Analytical Solution for Duct Mode Propagation in Uniform Flow

Swirl Validation

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This document shows the analytical duct mode solution as well as a numerical comparison.

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October 25, 2022

0.1 Introduction - Turbomachinery Noise

Turbomachinery noise generation occurs from pressure fluctuations from the series of fans within it's annular duct. While the jet that is produced from this stream of air freely radiates to the observer, the pressure fluctuations produced from the rotor may or may not propagate out of the inlet and exhaust and radiate to the observer. The production of this propagation can be characterized by standing waves referred to as modes, in particular, duct modes because the mode itself is dependent on the geometry of the column of air within the annular duct, as well as the speed of the flow moving through it

0.2 Duct Mode Theory

The pressure field within a duct is governed by the convective wave equation, a second order ODE as a function of radius.

The solution of the convective wave equation are eigenvalues and eigenvectors which may or may not correspond to acoustic disturbances fall into two groups. One group corresponding to the acoustics propagation and the other group corresponding to the convection speed of the flow. Both are modes that are a result from the pressure distribution from within the cylindrical domain.

0.2.1 Analytic Solution: Axial wavenumbers and Pressure Modes

Modes can be categorized based on the sign of the axial wavenumber and if it is complex in value. For example, for the uniform axial flow case, propagating modes are defined by axial wavenumbers, k_x, that have a real-part only, yielding the assumed fluctuation to resemble Euler's Formula ($e^{ik_x x}$). On the other hand, if the k_x is complex, then the mode will resemble an exponentially decaying function since the imaginary number cancels, leaving a minus sign in front of the axial wavenumber. These two distinctions are referred to as "cut-on" and "cut-off" in the field of ducted sound propagation. Furthermore, the sign of the imaginary part will change the direction of the mode's decay. If k_x is positive, the decay rate occurs in the negative direction. Conversely, if k_x is negative, the decay occurs in the positive direction. The axial wavenumber for uniform axial flow in a hollow duct is,

$$k_x = \frac{-M_x k \pm \sqrt{k^2 - (1 - M_x^2) J'^2_{m,n}}}{(1 - M_x^2)}. \tag{1}$$

where M_x is the axial Mach number, k is the temporal (referred to as reduced) frequency, and $J'_{m,n}$ is the derivative of the Bessel function of the first kind. The $\pm$ accounts for both upstream and downstream modes.

The condition for propagation is such that the axial wavenumber is larger than a "cut-off" value

$$k_{x,real} = \frac{\pm M_x k}{(M_x^2 - 1)}. \tag{2}$$

Every term that is being raised to the one half i.e. square rooted must be larger than zero to keep axial wavenumber from being imaginary. The mode will propagate or decay based on this condition.

0.2.2 Methods

Bessel's Function,

$$R(r) = AJ_0(k_r r) + BY_0(k_r r) \tag{3}$$

0.3 Analytical Test Case

The results present a comparison of analytical and numerical solutions for a uniform flow, hard-wall, cylindrical duct for six grids. The physical parameters for this test case are reporte in Table 1. The grid points were doubled each iteration and a starting grid of 33 points was chosen. The first five radial modes were chosen in order to get a group of modes that were cut on and cut off. The case number's are used as identifiers for each mode propagating up or downstream. In the event that a spurious mode is identified, the case number will index it. The number of cut off modes was also constrained by setting a maximum value for the imaginary part of the axial wavenumber. While this magnitude is currently arbitrary, the value could be correlated to a desired decay rate for the corresponding mode. Each of the axial wavenumber's radial mode for each grid in also reported. The same test case was ran twice using second and fourth order central schemes for the radial derivatives. The approximate rate of convergence is also reported for both numerical schemes.

σ	*0.0*
k	*10*
m	*2*
M_x	*0.3*

Table 1: Validation test case parameters, Uniform Flow Hard-wall CylindricalDuct

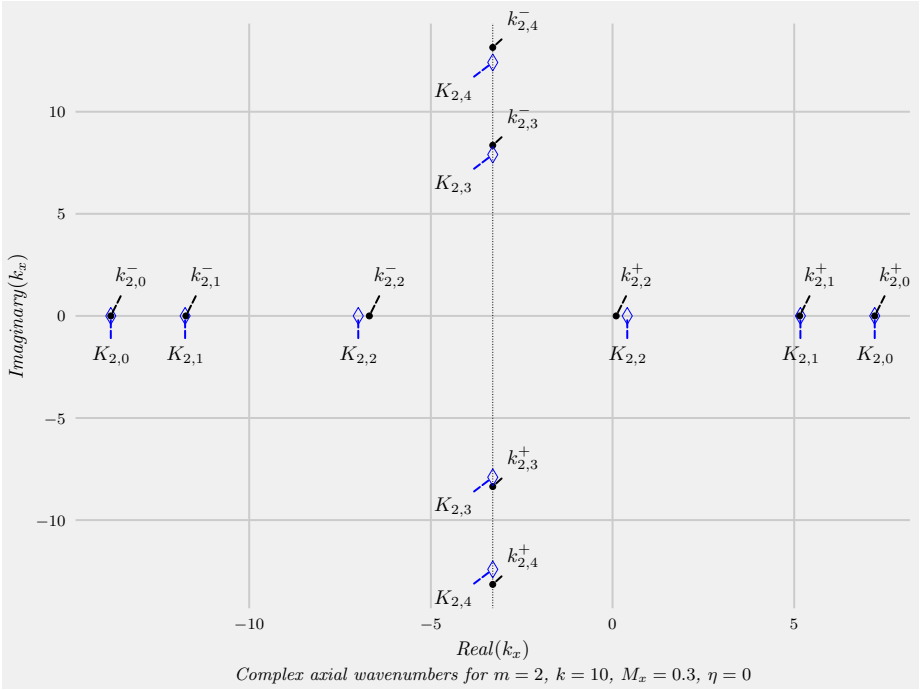

Figure 1: Second Order, Analytical Solution vs Numerical Approximation using 33 points

0.3.1 Axial Wavenumber

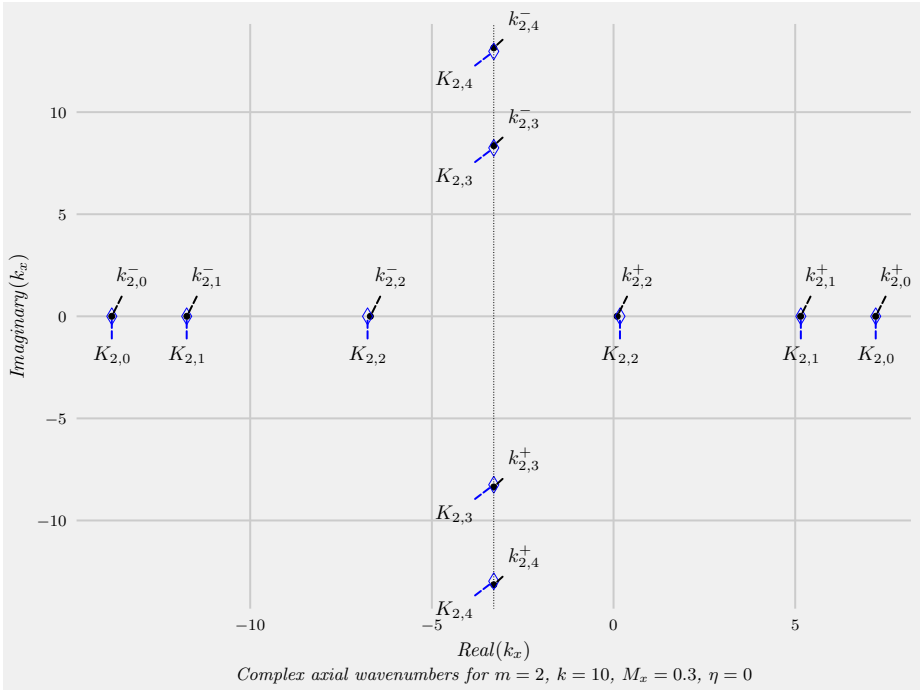

Figure 2: Second Order, Analytical Solution vs Numerical Approximation using 66 points

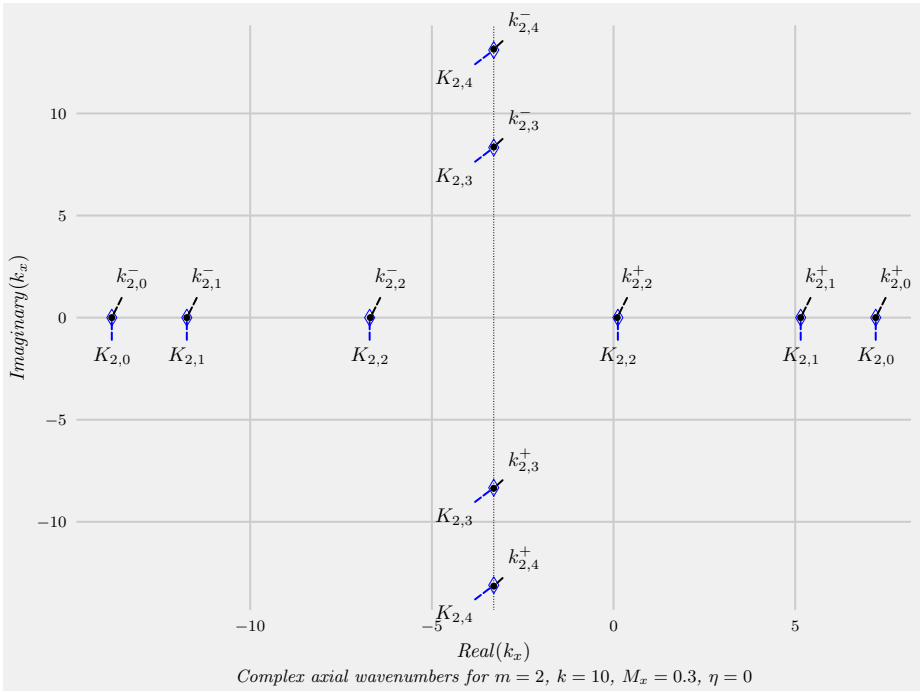

Figure 3: Second Order, Analytical Solution vs Numerical Approximation using 132 points

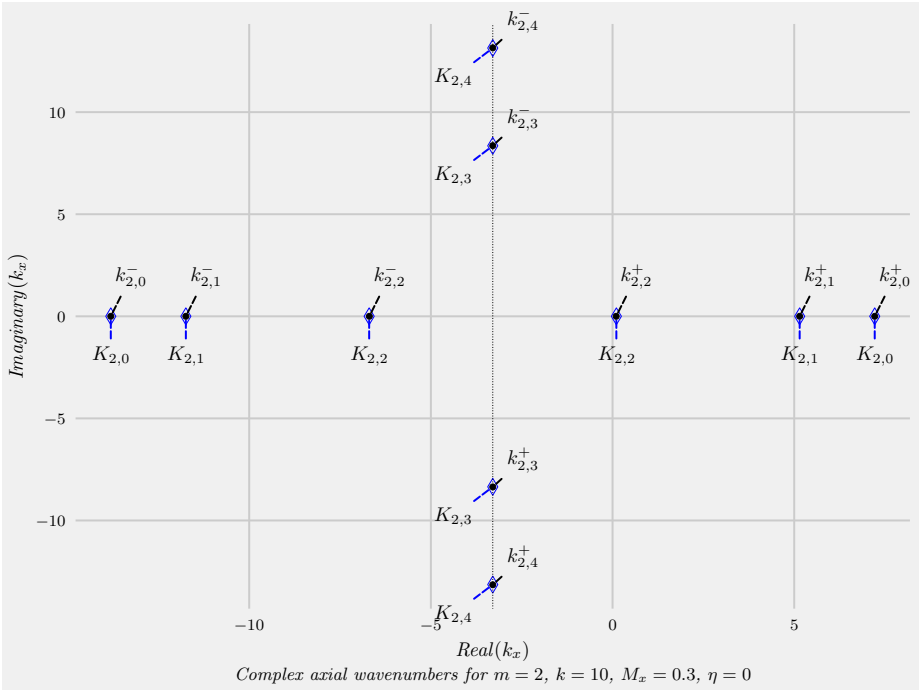

Figure 4: Second Order, Analytical Solution vs Numerical Approximation using 264 points

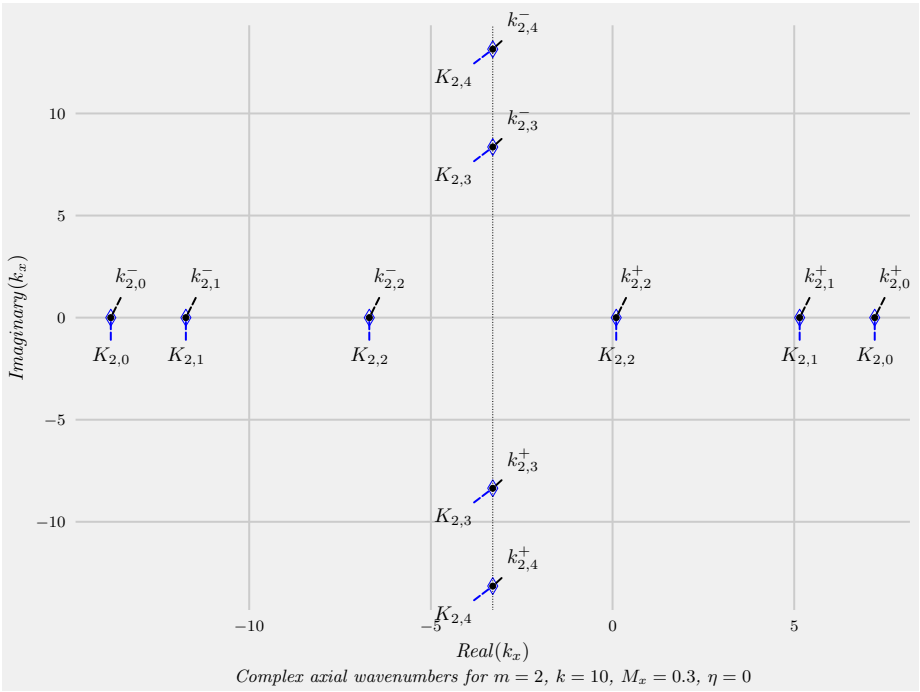

Figure 5: Second Order, Analytical Solution vs Numerical Approximation using 528 points

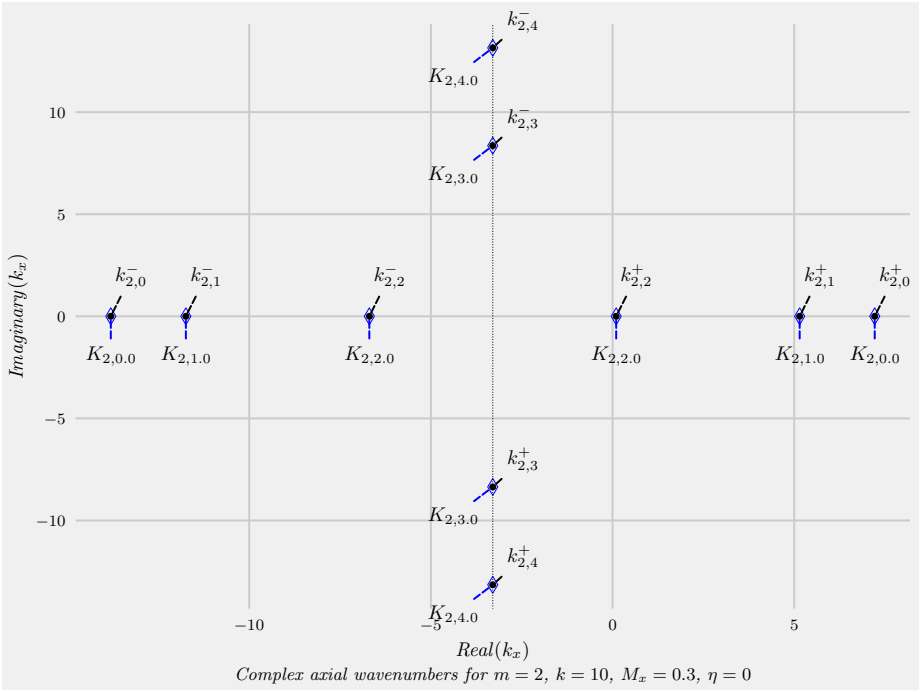

Figure 6: Second Order, Analytical Solution vs Numerical Approximation using 1056 points

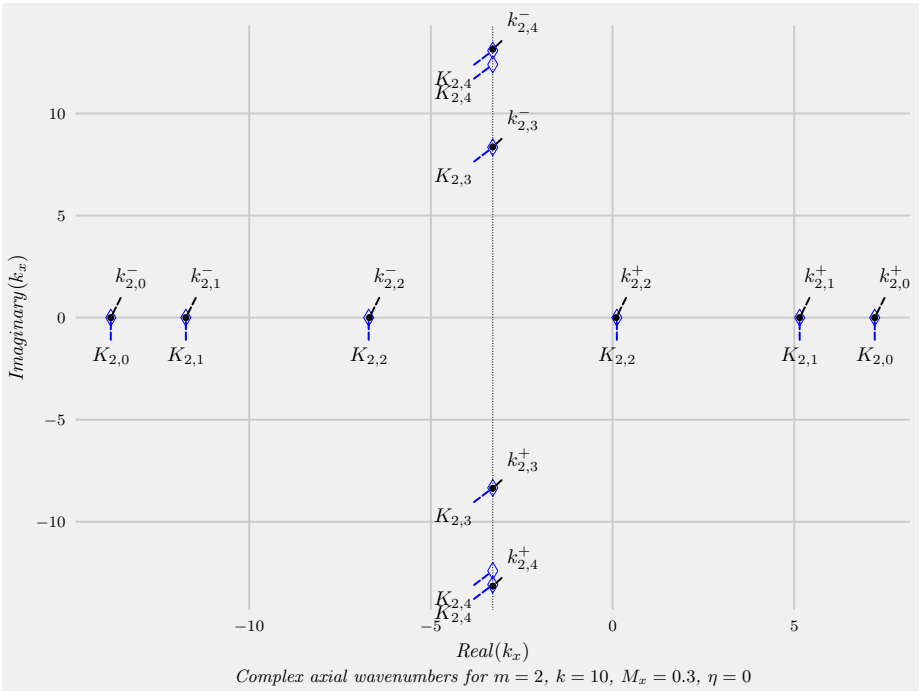

Figure 7: Fourth Order, Analytical Solution vs Numerical Approximation using 33 points

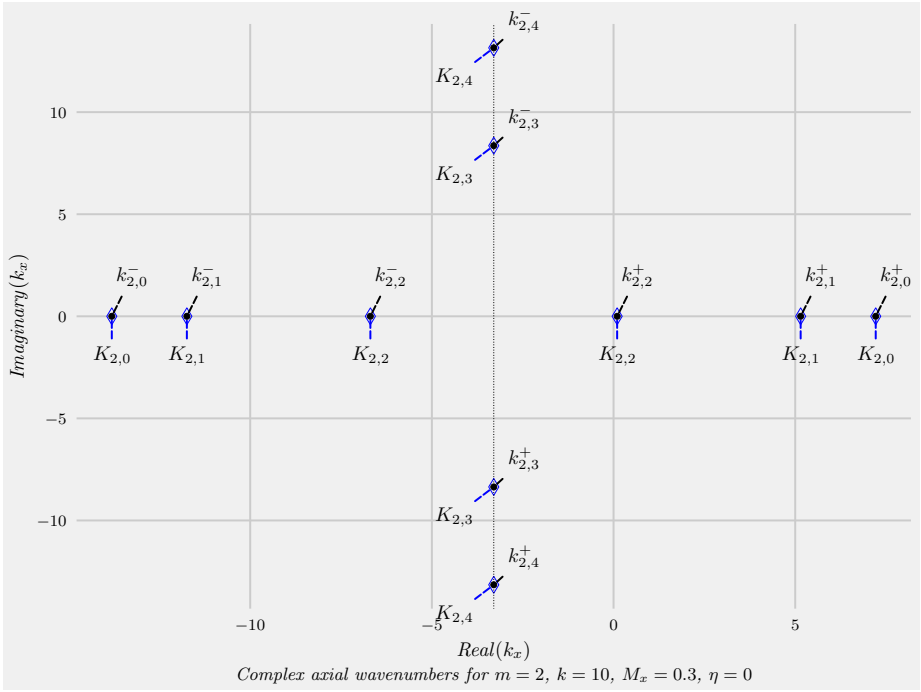

Figure 8: Fourth Order, Analytical Solution vs Numerical Approximation using 66 points

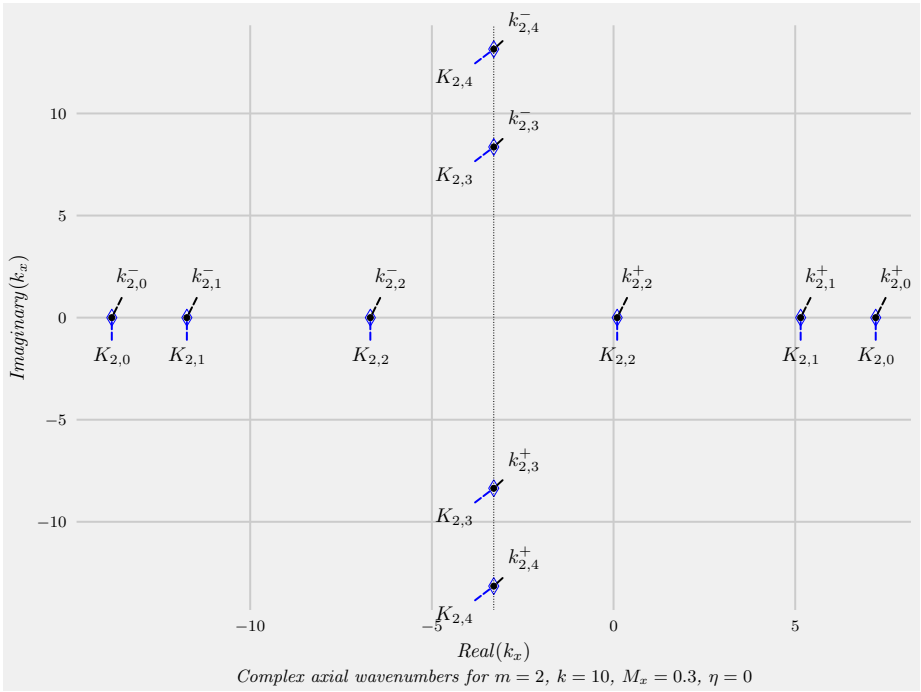

Figure 9: Fourth Order, Analytical Solution vs Numerical Approximation using 132 points

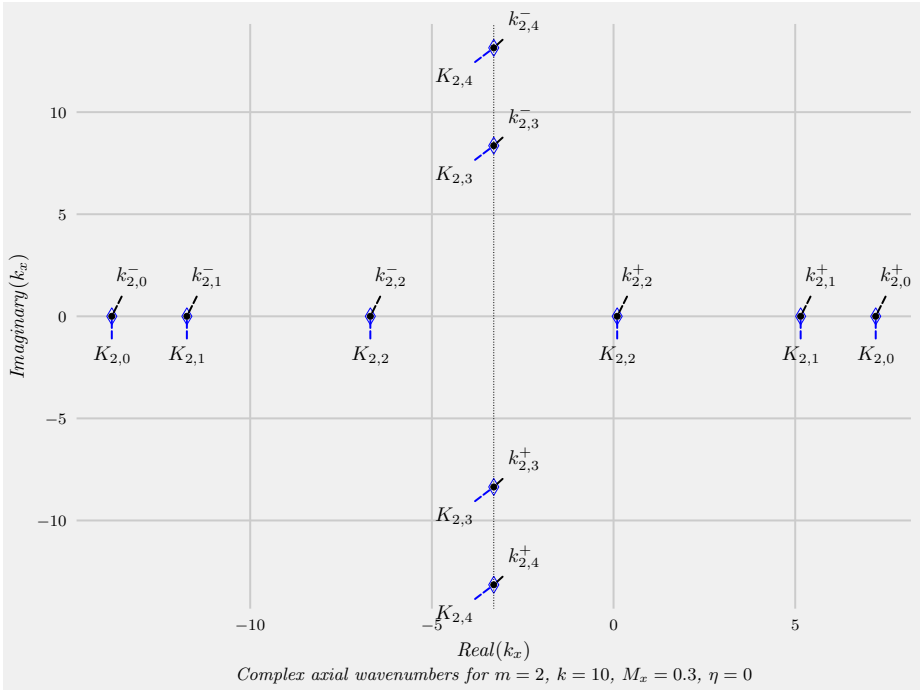

Figure 10: Fourth Order, Analytical Solution vs Numerical Approximation using 264 points

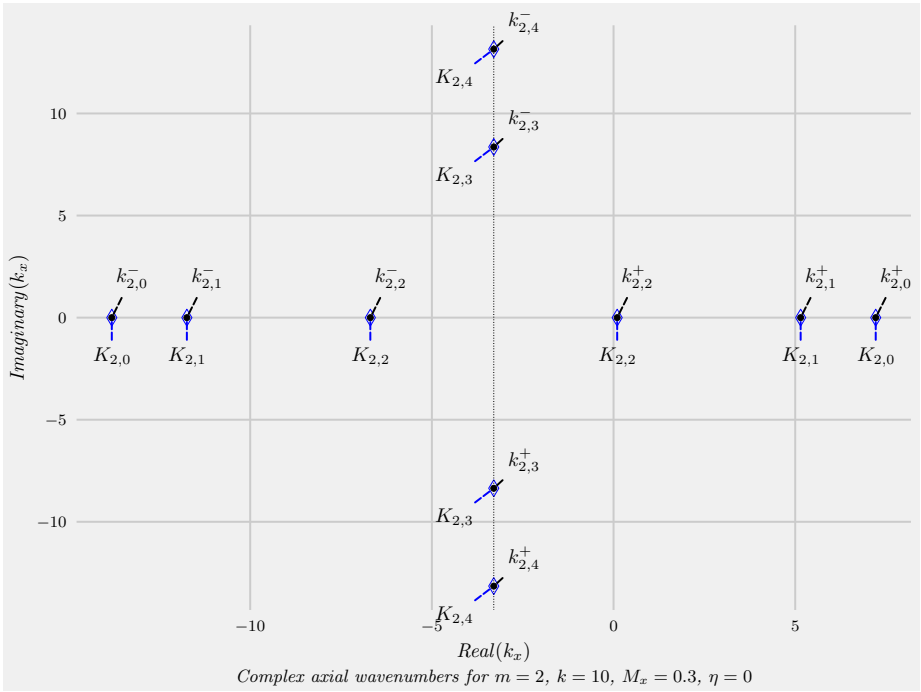

Figure 11: Fourth Order, Analytical Solution vs Numerical Approximation using 528 points

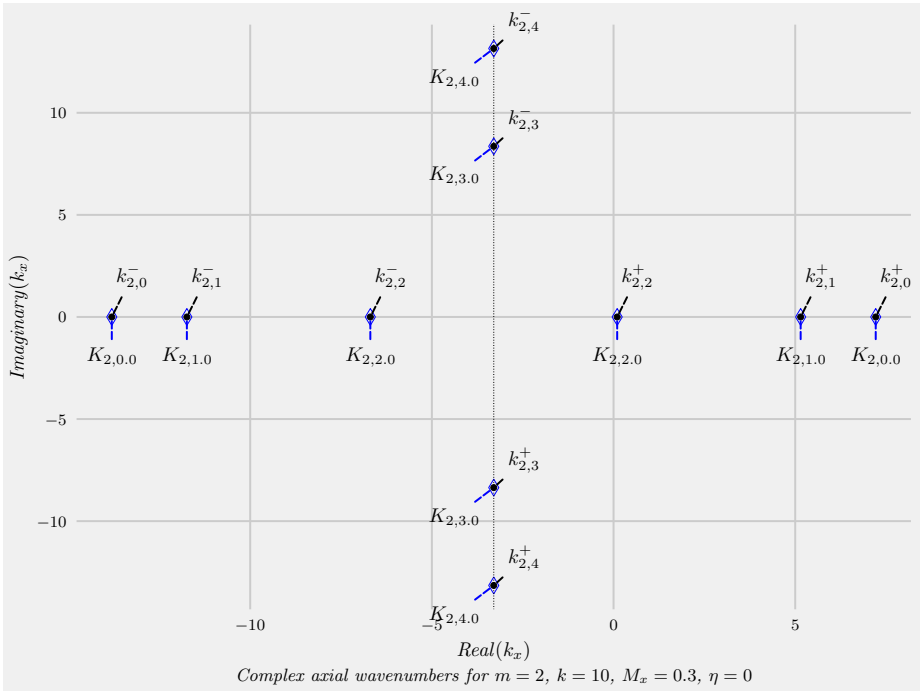

Figure 12: Fourth Order, Analytical Solution vs Numerical Approximation using 1056 points

