533E+01	-0.241869622526E+03	-0.301859603178E+00	-0.806232075085E+01	-0.463742052199E-02	0.123860136670E+00
81	-0.878788292286E+01	0.221853292690E+03	-0.292929430762E+00	0.739510975633E+01	-0.534801804324E-02	-0.135012655798E+00
82	-0.903634292032E+01	-0.232026944498E+03	-0.301211430677E+00	-0.773423148326E+01	-0.502781058034E-02	0.129099519214E+00
83	-0.876950546294E+01	0.211990594959E+03	-0.292316848765E+00	0.706635316531E+01	-0.584414002315E-02	-0.141273955044E+00
84	-0.905240047945E+01	-0.222164468182E+03	-0.301746682648E+00	-0.740548227275E+01	-0.549307290878E-02	0.134811271799E+00
85	-0.875952082791E+01	0.202109640725E+03	-0.291984027597E+00	0.673698802415E+01	-0.642114583187E-02	-0.148155989650E+00
86	-0.903511935829E+01	-0.212310101908E+03	-0.301170645276E+00	-0.707700339695E+01	-0.600244267121E-02	0.141047302718E+00
87	-0.873921254405E+01	0.192229591713E+03	-0.291307084802E+00	0.640765305709E+01	-0.708037702703E-02	-0.155741490233E+00
88	-0.904877568037E+01	-0.202427026509E+03	-0.301625856012E+00	-0.674756755031E+01	-0.661160861229E-02	0.147906006194E+00
89	-0.872800653443E+01	0.182322226120E+03	-0.290933551148E+00	0.607740753734E+01	-0.785892183806E-02	-0.164167627369E+00
90	0.233541341856E+03	-0.415769538715E-01	0.778471139520E+01	-0.138589846238E-02	0.128456909645E+00	0.228689574374E-04
91	0.227775772988E+03	-0.248422186077E-01	0.759252576626E+01	-0.828073953588E-03	0.131708474741E+00	0.143647003326E-04
92	-0.903398120698E+01	-0.192555515388E+03	-0.301132706899E+00	-0.641851717961E+01	-0.729346028465E-02	0.155457042903E+00
93	-0.904757920794E+01	-0.182646798584E+03	-0.301585973598E+00	-0.608822661946E+01	-0.811643191268E-02	0.163849386748E+00
94	-0.870348120172E+01	0.172416734835E+03	-0.290116040057E+00	0.574722449451E+01	-0.876092923237E-02	-0.173554785420E+00
95	-0.903366807175E+01	-0.172750232910E+03	-0.301122269058E+00	-0.575834109700E+01	-0.905653578557E-02	0.173187530678E+00
96	-0.868947465762E+01	0.162473437250E+03	-0.289649155254E+00	0.541578124166E+01	-0.984712767682E-02	-0.184118919006E+00
97	-0.904711042114E+01	-0.162806484361E+03	-0.301570347371E+00	-0.542688281203E+01	-0.102081893478E-01	0.183700578642E+00
98	-0.865952770748E+01	0.152530586203E+03	-0.288650923583E+00	0.508435287342E+01	-0.111302278811E-01	-0.196049974158E+00
99	-0.903422219812E+01	-0.152873108479E+03	-0.301140739937E+00	-0.509577028263E+01	-0.115567510521E-01	0.195558224992E+00
100	-0.864182197255E+01	0.142536668707E+03	-0.288060732418E+00	0.475122229023E+01	-0.127139321060E-01	-0.209701326215E+00
101	-0.904875383972E+01	-0.142879926638E+03	-0.301625127991E+00	-0.476266422125E+01	-0.132443055246E-01	0.209127735734E+00
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106	-0.852885156158E+01	0.112375755408E+03	-0.284295052053E+00	0.374585851361E+01	-0.201452251859E-01	-0.265432559330E+00

107	-0.904153907991E+01	-0.112743531948E+03	-0.301384635997E+00	-0.375811773159E+01	-0.212029453779E-01	0.264390269009E+00
108	-0.849424202667E+01	0.102183778772E+03	-0.283141400889E+00	0.340612595906E+01	-0.242376920890E-01	-0.291573863634E+00
109	-0.906229141429E+01	-0.102556395545E+03	-0.302076380476E+00	-0.341854651818E+01	-0.256481412158E-01	0.290255609236E+00
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111	0.189759766976E+03	0.138129334324E-01	0.632532556587E+01	0.460431114414E-03	0.158094628377E+00	-0.115079745965E-04
112	-0.842434935075E+01	0.919353541009E+02	-0.280811645025E+00	0.306451180336E+01	-0.296524859915E-01	-0.323599091882E+00
113	-0.905454087297E+01	-0.923221185214E+02	-0.301818029099E+00	-0.307740395071E+01	-0.315659403585E-01	0.321853369254E+00
114	-0.836955207578E+01	0.815256520043E+02	-0.278985069193E+00	0.271752173348E+01	-0.373836462408E-01	-0.364144473502E+00
