

# Documentation for the SWIRL code

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# 1 Introduction

This writeup is my notes for the SWIRL code, based on the Kousen technical report and on the code itself.

## 2 Notes on the Kousen report

From the Summary: there are two types of eigenvalues: first, the acoustic modes (discrete values). Second, the vortical modes, which are convecting with phase velocities that 'correspond to the local mean flow velocities'.

From the Introduction: when the mean flow is nonuniform (swirling or not), there is a question about whether an exponential description of the axial disturbance is correct or not. The SWIRL code assumes an exponential function for the axial variation.

## 3 Aerodynamic Model

The cylindrical coordinate Euler equations are:

$$\begin{aligned}\frac{\partial \rho}{\partial t} + v_r \frac{\partial \rho}{\partial r} + \frac{v_\theta}{r} \frac{\partial \rho}{\partial \theta} + v_x \frac{\partial \rho}{\partial x} + \rho \left( \frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x} \right) &= 0 \\ \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_x \frac{\partial v_r}{\partial x} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} \\ \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_x \frac{\partial v_\theta}{\partial x} &= -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} \\ \frac{\partial v_x}{\partial t} + v_r \frac{\partial v_x}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_x}{\partial \theta} + v_x \frac{\partial v_x}{\partial x} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial t} + v_r \frac{\partial p}{\partial r} + \frac{v_\theta}{r} \frac{\partial p}{\partial \theta} + v_x \frac{\partial p}{\partial x} + \gamma p \left( \frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_x}{\partial x} \right) &= 0\end{aligned}$$

### 3.1 Mean Flow

For a steady axisymmetric mean flow with no radial velocity, the Euler equations simplify to:

$$\begin{aligned}
\frac{\partial}{\partial x} (\bar{\rho} V_x) &= 0 \\
-\frac{V_\theta^2}{r} &= -\frac{1}{\bar{\rho}} \frac{\partial P}{\partial r} \\
\frac{\partial}{\partial x} (V_\theta) &= 0 \\
\frac{\partial}{\partial x} (V_x) &= 0
\end{aligned}$$

The radial momentum equation can be integrated to obtain the mean pressure distribution:

$$P = \int_{r_{min}}^{r_{max}} \frac{\bar{\rho} V_\theta^2}{r} dr$$

Defining

$$\begin{aligned}
\tilde{r} &= \frac{r}{r_{max}} \\
dr &= \frac{\partial r}{\partial \tilde{r}} d\tilde{r} \\
&= r_{max} d\tilde{r}
\end{aligned}$$

and substituting in gives:

$$P(\tilde{r}) = P(1) - \int_{\tilde{r}}^1 \frac{\bar{\rho} V_\theta^2}{\tilde{r}} d\tilde{r}$$

For reference, the minimum value of  $\tilde{r}$  is

$$\sigma = \frac{r_{min}}{r_{max}}$$

For isentropic flow,

$$\begin{aligned}
S &= \frac{P}{\bar{\rho}^\gamma} \\
\vec{\nabla} S &= 0 \\
\vec{\nabla} P &= A^2 \vec{\nabla} \bar{\rho}
\end{aligned}$$

where  $A$  is the local mean speed of sound:

$$A^2 = \frac{\gamma P}{\bar{\rho}}$$

Then,

$$\begin{aligned} \frac{\partial A^2}{\partial r} &= \frac{\partial}{\partial r} \left( \frac{\gamma P}{\bar{\rho}} \right) \\ &= \frac{\gamma}{\bar{\rho}} \frac{\partial P}{\partial r} - \frac{\gamma P}{\bar{\rho}^2} \frac{\partial \bar{\rho}}{\partial r} \\ &= \frac{\gamma}{\bar{\rho}} \frac{\partial P}{\partial r} - \frac{A^2}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial r} \\ &= \frac{\gamma}{\bar{\rho}} \frac{\partial P}{\partial r} - \frac{1}{\bar{\rho}} \frac{\partial P}{\partial r} \\ &= \frac{\gamma - 1}{\bar{\rho}} \frac{\partial P}{\partial r} \end{aligned}$$

Then,

$$\begin{aligned} \frac{\partial P}{\partial r} &= \frac{\bar{\rho} V_\theta^2}{r} \\ \frac{\bar{\rho}}{\gamma - 1} \frac{\partial A^2}{\partial r} &= \frac{\bar{\rho} V_\theta^2}{r} \\ \frac{\partial A^2}{\partial r} &= (\gamma - 1) \frac{V_\theta^2}{r} \\ \frac{1}{A^2} \frac{\partial A^2}{\partial r} &= (\gamma - 1) \frac{M_\theta^2}{r} \end{aligned}$$

Integrating, and referencing to the conditions at  $r = r_{max}$ ,

$$\begin{aligned} \ln(A^2(r)) &= \ln(A^2(r_{max})) - \int_r^{r_{max}} (\gamma - 1) \frac{M_\theta^2}{r} dr \\ \ln(A^2(r)) - \ln(A^2(r_{max})) &= - \int_r^{r_{max}} (\gamma - 1) \frac{M_\theta^2}{r} dr \\ \ln\left(\frac{A^2(r)}{A^2(r_{max})}\right) &= - \int_r^{r_{max}} (\gamma - 1) \frac{M_\theta^2}{r} dr \end{aligned}$$

Defining

$$\tilde{A}(r) = \frac{A(r)}{A(r_{max})}$$

gives:

$$\begin{aligned}\ln(\tilde{A}^2(r)) &= (1-\gamma) \int_r^{r_{max}} \frac{M_\theta^2}{r} dr \\ 2 \ln(\tilde{A}(r)) &= (1-\gamma) \int_r^{r_{max}} \frac{M_\theta^2}{r} dr \\ \ln(\tilde{A}(r)) &= \frac{1-\gamma}{2} \int_r^{r_{max}} \frac{M_\theta^2}{r} dr \\ \ln(\tilde{A}(\tilde{r})) &= \frac{1-\gamma}{2} \int_{\tilde{r}}^1 \frac{M_\theta^2}{\tilde{r}} d\tilde{r}\end{aligned}$$

This gives the general result:

$$\tilde{A}(\tilde{r}) = \exp\left(\frac{1-\gamma}{2} \int_{\tilde{r}}^1 \frac{M_\theta^2}{\tilde{r}} d\tilde{r}\right)$$

which is Eq. (2.6) in the Kousen paper.

## 4 Linearized Perturbation Equations

The linearized perturbation equations are:

$$\begin{aligned}\frac{\partial v'_r}{\partial t} + \frac{V_\theta}{r} \frac{\partial v'_r}{\partial \theta} - \frac{2V_\theta v'_\theta}{r} + V_x \frac{\partial v'_r}{\partial x} &= -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial r} + \frac{\rho'}{\bar{\rho}^2} \frac{\partial P}{\partial r} \\ \frac{\partial v'_\theta}{\partial t} + v'_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial v'_\theta}{\partial \theta} + \frac{v'_r V_\theta}{r} + V_x \frac{\partial v'_\theta}{\partial x} &= -\frac{1}{\bar{\rho} r} \frac{\partial p'}{\partial \theta} \\ \frac{\partial v'_x}{\partial t} + v'_r \frac{\partial V_x}{\partial r} + \frac{V_\theta}{r} \frac{\partial v'_x}{\partial \theta} + V_x \frac{\partial v'_x}{\partial x} &= -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x} \\ \frac{\partial p'}{\partial t} + v'_r \frac{\partial P}{\partial r} + \frac{V_\theta}{r} \frac{\partial p'}{\partial \theta} + V_x \frac{\partial p'}{\partial x} + \gamma P \left( \frac{1}{r} \frac{\partial (r v'_r)}{\partial r} + \frac{1}{r} \frac{\partial v'_\theta}{\partial \theta} + \frac{\partial v'_x}{\partial x} \right) &= 0\end{aligned}$$

Remembering that:

$$\begin{aligned}\frac{\partial P}{\partial r} &= \frac{\bar{\rho} V_\theta^2}{r} \\ \gamma P &= \bar{\rho} A^2 \\ \rho' &= \frac{1}{A^2} p'\end{aligned}$$

and substituting in gives Eqs. (2.33-2.36) in the Kousen report:

$$\begin{aligned}\frac{\partial v'_r}{\partial t} + \frac{V_\theta}{r} \frac{\partial v'_r}{\partial \theta} + V_x \frac{\partial v'_r}{\partial x} - \frac{2V_\theta}{r} v'_\theta &= -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial r} + \frac{V_\theta^2}{\bar{\rho} r A^2} p' \\ \frac{\partial v'_\theta}{\partial t} + \frac{V_\theta}{r} \frac{\partial v'_\theta}{\partial \theta} + V_x \frac{\partial v'_\theta}{\partial x} + \left( \frac{V_\theta}{r} + \frac{\partial V_\theta}{\partial r} \right) v'_r &= -\frac{1}{\bar{\rho} r} \frac{\partial p'}{\partial \theta} \\ \frac{\partial v'_x}{\partial t} + \frac{V_\theta}{r} \frac{\partial v'_x}{\partial \theta} + V_x \frac{\partial v'_x}{\partial x} + \frac{\partial V_x}{\partial r} v'_r &= -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x} \\ \frac{1}{\bar{\rho} A^2} \left( \frac{\partial p'}{\partial t} + \frac{V_\theta}{r} \frac{\partial p'}{\partial \theta} + V_x \frac{\partial p'}{\partial x} \right) + \frac{V_\theta^2}{A^2 r} v'_r + \frac{\partial v'_r}{\partial r} + \frac{v'_r}{r} + \frac{1}{r} \frac{\partial v'_\theta}{\partial \theta} + \frac{\partial v'_x}{\partial x} &= 0\end{aligned}$$

Defining the perturbation variables as:

$$\begin{aligned}v'_r &= v_r(r) e^{i(k_x x + m\theta - \omega t)} \\ v'_\theta &= v_\theta(r) e^{i(k_x x + m\theta - \omega t)} \\ v'_x &= v_x(r) e^{i(k_x x + m\theta - \omega t)} \\ p' &= p(r) e^{i(k_x x + m\theta - \omega t)}\end{aligned}$$

and substituting into the perturbation equations gives:

$$\begin{aligned}\left( -i\omega + \frac{imV_\theta}{r} + ik_x V_x \right) v_r - \frac{2V_\theta}{r} v_\theta &= -\frac{1}{\bar{\rho}} \frac{\partial p}{\partial r} + \frac{V_\theta^2}{\bar{\rho} r A^2} p \\ \left( -i\omega + \frac{imV_\theta}{r} + ik_x V_x \right) v_\theta + \left( \frac{V_\theta}{r} + \frac{\partial V_\theta}{\partial r} \right) v_r &= -\frac{im}{\bar{\rho} r} p \\ \left( -i\omega + \frac{imV_\theta}{r} + ik_x V_x \right) v_x + \frac{\partial V_x}{\partial r} v_r &= -\frac{ik_x}{\bar{\rho}} p \\ \frac{1}{\bar{\rho} A^2} \left( -i\omega + \frac{imV_\theta}{r} + ik_x V_x \right) p + \frac{V_\theta^2}{A^2 r} v_r + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{im}{r} v_\theta + ik_x v_x &= 0\end{aligned}$$

Defining

$$\begin{aligned}
r_T &= r_{max} \\
A_T &= A(r_{max}) \\
k &= \frac{\omega r_T}{A_T} \\
\bar{\gamma} &= k_x r_T \\
\tilde{r} &= \frac{r}{r_T} \\
\frac{\partial}{\partial r} &= \frac{\partial \tilde{r}}{\partial r} \frac{\partial}{\partial \tilde{r}} \\
&= \frac{1}{r_T} \frac{\partial}{\partial \tilde{r}} \\
V_\theta &= M_\theta A \\
V_x &= M_x A \\
\tilde{A} &= \frac{A}{A_T} \\
v_x &= \tilde{v}_x A \\
v_r &= \tilde{v}_r A \\
v_\theta &= \tilde{v}_\theta A \\
p &= \tilde{p} \bar{\rho} A^2
\end{aligned}$$

gives:

$$\begin{aligned}
&-i \left( k \frac{A_T}{r_T} - \frac{m M_\theta A}{\tilde{r} r_T} - \frac{\bar{\gamma}}{r_T} M_x A \right) \tilde{v}_r A - \frac{2 M_\theta A^2}{\tilde{r} r_T} \tilde{v}_\theta = -\frac{1}{\bar{\rho}} \frac{1}{r_T} \frac{\partial (\tilde{p} \bar{\rho} A^2)}{\partial \tilde{r}} + \frac{M_\theta^2 A^2}{\bar{\rho} \tilde{r} r_T A^2} \tilde{p} \bar{\rho} A^2 \\
&-i \left( k \frac{A_T}{r_T} - \frac{m M_\theta A}{\tilde{r} r_T} - \frac{\bar{\gamma}}{r_T} M_x A \right) \tilde{v}_\theta A + \left( \frac{M_\theta A}{\tilde{r} r_T} + \frac{1}{r_T} \frac{\partial (M_\theta A)}{\partial \tilde{r}} \right) \tilde{v}_r A = -\frac{i m \bar{\rho} A^2}{\bar{\rho} \tilde{r} r_T} \tilde{p} \\
&-i \left( k \frac{A_T}{r_T} - \frac{m M_\theta A}{\tilde{r} r_T} - \frac{\bar{\gamma}}{r_T} M_x A \right) \tilde{v}_x A + \frac{1}{r_T} \frac{\partial (M_x A)}{\partial \tilde{r}} \tilde{v}_r A = -\frac{i \bar{\gamma} \bar{\rho} A^2}{\bar{\rho} r_T} \tilde{p} \\
&\frac{-i}{\bar{\rho} A^2} \left( k \frac{A_T}{r_T} - \frac{m M_\theta A}{\tilde{r} r_T} - \frac{\bar{\gamma}}{r_T} M_x A \right) \bar{\rho} A^2 \tilde{p} + \frac{M_\theta^2 A^2}{A^2 \tilde{r} r_T} \tilde{v}_r A + \frac{1}{r_T} \frac{\partial (\tilde{v}_r A)}{\partial \tilde{r}} + \frac{\tilde{v}_r A}{\tilde{r} r_T} + \frac{i m}{\tilde{r} r_T} \tilde{v}_\theta A + \frac{i \bar{\gamma}}{r_T} \tilde{v}_x A = 0
\end{aligned}$$

which becomes

$$-i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} - \bar{\gamma} M_x \right) \tilde{v}_r - \frac{2 M_\theta}{\tilde{r}} \tilde{v}_\theta = -\frac{1}{\bar{\rho} A^2} \frac{\partial (\tilde{p} \bar{\rho} A^2)}{\partial \tilde{r}} + \frac{M_\theta^2}{\tilde{r}} \tilde{p}$$

$$\begin{aligned}
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_\theta + \left( \frac{M_\theta}{\tilde{r}} + \frac{1}{A} \frac{\partial (M_\theta A)}{\partial \tilde{r}} \right) \tilde{v}_r = -\frac{im}{\tilde{r}} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_x + \frac{1}{A} \frac{\partial (M_x A)}{\partial \tilde{r}} \tilde{v}_r = -i\bar{\gamma} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{p} + \frac{M_\theta^2}{\tilde{r}} \tilde{v}_r + \frac{1}{A} \frac{\partial (\tilde{v}_r A)}{\partial \tilde{r}} + \frac{\tilde{v}_r}{\tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_\theta + i\bar{\gamma} \tilde{v}_x = 0
\end{aligned}$$

