

A Thesis

entitled

Verification and Validation Method for
an Acoustic Mode Prediction Code for Turbomachinery Noise

by

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Submitted to the Graduate Faculty as partial fulfillment of the requirements for the
Masters of Science Degree in Mechanical Engineering

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Over the last 20 years, there has been an increase in computational fluid dynamic codes that have made numerical analysis more and more readily available, allowing turbomachine designers to create more novel designs. However, as airport noise limitations become more restrictive over time, reducing aircraft takeoff and landing noise remains a prominent issue in the aviation community. One popular method to reduce aircraft noise is using acoustic liners placed on the walls of the engine inlet and exhaust ducts. These liners are designed to reduce the amplitude of acoustic modes emanating from the bypass fan as they propagate through the engine. The SWIRL code is a frequency-domain linearized Euler equation solver that is designed to predict the effect of acoustic liners on acoustic modes propagating in realistic sheared and swirling mean flows, guiding the design of more efficient liner configurations. The purpose of this study is to validate SWIRL using the Method Of Manufactured Solutions (MMS). This study also investigated the effect of the integration and spatial differencing methods on the convergence for a given Manufactured Solution. In addition, the effect of boundary condition implementation was tested. The improved MMS convergence rates shown for these tests suggest that the revised SWIRL code provides more accurate solutions with less computational effort than the original formulation.

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List of Symbols

A	mean flow speed of sound
A_T	speed of sound at the duct radius
\tilde{A}	dimensionless speed of sound, $\frac{A}{A_T}$
D/Dt	material derivative, $\partial/\partial t + V \cdot \nabla$
D_N	derivative matrix using N points
$\mathbf{e}_x, \mathbf{e}_\theta$	s
k_x	perturbation axial wavenumber
k	reduced frequency, $\omega r_{max}/A_T$
m	number of nodal diameters, i.e. azimuthal mode number
M_x	axial Mach number
M_θ	tangential Mach number
P	mean pressure
p'	perturbation pressure
r	radial coordinate
r_{min}	hub radius, i.e. minimum radius
r_{max}	hub radius, i.e. maximum radius
\bar{r}	dimensionless radial coordinate, r/r_{max}
S	mean entropy
s'	perturbation entropy
t	time
\vec{V}	mean flow velocity vector
V	mean flow velocity
v'	perturbation flow velocity
V_x	axial component of mean flow velocity
V_θ	tangential component of mean flow velocity
v'_r	axial component of perturbation velocity
v'_x	axial component of perturbation velocity
v'_θ	tangential component of perturbation flow velocity
v_ϕ	phase velocity, $k/\bar{\gamma}$
v_g	group velocity, $dk/d\bar{\gamma}$
x	axial coordinate

Greek Symbols

$\bar{\gamma}$	dimensionless axial wavenumber, $k_x r_{max}$
Γ	free vortex strength
$\bar{\Gamma}$	$\Gamma/(r_T A_T)$
δ	Kronecker delta
η_H	hub acoustic liner admittance (at r_{min})
η_T	tip acoustic liner admittance (at r_{max})
Θ	circumferential/azimuthal coordinate
κ	ratio of specific heats
$\kappa_{m\mu}$	modal separation constant
λ	eigenvalue, $-i\bar{\gamma}$
μ	radial mode index
$\bar{\rho}$	mean density
ρ'	perturbation density
σ	hub-to-tip radius ratio, r_{min}/r_{max}
Ω	angular frequency for solid body swirl
$\bar{\Omega}$	$\Omega r_T/A_T$
ω	perturbation angular frequency

List of Abbreviations

CAA	Computational Aeroacoustics
CFD	Computational Fluid Dynamics
MMS	Method of Manufactured Solutions
TSM	Tanh Summation Method
NS	Navier-Stokes
RK	Runge-Kutta