115	-0.908436302123E+01	-0.819214231273E+02	-0.302812100708E+00	-0.273071410424E+01	-0.401155612140E-01	0.361756113942E+00
116	-0.826549491684E+01	0.709726958254E+02	-0.275516497228E+00	0.236575652751E+01	-0.485687330302E-01	-0.417041441639E+00
117	-0.908860924559E+01	-0.713869169742E+02	-0.302953641520E+00	-0.237956389914E+01	-0.526499999363E-01	0.413541948232E+00
118	-0.816475628873E+01	0.601070472972E+02	-0.272158542958E+00	0.200356824324E+01	-0.665691892863E-01	-0.490066974135E+00
119	-0.914261469033E+01	-0.605381107644E+02	-0.304753823011E+00	-0.201793702548E+01	-0.731711565606E-01	0.484505114857E+00
120	-0.409294296057E+02	-0.102275549244E+00	-0.136431432019E+01	-0.340918497480E-02	-0.732964347809E+00	0.183155084179E-02
121	-0.376998993367E+02	-0.478185598414E+00	-0.125666331122E+01	-0.159395199471E-01	-0.795630088858E+00	0.100917736347E-01
122	-0.390495859743E+02	0.402298805433E-01	-0.130165286581E+01	0.134099601811E-02	-0.768253168647E+00	-0.791474030532E-03
123	-0.327986307863E+02	-0.133296384135E+01	-0.109328769288E+01	-0.444321280451E-01	-0.913164075960E+00	0.371117533048E-01
124	-0.797965232317E+01	0.488247061987E+02	-0.265988410772E+00	0.162749020662E+01	-0.978087639356E-01	-0.598457673268E+00
125	-0.919278267170E+01	-0.492832734309E+02	-0.306426089057E+00	-0.164277578103E+01	-0.109727522001E+00	0.588258382994E+00
126	-0.208223941310E+02	-0.275985168587E+01	-0.694079804366E+00	-0.919950561958E-01	-0.141588297921E+01	0.187664636572E+00
127	-0.935159157686E+01	-0.371328773015E+02	-0.311719719229E+00	-0.123776257672E+01	-0.191330016778E+00	0.759724585779E+00
128	-0.978428784055E+01	-0.228488803982E+02	-0.326142928018E+00	-0.761629346605E+00	-0.475116402390E+00	0.110952151350E+01
129	-0.771927763371E+01	0.366406367650E+02	-0.257309254457E+00	0.122135455883E+01	-0.165162574769E+00	-0.783967386126E+00
130	-0.706875252877E+01	0.223114268841E+02	-0.235625084292E+00	0.743714229471E+00	-0.387140086033E+00	-0.122194795875E+01
131	0.429293559785E+01	0.218566893343E+01	0.143097853262E+00	0.728556311144E-01	0.554966818108E+01	-0.282551113516E+01
132	0.172989483152E+03	0.954016363547E-04	0.576631610508E+01	0.318005454516E-05	0.173420947062E+00	-0.956395835537E-07
133	0.170155153961E+03	-0.150075958345E-02	0.567183846537E+01	-0.500253194484E-04	0.176309675607E+00	0.155504214338E-05
134	0.168036436292E+02	0.941135917953E+00	0.560121454306E+00	0.313711972651E-01	0.177974423679E+01	-0.996796446639E-01
135	0.220780843504E+02	0.789988134835E+00	0.735936145015E+00	0.263329378278E-01	0.135707605121E+01	-0.485582880068E-01
136	0.236168578482E+02	0.112727201685E+00	0.787228594938E+00	0.375757338951E-02	0.127025012027E+01	-0.606311569553E-02
137	0.243048665808E+02	0.802396851462E-02	0.810162219361E+00	0.267465617154E-03	0.123432056829E+01	-0.407496553991E-03
138	0.160262305481E+03	0.166959025035E-02	0.534207684938E+01	0.556530083449E-04	0.187193113856E+00	-0.195015163975E-05
139	0.158046662603E+03	0.429825522785E-02	0.526822208678E+01	0.143275174262E-03	0.189817358264E+00	-0.516229472395E-05
140	0.151166680900E+03	0.127309181889E-03	0.503888936335E+01	0.424363939628E-05	0.198456431148E+00	-0.167135546930E-06
141	0.149249629763E+03	-0.353596334592E-03	0.497498765877E+01	-0.117865444864E-04	0.201005523749E+00	0.476214356733E-06
142	0.144267122962E+03	0.370219381443E-03	0.480890409872E+01	0.123406460481E-04	0.207947586283E+00	-0.533636667771E-06
143	0.142629866522E+03	0.138941377248E-02	0.475432888408E+01	0.463137924161E-04	0.210334628564E+00	-0.204895255729E-05
144	0.137304249568E+03	0.776168320958E-05	0.457680831892E+01	0.258722773653E-06	0.218492873269E+00	-0.123512015921E-07
145	0.138764932108E+03	-0.314488861161E-06	0.462549773694E+01	-0.104829620387E-07	0.216192949791E+00	0.489967267218E-09
146	0.134248376559E+03	0.737057017666E-04	0.447494588530E+01	0.245685672555E-05	0.223466389456E+00	-0.122688612543E-06