Expanding the mean flow derivatives gives:

$$\begin{aligned}
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_r - \frac{2M_\theta}{\tilde{r}} \tilde{v}_\theta = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{\tilde{p}}{\bar{\rho}A^2} \frac{\partial (\bar{\rho}A^2)}{\partial \tilde{r}} + \frac{M_\theta^2}{\tilde{r}} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_\theta + \left( \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} + \frac{M_\theta}{A} \frac{\partial A}{\partial \tilde{r}} \right) \tilde{v}_r = -\frac{im}{\tilde{r}} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_x + \frac{\partial M_x}{\partial \tilde{r}} \tilde{v}_r + \frac{M_x}{A} \frac{\partial A}{\partial \tilde{r}} \tilde{v}_r = -i\bar{\gamma} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{p} + \frac{M_\theta^2}{\tilde{r}} \tilde{v}_r + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \frac{1}{A} \frac{\partial A}{\partial \tilde{r}} \tilde{v}_r + \frac{\tilde{v}_r}{\tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_\theta + i\bar{\gamma} \tilde{v}_x = 0
\end{aligned}$$

The mean flow derivatives are:

$$\begin{aligned}
\frac{1}{A} \frac{\partial A}{\partial \tilde{r}} &= \frac{r_T}{A^2} \left( A \frac{\partial A}{\partial r} \right) \\
&= \frac{r_T}{2A^2} \frac{\partial A^2}{\partial r} \\
&= \frac{r_T}{A^2} \left( \frac{\gamma-1}{2} \right) \frac{V_\theta^2}{r} \\
&= \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}}
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{\bar{\rho}A^2} \frac{\partial (\bar{\rho}A^2)}{\partial \tilde{r}} &= \frac{\gamma}{\bar{\rho}A^2} \frac{\partial P}{\partial \tilde{r}} \\
&= \frac{r_T \gamma}{\bar{\rho}A^2} \gamma \frac{\partial P}{\partial r} \\
&= \frac{r_T \gamma}{\bar{\rho}A^2} \frac{\bar{\rho} V_\theta^2}{r}
\end{aligned}$$



$$\begin{aligned}
&= \frac{\gamma}{\bar{\rho} A^2} \frac{\bar{\rho} A^2 M_\theta^2}{\tilde{r}} \\
&= \frac{\gamma M_\theta^2}{\tilde{r}}
\end{aligned}$$

Substituting in,

$$\begin{aligned}
&-i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} - \bar{\gamma} M_x \right) \tilde{v}_r - \frac{2 M_\theta}{\tilde{r}} \tilde{v}_\theta = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{(\gamma - 1) M_\theta^2}{\tilde{r}} \tilde{p} \\
&-i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} - \bar{\gamma} M_x \right) \tilde{v}_\theta + \left( \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} + \left( \frac{\gamma - 1}{2} \right) \frac{M_\theta^3}{\tilde{r}} \right) \tilde{v}_r = -\frac{im}{\tilde{r}} \tilde{p} \\
&-i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} - \bar{\gamma} M_x \right) \tilde{v}_x + \left( \frac{\partial M_x}{\partial \tilde{r}} + \left( \frac{\gamma - 1}{2} \right) \frac{M_x M_\theta^2}{\tilde{r}} \right) \tilde{v}_r = -i \bar{\gamma} \tilde{p} \\
&-i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} - \bar{\gamma} M_x \right) \tilde{p} + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \left( \left( \frac{\gamma + 1}{2} \right) \frac{M_\theta^2}{\tilde{r}} + \frac{1}{\tilde{r}} \right) \tilde{v}_r + \frac{im}{\tilde{r}} \tilde{v}_\theta + i \bar{\gamma} \tilde{v}_x = 0
\end{aligned}$$

which are Eq. (2.38-2.41) in Kousen's report.

Defining:

$$\lambda = -i \bar{\gamma}$$

and defining:

$$\{\vec{x}\} = \begin{Bmatrix} \tilde{v}_r \\ \tilde{v}_\theta \\ \tilde{v}_x \\ \tilde{p} \end{Bmatrix}$$

and writing the equation in matrix form gives:

$$\begin{bmatrix}
-i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} \right) - \lambda M_x & -\frac{2 M_\theta}{\tilde{r}} & 0 & \frac{\partial}{\partial \tilde{r}} + \frac{(\gamma - 1) M_\theta^2}{\tilde{r}} \\
\frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} + \left( \frac{\gamma - 1}{2} \right) \frac{M_\theta^3}{\tilde{r}} & -i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} \right) - \lambda M_x & 0 & \frac{im}{\tilde{r}} \\
\frac{\partial M_x}{\partial \tilde{r}} + \left( \frac{\gamma - 1}{2} \right) \frac{M_x M_\theta^2}{\tilde{r}} & 0 & -i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} \right) - \lambda M_x & -\lambda \\
\frac{\partial}{\partial \tilde{r}} + \left( \frac{\gamma + 1}{2} \right) \frac{M_\theta^2}{\tilde{r}} + \frac{1}{\tilde{r}} & \frac{im}{\tilde{r}} & -\lambda & -i \left( \frac{k}{\tilde{A}} - \frac{m M_\theta}{\tilde{r}} \right) - \lambda M_x
\end{bmatrix} \{\vec{x}\} = 0$$

Following the Kousen report, this equation can be rewritten as:

$$[A] \{ \vec{x} \} = \lambda [B] \{ \vec{x} \}$$

where

$$[A] = \begin{bmatrix} -i \left( \frac{k}{A} - \frac{mM_\theta}{r} \right) & -\frac{2M_\theta}{r} & 0 & \frac{\partial}{\partial r} + \frac{(\gamma-1)M_\theta^2}{r} \\ \frac{M_\theta}{r} + \frac{\partial M_\theta}{\partial r} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^3}{r} & -i \left( \frac{k}{A} - \frac{mM_\theta}{r} \right) & 0 & \frac{im}{r} \\ \frac{\partial M_x}{\partial r} + \left( \frac{\gamma-1}{2} \right) \frac{M_x M_\theta^2}{r} & 0 & -i \left( \frac{k}{A} - \frac{mM_\theta}{r} \right) & 0 \\ \frac{\partial}{\partial r} + \left( \frac{\gamma+1}{2} \right) \frac{M_\theta^2}{r} + \frac{1}{r} & \frac{im}{r} & 0 & -i \left( \frac{k}{A} - \frac{mM_\theta}{r} \right) \end{bmatrix}$$

and

$$[B] = \begin{bmatrix} M_x & 0 & 0 & 0 \\ 0 & M_x & 0 & 0 \\ 0 & 0 & M_x & 1 \\ 0 & 0 & 1 & M_x \end{bmatrix}$$

From the SWIRL code: at  $\tilde{r} = 0$ ,

$$[A] = \begin{bmatrix} -i \left( \frac{k}{A} - mM_\theta \right) & -2M_\theta & 0 & \frac{\partial}{\partial r} + 2(\gamma-1)M_\theta \frac{\partial M_\theta}{\partial r} \\ \frac{\partial M_\theta}{\partial r} + \left( 1 + 3 \left( \frac{\gamma-1}{2} \right) M_\theta^2 \right) \frac{\partial M_\theta}{\partial r} & -i \left( \frac{k}{A} - mM_\theta \right) & 0 & 0 \\ \frac{\partial M_x}{\partial r} + \left( \frac{\gamma-1}{2} \right) M_\theta \left( M_\theta \frac{\partial M_x}{\partial r} + 2M_x \frac{\partial M_\theta}{\partial r} \right) & 0 & -i \left( \frac{k}{A} - mM_\theta \right) & 0 \\ \frac{\partial}{\partial r} + (\gamma+1)M_\theta \frac{\partial M_\theta}{\partial r} & 0 & 0 & -i \left( \frac{k}{A} - mM_\theta \right) \end{bmatrix}$$

## 5 Divergence and Vorticity?

The divergence and vorticity of the perturbation velocity in cylindrical coordinates is:

$$\begin{aligned} \vec{\nabla} \cdot \vec{v}' &= \frac{1}{r} \frac{\partial}{\partial r} (rv'_r) + \frac{1}{r} \frac{\partial v'_\theta}{\partial \theta} + \frac{\partial v'_x}{\partial x} \\ &= \frac{1}{r_T} \left( \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r}v'_r) + \frac{1}{\tilde{r}} \frac{\partial v'_\theta}{\partial \theta} + \frac{\partial v'_x}{\partial \tilde{x}} \right) \end{aligned}$$

$$\begin{aligned}
\vec{\nabla} \times \vec{v}' &= \begin{pmatrix} \left( \frac{1}{r} \frac{\partial v'_x}{\partial \theta} - \frac{\partial v'_\theta}{\partial x} \right) \vec{e}_r \\ + \left( \frac{\partial v'_r}{\partial x} - \frac{\partial v'_x}{\partial r} \right) \vec{e}_\theta \\ + \frac{1}{r} \left( \frac{\partial}{\partial r} (r v'_\theta) - \frac{\partial v'_r}{\partial \theta} \right) \vec{e}_x \end{pmatrix} \\
&= \frac{1}{r_T} \begin{pmatrix} \left( \frac{1}{\tilde{r}} \frac{\partial v'_x}{\partial \tilde{\theta}} - \frac{\partial v'_\theta}{\partial \tilde{x}} \right) \vec{e}_r \\ + \left( \frac{\partial v'_r}{\partial \tilde{x}} - \frac{\partial v'_x}{\partial \tilde{r}} \right) \vec{e}_\theta \\ + \frac{1}{\tilde{r}} \left( \frac{\partial}{\partial \tilde{r}} (\tilde{r} v'_\theta) - \frac{\partial v'_r}{\partial \tilde{\theta}} \right) \vec{e}_x \end{pmatrix}
\end{aligned}$$

Remembering the definitions:

$$\begin{aligned}
v'_r &= v_r(r) e^{i(k_x x + m\theta - \omega t)} \\
v'_\theta &= v_\theta(r) e^{i(k_x x + m\theta - \omega t)} \\
v'_x &= v_x(r) e^{i(k_x x + m\theta - \omega t)} \\
p' &= p(r) e^{i(k_x x + m\theta - \omega t)}
\end{aligned}$$

with the nondimensional counterparts:

$$\begin{aligned}
v'_r &= A \tilde{v}_r(\tilde{r}) e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)} \\
v'_\theta &= A \tilde{v}_\theta(\tilde{r}) e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)} \\
v'_x &= A \tilde{v}_x(\tilde{r}) e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)} \\
p' &= \bar{\rho} A^2 \tilde{p}(\tilde{r}) e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)}
\end{aligned}$$

where:

$$\bar{\gamma} = k_x r_T$$

The axial derivatives are:

$$\begin{aligned}
\frac{\partial v'_r}{\partial x} &= \frac{1}{r_T} \frac{\partial v'_r}{\partial \tilde{x}} \\
&= i \bar{\gamma} \tilde{v}_r \left( \frac{A}{r_T} e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)} \right) \\
\frac{\partial v'_\theta}{\partial x} &= i \bar{\gamma} \tilde{v}_\theta \left( \frac{A}{r_T} e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)} \right) \\
\frac{\partial v'_x}{\partial x} &= i \bar{\gamma} \tilde{v}_x \left( \frac{A}{r_T} e^{i(\tilde{\gamma} \tilde{x} + m\theta - \omega t)} \right)
\end{aligned}$$

The azimuthal derivatives are:

$$\begin{aligned}\frac{1}{r} \frac{\partial v'_r}{\partial \theta} &= \frac{im\tilde{v}_r}{\tilde{r}} \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right) \\ \frac{1}{r} \frac{\partial v'_\theta}{\partial \theta} &= \frac{im\tilde{v}_\theta}{\tilde{r}} \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right) \\ \frac{1}{r} \frac{\partial v'_x}{\partial \theta} &= \frac{im\tilde{v}_x}{\tilde{r}} \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right)\end{aligned}$$

To do the radial derivatives, note that:

$$\frac{\partial A}{\partial r} = \frac{A}{r_T} \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}}$$

