1 SWIRL Verification cases

In the Kousen report, there are a number of verification cases presented. In this section, the inputs for running those cases are determined.

1.1 Cylinder, Uniform Flow with Liner (Table 4.3)

In Table 4.3, a comparison is shown for the case of a cylinder (no centerbody) with uniform flow. For this test case,

$$\begin{aligned} m &= 2 \\ k &= \frac{\omega r_T}{A_T} = -1 \\ M_x &= 0.5 \\ \eta_T &= 0.72 + 0.42i \end{aligned}$$

Recall that in the SWIRL code, A_T is the speed of sound at the outer radius, r_T. Also, η_T is the admittance of the liner on the outer wall. The azimuthal mode number is m.

The input deck for this test case is:

input.data.Table4.3

The resulting γ values are given in 'gam.acc'. The data from left to right are:

1. Eigenvalue ID number, j.
2. Real part of the nondimensional axial wavenumber $\overline{\gamma}_j$
3. Imaginary part of the nondimensional axial wavenumber $\overline{\gamma}_j$
4. Real part of $\frac{k}{\overline{\gamma}_j}$
5. Imaginary part of $\frac{k}{\overline{\gamma}_j}$
6. Real part of $\frac{\overline{\gamma}_j}{k}$
7. Imaginary part of $\frac{\overline{\gamma}_j}{k}$

$\gamma_n^\pm$	Kousen Ref. [15]	Kousen report	current	index
γ_0^+	$0.620 - 5.014i$	$0.6195 - 5.0139i$	$0.61954 - 5.01386i$	60
γ_1^+	$-5.820 - 3.897i$	$-5.8195 - 3.8968i$	$-5.81953 - 3.89677i$	58
γ_2^+	$0.445 - 9.187i$	$0.4453 - 9.1868i$	$0.44533 - 9.18684i$	59
γ_3^+	$0.453 - 13.062i$	$0.4539 - 13.062i$	$0.45389 - 13.0615i$	57
γ_4^+	$0.480 - 16.822i$	$0.4795 - 16.822i$	$0.47952 - 16.8216i$	55
γ_5^+	$0.503 - 20.531i$	$0.5029 - 20.531i$	$0.50287 - 20.5307i$	51
γ_6^+	$0.522 - 24.213i$	$0.5220 - 24.213i$	$0.52202 - 24.2129i$	50
γ_7^+	$0.538 - 27.880i$	$0.5376 - 27.880i$	$0.53754 - 27.8800i$	48
γ_8^+	$0.550 - 31.537i$	$0.5502 - 31.537i$	$0.55024 - 31.5368i$	47
γ_9^+	$0.589 - 49.75i$	$0.5891 - 49.754i$	$0.58745 - 49.7669i$	33
γ_0^-	$0.410 + 1.290i$	$0.4101 + 1.2904i$	$0.41009 + 1.29037i$	64
γ_1^-	$1.259 + 6.085i$	$1.2595 + 6.0852i$	$1.25949 + 6.08517i$	63
γ_2^-	$1.146 + 9.668i$	$1.1457 + 9.6679i$	$1.14567 + 9.66787i$	62
γ_3^-	$1.022 + 13.315i$	$1.0218 + 13.315i$	$1.02183 + 13.3150i$	61
γ_4^-	$0.943 + 16.977i$	$0.9425 + 16.977i$	$0.94250 + 16.9767i$	56
γ_5^-	$0.891 + 20.635i$	$0.8908 + 20.635i$	$0.89075 + 20.6353i$	54
γ_6^-	$0.855 + 24.288i$	$0.8549 + 24.288i$	$0.85490 + 24.2883i$	53
γ_7^-	$0.829 + 27.937i$	$0.8288 + 27.937i$	$0.82877 + 27.9369i$	52
γ_8^-	$0.809 + 31.581i$	$0.8089 + 31.581i$	$0.80891 + 31.5812i$	49
γ_9^-	$0.755 + 49.77i$	$0.7547 + 49.772i$	$0.75658 + 49.7851i$	39

Table 1: Table 4.3 data

(I have no idea why the last two columns are needed.)

The output in 'gam.acc' for this test case is:

gam.acc.Table4.3

Comparing to the data in Table 4.3 of the paper, the first eigenvalues are well predicted. However, there are a number of eigenvalues that lie between the tabulated results for γ_8^+ and γ_9^+, and between γ_8^- and γ_9^-.