147	0.132946552481E+03	0.528471004459E-03	0.443155174937E+01	0.176157001486E-04	0.225654591561E+00	-0.896991358086E-06
148	0.130445829958E+03	-0.692348900856E-05	0.434819433192E+01	-0.230782966952E-06	0.229980521491E+00	0.122063511976E-07
149	0.129262626360E+03	0.121542304264E-03	0.430875421200E+01	0.405141014214E-05	0.232085644898E+00	-0.218224129139E-06
150	0.127188726737E+03	0.156962828962E-04	0.423962422458E+01	0.523209429874E-06	0.235869960880E+00	-0.291085674624E-07
151	0.126109627794E+03	0.253756296271E-03	0.420365425980E+01	0.845854320905E-05	0.237888260592E+00	-0.478675934505E-06
152	0.124356573422E+03	-0.178698647906E-05	0.414521911405E+01	-0.595662159686E-07	0.241241770938E+00	0.346661033662E-08
153	0.123362580297E+03	0.128259639883E-03	0.411208600989E+01	0.427532132944E-05	0.243185574814E+00	-0.252839184908E-06
154	0.121865179852E+03	0.412215770221E-05	0.406217266172E+01	0.137405256740E-06	0.246173681740E+00	-0.832696213554E-08
155	0.120945513396E+03	0.150659799065E-03	0.403151711319E+01	0.502199330217E-05	0.248045579845E+00	-0.308986221723E-06
156	0.119651213632E+03	0.989427718317E-07	0.398837378774E+01	0.329809239439E-08	0.250728756435E+00	-0.207334279248E-09
157	0.118795062752E+03	0.106377713959E-03	0.395983542508E+01	0.354592379862E-05	0.252535747740E+00	-0.226138821892E-06
158	0.117667077025E+03	0.143161281942E-05	0.392223590084E+01	0.477204273142E-07	0.254956617929E+00	-0.310196506833E-08
159	0.116866665381E+03	0.102602616175E-03	0.389555551269E+01	0.342008720584E-05	0.256702798033E+00	-0.225371183236E-06
160	0.115875620238E+03	0.413461985341E-06	0.386252067459E+01	0.137820661780E-07	0.258898290585E+00	-0.923788809132E-09
161	0.115123919689E+03	0.830439808969E-04	0.383746398964E+01	0.276813269656E-05	0.260588764533E+00	-0.187974214587E-06
162	0.114247630554E+03	0.650955917718E-06	0.380825435182E+01	0.216985305906E-07	0.262587502729E+00	-0.149616134698E-08
163	0.113538990550E+03	0.750967029457E-04	0.378463301835E+01	0.250322343152E-05	0.264226411161E+00	-0.174764036682E-06
164	0.112759648445E+03	0.352366833181E-06	0.375865494815E+01	0.117455611060E-07	0.266052620896E+00	-0.831397763100E-09
165	0.112089108765E+03	0.641198882352E-04	0.373630362549E+01	0.213732960784E-05	0.267644201391E+00	-0.153104226353E-06
166	0.111392657092E+03	0.359098059952E-06	0.371308856975E+01	0.119699353317E-07	0.269317572478E+00	-0.868202808984E-09
167	0.110756013046E+03	0.568993187026E-04	0.369186710155E+01	0.189664395675E-05	0.270865654828E+00	-0.139153358772E-06
168	0.110131033338E+03	0.250199442928E-06	0.367103444460E+01	0.833998143094E-08	0.272402783218E+00	-0.618853946499E-09
169	0.109524587210E+03	0.495777166096E-04	0.365081957368E+01	0.165259055365E-05	0.273911098540E+00	-0.123989390561E-06
170	0.100558158854E+03	0.964210740509E-05	0.335193862845E+01	0.321403580170E-06	0.298334817801E+00	-0.286060961011E-07
171	0.100556911337E+03	-0.388860050521E-05	0.335189704456E+01	-0.129620016840E-06	0.298338518965E+00	0.115369426144E-07
172	0.100869128876E+03	0.673338002964E-05	0.336230429587E+01	0.224446000988E-06	0.297415079661E+00	-0.198535347753E-07
173	0.108961826705E+03	0.220629301714E-06	0.363206089015E+01	0.735431005713E-08	0.275325780664E+00	-0.557488219213E-09
174	0.108382350509E+03	0.437500576956E-04	0.361274501696E+01	0.145833525652E-05	0.276797835248E+00	-0.111733333013E-06
175	0.101236931005E+03	0.215910010180E-07	0.337456436683E+01	0.719700033935E-09	0.296334546121E+00	-0.631998562526E-10
176	0.101494077854E+03	0.121871507074E-04	0.338313592847E+01	0.406238356914E-06	0.295583748671E+00	-0.354929446907E-07
177	0.107874200116E+03	0.169790916643E-06	0.359580667052E+01	0.565969722143E-08	0.278101714477E+00	-0.437724172882E-09
178	0.107318800233E+03	0.383481724095E-04	0.357729334109E+01	0.127827241365E-05	0.279540955871E+00	-0.998882278596E-07
