# Verification and Validation of an Acoustic Mode Prediction Code for Turbomachinery Noise

## Master’s Thesis Defense

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September 2022

# Outline

Section 1

# Results Update

# Motivation

## How is the analytical solution computed for Sound Prop. in Uniform Axial Flow

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5. On the other hand, if the $k_x$ is complex, then the mode will resemble an exponentially decaying function since the imaginary number cancels, leaving a minus sign in front of the axial wavenumber.

$$k_x = \frac{-M_x k \pm \sqrt{k^2 - (1 - M_x^2) J'^2_{m,n}}}{\left(1 - M_x^2\right)}. \tag{1}$$

where $M_x$ is the axial Mach number, $k$ is the temporal (referred to as reduced) frequency, and $J'_{m,n}$ is the derivative of the Bessel function of the first kind. The $\pm$ accounts for both upstream and downstream modes.

The condition for propagation is such that the axial wavenumber is larger than a “cut-off” value

$$k_{x,real} = \frac{\pm M_x k}{\left(M_x^2 - 1\right)}. \tag{2}$$

Every term that is being raised to the one half i.e. square rooted must be larger than zero to keep axial wavenumber from being imaginary. The mode will propagate or decay based on this condition. Recall thaT the mode is of the form

$$e^{ik_x x} \tag{3}$$

if $k_x$ has a real part, $k_{x,real}$ and an imaginary part $ik_{x,imag}$ then,

$$\begin{aligned} &= e^{ik_x x} && (4)\\ &= e^{i(k_{x,real}+ik_{x,imag})x} && (5)\\ &= \underbrace{e^{ik_{x,real}x}}_{amplitude}\ \underbrace{e^{-k_{x,imag}x}}_{exponential\ decay} && (6) \end{aligned}$$

Although the “cut-off” decay to nearly zero rapidly, the rate at which this occured was not much of a concern earlier on in turbomachinery design. As nacelles continue to grow shorter, a mode that is “cut-off” may make it outside the duct.
For this work a desired amplitude was arbitrarily chosen for a mode, $y_{desired}$ and then the axial location at which this occurred, $x_{desired}$ which can be compared against a desired length for a nacelle. Since SWIRL assumes an infinitely long duct, there is nothing limiting the modes propagation with respect to nacelle length. For example, if the desired amplitude is one percent, then $x_{desired}$,

$$
\begin{aligned}
0.01 &= e^{-10x_{desired}}, \\
-\frac{ln|0.01|}{10} &= x_{desired}, \\
-\frac{ln|0.01|}{10} &= 0.4605170185988091.
\end{aligned}
$$

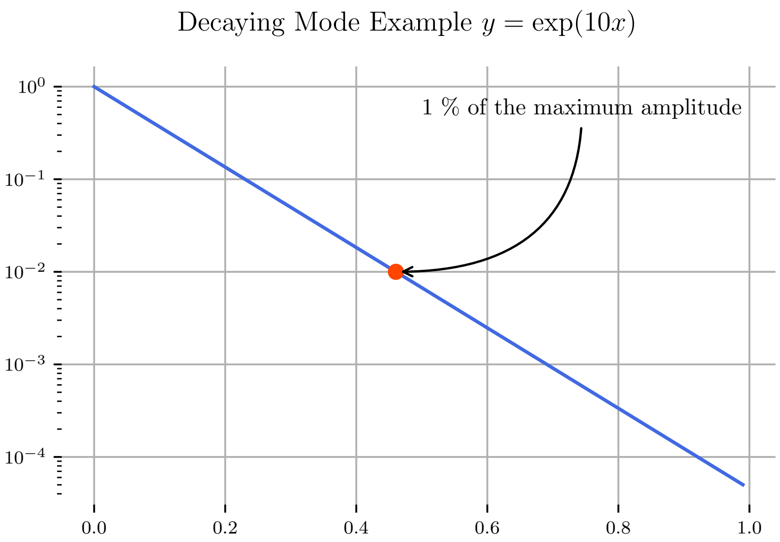



Figure: Decaying mode with $k_x = 0 + 10j$ and unit amplitude. One percent of the maximum amplitude is identified for nacelle length comparison

In general,

| | |
|---|---|
| $\sigma$ | *0.0* |
| $k$ | *10* |
| $m$ | *2* |
| $M_x$ | *0.3* |