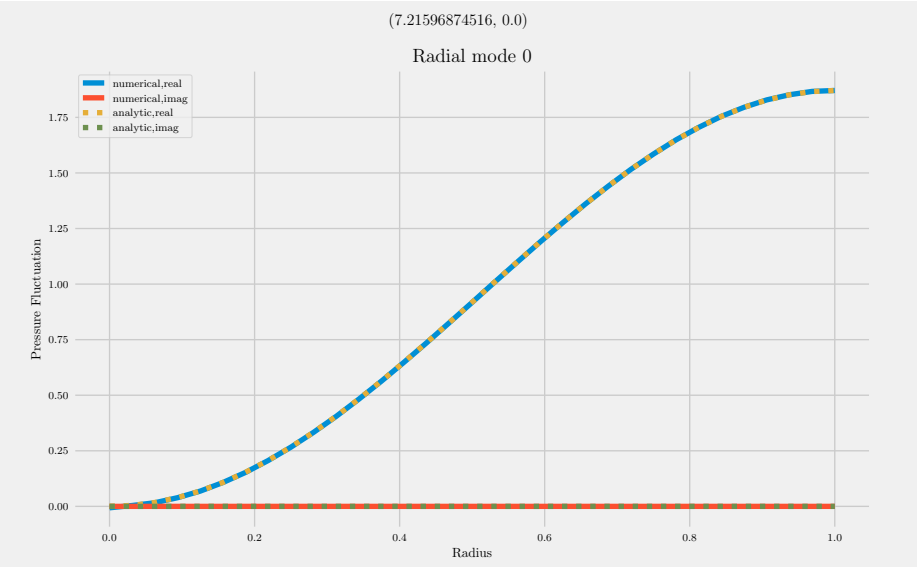

Figure 13: Second Order, Case number 0, 33points

0.3.2 Propagating Radial Modes

Second Order, Radial Mode 0

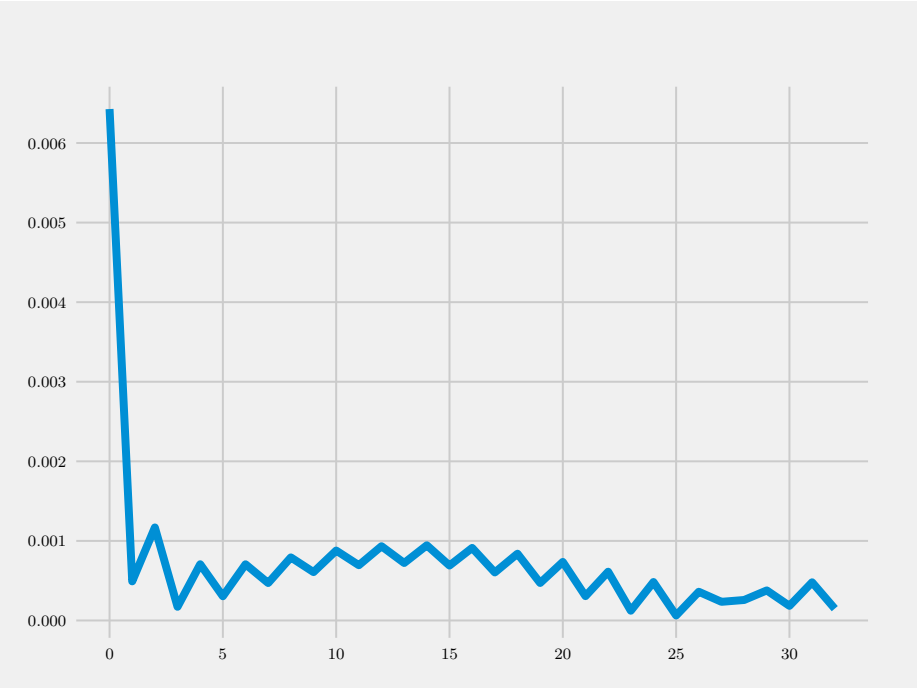

Figure 14: Second Order, Case number 0, 33points

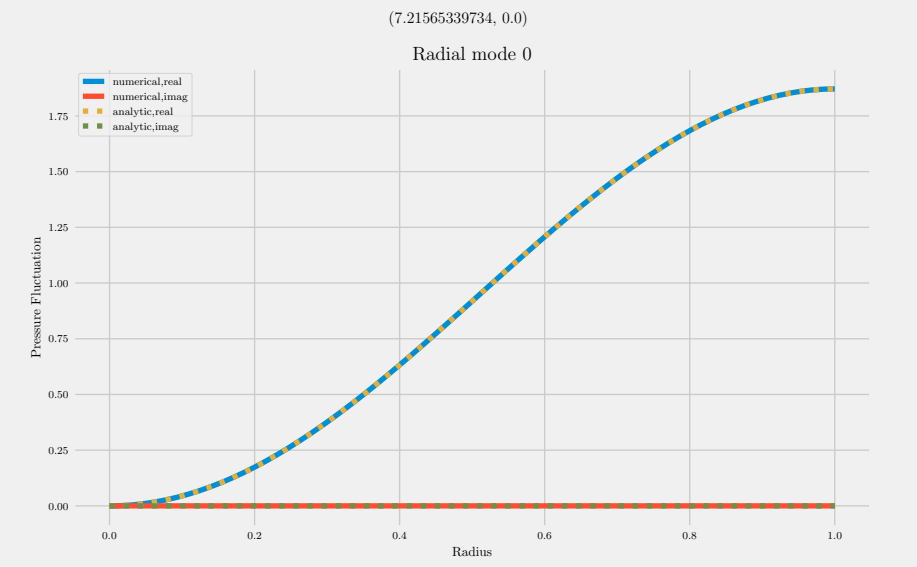

Figure 15: Second Order, Case number 0, 66points

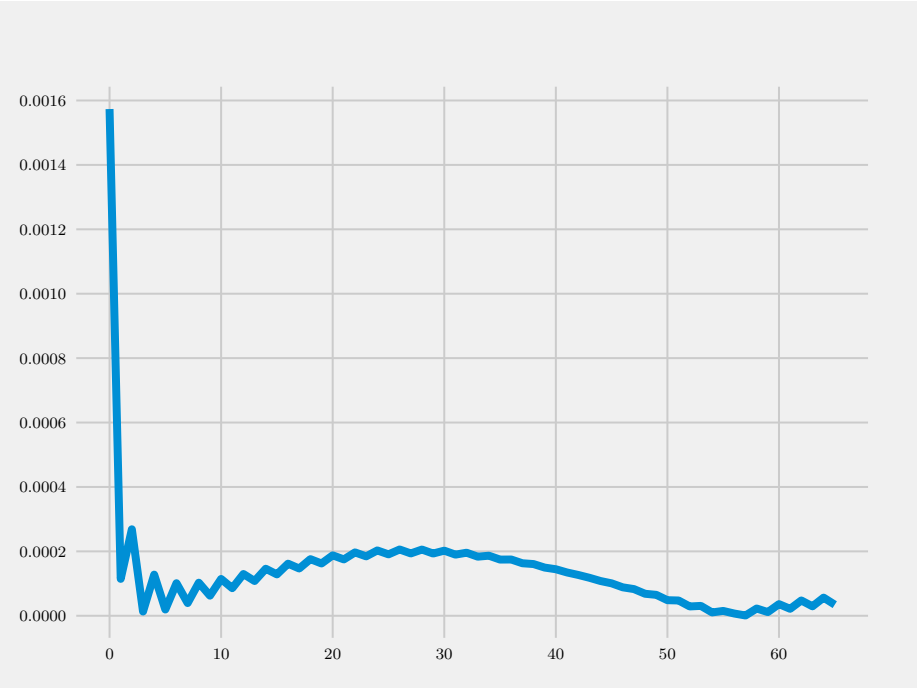

Figure 16: Second Order, Case number 0, 66points

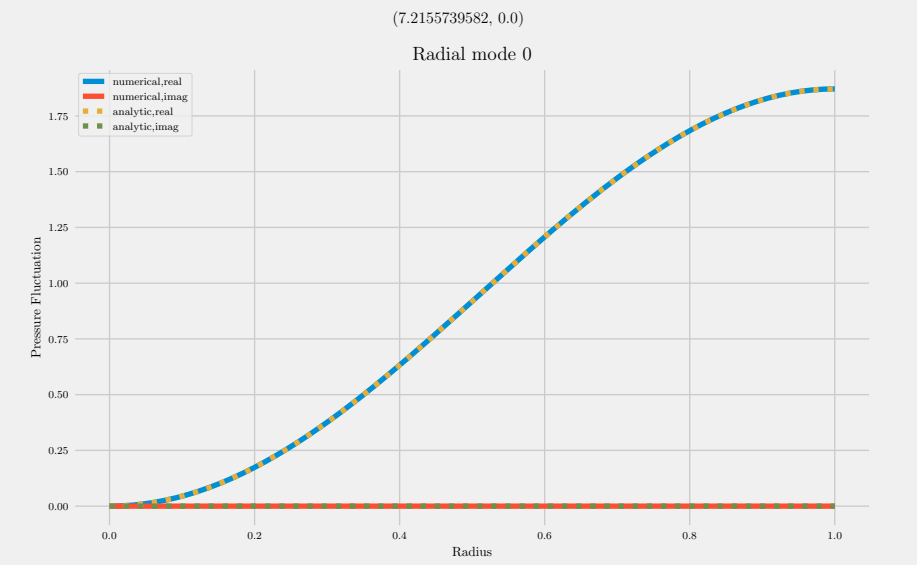

Figure 17: Second Order, Case number 0, 132points

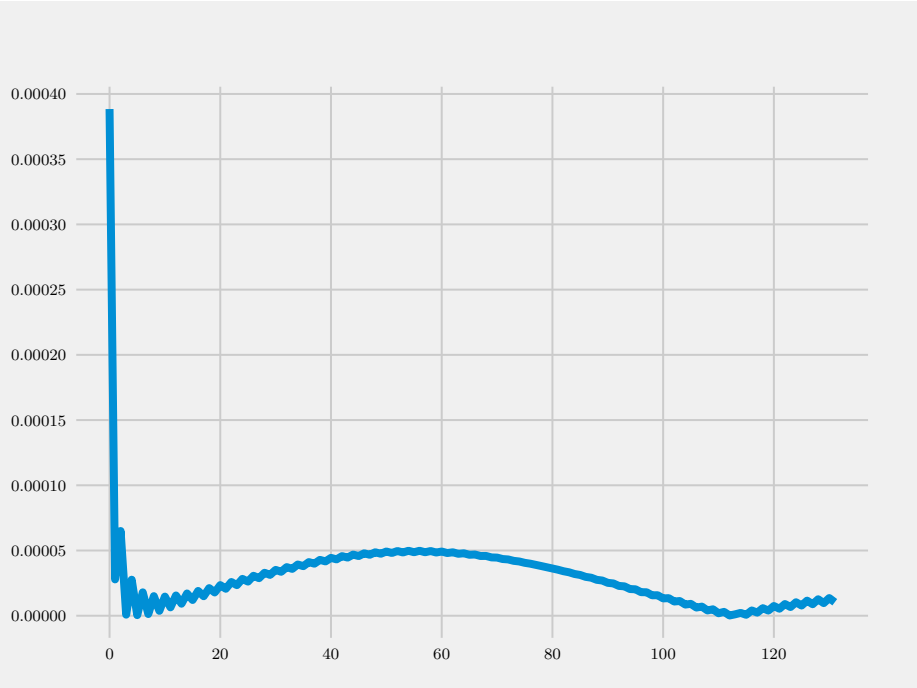

Figure 18: Second Order, Case number 0, 132points

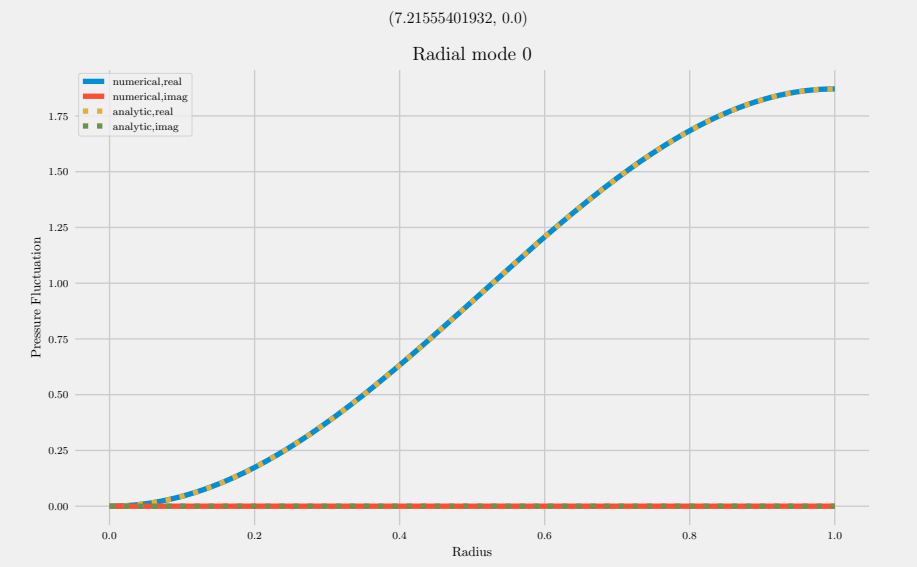

Figure 19: Second Order, Case number 0, 264points

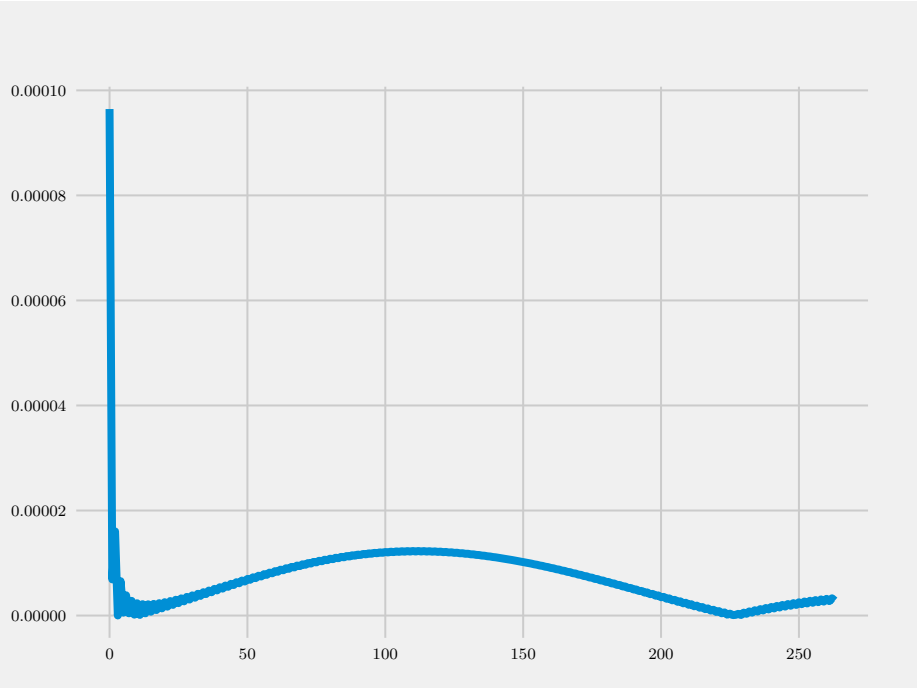

Figure 20: Second Order, Case number 0, 264points

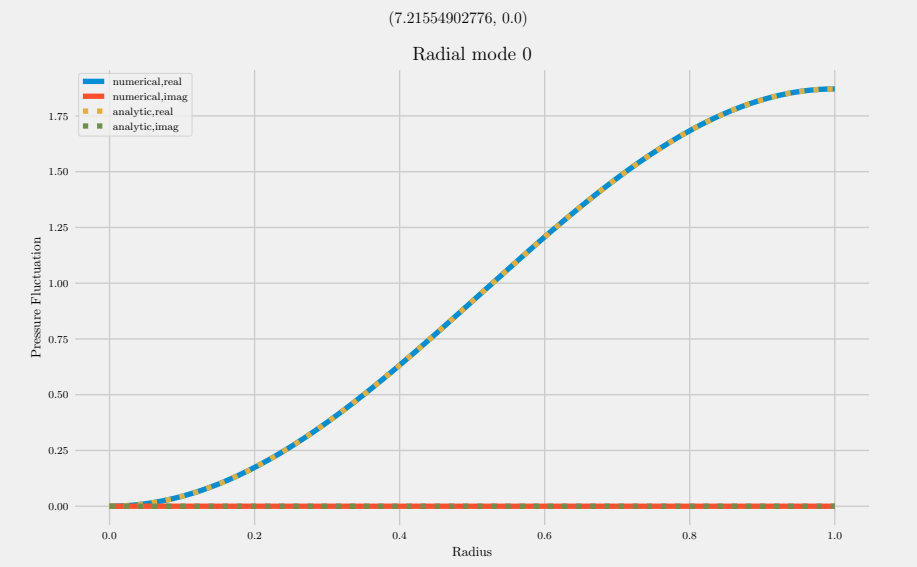

Figure 21: Second Order, Case number 0, 528points

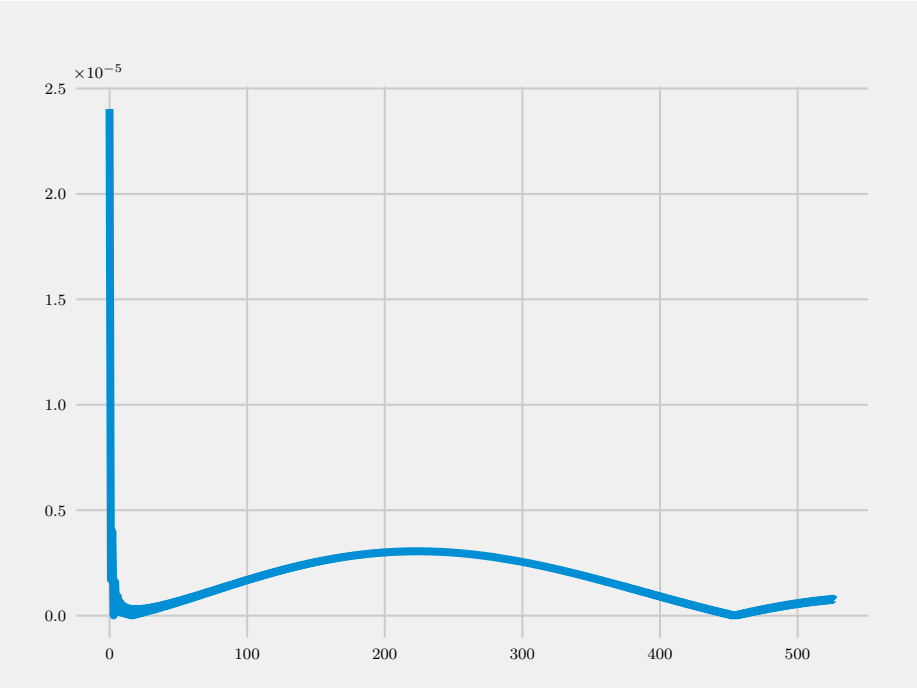

Figure 22: Second Order, Case number 0, 528points

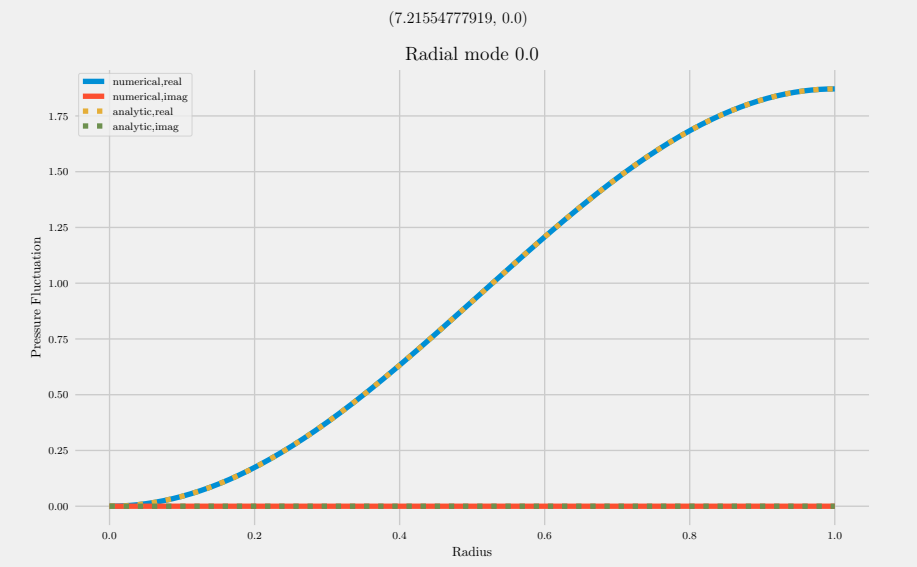

Figure 23: Second Order, Case number 0, 1056points

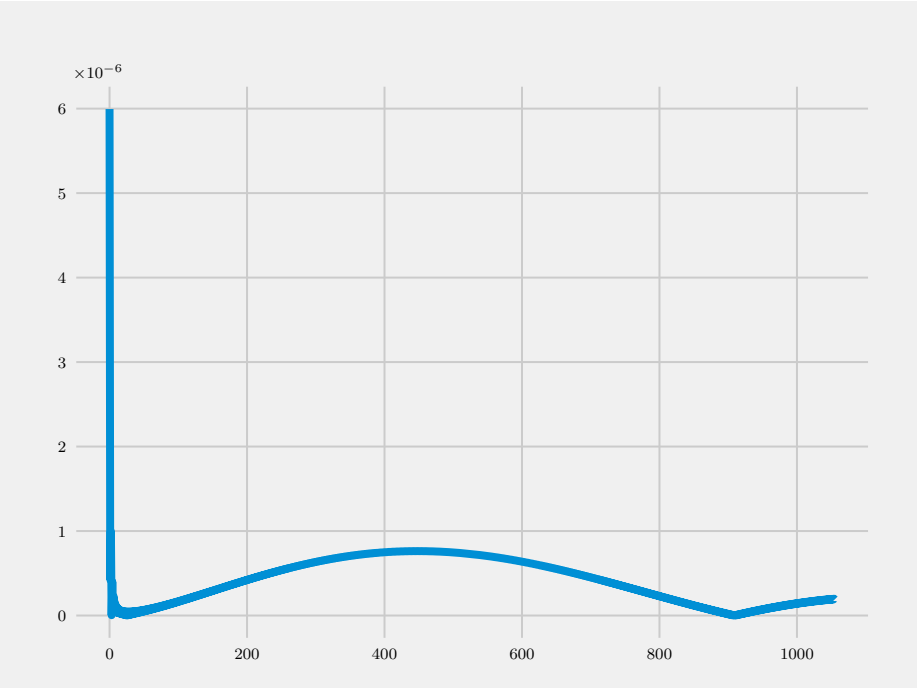

Figure 24: Second Order, Case number 0, 1056points

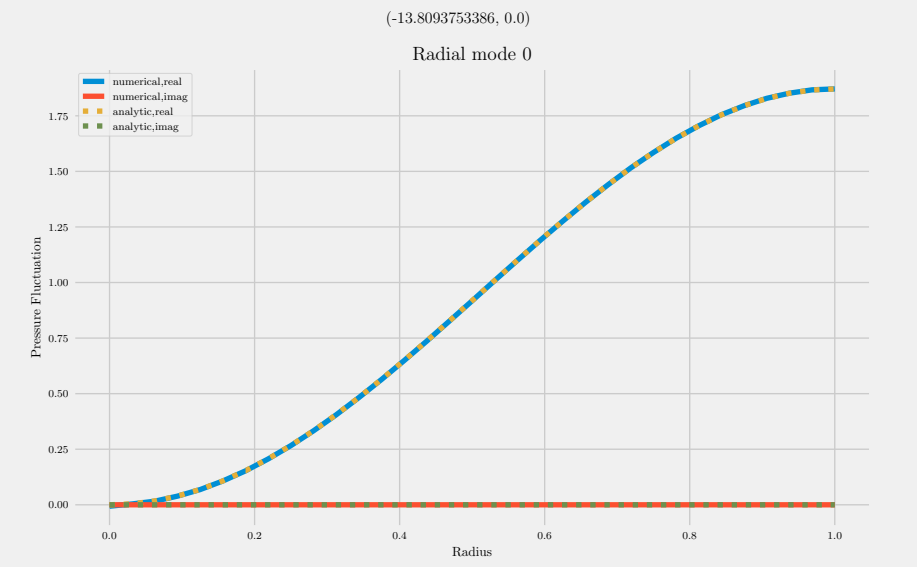

Figure 25: Second Order, Case number 1, 33points

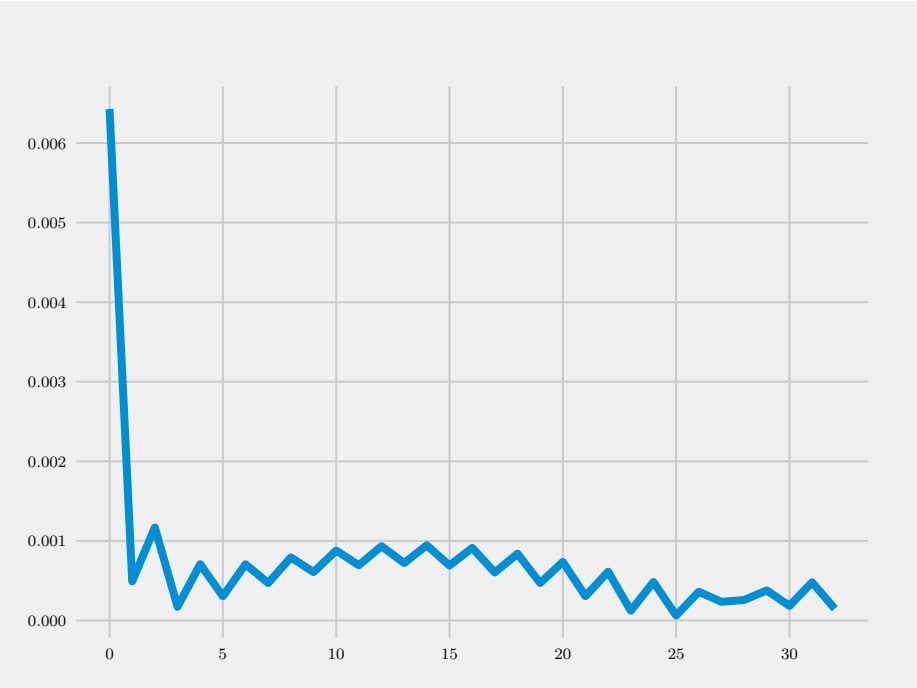

Figure 26: Second Order, Case number 1, 33points

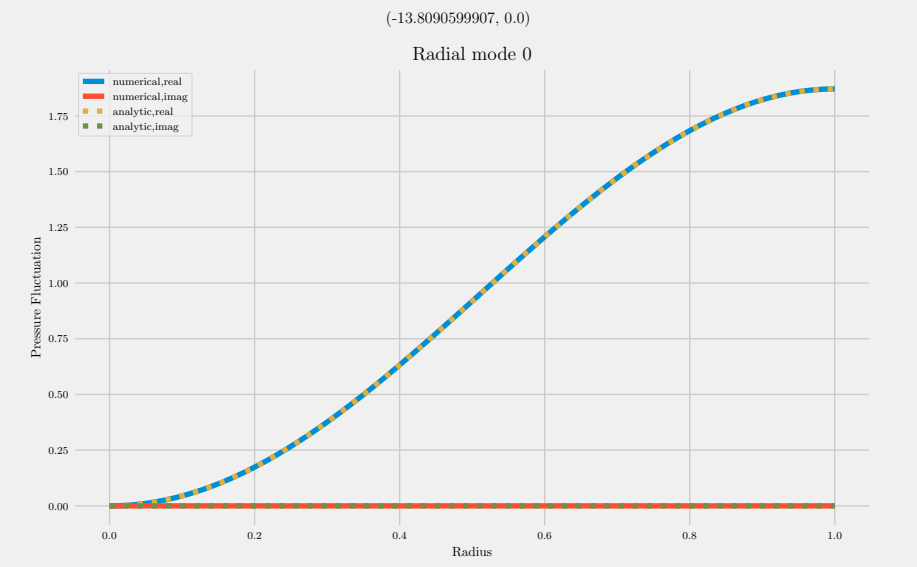

Figure 27: Second Order, Case number 1, 66points

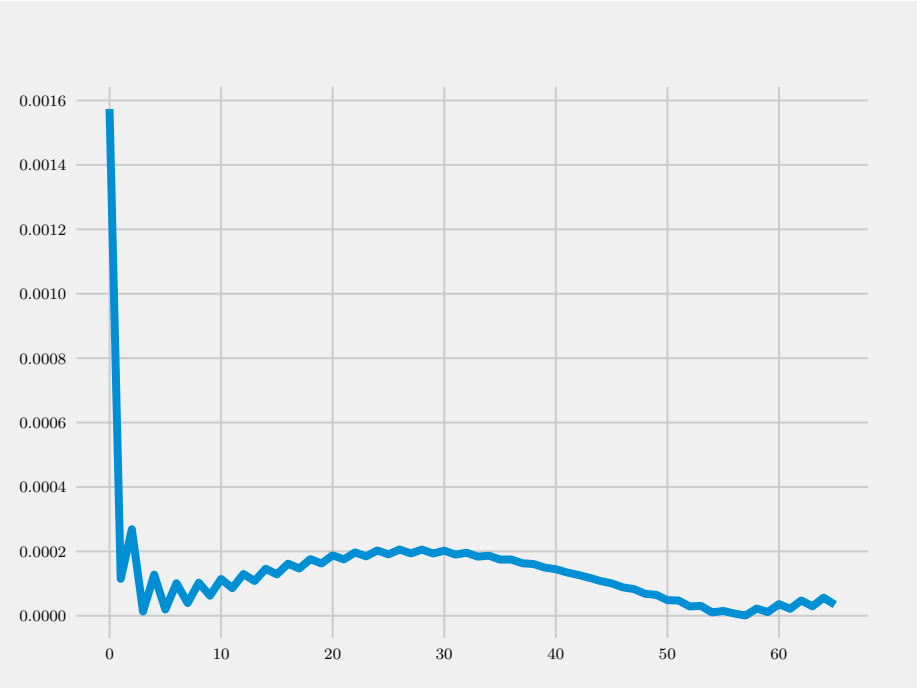

Figure 28: Second Order, Case number 1, 66points

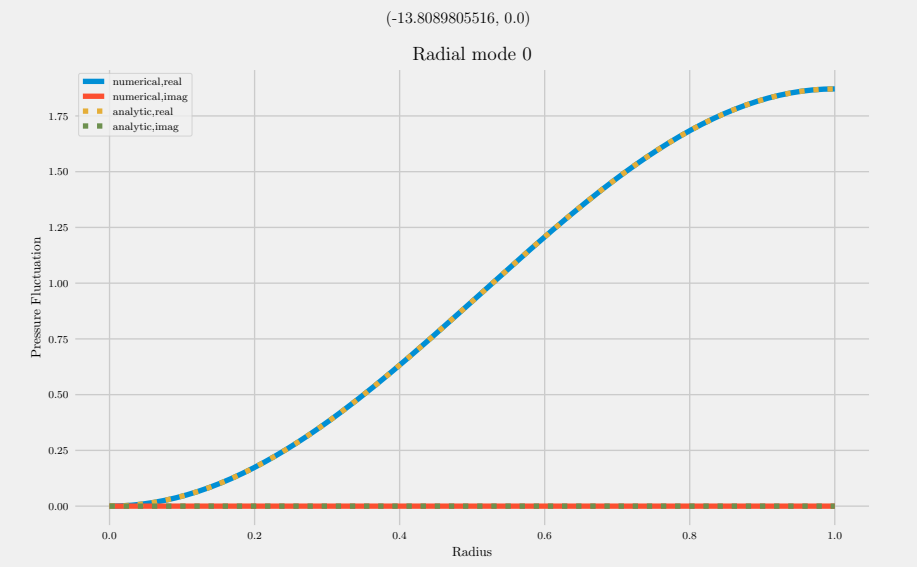

Figure 29: Second Order, Case number 1, 132points

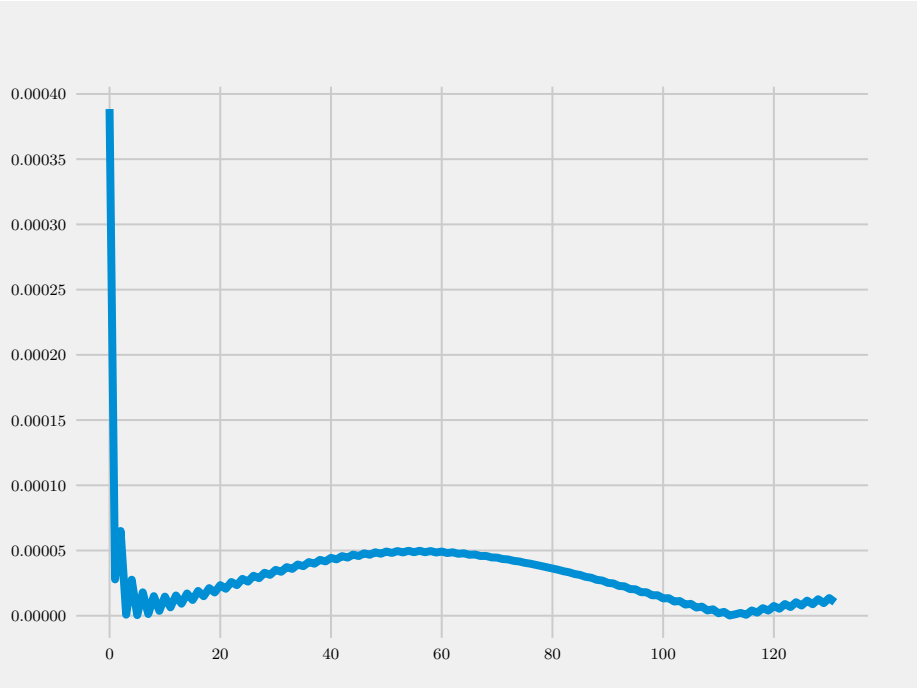

Figure 30: Second Order, Case number 1, 132points

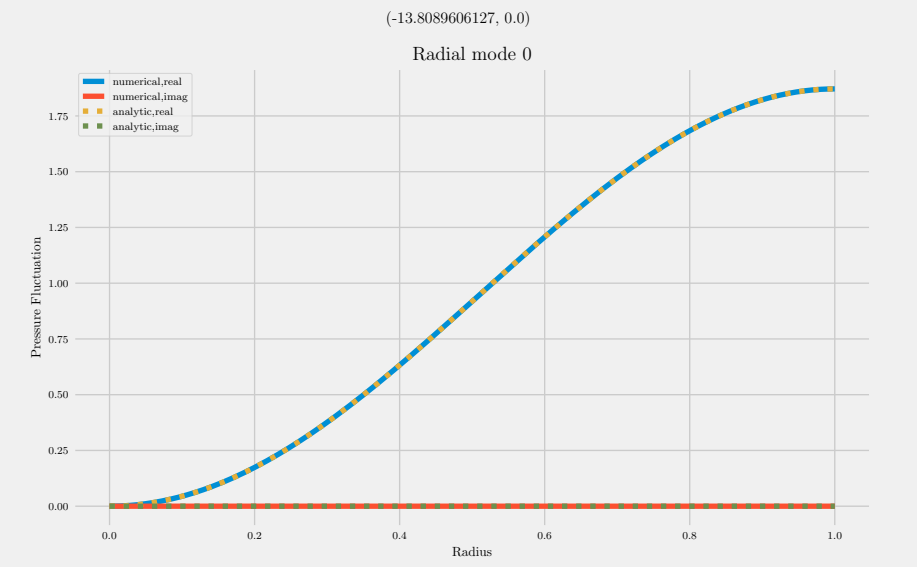

Figure 31: Second Order, Case number 1, 264points

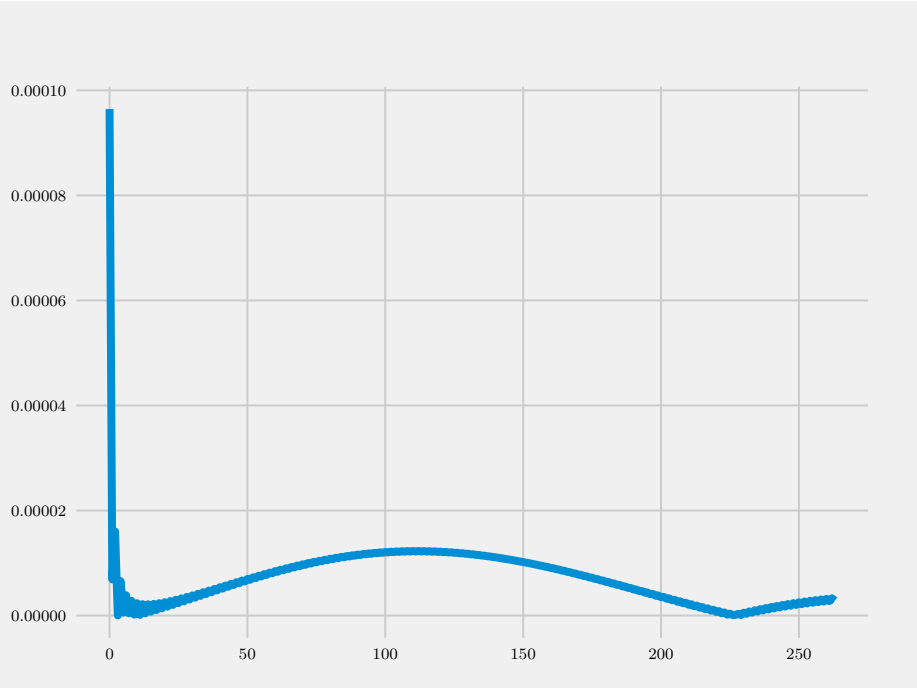

Figure 32: Second Order, Case number 1, 264points

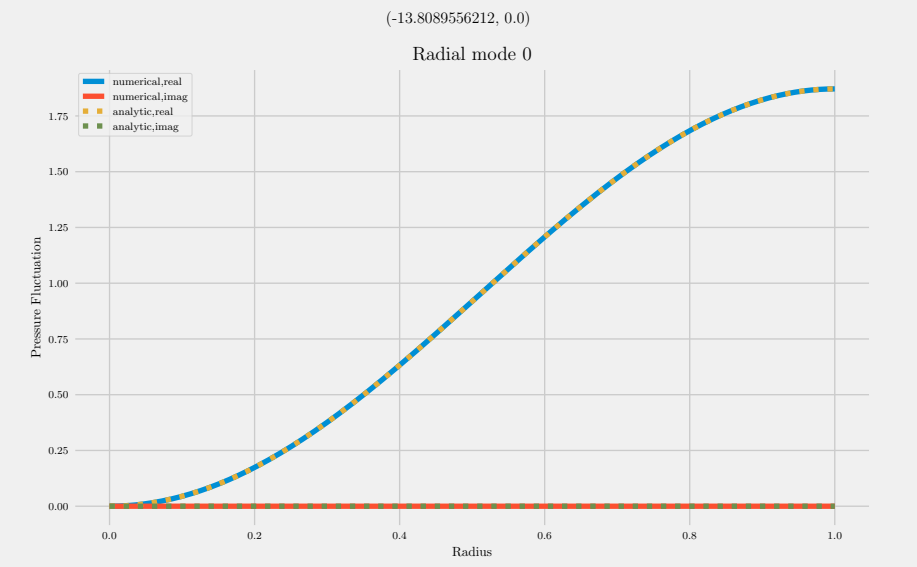

Figure 33: Second Order, Case number 1, 528points

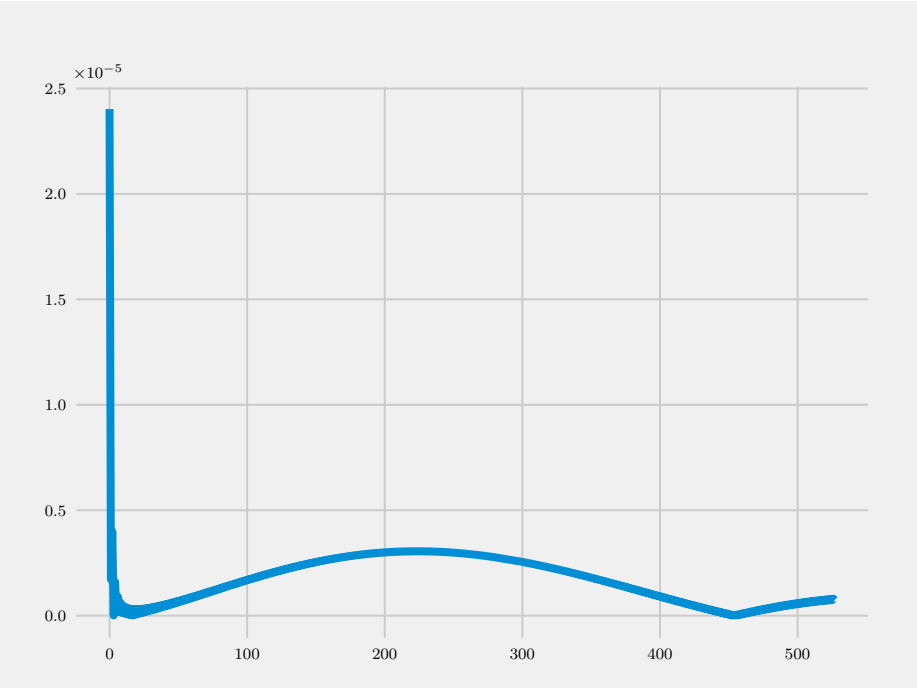

Figure 34: Second Order, Case number 1, 528points

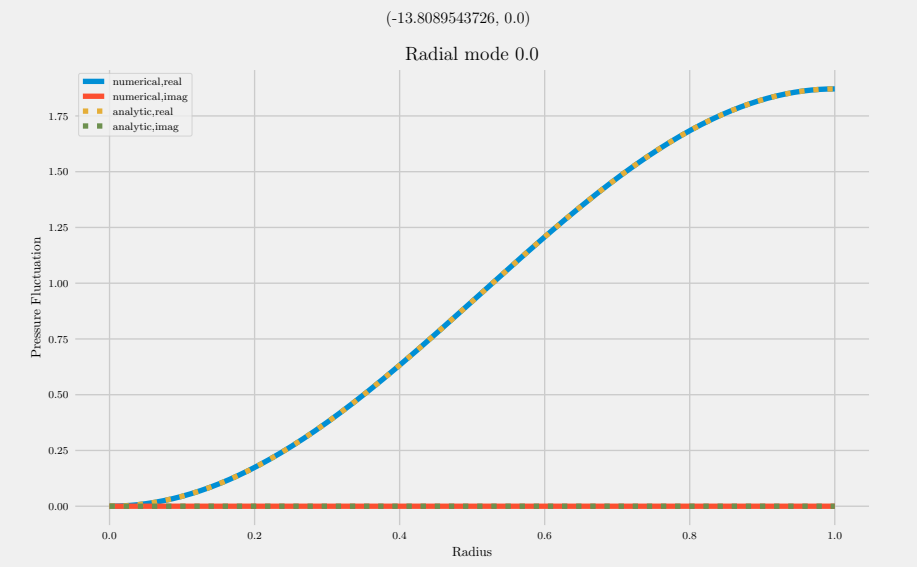

Figure 35: Second Order, Case number 1, 1056points

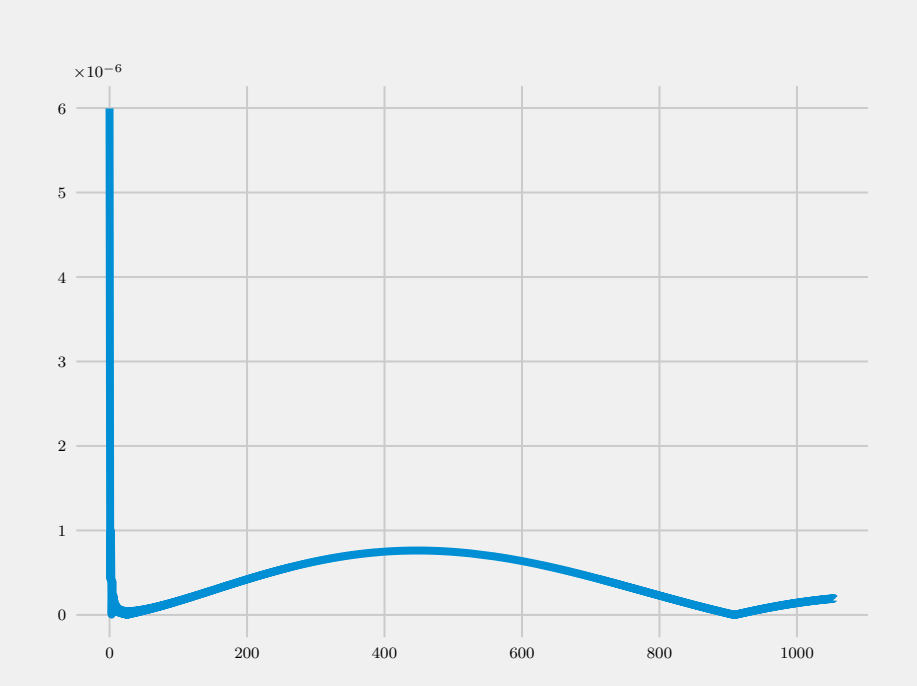

Figure 36: Second Order, Case number 1, 1056points

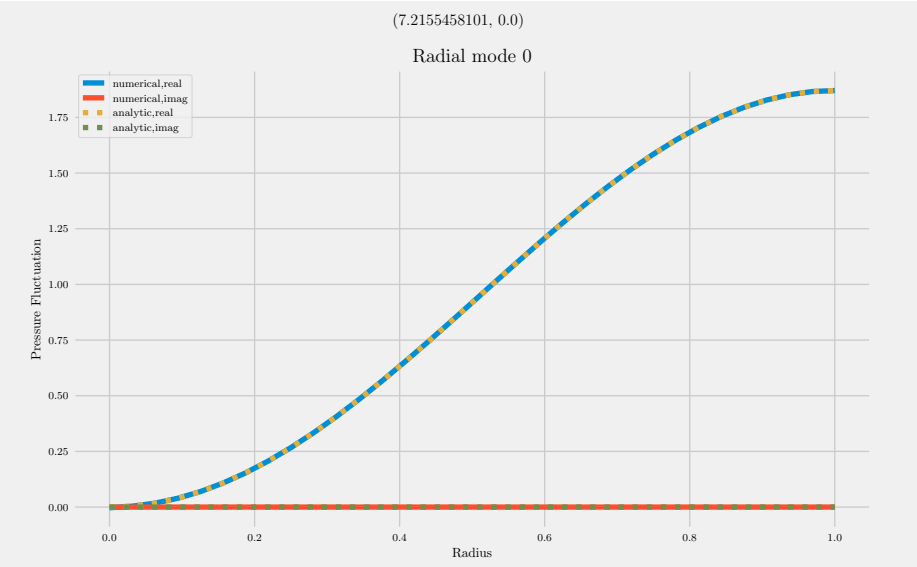

Figure 37: Fourth Order, Case number 0, 33points

Fourth Order, Radial Mode 0

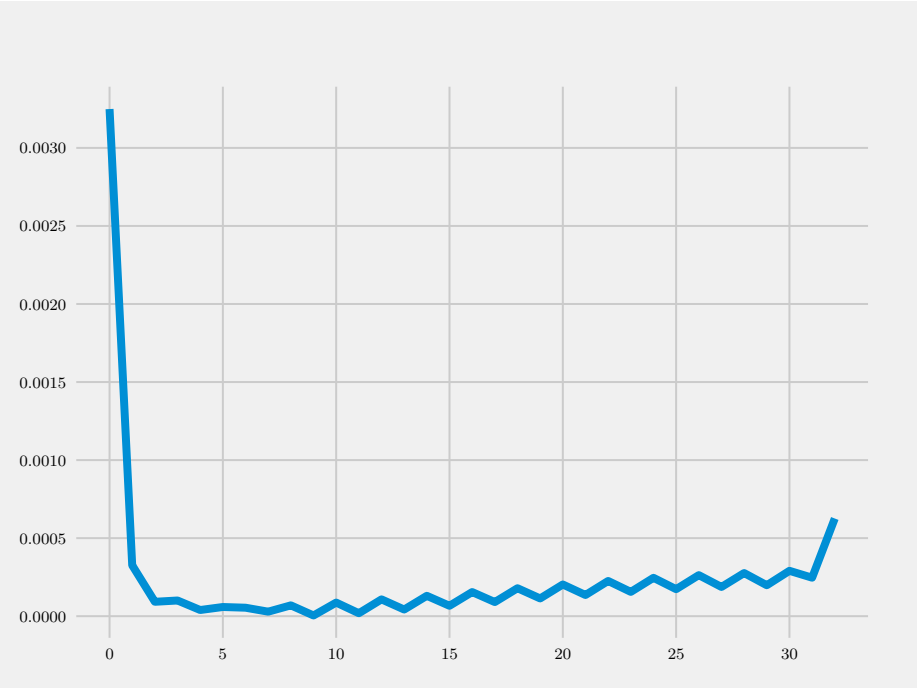

Figure 38: Fourth Order, Case number 0, 33points

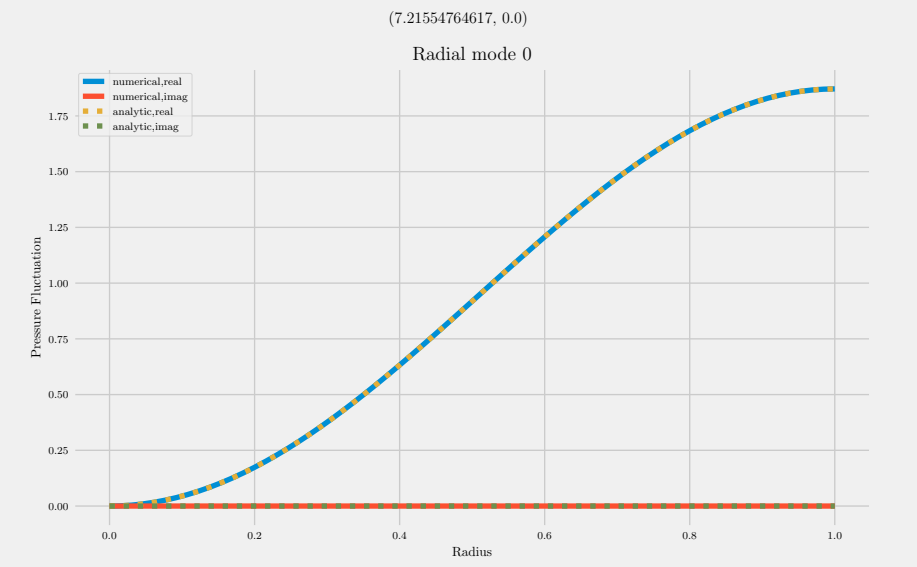

Figure 39: Fourth Order, Case number 0, 66points

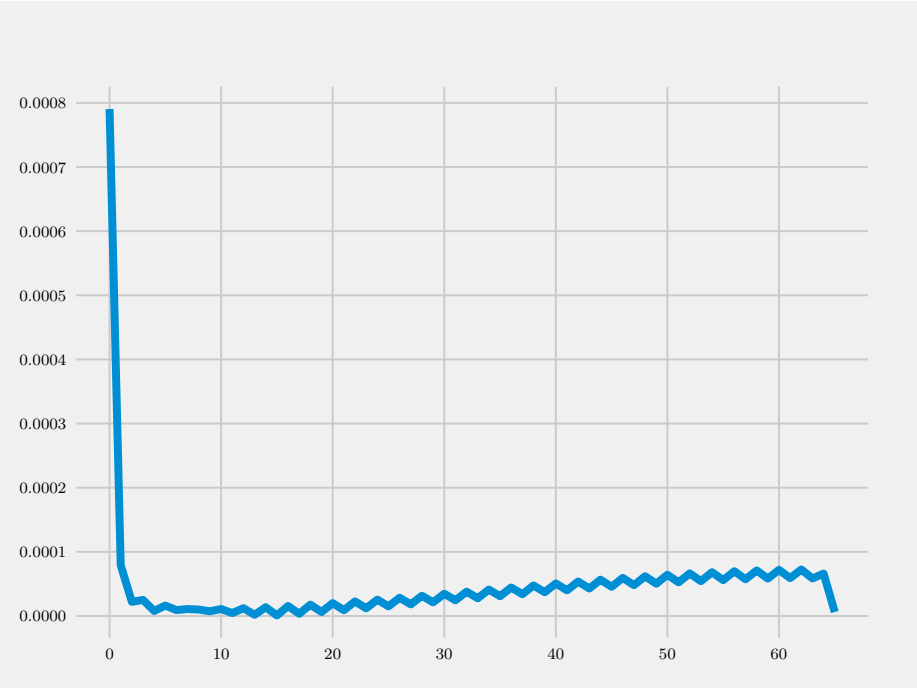

Figure 40: Fourth Order, Case number 0, 66points

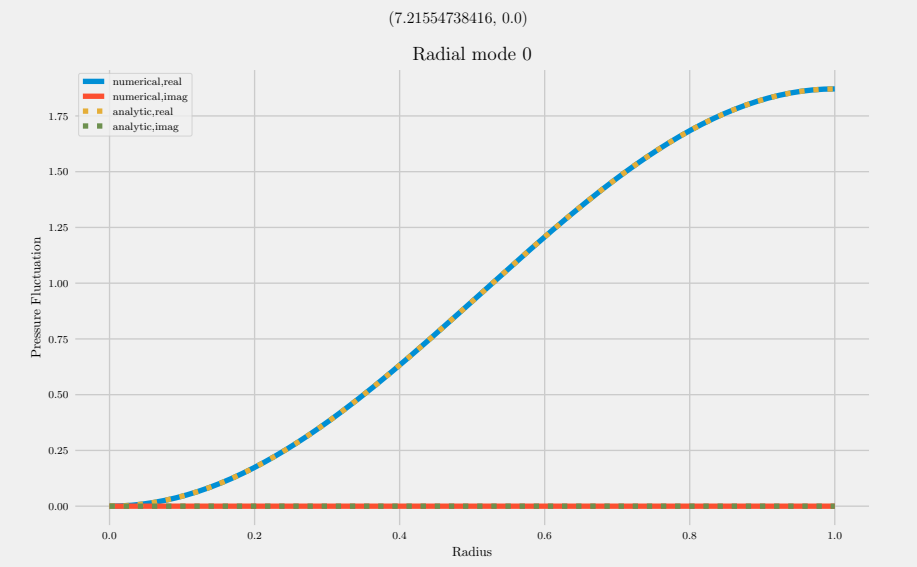

Figure 41: Fourth Order, Case number 0, 132points

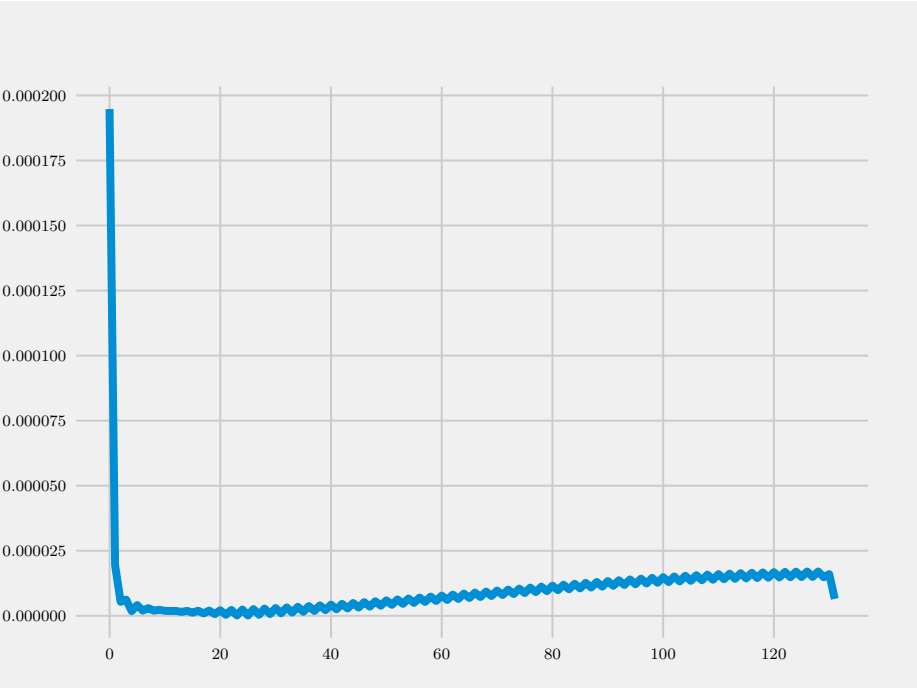

Figure 42: Fourth Order, Case number 0, 132points

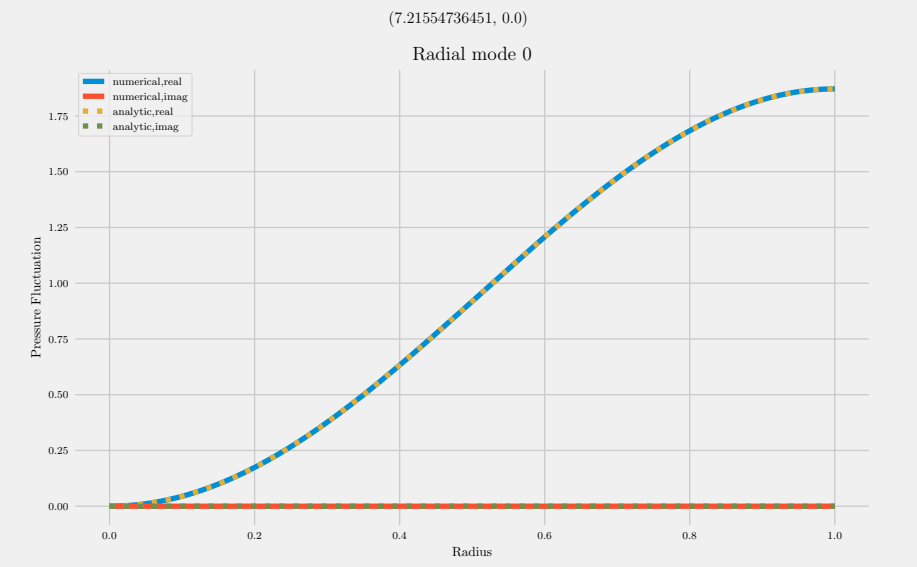

Figure 43: Fourth Order, Case number 0, 264points

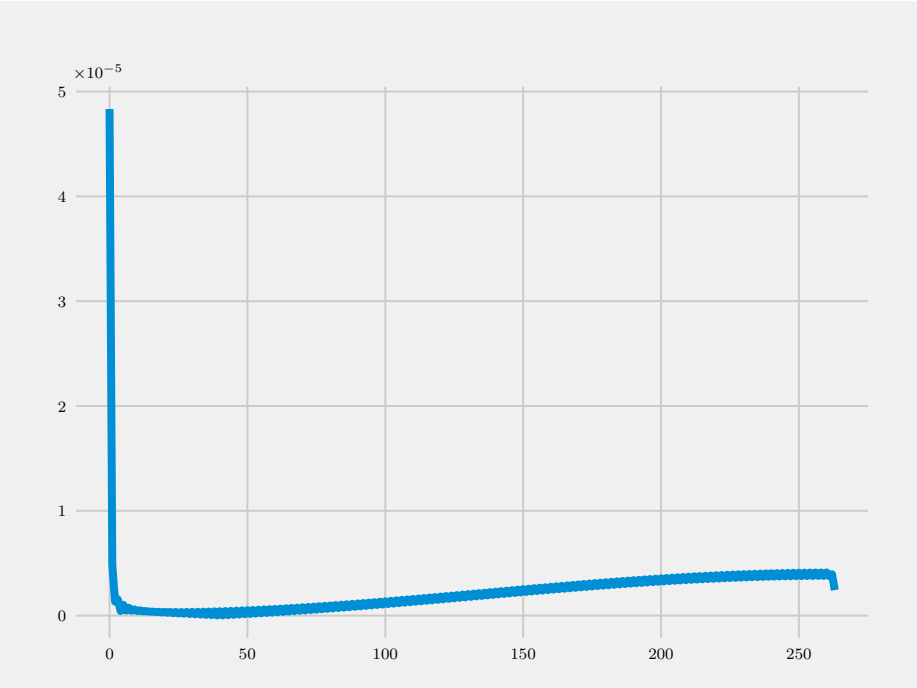

Figure 44: Fourth Order, Case number 0, 264points

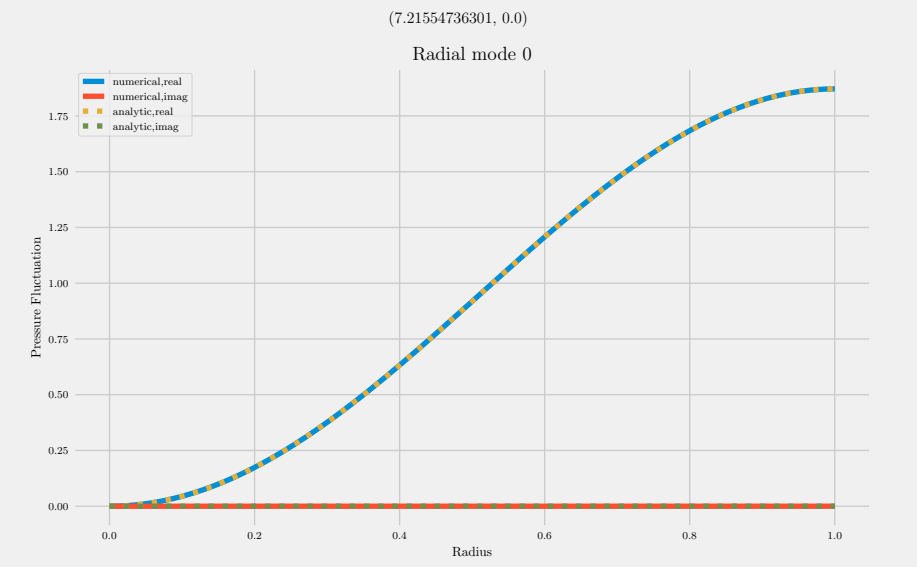

Figure 45: Fourth Order, Case number 0, 528points

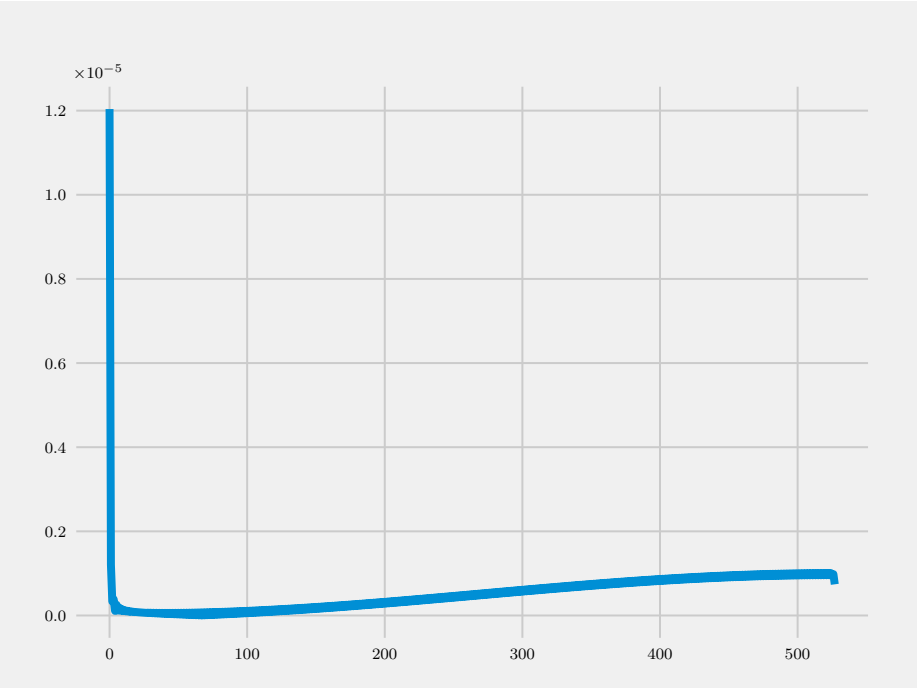

Figure 46: Fourth Order, Case number 0, 528points