The radial derivatives (remembering that  $A = A(r)$ !) are:

$$\begin{aligned}\frac{\partial v'_r}{\partial r} &= \left( \tilde{v}'_r \frac{\partial A}{\partial r} + \frac{A}{r_T} \frac{\partial v'_r}{\partial \tilde{r}} \right) e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \\ &= \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_r + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} \right) \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right) \\ \frac{\partial v'_\theta}{\partial r} &= \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_\theta + \frac{\partial \tilde{v}_\theta}{\partial \tilde{r}} \right) \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right) \\ \frac{\partial v'_x}{\partial r} &= \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_x + \frac{\partial \tilde{v}_x}{\partial \tilde{r}} \right) \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right)\end{aligned}$$

gives:

$$\begin{aligned}\vec{\nabla} \cdot \vec{v}' &= \left( \frac{\tilde{v}_r}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_r}{\tilde{r}} + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_\theta + i\tilde{\gamma} \tilde{v}_x \right) \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right) \\ \vec{\nabla} \times \vec{v}' &= \begin{pmatrix} \left( \frac{im\tilde{v}_x}{\tilde{r}} - i\tilde{\gamma} \tilde{v}_\theta \right) \vec{e}_r \\ + \left( i\tilde{\gamma} \tilde{v}_r - \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_x + \frac{\partial \tilde{v}_x}{\partial \tilde{r}} \right) \right) \vec{e}_\theta \\ + \left( \frac{\tilde{v}_\theta}{\tilde{r}} + \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_\theta + \frac{\partial \tilde{v}_\theta}{\partial \tilde{r}} \right) - \frac{im\tilde{v}_r}{\tilde{r}} \right) \vec{e}_x \end{pmatrix} \left( \frac{A}{r_T} e^{i(\tilde{\gamma}\tilde{x}+m\theta-\omega t)} \right)\end{aligned}$$

If all three vorticity components are zero, the perturbations will be irrotational. From the first two velocity components, this gives:

$$\begin{aligned}
\tilde{v}_\theta &= \frac{m}{\gamma \tilde{r}} \tilde{v}_x \\
\tilde{v}_r &= \frac{-i}{\gamma} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_x + \frac{\partial \tilde{v}_x}{\partial \tilde{r}} \right)
\end{aligned}$$

As a test, the last vorticity component is:

$$\begin{aligned}
\frac{\tilde{v}_\theta}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_\theta + \frac{\partial \tilde{v}_\theta}{\partial \tilde{r}} - \frac{im}{\tilde{r}} \tilde{v}_r &= \begin{pmatrix} \frac{m}{\gamma \tilde{r}} \left( \frac{1}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \right) \tilde{v}_x \\ + \frac{\partial}{\partial \tilde{r}} \left( \frac{m}{\gamma \tilde{r}} \tilde{v}_x \right) \\ - \frac{m}{\gamma \tilde{r}} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_x + \frac{\partial \tilde{v}_x}{\partial \tilde{r}} \right) \end{pmatrix} \\
&= \begin{pmatrix} \frac{m}{\gamma \tilde{r}} \left( \frac{1}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \right) \tilde{v}_x \\ - \frac{m}{\gamma \tilde{r}^2} \tilde{v}_x \\ + \frac{m}{\gamma \tilde{r}} \frac{\partial \tilde{v}_x}{\partial \tilde{r}} \\ - \frac{m}{\gamma \tilde{r}} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_x + \frac{\partial \tilde{v}_x}{\partial \tilde{r}} \right) \end{pmatrix} \\
&= 0
\end{aligned}$$

showing that these are the irrotational relations between the perturbation velocities.

## 5.1 Velocity decomposition

Let's decompose the perturbation velocity field into irrotational and divergence-free components:

$$\begin{aligned}
\tilde{v}_x &= \tilde{v}_{x,\omega} + \tilde{v}_{x,\phi} \\
\tilde{v}_r &= \tilde{v}_{r,\omega} + \tilde{v}_{r,\phi} \\
\tilde{v}_\theta &= \tilde{v}_{\theta,\omega} + \tilde{v}_{\theta,\phi}
\end{aligned}$$

with the relations:

$$\begin{aligned}
\tilde{v}_{\theta,\phi} &= \frac{m}{\gamma \tilde{r}} \tilde{v}_{x,\phi} \\
\tilde{v}_{r,\phi} &= \frac{-i}{\gamma} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} \right)
\end{aligned}$$

The velocity divergence gives one relation for the rotational components of the perturbation velocities:

$$\begin{aligned}\tilde{v}_{x,\omega} &= \frac{i}{\tilde{\gamma}} \left( \frac{\tilde{v}_{r,\omega}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\omega}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} \right) - \frac{m}{\tilde{\gamma} \tilde{r}} \tilde{v}_{\theta,\omega} \\ &= i \left( \frac{1+\Gamma}{\tilde{\gamma} \tilde{r}} \right) \tilde{v}_{r,\omega} + \frac{i}{\tilde{\gamma}} \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} - \frac{m}{\tilde{\gamma} \tilde{r}} \tilde{v}_{\theta,\omega}\end{aligned}$$

As a check, the velocity perturbations are put into the velocity divergence equation:

$$\begin{aligned}\vec{\nabla} \cdot \vec{v}' &= \left( \frac{\tilde{v}_r}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_r}{\tilde{r}} + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_\theta + i\tilde{\gamma} \tilde{v}_x \right) \\ &= \left( \frac{\tilde{v}_{r,\phi}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\phi}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\phi}}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_{\theta,\phi} + i\tilde{\gamma} \tilde{v}_{x,\phi} \right) \\ &\quad + \left( \frac{\tilde{v}_{r,\omega}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\omega}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_{\theta,\omega} + i\tilde{\gamma} \tilde{v}_{x,\omega} \right) \\ &= \left( \frac{1}{\tilde{r}} \left( 1 + \left( \frac{\gamma-1}{2} \right) M_\theta^2 \right) \frac{-i}{\tilde{\gamma}} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} \right) \right. \\ &\quad \left. + \frac{\partial}{\partial \tilde{r}} \left( \frac{-i}{\tilde{\gamma}} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} \right) \right) \right. \\ &\quad \left. + \frac{im^2}{\tilde{\gamma} \tilde{r}^2} \tilde{v}_{x,\phi} + i\tilde{\gamma} \tilde{v}_{x,\phi} \right. \\ &\quad \left. + \frac{\tilde{v}_{r,\omega}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\omega}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_{\theta,\omega} \right. \\ &\quad \left. + i\tilde{\gamma} \left( \frac{i}{\tilde{\gamma}} \left( \frac{\tilde{v}_{r,\omega}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\omega}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} \right) - \frac{m}{\tilde{\gamma} \tilde{r}} \tilde{v}_{\theta,\omega} \right) \right) \\ &= \left( \frac{-i}{\tilde{\gamma} \tilde{r}^2} \left( 1 + \left( \frac{\gamma-1}{2} \right) M_\theta^2 \right) \left( \left( \frac{\gamma-1}{2} \right) M_\theta^2 \tilde{v}_{x,\phi} \right. \right. \\ &\quad \left. \left. - \frac{i}{\tilde{\gamma} \tilde{r}} \left( 1 + \left( \frac{\gamma-1}{2} \right) M_\theta^2 \right) \left( \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} \right) \right. \right. \\ &\quad \left. \left. - \frac{i}{\tilde{\gamma}} \frac{\partial}{\partial \tilde{r}} \left( \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2}{\tilde{r}} \tilde{v}_{x,\phi} \right) \right. \right. \\ &\quad \left. \left. - \frac{i}{\tilde{\gamma}} \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial \tilde{r}^2} \right. \right. \\ &\quad \left. \left. + i\tilde{\gamma} \left( 1 + \frac{m^2}{\tilde{\gamma}^2 \tilde{r}^2} \right) \tilde{v}_{x,\phi} \right. \right. \\ &\quad \left. \left. + \frac{\tilde{v}_{r,\omega}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\omega}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_{\theta,\omega} \right. \right. \\ &\quad \left. \left. - \left( \frac{\tilde{v}_{r,\omega}}{\tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^2 \tilde{v}_{r,\omega}}{\tilde{r}} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} \right) - \frac{im}{\tilde{r}} \tilde{v}_{\theta,\omega} \right) \right)\end{aligned}$$

$$\begin{aligned}
&= \begin{pmatrix} i\bar{\gamma} \left( \left(1 + \frac{m^2}{\bar{\gamma}^2 \tilde{r}^2}\right) - \frac{1}{\bar{\gamma}^2 \tilde{r}^2} \left(1 + \left(\frac{\gamma-1}{2}\right) M_\theta^2\right) \left(\left(\frac{\gamma-1}{2}\right) M_\theta^2\right) \right) \tilde{v}_{x,\phi} \\ -\frac{i}{\bar{\gamma} \tilde{r}} \left(1 + \left(\frac{\gamma-1}{2}\right) M_\theta^2\right) \left(\frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}}\right) \\ +\frac{i}{\bar{\gamma}} \left(\left(\frac{\gamma-1}{2}\right) \frac{M_\theta^2}{\tilde{r}^2} \tilde{v}_{x,\phi}\right) \\ -\frac{i}{\bar{\gamma}} \frac{\partial M_\theta}{\partial \tilde{r}} \left(\left(\frac{\gamma-1}{2}\right) \frac{2M_\theta}{\tilde{r}} \tilde{v}_{x,\phi}\right) \\ -\frac{i}{\bar{\gamma} \tilde{r}} \left(\left(\frac{\gamma-1}{2}\right) M_\theta^2 \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}}\right) \\ -\frac{i}{\bar{\gamma}} \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial \tilde{r}^2} \end{pmatrix} \\
&= \begin{pmatrix} i \left(\bar{\gamma} + \frac{1}{\bar{\gamma} \tilde{r}^2} \left(m^2 - \Gamma^2 - \tilde{r} \frac{\partial \Gamma}{\partial \tilde{r}}\right)\right) \tilde{v}_{x,\phi} \\ -\frac{i}{\bar{\gamma} \tilde{r}} (1 + 2\Gamma) \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} \\ -\frac{i}{\bar{\gamma}} \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial \tilde{r}^2} \end{pmatrix}
\end{aligned}$$

where

$$\Gamma = \left(\frac{\gamma-1}{2}\right) M_\theta^2$$

Note that the divergence of velocity can be written solely in terms of  $\tilde{v}_{x,\phi}$ :

$$\left(\frac{\tilde{v}_r}{\tilde{r}} + \left(\frac{\gamma-1}{2}\right) \frac{M_\theta^2 \tilde{v}_r}{\tilde{r}} + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \frac{im}{\tilde{r}} \tilde{v}_\theta + i\bar{\gamma} \tilde{v}_x\right) = \Phi_0 \tilde{v}_{x,\phi} + \Phi_1 \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} + \Phi_2 \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial \tilde{r}^2}$$

where

$$\begin{aligned}
\Phi_0 &= i \left( \bar{\gamma} + \frac{1}{\bar{\gamma} \tilde{r}^2} \left( m^2 - \Gamma^2 - \tilde{r} \frac{\partial \Gamma}{\partial \tilde{r}} \right) \right) \\
\Phi_1 &= -\frac{i}{\bar{\gamma} \tilde{r}} (1 + 2\Gamma) \\
\Phi_2 &= -\frac{i}{\bar{\gamma}}
\end{aligned}$$

## 5.2 Back to business...

The SWIRL code equations are:

$$\begin{aligned}
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_r - \frac{2M_\theta}{\tilde{r}} \tilde{v}_\theta = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{(\gamma-1)M_\theta^2}{\tilde{r}} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_\theta + \left( \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^3}{\tilde{r}} \right) \tilde{v}_r = -\frac{im}{\tilde{r}} \tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{v}_x + \left( \frac{\partial M_x}{\partial \tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_x M_\theta^2}{\tilde{r}} \right) \tilde{v}_r = -i\bar{\gamma}\tilde{p} \\
& -i \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x \right) \tilde{p} + \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \left( \left( \frac{\gamma+1}{2} \right) \frac{M_\theta^2}{\tilde{r}} + \frac{1}{\tilde{r}} \right) \tilde{v}_r + \frac{im}{\tilde{r}} \tilde{v}_\theta + i\bar{\gamma}\tilde{v}_x = 0
\end{aligned}$$

Defining:

$$\alpha = \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} - \bar{\gamma}M_x$$

and expanding the pressure equation to isolate the divergence of velocity term gives:

$$\begin{aligned}
& -i\alpha \tilde{v}_r - \frac{2M_\theta}{\tilde{r}} \tilde{v}_\theta = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{(\gamma-1)M_\theta^2}{\tilde{r}} \tilde{p} \\
& -i\alpha \tilde{v}_\theta + \left( \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_\theta^3}{\tilde{r}} \right) \tilde{v}_r = -\frac{im}{\tilde{r}} \tilde{p} \\
& -i\alpha \tilde{v}_x + \left( \frac{\partial M_x}{\partial \tilde{r}} + \left( \frac{\gamma-1}{2} \right) \frac{M_x M_\theta^2}{\tilde{r}} \right) \tilde{v}_r = -i\bar{\gamma}\tilde{p} \\
& -i\alpha \tilde{p} + \frac{M_\theta^2}{\tilde{r}} \tilde{v}_r + \left( \Phi_0 \tilde{v}_{x,\phi} + \Phi_1 \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} + \Phi_2 \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial \tilde{r}^2} \right) = 0
\end{aligned}$$

Rewriting this in terms of the rotational and irrotational velocities,

$$\begin{aligned}
& -i\alpha (\tilde{v}_{r,\phi} + \tilde{v}_{r,\omega}) - \frac{2M_\theta}{\tilde{r}} (\tilde{v}_{\theta,\phi} + \tilde{v}_{\theta,\omega}) = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{2\Gamma}{\tilde{r}} \tilde{p} \\
& -i\alpha (\tilde{v}_{\theta,\phi} + \tilde{v}_{\theta,\omega}) + \left( (1+\Gamma) \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} \right) (\tilde{v}_{r,\phi} + \tilde{v}_{r,\omega}) = -\frac{im}{\tilde{r}} \tilde{p} \\
& -i\alpha (\tilde{v}_{x,\phi} + \tilde{v}_{x,\omega}) + \left( \frac{\partial M_x}{\partial \tilde{r}} + \Gamma \frac{M_x}{\tilde{r}} \right) (\tilde{v}_{r,\phi} + \tilde{v}_{r,\omega}) = -i\bar{\gamma}\tilde{p} \\
& -i\alpha \tilde{p} + \frac{M_\theta^2}{\tilde{r}} (\tilde{v}_{r,\phi} + \tilde{v}_{r,\omega}) + \left( \Phi_0 \tilde{v}_{x,\phi} + \Phi_1 \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} + \Phi_2 \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial \tilde{r}^2} \right) = 0
\end{aligned}$$



