Update Report

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1 Current Research Direction

The goal is to clarify what occurs when ducts are lined with sound absorbing material. Speciificaally, the description of acoustic liner in terms of the Linearized Euler Equations is investigated.

2 Research Performed

The acoustic absorbing material is called locally reacting and is often describes with "wall impedance" and will be denoted as $Z(\omega)$

The boundary condition is given as

$$i\omega p = -Z\frac{\partial p'}{\partial r}\mathrm{at}r = 1$$

A typical test case example is for the inlet of an aircraft turbojet engine. The general solution has a form similar to the hard wall case, except the eigenvalues k_r are now defined by

$$\frac{J_m(k_r)}{k_r J'_m(k_{r,mn})} = \frac{iZ}{\omega}$$

Which is related to k_x by

$$k_x^{\pm} = \pm\sqrt{k^2 - k_{r,mn}} = \pm\omega\zeta(k_{r,mn}/\omega)$$

While these "soft wall" modes are no longer orthoganal, the modes could be normalized in such a way that preserves the normalization relation,

$$\int_0^1 U_{mn}(r)\mathrm{conj}(U_{mn}(r))rd\Theta dr = 1$$

The normalization constan

$$N_{mn} = \frac{|Z|}{|J_m(k_{r,mn}|}\sqrt{\frac{Im(k_{r,mn}^2}{\omega Re(Z)}}$$

3 Issues and Concerns

The new function ζ is described in Rienstra's work in his appendix in a clever way...

He defines ζ as a function of an arbitrary number z.

$$\zeta(z) = \sqrt{1 - z^2}$$

where $Im(\zeta) \leq 0$, and $\zeta(0) = 1$

His description is,

" The sign choice is made such that always $Im(\zeta) \leq 0$. This implies that the branch cuts are located right along the curves where $Im(\zeta) = 0$. i.e. along the imaginary axis, and the real interval $|z| \leq 1$. For definiteness the branch cuts are included in the dommain of the definition as the limits from the $Re\zeta > 0$

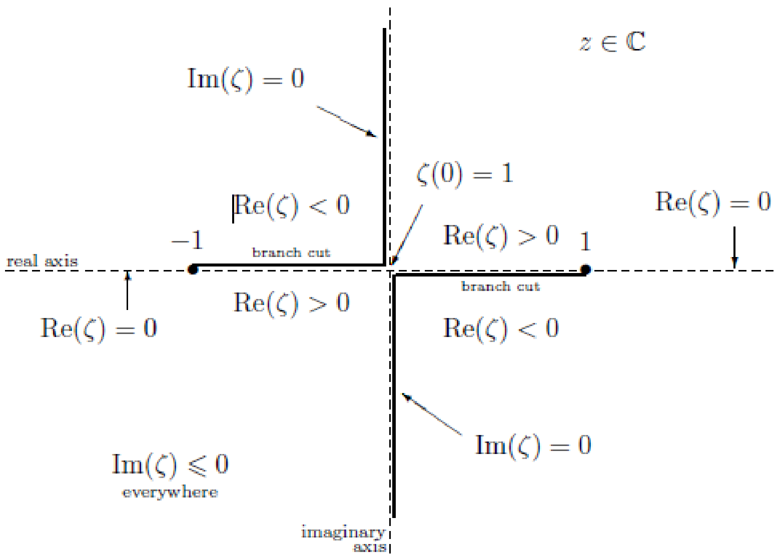

Figure 26: Branch cuts and signs of $\zeta(z) = \sqrt{1 - z^2}$ in complex z-plane.

Figure 1: Fiigure in appendix A.2 An Important Complex Square Root in Fundamentals of Duct Acoustics by Rienstra

side. If we take care along the branch cuts, where sign($Im\zeta$) is not defined, a simple (theres a footnote here) way to determine ζ is

$$\zeta(z) = -sign(Im\sqrt{1 - z^2})\sqrt{1 - z^2}$$

" There is a nice figure shown along with this description

4 Planned Research

I am slightly confused on how to use this information.. but my current understanding of this is that the value of Z changes which zero zrossings k_r are used.