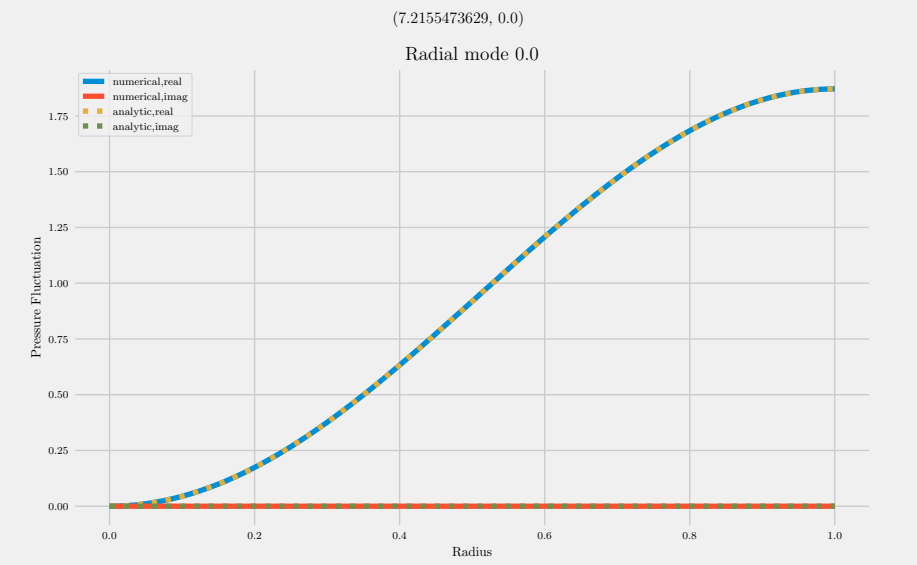

Figure 47: Fourth Order, Case number 0, 1056points

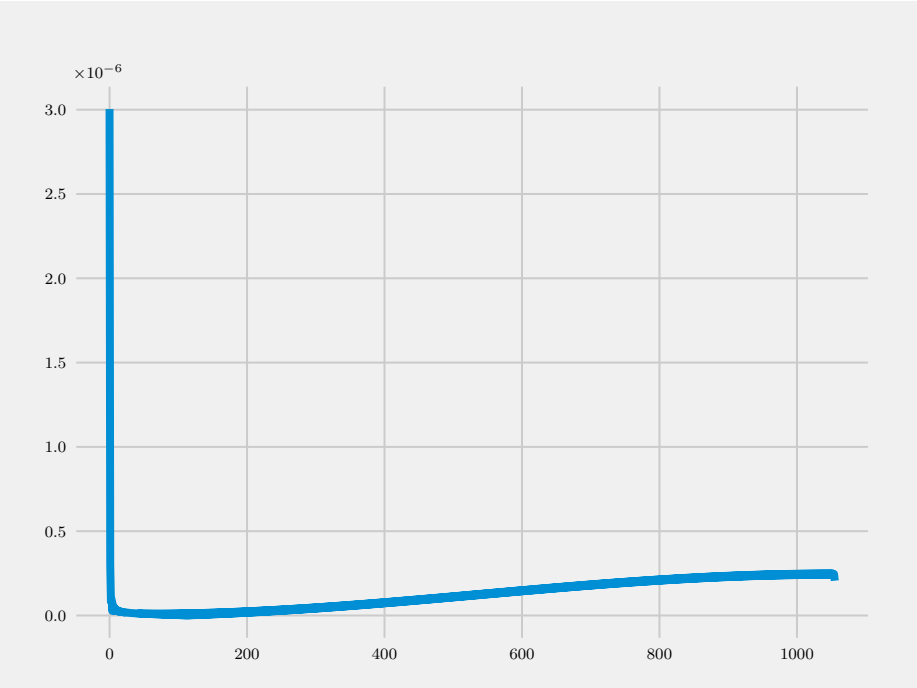

Figure 48: Fourth Order, Case number 0, 1056points

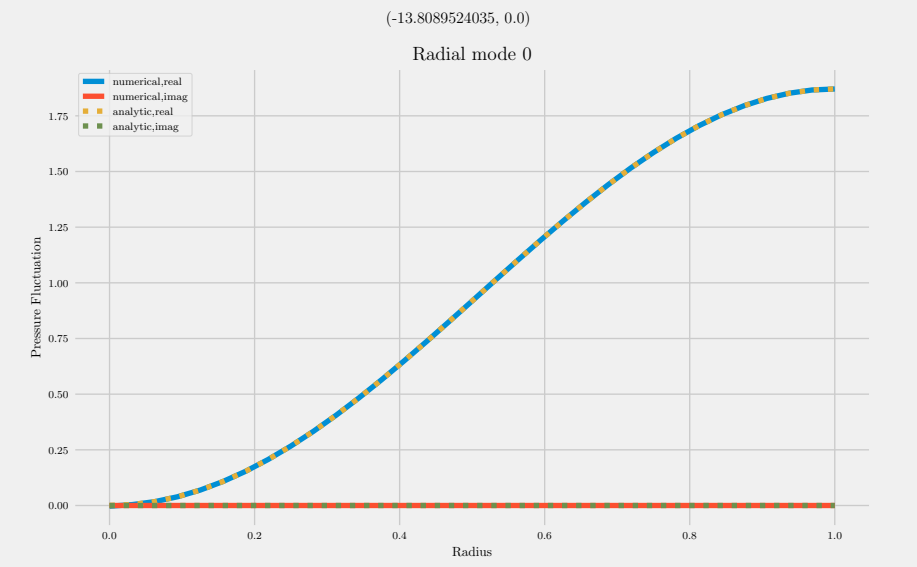

Figure 49: Fourth Order, Case number 1, 33points

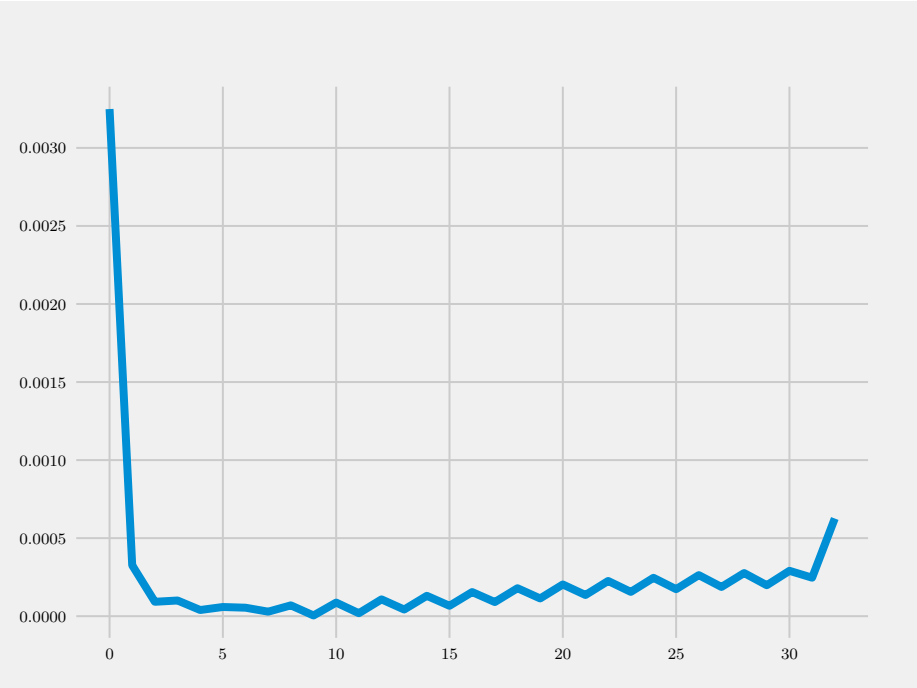

Figure 50: Fourth Order, Case number 1, 33points

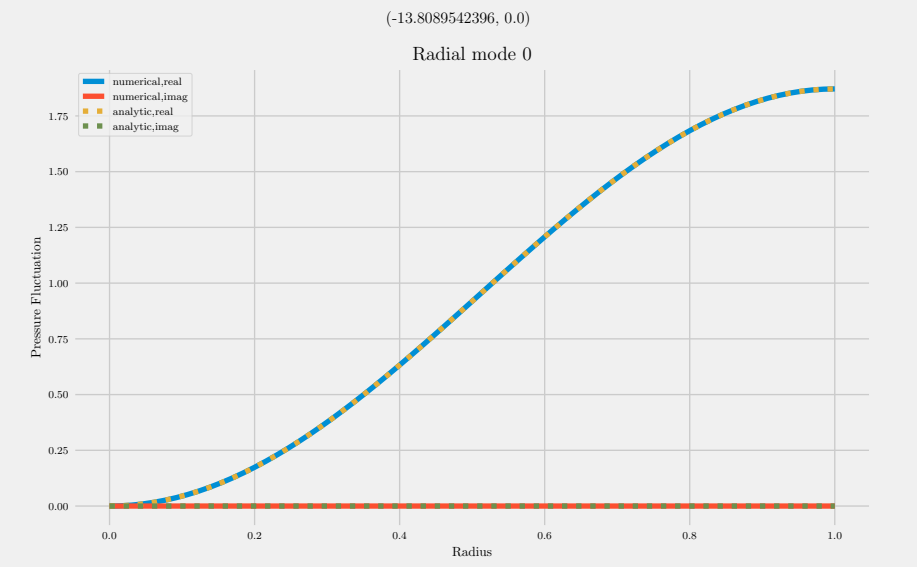

Figure 51: Fourth Order, Case number 1, 66points

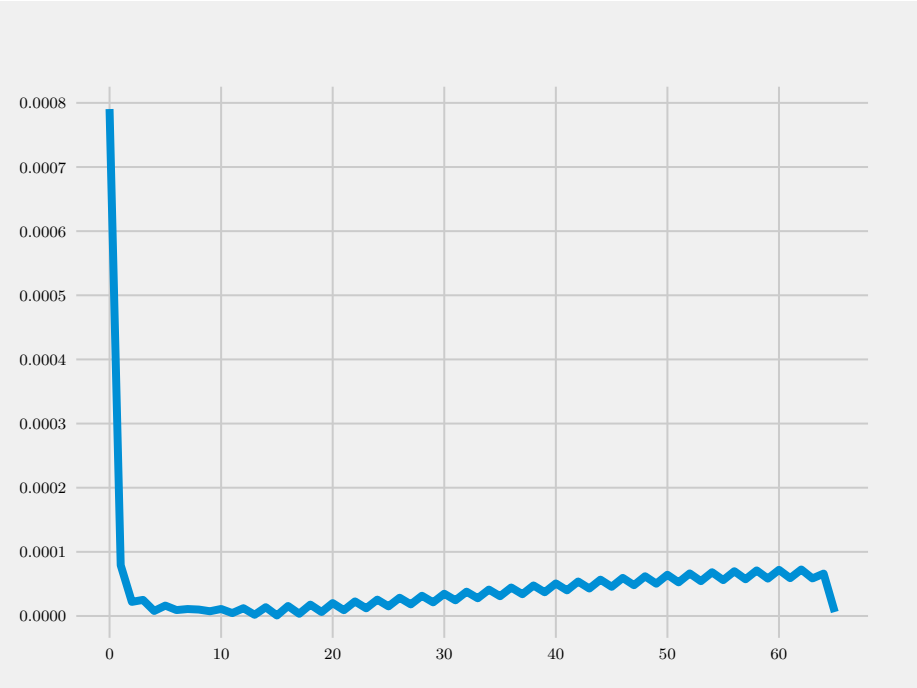

Figure 52: Fourth Order, Case number 1, 66points

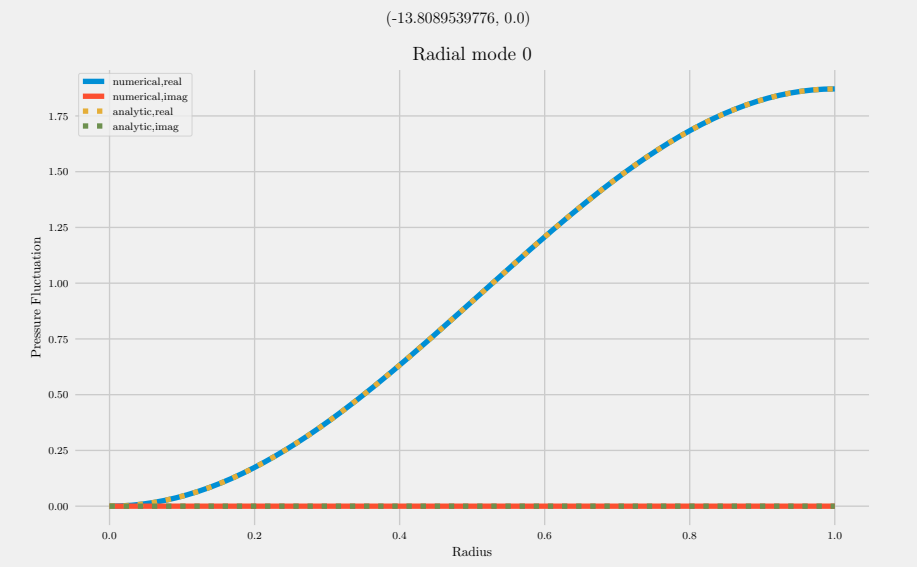

Figure 53: Fourth Order, Case number 1, 132points

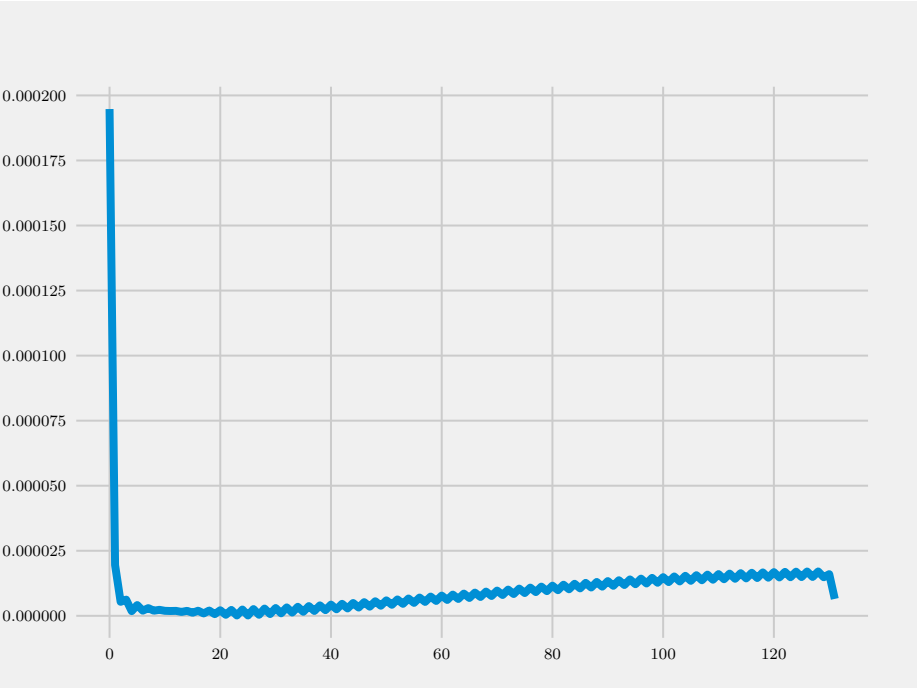

Figure 54: Fourth Order, Case number 1, 132points

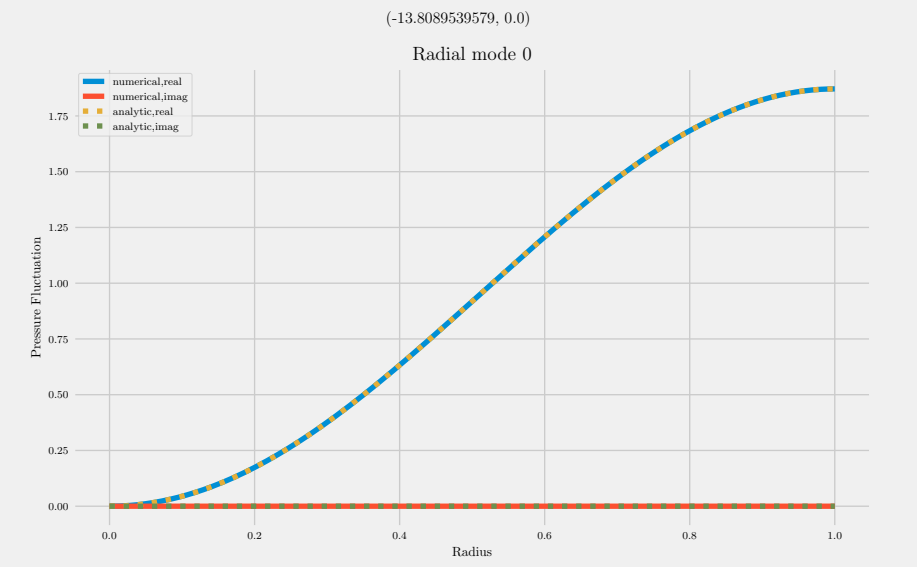

Figure 55: Fourth Order, Case number 1, 264points

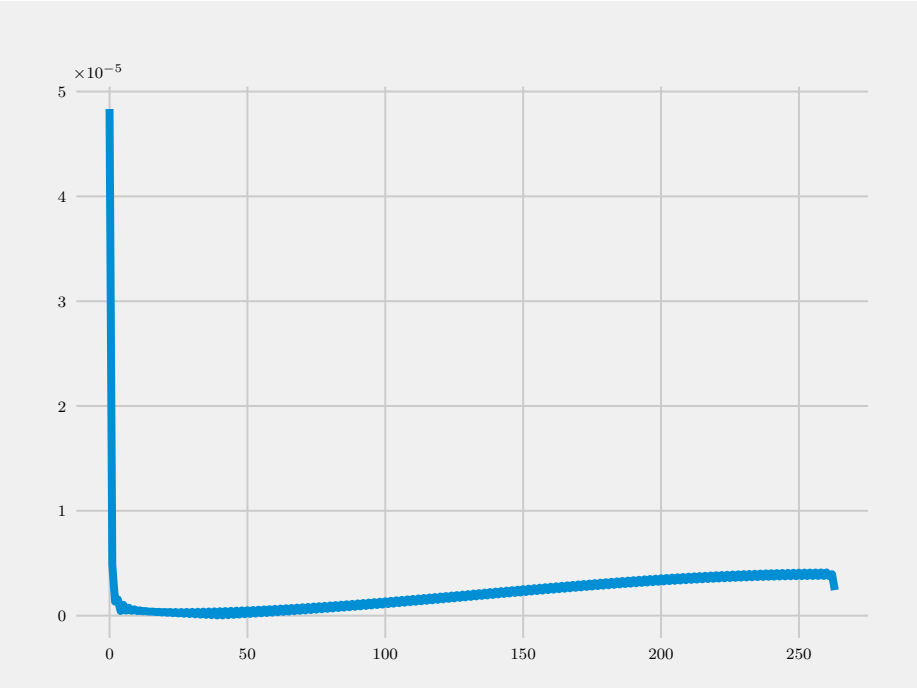

Figure 56: Fourth Order, Case number 1, 264points

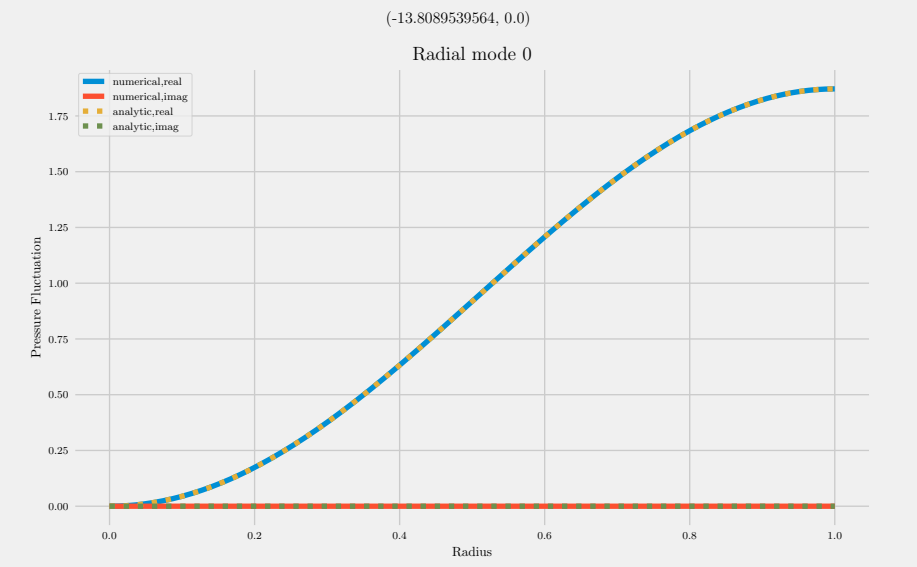

Figure 57: Fourth Order, Case number 1, 528points

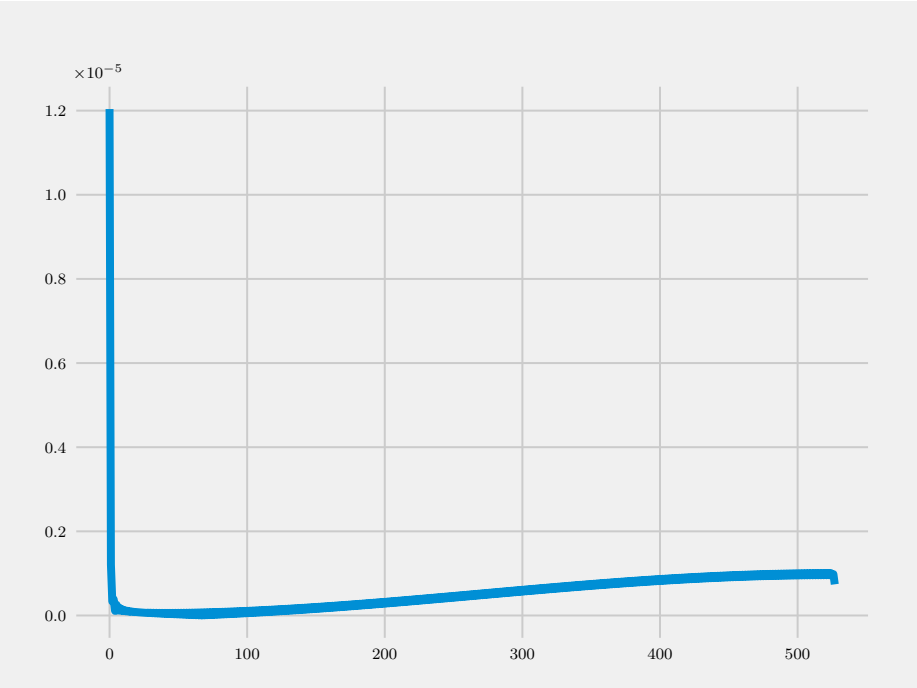

Figure 58: Fourth Order, Case number 1, 528points

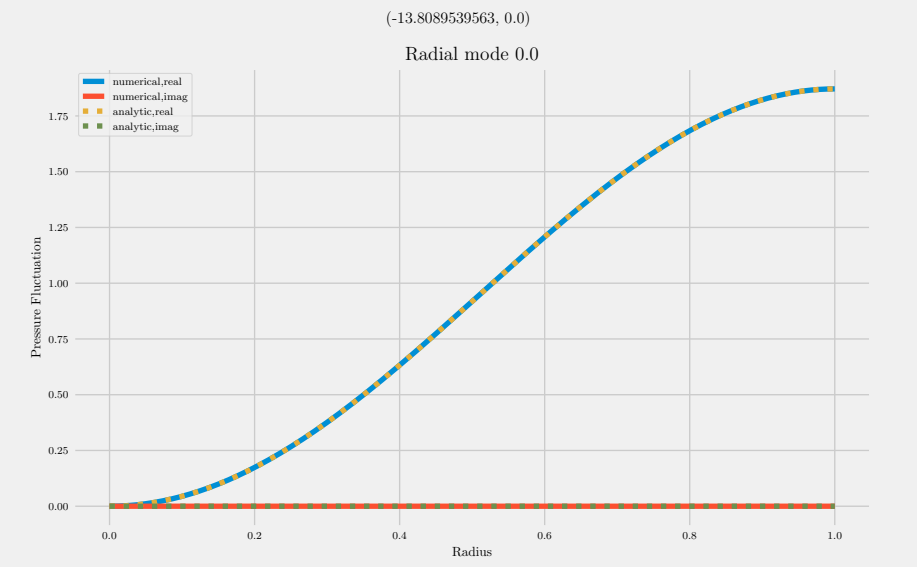

Figure 59: Fourth Order, Case number 1, 1056points

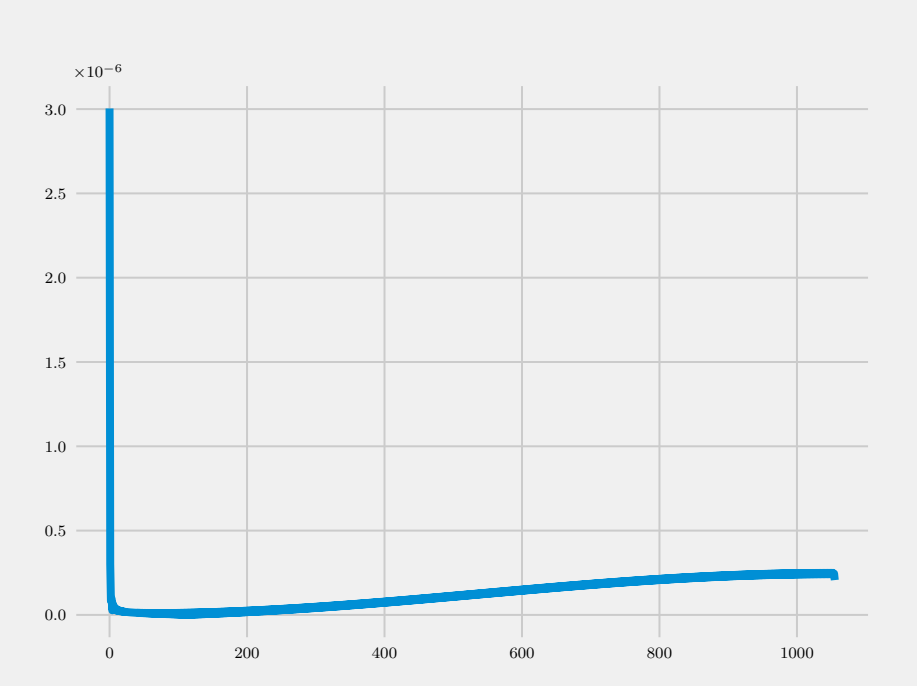

Figure 60: Fourth Order, Case number 1, 1056points

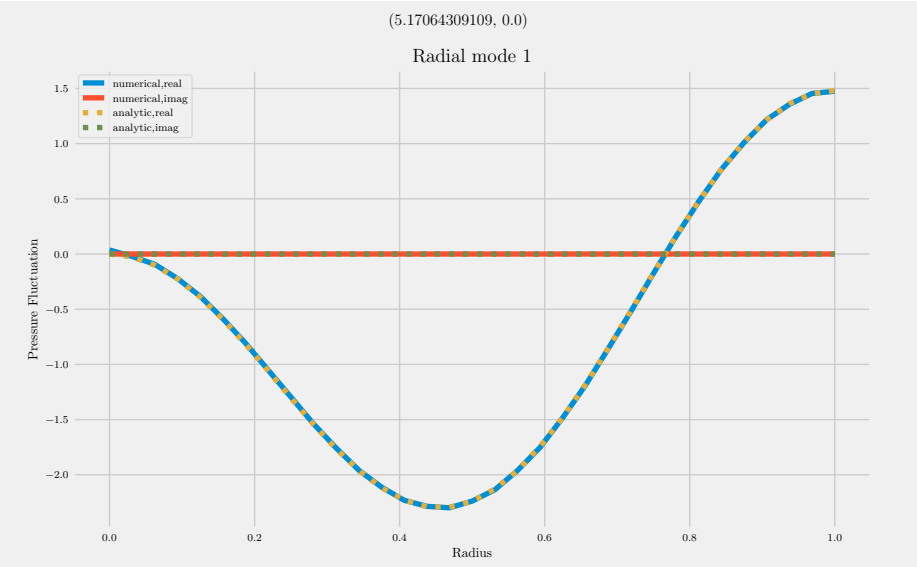

Figure 61: Second Order, Case number 0, 33points

Second Order, Radial Mode 1

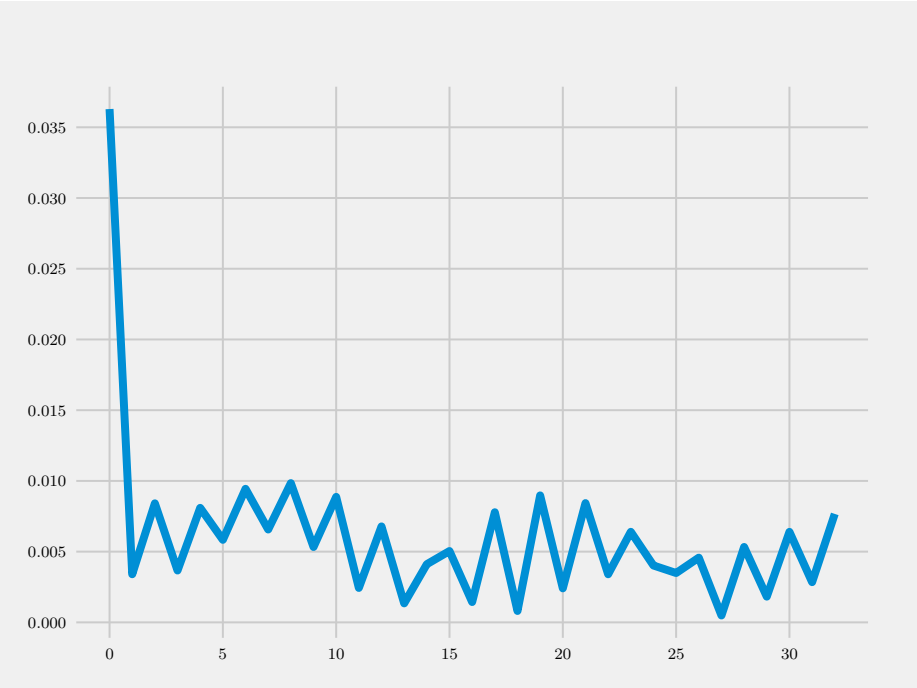

Figure 62: Second Order, Case number 0, 33points

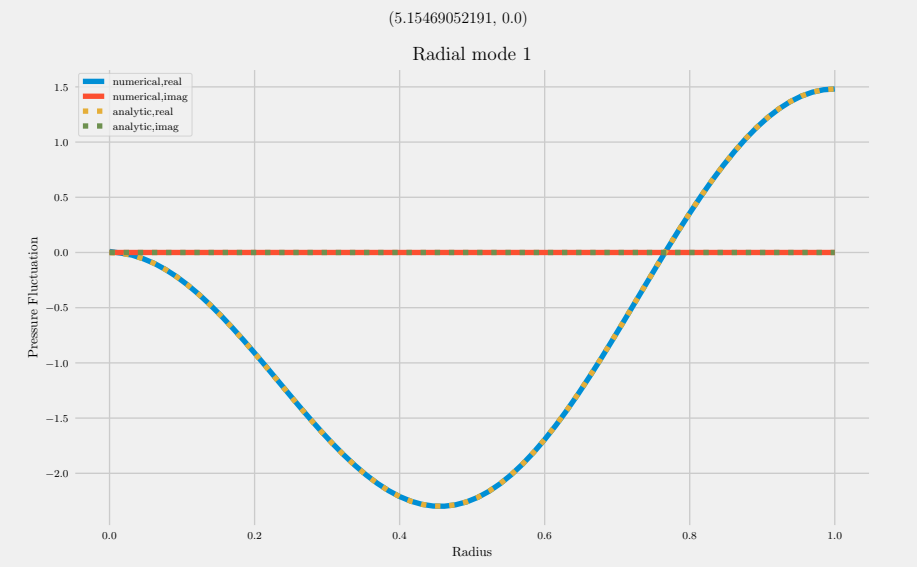

Figure 63: Second Order, Case number 0, 66points

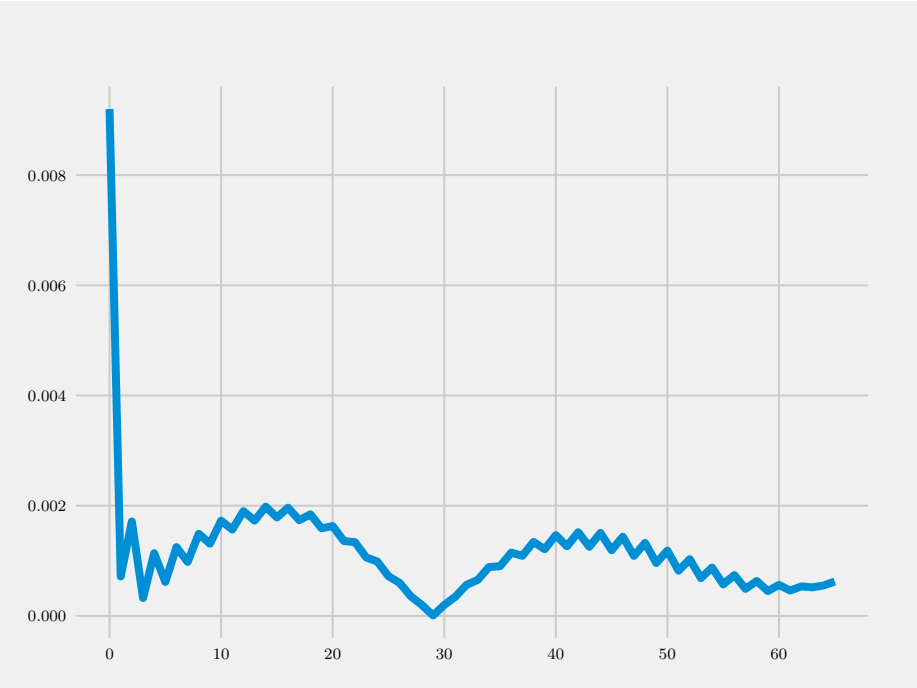

Figure 64: Second Order, Case number 0, 66points

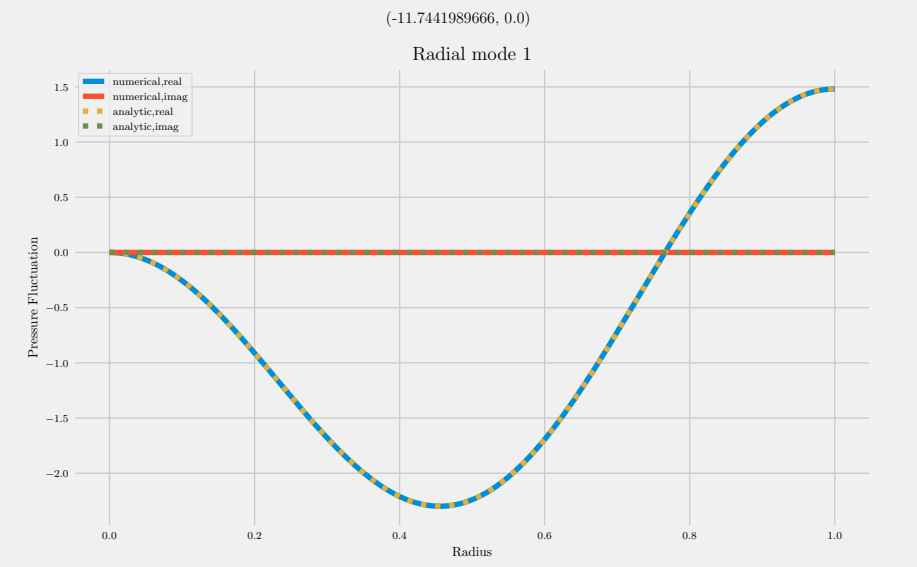

Figure 65: Second Order, Case number 0, 132points

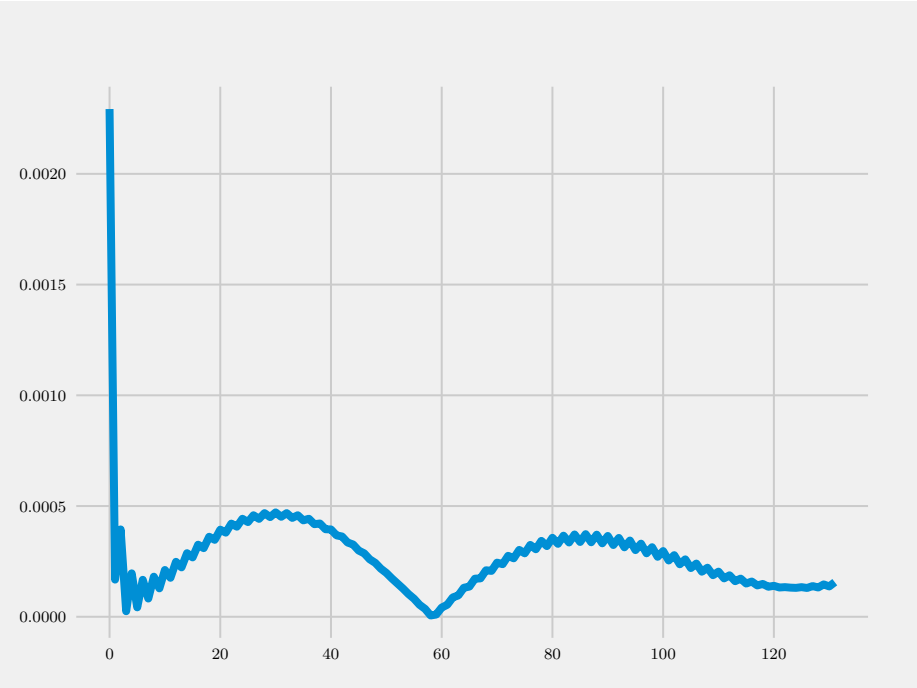

Figure 66: Second Order, Case number 0, 132points

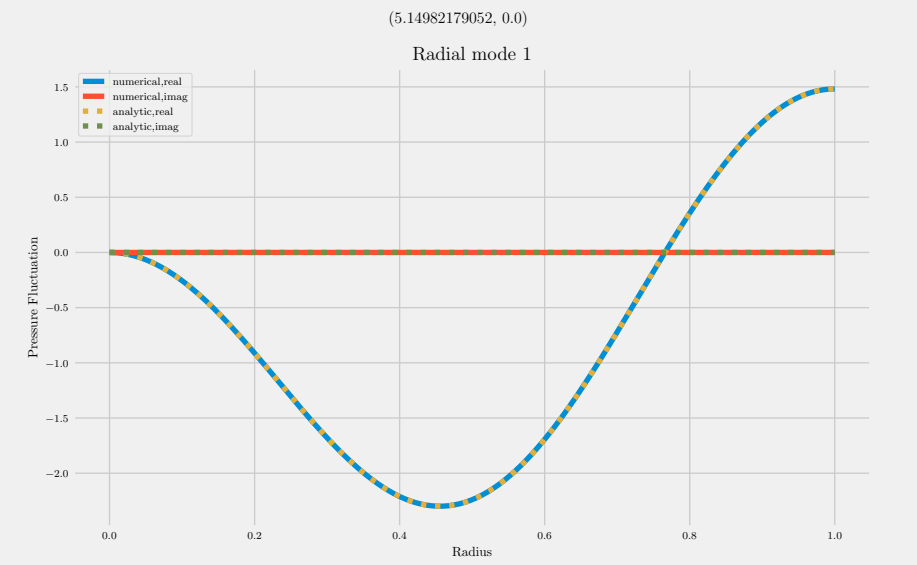

Figure 67: Second Order, Case number 0, 264points

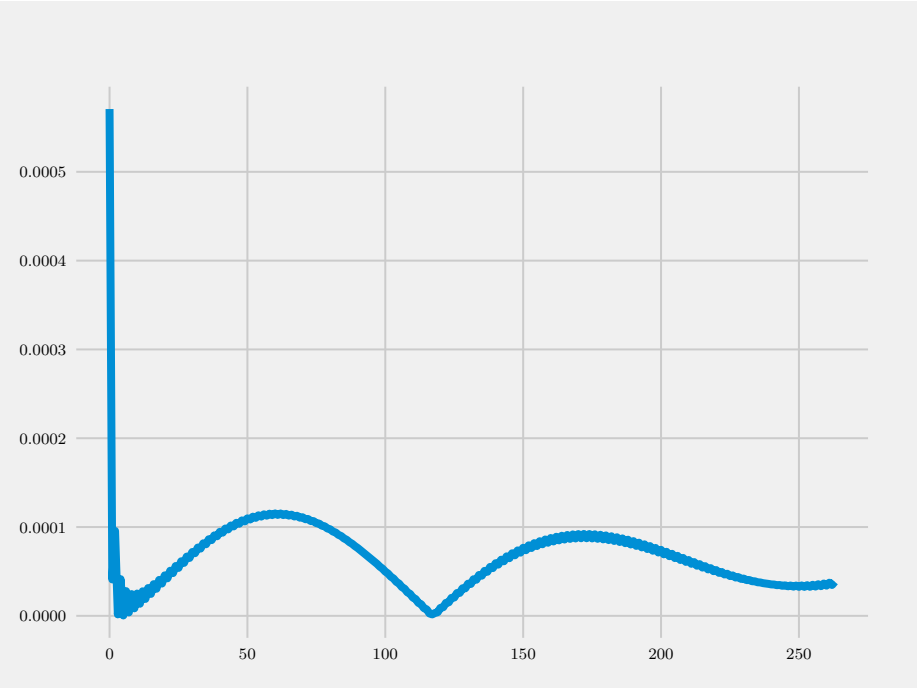

Figure 68: Second Order, Case number 0, 264points

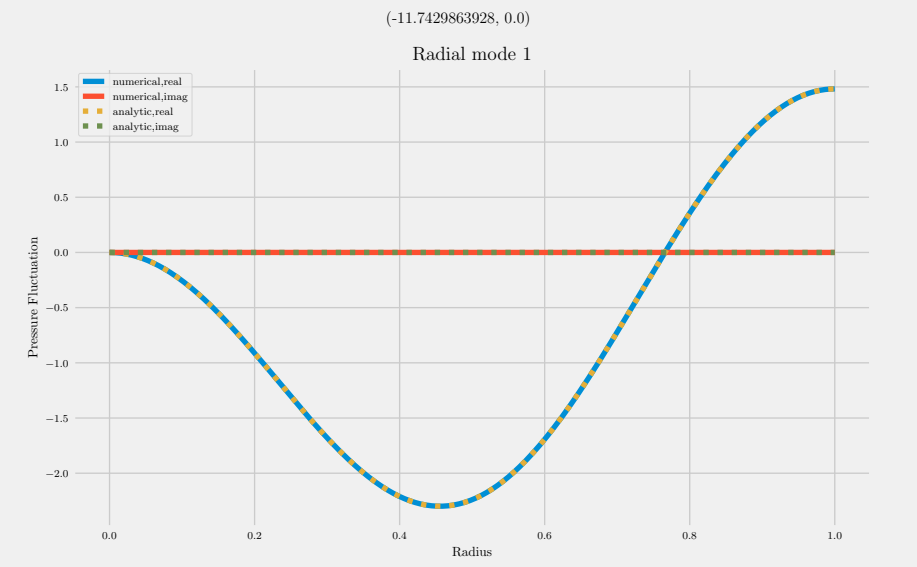

Figure 69: Second Order, Case number 0, 528points

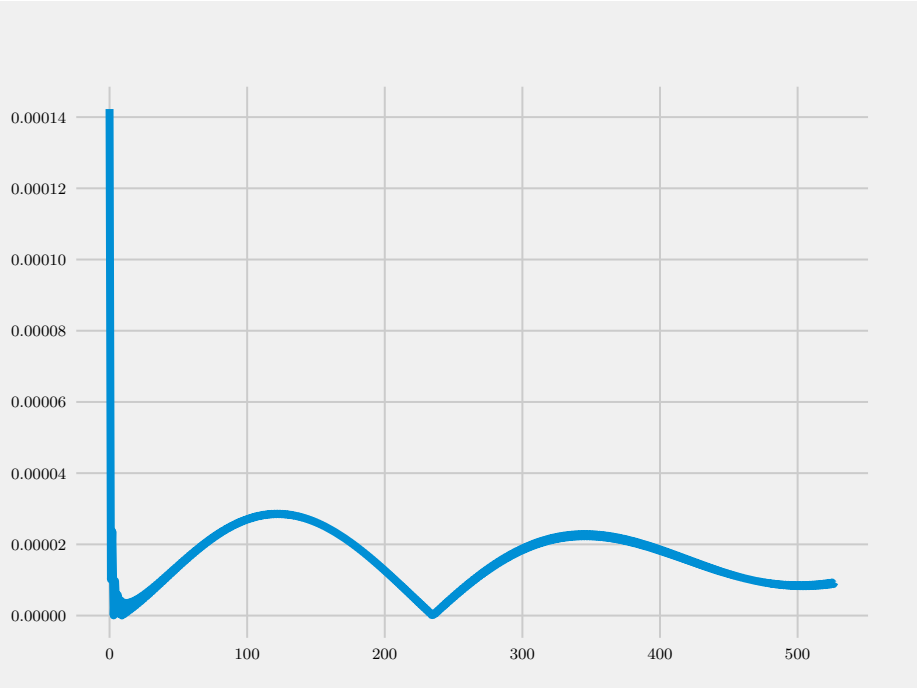

Figure 70: Second Order, Case number 0, 528points

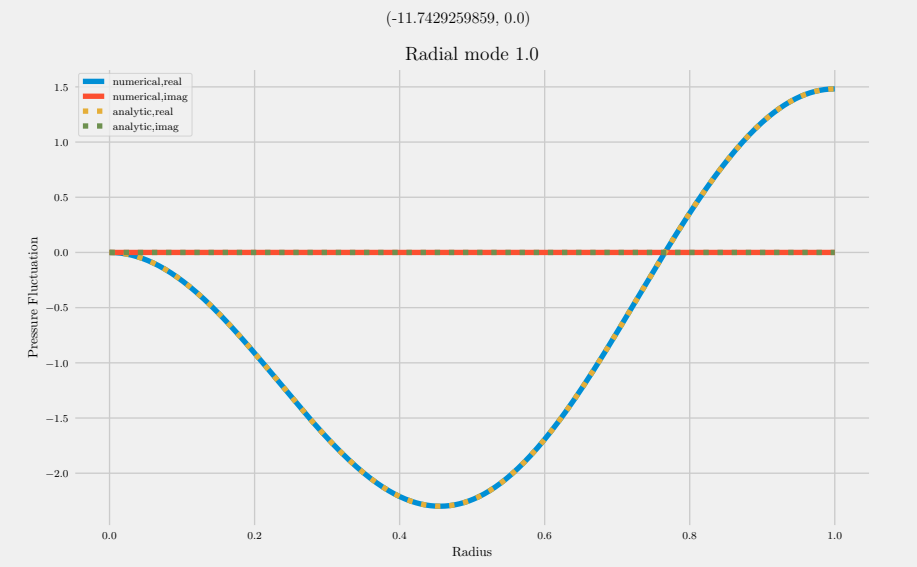

Figure 71: Second Order, Case number 0, 1056points

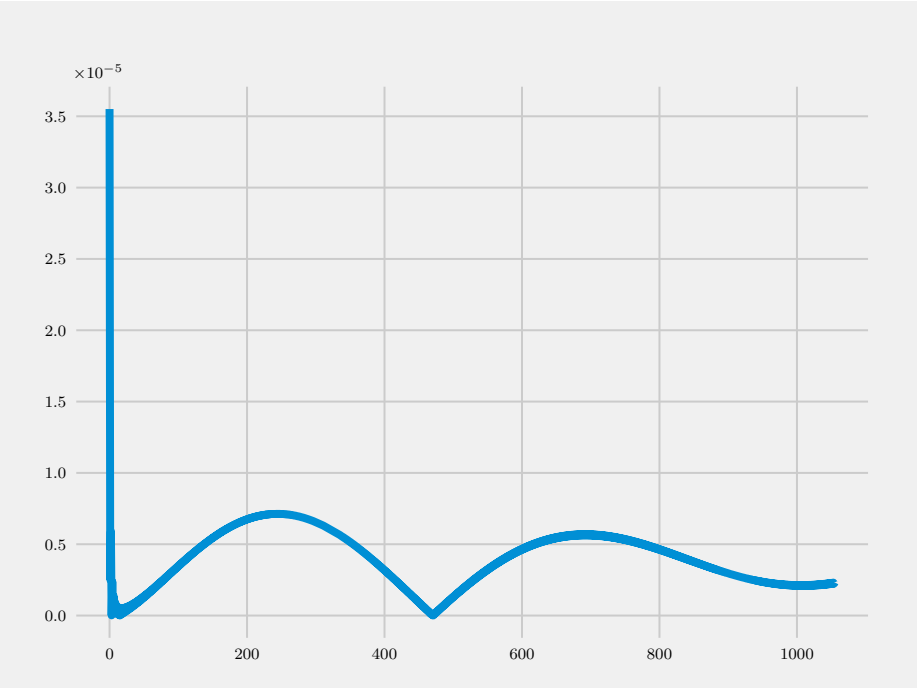

Figure 72: Second Order, Case number 0, 1056points

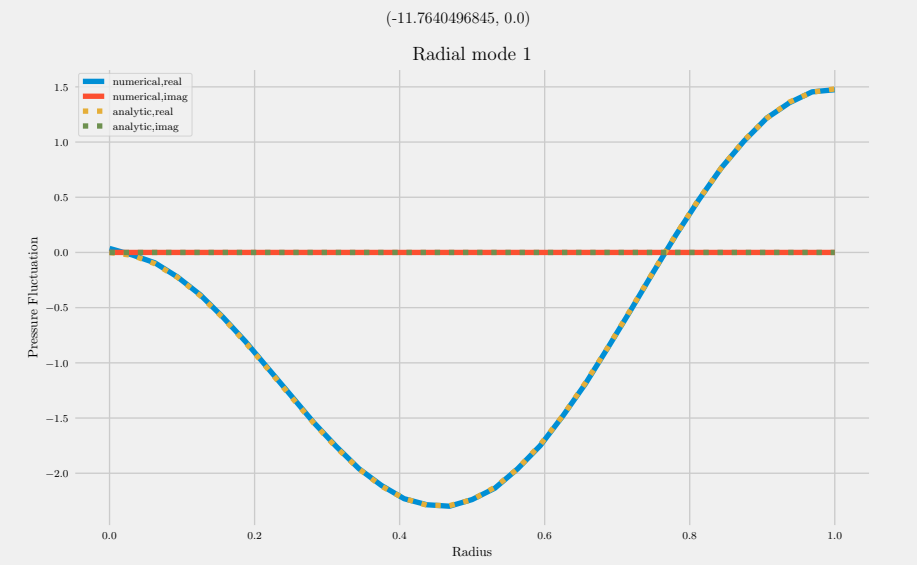

Figure 73: Second Order, Case number 1, 33points

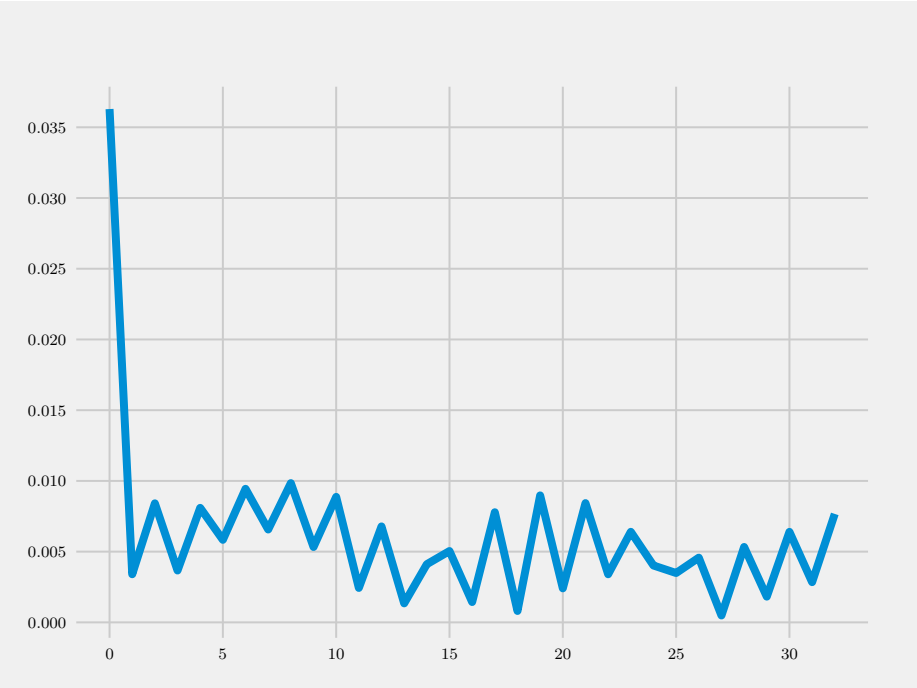

Figure 74: Second Order, Case number 1, 33points

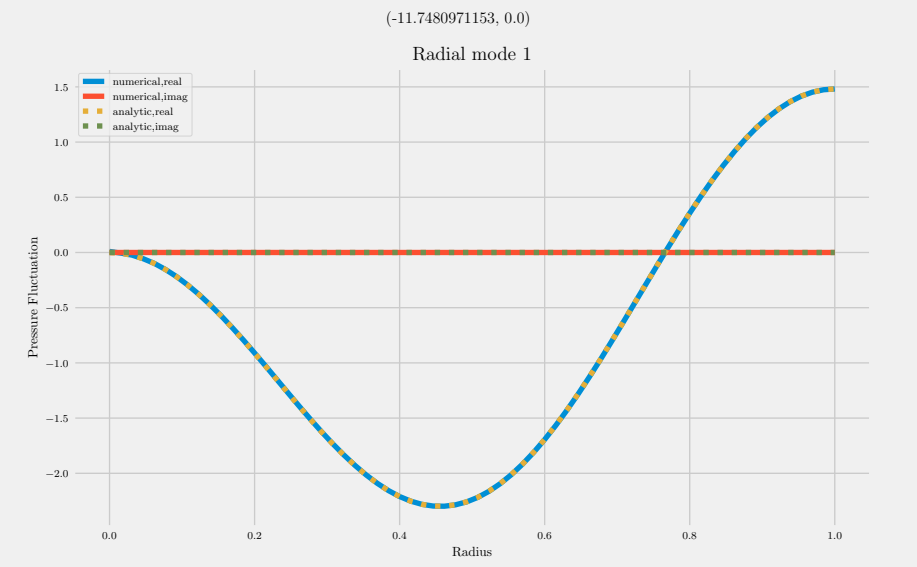

Figure 75: Second Order, Case number 1, 66points

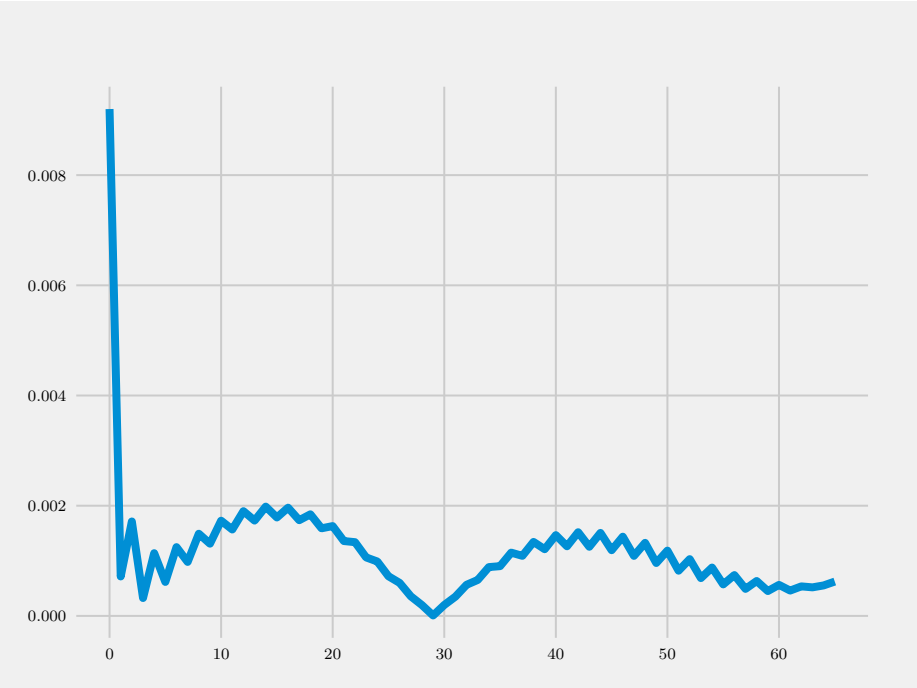

Figure 76: Second Order, Case number 1, 66points

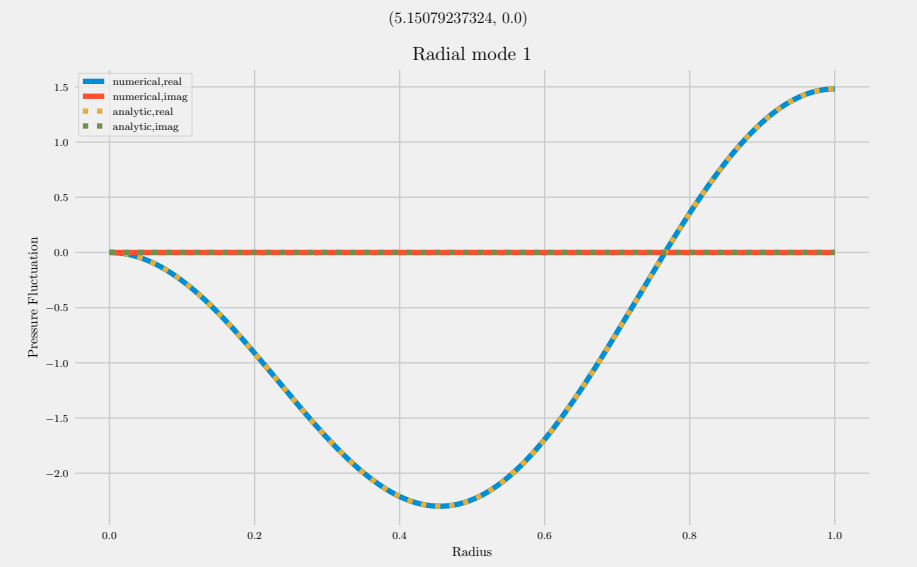

Figure 77: Second Order, Case number 1, 132points

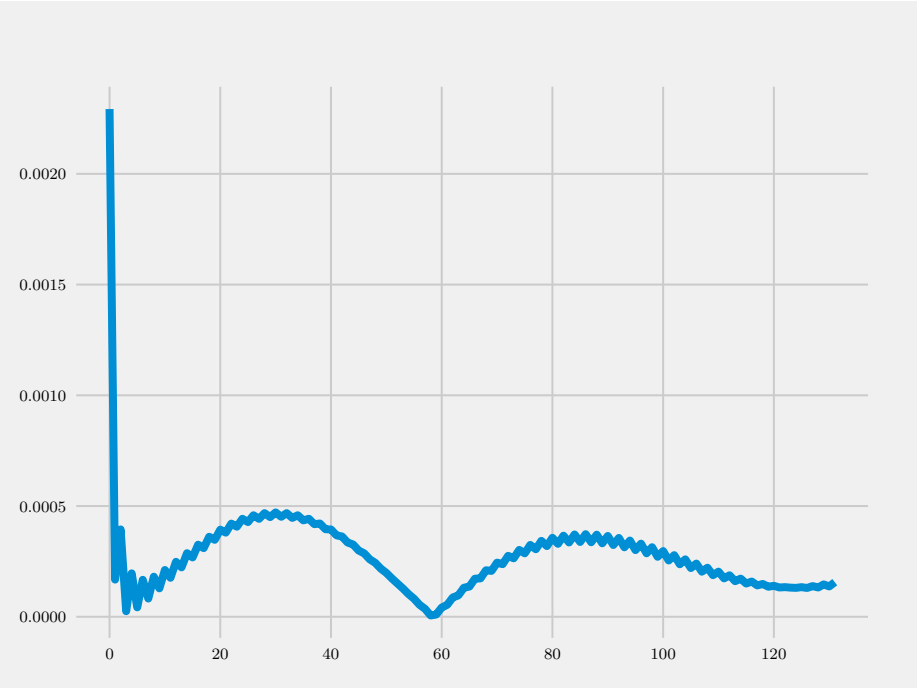

Figure 78: Second Order, Case number 1, 132points

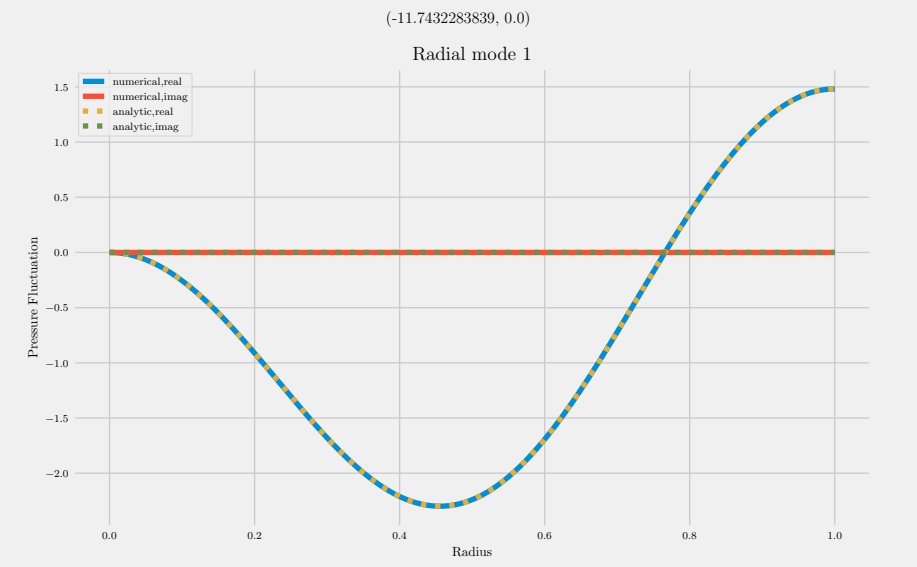

Figure 79: Second Order, Case number 1, 264points

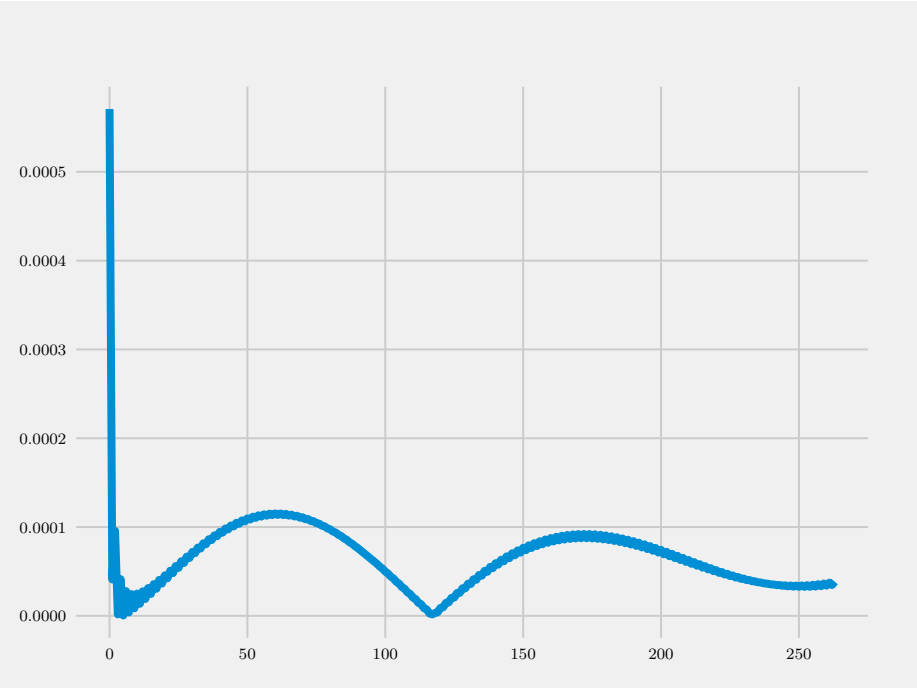

Figure 80: Second Order, Case number 1, 264points

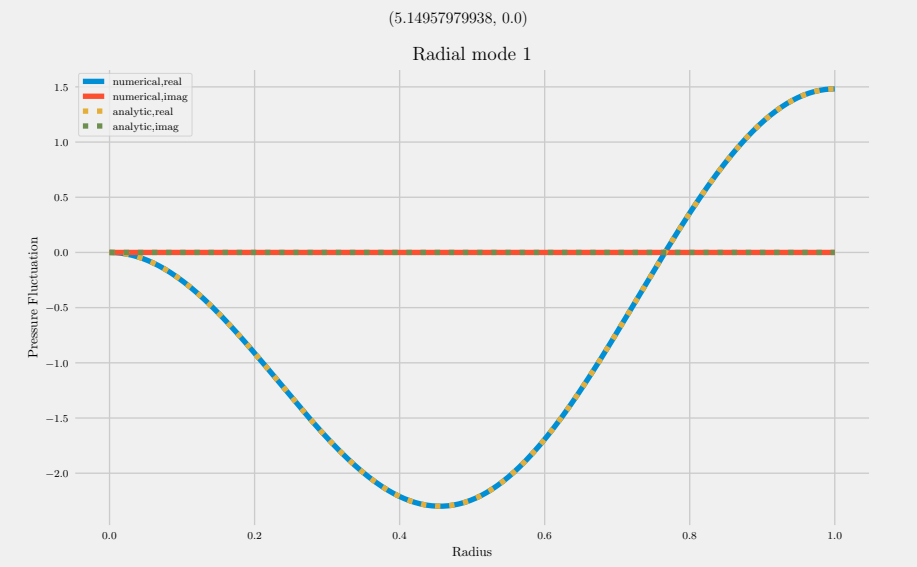

Figure 81: Second Order, Case number 1, 528points