Preface

Chapter 1

Introduction

1.1 Overview

1.1.1 Introduction of overall field and basic explanation of this topic (What)

Aircraft noise is one of the most environmentally detrimental consequences of commercial flight. Studies suggest that exposure to aircraft noise leads to diminished academic performance in youth, and could increase the risk of cardiovascular disease for populations close to airports [2]. Although the COVID-19 pandemic reduced air passenger traffic by 96% between 2019 and 2020, [3] the global aviation community has proven to be resilient during times of economic shock. The International Air Transport Association (IATA) has studied the resilience of the global air passenger markets after four notable shocks to the aviation economy; The impact of four notable events, (1979 oil shock, 2000-2001 dot-com bust, 9/11, and the 2008 financial crisis) was studied by a statistical analysis of the estimated 'passenger gap' from 1950-2014. Data shows that approximately 72% of the impact of the initial shock persists one year after the event and diminishes to just under one-fifth of the initial impact. Given these trends in air transportation, the need for aviation noise regulation persists and

will continue to be a concern so long as the aviation continues this rate of growth.

Since the dawn of commercial airlines in the early 20th century, the increased demand for aircraft transport introduced jet engines to support large cargo and passengers. Consequently, this rise in innovation resulted in high volume engine noise due to the frequency of flights. After 1975, efforts to reduce aircraft noise eliminated the noise pollution for 90% of the population [4]. However, given the rapid increase in aircraft movements and consequently increase in noise exposure to larger populations, the advancement in noise reduction technologies has only been moderately increasing, leaving a requirement for in aeroacoustic modeling techniques and treatment strategies to compete with the demand for quiet subsonic flight [5]. Between the 1950s and 1960's aero gas turbine designs shifted to higher by pass ratios with two or three shafts. The high by pass ratio (HBP) fan utilized multiple stages of fans and air streams [1]. The efficiency of these engines rose with the availability of materials that are able to cool flows passing over the turbofan, thus slowing the overall jet velocity but maintaining the efficiency of the engine. One of the most popular configurations is the geared turbofan (GTF), the largest contributor to the noise on modern aircraft has been the fan upon take-off and landing. Although the use of the GTF has reduced the noise emissions by 75% [6], further noise reduction technologies to mitigate the noise associated to the turbofans geometric configuration and operation speeds. One of the most common techniques to reduce fan noise is to include the use of acoustic treatment along the walls of the turbomachine's nacelle.

Due to the increase in high-bypass-ratio of turbomachines, the newest models of engines have a significantly larger diameter and a shorter nacelle, leaving less room to place acoustic treatments in regions where it will be effective [7]. Figure 1-1 shows the evolution of directivity for turbomachines as the use of HBR fans became more popular. As these engines continue to develop, an increased understanding of sound propagation within the interstage of the engine is going to be needed due to flow be-

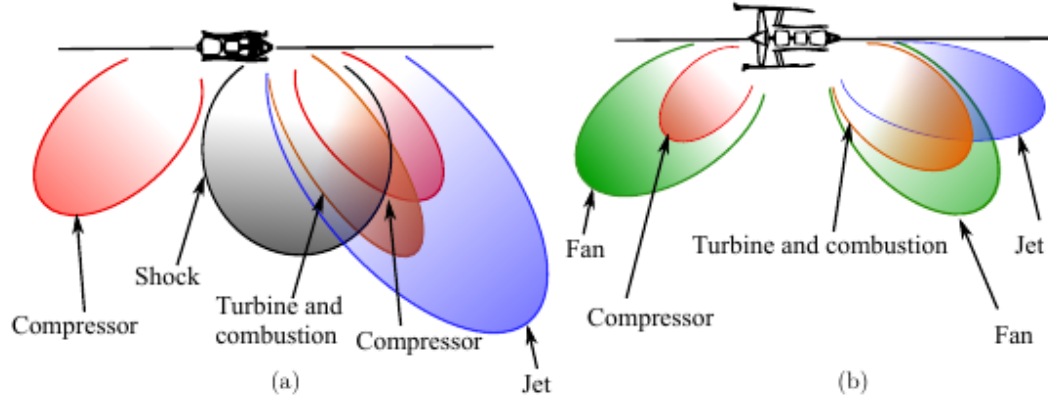


Figure 1-1: The evolution of the directivity and the relative levels of sources as a function of engine architecture (a) low bypass-ratio (b) high bypass ratio [1]

havior (high compressibility and rotational effects). While a turbomachine's general flow condition includes a series of axial, tangential, and radial velocity components that vary depending on the location of concern, the swirling flow between fan stages has been an area of interest due to the potential for acoustic treatment in a location previously avoided for its flow complexity. This work will explore how sound propagation is modeled and how the current state of code verification and validation (V&V) currently stands. This introduction will describe how fluid mechanics is utilized to establish an aeroacoustic model for various ducted flows. It will also discuss how code verification is used in the computational and numerical fields but will show the need for the use of these code verification techniques for a frequency domain CAA code.