Table: Validation test case parameters, Uniform Flow Annular Duct

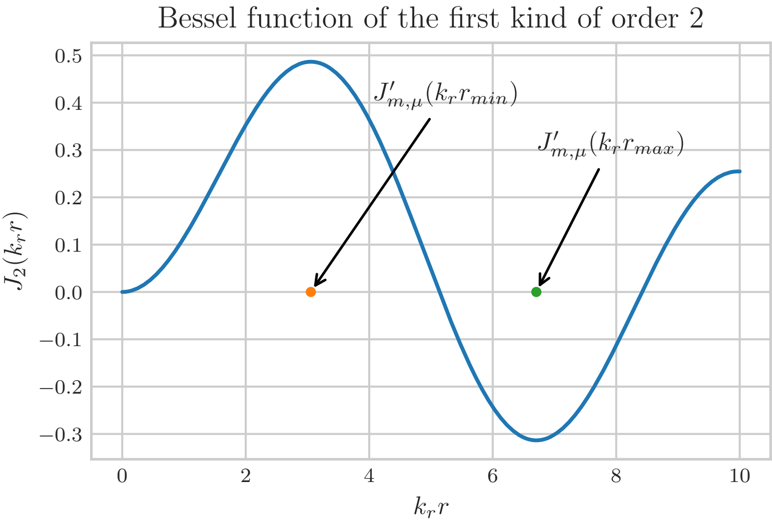
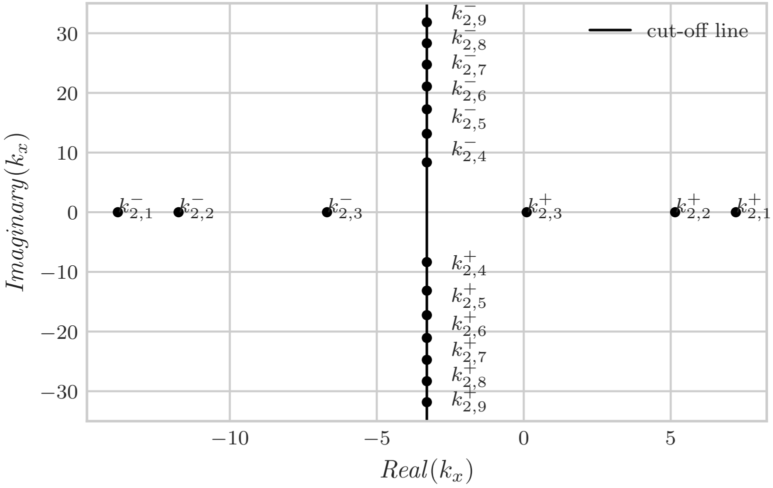



Figure: The Bessel function with the values of $J'_{m,\mu}$ labeled

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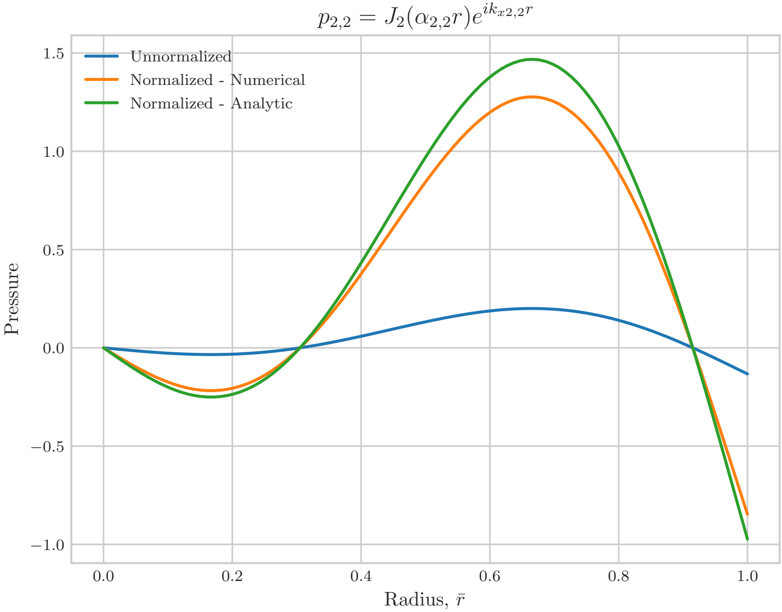
# Normalized Mode



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7. Check the analytical test case that has been reported in literature. The difference is that $\sigma = 0.25$