1.2 Cylinder, Shear Flow (Table 4.4)

This test case is given in Ref. 10 by Shankar. The data corresponds to the results given in Table 1 of Ref. 10. This is a hard-wall duct ($\eta = 0$), with an axial Mach number profile given by:

$$
\begin{aligned}
b &= r_T - r_H \\
\widetilde{r} &= \frac{r}{b} \\
M\left(\widetilde{r}\right) &= M_0\left(1-\widetilde{r}\right)^{\frac{1}{7}} \\
M_0 &= 0.3 \\
m &= 0 \\
kb &= 20
\end{aligned}
$$

Next, we translate Shankar's notation into the SWIRL notation. Comparing Shankar and SWIRL,

$$
\begin{aligned}
p_n\left(r,x,t\right) &= a_n\phi_n\left(r\right)e^{ik(\beta_n x - ct)} \\
\widetilde{p}_n\left(r,x,\theta,t\right) &= p_n\left(r\right)e^{i(k_x x + m\theta - \omega t)}
\end{aligned}
$$

Translating this into SWIRL inputs:

$$
\begin{aligned}
(kb)_{Shankar} &= \frac{\omega}{A_T} r_T \\
&= k_{SWIRL}
\end{aligned}
$$

The input deck for this test case is:

input.data.Table4.4

For the cylinder case of Shankar, the results in Table 4.4 are for $\frac{\overline{\gamma}_n}{k}$ in order to compare with the β_n results:

The output in 'gam.acc' for this test case (using 16 points) is:

gam.acc.Table4.4.16pt

The output in 'gam.acc' for this test case (using 32 points) is:

gam.acc.Table4.4.32pt

1.3 Annulus, Shear Flow (Table 4.5)

This test case is given in Ref. 10 by Shankar. In order to get the correct input data,

$\gamma_n^\pm$	Shankar β_n	$\frac{\overline{\gamma}_n}{k}$ (16 points)	index	$\frac{\overline{\gamma}_n}{k}$ (32 points)	index
γ_0^-	0.81500	0.816009	45	0.815493	69
γ_1^-	0.76944	0.769595	47	0.769520	71
γ_2^-	0.72751	0.727753	46	0.727642	70
γ_3^-	0.65329	0.653542	44	0.653472	68
γ_4^-	0.54028	0.540499	43	0.540524	67
γ_5^-	0.36933	0.369465	42	0.369746	66
γ_6^-	0.06361	0.064613	41	0.064754	65
γ_7^-	$-0.28313 + 0.48807i$	$-0.278964 + 0.487173i$	39	$-0.282743 + 0.486526i$	63
γ_8^-	$-0.28357 + 0.80635i$	$-0.289882 + 0.854887i$	29	$-0.283251 + 0.804397i$	62
γ_9^-	$-0.28410 + 1.05658i$	$-0.263260 + 1.090093i$	27	$-0.283670 + 1.053847i$	57
γ_{10}^-	$-0.28410 + 1.27947i$	$-0.315161 + 1.655351i$	25	$-0.284027 + 1.275635i$	50

Table 2: Table 4.4 data

$$
\begin{aligned}
\widetilde{r}_i &= 6.0 \\
\frac{r_i}{b} &= 6.0 \\
\frac{r_i}{r_o - r_i} &= 6.0 \\
r_i &= 6r_o - 6r_i \\
7r_i &= 6r_o \\
\frac{r_i}{r_o} &= \frac{6}{7} \\
\sigma &= \frac{6}{7} \\
b &= \frac{1}{7} \\
kb &= 10 \\
k &= 70
\end{aligned}
$$

The axial Mach number profile is given by:

$$
\begin{aligned}
M\left(\frac{r}{b}\right) &= M_0\left(1 - 2\left|\frac{r_i - r}{b} + 0.5\right|\right)^{\frac{1}{7}} \\
M_0 &= 0.3
\end{aligned}
$$