1.2 Statement of the of the research questions

1.2.1 What is already known?

In general, jet engine designers can model flow within a turbomachine with the Navier Stokes (N-S) Equations, a set of partial differential equations that describe

the mass, momentum and energy of a given viscous fluid, however these equations can be computationally expensive as they are used in the most general cases. For aeroacousticians, the N-S equations can be too complicated to identify sound generation and propagation because acoustic waves are low amplitude (only a fraction of atmospheric pressure) and are not strongly influenced by viscosity. As a result, it is common in practice to utilize the Linearized Euler equations (LEE), a closely related set of PDEs that model inviscid fluid, as they provide an approximation for higher Reynold number flows where viscosity does not play a critical role. A popular approach to modeling sound propagation within a flow is to “linearize” the Euler equations, which decomposes the flow solution into a mean and fluctuating component. The decomposition is done in a linear fashion because the sound propagation amplitude is small with respect to the mean flow, and their presence does not appreciably change the mean flow field. The LEE provides a system of linear equations where for uniform flow, the solution is a family of wavenumbers and radial mode shapes that arise from unsteady disturbances for flows within a cylindrical duct. Another method decomposes the flow into vortical and potential parts [8]. In either case, this presents an initial value problem which in limited cases can obtain analytical solutions for simplified mean flow. Once a mean flow contains a tangential component, the LEE equations must be solved numerically.

1.2.2 What is missing?/Why is it a problem?

For uniform flows in a hard wall duct, the waves are categorized as vortical, entropical, and acoustic waves. The vortical and entropic waves solely convect with the mean flow, whereas the acoustic wave can propagate without damping or decay exponentially. However, for swirling flows, the waves are partially coupled and are not easily categorized due to an additional category of “nearly convecting” modes [9],[10]. Therefore, the families of waves must be found numerically [11] making the

ducted acoustic propagation in swirling flow a problem without an analytical solution but has a framework for a numerical solution.

1.2.3 The goal and significance of the investigation

Swirling flow has been a difficult problem to investigate in comparison to flows parallel to the wall domain of a duct [12] because of the lack of an analytical solution and thus cannot be described from a single convective wave equation. However, the solution for sheared mean flows was first presented by Goldstein [13],[14]. Various special cases of swirling flow (free vortex and solid body swirl) was examined in [15] [9], [10]. In recent years V & V has been done given the rise in technologies capable of experimentally measuring the acoustic modes within a turbomachine [16]. This work aims offer additional insight to the V & V process by expanding on techniques used in this field.

Computational codes are widely used in the field of aeroacoustics and concerted effort has recently been made to improve V & V techniques by Ingraham and Hixon [17, 18]. The question of getting the correct answer was a large focus of this work.

This thesis aims to continue this effort by conducting code verification and validation on a frequency domain Linearized Euler equation approximation on previous work done by Pratt & Whitney and NASA Glenn Research Center through technical contract [19, 20].

1.3 Definition of the terms (if needed)

The algorithms presented in this thesis have been written using FORTRAN 77 and has been updated to FORTRAN 90, and implemented into a code named SWIRL. The foundational work has been started by Pratt and Whitney by Kenneth Kousen [19] and continued by Dr. Ray Hixon. SWIRL analyzes axial flows with mean shear

and swirl in hard wall and acoustically lined ducts. The Method of Manufactured Solutions (MMS) is a process for generating an analytical solution for a code that provides the numerical solution for a given domain. The goal of MMS is to establish a manufactured solution that can be used to establish the accuracy of the code within question. For this study, SWIRL, a code used to calculate the radial modes within an infinitely long duct is being validated through code verification. SWIRL accepts a given mean flow and uses numerical integration to obtain the speed of sound. The integration technique is found to be the composite trapezoidal rule through asymptotic error analysis.