Substituting in:

$$\begin{aligned}
& \left( \begin{array}{c} -i\alpha \left( \frac{-i}{\tilde{r}} \left( \frac{\Gamma}{r} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial r} \right) + \tilde{v}_{r,\omega} \right) \\ -\frac{2M_\theta}{\tilde{r}} \left( \frac{m}{\tilde{r}} \tilde{v}_{x,\phi} + \tilde{v}_{\theta,\omega} \right) \end{array} \right) = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{2\Gamma}{\tilde{r}} \tilde{p} \\
& \left( \begin{array}{c} -i\alpha \left( \frac{m}{\tilde{r}} \tilde{v}_{x,\phi} + \tilde{v}_{\theta,\omega} \right) \\ + \left( (1+\Gamma) \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} \right) \left( \frac{-i}{\tilde{r}} \left( \frac{\Gamma}{r} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial r} \right) + \tilde{v}_{r,\omega} \right) \end{array} \right) = -\frac{im}{\tilde{r}} \tilde{p} \\
& \left( \begin{array}{c} -i\alpha \left( \tilde{v}_{x,\phi} + i \left( \frac{1+\Gamma}{\tilde{r}} \right) \tilde{v}_{r,\omega} + \frac{i}{\tilde{r}} \frac{\partial \tilde{v}_{r,\omega}}{\partial r} - \frac{m}{\tilde{r}} \tilde{v}_{\theta,\omega} \right) \\ + \left( \frac{\partial M_x}{\partial \tilde{r}} + \Gamma \frac{M_x}{\tilde{r}} \right) \left( \frac{-i}{\tilde{r}} \left( \frac{\Gamma}{r} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial r} \right) + \tilde{v}_{r,\omega} \right) \end{array} \right) = -i\tilde{\gamma} \tilde{p} \\
& \left( \begin{array}{c} -i\alpha \tilde{p} \\ + \frac{M_\theta^2}{\tilde{r}} \left( \frac{-i}{\tilde{r}} \left( \frac{\Gamma}{r} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial r} \right) + \tilde{v}_{r,\omega} \right) \\ + \left( \Phi_0 \tilde{v}_{x,\phi} + \Phi_1 \frac{\partial \tilde{v}_{x,\phi}}{\partial r} + \Phi_2 \frac{\partial^2 \tilde{v}_{x,\phi}}{\partial r^2} \right) \end{array} \right) = 0
\end{aligned}$$

Defining:

$$\begin{aligned}
\Theta &= (1+\Gamma) \frac{M_\theta}{\tilde{r}} + \frac{\partial M_\theta}{\partial \tilde{r}} \\
\tau_\phi &= \frac{\Gamma}{\tilde{r}} \tilde{v}_{x,\phi} + \frac{\partial \tilde{v}_{x,\phi}}{\partial \tilde{r}} \\
\tau_\omega &= \frac{\Gamma}{\tilde{r}} \tilde{v}_{r,\omega} + \frac{\partial \tilde{v}_{r,\omega}}{\partial \tilde{r}} \\
S &= \Gamma \frac{M_x}{\tilde{r}} + \frac{\partial M_x}{\partial \tilde{r}}
\end{aligned}$$

and gathering the rotational and irrotational components together:

$$\begin{aligned}
& \left( \begin{array}{c} \frac{\alpha}{\tilde{r}} \tau_\phi - \frac{2mM_\theta}{\tilde{r}^2} \tilde{v}_{x,\phi} \\ -i\alpha \tilde{v}_{r,\omega} - \frac{2M_\theta}{\tilde{r}} \tilde{v}_{\theta,\omega} \end{array} \right) = -\frac{\partial \tilde{p}}{\partial \tilde{r}} - \frac{2\Gamma}{\tilde{r}} \tilde{p} \\
& \left( \begin{array}{c} -\frac{i\Theta}{\tilde{r}} \tau_\phi - \frac{im\alpha}{\tilde{r}} \tilde{v}_{x,\phi} \\ +\Theta \tilde{v}_{r,\omega} - i\alpha \tilde{v}_{\theta,\omega} \end{array} \right) = -\frac{im}{\tilde{r}} \tilde{p} \\
& \left( \begin{array}{c} -i\alpha \tilde{v}_{x,\phi} + \frac{\alpha}{\tilde{r}} \tau_\omega + \frac{\alpha}{\tilde{r}} \tilde{v}_{r,\omega} + i \frac{m\alpha}{\tilde{r}} \tilde{v}_{\theta,\omega} \\ -i \frac{S}{\tilde{r}} \tau_\phi + S \tilde{v}_{r,\omega} \end{array} \right) = -i\tilde{\gamma} \tilde{p}
\end{aligned}$$

$$\begin{pmatrix} -i\alpha\tilde{p} \\ -i\frac{M_\theta^2}{\tilde{r}}\tau_\phi \\ +\Phi_0\tilde{v}_{x,\phi} + \Phi_1\frac{\partial\tilde{v}_{x,\phi}}{\partial\tilde{r}} + \Phi_2\frac{\partial^2\tilde{v}_{x,\phi}}{\partial\tilde{r}^2} \\ -i\frac{M_\theta^2}{\tilde{r}}\tilde{v}_{r,\omega} \end{pmatrix} = 0$$

I can't help but notice these groupings:

$$\begin{aligned} A &= \frac{1}{\tilde{\gamma}}\tau_\phi - i\tilde{v}_{r,\omega} \\ B &= \frac{m}{\tilde{\gamma}\tilde{r}}\tilde{v}_{x,\phi} + \tilde{v}_{\theta,\omega} \end{aligned}$$

which gives:

$$\begin{aligned} \alpha A - \frac{2M_\theta}{\tilde{r}}B &= -\frac{\partial\tilde{p}}{\partial\tilde{r}} - \frac{2\Gamma}{\tilde{r}}\tilde{p} \\ i\Theta A - i\alpha B &= -\frac{im}{\tilde{r}}\tilde{p} \\ \begin{pmatrix} -i\alpha\tilde{v}_{x,\phi} + \frac{\alpha}{\tilde{\gamma}}\tau_\omega + \frac{\alpha}{\tilde{\gamma}\tilde{r}}\tilde{v}_{r,\omega} + i\frac{m\alpha}{\tilde{\gamma}\tilde{r}}\tilde{v}_{\theta,\omega} \\ -iSA \end{pmatrix} &= -i\tilde{\gamma}\tilde{p} \\ \begin{pmatrix} -i\alpha\tilde{p} \\ -i\frac{M_\theta^2}{\tilde{r}}A \\ +\Phi_0\tilde{v}_{x,\phi} + \Phi_1\frac{\partial\tilde{v}_{x,\phi}}{\partial\tilde{r}} + \Phi_2\frac{\partial^2\tilde{v}_{x,\phi}}{\partial\tilde{r}^2} \end{pmatrix} &= 0 \end{aligned}$$

I can't help but notice these groupings:

## 6 The SWIRL Code

In this section, the Fortran2008 version of the SWIRL code is discussed.

The main routine is 'swirl.f90'. The routines called (when Chebyshev polynomials are used) are:

1. input
2. grid
3. derivs

4. smachAndSndspd
5. rmach
6. machout
7. global
8. boundary
9. analysis
10. output

## 6.1 input

The subroutine 'input' is used to obtain the input data for the code. Input reads a NAMELIST input from the file 'input.data'. The NAMELIST entries are:

1. MM: circumferential mode number ( $m$  in report).
2. NPTS: number of radial mesh points.
3. SIG: hub-to-duct radius ratio ( $\sigma$  in report).
4. AKRE: real part of nondimensional frequency ( $k = \frac{\omega r_T}{A_T}$  in report).
5. AKIM: imaginary part of nondimensional frequency ( $k = \frac{\omega r_T}{A_T}$  in report).
6. IX: this parameter is not actually used in the code.
7. NX: this parameter is not actually used in the code.
8. IR: axial velocity distribution flag (used in rmachModule):
  - IR = 0: uniform axial flow velocity:

$$V_x = RMAX$$

- IR = 1: linear shear:

$$V_x = (RMAX - SLOPE)(\tilde{r} - 1.0)$$

- IR = 2: user axial velocity input from file 'mach.input'. See rma-  
chModule for file format.
- IR = 3: uniform flow plus sine wave boundary layers of thickness  
*SLOPE*.
- IR = 4: uniform flow plus linear boundary layers of thickness  
*SLOPE*.
- IR = 5: uniform flow plus  $\frac{1}{7}$  power boundary layers.
- IR = 6: hyperbolic secant profile.
- IR = 7: laminar mean flow:

$$V_x = \frac{-4RMAX}{(1 - \sigma)^2} (\tilde{r}^2 - (1 + \sigma)\tilde{r} + \sigma)$$

- IR = 8: wavy sinusoid for axial Mach number:

$$M_x = \frac{RMAX + SLOPE}{2} + \frac{RMAX - SLOPE}{2} \sin \left( 4\pi \left( \frac{2(\tilde{r} - \sigma)}{1 - \sigma} - 1 \right) \right)$$

- IR = 9: Hagen-Poiseuille flow:

$$K = \frac{\sigma^2 - 1}{\ln \sigma}$$

$$V_x = RMAX \left( \frac{1 + K \ln r - \tilde{r}^2}{1 + \frac{K}{2} \ln \frac{K}{2} - \frac{K}{2}} \right)$$

9. RMAX: maximum axial Mach number value.

10. SLOPE: slope of linear axial Mach number distribution.

11. IS: swirl Mach number distribution flag (0-6). See smach for details.

- IS = 0: no swirl.
- IS = 1: solid-body swirl with angular velocity ANGOM.

$$v_\theta = (ANGOM)\tilde{r}$$

- IS = 2: free-vortex swirl with strength  $GAM$ .

$$v_{\theta} = \frac{GAM}{\tilde{r}}$$

- IS = 3: combined solid-body and free-vortex swirl.

$$v_{\theta} = (ANGOM) \tilde{r} + \frac{GAM}{\tilde{r}}$$

- IS = 4: Stability test case:

$$v_{\theta} = \frac{1}{\tilde{r}^2}$$

- IS = 5: user-input azimuthal velocity profile from file 'swrl.input'.  
See smachAndSndspdModule for file format.
- IS = 6: constant swirl:

$$v_{\theta} = ANGOM$$

- IS = 7: trailing line vortex (NOT COMPLETED IN CODE):

$$v_{\theta} = \frac{GAM}{\tilde{r}} (1 - e^{-\tilde{r}^2})$$

12. ANGOM: magnitude of solid-body swirl.
13. GAM: magnitude of free-vortex swirl.
14. IREPEAT: this parameter is not actually used in the code.
15. IFD: flag to choose method for derivative calculation:
  - IFD=0: Chebyshev polynomials
  - IFD $\neq$  0: Finite differencing

16. ITEST: this parameter is not actually used in the code.
17. ETAHR: real part of hub liner admittance ( $\eta_H$  in report).
18. ETAHI: imaginary part of hub liner admittance ( $\eta_H$  in report).
19. ETADR: real part of duct liner admittance ( $\eta_D$  in report).
20. ETADI: imaginary part of duct liner admittance ( $\eta_D$  in report).
21. ED2: second derivative smoothing coefficient.
22. ED4: fourth derivative smoothing coefficient.
23. ICMPR: flag to perform consistency test on selected modes (currently does not work):
  - ICMPR=1: Perform consistency test.
  - ICMPR  $\neq$  1: Do not perform consistency test.

## **6.2 grid**

## **6.3 derivs**

## **6.4 smachAndSndspd**

## **6.5 rmach**

## **6.6 machout**

## **6.7 globalM**

## **6.8 boundary**

## **6.9 analysis**

In this module, the eigendecomposition is actually performed.

The convected wavenumbers are computed as:

$$\lambda_{cvct}(\tilde{r}) = \frac{1}{M_x} \left( \frac{k}{\tilde{A}} - \frac{mM_\theta}{\tilde{r}} \right)$$

(noting that this is a range of convection speeds, due to the different axial and azimuthal mean velocities)

The routine ZGEGV is used to obtain the eigenvalues and eigenvectors. ZGEGV is a LAPACK routine that calculates the generalized eigenvalues  $(\alpha, \beta)$  and the generalized eigenvector  $\vec{r}$  that satisfy the equation:

$$\begin{aligned} ([A] - w [B]) \vec{r} &= 0 \\ w &= \frac{\alpha}{\beta} \end{aligned}$$

For SWIRL, the result is:

$$\begin{aligned} \lambda &= \frac{\alpha}{\beta} \\ \bar{\gamma} &= \frac{i\alpha}{\beta} \\ k_x &= \frac{1}{r_t} \frac{i\alpha}{\beta} \end{aligned}$$

The  $\bar{\gamma}$  values are printed to the display, with:

$$\dots \tag{1}$$

## 6.10 output

## 7 SWIRL Verification cases

In the Kousen report, there are a number of verification cases presented. In this section, the inputs for running those cases are determined.

### 7.1 Cylinder, Uniform Flow with Liner (Table 4.3)

In Table 4.3, a comparison is shown for the case of a cylinder (no centerbody) with uniform flow. For this test case,

$$\begin{aligned}
m &= 2 \\
k &= \frac{\omega r_T}{A_T} = -1 \\
M_x &= 0.5 \\
\eta_T &= 0.72 + 0.42i
\end{aligned}$$

Recall that in the SWIRL code,  $A_T$  is the speed of sound at the outer radius,  $r_T$ . Also,  $\eta_T$  is the admittance of the liner on the outer wall. The azimuthal mode number is  $m$ .