$\gamma_n^{\pm}$	Shankar β_n	Kousen report	$\frac{\overline{\gamma}_n}{k}$ (current)	index
γ_0^-	0.79293	0.79353	0.793478	134
γ_1^-	0.75075	0.75292	0.752847	132
γ_2^-	0.57143	0.57320	0.573124	131
γ_3^-	-0.00969	0.16437	0.164294	130
γ_4^-	$-0.28733 + 0.73219i$	$-0.28357 + 0.73425i$	$-0.283637 + 0.734325i$	127
γ_5^-	$-0.29118 + 1.21721i$	$-0.28622 + 1.2198i$	$-0.286225 + 1.219841i$	125
γ_6^-	$-0.29248 + 1.62569i$	$-0.28766 + 1.6281i$	$-0.287553 + 1.628282i$	122
γ_7^-	$-0.29519 + 2.00221i$	$-0.28947 + 2.0055i$	$-0.289320 + 2.005620i$	120
γ_8^-	$-0.29567 + 2.36511i$	$-0.29035 + 2.3683i$	$-0.289988 + 2.368514i$	114
γ_9^-	$-0.29768 + 2.71665i$	$-0.29167 + 2.7209i$	$-0.291333 + 2.720846i$	112
γ_{10}^-	$-0.29776 + 3.06414i$	$-0.29243 + 3.0679i$	$-0.291720 + 3.068215i$	110

Table 3: Table 4.5 data

The input data for SWIRL is then:

input.data.Table4.5

The output in 'gam.acc' for this test case is:

gam.acc.Table4.5

The comparison with the original data is given in the table. Note that, in order to make sure of a valid comparison, 64 radial grid points were run.

In the data, note the propagating mode at index 133. This needs to be investigated further.

1.4 Lined Annulus, Shear Flow (Table 4.6)

This test case is given in Ref. 10 by Shankar. In order to get the correct input data,

$$\begin{aligned}
\tilde{r}_i &= 2.0 \\
\frac{r_i}{b} &= 2.0 \\
\frac{r_i}{r_o - r_i} &= 2.0 \\
r_i &= 2r_o - 2r_i \\
3r_i &= 2r_o \\
\frac{r_i}{r_o} &= \frac{2}{3}
\end{aligned}$$

$$\begin{aligned}\sigma &= \frac{2}{3} \\ b &= \frac{1}{3} \\ kb &= 10 \\ k &= 30\end{aligned}$$

The axial Mach number profile is again given by:

$$\begin{aligned}M\left(\frac{r}{b}\right) &= M_0\left(1-2\left|\frac{r_i-r}{b}+0.5\right|\right)^{\frac{1}{7}} \\ M_0 &= 0.3\end{aligned}$$

The admittance is given as:

$$\eta = 0.3+0.1i$$

It is not clear from Shankar whether both walls are lined or only one. Running the code for each case, it appears that the outer wall is lined and the inner wall is hard.

The input data for SWIRL is then:

input.data.Table4.6

The output in 'gam.acc' for this test case is:

gam.acc.Table4.6

The comparison with the original data is given in the table. Note that, in order to make sure of a valid comparison, 64 radial grid points were run (though the 32 point dataset looked good too).

In the data, note the (very nearly) propagating mode at index 137. This needs to be investigated further – are these modes real?

$\gamma_n^{\pm}$	Shankar β_n	Kousen report	$\frac{\overline{\gamma}_n}{k}$ (current)	index
γ_0^-	$0.78698 + 0.00400i$	$0.78093 + 0.00913i$	$0.787228 + 0.003757i$	136
γ_1^-	$0.73438 + 0.02541i$	$0.75079 + 0.03387i$	$0.735936 + 0.026332i$	135
γ_2^-	$0.55840 + 0.03148i$	$0.57267 + 0.03246i$	$0.560121 + 0.031371i$	134
γ_3^-	$0.14308 + 0.07638i$	$0.16875 + 0.06982i$	$0.143097 + 0.072855i$	131
γ_4^-	$-0.23900 + 0.74173i$	$-0.23734 + 0.72727i$	$-0.235625 + 0.743714i$	130
γ_5^-	$-0.26149 + 1.21973i$	$-0.25993 + 1.2120i$	$-0.257309 + 1.221354i$	129
γ_6^-	$-0.26996 + 1.62627i$	$-0.26860 + 1.6207i$	$-0.265988 + 1.627490i$	124
γ_7^-	$-0.27669 + 2.00192i$	$-0.27468 + 1.9983i$	$-0.272158 + 2.003568i$	118
γ_8^-	$-0.27974 + 2.36439i$	$-0.27813 + 2.3612i$	$-0.275516 + 2.365756i$	116
γ_9^-	$-0.28359 + 2.71568i$	$-0.28147 + 2.7139i$	$-0.278985 + 2.717521i$	114
γ_{10}^-	$-0.28502 + 3.06304i$	$-0.28361 + 3.0610i$	$-0.280811 + 3.064511i$	112

Table 4: Table 4.6 data