Prior work has largely depended on conducting code validation through exact solutions to the partial differential equation (PDE), the convective wave equation. Such a procedure formally known as the method of exact solutions (MES) [21]. MES has the advantage in that the output of SWIRL can be directly compared to the solution of the PDE, however available solutions are limited for sound propagation in ducted flows.

1.4 Organization of research (Structure)

The literature review in the next chapter will discuss the governing equations that are used to predict the acoustic behavior in a fluid flowing internally, followed by the research problem that arises in swirling flow, the objectives and questions, the significance and finally the limitations. The proposed research aims to determine the impact of the numerical schemes used in the swirling flow problem and how it affects the family of waves that are produced from the problem formulation so a better understanding of the acoustic phenomena as the flow under goes a compressible rotational flow. The use of the method of manufactured solutions is used as a means of ensuring the code is correctly approximating the governing equations and will

check the effect of the numerical schemes on the axial wavenumbers produced.

1.5 Research Questions and Hypothesis

The use of numerical approximations are powerful for cases where there is no analytical solution, thus leaving multiple ways of arriving at a numerical solution. The use of MMS allows the user to obtain various metrics such as the approximated asymptotic rate of convergence, or the grid convergence index, to identify if the code is performing as intended , i.e. using the right equations.

The goal of this research is to conduct component level code verification tests for the problem of characterizing the duct acoustics for flow using a LEE model by looking at the two numerical components for SWIRL.

The goal o The proposed component verification from Kleb and Wood will be presented to address the characterization of modes and the presence of numerical ones.

Hypothesis. further improve the result.

Chapter 2

Chapter 2: Literature Review

2.1 Introduction

2.1.1 The problem to be addressed and its significance

The impact of swirling flow has been , there has been a concerted effort to model sound propagation under such conditions. This has lead to a number of publications that attempt to discern the acoustic signature of swirling flow through a cylindrical domain [9] , [10], [19], [20], [8] ,[22],[23], [24], [25], [26], [12], [27], [28], [29].

2.1.2 Establish reasoning - i.e. point - of - view for reviewing the literature

A large amount of aircraft noise was reduced from 1975-2000, effectively eliminating the noise pollution for 90% of the population [4]. Since the early 2000's, the advancement in noise reduction technologies has been gradual, leaving a requirement for drastic improvement in aeroacoustic treatment strategies to compete with the demand of quiet subsonic flight. A previous theoretical review by Envia has been suggested that a non linear time domain computation could capture the source gener-

ation (incident turbulence) in addition to the broadband noise. Such a process would have the capabilities of solving all components of noise generation in an individual calculation [11]. This would at minimum, require an “LES-type” fidelity code, which can tend to be computationally expensive. A promising option is NASA GRC’s Broadband Aeroacoustic Stator Simulation (BASS). BASS is a high-order, high accuracy computational aeroacoustics (CAA) code which has been used to study non linear provides mean an extensive study of non linear phenomena in turbomachinery flow, and in particular, mechanisms of noise generation that are produced from unsteady disturbances. This code allows for a wide variety of finite differencing and time marching schemes as well as artificial dissipation methods. This effective computational tool allows for an in-depth modeling of realistic velocity profiles that would be representative of flow produced from a rotor blade row. In recent work, a new method of implementing realistic, three-dimensional rotor wakes free from acoustics was validated [30]. This provides a means of studying acoustic responses non linear swirling flows within pragmatic geometric configurations, while simultaneously allow for the modeling of sources generation produced from the incident turbulence.

Over the last 40 years, several studies have investigated the impact of swirling flow on the sound propagation with the use of the 3D linearized Euler equations, directly solving the unsteady equations as opposed to approximating the solutions to both steady and unsteady flow problems. Another NASA code, LINFLUX predict acoustic disturbances from blade movements in subsonic flows. While LINFLUX demonstrates capabilities to model three dimensional steady and unsteady flows, the computational time is larger compared to lower fidelity codes that only compute the unsteady portion of the problem. The benefit of such a code is beneficial to engine and liner designers who are interested in a wide range of configurations, requiring a parametric study. SWIRL [20], is a code that conducts an eigenmode analysis by assuming a constant radius annular or cylindrical duct with acoustically lined walls

using the linearized unsteady Euler equations. Such a code is needed to identify the modal content and can be used as a boundary condition with LINFLUX by using a mode matching technique. The goal of this theoretical review is to see the current state of low-fidelity frequency domain LEE codes and where the foundational model differs depending on the problem formulation.