The input deck for this test case is:

```

&INPUTS
MM      = 2,
NPTS    = 32,
SIG      = 0.00,
AKRE     = -1.,
AKIM     = 0.,
IX       = 0,
NX       = 0,
IR       = 0,
RMAX     = 0.50,
SLOPE    = 0.00,
IS       = 0,
ANGOM    = 0.00,
GAM      = 0.00,
IREPEAT  = 0,
IFD      = 0,
ITEST    = 0,
ETAHR    = 0.00,
ETAHI    = 0.00,
ETADR    = 0.72,
ETADI    = 0.42,
ED2      = 0.0,
ED4      = 0.0,
&END

```

The resulting  $\gamma$  values are given in 'gam.acc'. The data from left to right are:

1. Eigenvalue ID number, j.
2. Real part of the nondimensional axial wavenumber  $\bar{\gamma}_j$
3. Imaginary part of the nondimensional axial wavenumber  $\bar{\gamma}_j$
4. Real part of  $\frac{k}{\bar{\gamma}_j}$
5. Imaginary part of  $\frac{k}{\bar{\gamma}_j}$
6. Real part of  $\frac{\bar{\gamma}_j}{k}$



## 7. Imaginary part of $\frac{\bar{\gamma}_j}{k}$

(I have no idea why the last two columns are needed.)

The output in 'gam.acc' for this test case is:

#	j	Re{gam}	Im{gam}	Re{gam/ak}	Im{gam/ak}	nz	
1	0.666661832386E+00	-0.863181766372E+03	-0.666661832386E+00	0.863181766372E+03	-0.894748166120E-06	-0.115850385453E-02	
2	0.666671535394E+00	0.863181766432E+03	-0.666671535394E+00	-0.863181766432E+03	-0.894761188697E-06	0.115850385443E-02	
3	0.486904334131E+03	0.711226578236E-05	-0.486904334131E+03	-0.711226578236E-05	-0.205379153542E-02	0.299999614658E-10	
4	-0.485571000887E+03	-0.727158271276E-05	0.485571000887E+03	0.727158271276E-05	0.205943105781E-02	-0.308406458597E-10	
5	0.666186196644E+00	-0.270964395496E+03	-0.666186196644E+00	0.270964395496E+03	-0.907336845842E-05	-0.369049946071E-02	
6	0.667159178385E+00	0.270964411411E+03	-0.667159178385E+00	-0.270964411411E+03	-0.908661893262E-05	0.369049914192E-02	
7	-0.152502349084E+00	-0.235471154884E+03	0.152502349084E+00	0.235471154884E+03	0.275043207226E-05	-0.424680288779E-02	
8	0.150782379084E+01	0.235508741734E+03	-0.150782379084E+01	-0.235508741734E+03	-0.271843357482E-04	0.424595284000E-02	
9	0.654577649002E+00	-0.154440881062E+03	-0.654577649002E+00	0.154440881062E+03	-0.274428258331E-04	-0.647485322322E-02	
10	0.67936066482E+00	0.154441666296E+03	-0.67936066482E+00	-0.154441666296E+03	-0.284815117974E-04	0.647481133017E-02	
11	0.203817254085E+00	-0.147580882502E+03	-0.203817254085E+00	0.147580882502E+03	-0.935793257979E-05	-0.677593246321E-02	
12	0.114908693497E+01	0.147615287403E+03	-0.114908693497E+01	-0.147615287403E+03	-0.527307439101E-04	0.677395563416E-02	
13	0.659171278977E+00	-0.109556505279E+03	-0.659171278977E+00	0.109556505279E+03	-0.549169253183E-04	-0.912737949970E-02	
14	0.674649119395E+00	0.109557229559E+03	-0.674649119395E+00	-0.109557229559E+03	-0.562055764807E-04	0.912730346485E-02	
15	0.398023285909E+00	-0.101971304048E+03	-0.398023285909E+00	0.101971304048E+03	-0.382777073285E-04	-0.980653110120E-02	
16	0.952415052147E+00	0.102000245487E+03	-0.952415052147E+00	-0.102000245487E+03	-0.915347402439E-04	0.980304327863E-02	
17	0.631981427627E+00	-0.878632034233E+02	-0.631981427627E+00	0.878632034233E+02	-0.818592537062E-04	-0.113807399174E-01	
18	0.704204711787E+00	0.878674124623E+02	-0.704204711787E+00	-0.878674124623E+02	-0.912043090777E-04	0.113800525755E-01	
19	0.512907574867E+00	-0.842237595480E+02	-0.512907574867E+00	0.842237595480E+02	-0.723025742788E-04	-0.118726938910E-01	
20	0.831861539314E+00	0.842442317160E+02	-0.831861539314E+00	-0.842442317160E+02	-0.117200200343E-03	0.118690916315E-01	
21	0.631266364148E+00	-0.757541009147E+02	-0.631266364148E+00	0.757541009147E+02	-0.109994304117E-03	-0.13199688911E-01	
22	0.705249647838E+00	0.757592371570E+02	-0.705249647838E+00	-0.757592371570E+02	-0.122866661541E-03	0.131985667472E-01	
23	0.584693633914E+00	-0.720556587549E+02	-0.584693633914E+00	0.720556587549E+02	-0.112606532211E-03	-0.138772467972E-01	
24	0.756173116912E+00	0.720683291681E+02	-0.756173116912E+00	-0.720683291681E+02	-0.145574235158E-03	0.138741931750E-01	
25	0.608943525213E+00	-0.679734189871E+02	-0.608943525213E+00	0.679734189871E+02	-0.131784378831E-03	-0.147104524924E-01	
26	0.729943721425E+00	0.679829463997E+02	-0.729943721425E+00	-0.679829463997E+02	-0.157920803754E-03	0.147078757196E-01	
27	0.603587261705E+00	-0.643648950046E+02	-0.603587261705E+00	0.643648950046E+02	-0.145681278991E-03	-0.155350532082E-01	
28	0.736150415165E+00	0.643759364750E+02	-0.736150415165E+00	-0.643759364750E+02	-0.177608053949E-03	0.155317236301E-01	
29	0.602714553949E+00	-0.606583474684E+02	-0.602714553949E+00	0.606583474684E+02	-0.163790102422E-03	-0.164841497181E-01	
30	0.598330594187E+00	-0.570207468632E+02	-0.598330594187E+00	0.570207468632E+02	-0.184004138383E-03	-0.17535455637E-01	
31	0.593992620827E+00	-0.533940329180E+02	-0.593992620827E+00	0.533940329180E+02	-0.208325223037E-03	-0.187263670060E-01	
32	0.668203564553E+00	-0.503674749089E+02	-0.668203564553E+00	0.503674749089E+02	-0.263349198330E-03	-0.19850587169E-01	
33	0.587454135440E+00	-0.497669424711E+02	-0.587454135440E+00	0.497669424711E+02	-0.237154591154E-03	-0.200908601756E-01	
34	0.737556431878E+00	0.606702564914E+02	-0.737556431878E+00	-0.606702564914E+02	-0.200345417813E-03	0.164801056031E-01	
35	0.583528302269E+00	-0.461144238809E+02	-0.583528302269E+00	0.461144238809E+02	-0.274358750369E-03	-0.216817173405E-01	
36	0.742927474892E+00	0.570343361984E+02	-0.742927474892E+00	-0.570343361984E+02	-0.228349441645E-03	0.175303233029E-01	
37	0.748373723996E+00	-0.534095424830E+02	-0.748373723996E+00	0.534095424830E+02	-0.262298348430E-03	-0.187195457917E-01	
38	0.664837047477E+00	0.503671839626E+02	-0.664837047477E+00	-0.503671839626E+02	-0.262025892761E-03	0.198507384535E-01	
39	0.756588821850E+00	-0.497851192860E+02	-0.756588821850E+00	0.497851192860E+02	-0.305183132961E-03	-0.200816853749E-01	
40	0.761911557798E+00	0.461352109598E+02	-0.761911557798E+00	-0.461352109598E+02	-0.357866551181E-03	0.216695083113E-01	
41	0.577042153605E+00	-0.424768910577E+02	-0.577042153605E+00	0.424768910577E+02	-0.319758393967E-03	-0.235378687336E-01	
42	0.569568800457E+00	-0.388329943784E+02	-0.569568800457E+00	0.388329943784E+02	-0.377616545059E-03	-0.257457591773E-01	
43	0.770559879483E+00	0.425014080621E+02	-0.770559879483E+00	-0.425014080621E+02	-0.426439106763E-03	0.235209008057E-01	
44	0.780827155751E+00	-0.388623195128E+02	-0.780827155751E+00	0.388623195128E+02	-0.516799620431E-03	-0.257214824374E-01	
45	0.560740641222E+00	-0.351878517665E+02	-0.560740641222E+00	0.351878517665E+02	-0.452758126067E-03	-0.284116838604E-01	
46	0.793358476458E+00	0.352235698890E+02	-0.793358476458E+00	-0.352235698890E+02	-0.639120079320E-03	0.283756857076E-01	
47	0.550240023912E+00	-0.315368072415E+02	-0.550240023912E+00	0.315368072415E+02	-0.553075656592E-03	-0.316993304992E-01	
48	0.537545130773E+00	-0.278800642562E+02	-0.537545130773E+00	0.278800642562E+02	-0.691299075016E-03	-0.358545943927E-01	
49	0.808917325270E+00	0.315812623997E+02	-0.808917325270E+00	-0.315812623997E+02	-0.810513646490E-03	0.316435850101E-01	
50	0.522026084592E+00	-0.242129844673E+02	-0.522026084592E+00	0.242129844673E+02	-0.890007639072E-03	-0.412809661752E-01	
51	0.502872959887E+00	-0.205307641185E+02	-0.502872959887E+00	0.205307641185E+02	-0.119230560451E-02	-0.486781892764E-01	
52	0.828772145700E+00	0.279369289497E+02	-0.828772145700E+00	-0.279369289497E+02	-0.106095211305E-02	0.357634410798E-01	
53	0.854906548852E+00	0.242883150612E+02	-0.854906548852E+00	-0.242883150612E+02	-0.144739192767E-02	0.411211157565E-01	
54	0.890752800052E+00	0.206353530687E+02	-0.890752800052E+00	-0.206353530687E+02	-0.208797313144E-02	0.483703927316E-01	
55	0.479527868207E+00	-0.168216752549E+02	-0.479527868207E+00	0.168216752549E+02	-0.169325586289E-02	-0.593988423559E-01	
56	0.942504798216E+00	0.169767389380E+02	-0.942504798216E+00	-0.169767389380E+02	-0.326015697984E-02	0.587231322838E-01	
57	0.453892583503E+00	-0.130615530535E+02	-0.453892583503E+00	0.130615530535E+02	-0.265729224401E-02	-0.764682324278E-01	
58	-0.581953517139E+01	-0.389677944159E+01	0.581953517139E+01	0.389677944159E+01	0.118640411246E+00	-0.794420004123E-01	
59	0.445334863371E+00	-0.918684055908E+01	-0.445334863371E+00	0.918684055908E+01	-0.526423181437E-02	-0.108596165094E+00	
60	0.619541702280E+00	-0.501386488846E+01	0.619541702280E+00	0.501386488846E+01	-0.242741699232E-01	-0.196447483400E+00	
61	0.102183778539E+01	0.133150046366E+02	-0.102183778539E+01	-0.133150046366E+02	-0.572992612188E-02	0.746635072323E-01	
62	0.114567394058E+01	0.966787961563E+01	-0.114567394058E+01	-0.966787961563E+01	-0.120876589928E-01	0.102002871705E+00	
63	0.125949805048E+01	0.608517533075E+01	-0.125949805048E+01	-0.608517533075E+01	-0.326162219368E-01	0.157582958573E+00	
64	0.410097973544E+00	0.129037864478E+01	-0.410097973544E+00	-0.129037864478E+01	-0.223699069595E+00	0.703872052248E+00	
65	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00	
66	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00	
67	0.000000000000E+00	0.000000000000E+00	-0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	
68	-0.200000000000E+01	0.000000000000E+00	0.200000000000E+01	-0.000000000000E+00	0.500000000000E+00	-0.000000000000E+00	

Comparing to the data in Table 4.3 of the paper, the first eigenvalues are well predicted. However, there are a number of eigenvalues that lie between the tabulated results for  $\gamma_8^+$  and  $\gamma_9^+$ , and between  $\gamma_8^-$  and  $\gamma_9^-$ .