179	0.101918731376E+03	0.335842349200E-07	0.339729104586E+01	0.111947449733E-08	0.294352172510E+00	-0.969948544036E-10
180	0.102174169275E+03	0.156223075765E-04	0.340580564250E+01	0.520743585883E-06	0.293616284946E+00	-0.448935767762E-07
181	0.106859007598E+03	0.142263031289E-06	0.356196691995E+01	0.474210104297E-08	0.280743763902E+00	-0.373758467028E-09
182	0.106325124390E+03	0.336931981006E-04	0.354417081300E+01	0.112310660335E-05	0.282153443715E+00	-0.894111521467E-07
183	0.102632210246E+03	0.448204618613E-07	0.342107367488E+01	0.149401539538E-08	0.292305894300E+00	-0.127652762772E-09

184	0.102913849259E+03	0.188349638447E-04	0.343046164197E+01	0.627832128156E-06	0.291505955865E+00	-0.533504885763E-07
185	0.105908484816E+03	0.113250113674E-06	0.353028282719E+01	0.377500378912E-08	0.283263423627E+00	-0.302899385079E-09
186	0.105393684908E+03	0.294503903255E-04	0.351312283027E+01	0.981679677518E-06	0.284647035790E+00	-0.795395503661E-07
187	0.103382582416E+03	0.585201176990E-07	0.344608608055E+01	0.195067058997E-08	0.290184277649E+00	-0.164259952552E-09
188	0.103692628661E+03	0.221204956468E-04	0.345642095537E+01	0.737349854895E-06	0.289316611869E+00	-0.617192073924E-07
189	0.105015955071E+03	0.925692874457E-07	0.350053183569E+01	0.308564291486E-08	0.285670877152E+00	-0.251812684313E-09
190	0.104518245558E+03	0.256394092275E-04	0.348394151860E+01	0.854646974249E-06	0.287031224451E+00	-0.704117351516E-07
191	0.104175713449E+03	0.736346384247E-07	0.347252378162E+01	0.245448794749E-08	0.287974989629E+00	-0.203549690563E-09
192	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
193	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
194	0.243390630905E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
195	0.199817474061E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
196	0.178185978876E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
197	0.164409151074E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
198	0.154583697145E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
199	0.147102873902E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
200	0.141157940444E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
201	0.136287655966E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00
202	0.132205449658E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
203	0.128721837471E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
204	0.125705485746E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
205	0.123061956838E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
206	0.120721316743E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
207	0.118630516330E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
208	0.116748499344E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
209	0.115042945804E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
210	0.113488037291E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
211	0.112062883472E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
212	0.110750389451E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
213	0.109536424701E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
214	0.108409202987E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
215	0.107358812848E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
216	0.106376857366E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
217	0.105456174534E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
218	0.104590617860E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
219	0.103774882579E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
220	0.103004366779E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
221	0.102275059553E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
222	0.101583450255E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