2.1.3 Explain the order/sequence of the review

This review will compare studies that have considered the applicability of unsteady linearized Euler equations on cylindrical and annular ducts. First the work of Kousen will be briefly summarized [20, 19]. Secondly, the alternate approach shown by Golubev and Atassi used in [22, 23] were also compared. These works have utilized a standard normal mode approach to determine the modal response of inviscid, compressible, swirling flow within a cylindrical and annular ducts. Results and findings have revealed three categories of associated wave - modes, acoustic, nearly convected, and nearly sonic. Detailed examination of the literature indicate that the hierarchical system was not initially apparent. A qualitative description of the numerical methods used to evaluate the eigensystem of solutions will be described. The use of the Method of Manufactured Solutions will also be described. While its use is widespread in the verification and validation community, very little aeroacoustic codes utilize the MMS to apply code verification. This review will discuss the procedures and guidelines associated with the MMS, and some specific measures that were taken to ensure that the guidelines were met and that the suggestions used in literature were expanded upon.

2.1.4 The theoretical foundation of conceptual framework

One key question that remains, is how well do unsteady linearized equations capture the mechanisms of noise generation within realistic turbomachinery flow? There exists a copious amount of published work describing the use of wave equations to describe the governing behavior of ducted sound propagation. However, turbomachines inherently rely on high velocities, high temperatures to maximize efficiencies. Such mechanisms need a more thorough formulation so that a complete acoustic description of the flow can be provided.

To begin exploring various geometries, we must first ask: what are the central theories that have been used to explain the acoustic behavior of unsteady swirling flows? Do the linearized Euler equations provide an adequate means of capturing the mechanisms of noise generation within realistic turbomachinery flow? The study was expanded by [10, 31] who included cases of solid body swirl. Wundrow studied the swirling potential flows using Goldstein’s disturbance velocity decomposition [13] using a numerical approach and found that the solutions were more accurate and found more efficiently [10]. Kousen expanded these efforts by including [19, 20] the effect unsteady disturbances in the presence of forced solid body-swirl and free-vortex flow without the use of potential flow theory. Using normal mode analysis along with a radial spatial differencing scheme, the wave modes produced within cylindrical and annular ducts was reported. Results show (figure 4.7) two distinct families of modes, purely convected and acoustic modes. The presence of axial shear in combination solid body flows can endue coupling between these modes, which in theory would not appear unless viscous terms were present in the eigenvalue analysis. Lack of available swirl flow results “hampered” validation attempts. However, this eigenvalue approach was utilized with a new quasi-3D formulation to find the axial wavenumbers from solid body flow was proposed in his next work .

In [20], A quasi 3D formulation was validated and used to predict the appearance of modes due to the interaction of a rotor with spatially uniform steady and unsteady flow. These modes were first classified by [32] as “spinning modes”. In addition, further investigation was done on the results shown in [20]. The axial wavenumbers that were previously found to be purely convective were shown to be in part , “nearly convective” (shear) pressure modes. These were found to propagate in the axial direction with no loss in amplitude, thus never satisfying the cut-off condition.

2.2 Review of the assessment of the numerical techniques

Kousen assessed the accuracy of the numerical discretization technique for a series of test cases using three sources [33] [34] and [35] ([10] [15] and [45] in Kousen’s work [19] respectively). The results were assessed by using various literature comparisons. The methodology was presented of uniform mean axial flows, but results for hard wall cases were presented by computing the order of accuracy for the first four radial modes (See Figures 4.1-4.4), . (Explain why this is MES and offer MMS as a source of verification and explain that MES is validation [36] [37]). For a uniform axial flow, the axial wavenumbers can be computed from an analytical solution, where one of the key input parameters are the zero crossings of the derivative of the Bessel Function of the first kind . The values are presented [38] and are often referred to as separation constants for a circumferential and radial mode pair ; which are needed to compute the solution of the convective wave equation.