This test case is given in Ref. 10 by Shankar. The data corresponds to the results given in Table 1 of Ref. 10. This is a hard-wall duct ( $\eta = 0$ ), with an

$\gamma_n^\pm$	Kousen Ref. [15]	Kousen report	current	index
$\gamma_0^+$	$0.620 - 5.014i$	$0.6195 - 5.0139i$	$0.61954 - 5.01386i$	60
$\gamma_1^+$	$-5.820 - 3.897i$	$-5.8195 - 3.8968i$	$-5.81953 - 3.89677i$	58
$\gamma_2^+$	$0.445 - 9.187i$	$0.4453 - 9.1868i$	$0.44533 - 9.18684i$	59
$\gamma_3^+$	$0.453 - 13.062i$	$0.4539 - 13.062i$	$0.45389 - 13.0615i$	57
$\gamma_4^+$	$0.480 - 16.822i$	$0.4795 - 16.822i$	$0.47952 - 16.8216i$	55
$\gamma_5^+$	$0.503 - 20.531i$	$0.5029 - 20.531i$	$0.50287 - 20.5307i$	51
$\gamma_6^+$	$0.522 - 24.213i$	$0.5220 - 24.213i$	$0.52202 - 24.2129i$	50
$\gamma_7^+$	$0.538 - 27.880i$	$0.5376 - 27.880i$	$0.53754 - 27.8800i$	48
$\gamma_8^+$	$0.550 - 31.537i$	$0.5502 - 31.537i$	$0.55024 - 31.5368i$	47
$\gamma_9^+$	$0.589 - 49.75i$	$0.5891 - 49.754i$	$0.58745 - 49.7669i$	33
$\gamma_0^-$	$0.410 + 1.290i$	$0.4101 + 1.2904i$	$0.41009 + 1.29037i$	64
$\gamma_1^-$	$1.259 + 6.085i$	$1.2595 + 6.0852i$	$1.25949 + 6.08517i$	63
$\gamma_2^-$	$1.146 + 9.668i$	$1.1457 + 9.6679i$	$1.14567 + 9.66787i$	62
$\gamma_3^-$	$1.022 + 13.315i$	$1.0218 + 13.315i$	$1.02183 + 13.3150i$	61
$\gamma_4^-$	$0.943 + 16.977i$	$0.9425 + 16.977i$	$0.94250 + 16.9767i$	56
$\gamma_5^-$	$0.891 + 20.635i$	$0.8908 + 20.635i$	$0.89075 + 20.6353i$	54
$\gamma_6^-$	$0.855 + 24.288i$	$0.8549 + 24.288i$	$0.85490 + 24.2883i$	53
$\gamma_7^-$	$0.829 + 27.937i$	$0.8288 + 27.937i$	$0.82877 + 27.9369i$	52
$\gamma_8^-$	$0.809 + 31.581i$	$0.8089 + 31.581i$	$0.80891 + 31.5812i$	49
$\gamma_9^-$	$0.755 + 49.77i$	$0.7547 + 49.772i$	$0.75658 + 49.7851i$	39

Table 1: Table 4.3 data

axial Mach number profile given by:

$$\begin{aligned}
b &= r_T - r_H \\
\tilde{r} &= \frac{r}{b} \\
M(\tilde{r}) &= M_0 (1 - \tilde{r})^{\frac{1}{7}} \\
M_0 &= 0.3 \\
m &= 0 \\
kb &= 20
\end{aligned}$$

Next, we translate Shankar's notation into the SWIRL notation. Comparing Shankar and SWIRL,

$$\begin{aligned}
p_n(r, x, t) &= a_n \phi_n(r) e^{ik(\beta_n x - ct)} \\
\tilde{p}_n(r, x, \theta, t) &= p_n(r) e^{i(k_x x + m\theta - \omega t)}
\end{aligned}$$

Translating this into SWIRL inputs:

$$\begin{aligned}
(kb)_{Shankar} &= \frac{\omega}{A_T} r_T \\
&= k_{SWIRL}
\end{aligned}$$

The input deck for this test case is:

```

&INPUTS
MM      = 0,
NPTS    = 32,
SIG     = 0.00,
AKRE    = 20.,
AKIM    = 0.,
IX      = 0,
NX      = 0,
IR      = 5,
RMAX    = 0.30,
SLOPE   = 0.00,
IS      = 0,
ANGOM   = 0.00,
GAM     = 0.00,
IREPEAT = 0,
IFD     = 0,
ITEST   = 0,
ETAHR   = 0.00,
ETAHI   = 0.00,
ETADR   = 0.00,
ETADI   = 0.00,
ED2     = 0.0,
ED4     = 0.0,
&END

```

$\gamma_n^\pm$	Shankar $\beta_n$	$\bar{\gamma}_n$ (16 points)	index	$\bar{\gamma}_n$ (32 points)	index
$\gamma_0^-$	0.81500	0.816009	45	0.815493	69
$\gamma_1^-$	0.76944	0.769595	47	0.769520	71
$\gamma_2^-$	0.72751	0.727753	46	0.727642	70
$\gamma_3^-$	0.65329	0.653542	44	0.653472	68
$\gamma_4^-$	0.54028	0.540499	43	0.540524	67
$\gamma_5^-$	0.36933	0.369465	42	0.369746	66
$\gamma_6^-$	0.06361	0.064613	41	0.064754	65
$\gamma_7^-$	$-0.28313 + 0.48807i$	$-0.278964 + 0.487173i$	39	$-0.282743 + 0.486526i$	63
$\gamma_8^-$	$-0.28357 + 0.80635i$	$-0.289882 + 0.854887i$	29	$-0.283251 + 0.804397i$	62
$\gamma_9^-$	$-0.28410 + 1.05658i$	$-0.263260 + 1.090093i$	27	$-0.283670 + 1.053847i$	57
$\gamma_{10}^-$	$-0.28410 + 1.27947i$	$-0.315161 + 1.655351i$	25	$-0.284027 + 1.275635i$	50

Table 2: Table 4.4 data

For the cylinder case of Shankar, the results in Table 4.4 are for  $\bar{\gamma}_n$  in order to compare with the  $\beta_n$  results:

The output in 'gam.acc' for this test case (using 16 points) is:

#	j	Re{gam}	Im{gam}	Re{gam/ak}	Im{gam/ak}	nz
1	0.163319288641E+03	0.000000000000E+00	0.816596443205E+01	0.000000000000E+00	0.122459509629E+00	0.000000000000E+00
2	-0.117303925650E+03	0.000000000000E+00	-0.586519628252E+01	0.000000000000E+00	-0.170497277811E+00	-0.000000000000E+00
3	-0.190345763833E+02	-0.825374426224E+02	-0.951728819167E+00	-0.412687213112E+01	-0.530599418459E-01	0.230077718435E+00
4	-0.190345763833E+02	0.825374426224E+02	-0.951728819167E+00	0.412687213112E+01	-0.530599418459E-01	-0.230077718435E+00
5	-0.662048021457E+01	-0.743548673755E+02	-0.331024010729E+00	-0.371774336877E+01	-0.237613547529E-01	0.266864687160E+00
6	-0.662048021457E+01	0.743548673755E+02	-0.331024010729E+00	0.371774336877E+01	-0.237613547529E-01	-0.266864687160E+00
7	0.105980545858E+03	0.000000000000E+00	0.529902729289E+01	0.000000000000E+00	0.188713879874E+00	0.000000000000E+00
8	0.104346612102E+03	0.000000000000E+00	0.521733060512E+01	0.000000000000E+00	0.191668896546E+00	0.000000000000E+00
9	0.939100552891E+02	0.000000000000E+00	0.469550276446E+01	0.000000000000E+00	0.212969739379E+00	0.000000000000E+00
10	0.865107830636E+02	0.000000000000E+00	0.432553915318E+01	0.000000000000E+00	0.231185053374E+00	0.000000000000E+00
11	0.814508584375E+02	0.000000000000E+00	0.407254292187E+01	0.000000000000E+00	0.245546828894E+00	0.000000000000E+00
12	0.777325529553E+02	0.000000000000E+00	0.388662764776E+01	0.000000000000E+00	0.257292462934E+00	0.000000000000E+00
13	0.748921084828E+02	0.000000000000E+00	0.374460542414E+01	0.000000000000E+00	0.267050833595E+00	0.000000000000E+00
14	0.726768738467E+02	0.000000000000E+00	0.363384369234E+01	0.000000000000E+00	0.275190702921E+00	0.000000000000E+00
15	0.709361335325E+02	0.000000000000E+00	0.354680667662E+01	0.000000000000E+00	0.281943757068E+00	0.000000000000E+00
16	0.695750820389E+02	0.000000000000E+00	0.347875410194E+01	0.000000000000E+00	0.287459237042E+00	0.000000000000E+00
17	0.685320076963E+02	0.000000000000E+00	0.342660038481E+01	0.000000000000E+00	0.291834438714E+00	0.000000000000E+00
18	0.677612959441E+02	0.000000000000E+00	0.338806479721E+01	0.000000000000E+00	0.295153741105E+00	0.000000000000E+00
19	0.672124795891E+02	0.000000000000E+00	0.336062397946E+01	0.000000000000E+00	0.297563787592E+00	0.000000000000E+00
20	0.665514226573E+02	0.000000000000E+00	0.332757113286E+01	0.000000000000E+00	0.300519496074E+00	0.000000000000E+00
21	0.668152109662E+02	0.000000000000E+00	0.334076054831E+01	0.000000000000E+00	0.299333036756E+00	0.000000000000E+00
22	-0.471272186164E+01	-0.379656309224E+02	-0.235636093082E+00	-0.189828154612E+01	-0.643990909023E-01	0.518798305675E+00
23	-0.630323705080E+01	-0.331070325502E+02	-0.315161852540E+00	-0.165535162751E+01	-0.110991398568E+00	0.582969641721E+00
24	-0.471272186164E+01	0.379656309224E+02	-0.235636093082E+00	0.189828154612E+01	-0.643990909023E-01	-0.518798305675E+00
25	-0.630323705080E+01	0.331070325502E+02	-0.315161852540E+00	0.165535162751E+01	-0.110991398568E+00	-0.582969641721E+00
26	-0.526520450645E+01	-0.218018701463E+02	-0.263260225323E+00	-0.109009350732E+01	-0.209333789485E+00	0.866797878412E+00
27	-0.526520450645E+01	0.218018701463E+02	-0.263260225323E+00	0.109009350732E+01	-0.209333789485E+00	-0.866797878412E+00
28	-0.579764528750E+01	-0.170977580948E+02	-0.289882264375E+00	-0.854887904738E+00	-0.355742661846E+00	0.104911592114E+01
29	-0.579764528750E+01	0.170977580948E+02	-0.289882264375E+00	0.854887904738E+00	-0.355742661846E+00	-0.104911592114E+01
30	-0.277384694658E+02	0.000000000000E+00	-0.138692347284E+01	0.000000000000E+00	0.721020315527E+00	-0.000000000000E+00
31	-0.265814970586E+02	0.000000000000E+00	-0.132907485293E+01	0.000000000000E+00	-0.752403070299E+00	-0.000000000000E+00
32	-0.254247235137E+02	0.000000000000E+00	-0.127123617569E+01	0.000000000000E+00	-0.786635889638E+00	-0.000000000000E+00
33	-0.240697952822E+02	0.000000000000E+00	-0.120348976411E+01	0.000000000000E+00	-0.830916913316E+00	-0.000000000000E+00
34	-0.219589611241E+02	0.000000000000E+00	-0.109794805621E+01	0.000000000000E+00	-0.910789899711E+00	-0.000000000000E+00
35	-0.186181534184E+02	0.000000000000E+00	-0.930907670922E+00	0.000000000000E+00	-0.1074220338859E+01	-0.000000000000E+00
36	-0.125335327984E+02	0.000000000000E+00	-0.626676639920E+00	0.000000000000E+00	-0.159571928535E+01	-0.000000000000E+00
37	-0.557929881970E+01	-0.974346480870E+01	-0.278964940985E+00	-0.487173240435E+00	-0.885155619784E+00	0.154580045097E+01
38	-0.543485433791E+01	-0.550803802019E+01	-0.271742716896E+00	-0.275401901009E+00	-0.181536610218E+01	0.183981113194E+01
39	-0.557929881970E+01	0.974346480870E+01	-0.278964940985E+00	0.487173240435E+00	-0.885155619784E+00	-0.154580045097E+01

40	-0.543485433791E+01	0.550803802019E+01	-0.271742716896E+00	0.275401901009E+00	-0.181536610218E+01	-0.18398113194E+01
41	0.129227898584E+01	0.000000000000E+00	0.646139492918E-01	0.000000000000E+00	0.154765342617E+02	0.000000000000E+00
42	0.738930776482E+01	0.000000000000E+00	0.369465388241E+00	0.000000000000E+00	0.270661347944E+01	0.000000000000E+00
43	0.108099837563E+02	0.000000000000E+00	0.540499187815E+00	0.000000000000E+00	0.185014154053E+01	0.000000000000E+00
44	0.130708424342E+02	0.000000000000E+00	0.653542121712E+00	0.000000000000E+00	0.153012325721E+01	0.000000000000E+00
45	0.163201998955E+02	0.000000000000E+00	0.816009994775E+00	0.000000000000E+00	0.122547518585E+01	0.000000000000E+00
46	0.145550786146E+02	0.000000000000E+00	0.727753930731E+00	0.000000000000E+00	0.137409082627E+01	0.000000000000E+00
47	0.153919047635E+02	0.000000000000E+00	0.769595238174E+00	0.000000000000E+00	0.129938433919E+01	0.000000000000E+00
48	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
49	0.66666666667E+02	0.000000000000E+00	0.333333333333E+01	0.000000000000E+00	0.300000000000E+00	0.000000000000E+00
50	0.667724010447E+02	0.000000000000E+00	0.333862005224E+01	0.000000000000E+00	0.299524948737E+00	0.000000000000E+00
51	0.670926323634E+02	0.000000000000E+00	0.335463161817E+01	0.000000000000E+00	0.298095324263E+00	0.000000000000E+00
52	0.676368394188E+02	0.000000000000E+00	0.338184197094E+01	0.000000000000E+00	0.295696844676E+00	0.000000000000E+00
53	0.684221959778E+02	0.000000000000E+00	0.342110979889E+01	0.000000000000E+00	0.292302807798E+00	0.000000000000E+00
54	0.694758827877E+02	0.000000000000E+00	0.347379413939E+01	0.000000000000E+00	0.287869677901E+00	0.000000000000E+00
55	0.708389526804E+02	0.000000000000E+00	0.354194763402E+01	0.000000000000E+00	0.282330543341E+00	0.000000000000E+00
56	0.725727534440E+02	0.000000000000E+00	0.362863767220E+01	0.000000000000E+00	0.275585520059E+00	0.000000000000E+00
57	0.747700003610E+02	0.000000000000E+00	0.373850001805E+01	0.000000000000E+00	0.267486958719E+00	0.000000000000E+00
58	0.775750782186E+02	0.000000000000E+00	0.387875391093E+01	0.000000000000E+00	0.257814757771E+00	0.000000000000E+00
59	0.812245018074E+02	0.000000000000E+00	0.406122509037E+01	0.000000000000E+00	0.246231119366E+00	0.000000000000E+00
60	0.861370575300E+02	0.000000000000E+00	0.430685287650E+01	0.000000000000E+00	0.232188103164E+00	0.000000000000E+00
61	0.931487672395E+02	0.000000000000E+00	0.465743836198E+01	0.000000000000E+00	0.214710302591E+00	0.000000000000E+00
62	0.104294381450E+03	0.000000000000E+00	0.521471907251E+01	0.000000000000E+00	0.191764884377E+00	0.000000000000E+00
63	0.126919835334E+03	0.000000000000E+00	0.634599176670E+01	0.000000000000E+00	0.157579782131E+00	0.000000000000E+00
64	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00