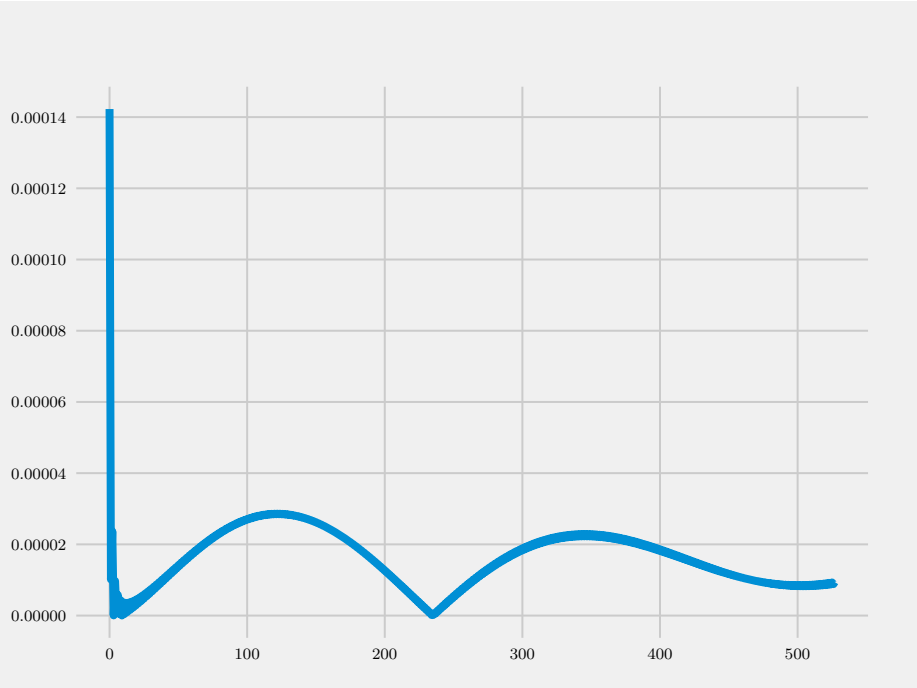

Figure 82: Second Order, Case number 1, 528points

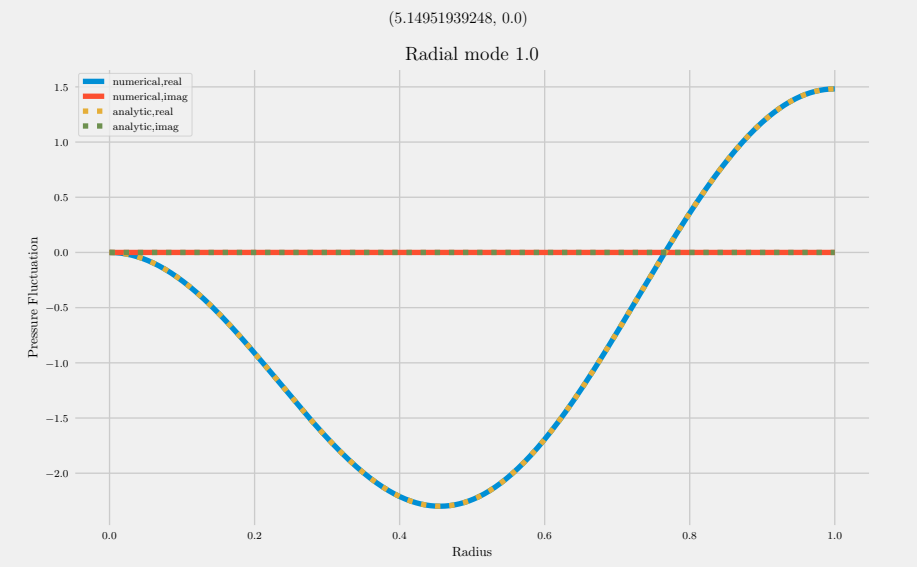

Figure 83: Second Order, Case number 1, 1056points

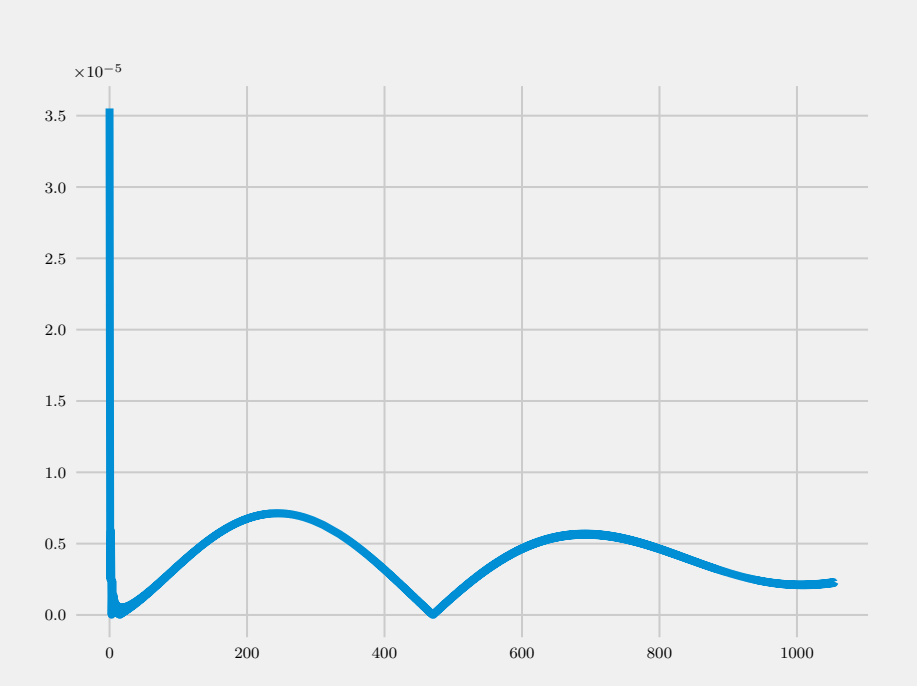

Figure 84: Second Order, Case number 1, 1056points

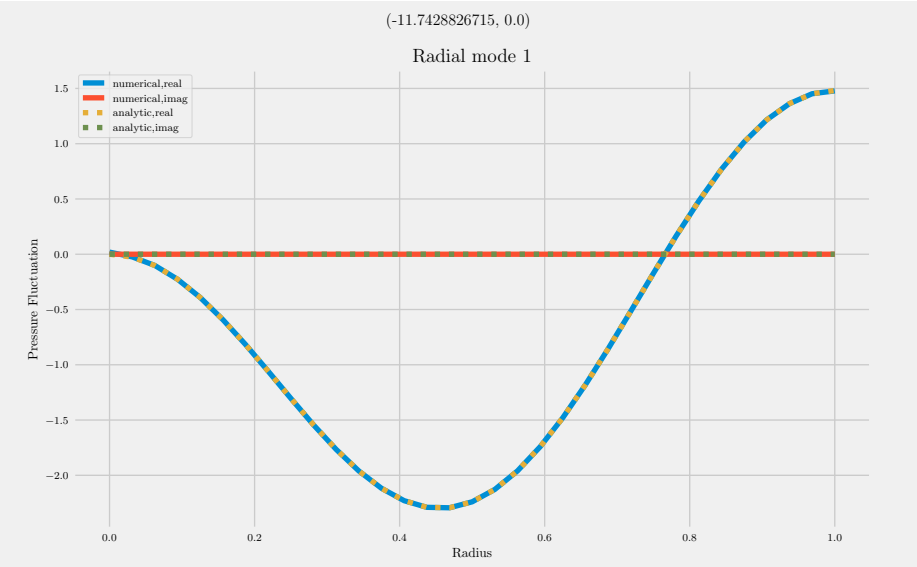

Figure 85: Fourth Order, Case number 0, 33points

Fourth Order, Radial Mode 1

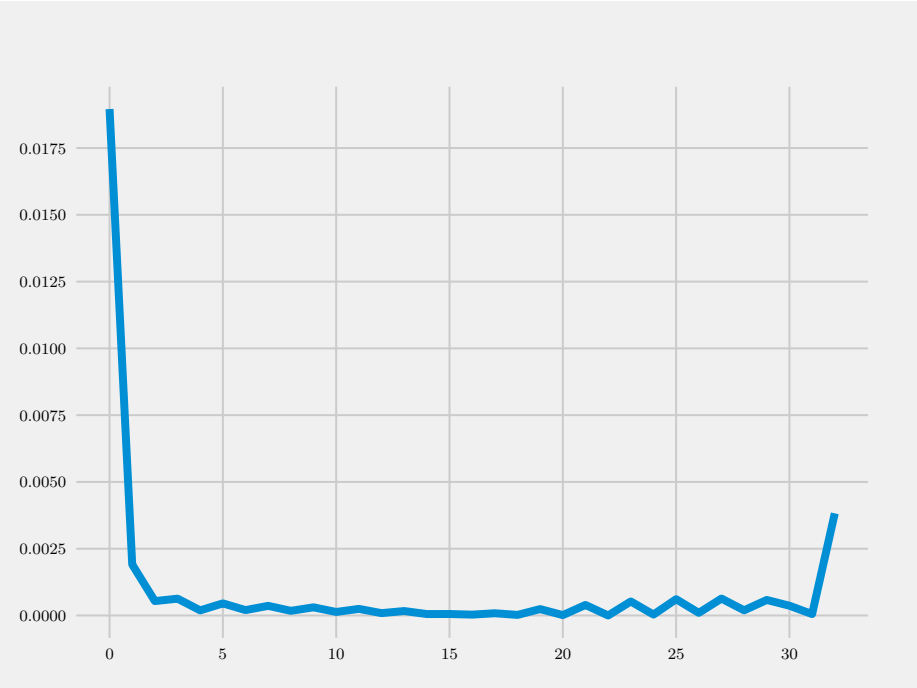

Figure 86: Fourth Order, Case number 0, 33points

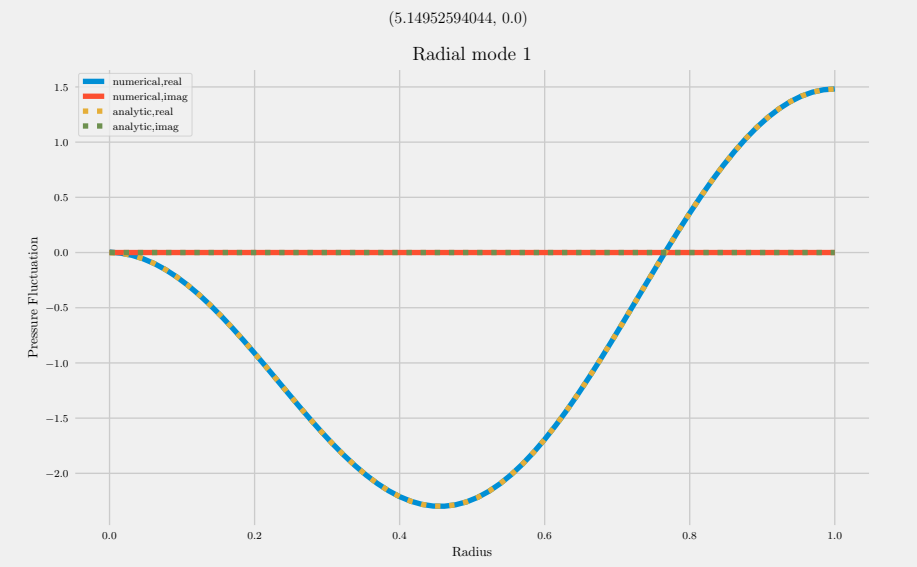

Figure 87: Fourth Order, Case number 0, 66points

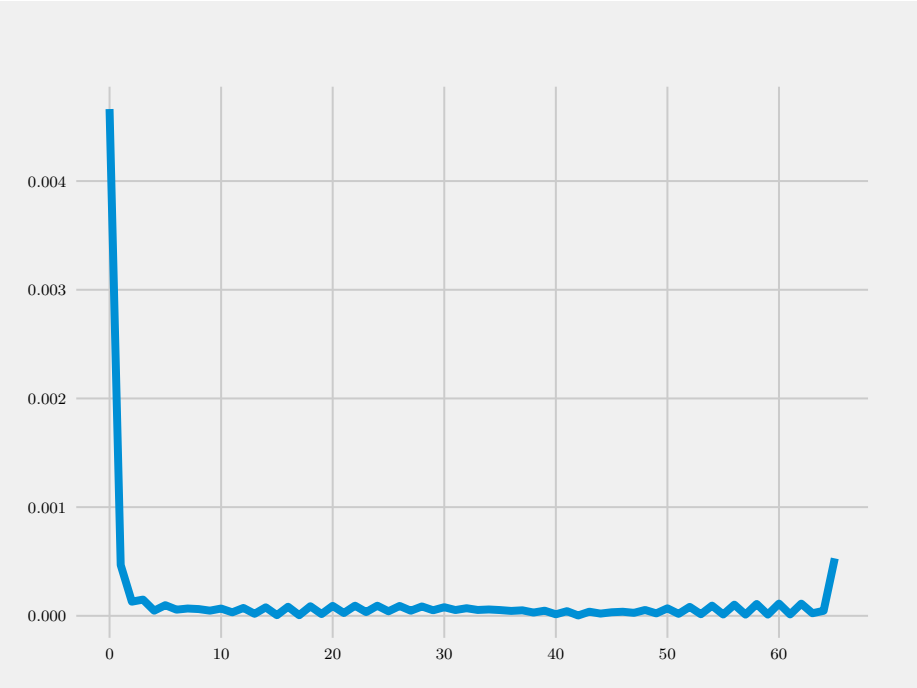

Figure 88: Fourth Order, Case number 0, 66points

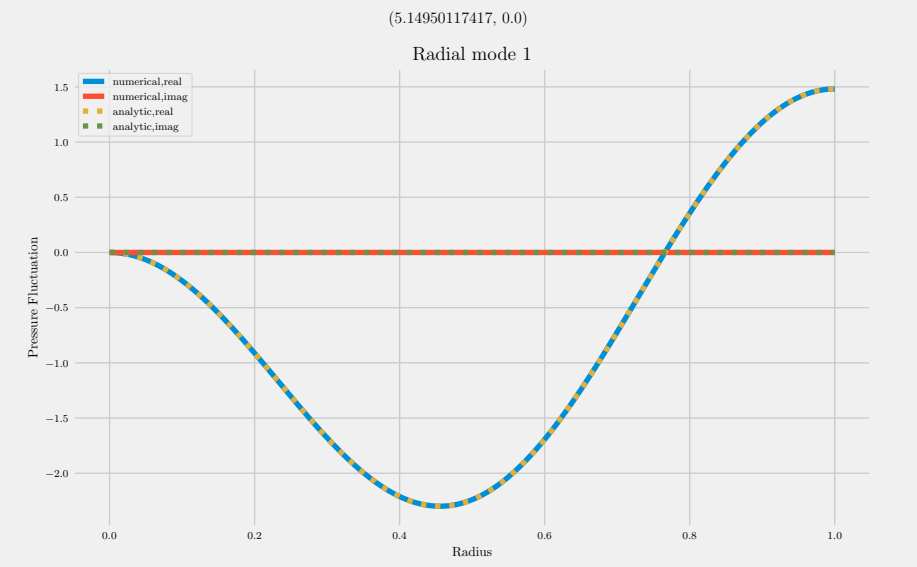

Figure 89: Fourth Order, Case number 0, 132points

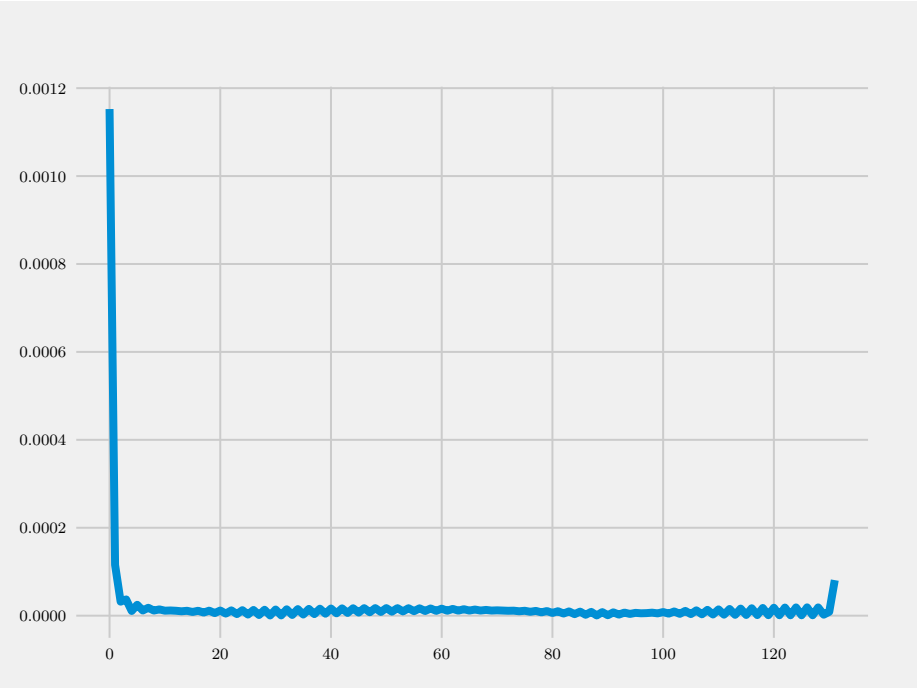

Figure 90: Fourth Order, Case number 0, 132points

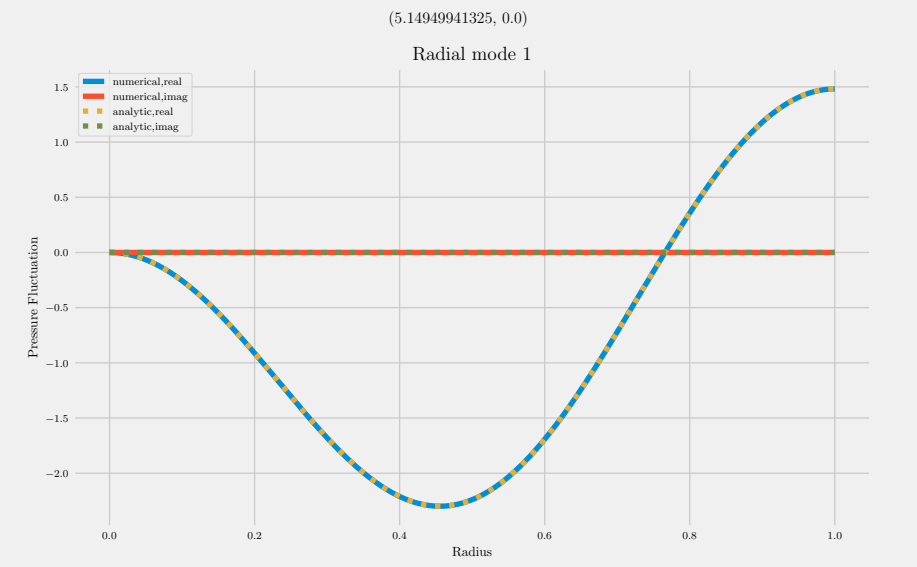

Figure 91: Fourth Order, Case number 0, 264points

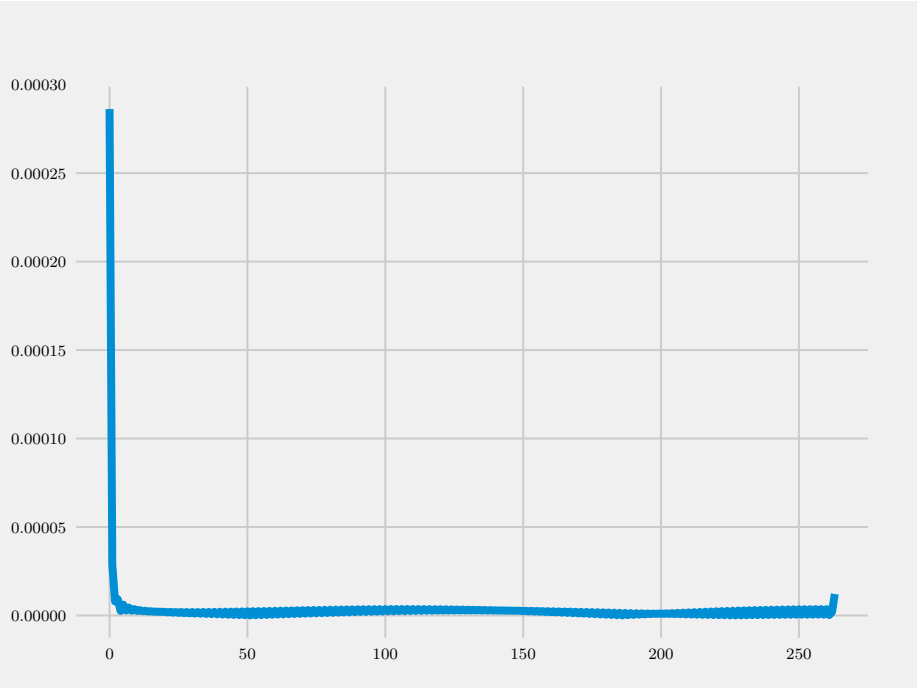

Figure 92: Fourth Order, Case number 0, 264points

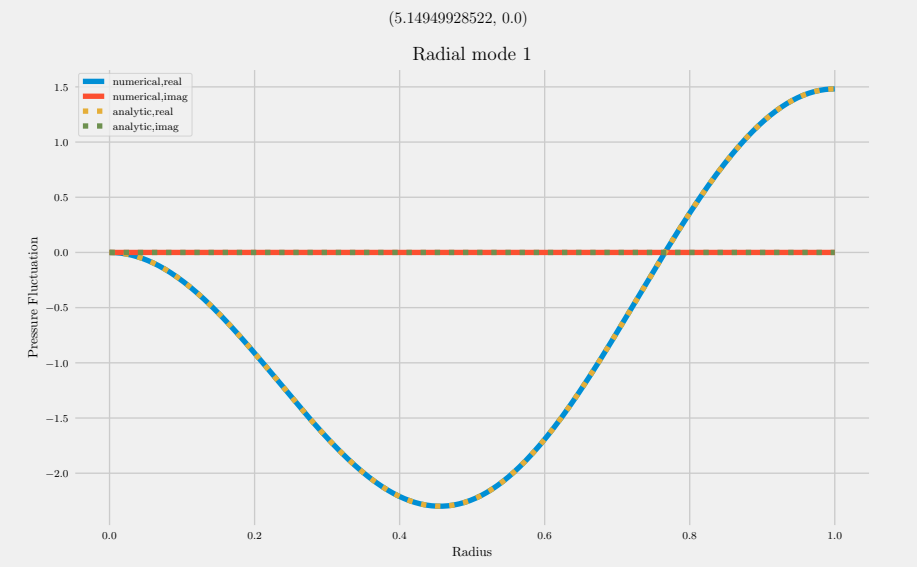

Figure 93: Fourth Order, Case number 0, 528points

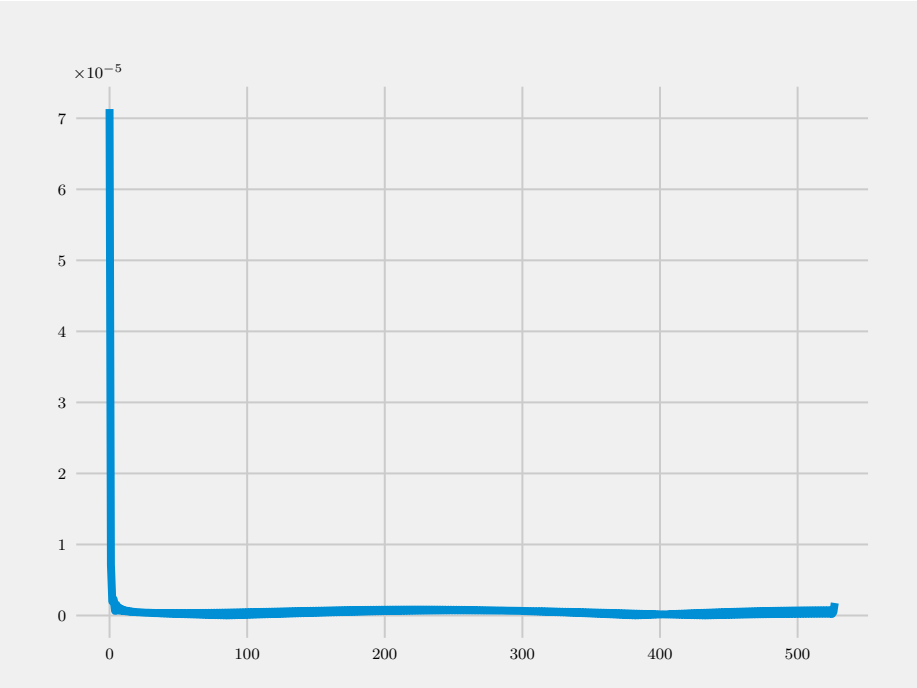

Figure 94: Fourth Order, Case number 0, 528points

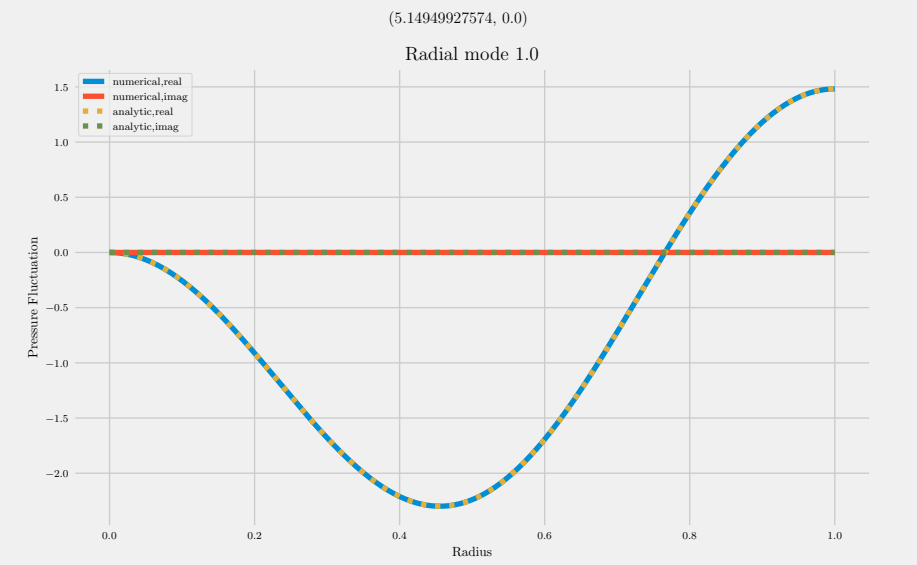

Figure 95: Fourth Order, Case number 0, 1056points

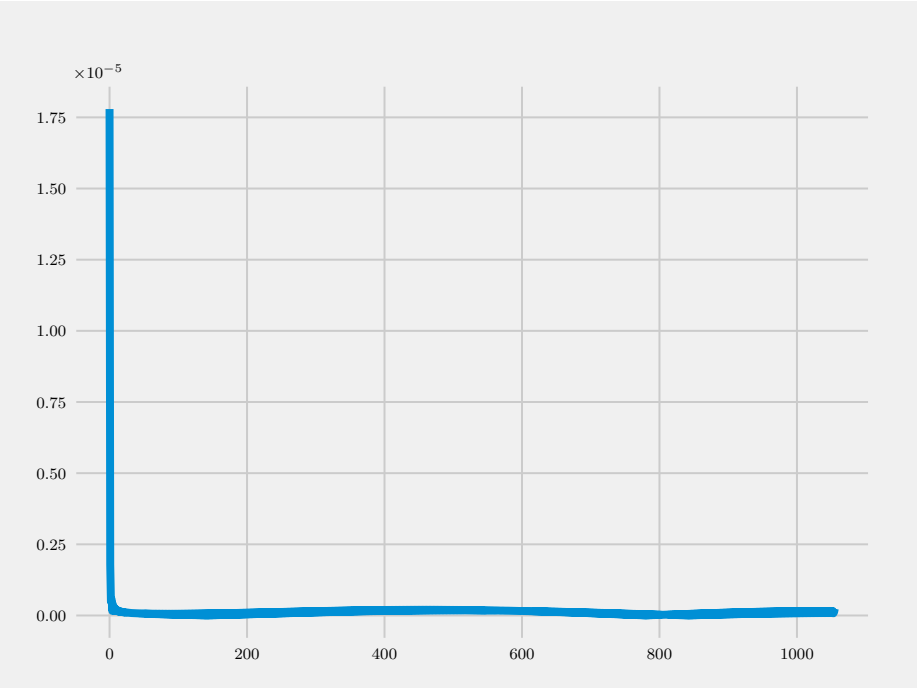

Figure 96: Fourth Order, Case number 0, 1056points

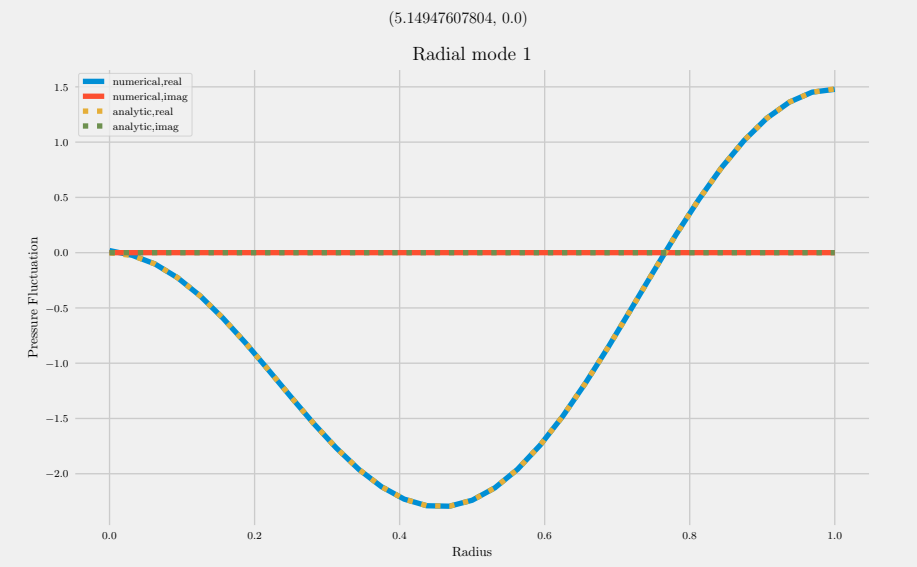

Figure 97: Fourth Order, Case number 1, 33points

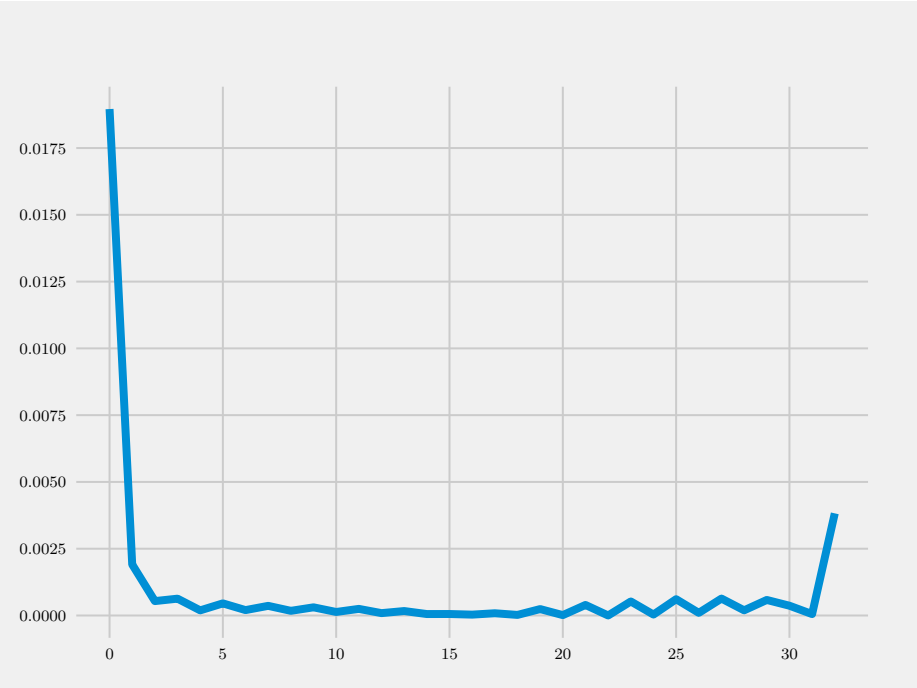

Figure 98: Fourth Order, Case number 1, 33points

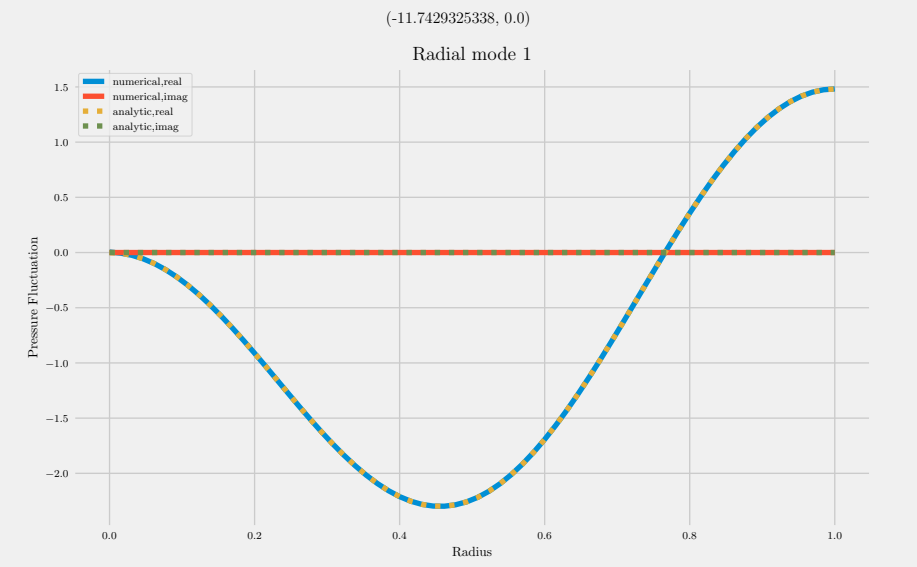

Figure 99: Fourth Order, Case number 1, 66points

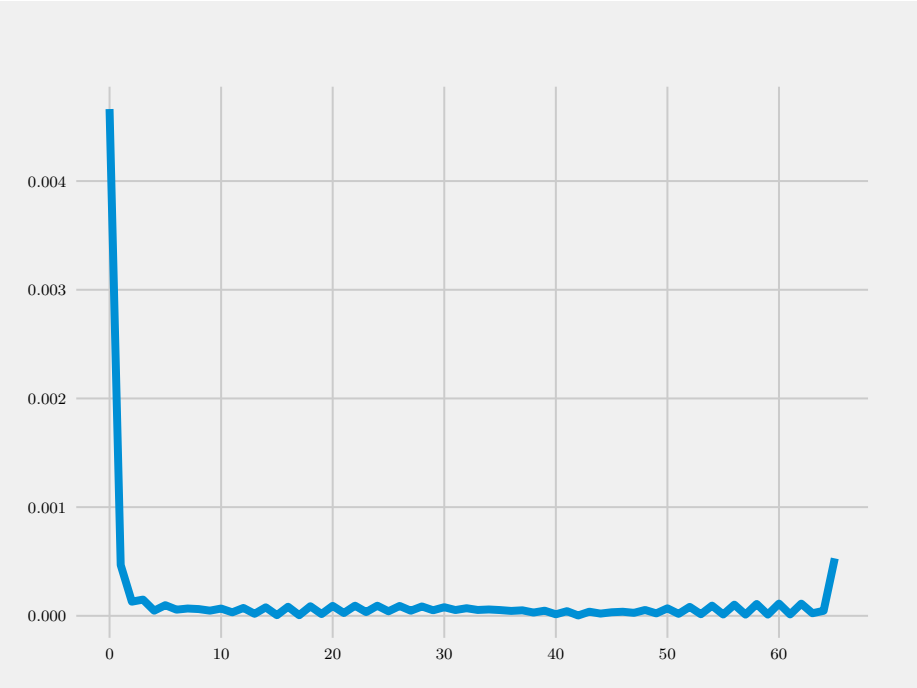

Figure 100: Fourth Order, Case number 1, 66points

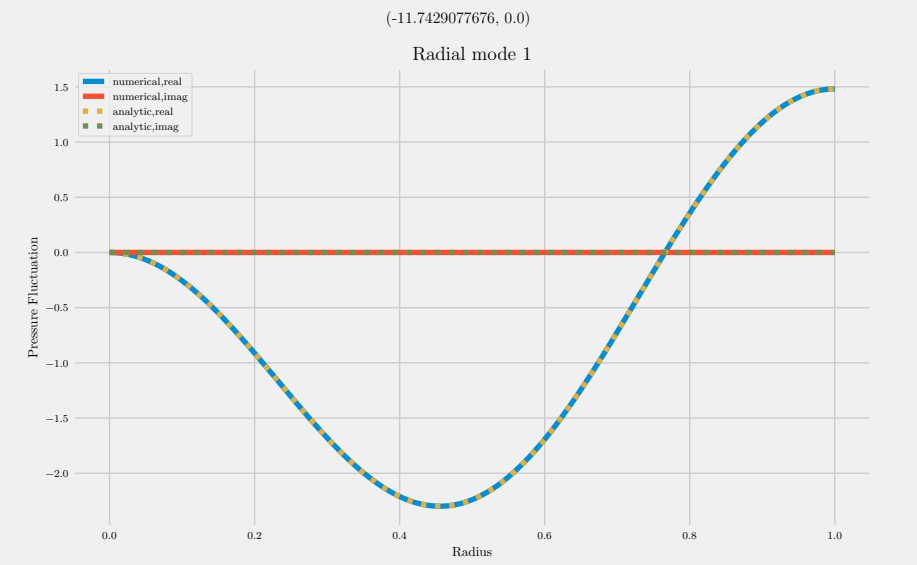

Figure 101: Fourth Order, Case number 1, 132points

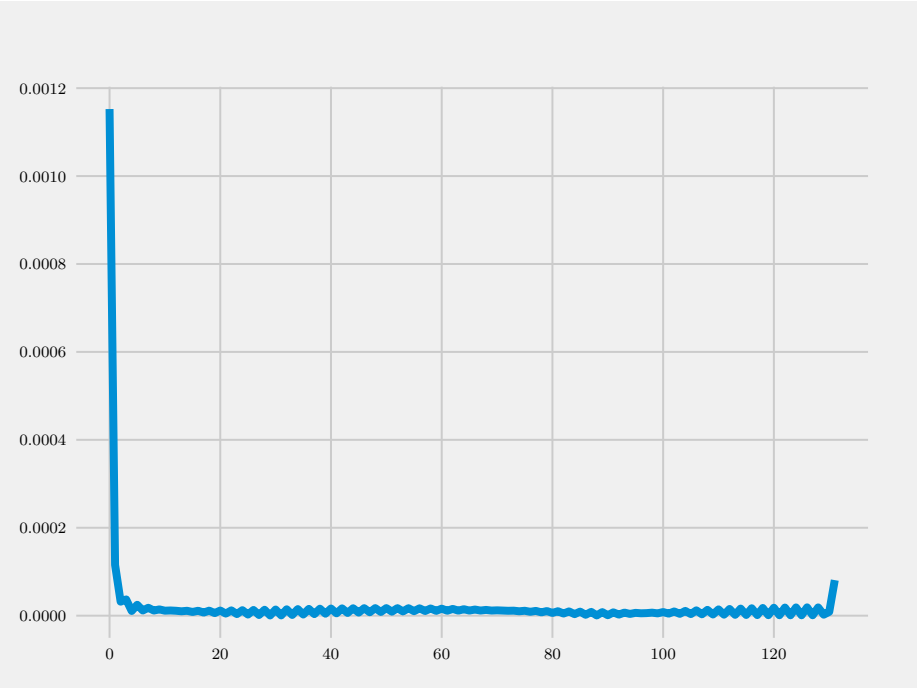

Figure 102: Fourth Order, Case number 1, 132points

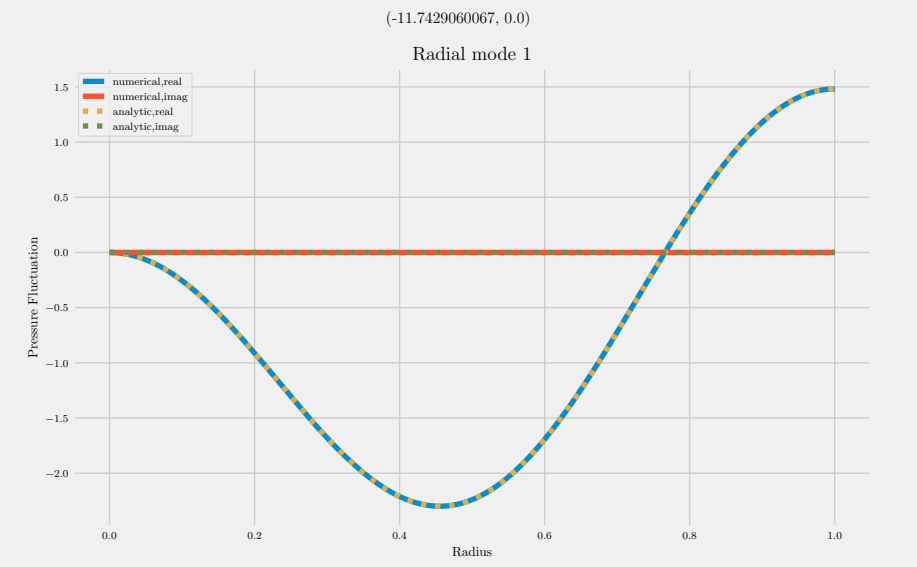

Figure 103: Fourth Order, Case number 1, 264points

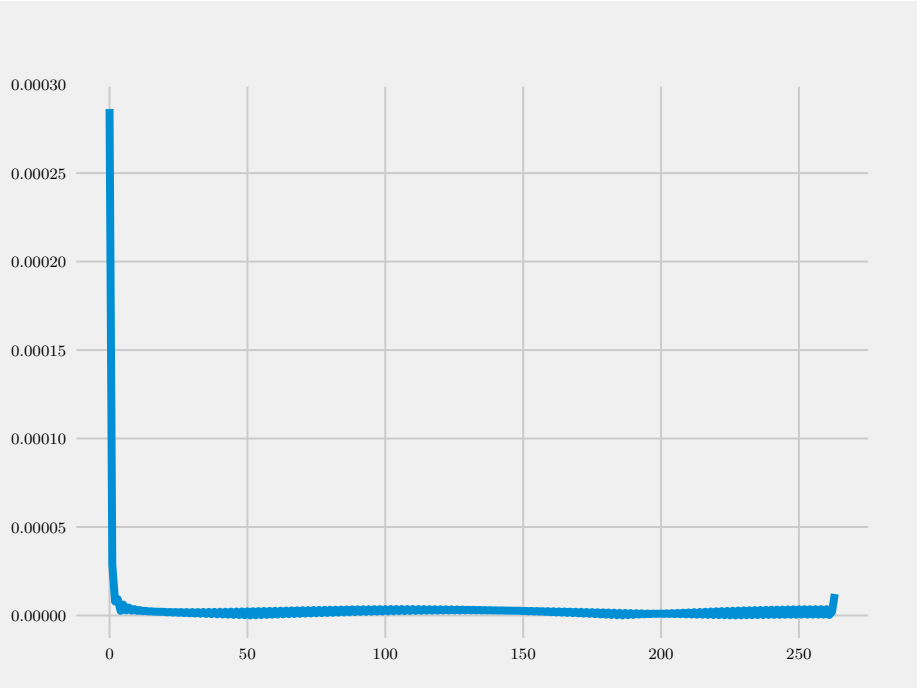

Figure 104: Fourth Order, Case number 1, 264points

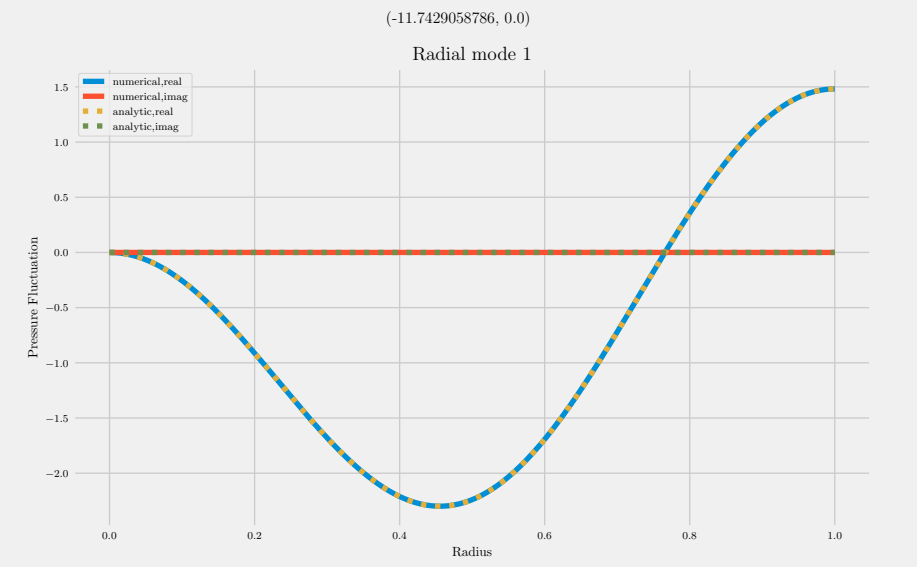

Figure 105: Fourth Order, Case number 1, 528points

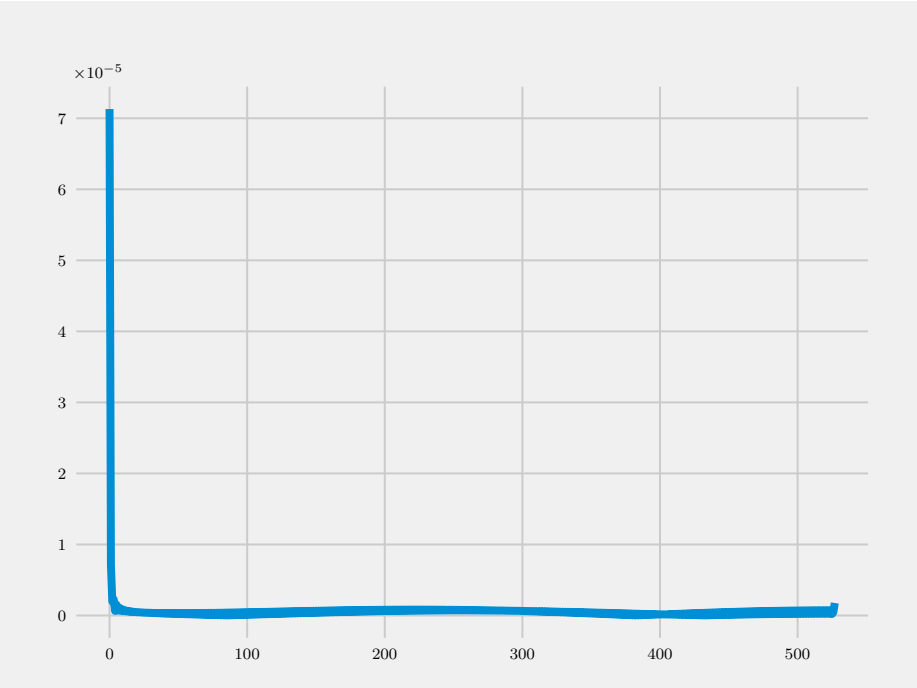

Figure 106: Fourth Order, Case number 1, 528points

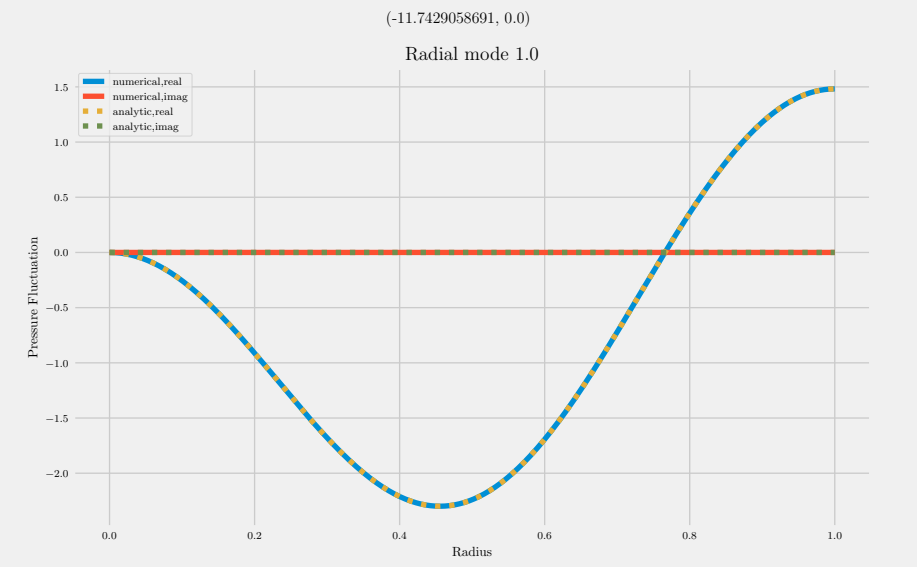

Figure 107: Fourth Order, Case number 1, 1056points

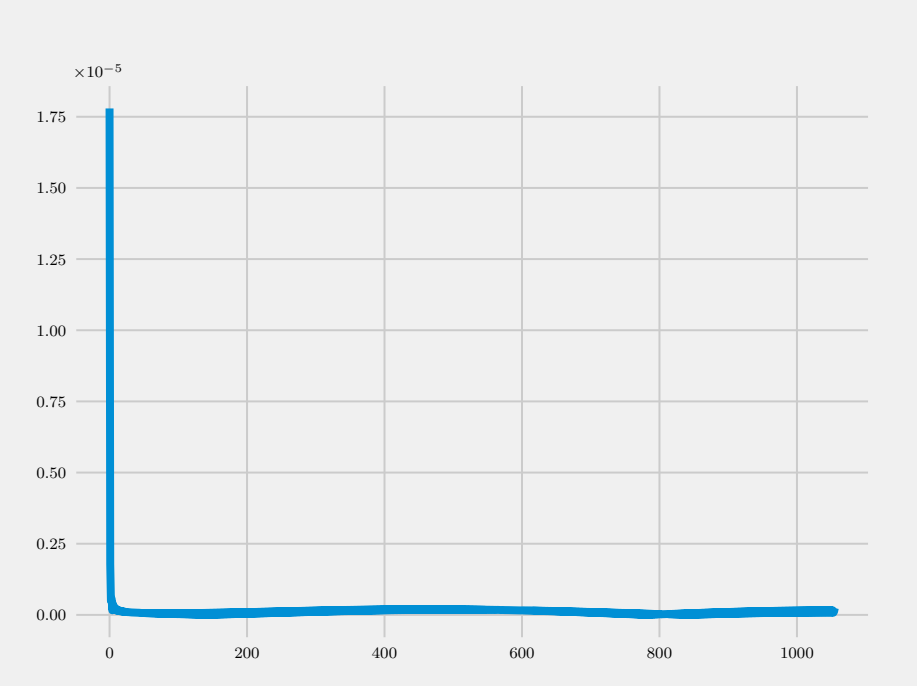

Figure 108: Fourth Order, Case number 1, 1056points

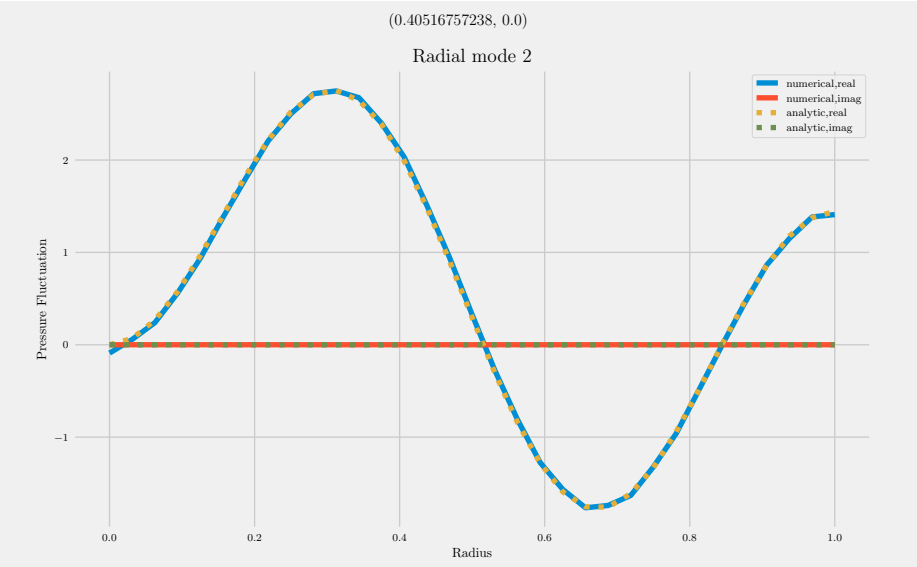

Figure 109: Second Order, Case number 0, 33points

Second Order, Radial Mode 2

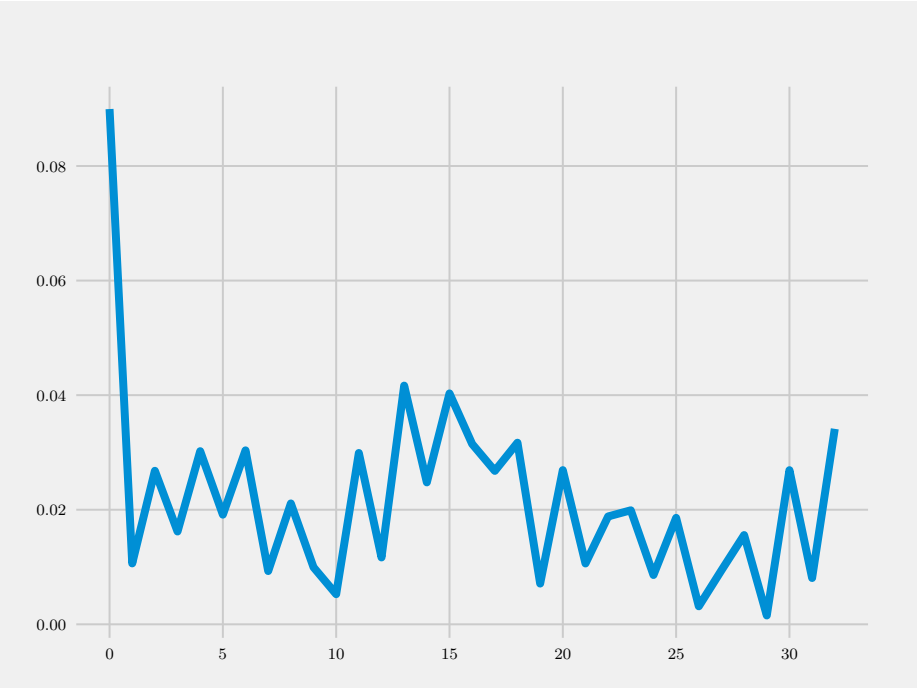

Figure 110: Second Order, Case number 0, 33points

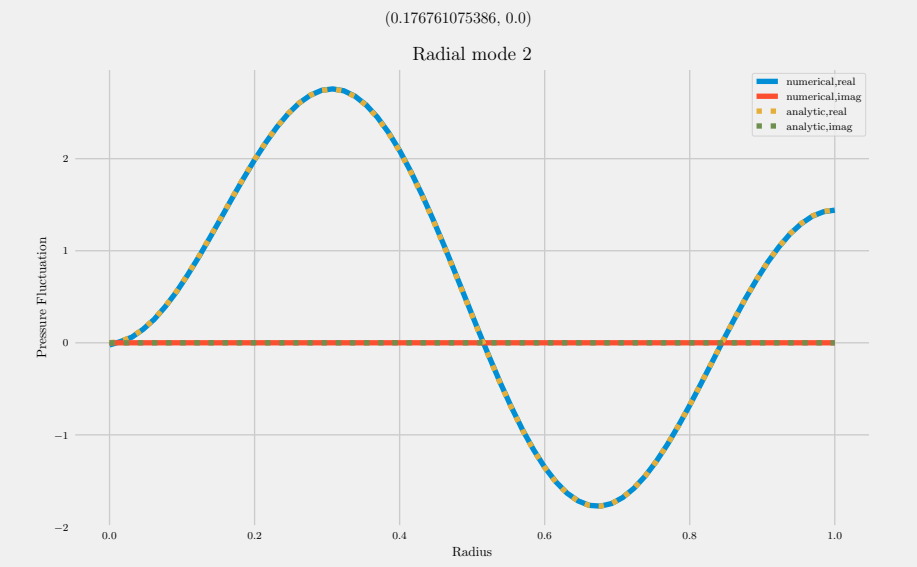

Figure 111: Second Order, Case number 0, 66points

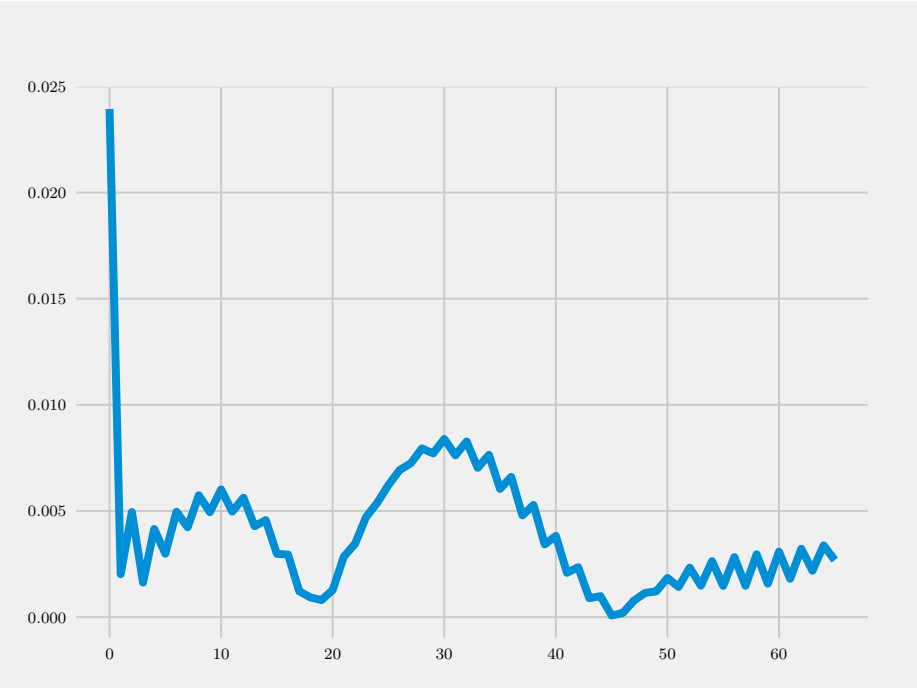

Figure 112: Second Order, Case number 0, 66points

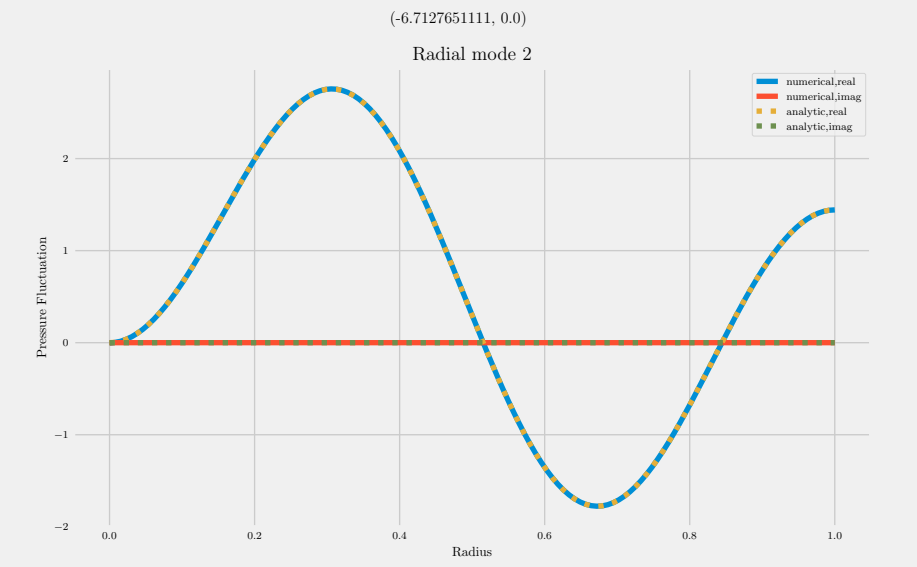

Figure 113: Second Order, Case number 0, 132points

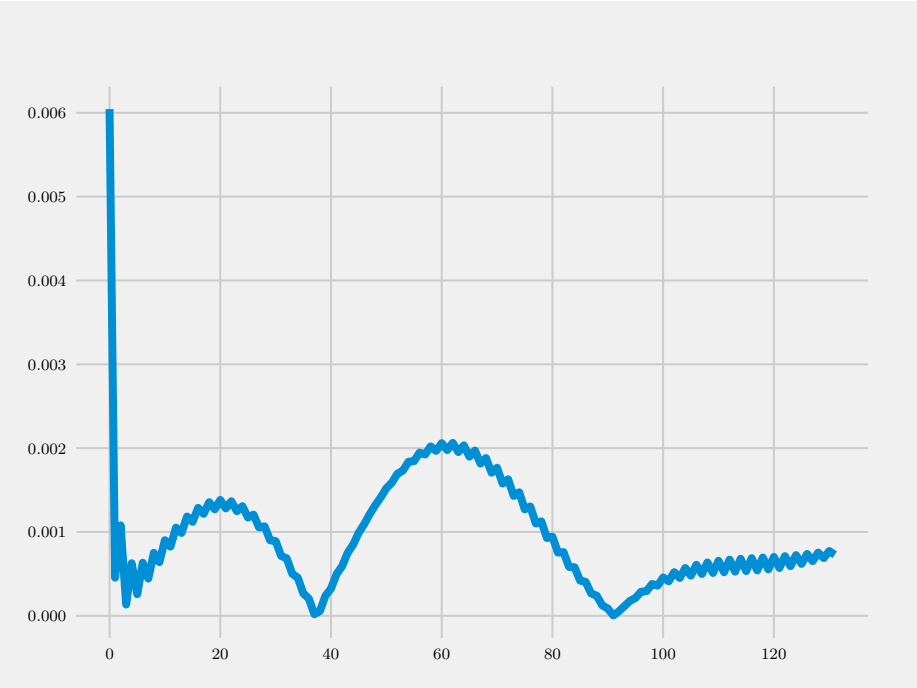

Figure 114: Second Order, Case number 0, 132points

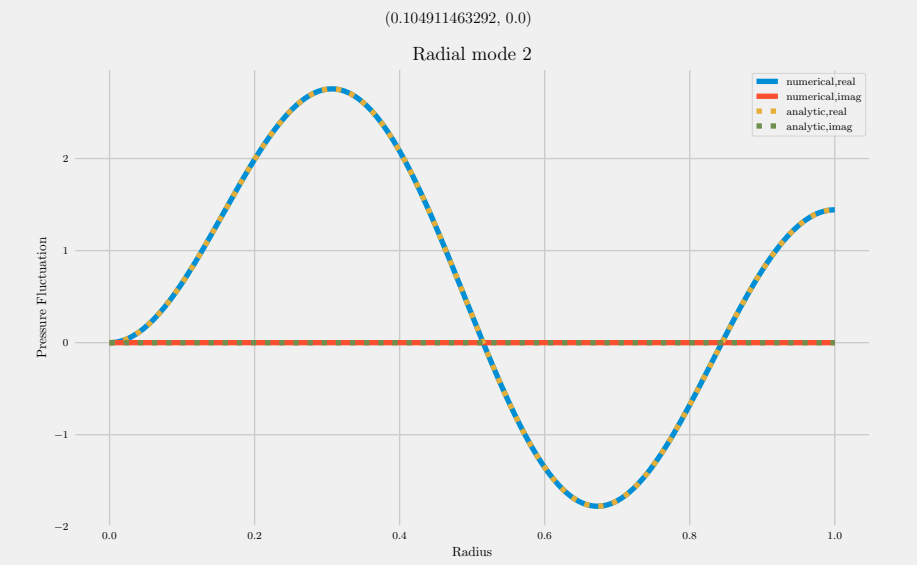

Figure 115: Second Order, Case number 0, 264points