The axial wavenumber is found by using second-order differential convective wave equation for pressure using a fourth order accurate Runge-Kutta(RK) method [20] which was done to check against the results in [39] (Table 4.1 and 4.2 in [20]). The output parameter was γ/k . Each axial wave number can then be used to compute

the analytical radial mode using the exponential assumption. Axial wavenumbers from annular and cylindrical ducts with lined walls were compared to findings from Astley and Eversman [35] for uniform and sheared axial flow with liner (Table 4.3). The results taken were from a “high-order” RK scheme used. Axial wavenumbers from cylindrical ducts with hard walls were compared to findings from Shankar [33] in Table 4.4 of [20].

In recent years Maldonado et. al, [16] has made significant contributions in solution verification given the recent improvements in experimental measurement techniques. The work has presented test cases for lined ducts that have been compared to Kousen [20], Nijboer [25] and Peake [28] and show excellent comparison. The goal of this work is to contribute these efforts by conducting the method of manufactured solutions to offer clarity in using techniques often used in other verification and validation (V&V) studies.

2.2.1 The research questions, hypotheses, foreshadowed problems, or conjectures

While these results confirm the findings of SWIRL and other LEE codes, this does not check if the equations that were programmed were entered correctly. While an emphasis on solution verification is vital, it should be coupled with code verification to determine the robustness and consistency of the algorithm. The method of manufactured solutions combined with order of accuracy verification is often used as a gold standard of code verification [36] and has been shown to provide an estimate of discretization error that can be computed before the final answer is obtained. Since the mid 2000’s, code verification literature has grown popular in the field of computational mathematics and physics due to its ability to conduct tests for numerical approximations of partial and ordinary differential equations. The MMS offers

a means of “manufacturing” an arbitrary solution by define functions for each term in the governing equation. Common practices and guidelines are offered in [37] to choose the functions, but are phrased such that MMS can be widely applied. Since various numerical problems are unique in their treatment of spatial discretization, and boundary conditions, this work will describe the use of these guidelines and the nuances that have been taken to check the boundary condition and radial derivatives used in SWIRL.

Knupp in Code Verification by the MMS [37] provides guidelines for creating manufactured solutions which states,

1. The manufactured solutions should be composed of smooth analytic functions
2. The manufactured solutions should exercise every term in the governing equation that is being tested,
3. The solution should have non trivial derivatives.
4. The solution derivatives should be bounded by a small constant. In this case this constant should prevent the function from becoming greater than one.
5. The solution should not prevent the code from running
6. The solution should be defined on a connected subset of two- or three- dimensional space to allow flexibility in choosing the domain of the PDE.
7. The solution should coincide with the differential operators of the PDE. For example, the flux term in Fourier’s law of conduction requires T to be differentiable.

The goal is then to calculate an observed order of accuracy.

2.2.2 Calculation of Observed Order-of-Accuracy

To begin the method of manufactured solutions, the discretization error, ϵ is defined as a function of radial grid spacing, Δr

$$\epsilon = \epsilon(\Delta r)$$

The discretization error in the solution should be proportional to $(\Delta r)^\alpha$ where $\alpha > 0$ is the theoretical order for the computational method. The error for each grid is expressed as

$$\epsilon_{M_\theta}(\Delta r) = |M_{\theta,Analytic} - M_{\theta,calc}|$$

where $M_{\theta,Analytic}$ is the tangential mach number that is defined from the speed of sound we also defined and the $M_{\theta,calc}$ is the result from SWIRL. The Δr is to indicate that this is a discretization error for a specific grid spacing. Applying the same concept to to the speed of sound,

If we define this error on various grid sizes and compute ϵ for each grid, the observed order of accuracy can be estimated and compared to the theoretical order of accuracy. For instance, if the numerical soution is second-order accurate and the error is converging to a value, the L2 norm of the error will decrease by a factor of 4 for every halving of the grid cell size. Since the input variables should remain unchanged, the error for the axial and tangential mach number should be zero. As for the speed of sound, since we are using an analytic expression for the tangential mach number, we know what the theoretical result would be from the numerical integration technique as shown above. Similarly we define the discretization error for the speed of sound.

$$\epsilon_A(\Delta r) = |A_{Analytic} - A_{calc}|$$

For a perfect answer, we expect ϵ to be zero. Since a Taylor series can be used to derive the numerical schemes, we know that the truncation of higher order terms is what indicates the error we expect from using a scheme that is constructed with such truncated Taylor series.