The output in 'gam.acc' for this test case (using 32 points) is:

#	j	Re{gam}	Im{gam}	Re{gam/ak}	Im{gam/ak}	nz
1	-0.419148876321E+03	0.000000000000E+00	-0.209574438160E+02	0.000000000000E+00	-0.477157428539E-01	-0.000000000000E+00
2	0.405930803348E+03	0.000000000000E+00	0.202965401674E+02	0.000000000000E+00	0.492694809930E-01	0.000000000000E+00
3	-0.658653414210E+03	-0.290862176480E+03	-0.329326707105E+00	-0.145431088240E+02	-0.155628739242E-02	-0.687258470725E-01
4	-0.658653414210E+03	0.290862176480E+03	-0.329326707105E+00	0.145431088240E+02	-0.155628739242E-02	-0.687258470725E-01
5	-0.784863705464E+02	-0.247706992620E+03	-0.392431852732E+01	-0.123853496310E+02	-0.232487234685E-01	-0.733741582460E-01
6	-0.784863705464E+02	0.247706992620E+03	-0.392431852732E+01	0.123853496310E+02	-0.232487234685E-01	-0.733741582460E-01
7	0.309500174533E+03	0.000000000000E+00	0.154750087267E+02	0.000000000000E+00	0.646203189712E-01	0.000000000000E+00
8	-0.466033764842E+01	-0.153375838349E+03	-0.233016882421E+00	-0.766879191744E+01	-0.395851869904E-02	0.130278355322E+00
9	-0.653017072648E+01	-0.145048816442E+03	-0.326508536324E+00	-0.725244082209E+01	-0.619507823807E-02	-0.137605708003E+00
10	-0.466033764842E+01	0.153375838349E+03	-0.233016882421E+00	0.766879191744E+01	-0.395851869904E-02	-0.130278355322E+00
11	-0.653017072648E+01	0.145048816442E+03	-0.326508536324E+00	0.725244082209E+01	-0.619507823807E-02	-0.137605708003E+00
12	0.134486754616E+03	0.000000000000E+00	0.672433773078E+01	0.000000000000E+00	0.148713529873E+00	0.000000000000E+00
13	-0.742683514153E+01	0.101949221883E+03	-0.371341757077E+00	0.509746109414E+01	-0.142156686262E-01	-0.195140504315E+00
14	-0.657782693991E+01	0.982123578086E+02	-0.328891346996E+00	0.491061789043E+01	-0.135780184656E-01	-0.202730965722E+00
15	-0.742683514153E+01	-0.101949221883E+03	-0.371341757077E+00	-0.509746109414E+01	-0.142156686262E-01	-0.195140504315E+00
16	-0.657782693991E+01	-0.982123578086E+02	-0.328891346996E+00	0.491061789043E+01	-0.135780184656E-01	-0.202730965722E+00
17	0.115944495642E+03	0.000000000000E+00	0.579722478208E+01	0.000000000000E+00	0.172496330156E+00	0.000000000000E+00
18	-0.518443551179E+01	-0.789084126835E+02	-0.259221775589E+00	-0.394542063418E+01	-0.165811311507E-01	-0.252368987255E+00
19	-0.610033946865E+01	-0.762435857044E+02	-0.305016973433E+00	-0.381217928522E+01	-0.208547972923E-01	-0.260648531589E+00
20	0.105446208485E+03	0.000000000000E+00	0.527231042423E+01	0.000000000000E+00	0.189670167258E+00	0.000000000000E+00
21	-0.518443551179E+01	0.789084126835E+02	-0.259221775589E+00	0.394542063418E+01	-0.165811311507E-01	-0.252368987255E+00
22	-0.610033946865E+01	0.762435857044E+02	-0.305016973433E+00	0.381217928522E+01	-0.208547972923E-01	-0.260648531589E+00
23	0.986084901449E+02	0.000000000000E+00	0.493042450725E+01	0.000000000000E+00	0.202822292184E+00	0.000000000000E+00
24	-0.570634712692E+01	-0.665650030025E+02	-0.285317356346E+00	-0.332825015012E+01	-0.255691528167E-01	-0.298266245666E+00
25	-0.570634712692E+01	0.665650030025E+02	-0.285317356346E+00	0.332825015012E+01	-0.255691528167E-01	-0.298266245666E+00
26	-0.587913448638E+01	0.636369771958E+02	-0.293956724319E+00	0.318184885979E+01	-0.287894452460E-01	-0.311622956549E+00
27	-0.587913448637E+01	-0.636369771958E+02	-0.293956724319E+00	-0.318184885979E+01	-0.287894452460E-01	-0.311622956549E+00
28	0.935764252574E+02	0.000000000000E+00	0.467882126287E+01	0.000000000000E+00	0.213729044949E+00	0.000000000000E+00
29	-0.569095796687E+01	0.588308495638E+02	-0.284547898344E+00	0.294154247819E+01	-0.325806776446E-01	-0.336806027448E+00
30	-0.569095796687E+01	-0.588308495638E+02	-0.284547898344E+00	-0.294154247819E+01	-0.325806776446E-01	-0.336806027448E+00
31	-0.571582832349E+01	-0.554812940612E+02	-0.285791416174E+00	-0.277406470306E+01	-0.367477576278E-01	-0.356696008285E+00
32	-0.571582832349E+01	0.554812940612E+02	-0.285791416174E+00	0.277406470306E+01	-0.367477576278E-01	-0.356696008285E+00
33	-0.571740595343E+01	-0.518791633984E+02	-0.285870297671E+00	-0.259395816992E+01	-0.419759165456E-01	-0.380885221550E+00
34	-0.571740595343E+01	0.518791633984E+02	-0.285870297671E+00	0.259395816992E+01	-0.419759165456E-01	-0.380885221550E+00
35	-0.571464946639E+01	-0.483366390993E+02	-0.285732473319E+00	-0.241683195497E+01	-0.482434566599E-01	-0.408061171064E+00
36	-0.571464946639E+01	0.483366390993E+02	-0.285732473319E+00	0.241683195497E+01	-0.482434566599E-01	-0.408061171064E+00
37	-0.570863182790E+01	-0.447486900786E+02	-0.285431591395E+00	-0.223743450393E+01	-0.561035594901E-01	-0.439783274104E+00
38	-0.570863182790E+01	0.447486900786E+02	-0.285431591395E+00	0.223743450393E+01	-0.561035594901E-01	-0.439783274104E+00
39	-0.570360025930E+01	-0.411061047912E+02	-0.285180012965E+00	-0.205530523956E+01	-0.662345623378E-01	-0.477355483638E+00
40	-0.687998218961E+01	-0.380630218933E+02	-0.343999109480E+00	-0.190315109466E+01	-0.919705152587E-01	-0.508820464843E+00
41	-0.569664019370E+01	-0.373916704144E+02	-0.284832009685E+00	-0.186958352072E+01	-0.796405017132E-01	-0.522745213045E+00
42	-0.570360025930E+01	0.411061047912E+02	-0.285180012965E+00	0.205530523956E+01	-0.662345623378E-01	-0.477355483638E+00
43	-0.687998218961E+01	0.380630218933E+02	-0.343999109480E+00	0.190315109466E+01	-0.919705152587E-01	-0.508820464843E+00
44	-0.569664019370E+01	0.373916704144E+02	-0.284832009685E+00	0.186958352072E+01	-0.796405017132E-01	-0.522745213045E+00
45	-0.569255861892E+01	-0.335842778282E+02	-0.284627930946E+00	-0.167921389141E+01	-0.981214298780E-01	-0.578885099394E+00
46	-0.569255861892E+01	0.335842778282E+02	-0.284627930946E+00	0.167921389141E+01	-0.981214298780E-01	-0.578885099394E+00