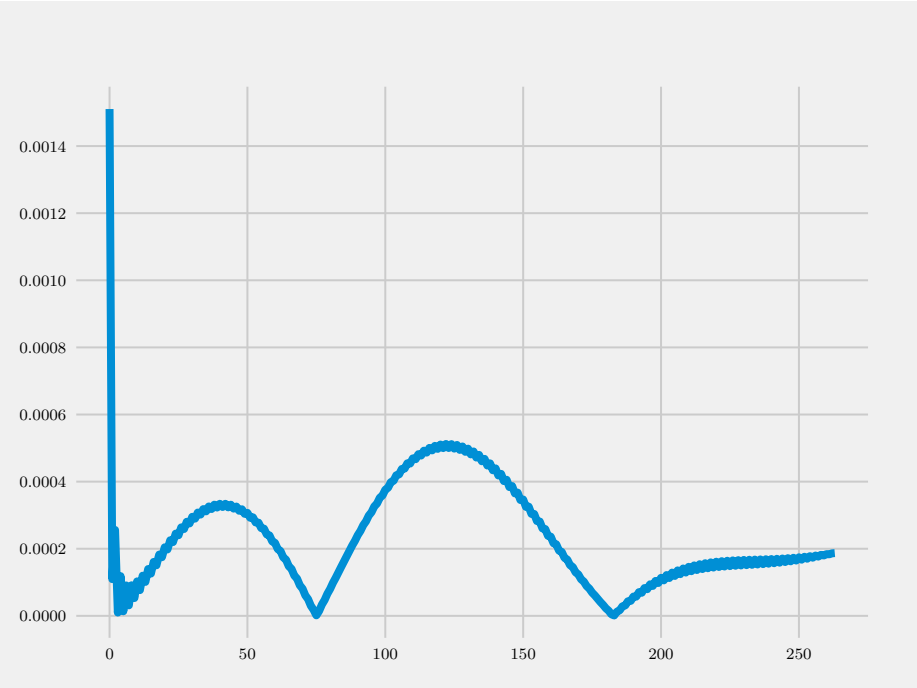

Figure 116: Second Order, Case number 0, 264points

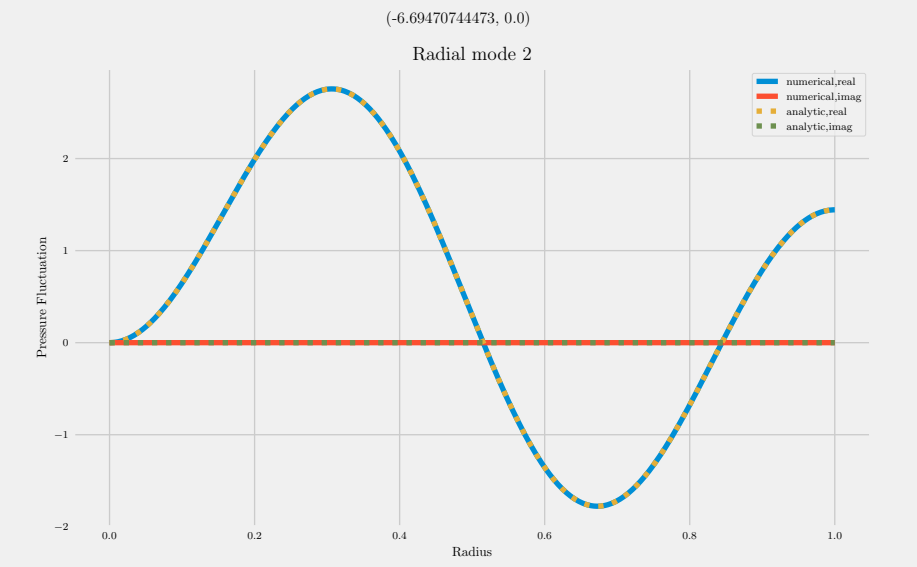

Figure 117: Second Order, Case number 0, 528points

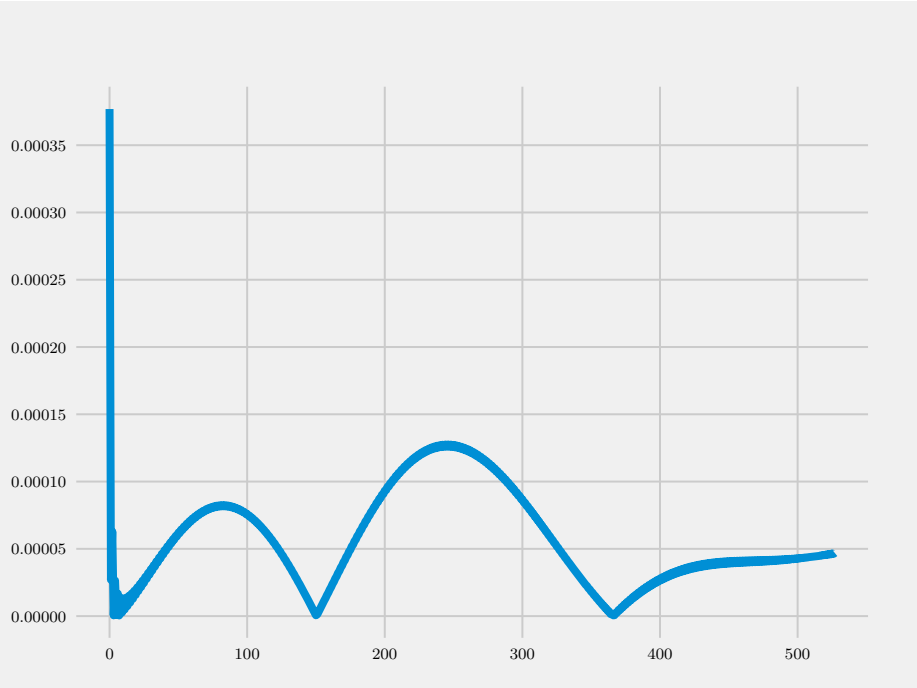

Figure 118: Second Order, Case number 0, 528points

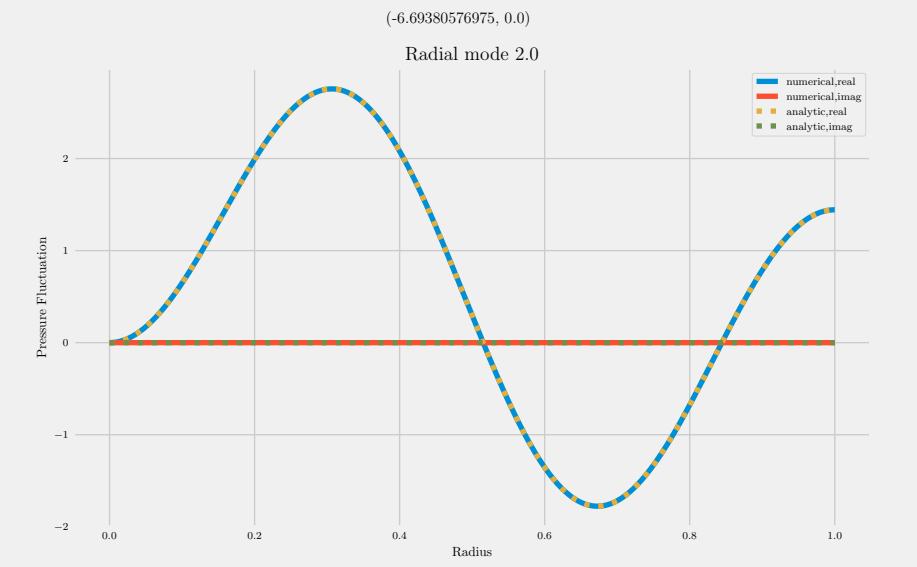

Figure 119: Second Order, Case number 0, 1056points

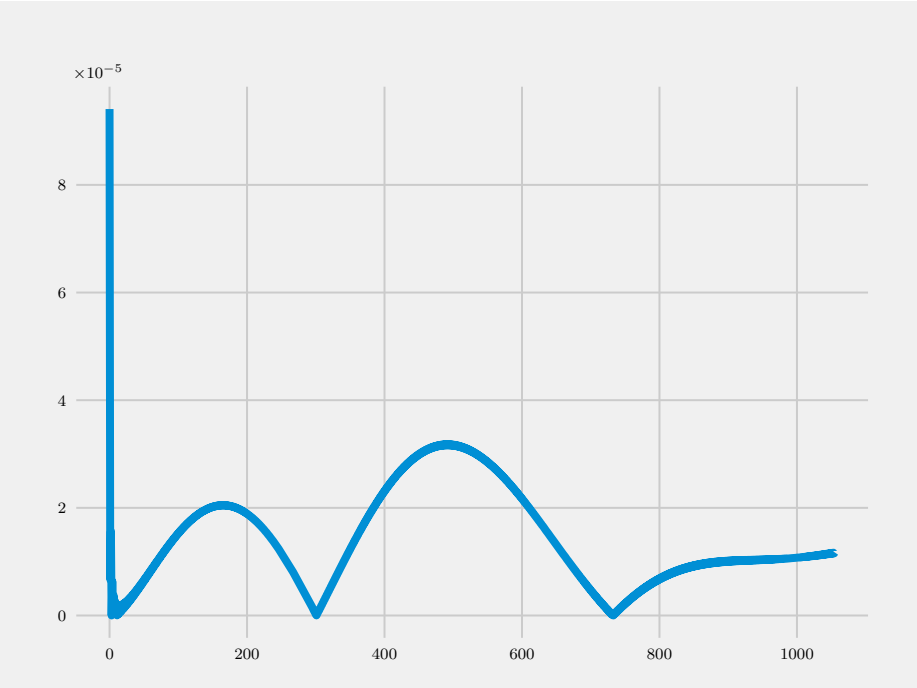

Figure 120: Second Order, Case number 0, 1056points

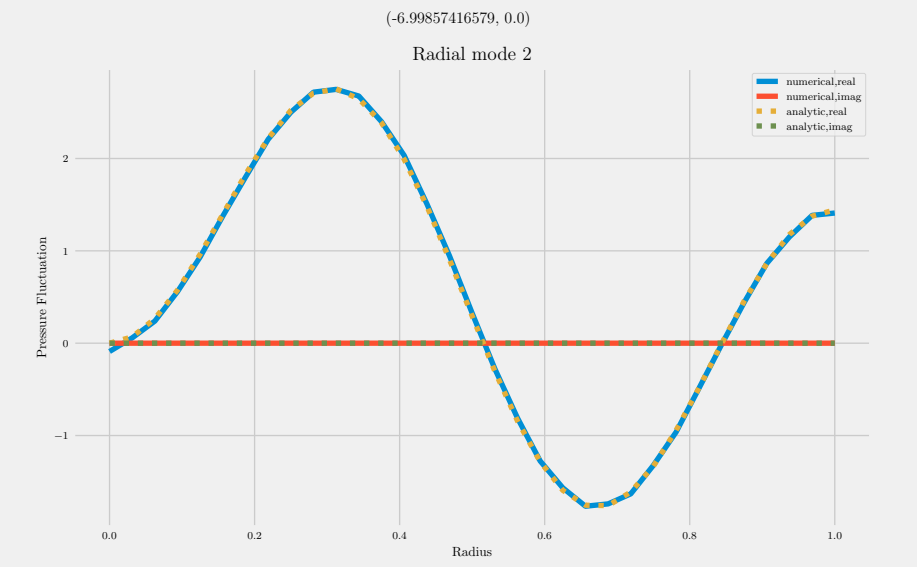

Figure 121: Second Order, Case number 1, 33points

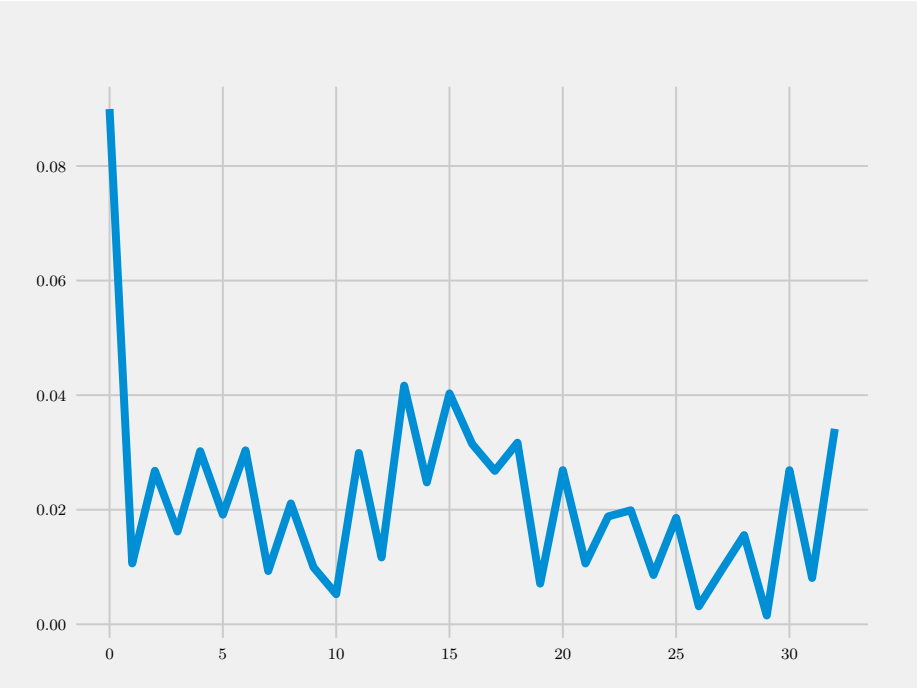

Figure 122: Second Order, Case number 1, 33points

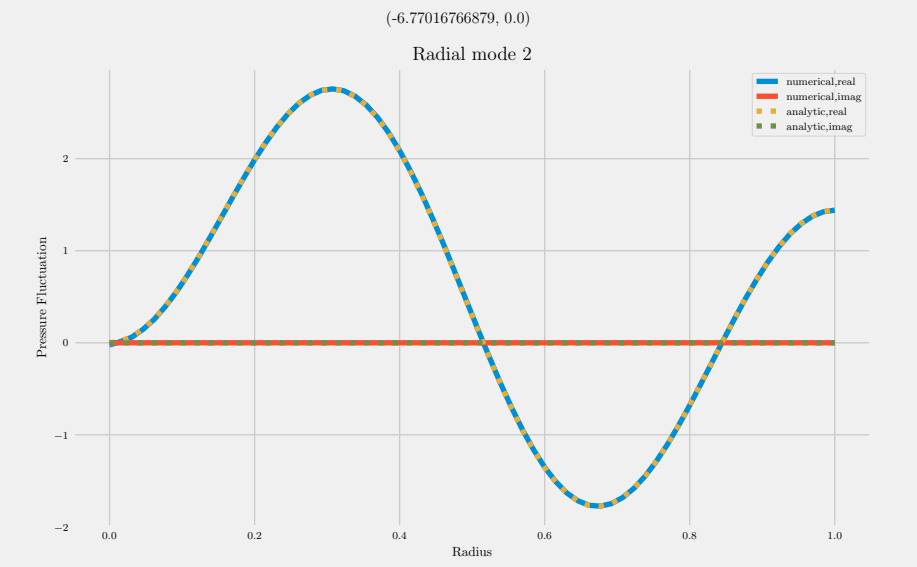

Figure 123: Second Order, Case number 1, 66points

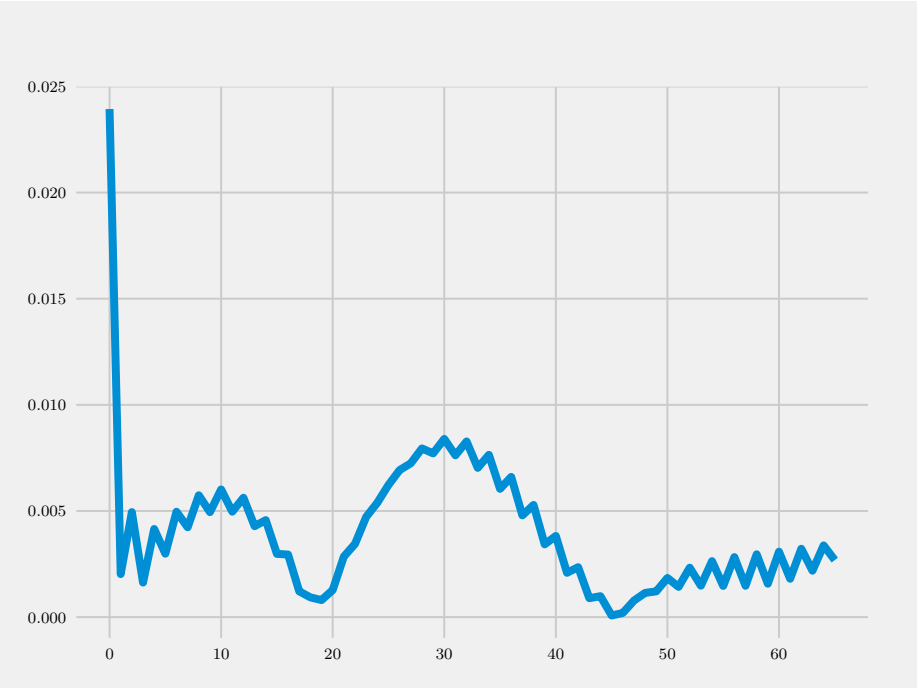

Figure 124: Second Order, Case number 1, 66points

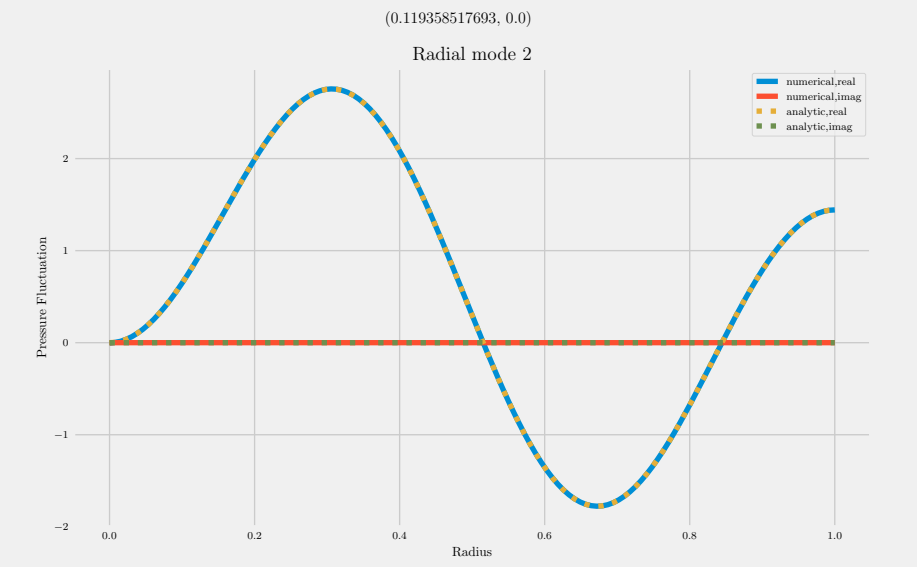

Figure 125: Second Order, Case number 1, 132points

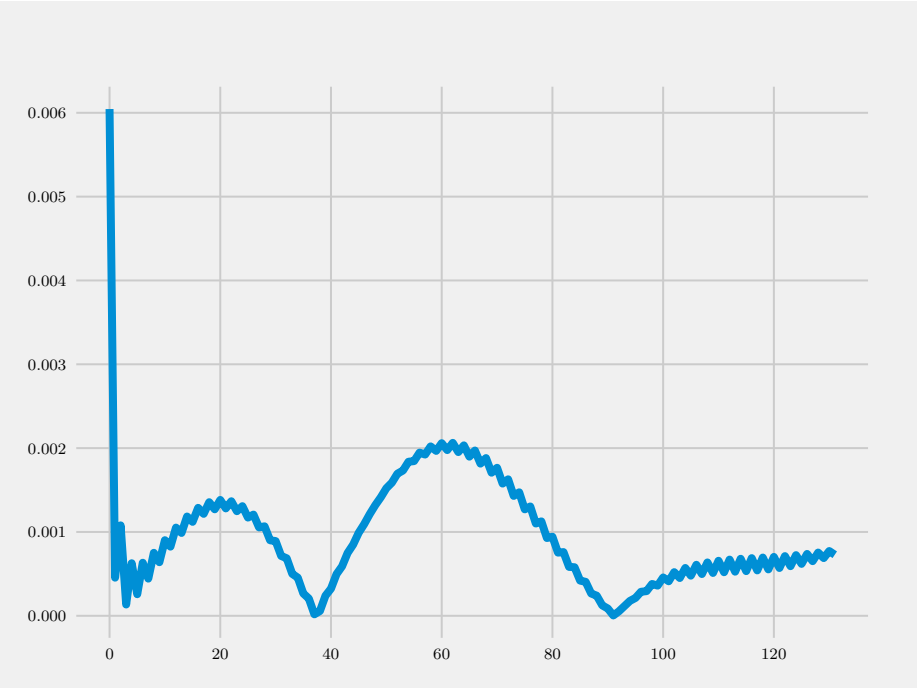

Figure 126: Second Order, Case number 1, 132points

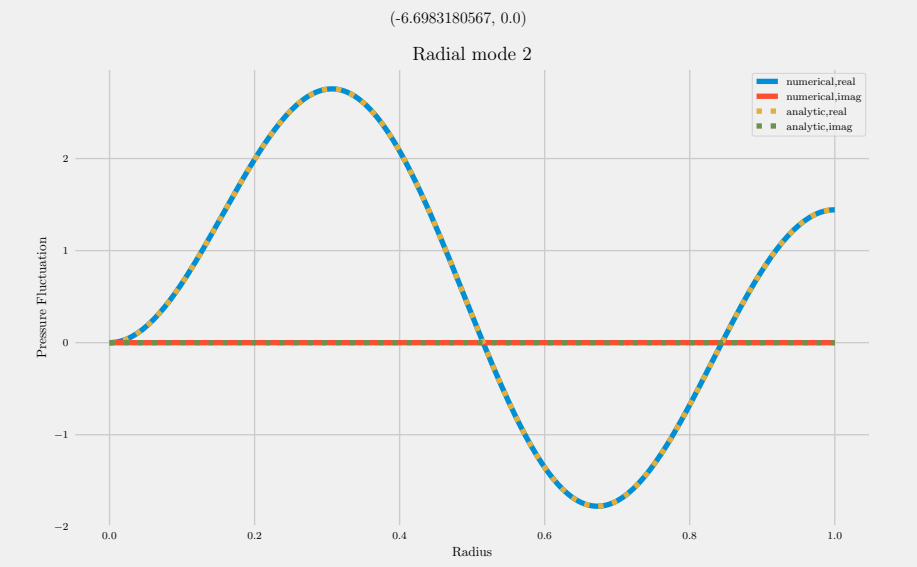

Figure 127: Second Order, Case number 1, 264points

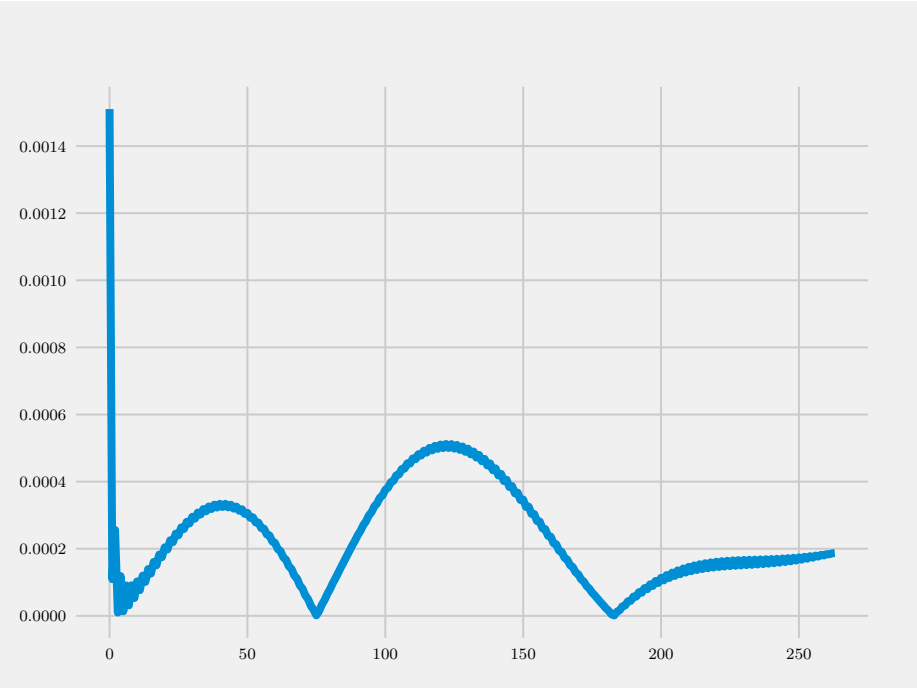

Figure 128: Second Order, Case number 1, 264points

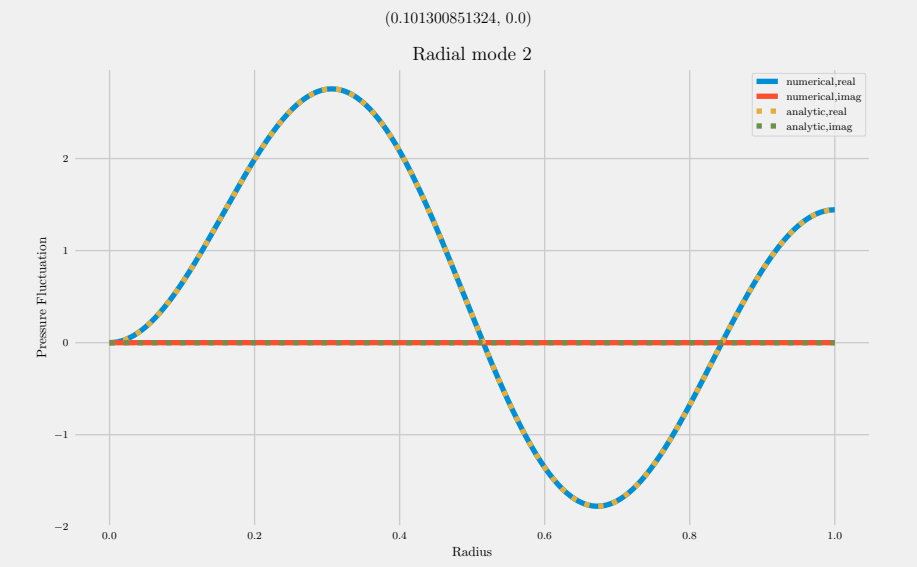

Figure 129: Second Order, Case number 1, 528points

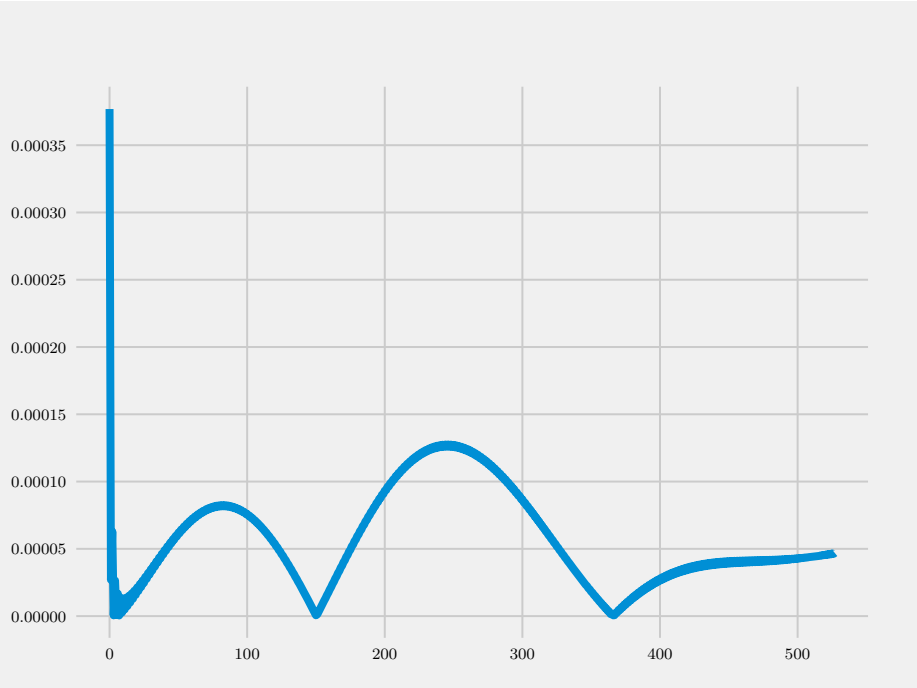

Figure 130: Second Order, Case number 1, 528points

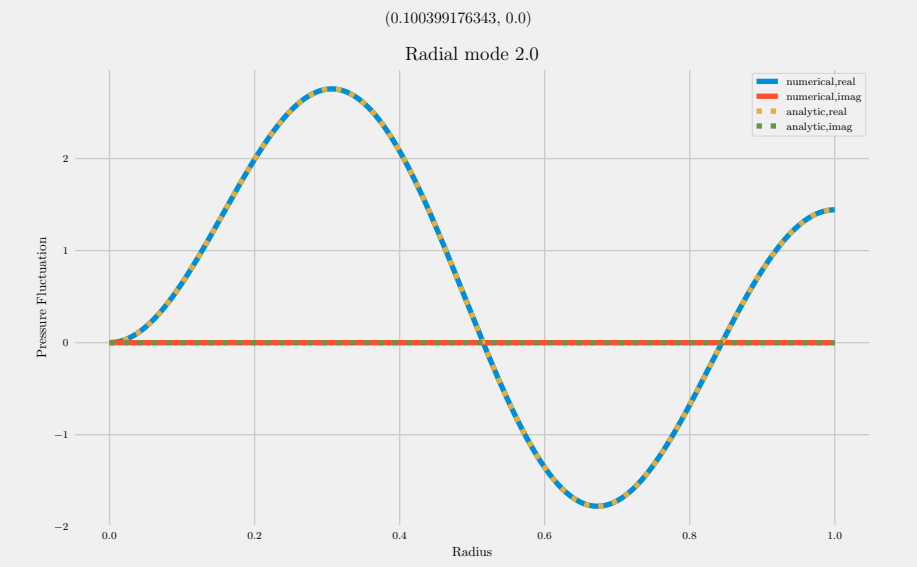

Figure 131: Second Order, Case number 1, 1056points

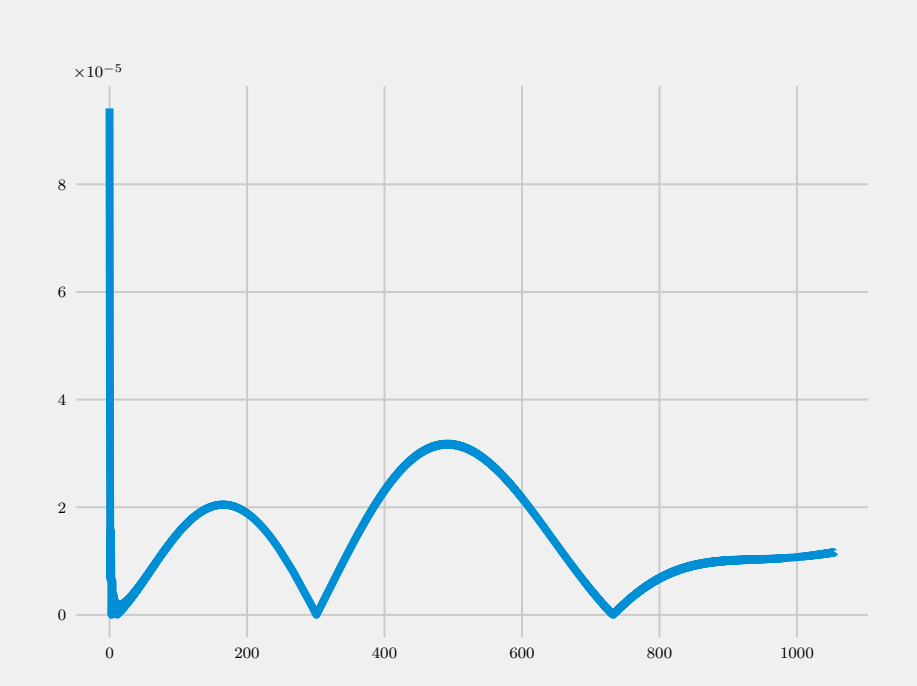

Figure 132: Second Order, Case number 1, 1056points

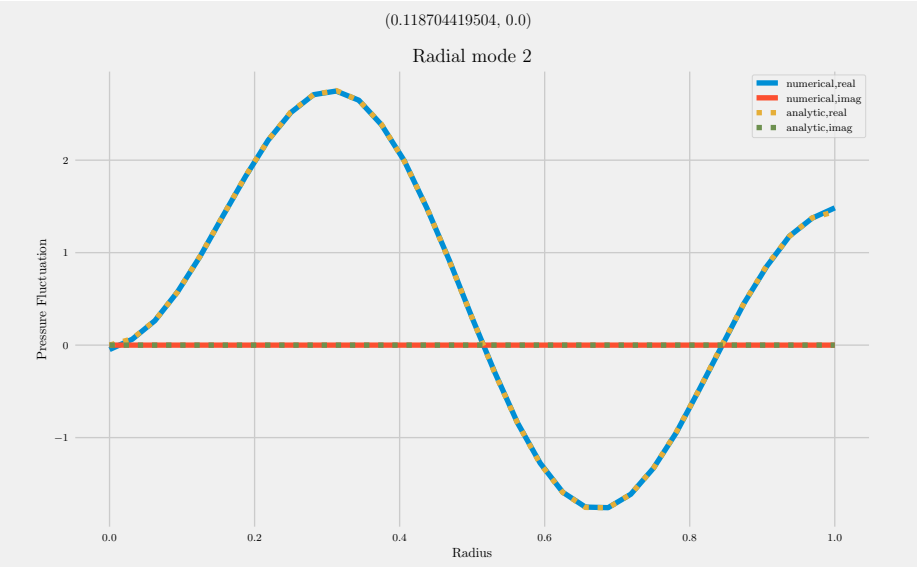

Figure 133: Fourth Order, Case number 0, 33points

Fourth Order, Radial Mode 2

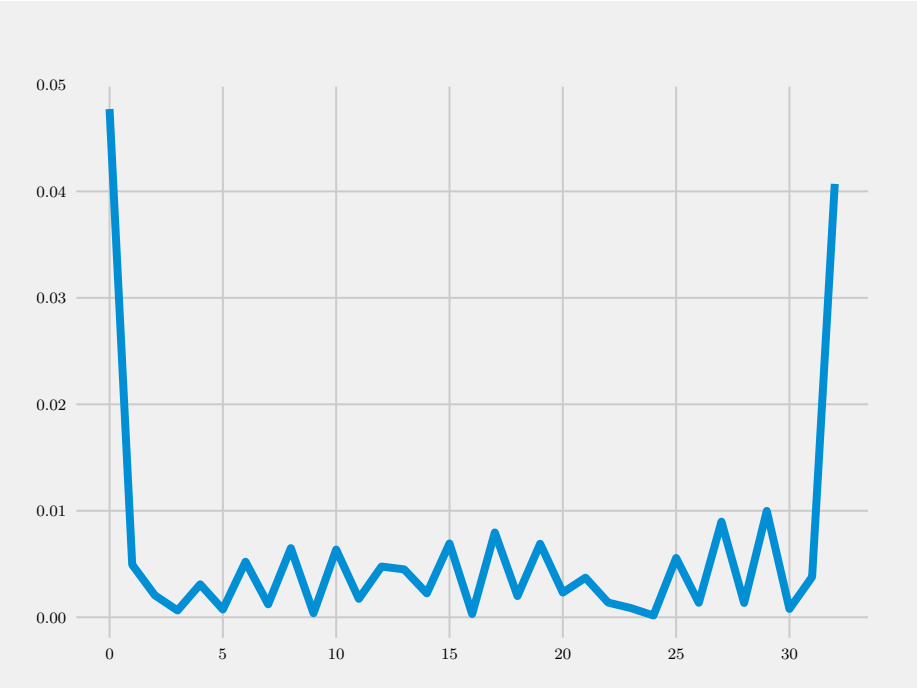

Figure 134: Fourth Order, Case number 0, 33points

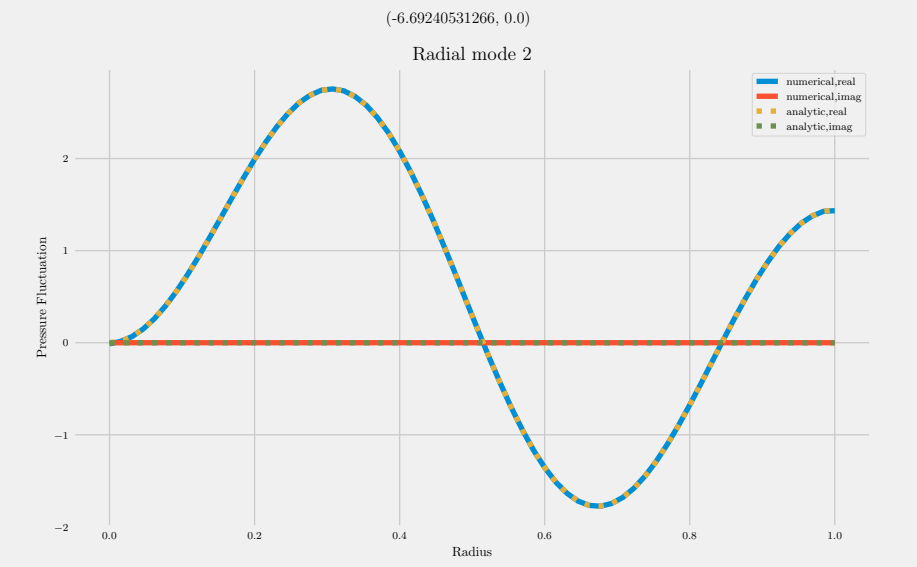

Figure 135: Fourth Order, Case number 0, 66points

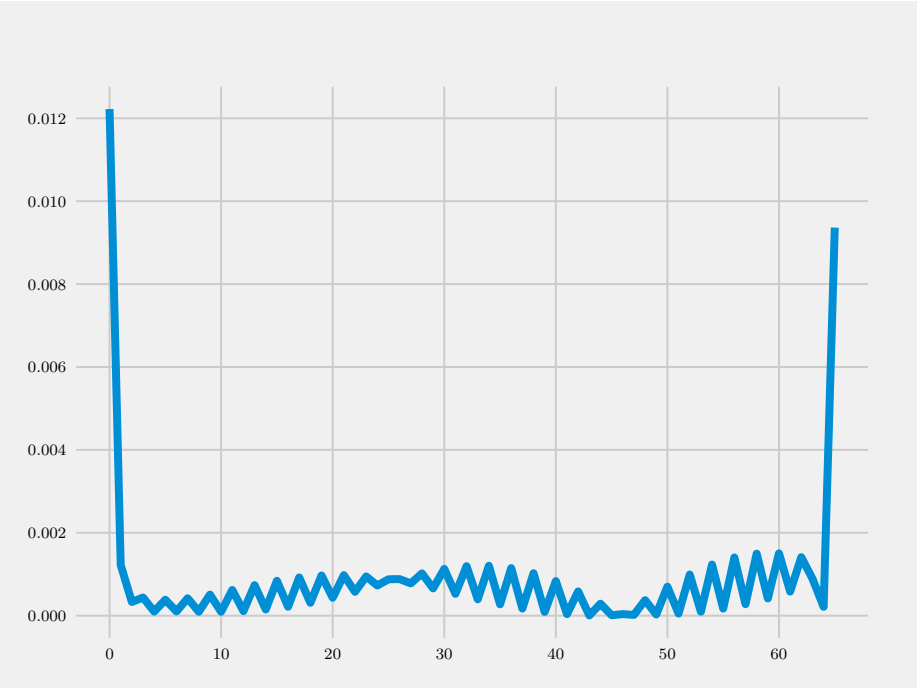

Figure 136: Fourth Order, Case number 0, 66points

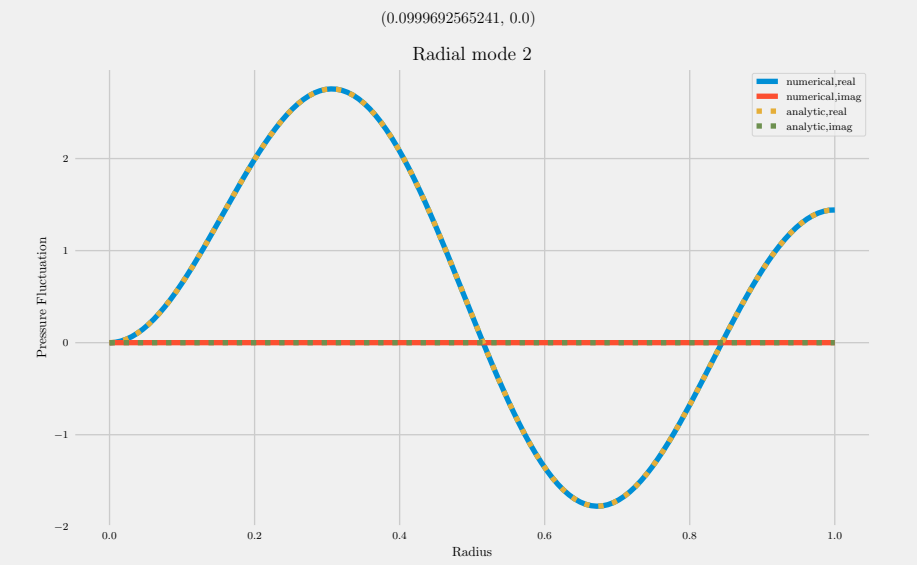

Figure 137: Fourth Order, Case number 0, 132points

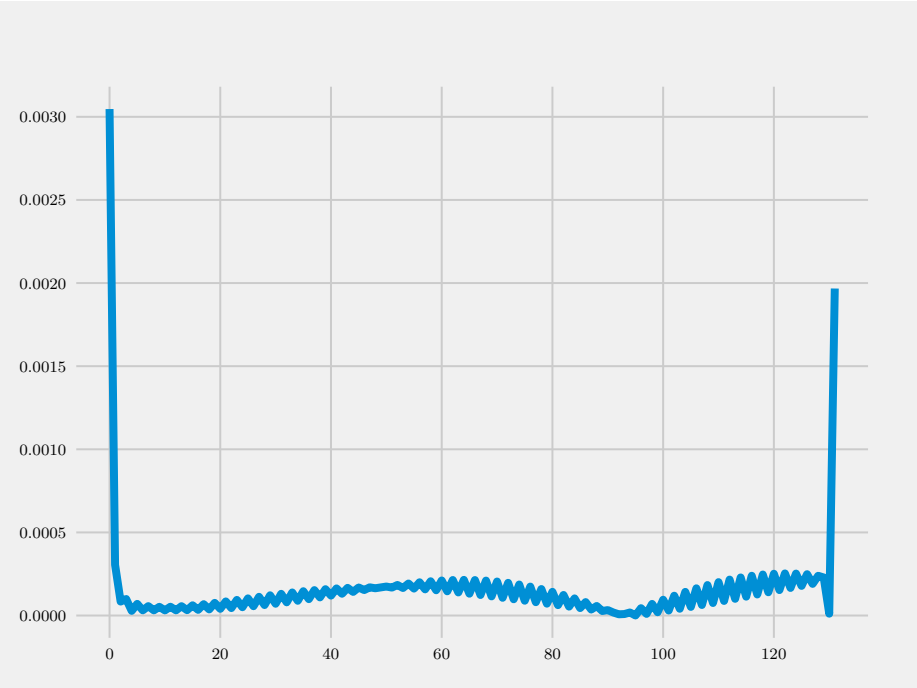

Figure 138: Fourth Order, Case number 0, 132points

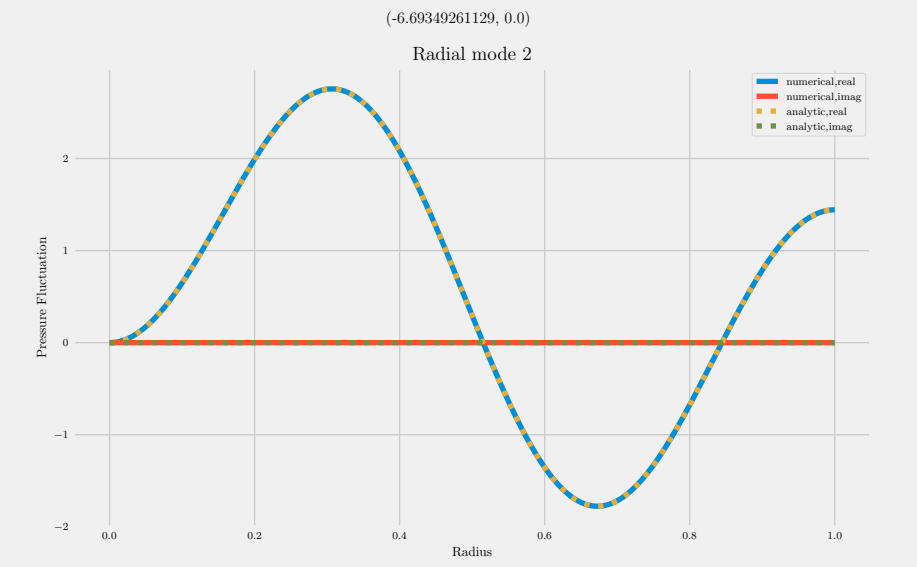

Figure 139: Fourth Order, Case number 0, 264points

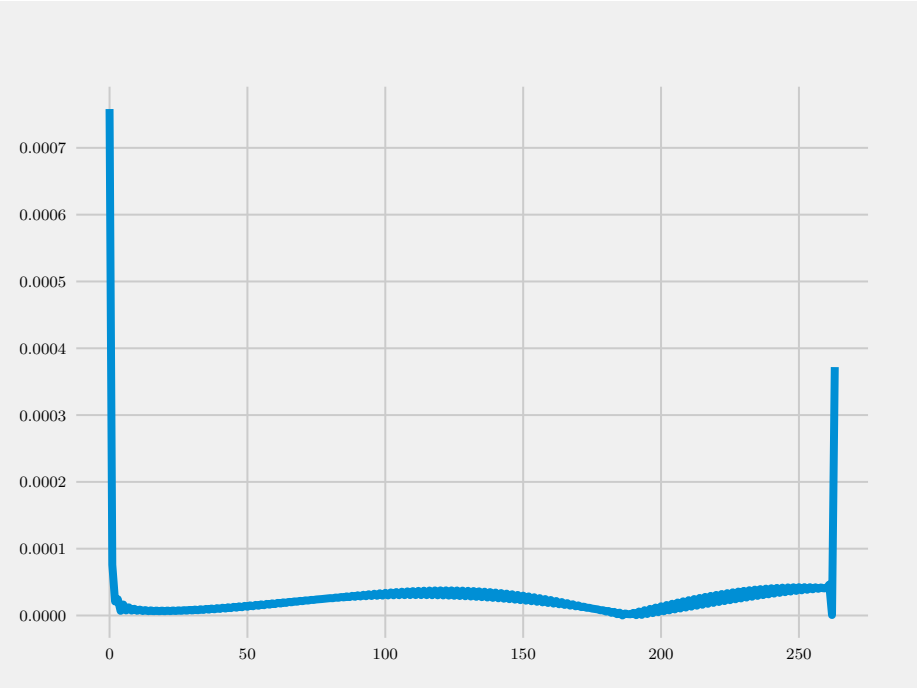

Figure 140: Fourth Order, Case number 0, 264points

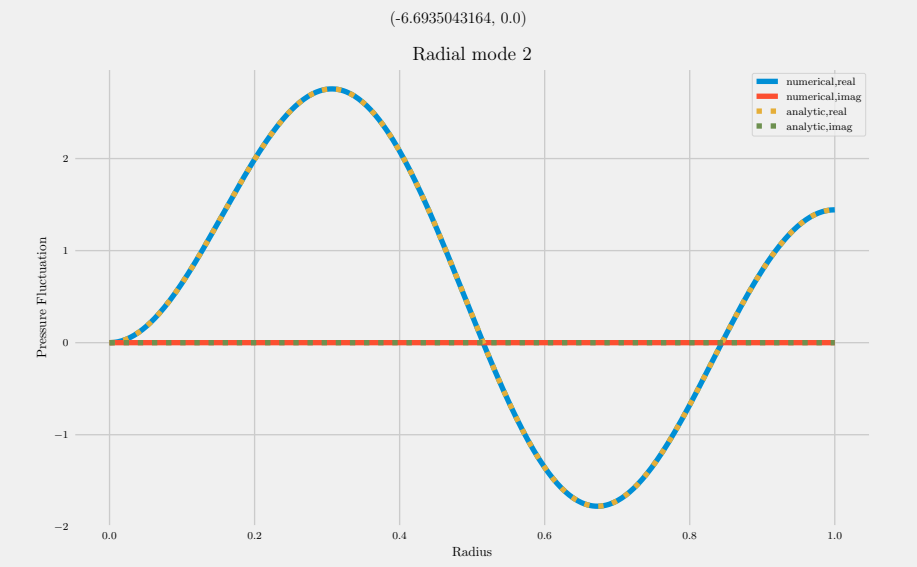

Figure 141: Fourth Order, Case number 0, 528points

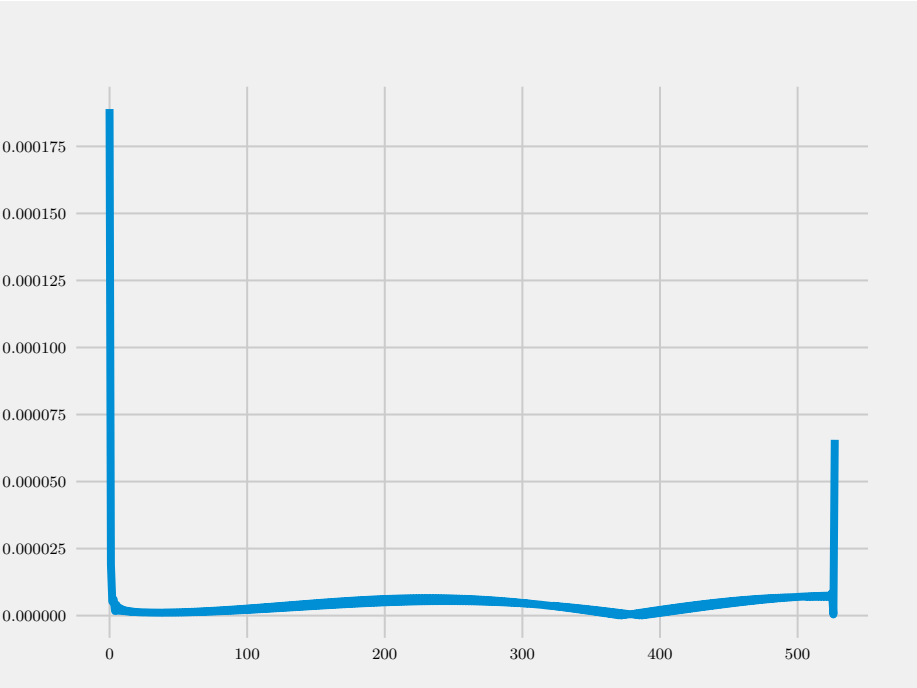

Figure 142: Fourth Order, Case number 0, 528points

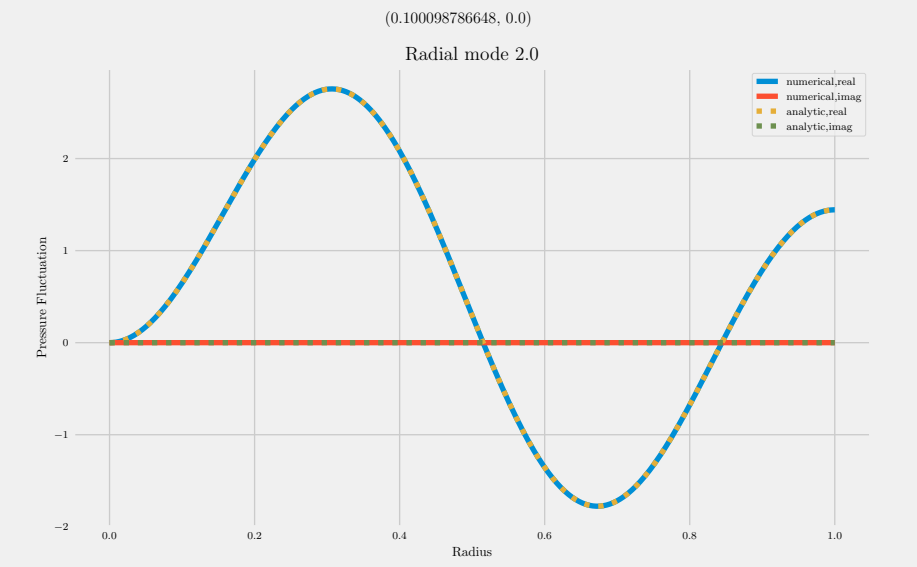

Figure 143: Fourth Order, Case number 0, 1056points

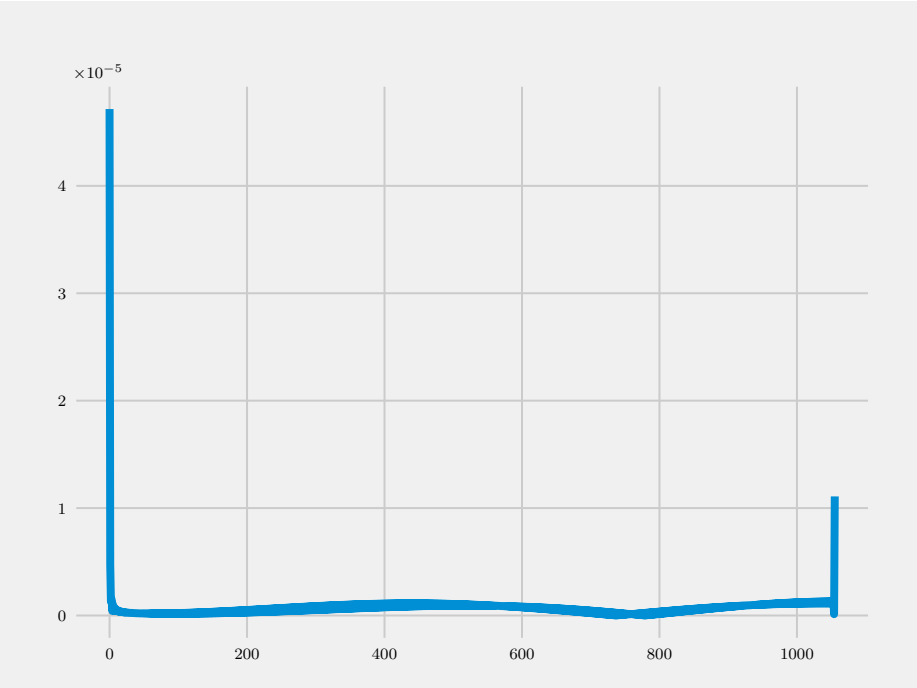

Figure 144: Fourth Order, Case number 0, 1056points

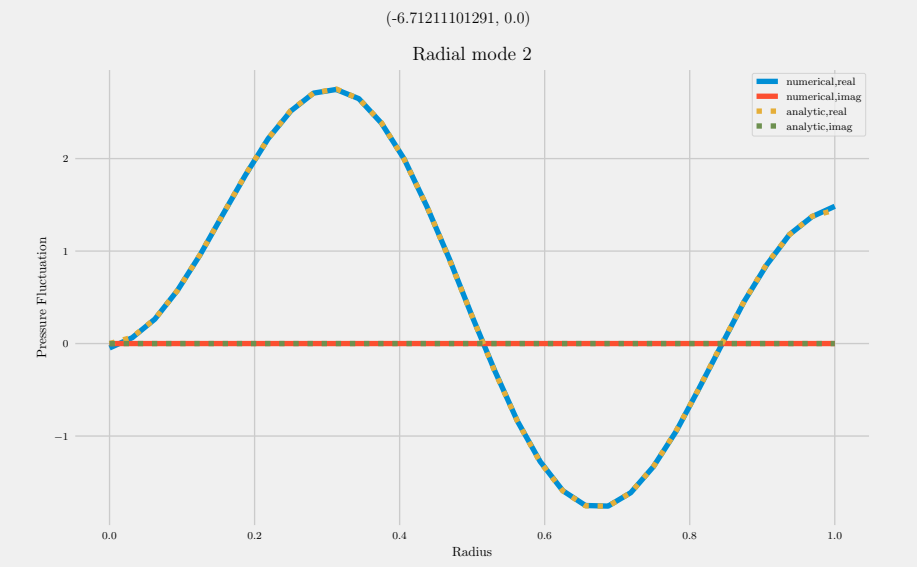

Figure 145: Fourth Order, Case number 1, 33points

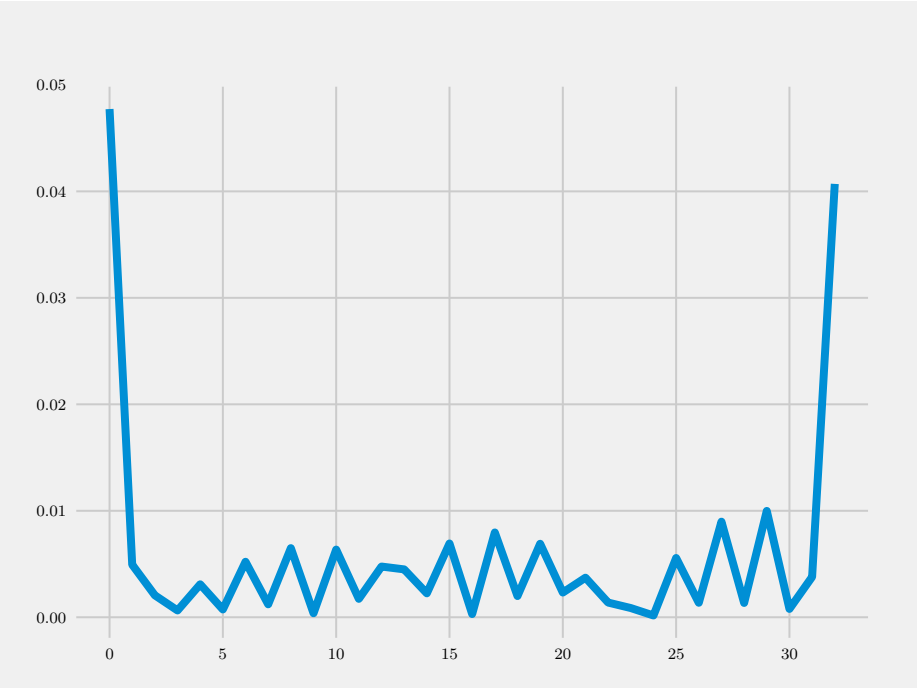

Figure 146: Fourth Order, Case number 1, 33points

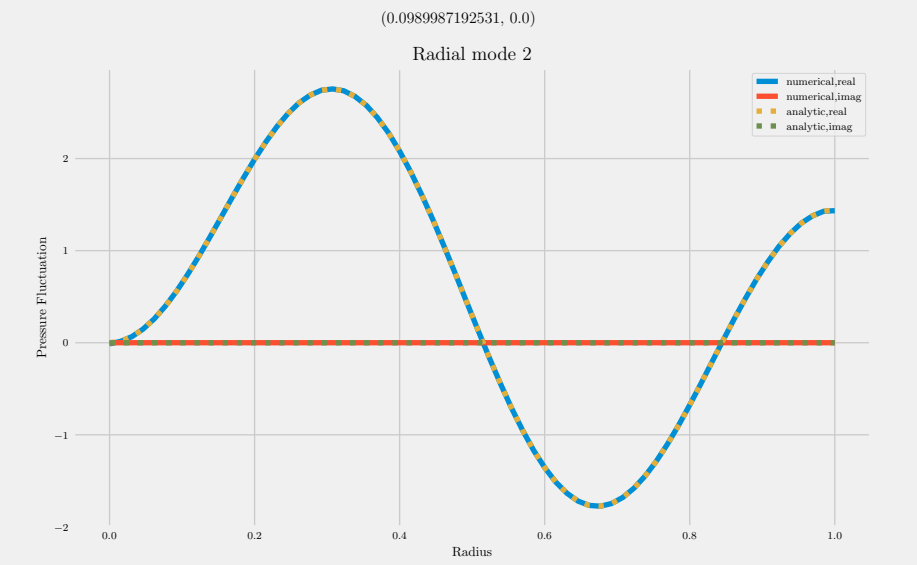

Figure 147: Fourth Order, Case number 1, 66points

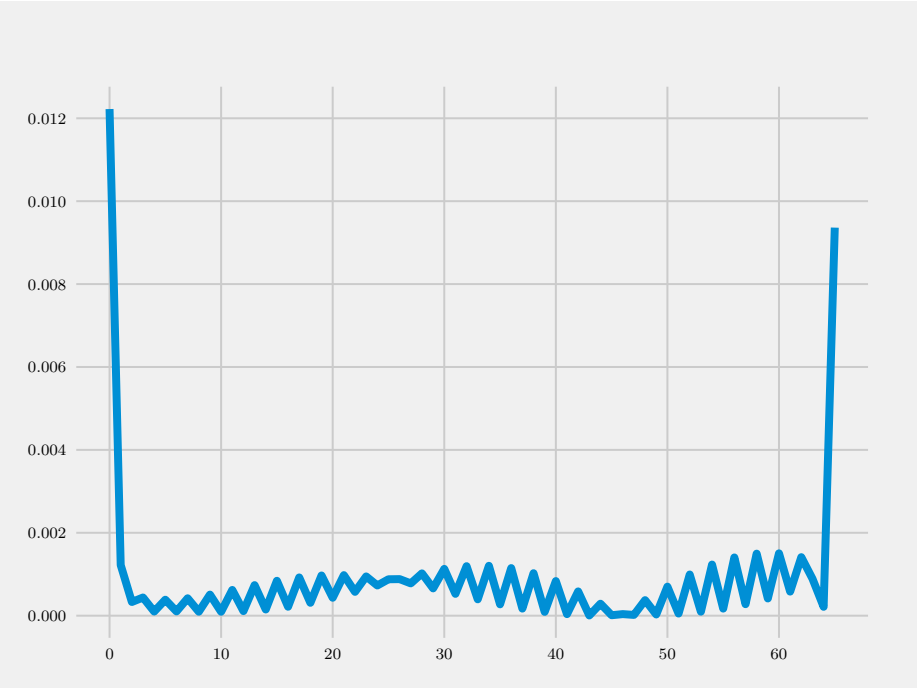

Figure 148: Fourth Order, Case number 1, 66points

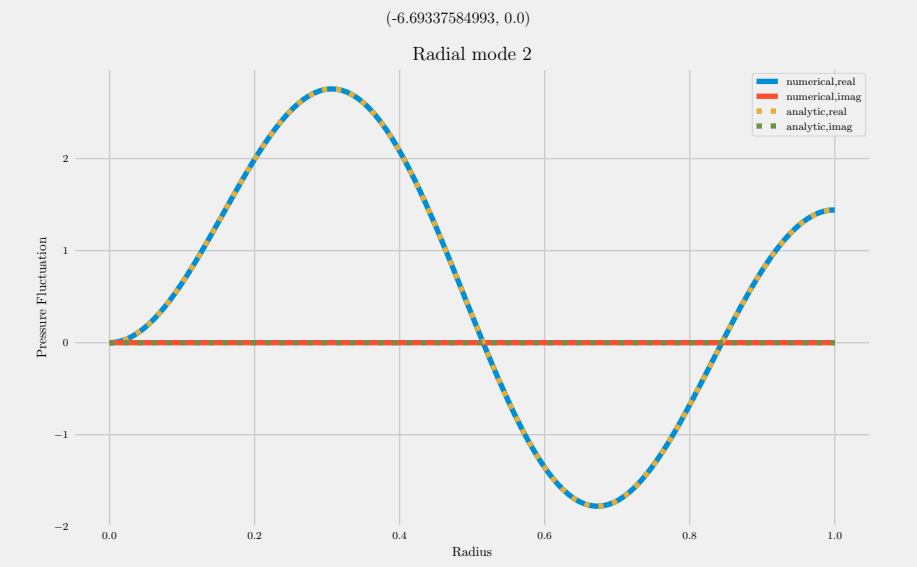

Figure 149: Fourth Order, Case number 1, 132points

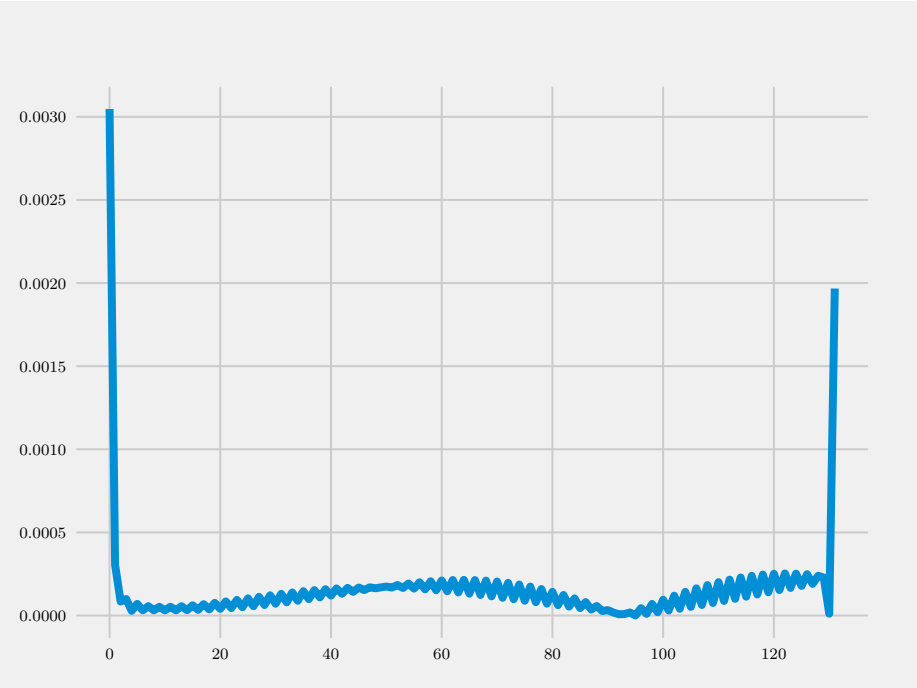