The expectation is that the error at each grid point j to satisfy the following,

$$0 = |A_{Analytic}(r_j) - A_{calc}(r_j)|$$

$$\tilde{A}_{Analytic}(r_j) = \tilde{A}_{calc}(r_j) + (\Delta r)^\alpha \beta(r_j) + H.O.T$$

where the value of $\beta(r_j)$ does not change with grid spacing, and α is the asymptotic order of accuracy of the method. It is important to note that the numerical method recovers the original equations as the grid spacing approached zero.

Subtracting $A_{Analytic}$ from both sides gives

$$A_{calc}(r_j) - A_{Analytic}(r_j) = A_{Analytic}(r_j) - A_{Analytic}(r_j) + \beta(r_j)(\Delta r)^\alpha$$

$$\epsilon_A(r_j)(\Delta r) = \beta(r_j)(\Delta r)^\alpha$$

To estimate the order of accuracy of the accuracy, the global errors are calculated by taking the L2 Norm of the error which is denoted as $\hat{\epsilon}_A$

$$\hat{\epsilon}_A = \sqrt{\frac{1}{N} \sum_{j=1}^N \epsilon(r_j)^2}$$

$$\hat{\beta}_A(r_j) = \sqrt{\frac{1}{N} \sum_{j=1}^N \beta(r_j)^2}$$

As the grid density increases, $\hat{\beta}$ should asymptote to a constant value. Given two grid densities, Δr and $\sigma\Delta r$, and assuming that the leading error term is much larger than any other error term,

$$\begin{aligned}\hat{\epsilon}_{grid1} &= \hat{\epsilon}(\Delta r) = \hat{\beta}(\Delta r)^\alpha \\ \hat{\epsilon}_{grid2} &= \hat{\epsilon}(\sigma\Delta r) = \hat{\beta}(\sigma\Delta r)^\alpha \\ &= \hat{\beta}(\Delta r)^\alpha \sigma^\alpha\end{aligned}$$

The ratio of two errors is given by,

$$\begin{aligned}\frac{\hat{\epsilon}_{grid2}}{\hat{\epsilon}_{grid1}} &= \frac{\hat{\beta}(\Delta r)^\alpha}{\hat{\beta}(\Delta r)^\alpha} \sigma^\alpha \\ &= \sigma^\alpha\end{aligned}$$

Thus, α , the asymptotic rate of convergence is computed as follows

$$\alpha = \frac{\ln \frac{\hat{\epsilon}_{grid2}}{\hat{\epsilon}_{grid1}}}{\ln(\sigma)}$$

For example, a doubling of grid points has $\sigma = 1/2$,

$$\alpha = \frac{\ln(\hat{\epsilon}(\frac{1}{2}\Delta r)) - \ln(\hat{\epsilon}(\Delta r))}{\ln(\frac{1}{2})}$$

2.3 Conclusion

This review discusses the development of the unsteady linearized equations, and how improvements in the modal analysis capture more families of mechanisms of noise generation within non-uniform swirling flow turbomachinery flow. While the literature presented offers a measure of verification and validation through the use of the Method of Exact Solutions, the Method of Manufactured Solutions offers a level of code verification which allows for error checking by computing the approximate order of accuracy for a given numerical scheme, which is independent of the final answer, which in this case is the axial wavenumber. The next chapter will outline the methodology and techniques used when applying MMS to SWIRL. The methods consist of unique treatment of boundary conditions using fairing functions as well as an example of using a summation to generate arbitrary functions as manufactured solutions which has the dual benefit of giving a large number of derivatives but allows for high gradients in specific locations along the domain of the MS. The use of open-source widely available functions in Python were used to symbolically create the MS and then used to generate FORTRAN code that will compute the MS for code comparison.

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