223	0.100926454403E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
224	0.100301352829E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
225	0.100301352829E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
226	0.100926454403E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
227	0.101583450255E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
228	0.102275059553E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
229	0.103004366779E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
230	0.103774882579E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
231	0.104590617860E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
232	0.105456174534E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
233	0.106376857366E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
234	0.107358812848E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
235	0.108409202987E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
236	0.109536424701E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
237	0.110750389451E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
238	0.112062883472E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
239	0.113488037291E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
240	0.115042945804E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
241	0.116748499344E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
242	0.118630516330E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
243	0.120721316743E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
244	0.123061956838E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
245	0.125705485746E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
246	0.128721837471E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
247	0.132205449658E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
248	0.136287655966E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00
249	0.141157940444E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
250	0.147102873902E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
251	0.154583697145E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
252	0.164409151074E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
253	0.178185978876E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
254	0.199817474061E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
255	0.243390630905E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
256	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00

The comparison with the original data is given in the table. Note that,

$\gamma_n^{\pm}$	Shankar β_n	Kousen report	$\frac{\overline{\gamma}_n}{k}$ (current)	index
γ_0^-	$0.78698 + 0.00400i$	$0.78093 + 0.00913i$	$0.787228 + 0.003757i$	136
γ_1^-	$0.73438 + 0.02541i$	$0.75079 + 0.03387i$	$0.735936 + 0.026332i$	135
γ_2^-	$0.55840 + 0.03148i$	$0.57267 + 0.03246i$	$0.560121 + 0.031371i$	134
γ_3^-	$0.14308 + 0.07638i$	$0.16875 + 0.06982i$	$0.143097 + 0.072855i$	131
γ_4^-	$-0.23900 + 0.74173i$	$-0.23734 + 0.72727i$	$-0.235625 + 0.743714i$	130
γ_5^-	$-0.26149 + 1.21973i$	$-0.25993 + 1.2120i$	$-0.257309 + 1.221354i$	129
γ_6^-	$-0.26996 + 1.62627i$	$-0.26860 + 1.6207i$	$-0.265988 + 1.627490i$	124
γ_7^-	$-0.27669 + 2.00192i$	$-0.27468 + 1.9983i$	$-0.272158 + 2.003568i$	118
γ_8^-	$-0.27974 + 2.36439i$	$-0.27813 + 2.3612i$	$-0.275516 + 2.365756i$	116
γ_9^-	$-0.28359 + 2.71568i$	$-0.28147 + 2.7139i$	$-0.278985 + 2.717521i$	114
γ_{10}^-	$-0.28502 + 3.06304i$	$-0.28361 + 3.0610i$	$-0.280811 + 3.064511i$	112

Table 4: Table 4.6 data

in order to make sure of a valid comparison, 64 radial grid points were run (though the 32 point dataset looked good too).

In the data, note the (very nearly) propagating mode at index 137. This needs to be investigated further – are these modes real?