Figure 150: Fourth Order, Case number 1, 132points

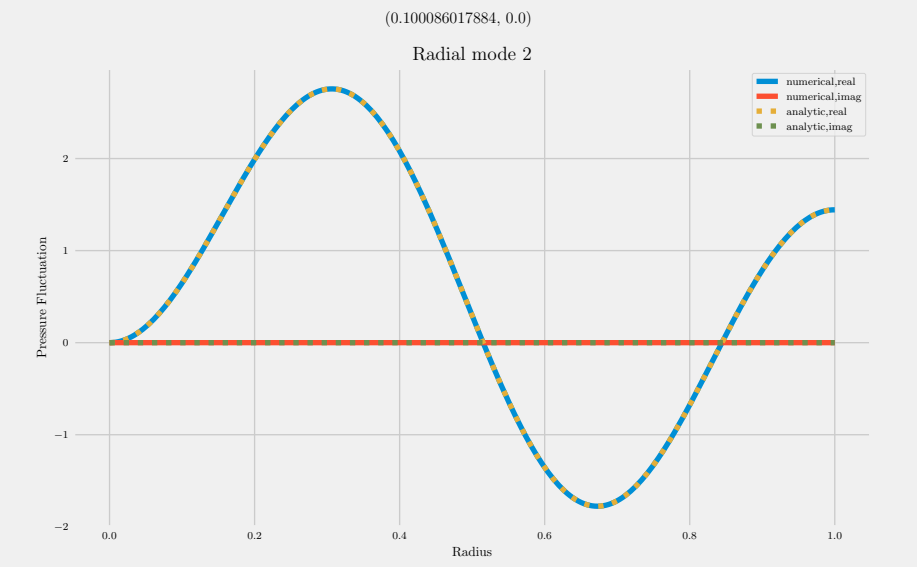

Figure 151: Fourth Order, Case number 1, 264points

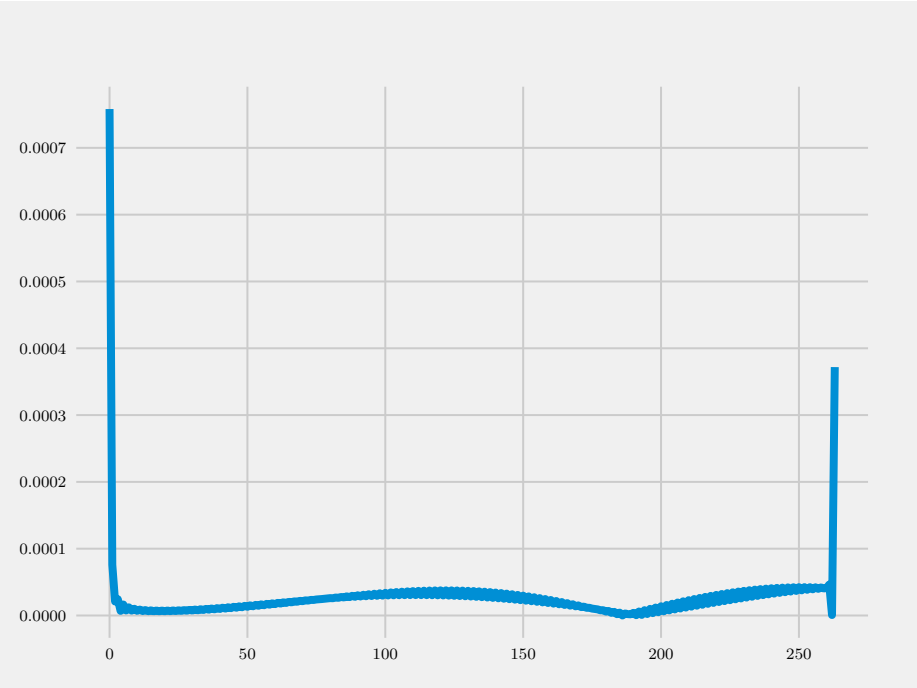

Figure 152: Fourth Order, Case number 1, 264points

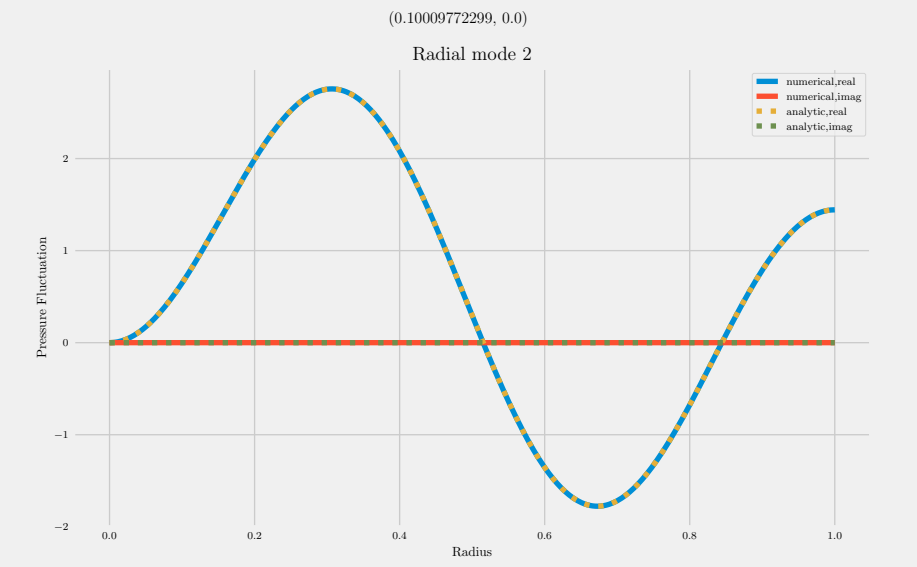

Figure 153: Fourth Order, Case number 1, 528points

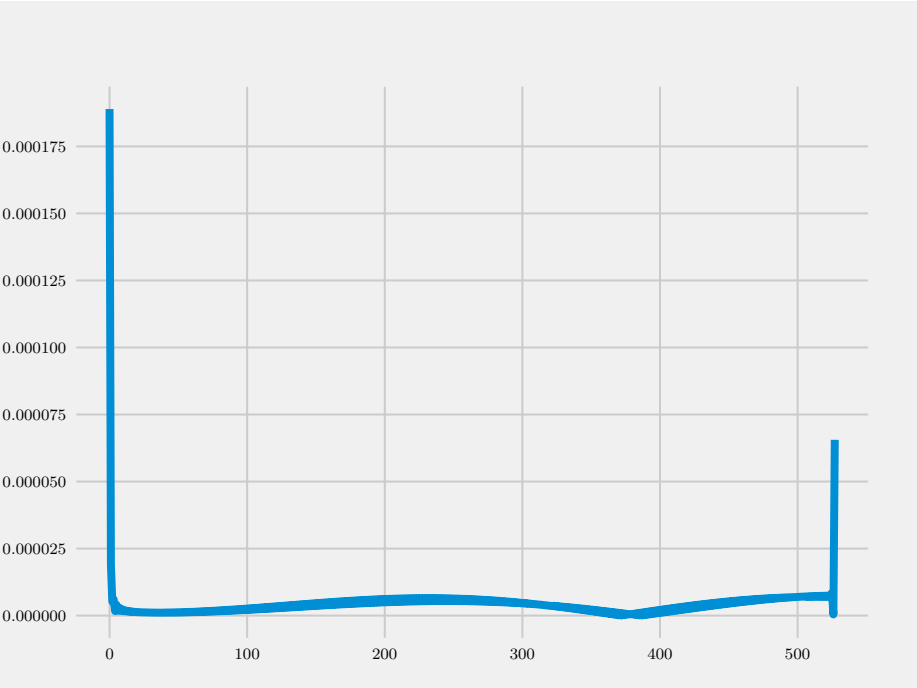

Figure 154: Fourth Order, Case number 1, 528points

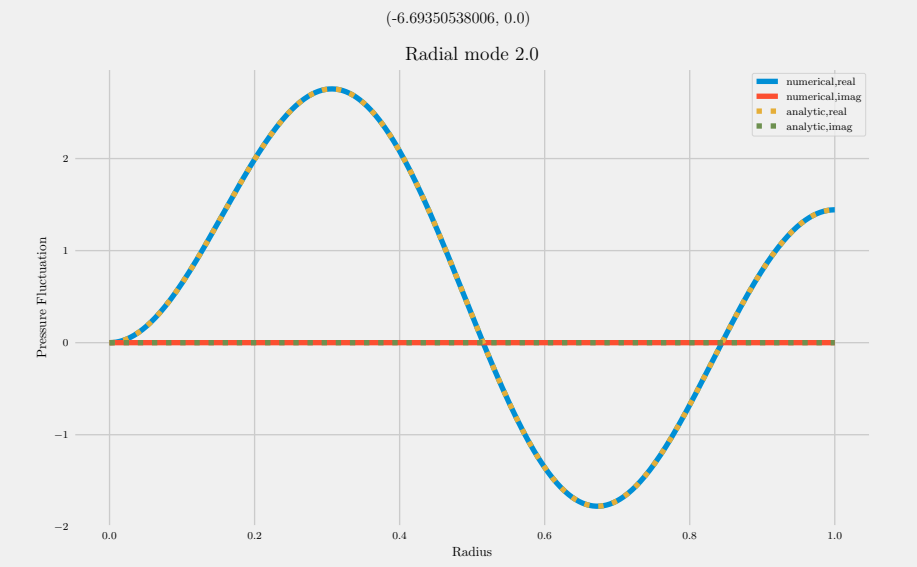

Figure 155: Fourth Order, Case number 1, 1056points

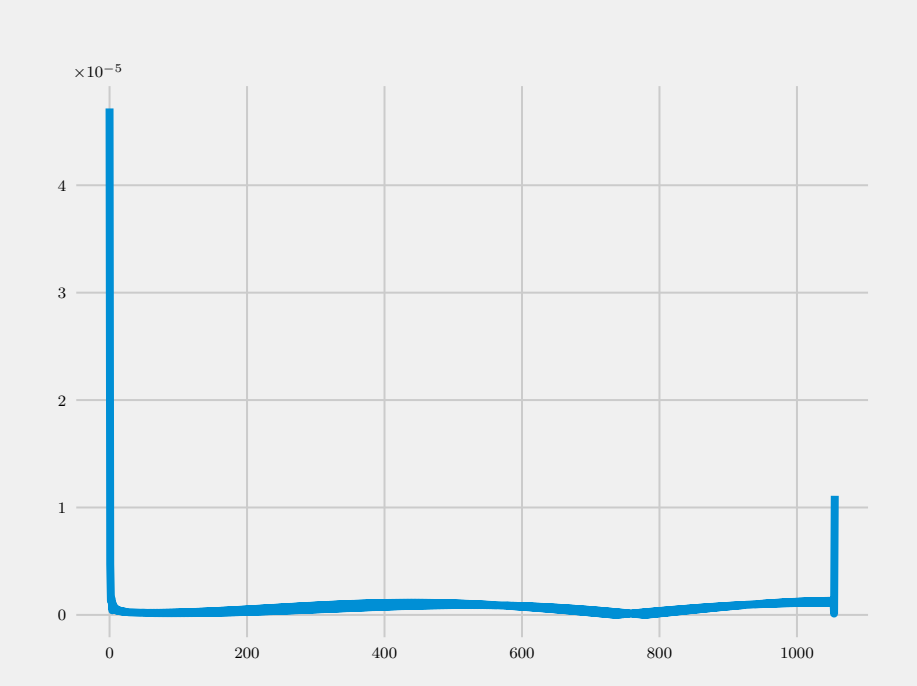

Figure 156: Fourth Order, Case number 1, 1056points

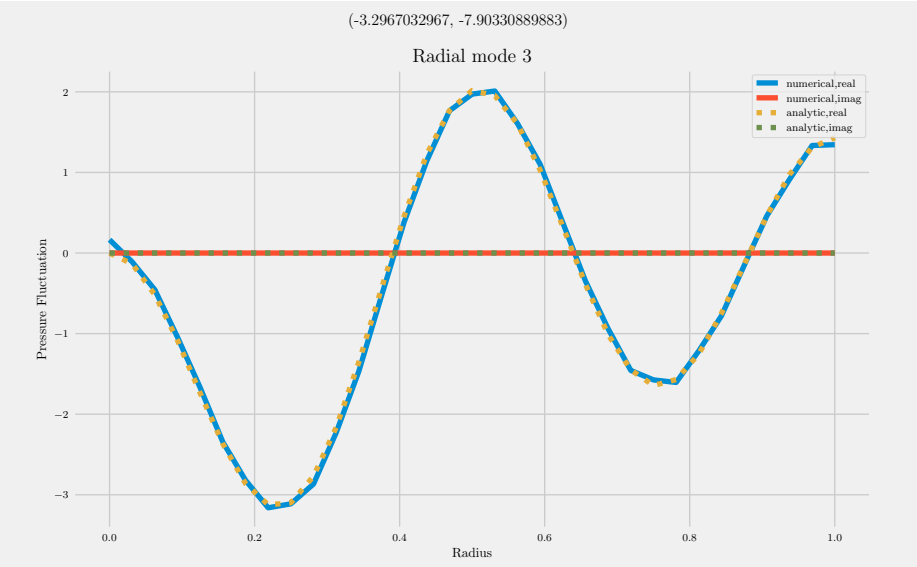

Figure 157: Second Order, Case number 0, 33points

Second Order, Radial Mode 3

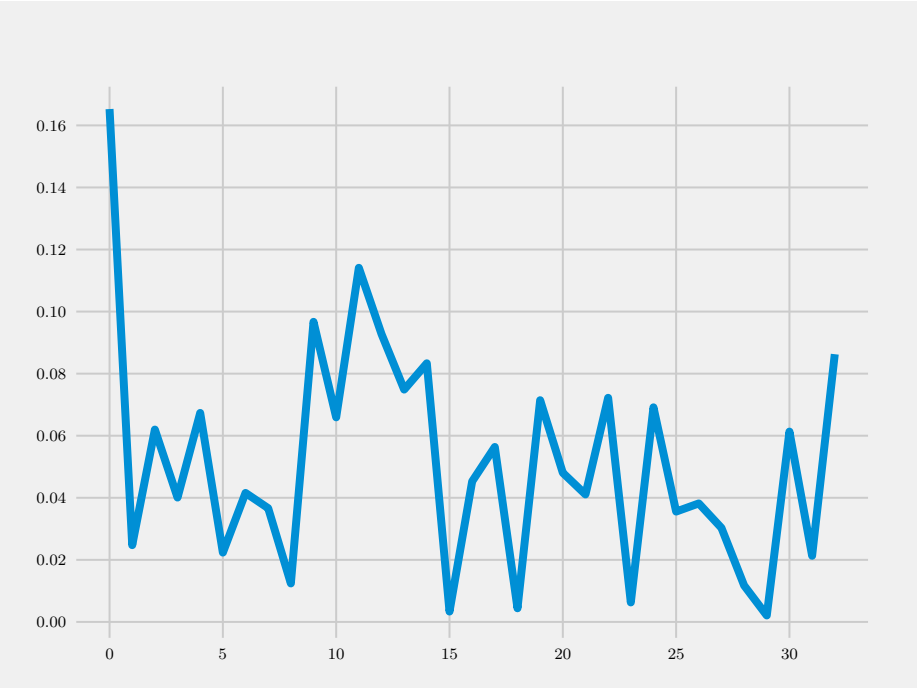

Figure 158: Second Order, Case number 0, 33points

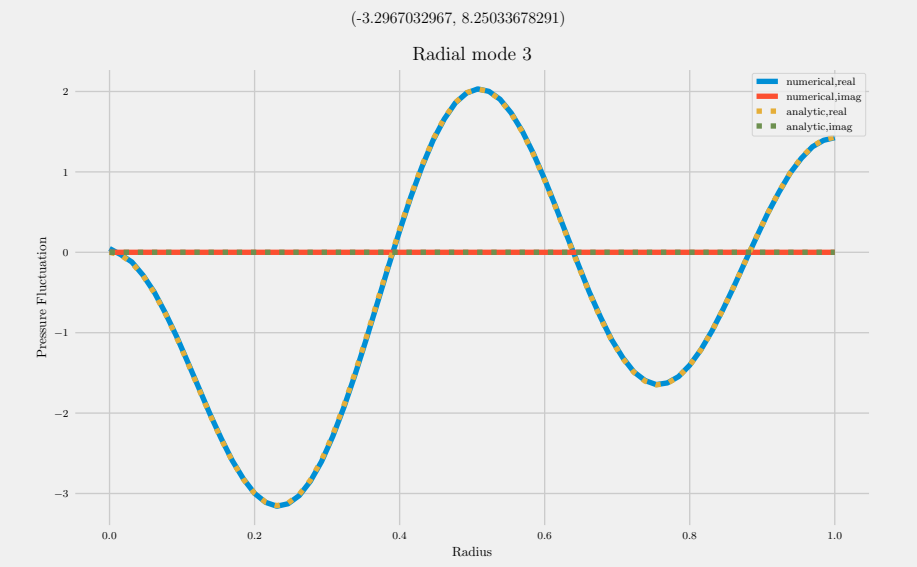

Figure 159: Second Order, Case number 0, 66points

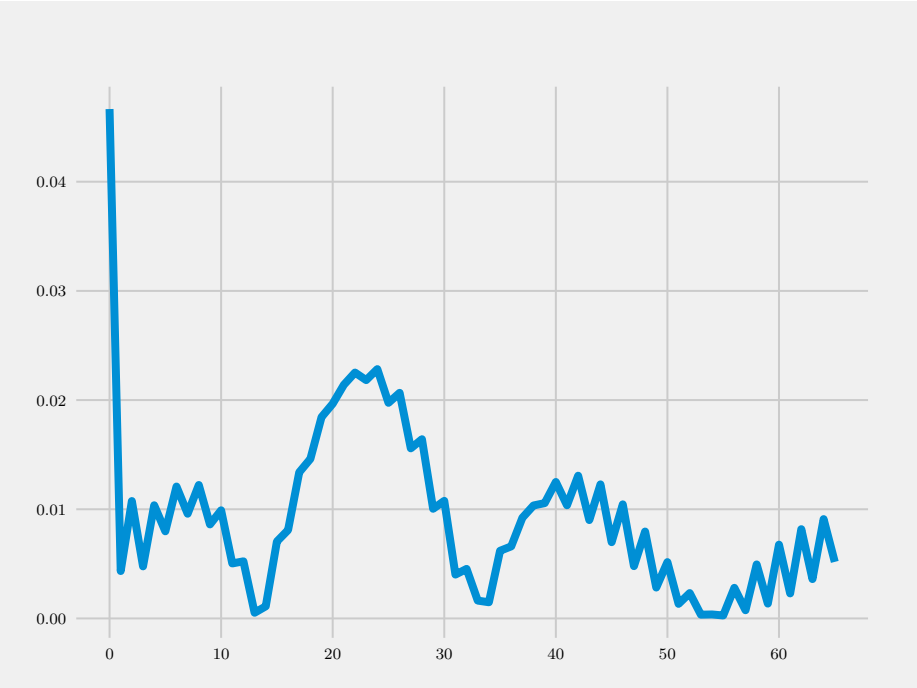

Figure 160: Second Order, Case number 0, 66points

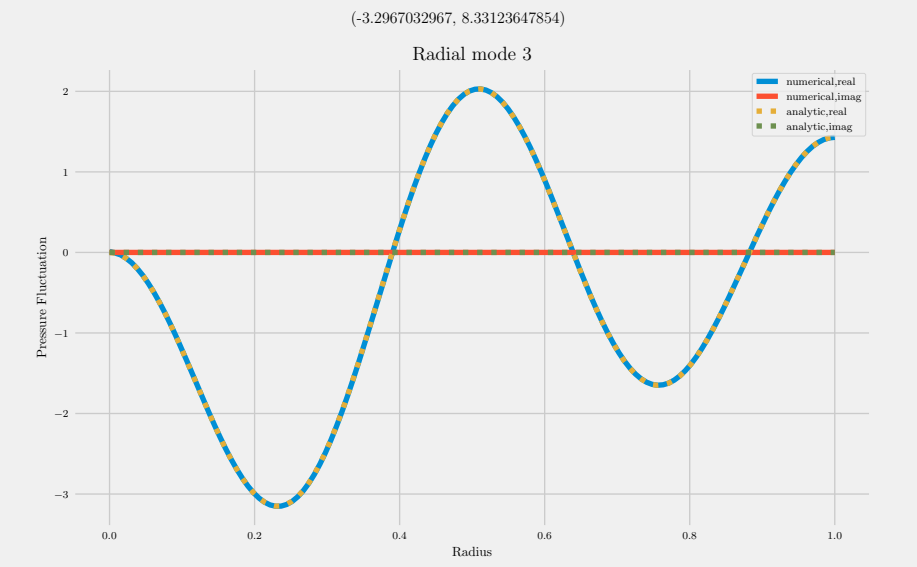

Figure 161: Second Order, Case number 0, 132points

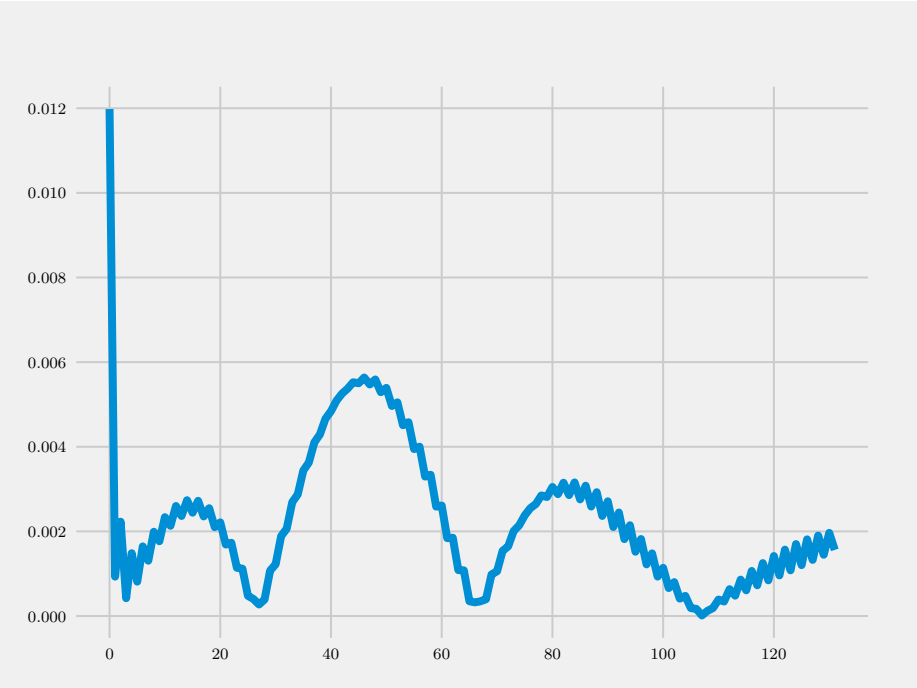

Figure 162: Second Order, Case number 0, 132points

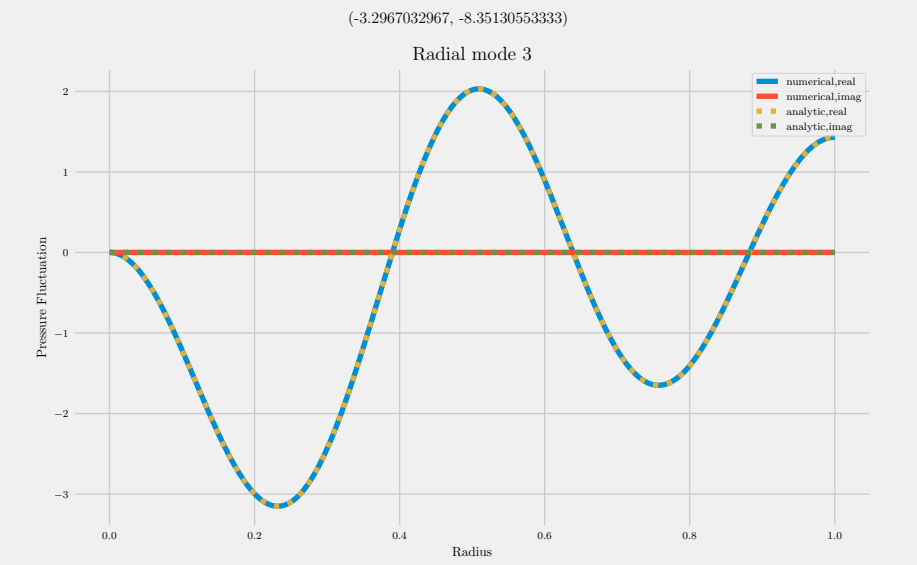

Figure 163: Second Order, Case number 0, 264points

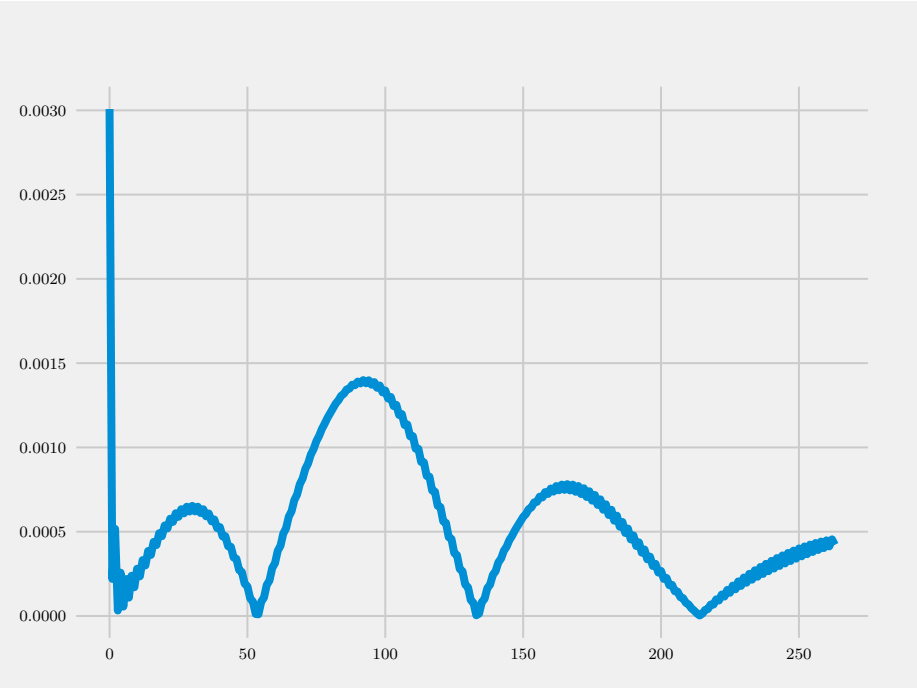

Figure 164: Second Order, Case number 0, 264points

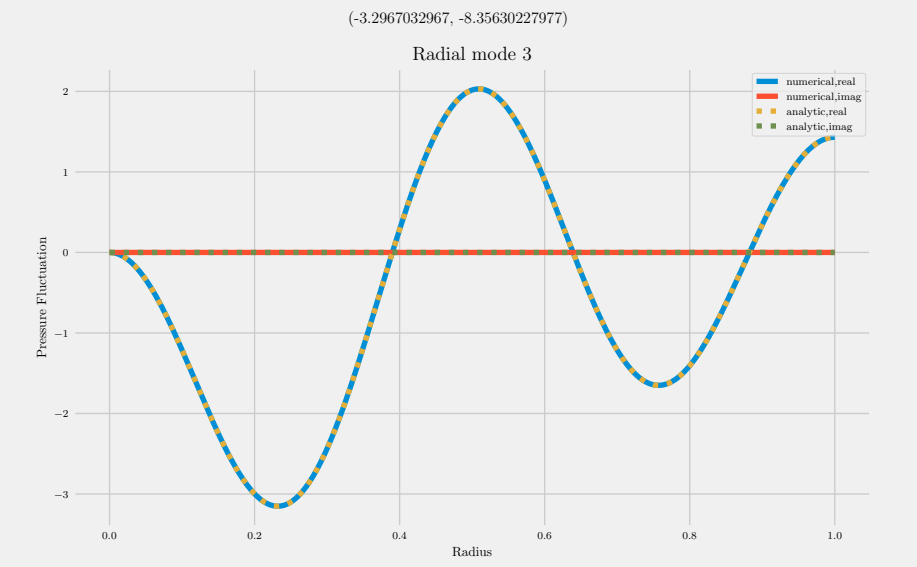

Figure 165: Second Order, Case number 0, 528points

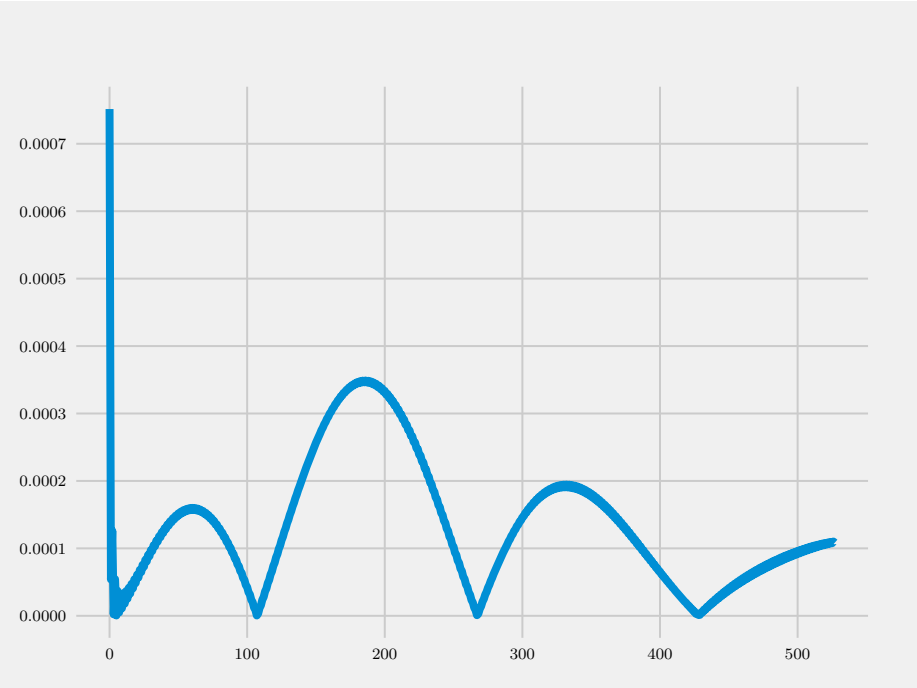

Figure 166: Second Order, Case number 0, 528points

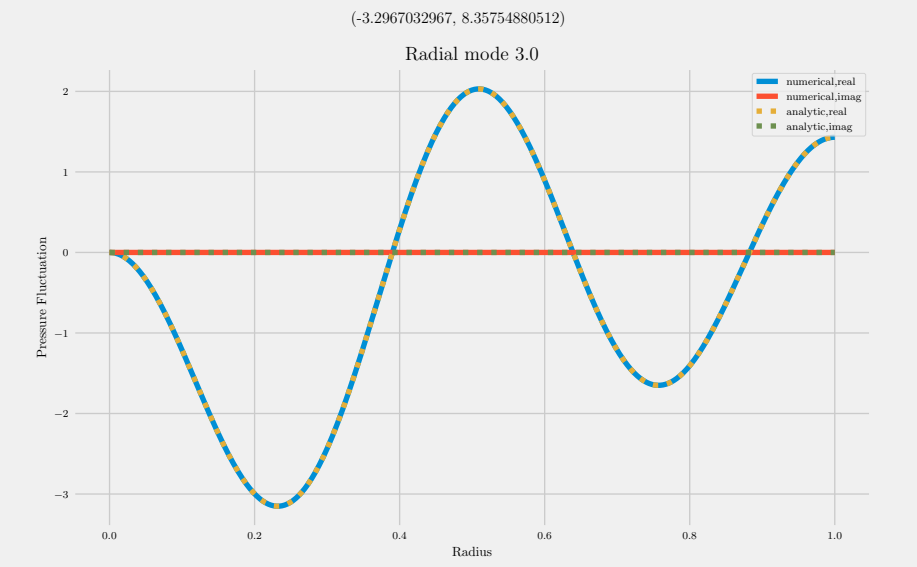

Figure 167: Second Order, Case number 0, 1056points

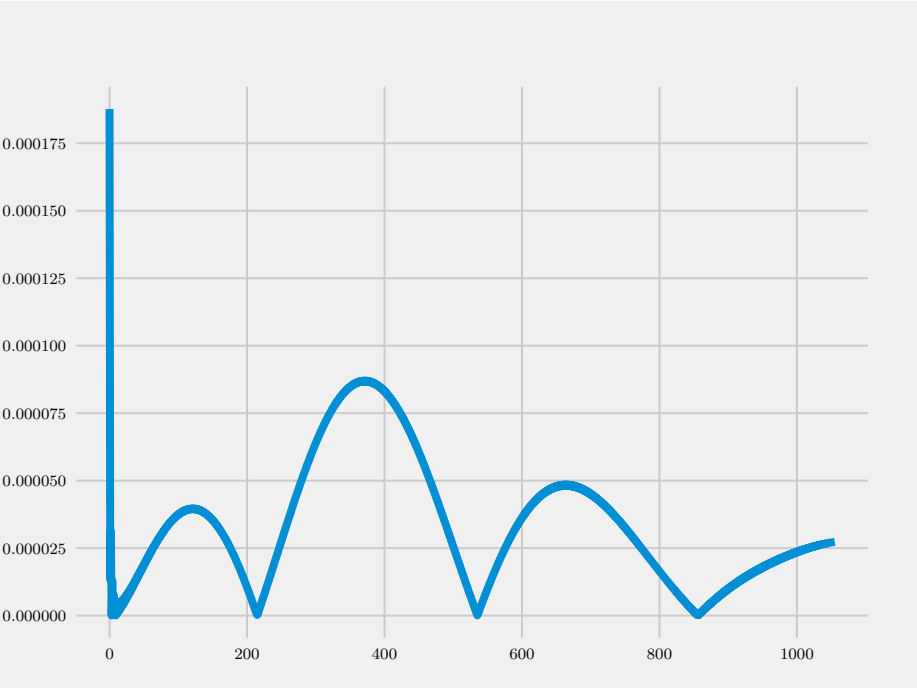

Figure 168: Second Order, Case number 0, 1056points

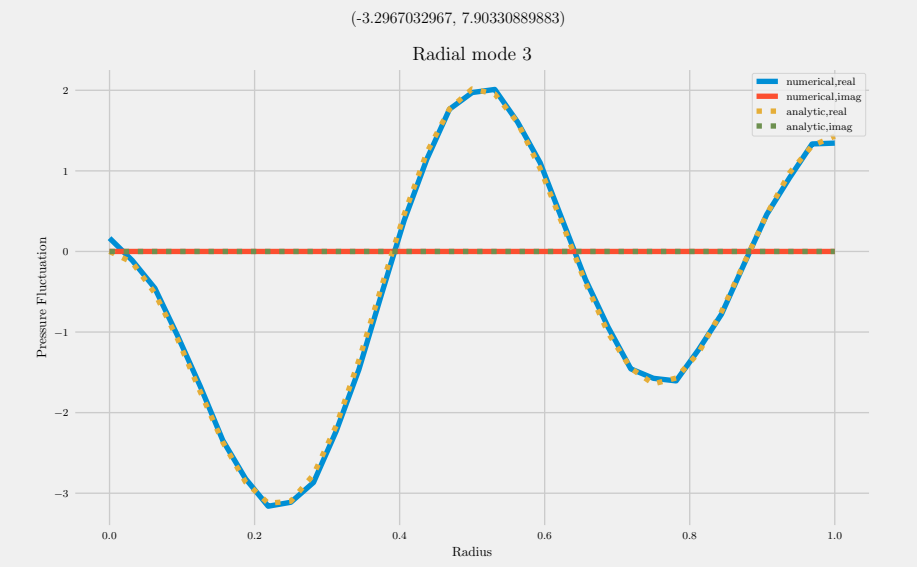

Figure 169: Second Order, Case number 1, 33points

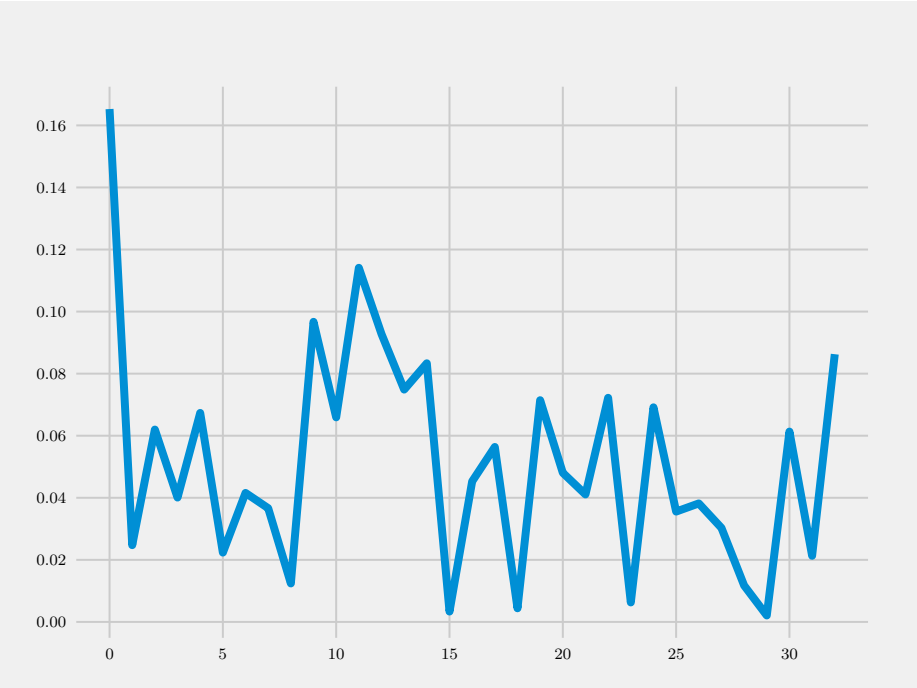

Figure 170: Second Order, Case number 1, 33points

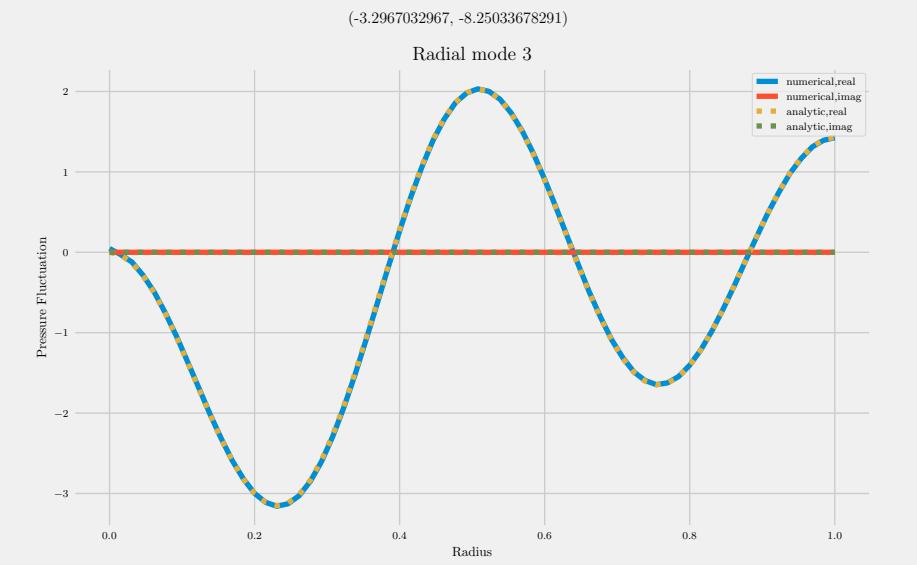

Figure 171: Second Order, Case number 1, 66points

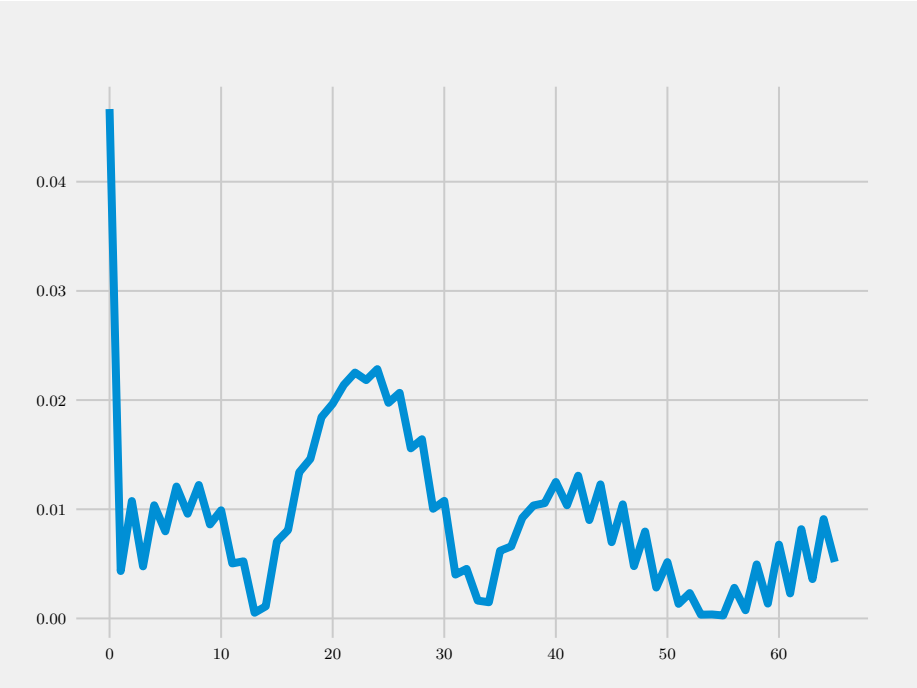

Figure 172: Second Order, Case number 1, 66points

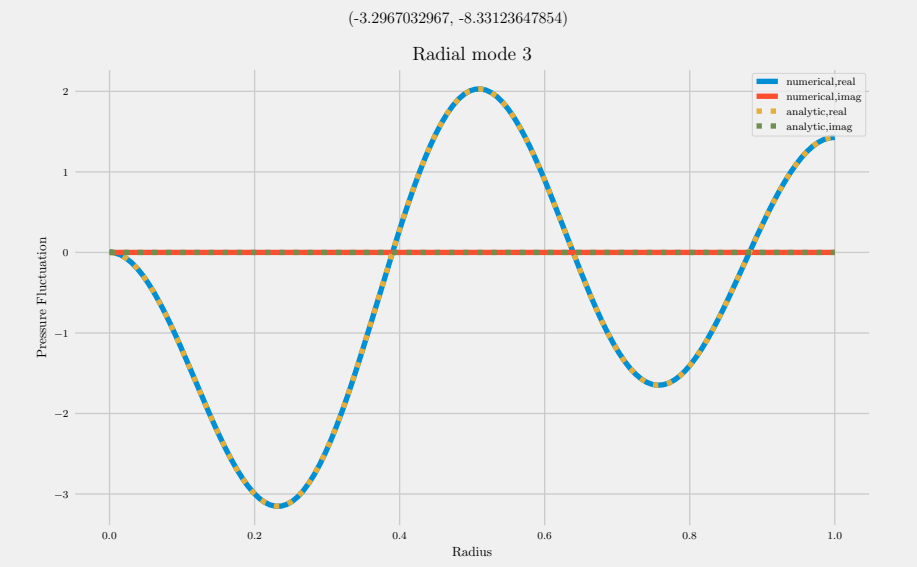

Figure 173: Second Order, Case number 1, 132points

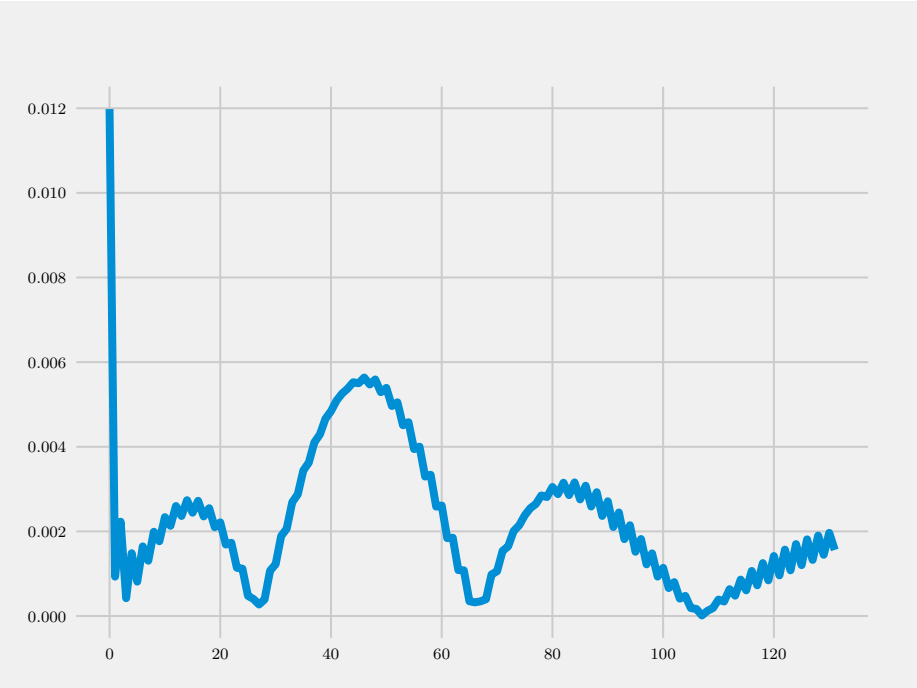

Figure 174: Second Order, Case number 1, 132points

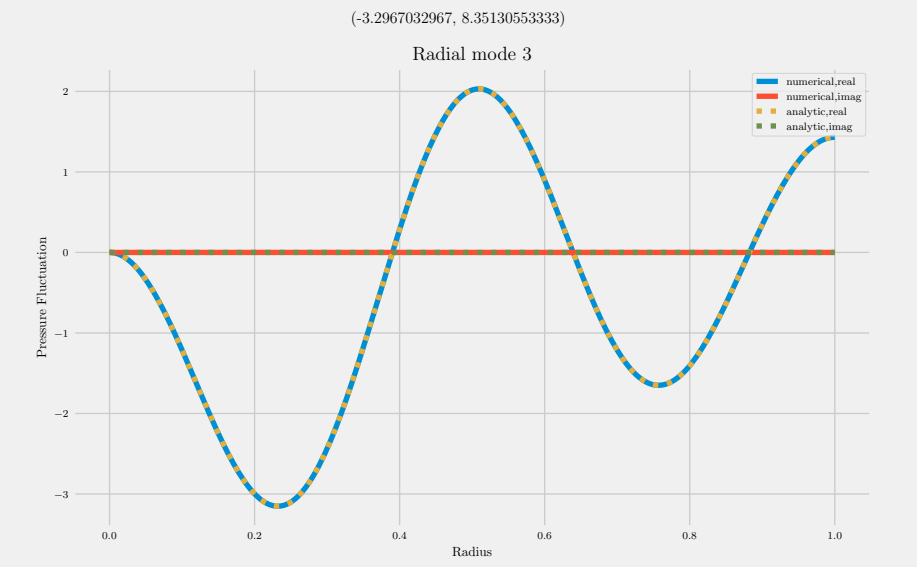

Figure 175: Second Order, Case number 1, 264points

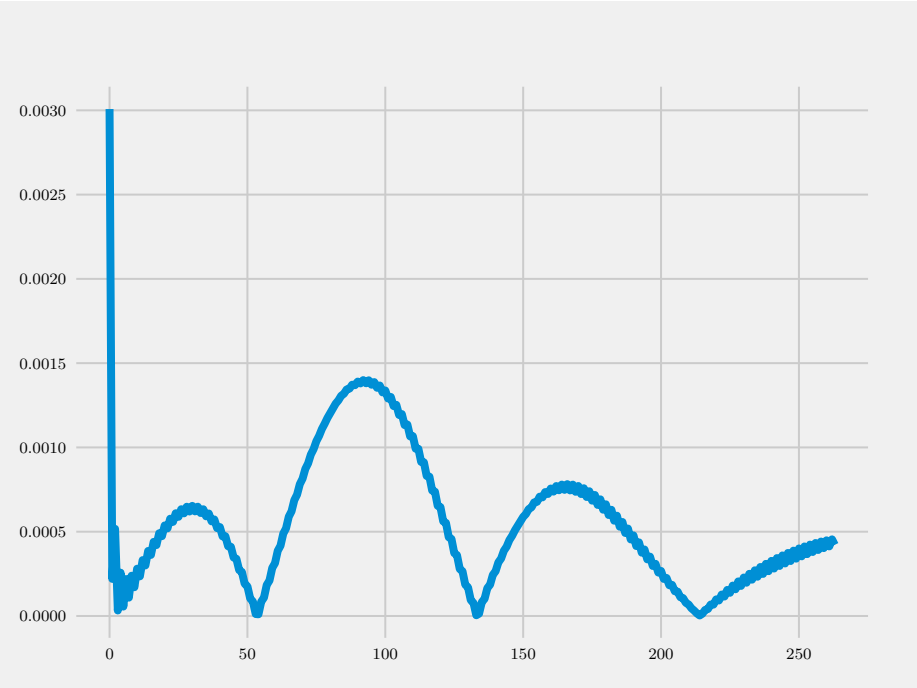

Figure 176: Second Order, Case number 1, 264points

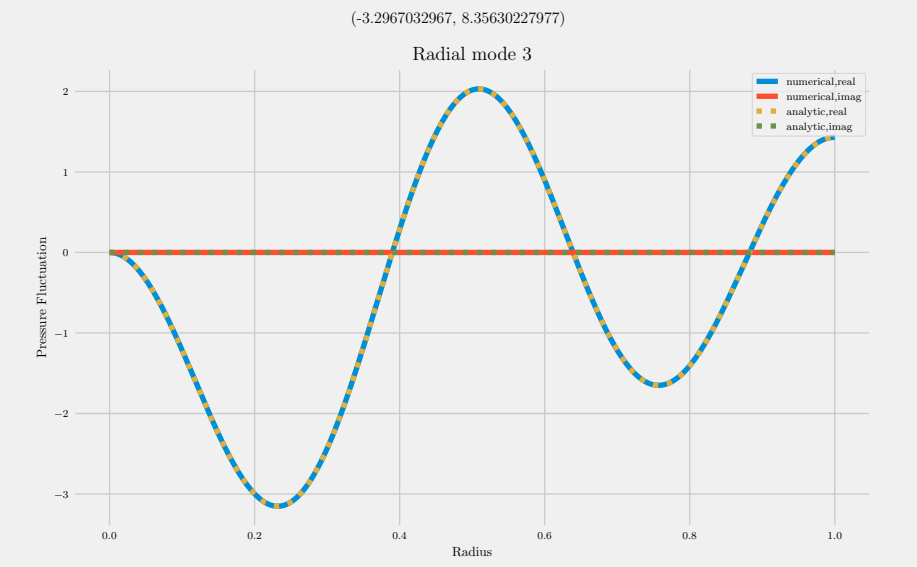

Figure 177: Second Order, Case number 1, 528points

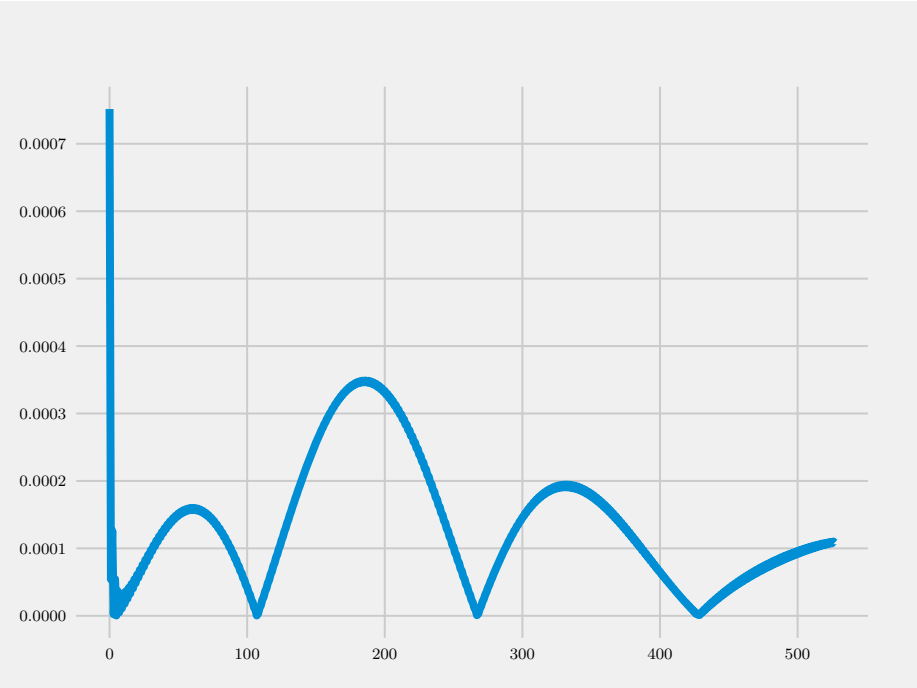

Figure 178: Second Order, Case number 1, 528points

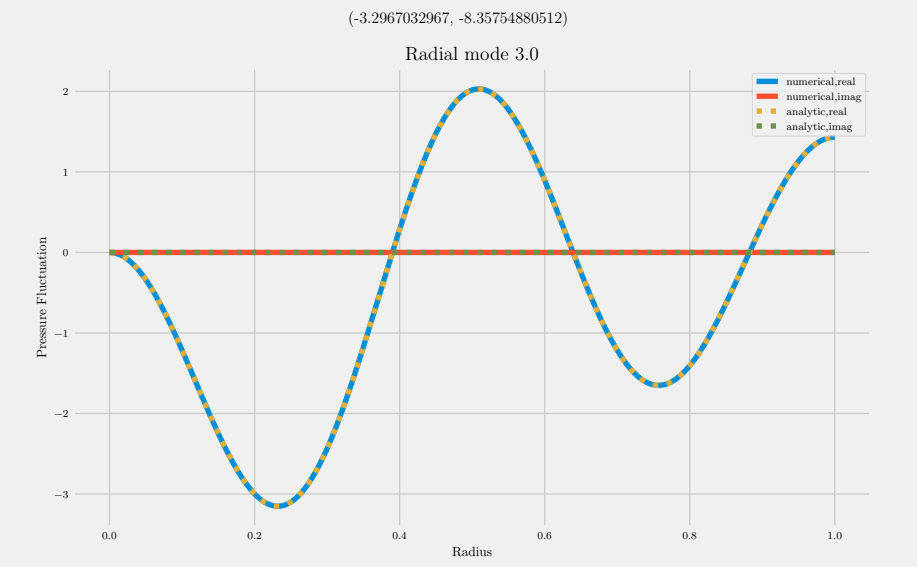

Figure 179: Second Order, Case number 1, 1056points

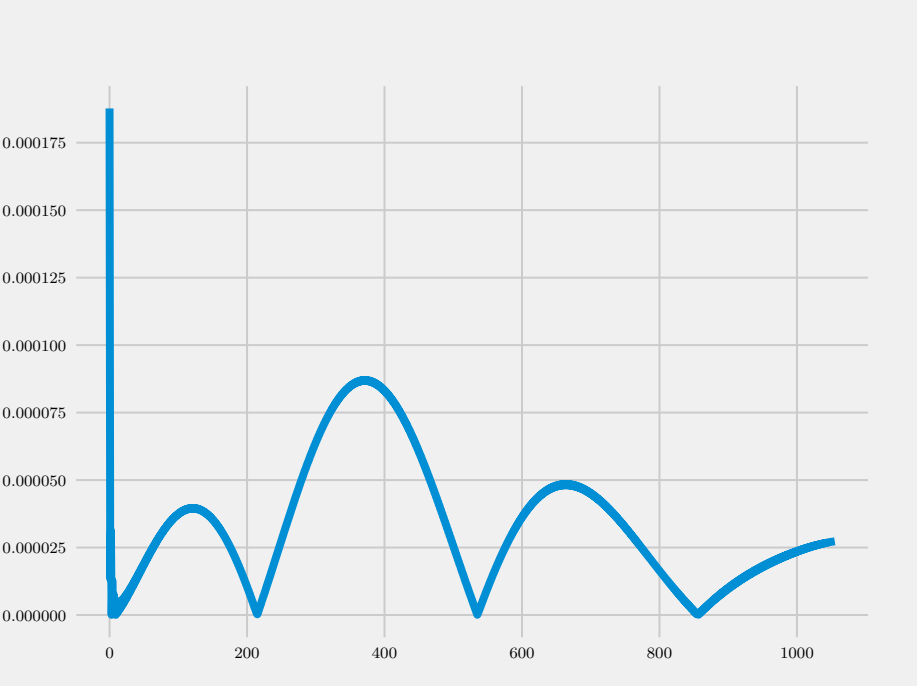

Figure 180: Second Order, Case number 1, 1056points

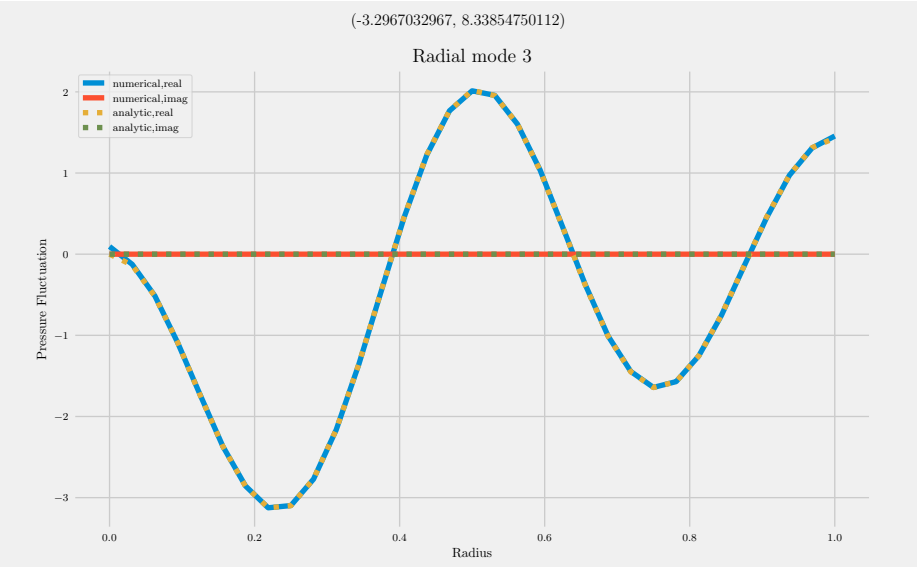

Figure 181: Fourth Order, Case number 0, 33points

Fourth Order, Radial Mode 3

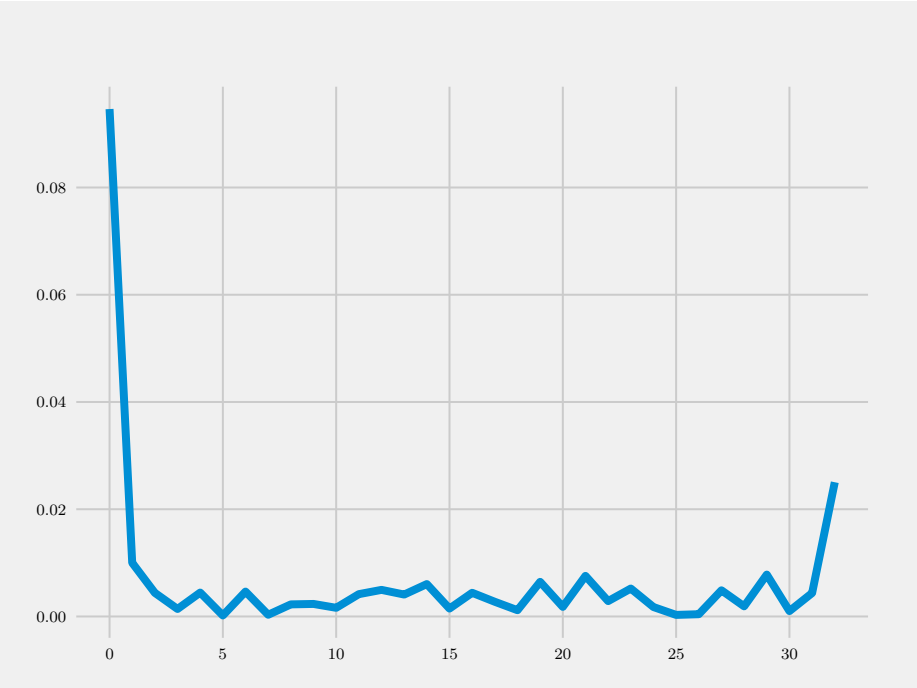

Figure 182: Fourth Order, Case number 0, 33points

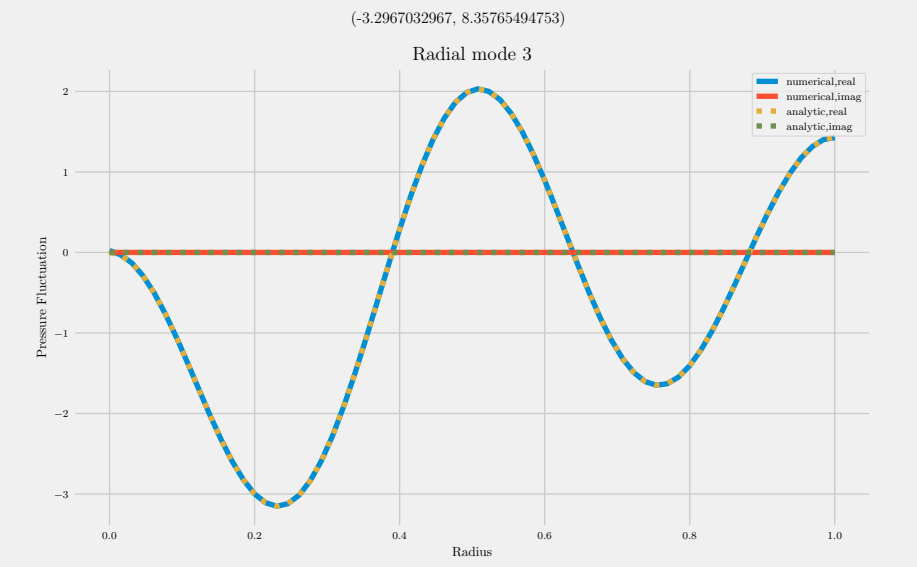

Figure 183: Fourth Order, Case number 0, 66points

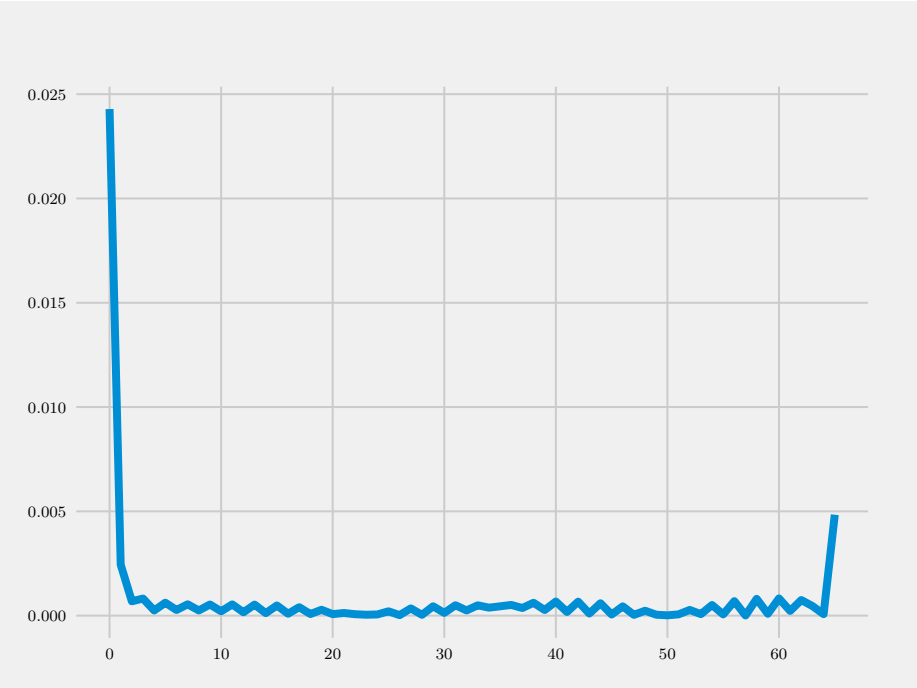

Figure 184: Fourth Order, Case number 0, 66points

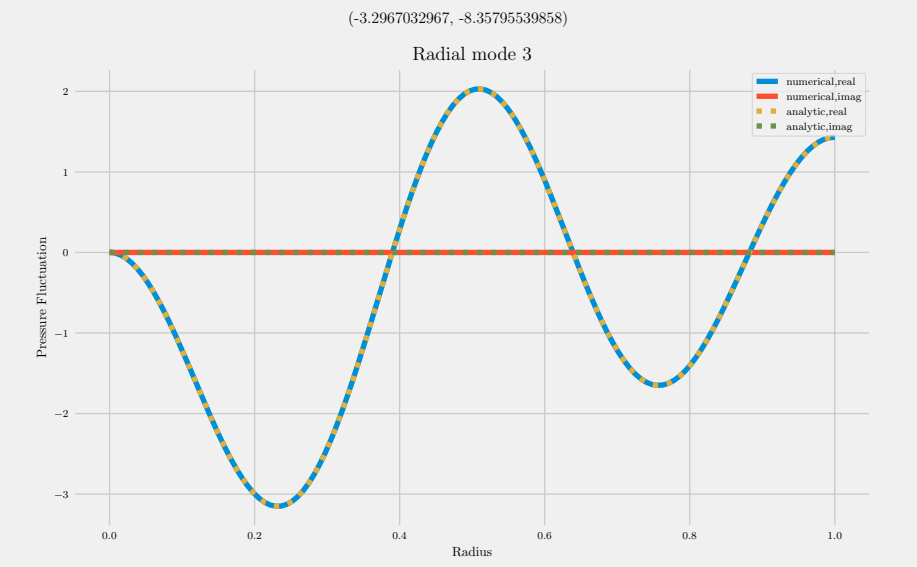

Figure 185: Fourth Order, Case number 0, 132points

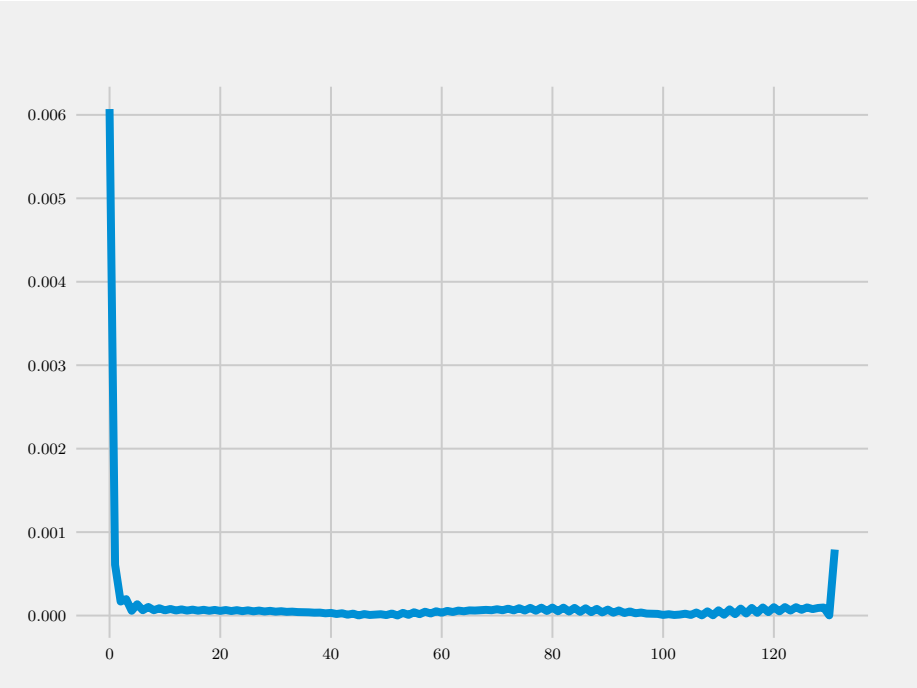

Figure 186: Fourth Order, Case number 0, 132points

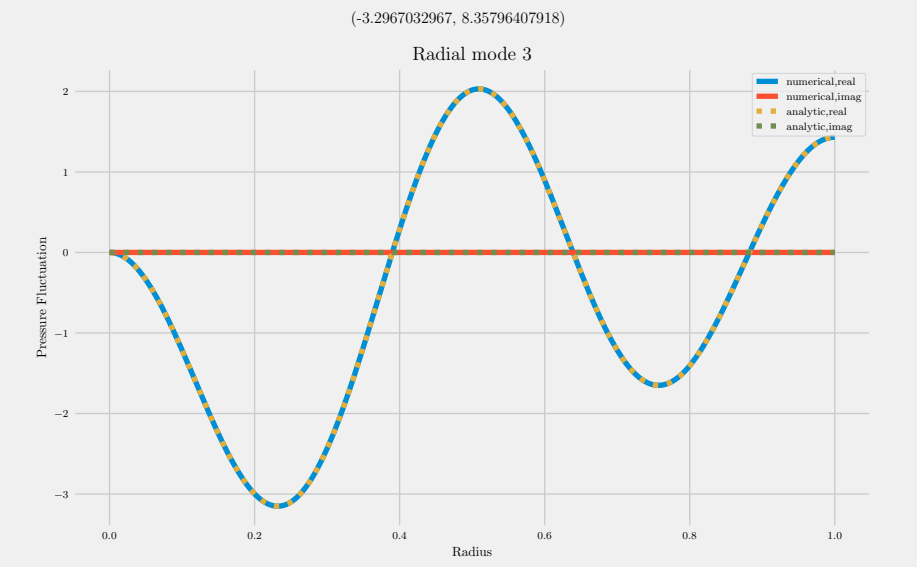

Figure 187: Fourth Order, Case number 0, 264points

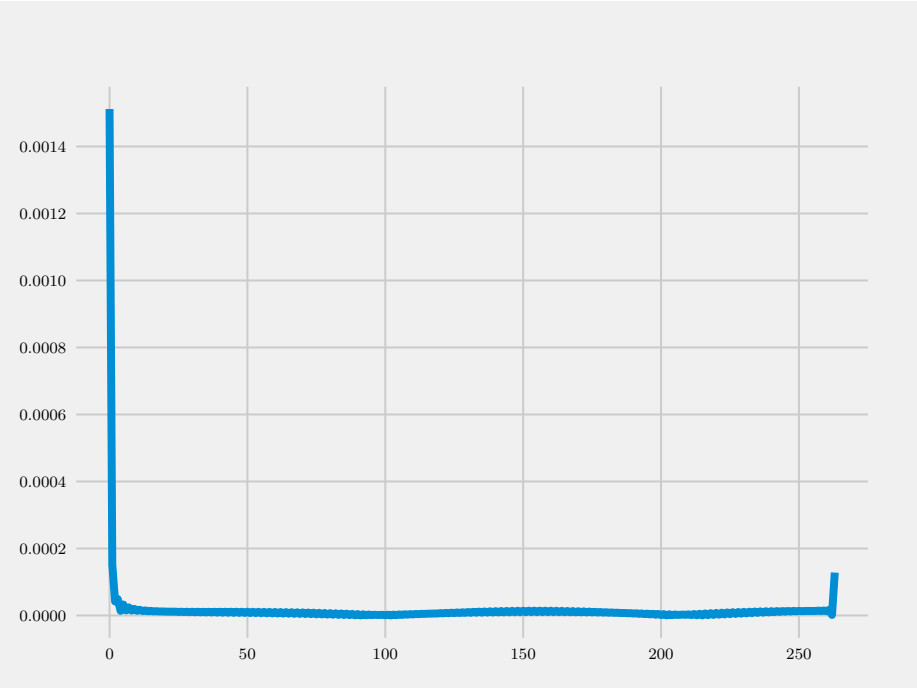

Figure 188: Fourth Order, Case number 0, 264points

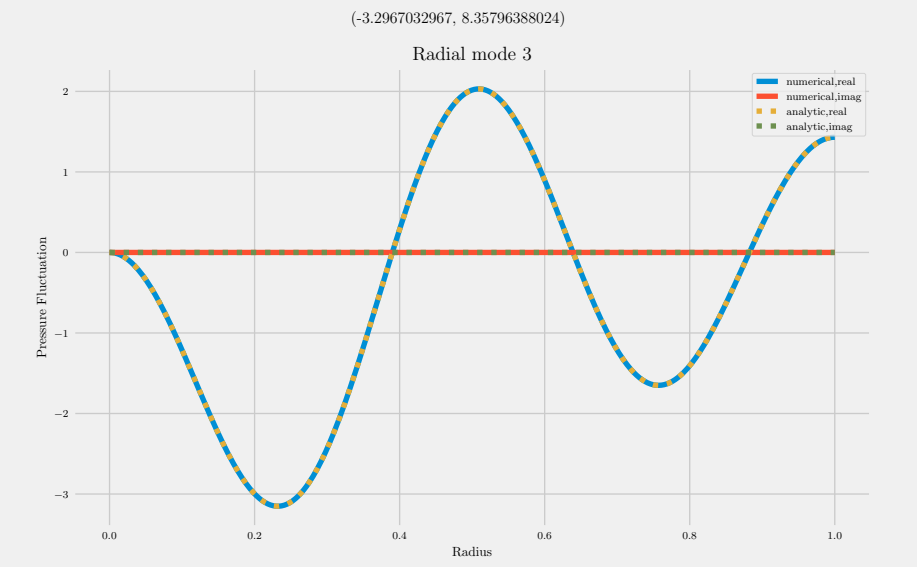

Figure 189: Fourth Order, Case number 0, 528points

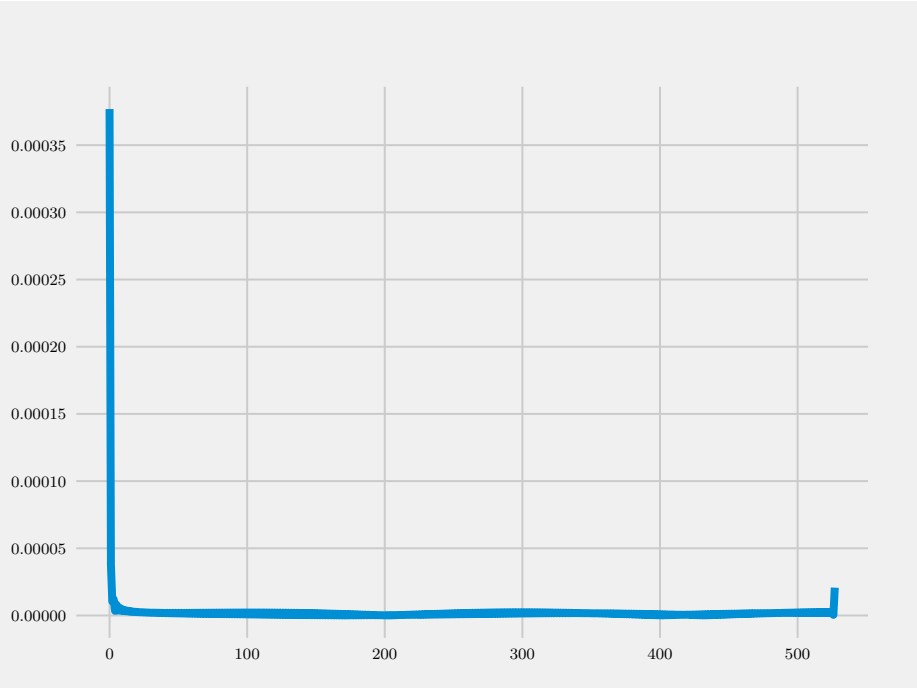

Figure 190: Fourth Order, Case number 0, 528points

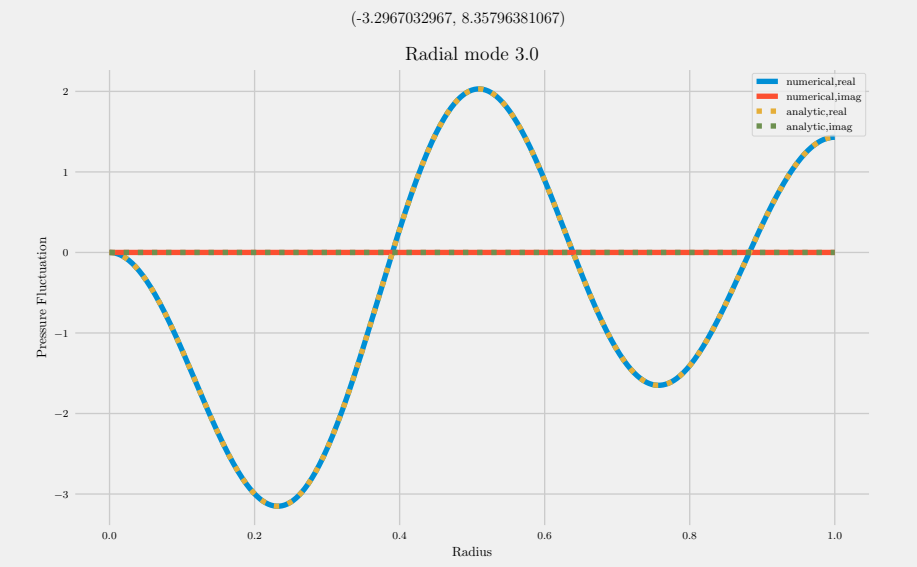

Figure 191: Fourth Order, Case number 0, 1056points

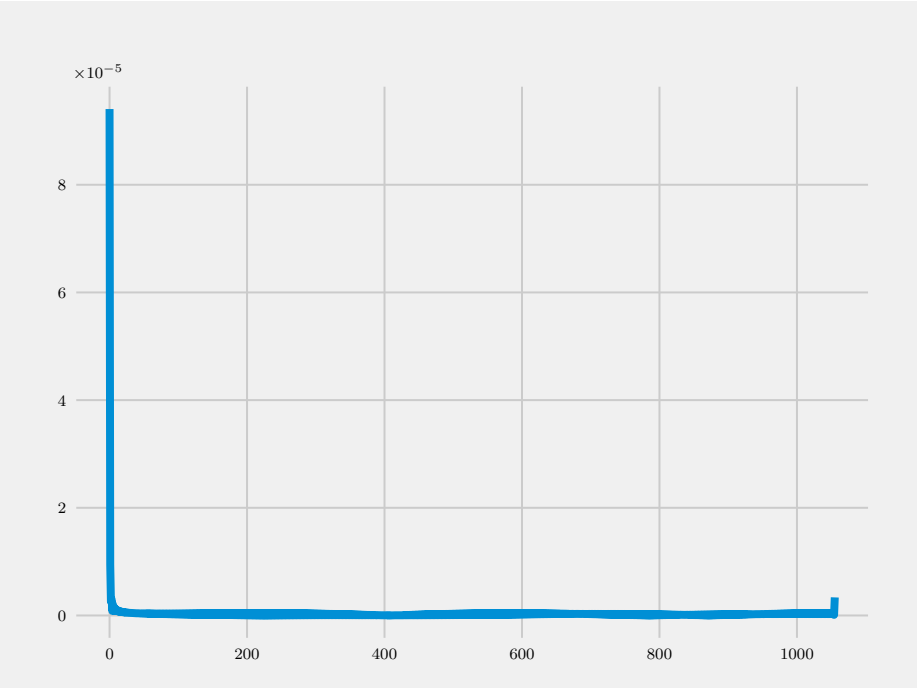

Figure 192: Fourth Order, Case number 0, 1056points

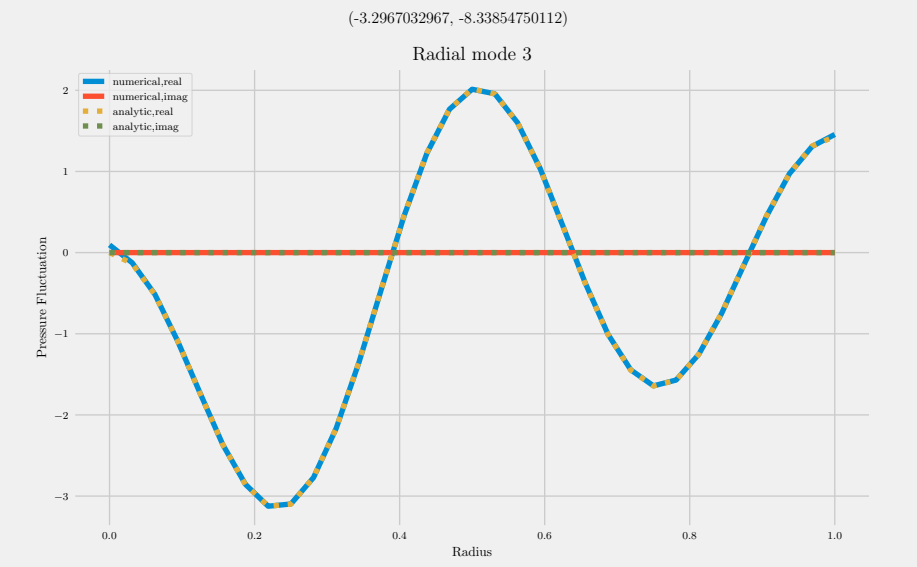

Figure 193: Fourth Order, Case number 1, 33points

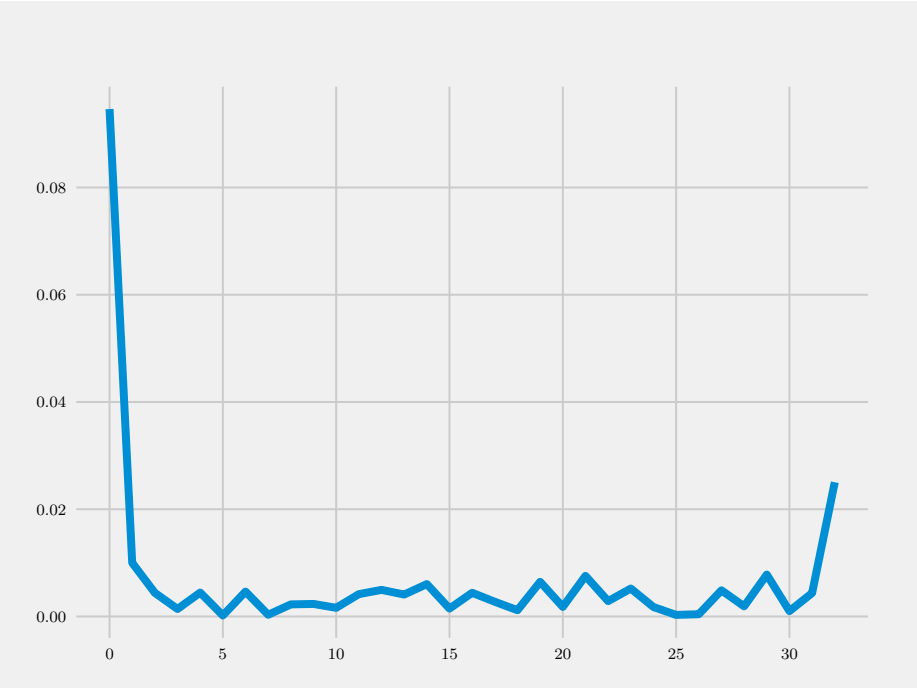

Figure 194: Fourth Order, Case number 1, 33points

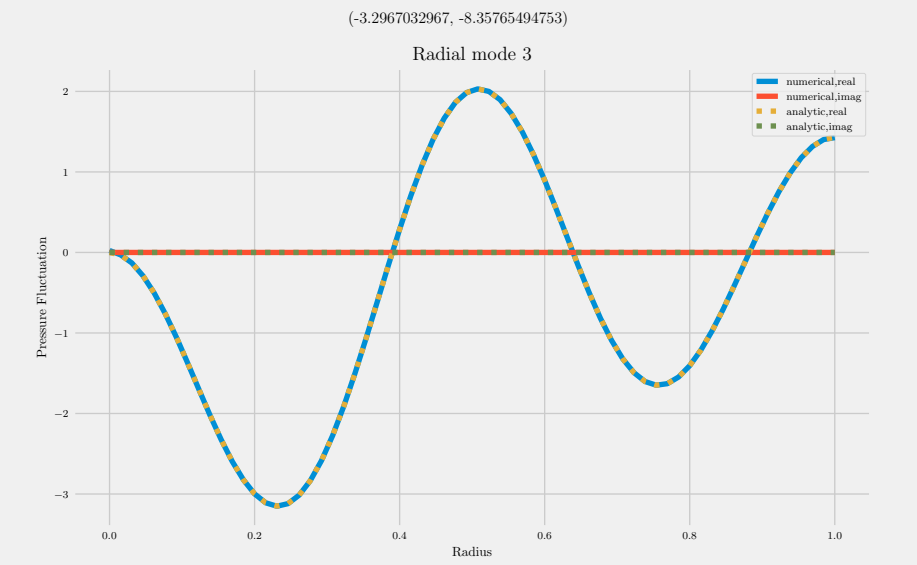

Figure 195: Fourth Order, Case number 1, 66points

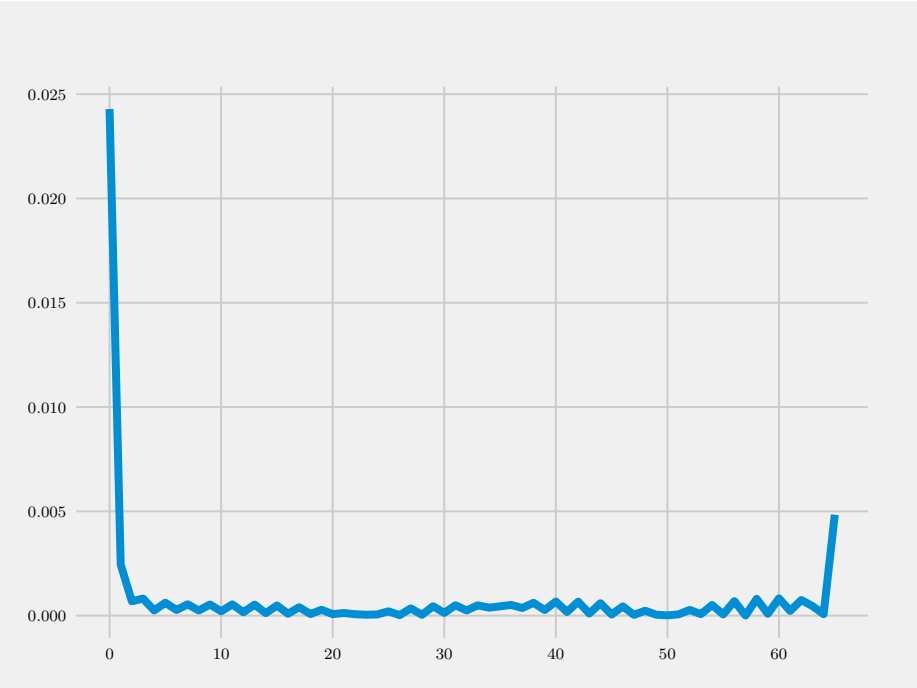

Figure 196: Fourth Order, Case number 1, 66points

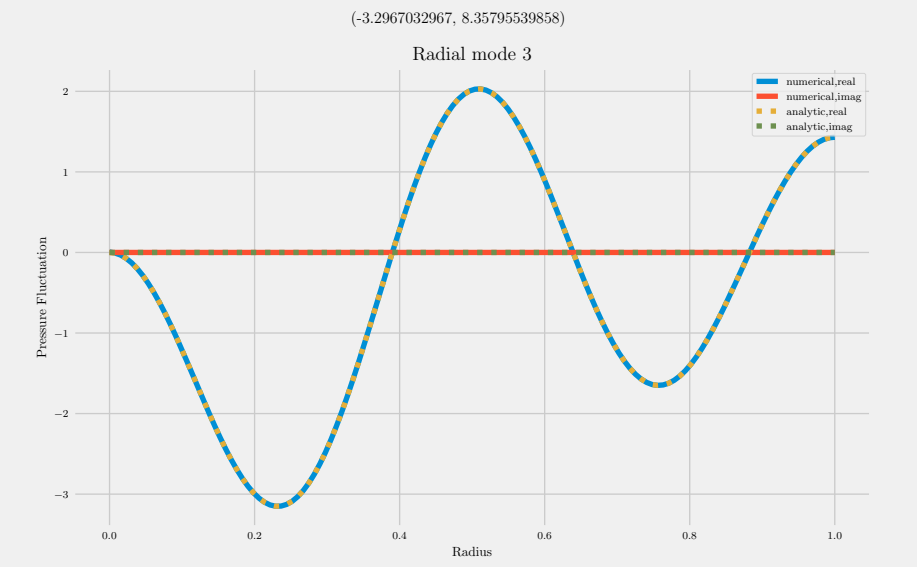

Figure 197: Fourth Order, Case number 1, 132points

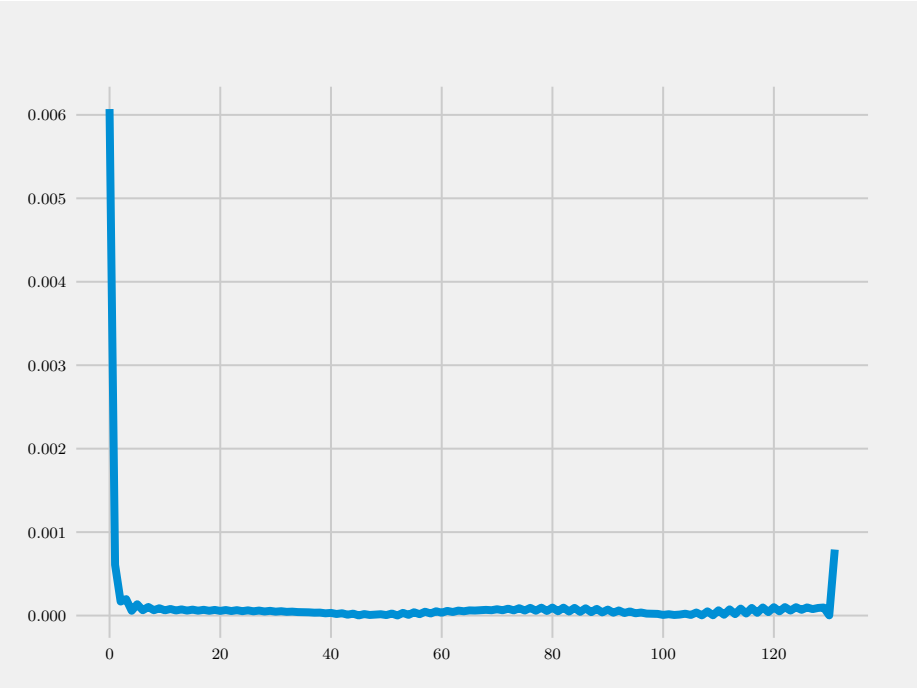

Figure 198: Fourth Order, Case number 1, 132points

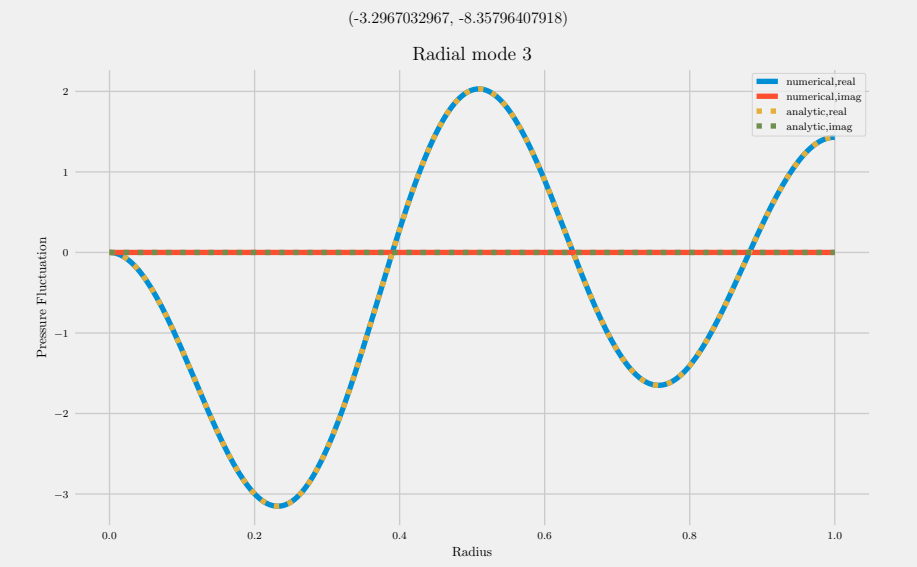

Figure 199: Fourth Order, Case number 1, 264points

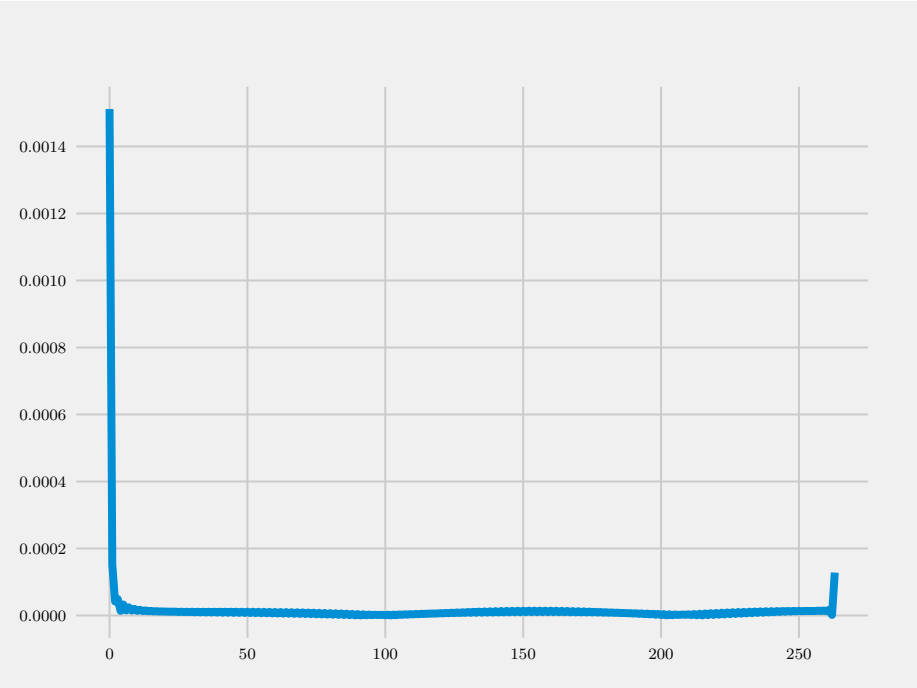

Figure 200: Fourth Order, Case number 1, 264points

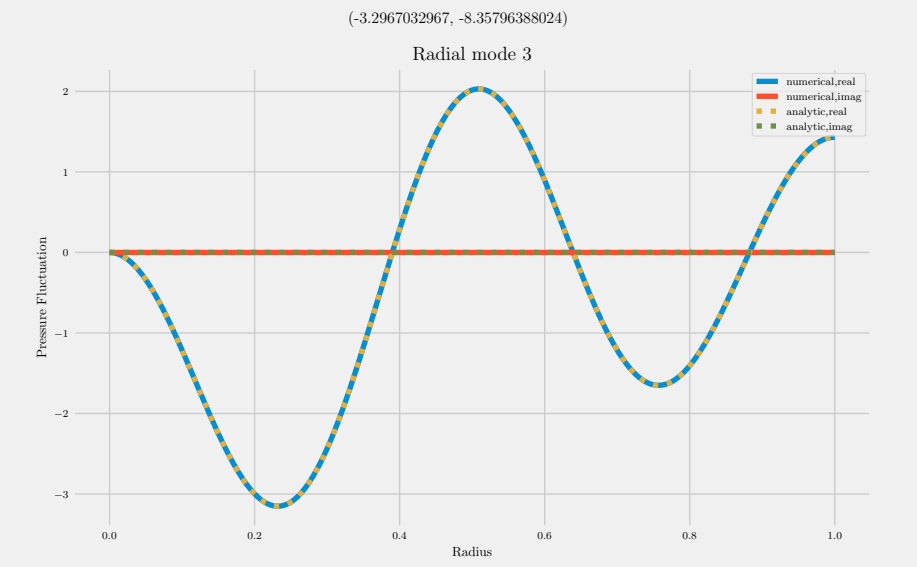

Figure 201: Fourth Order, Case number 1, 528points

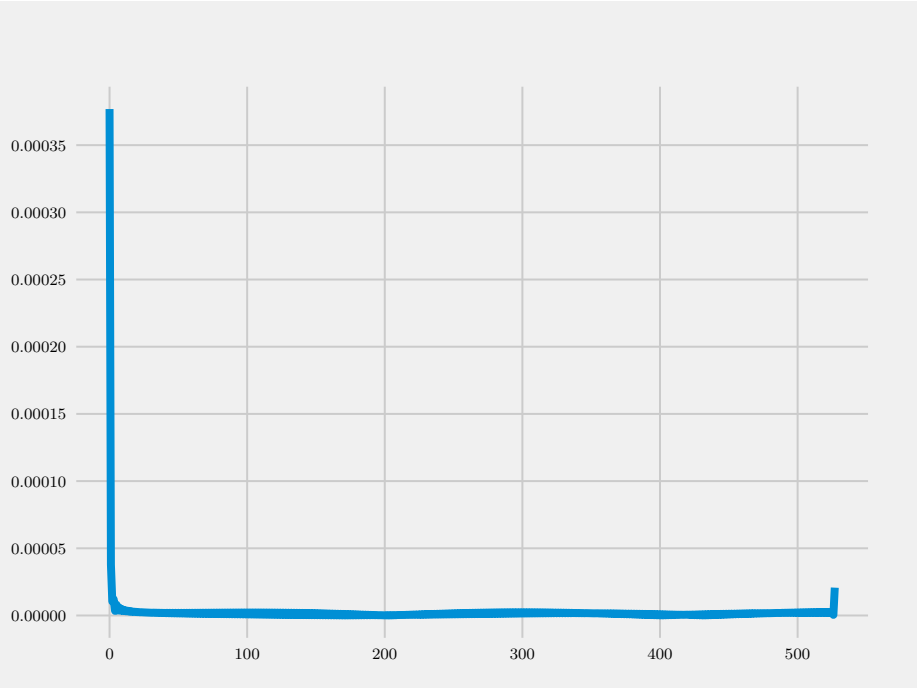

Figure 202: Fourth Order, Case number 1, 528points

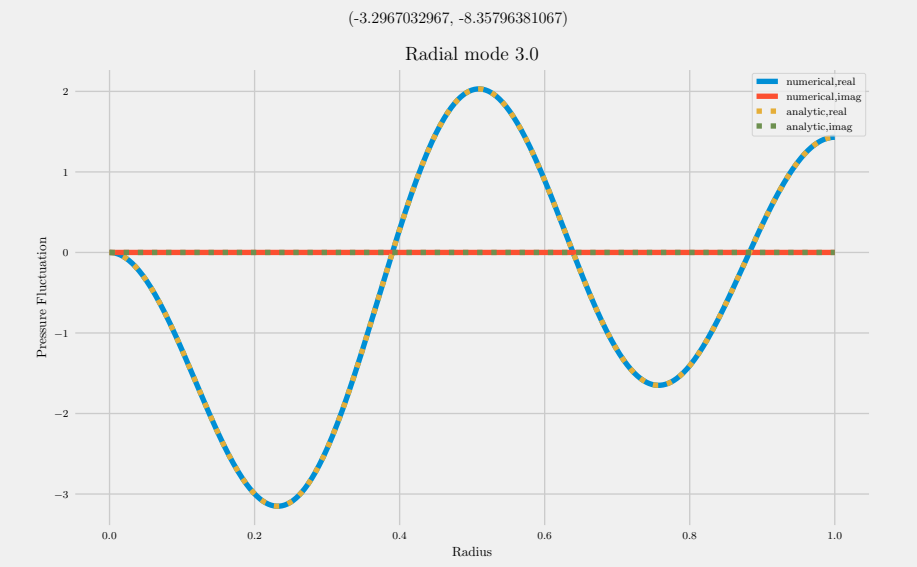

Figure 203: Fourth Order, Case number 1, 1056points

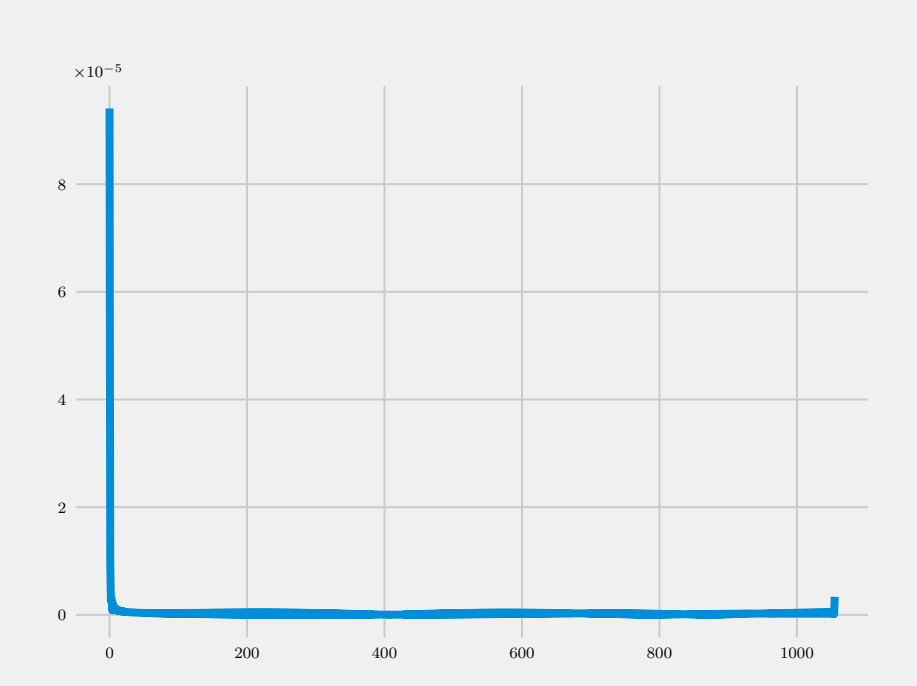

Figure 204: Fourth Order, Case number 1, 1056points

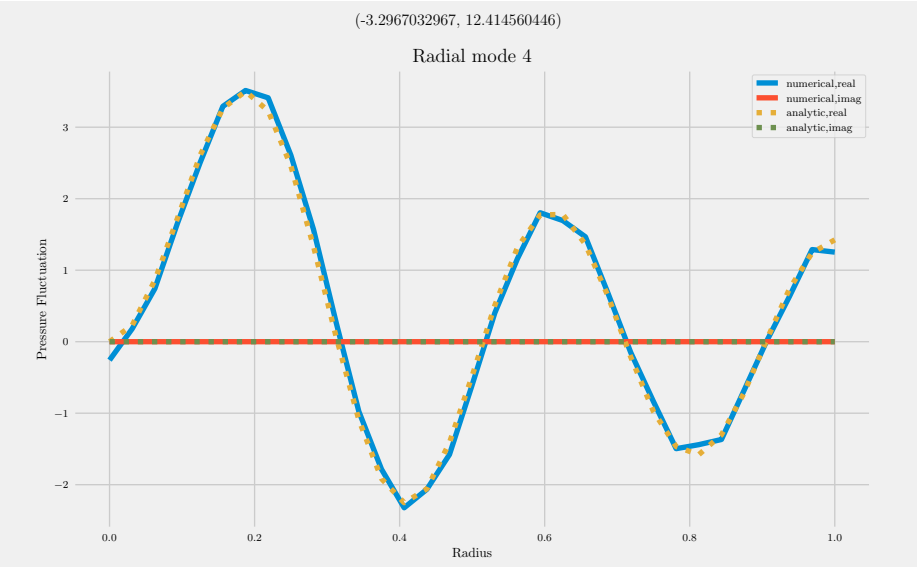

Figure 205: Second Order, Case number 0, 33points

Second Order, Radial Mode 4

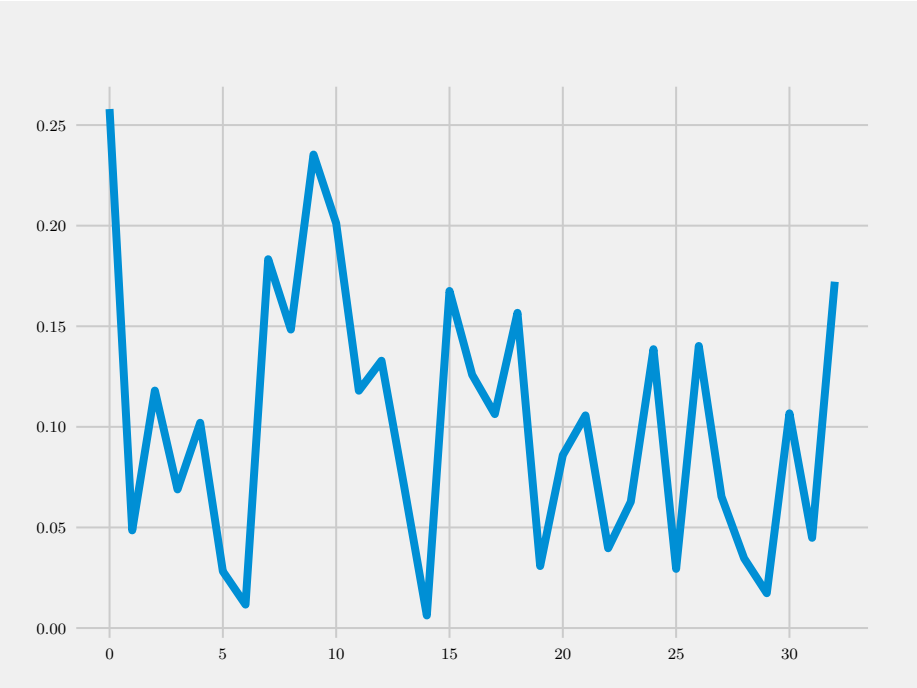

Figure 206: Second Order, Case number 0, 33points

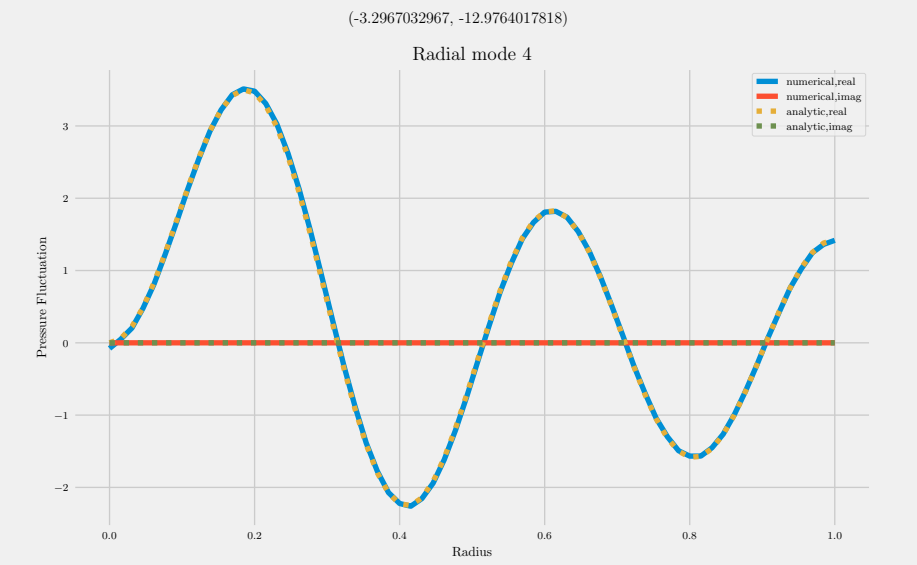

Figure 207: Second Order, Case number 0, 66points

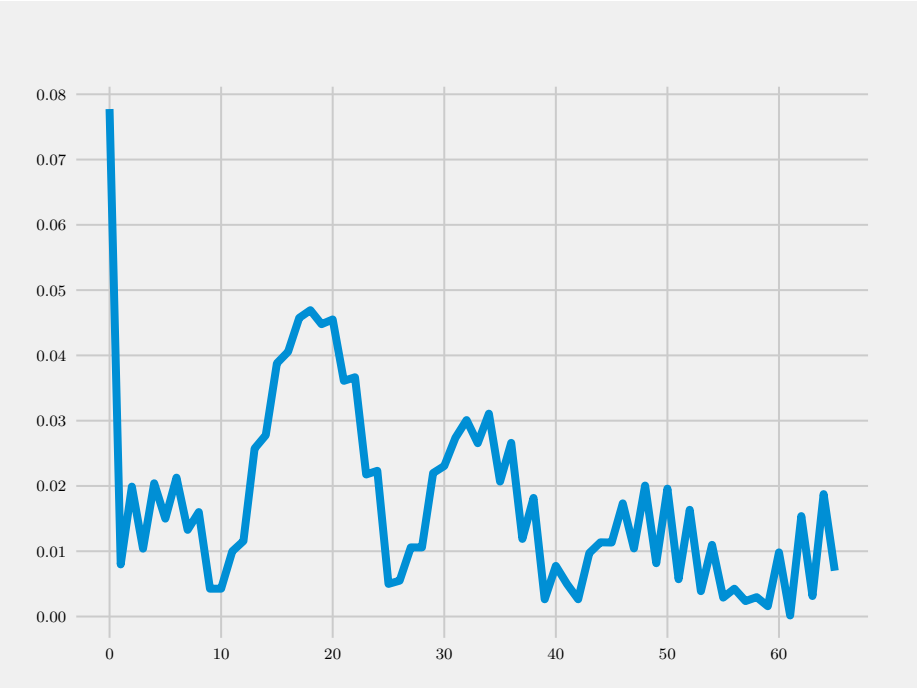

Figure 208: Second Order, Case number 0, 66points

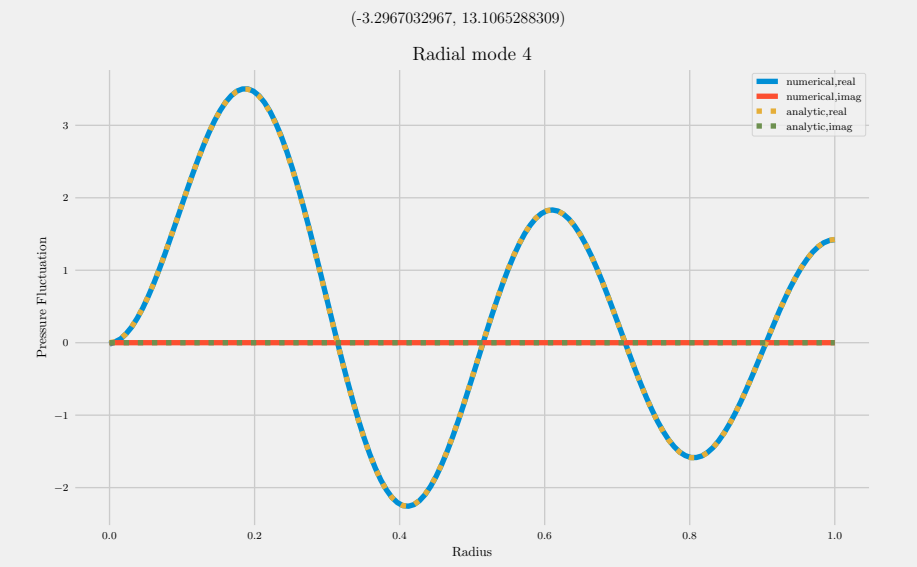

Figure 209: Second Order, Case number 0, 132points

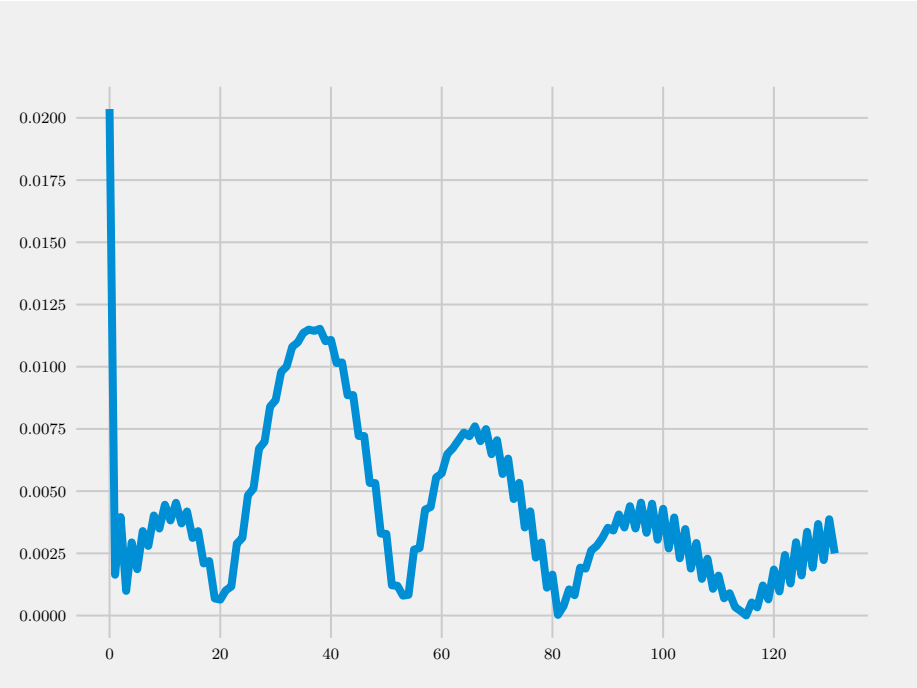

Figure 210: Second Order, Case number 0, 132points

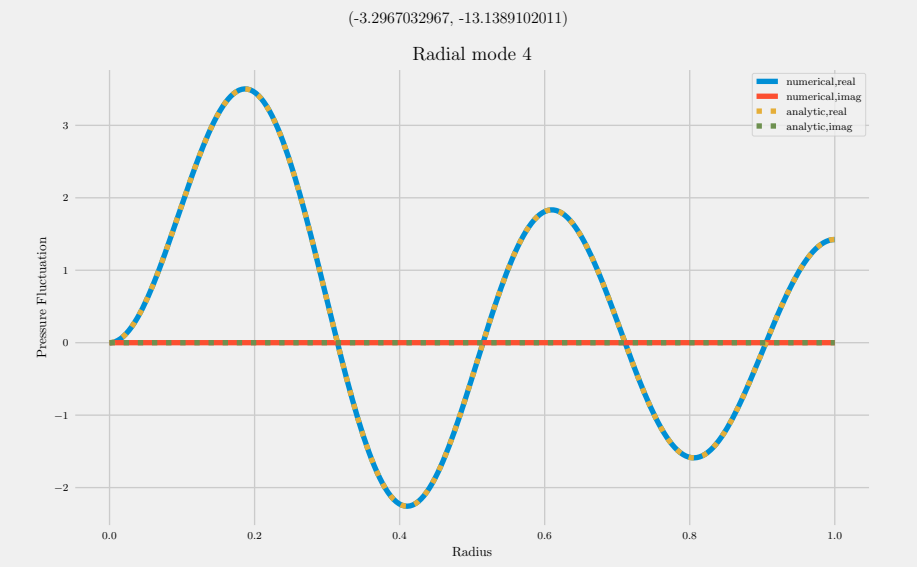

Figure 211: Second Order, Case number 0, 264points

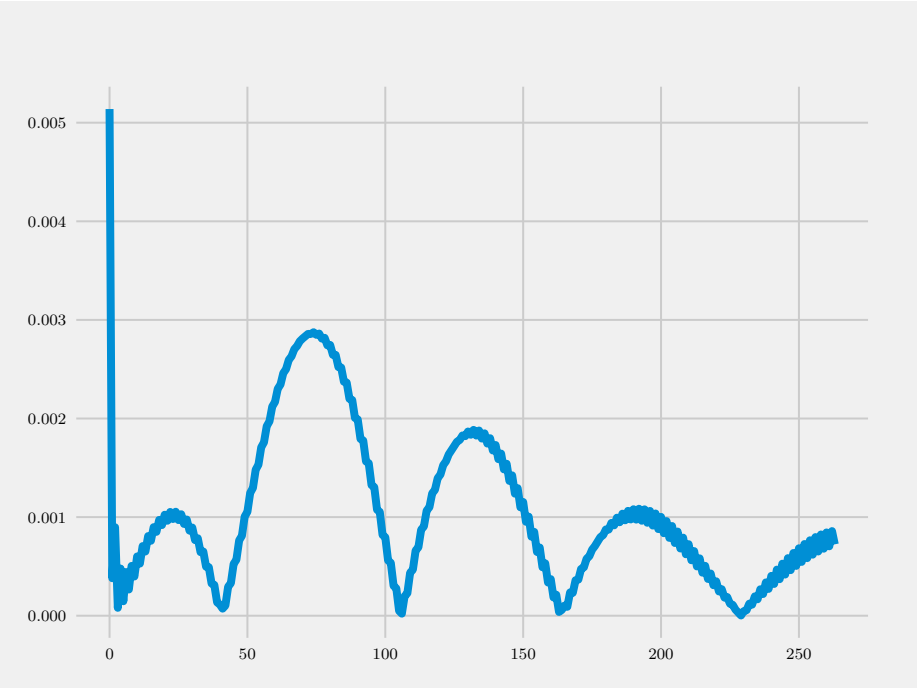

Figure 212: Second Order, Case number 0, 264points

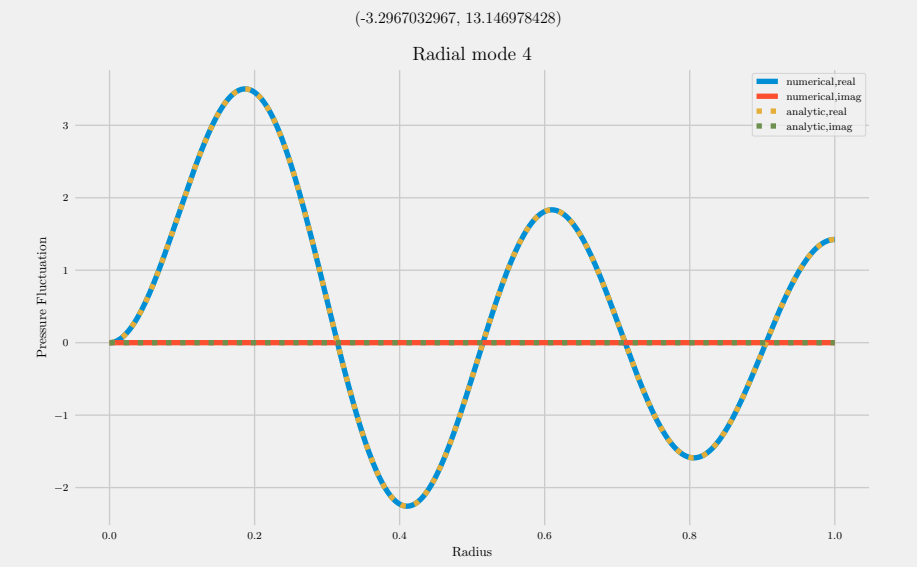

Figure 213: Second Order, Case number 0, 528points

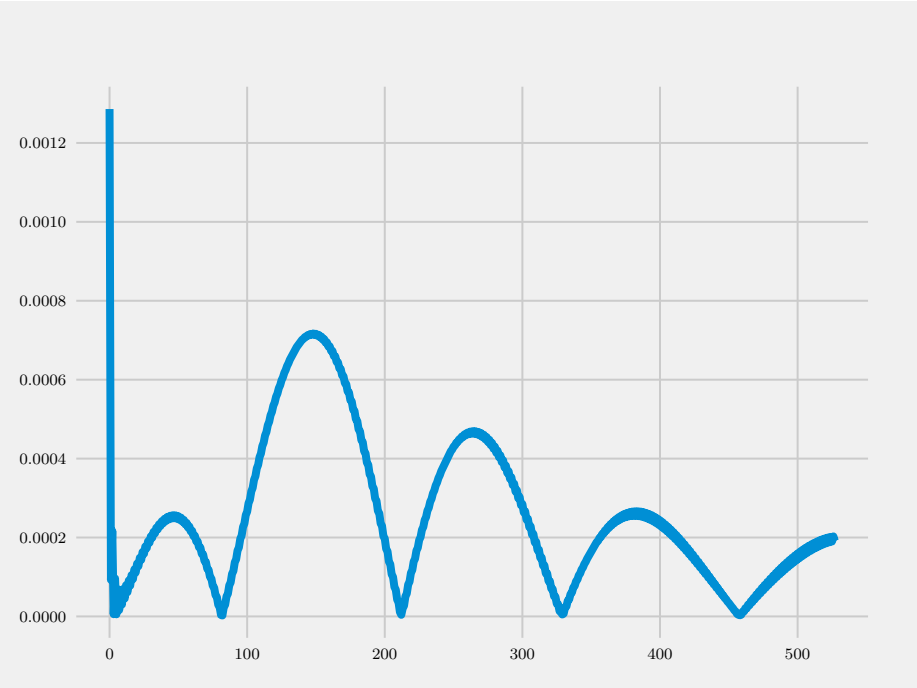

Figure 214: Second Order, Case number 0, 528points

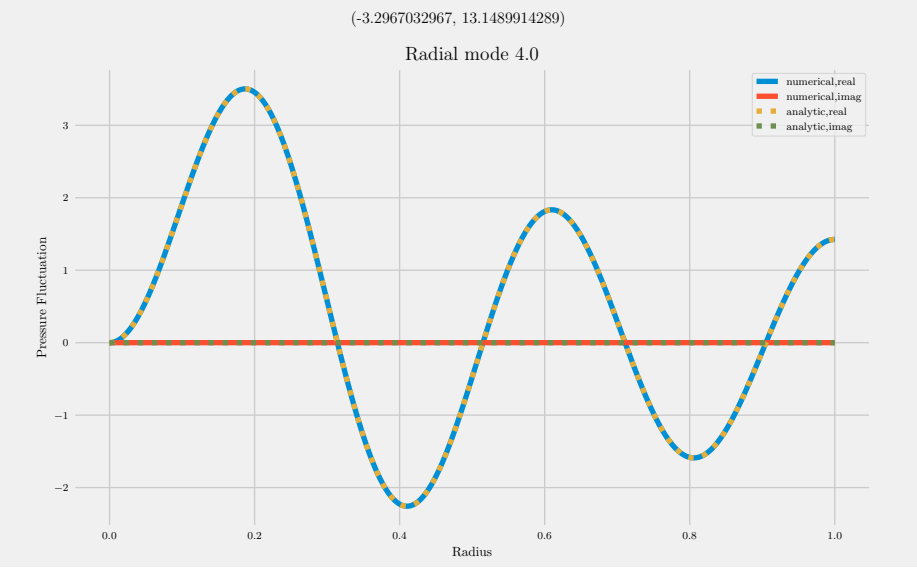

Figure 215: Second Order, Case number 0, 1056points

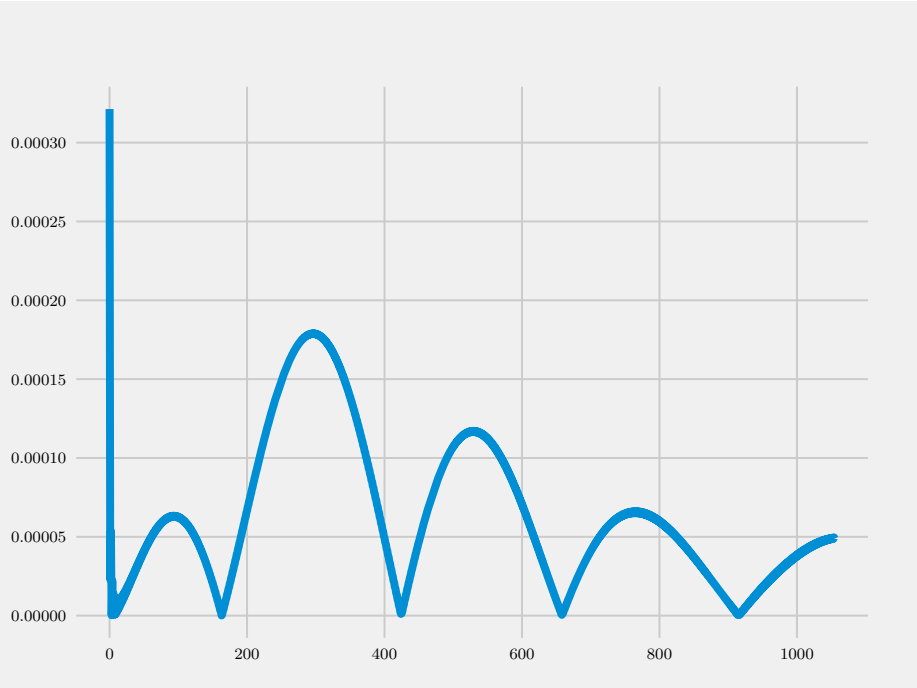

Figure 216: Second Order, Case number 0, 1056points

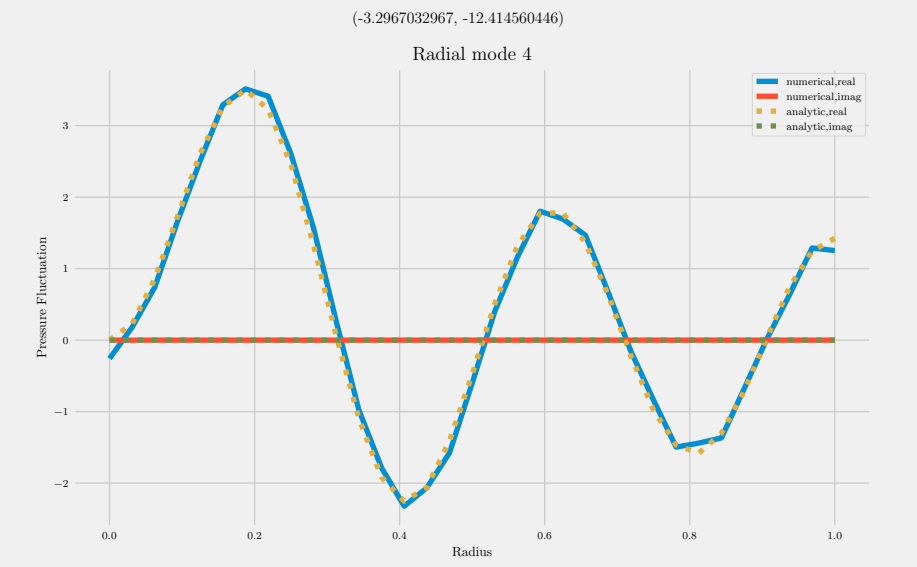

Figure 217: Second Order, Case number 1, 33points

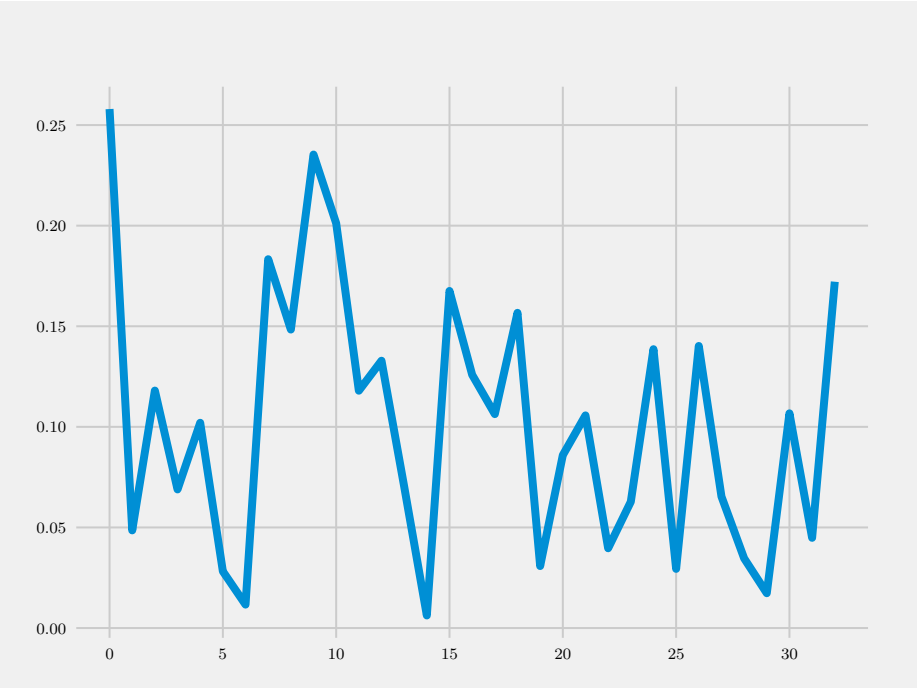

Figure 218: Second Order, Case number 1, 33points

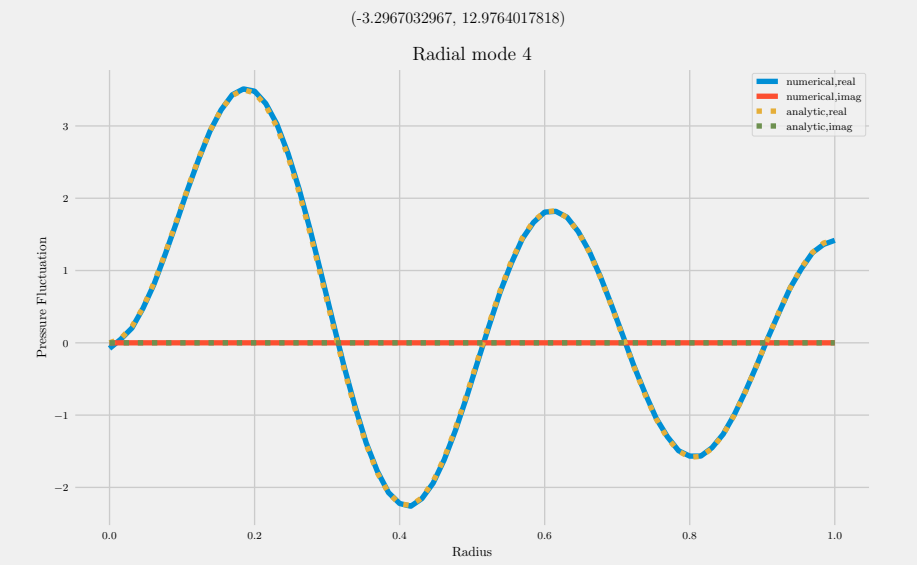

Figure 219: Second Order, Case number 1, 66points

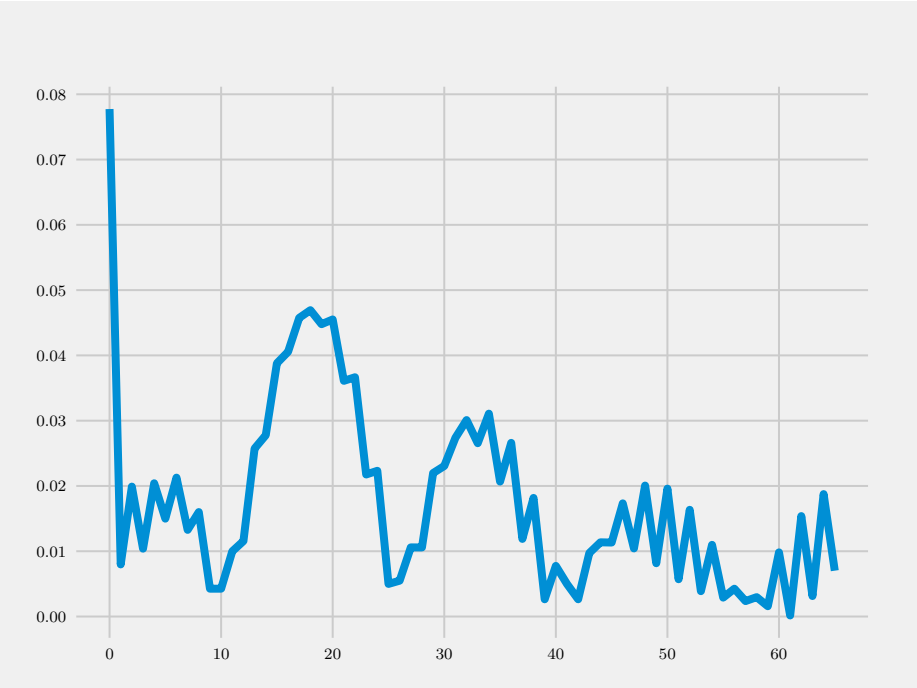

Figure 220: Second Order, Case number 1, 66points

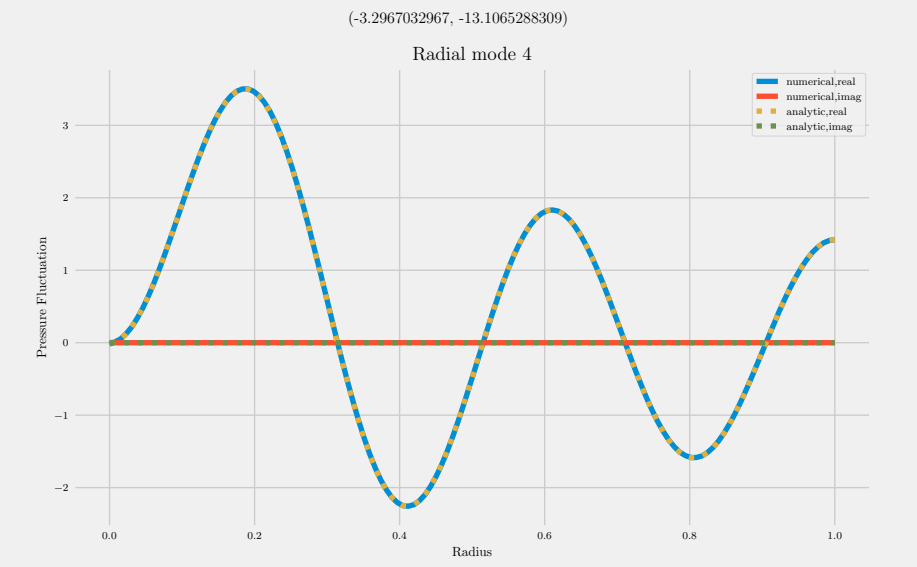

Figure 221: Second Order, Case number 1, 132points

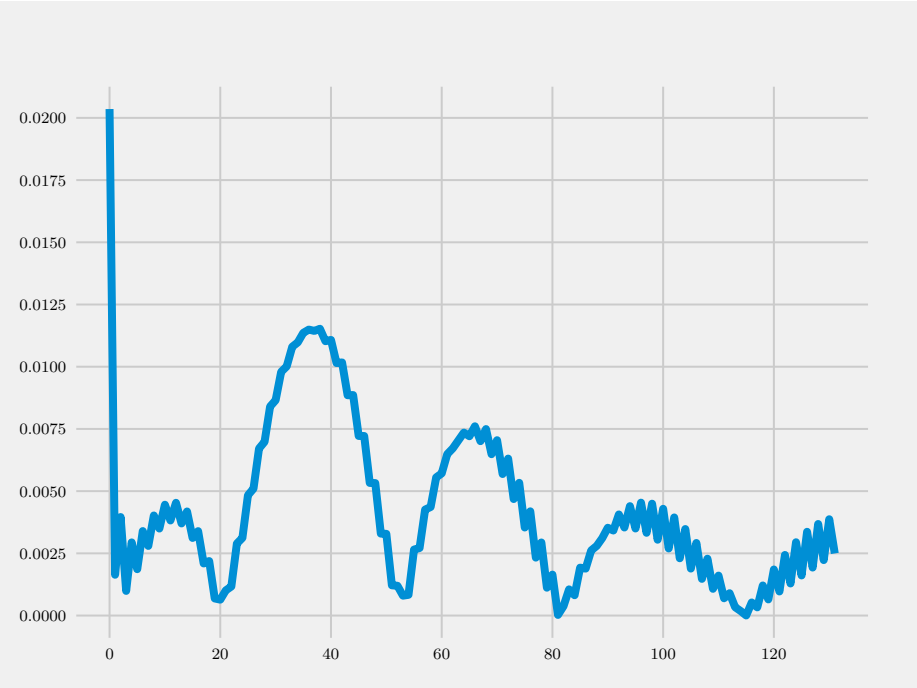

Figure 222: Second Order, Case number 1, 132points

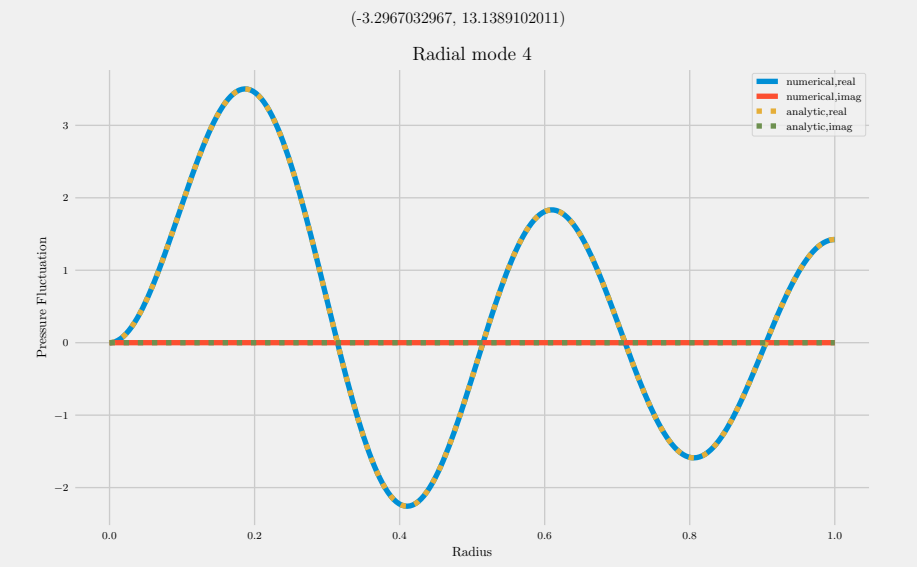

Figure 223: Second Order, Case number 1, 264points

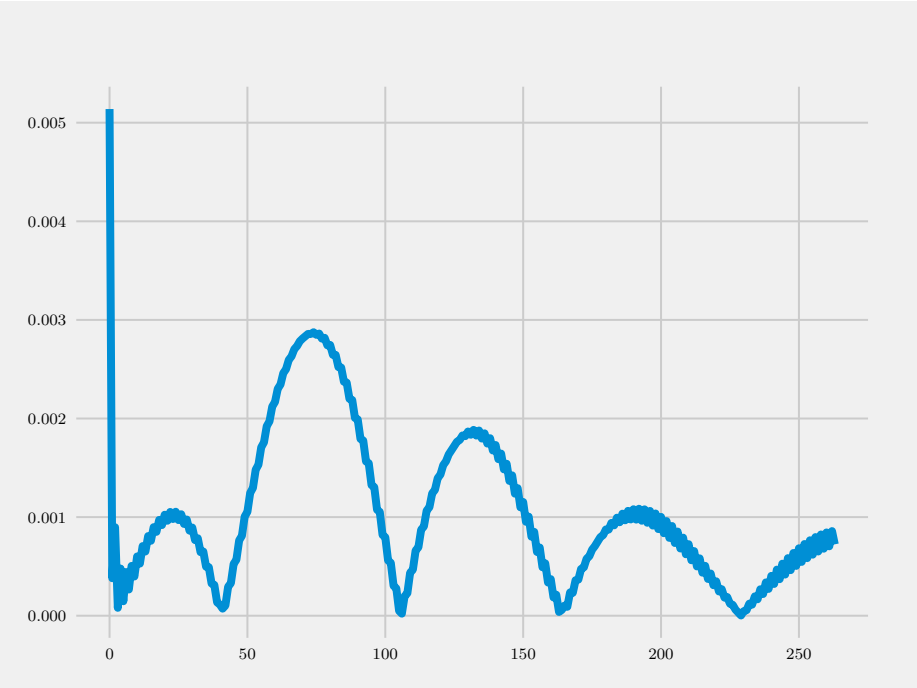

Figure 224: Second Order, Case number 1, 264points

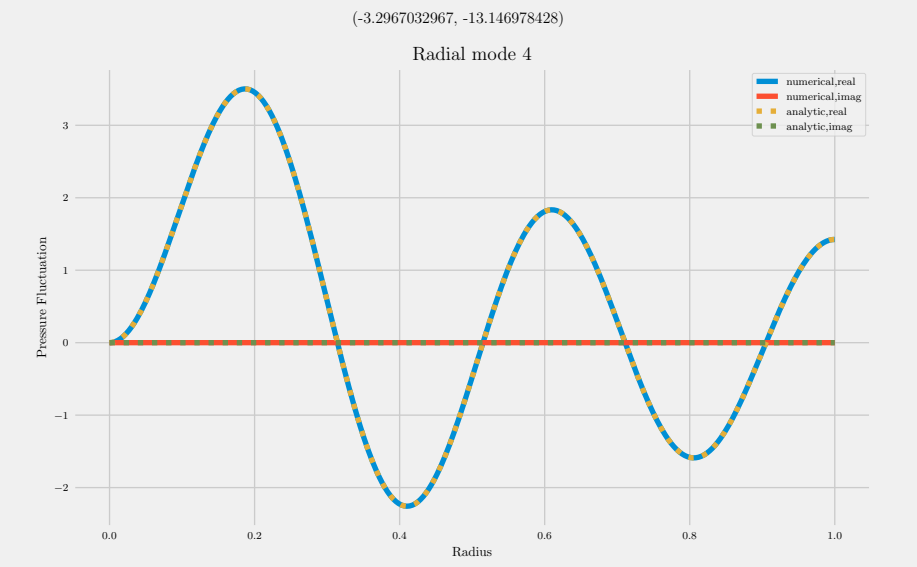

Figure 225: Second Order, Case number 1, 528points

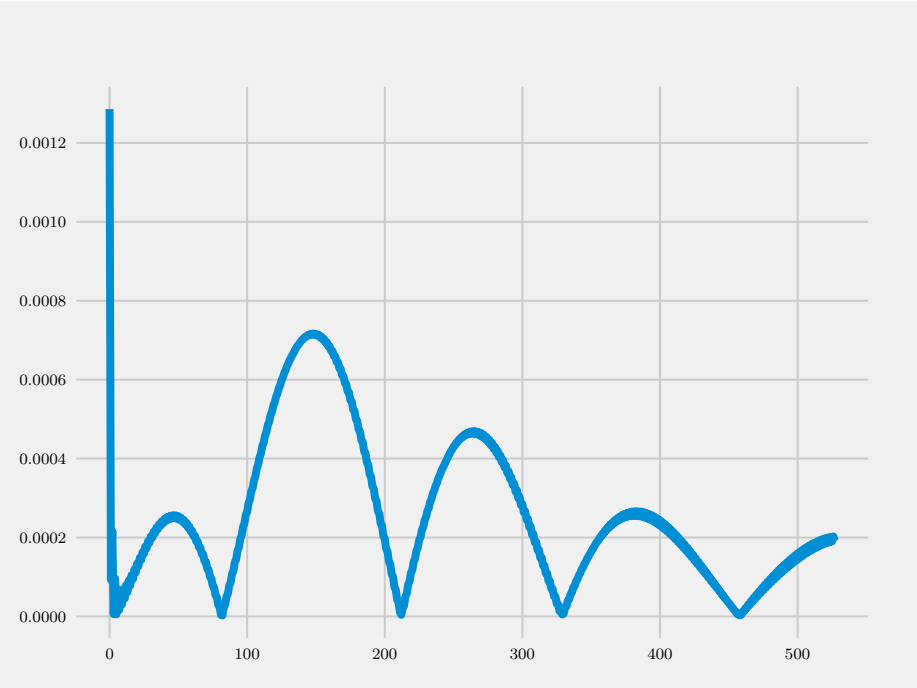

Figure 226: Second Order, Case number 1, 528points

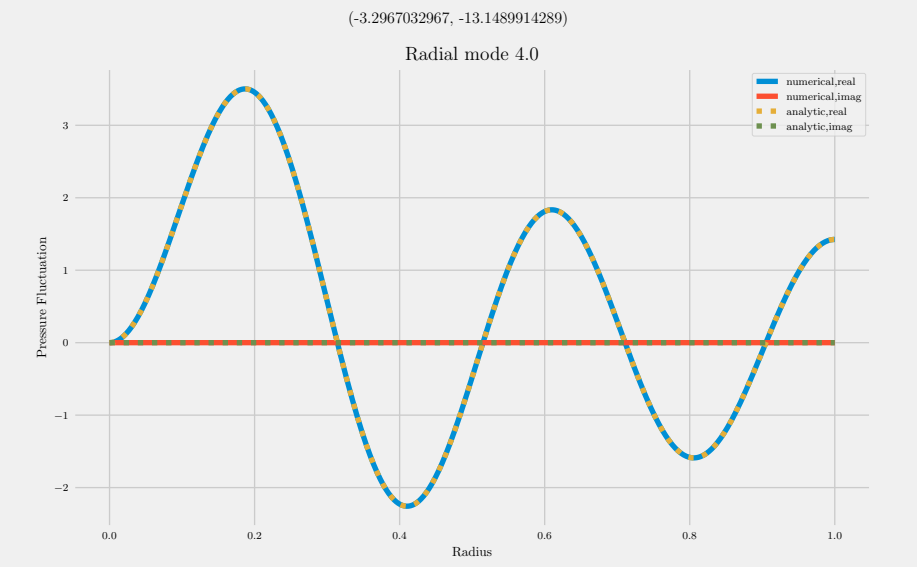

Figure 227: Second Order, Case number 1, 1056points

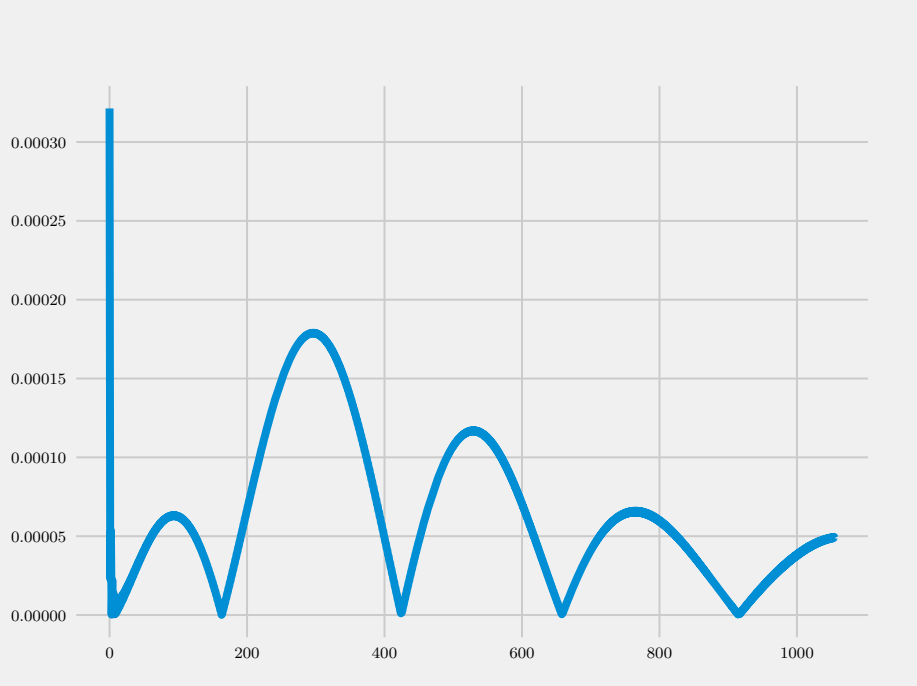

Figure 228: Second Order, Case number 1, 1056points

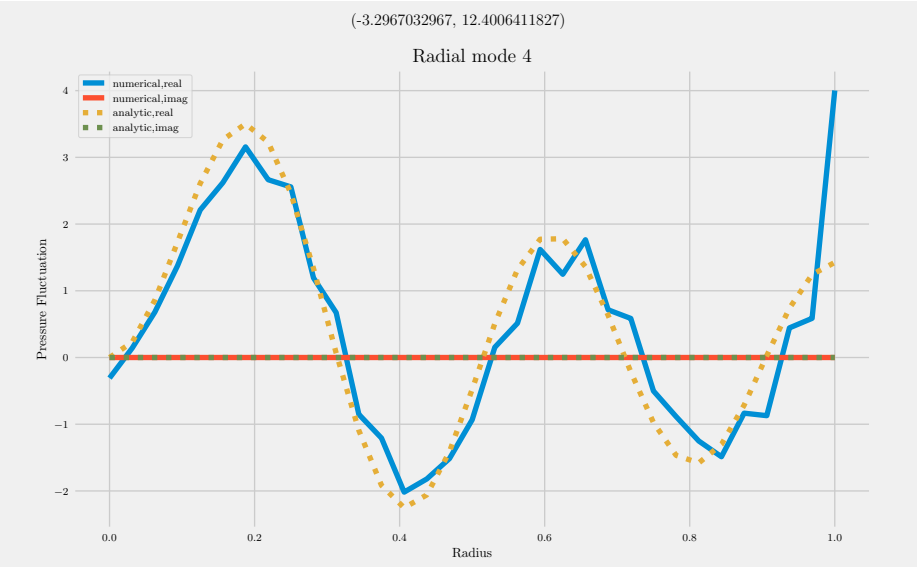

Figure 229: Fourth Order, Case number 0, 33points

Fourth Order, Radial Mode 4

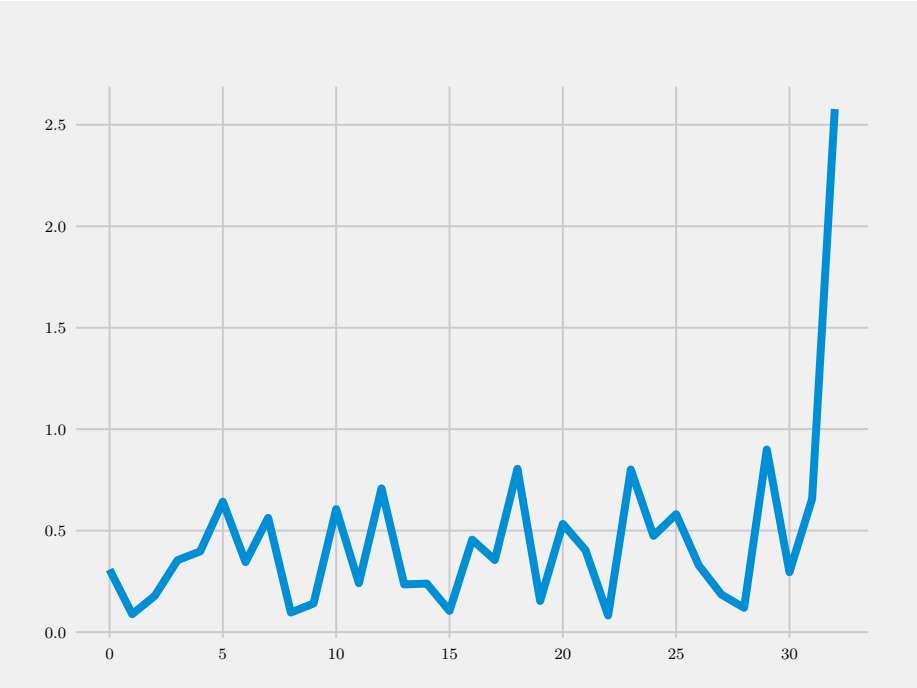

Figure 230: Fourth Order, Case number 0, 33points

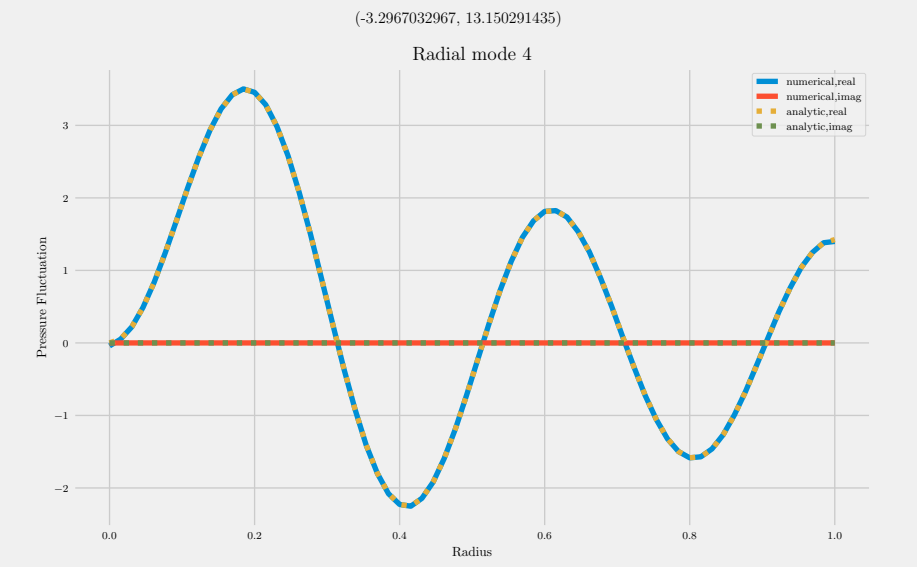

Figure 231: Fourth Order, Case number 0, 66points

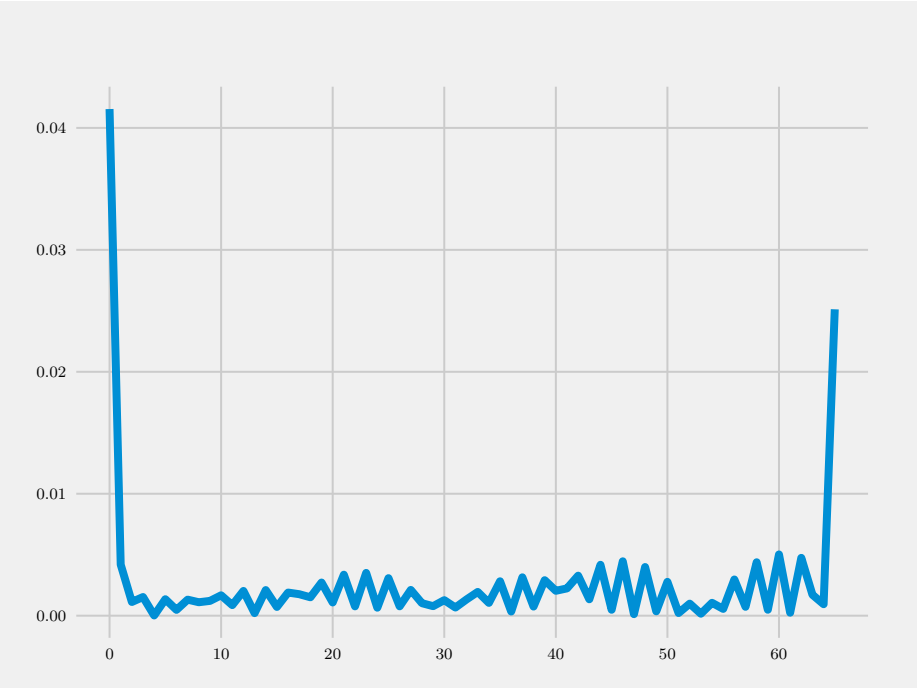

Figure 232: Fourth Order, Case number 0, 66points

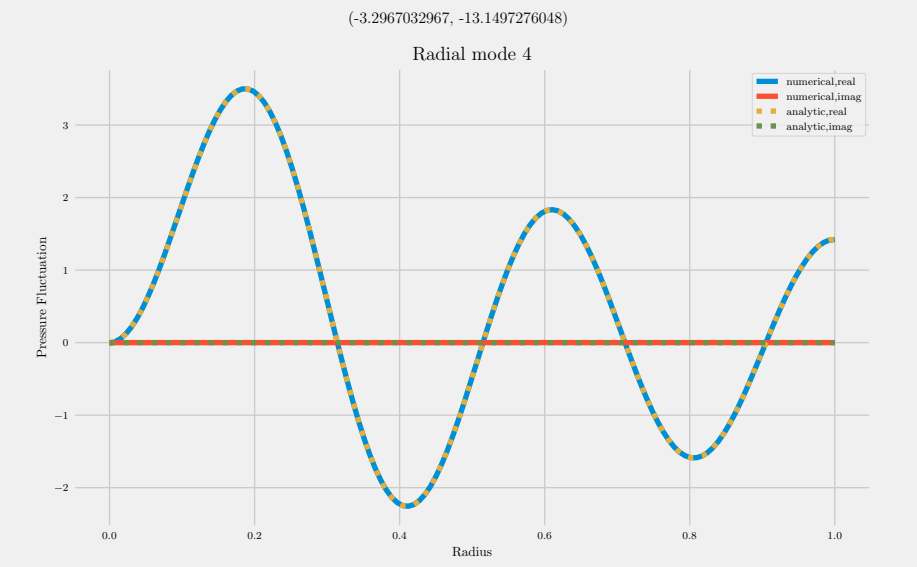

Figure 233: Fourth Order, Case number 0, 132points

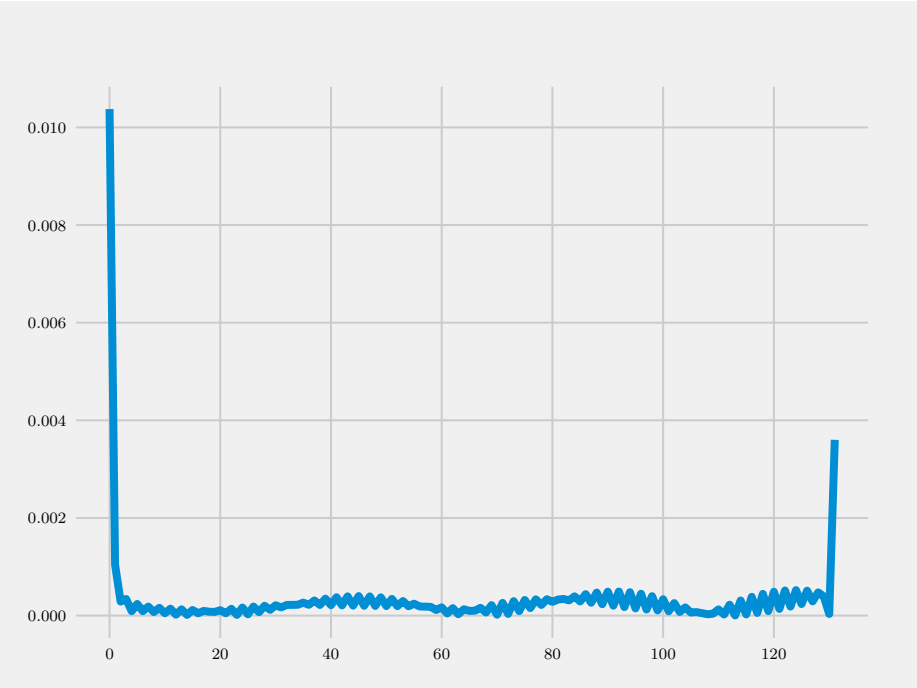

Figure 234: Fourth Order, Case number 0, 132points

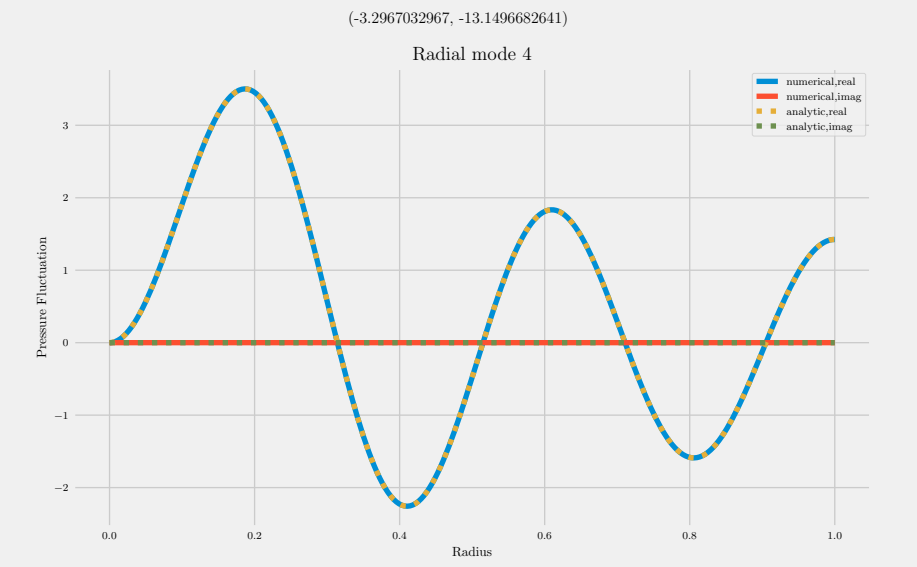

Figure 235: Fourth Order, Case number 0, 264points

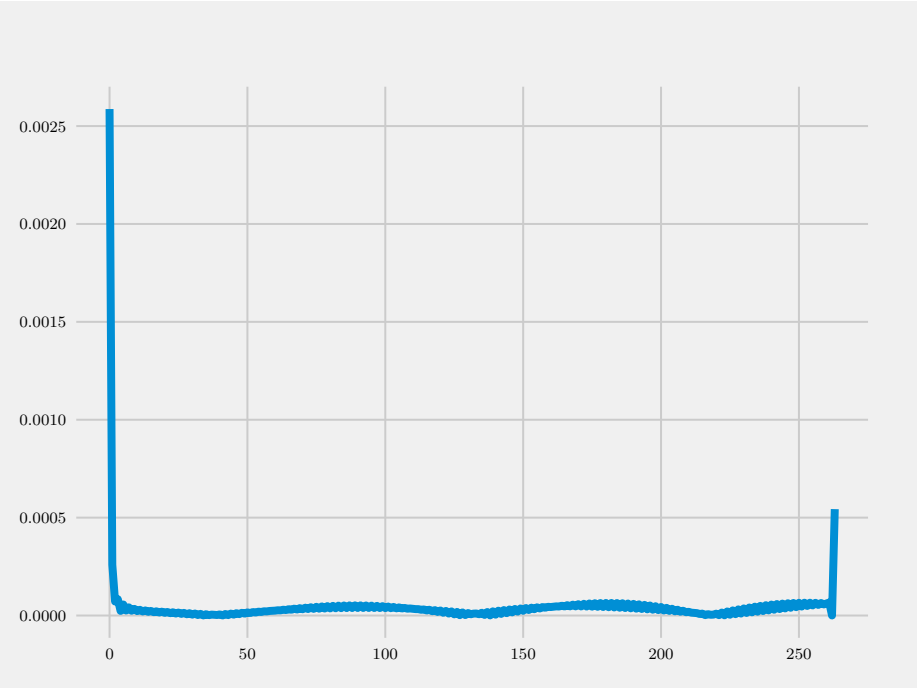

Figure 236: Fourth Order, Case number 0, 264points

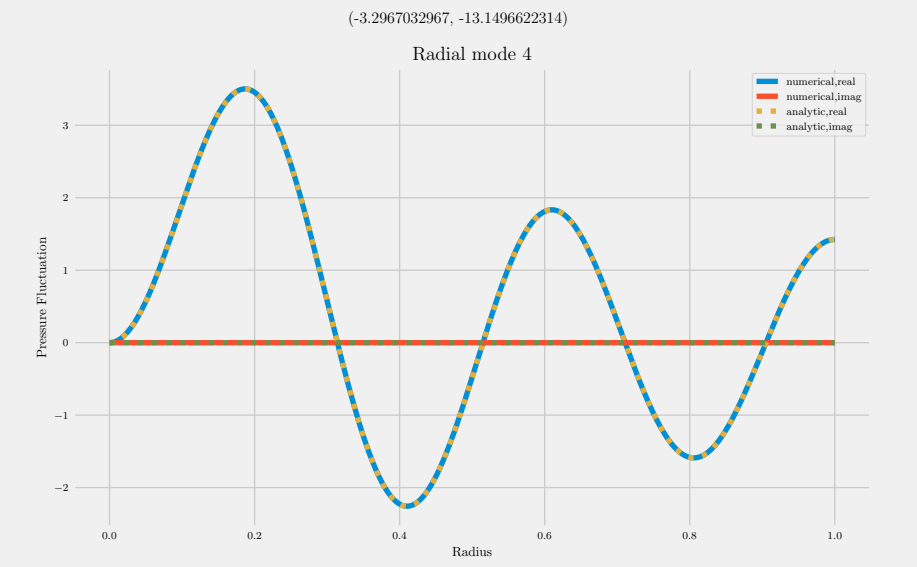

Figure 237: Fourth Order, Case number 0, 528points

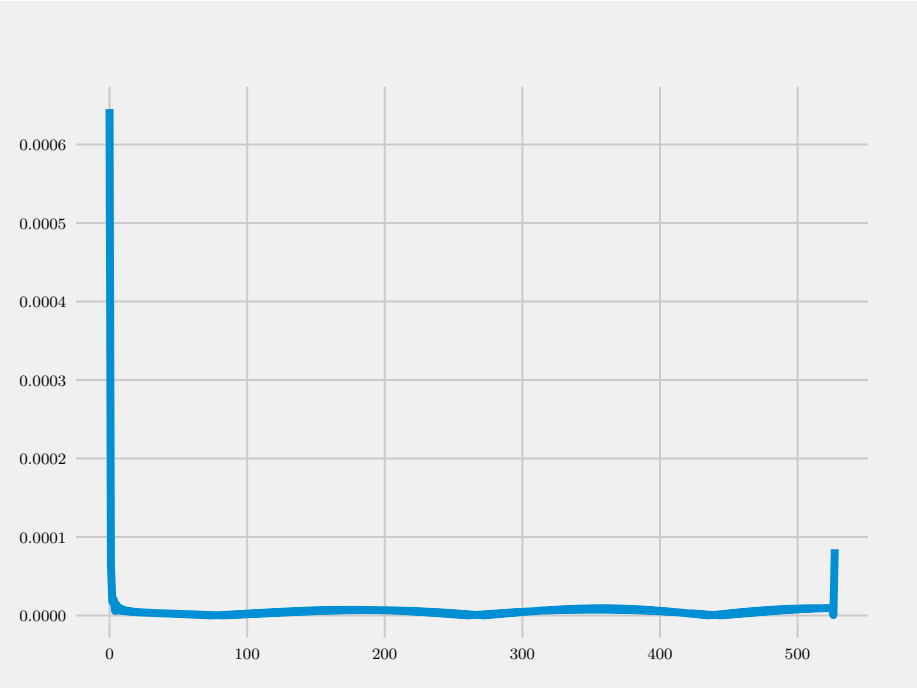

Figure 238: Fourth Order, Case number 0, 528points

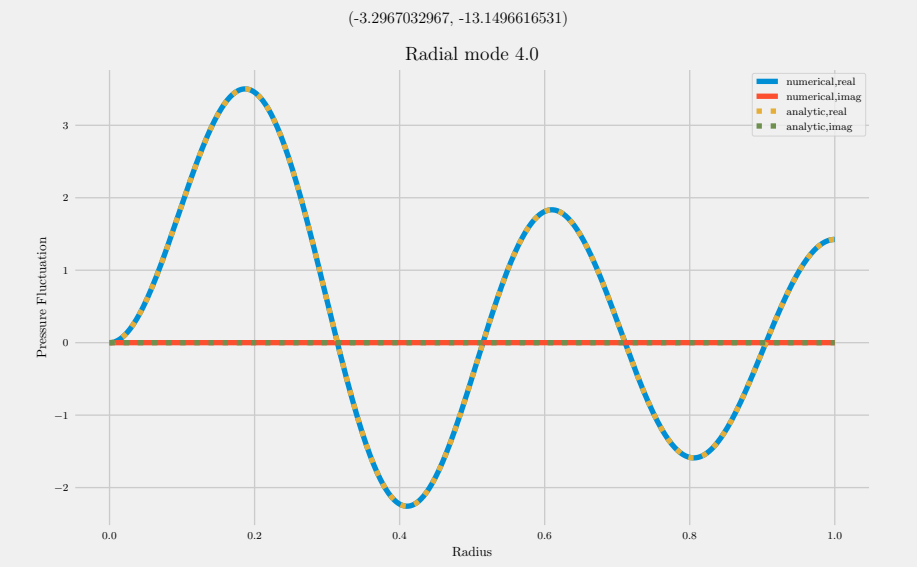

Figure 239: Fourth Order, Case number 0, 1056points

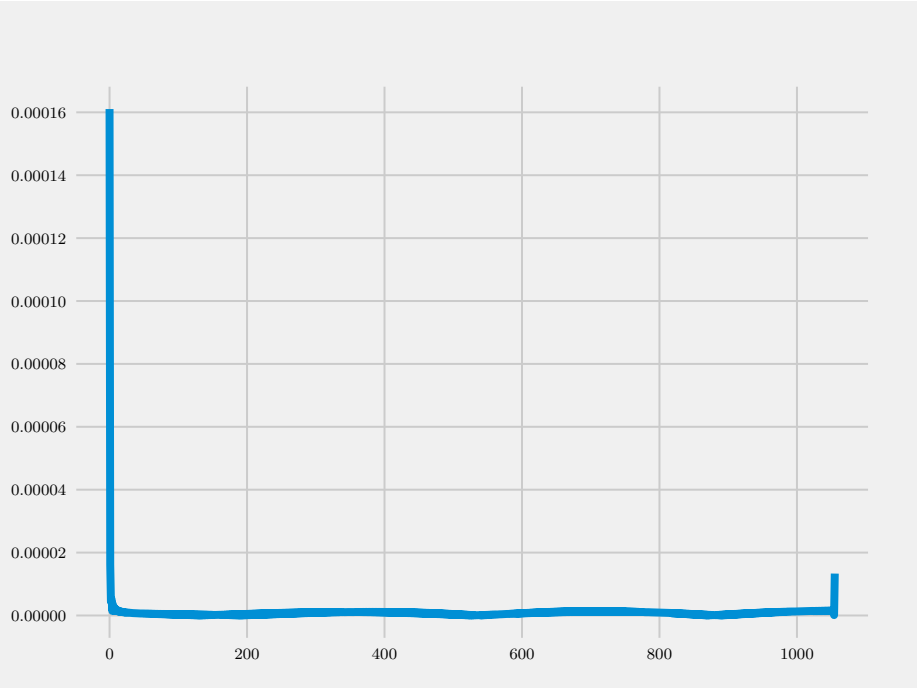

Figure 240: Fourth Order, Case number 0, 1056points

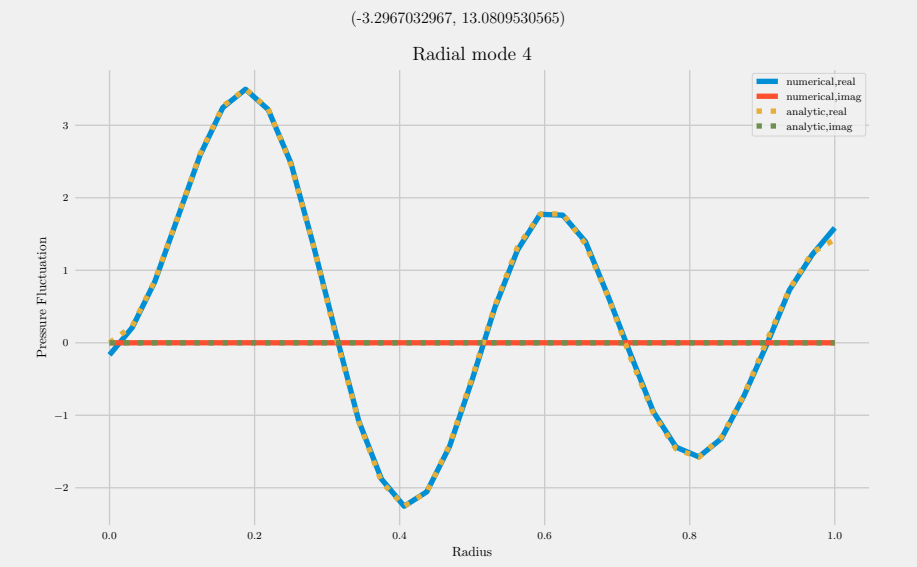

Figure 241: Fourth Order, Case number 1, 33points

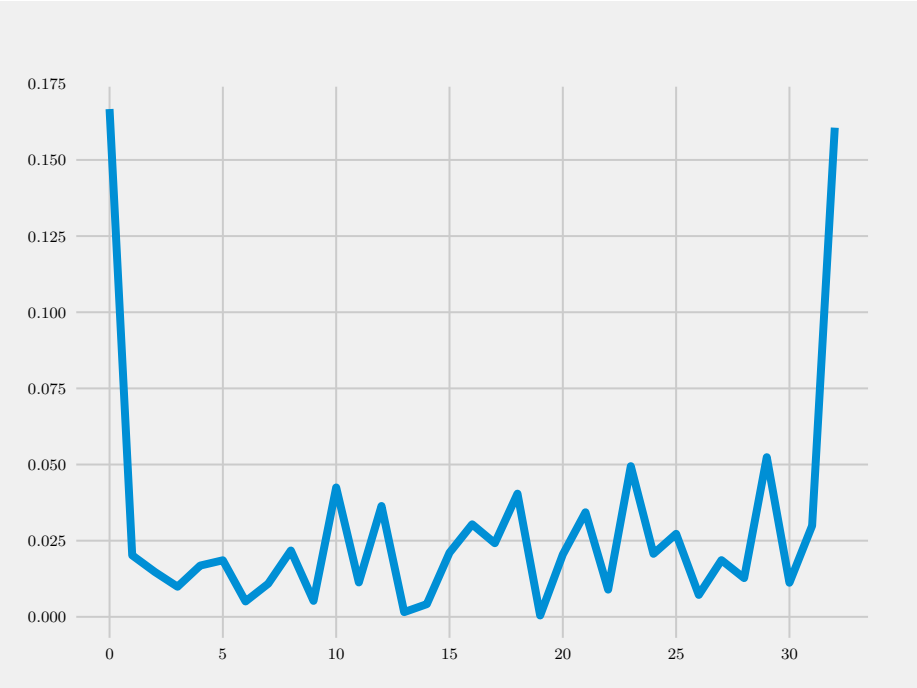

Figure 242: Fourth Order, Case number 1, 33points

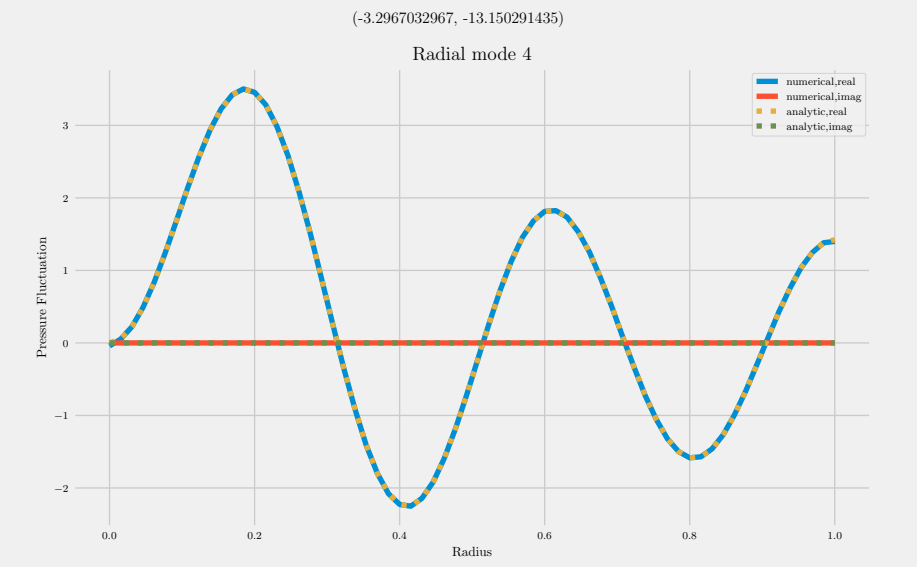

Figure 243: Fourth Order, Case number 1, 66points

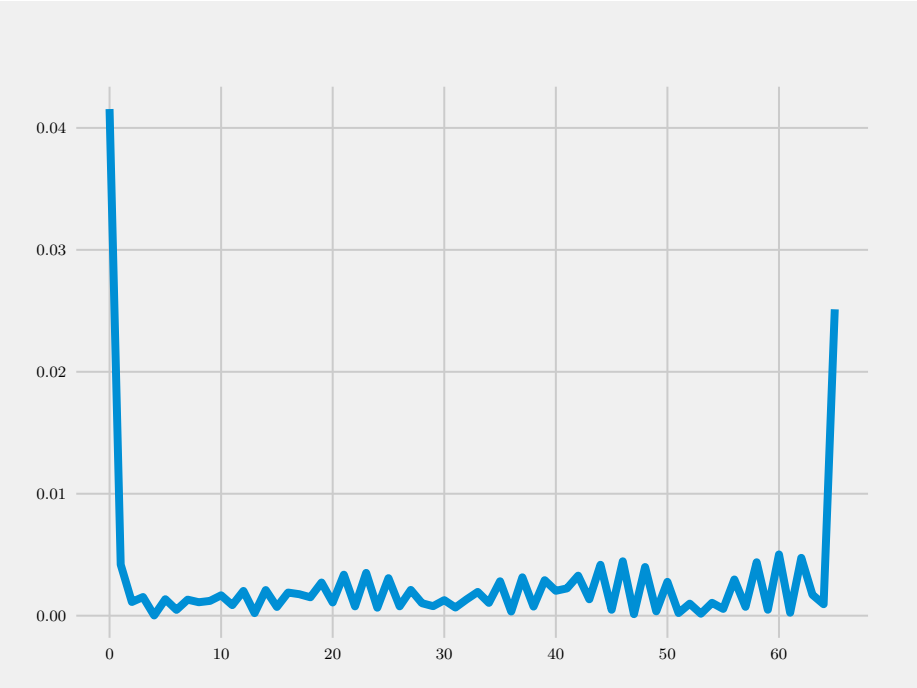

Figure 244: Fourth Order, Case number 1, 66points

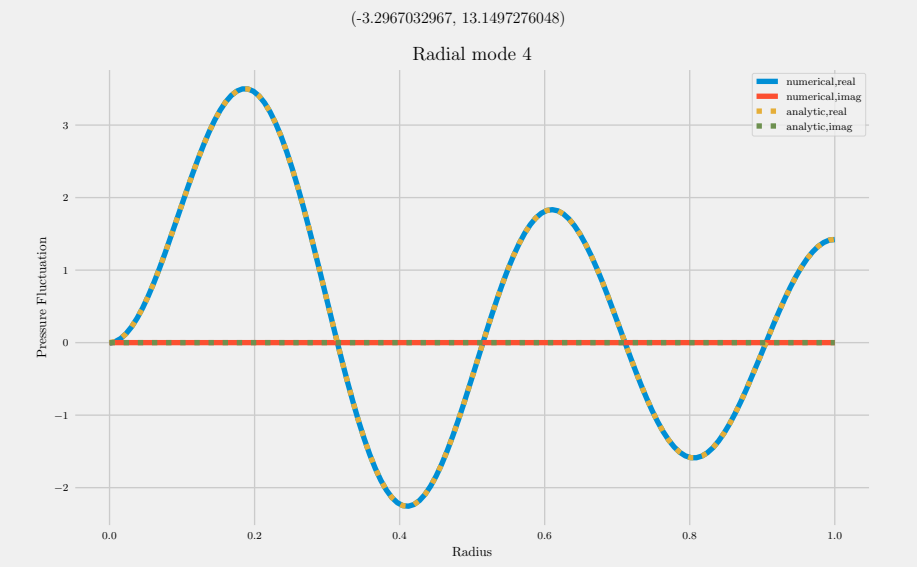

Figure 245: Fourth Order, Case number 1, 132points

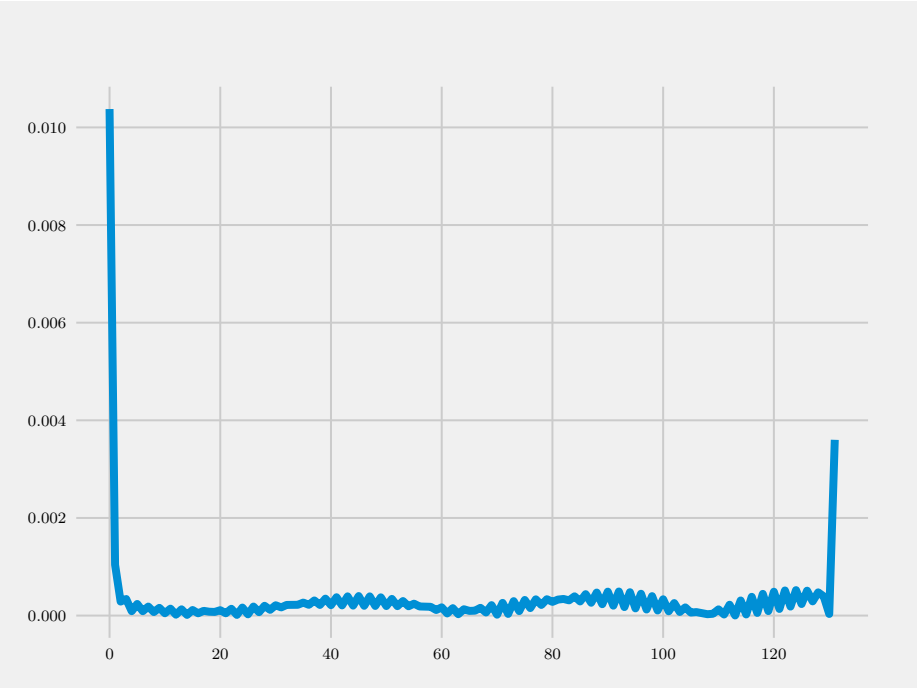

Figure 246: Fourth Order, Case number 1, 132points

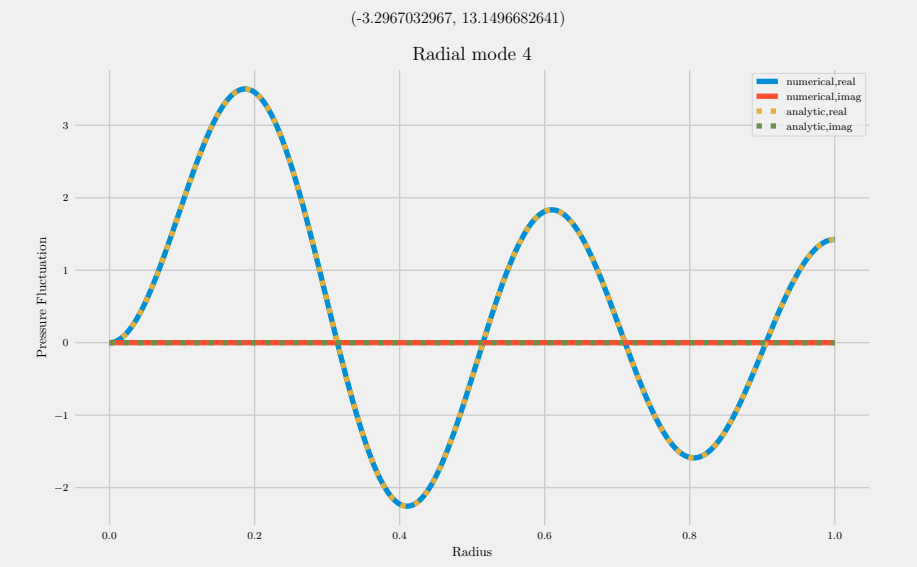

Figure 247: Fourth Order, Case number 1, 264points

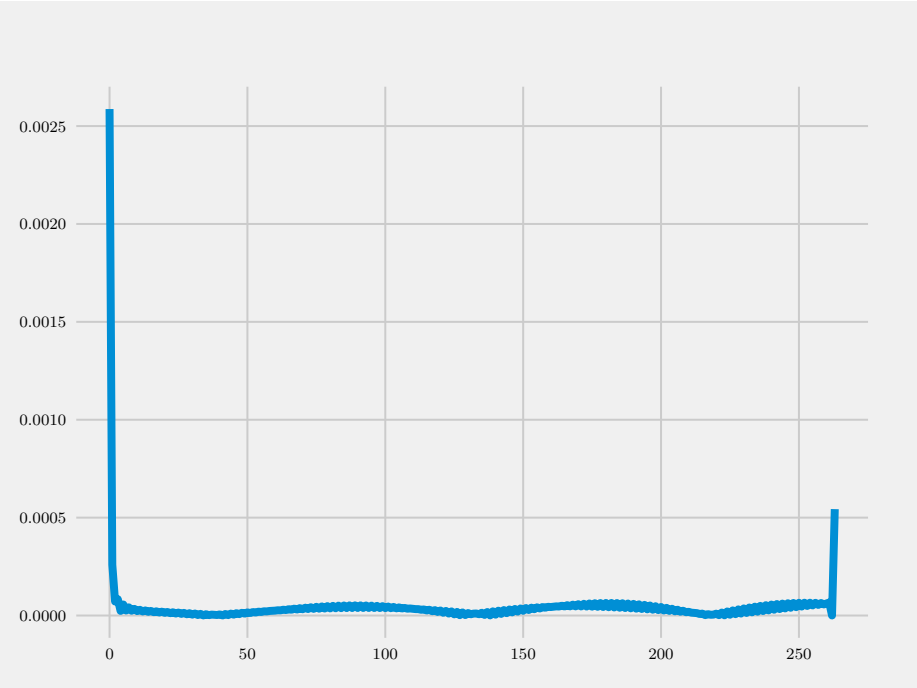

Figure 248: Fourth Order, Case number 1, 264points

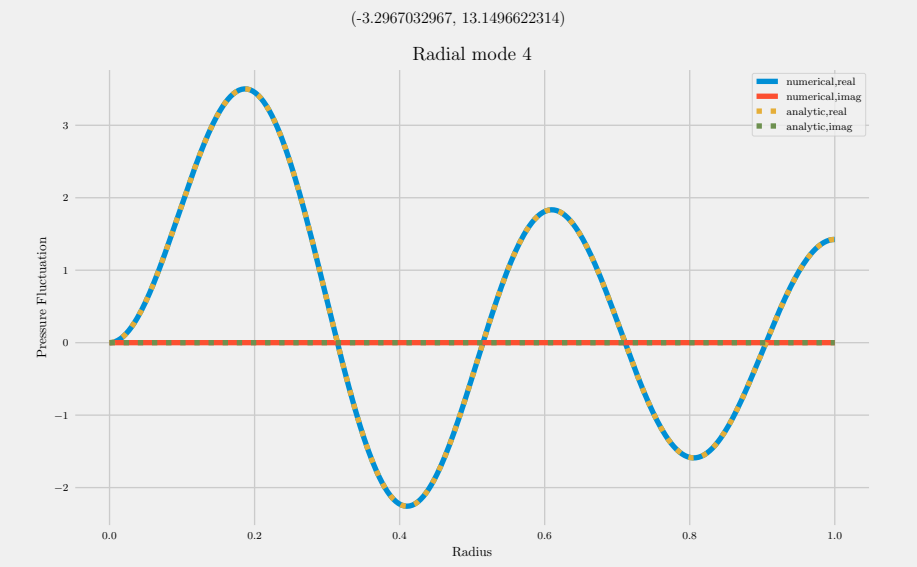

Figure 249: Fourth Order, Case number 1, 528points

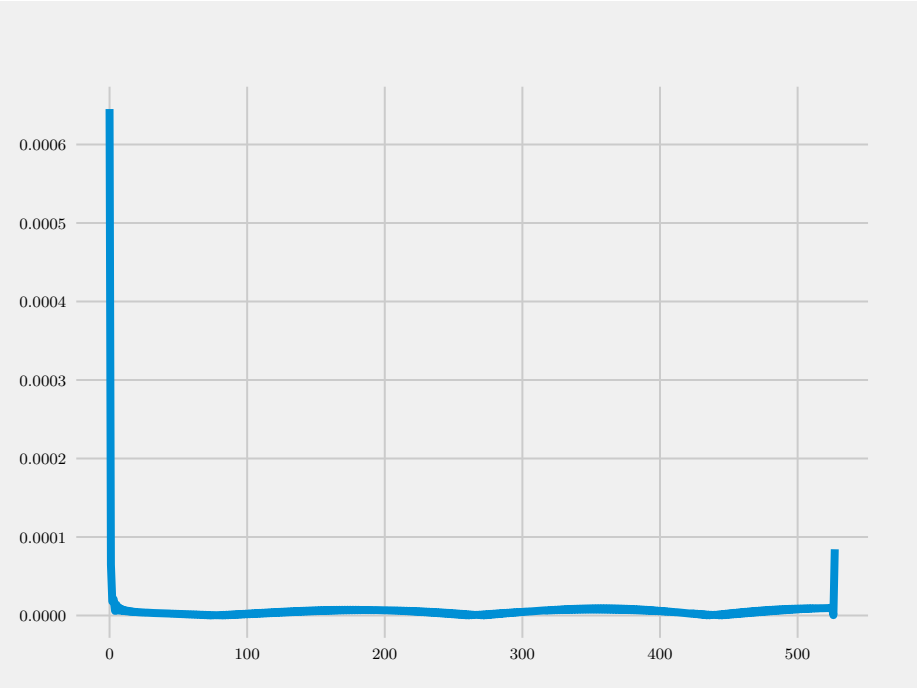

Figure 250: Fourth Order, Case number 1, 528points

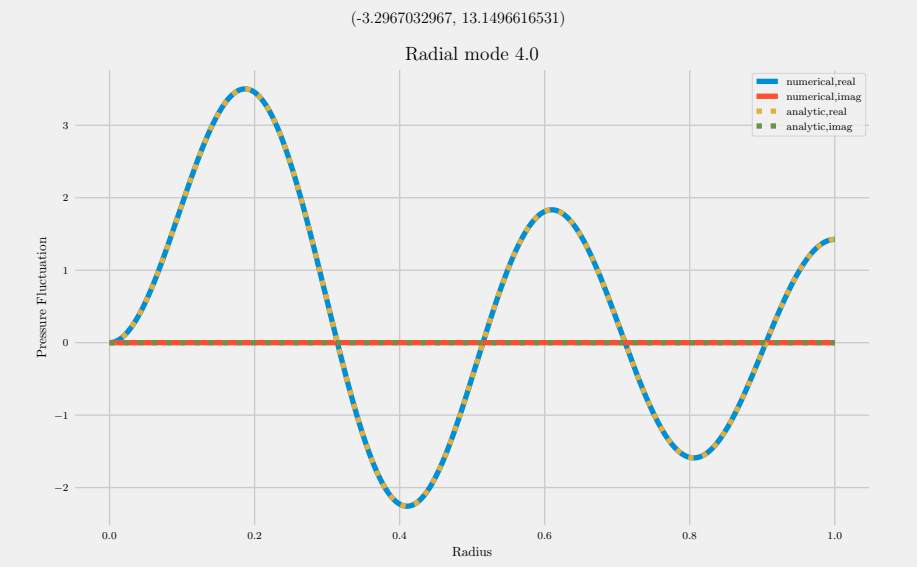

Figure 251: Fourth Order, Case number 1, 1056points

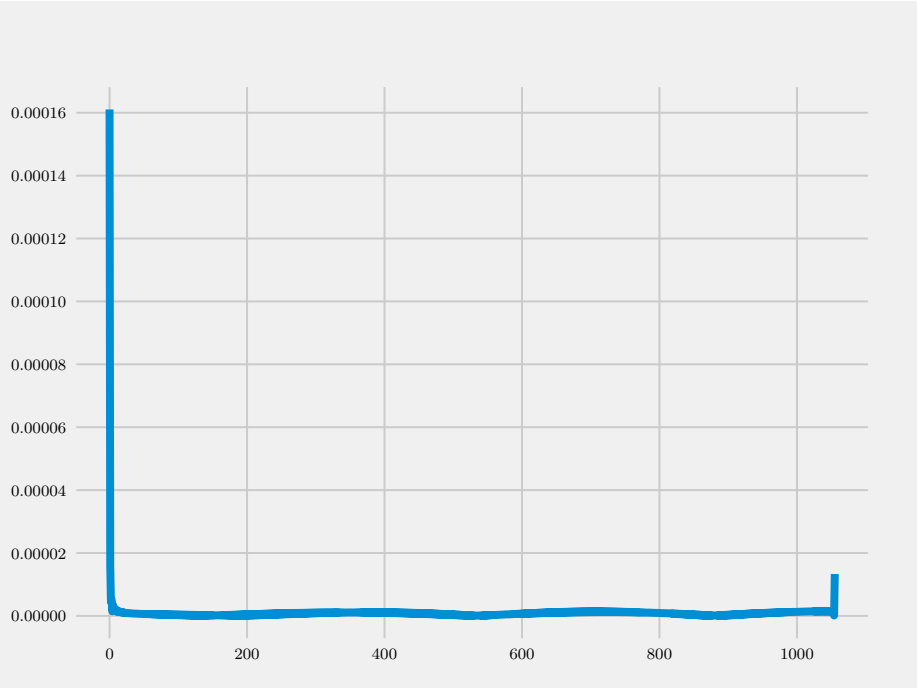

Figure 252: Fourth Order, Case number 1, 1056points

0.4 Discussion