47 -0.568686921779E+01 -0.296444024663E+02 -0.284343460889E+00 -0.148222012331E+01 -0.124830978733E+00 0.650716524698E+00  
48 -0.568686921779E+01 0.296444024663E+02 -0.284343460889E+00 0.148222012331E+01 -0.124830978733E+00 -0.650716524698E+00  
49 -0.568055017101E+01 -0.255127184518E+02 -0.284027508550E+00 -0.127563592259E+01 -0.166300374694E+00 0.746895020775E+00  
50 -0.568055017101E+01 0.255127184518E+02 -0.284027508550E+00 0.127563592259E+01 -0.166300374694E+00 -0.746895020775E+00  
51 -0.277387784651E+02 0.000000000000E+00 -0.138693892325E+01 0.000000000000E+00 -0.721012283406E+00 -0.000000000000E+00  
52 -0.265826768992E+02 0.000000000000E+00 -0.132913384496E+01 0.000000000000E+00 -0.752369675780E+00 -0.000000000000E+00  
53 -0.254341931169E+02 0.000000000000E+00 -0.127170965584E+01 0.000000000000E+00 -0.786343011084E+00 -0.000000000000E+00  
54 -0.240896983111E+02 0.000000000000E+00 -0.120448491555E+01 0.000000000000E+00 -0.830230405617E+00 -0.000000000000E+00  
55 -0.219801383439E+02 0.000000000000E+00 -0.109900691719E+01 0.000000000000E+00 -0.909912380309E+00 -0.000000000000E+00  
56 -0.186373158152E+02 0.000000000000E+00 -0.931865790762E+00 0.000000000000E+00 -0.107311590351E+01 -0.000000000000E+00  
57 -0.567340162149E+01 0.210769456156E+02 -0.283670081075E+00 0.105384728078E+01 -0.238165510351E+00 -0.884795726456E+00  
58 -0.567340162149E+01 -0.210769456156E+02 -0.283670081075E+00 -0.105384728078E+01 -0.238165510351E+00 0.884795726456E+00  
59 -0.125777769077E+02 0.000000000000E+00 -0.62888845383E+00 0.000000000000E+00 -0.159010611707E+01 -0.000000000000E+00  
60 -0.566503698651E+01 -0.160879564984E+02 -0.283251849325E+00 -0.804397824922E+00 -0.389463397438E+00 0.110602458742E+01  
61 -0.565486924799E+01 -0.973053693345E+01 -0.282743462489E+00 -0.486526846673E+00 -0.892914776707E+00 0.153647057596E+01  
62 -0.566503698651E+01 0.160879564984E+02 -0.283251849325E+00 0.804397824922E+00 -0.389463397438E+00 -0.110602458742E+01  
63 -0.565486924799E+01 0.973053693345E+01 -0.282743462489E+00 0.486526846673E+00 -0.892914776707E+00 -0.153647057596E+01  
64 0.896639357693E+02 0.000000000000E+00 0.448319678847E+01 0.000000000000E+00 0.223055120528E+00 0.000000000000E+00  
65 0.129509349093E+01 0.000000000000E+00 0.647546745465E-01 0.000000000000E+00 0.154429005628E+02 0.000000000000E+00  
66 0.739493575533E+01 0.000000000000E+00 0.369746787766E+00 0.000000000000E+00 0.270455358393E+01 0.000000000000E+00  
67 0.108104868030E+02 0.000000000000E+00 0.540524340149E+00 0.000000000000E+00 0.185005544750E+01 0.000000000000E+00  
68 0.130694531452E+02 0.000000000000E+00 0.653472657260E+00 0.000000000000E+00 0.153028591004E+01 0.000000000000E+00  
69 0.163098658470E+02 0.000000000000E+00 0.815493292350E+00 0.000000000000E+00 0.122625165575E+01 0.000000000000E+00  
70 0.14552853481E+02 0.000000000000E+00 0.727642767404E+00 0.000000000000E+00 0.137430074866E+01 0.000000000000E+00  
71 0.153904181199E+02 0.000000000000E+00 0.769520905996E+00 0.000000000000E+00 0.129950985374E+01 0.000000000000E+00  
72 0.865054770642E+02 0.000000000000E+00 0.432527385321E+01 0.000000000000E+00 0.231199233607E+00 0.000000000000E+00  
73 0.838882082668E+02 0.000000000000E+00 0.419441041334E+01 0.000000000000E+00 0.238412530357E+00 0.000000000000E+00  
74 0.816766572258E+02 0.000000000000E+00 0.408383286129E+01 0.000000000000E+00 0.244868003654E+00 0.000000000000E+00  
75 0.797794690015E+02 0.000000000000E+00 0.398897345007E+01 0.000000000000E+00 0.250691064384E+00 0.000000000000E+00  
76 0.781323134320E+02 0.000000000000E+00 0.390661567160E+01 0.000000000000E+00 0.255976037589E+00 0.000000000000E+00  
77 0.766883760716E+02 0.000000000000E+00 0.383441880358E+01 0.000000000000E+00 0.260795716698E+00 0.000000000000E+00  
78 0.754127097705E+02 0.000000000000E+00 0.377063548852E+01 0.000000000000E+00 0.265207284831E+00 0.000000000000E+00  
79 0.742787034150E+02 0.000000000000E+00 0.371393517075E+01 0.000000000000E+00 0.269256180850E+00 0.000000000000E+00  
80 0.732657804025E+02 0.000000000000E+00 0.366328902012E+01 0.000000000000E+00 0.272978734276E+00 0.000000000000E+00  
81 0.723578447929E+02 0.000000000000E+00 0.361789223964E+01 0.000000000000E+00 0.276404031342E+00 0.000000000000E+00  
82 0.715421978123E+02 0.000000000000E+00 0.357710989062E+01 0.000000000000E+00 0.279555292003E+00 0.000000000000E+00  
83 0.708087562810E+02 0.000000000000E+00 0.354043781405E+01 0.000000000000E+00 0.282450943223E+00 0.000000000000E+00  
84 0.701494654443E+02 0.000000000000E+00 0.350747327222E+01 0.000000000000E+00 0.285105522520E+00 0.000000000000E+00  
85 0.695578365653E+02 0.000000000000E+00 0.347789182827E+01 0.000000000000E+00 0.287530560807E+00 0.000000000000E+00  
86 0.690285708194E+02 0.000000000000E+00 0.345142854097E+01 0.000000000000E+00 0.289735101895E+00 0.000000000000E+00  
87 0.685572669257E+02 0.000000000000E+00 0.342786334628E+01 0.000000000000E+00 0.291726915276E+00 0.000000000000E+00  
88 0.681402464491E+02 0.000000000000E+00 0.340701232245E+01 0.000000000000E+00 0.293512293281E+00 0.000000000000E+00  
89 0.677745443370E+02 0.000000000000E+00 0.338872721685E+01 0.000000000000E+00 0.295096045214E+00 0.000000000000E+00  
90 0.674580039506E+02 0.000000000000E+00 0.337290019753E+01 0.000000000000E+00 0.296480758589E+00 0.000000000000E+00  
91 0.671895173396E+02 0.000000000000E+00 0.335947586698E+01 0.000000000000E+00 0.297665481044E+00 0.000000000000E+00  
92 0.666544306987E+02 0.000000000000E+00 0.333272153493E+01 0.000000000000E+00 0.300055071964E+00 0.000000000000E+00  
93 0.666943560726E+02 0.000000000000E+00 0.333471780363E+01 0.000000000000E+00 0.299875449404E+00 0.000000000000E+00  
94 0.668000530852E+02 0.000000000000E+00 0.334000265426E+01 0.000000000000E+00 0.299400959674E+00 0.000000000000E+00  
95 0.669693098976E+02 0.000000000000E+00 0.334846549488E+01 0.000000000000E+00 0.298644260044E+00 0.000000000000E+00  
96 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00  
97 0.666666666667E+02 0.000000000000E+00 0.333333333333E+01 0.000000000000E+00 0.300000000000E+00 0.000000000000E+00  
98 0.666944512249E+02 0.000000000000E+00 0.333472256125E+01 0.000000000000E+00 0.299875021575E+00 0.000000000000E+00  
99 0.667776922150E+02 0.000000000000E+00 0.333888461075E+01 0.000000000000E+00 0.299501215700E+00 0.000000000000E+00  
100 0.669160870679E+02 0.000000000000E+00 0.334580435339E+01 0.000000000000E+00 0.298881791754E+00 0.000000000000E+00  
101 0.671092425331E+02 0.000000000000E+00 0.33546212666E+01 0.000000000000E+00 0.298021542862E+00 0.000000000000E+00  
102 0.673568188899E+02 0.000000000000E+00 0.336784094449E+01 0.000000000000E+00 0.296926136502E+00 0.000000000000E+00  
103 0.676586963257E+02 0.000000000000E+00 0.338293481629E+01 0.000000000000E+00 0.295601320837E+00 0.000000000000E+00  
104 0.680151441632E+02 0.000000000000E+00 0.340075720816E+01 0.000000000000E+00 0.294052159208E+00 0.000000000000E+00  
105 0.684269818625E+02 0.000000000000E+00 0.342134909312E+01 0.000000000000E+00 0.292282363706E+00 0.000000000000E+00  
106 0.688957299739E+02 0.000000000000E+00 0.344478649869E+01 0.000000000000E+00 0.290293752713E+00 0.000000000000E+00  
107 0.694237569101E+02 0.000000000000E+00 0.347118784551E+01 0.000000000000E+00 0.288085820908E+00 0.000000000000E+00  
108 0.700144328307E+02 0.000000000000E+00 0.350072164153E+01 0.000000000000E+00 0.285655388345E+00 0.000000000000E+00  
109 0.706723056747E+02 0.000000000000E+00 0.353361528373E+01 0.000000000000E+00 0.282996285590E+00 0.000000000000E+00  
110 0.714033176781E+02 0.000000000000E+00 0.357016588390E+01 0.000000000000E+00 0.280099029714E+00 0.000000000000E+00  
111 0.722150849613E+02 0.000000000000E+00 0.361075424807E+01 0.000000000000E+00 0.276950446167E+00 0.000000000000E+00  
112 0.731172694815E+02 0.000000000000E+00 0.365586347408E+01 0.000000000000E+00 0.273533190474E+00 0.000000000000E+00  
113 0.741220836745E+02 0.000000000000E+00 0.370610418372E+01 0.000000000000E+00 0.269825118352E+00 0.000000000000E+00  
114 0.752449862794E+02 0.000000000000E+00 0.376224931397E+01 0.000000000000E+00 0.265798440387E+00 0.000000000000E+00  
115 0.765056579357E+02 0.000000000000E+00 0.382528289678E+01 0.000000000000E+00 0.261418573994E+00 0.000000000000E+00  
116 0.779293959244E+02 0.000000000000E+00 0.389646979622E+01 0.000000000000E+00 0.256642564244E+00 0.000000000000E+00  
117 0.795491556641E+02 0.000000000000E+00 0.397745778321E+01 0.000000000000E+00 0.251416873417E+00 0.000000000000E+00  
118 0.814086258893E+02 0.000000000000E+00 0.407043129447E+01 0.000000000000E+00 0.245674211811E+00 0.000000000000E+00  
119 0.835670258134E+02 0.000000000000E+00 0.417835129067E+01 0.000000000000E+00 0.239328847776E+00 0.000000000000E+00  
120 0.861069152010E+02 0.000000000000E+00 0.430534576005E+01 0.000000000000E+00 0.232269382236E+00 0.000000000000E+00  
121 0.891475960278E+02 0.000000000000E+00 0.445737980139E+01 0.000000000000E+00 0.224347047942E+00 0.000000000000E+00  
122 0.928696688792E+02 0.000000000000E+00 0.464348344396E+01 0.000000000000E+00 0.215355564862E+00 0.000000000000E+00  
123 0.975639677421E+02 0.000000000000E+00 0.487819838710E+01 0.000000000000E+00 0.204993712975E+00 0.000000000000E+00

124	0.103740573661E+03	0.000000000000E+00	0.518702868305E+01	0.000000000000E+00	0.192788600392E+00	0.000000000000E+00
125	0.112412971203E+03	0.000000000000E+00	0.562064856014E+01	0.000000000000E+00	0.177915411238E+00	0.000000000000E+00
126	0.126043252760E+03	0.000000000000E+00	0.630216263801E+01	0.000000000000E+00	0.158675689194E+00	0.000000000000E+00
127	0.153516836704E+03	0.000000000000E+00	0.767584183522E+01	0.000000000000E+00	0.130278869923E+00	0.000000000000E+00
128	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00

### 7.3 Annulus, Shear Flow (Table 4.5)

This test case is given in Ref. 10 by Shankar. In order to get the correct input data,

$$\begin{aligned}
\tilde{r}_i &= 6.0 \\
\frac{r_i}{b} &= 6.0 \\
\frac{r_i}{r_o - r_i} &= 6.0 \\
r_i &= 6r_o - 6r_i \\
7r_i &= 6r_o \\
\frac{r_i}{r_o} &= \frac{6}{7} \\
\sigma &= \frac{6}{7} \\
b &= \frac{1}{7} \\
kb &= 10 \\
k &= 70
\end{aligned}$$

The axial Mach number profile is given by:

$$\begin{aligned}
M\left(\frac{r}{b}\right) &= M_0 \left(1 - 2 \left| \frac{r_i - r}{b} + 0.5 \right| \right)^{\frac{1}{7}} \\
M_0 &= 0.3
\end{aligned}$$

The input data for SWIRL is then:

```

&INPUTS
MM      = 0,
NPTS    = 64,
SIG     = 0.857142857143,
AKRE    = 70.,
AKIM    = 0.,
IX      = 0,
NX      = 0,
IR      = 5,

```



```

RMAX      = 0.30,
SLOPE     = 0.00,
IS        = 0,
ANGOM     = 0.00,
GAM       = 0.00,
IREPEAT   = 0,
IFD       = 0,
ITEST     = 0,
ETAHR     = 0.00,
ETAHI     = 0.00,
ETADR     = 0.00,
ETADI     = 0.00,
ED2       = 0.0,
ED4       = 0.0,
&END

```

The output in 'gam.acc' for this test case is:

#	j	Re{gam}	Im{gam}	Re{gam/ak}	Im{gam/ak}	nz
1	-0.120741787661E+04	-0.349111287122E+04	-0.172488268087E+02	-0.498730410175E+02	-0.619382545985E-02	0.179087490784E-01
2	-0.120705227993E+04	-0.347634030347E+04	-0.172436039989E+02	-0.496620043353E+02	-0.623941639104E-02	0.179696729222E-01
3	-0.120741787661E+04	0.349111287122E+04	-0.172488268087E+02	0.498730410175E+02	-0.619382545984E-02	-0.179087490784E-01
4	-0.120705227993E+04	0.347634030347E+04	-0.172436039989E+02	0.496620043353E+02	-0.623941639104E-02	-0.179696729222E-01
5	0.363090674699E+03	-0.311800080711E+04	0.518700963856E+01	-0.445428686729E+02	0.257935420021E-02	0.221499174682E-01
6	0.364635350207E+03	-0.310984927648E+04	0.520907643153E+01	-0.444264182354E+02	0.260344331880E-02	0.222038711187E-01
7	0.364635350207E+03	0.310984927648E+04	0.520907643153E+01	0.444264182354E+02	0.260344331880E-02	-0.222038711187E-01
8	0.363090674699E+03	0.311800080711E+04	0.518700963856E+01	0.445428686729E+02	0.257935420021E-02	-0.221499174682E-01
9	0.276837698579E+04	0.000000000000E+00	0.395482426541E+02	0.000000000000E+00	0.252855735904E-01	0.000000000000E+00
10	0.271834068634E+04	0.000000000000E+00	0.388334383762E+02	0.000000000000E+00	0.257510033058E-01	0.000000000000E+00
11	-0.283785246651E+03	-0.198121755313E+04	-0.405407495216E+01	-0.283031079018E+02	-0.495910435226E-02	0.346214777070E-01
12	-0.288389122393E+03	-0.196261386206E+04	-0.411984460562E+01	-0.280373408866E+02	-0.513014701083E-02	0.349128897591E-01
13	-0.283785246651E+03	0.198121755313E+04	-0.405407495216E+01	0.283031079018E+02	-0.495910435226E-02	-0.346214777070E-01
14	-0.288389122393E+03	0.196261386206E+04	-0.411984460562E+01	0.280373408866E+02	-0.513014701082E-02	-0.349128897591E-01
15	0.359086850807E+02	-0.163143883751E+04	0.512981215439E+00	-0.233062691073E+02	0.943942633522E-03	0.428861337933E-01
16	0.368238415522E+02	-0.161980711915E+04	0.526054879317E+00	-0.231401017021E+02	0.981920068708E-03	0.431926993677E-01
17	0.359086850806E+02	0.163143883751E+04	0.512981215438E+00	0.233062691073E+02	0.943942633520E-03	-0.428861337933E-01
18	0.368238415522E+02	0.161980711915E+04	0.526054879318E+00	0.231401017021E+02	0.981920068709E-03	-0.431926993677E-01
19	-0.742193248141E+02	-0.136832783077E+04	-0.106027606877E+01	-0.195475404395E+02	-0.276667959964E-02	0.510072640041E-01
20	-0.771377775956E+02	-0.135027287940E+04	-0.110196825137E+01	-0.192896125629E+02	-0.295193666861E-02	0.516727361039E-01
21	-0.742193248141E+02	0.136832783077E+04	-0.106027606877E+01	0.195475404395E+02	-0.276667959963E-02	-0.510072640041E-01
22	-0.771377775956E+02	0.135027287940E+04	-0.110196825137E+01	0.192896125629E+02	-0.295193666861E-02	-0.516727361039E-01
23	-0.291264334872E+01	-0.122047595092E+04	-0.416091906960E-01	-0.174353707274E+02	-0.136875091649E-03	0.573543471126E-01
24	-0.299392234434E+01	-0.120409350298E+04	-0.427703192049E-01	-0.172013357568E+02	-0.144549122790E-03	0.581346606872E-01
25	-0.291264334873E+01	0.122047595092E+04	-0.416091906962E-01	0.174353707274E+02	-0.136875091650E-03	-0.573543471126E-01
26	-0.299392234443E+01	0.120409350298E+04	-0.427703192061E-01	0.172013357568E+02	-0.144549122794E-03	-0.581346606872E-01
27	-0.334118131995E+02	-0.112830252029E+04	-0.477311617136E+00	-0.161186074327E+02	-0.183555035562E-02	0.619857437847E-01
28	-0.351299647894E+02	-0.110503918080E+04	-0.501856639849E+00	-0.157862740114E+02	-0.201178563818E-02	0.632822141121E-01
29	-0.334118131996E+02	0.112830252029E+04	-0.477311617136E+00	0.161186074327E+02	-0.183555035562E-02	-0.619857437847E-01
30	-0.351299647894E+02	0.110503918080E+04	-0.501856639848E+00	0.157862740114E+02	-0.201178563818E-02	-0.632822141121E-01
31	-0.171140520383E+02	-0.107070602389E+04	-0.244486457690E+00	-0.152958003413E+02	-0.104471890720E-02	0.653607237323E-01
32	-0.171140520383E+02	0.107070602389E+04	-0.244486457690E+00	0.152958003413E+02	-0.104471890720E-02	-0.653607237323E-01
33	-0.167024565682E+02	-0.104557021514E+04	-0.238606522403E+00	-0.149367173591E+02	-0.106920547464E-02	0.669320344334E-01
34	-0.167024565682E+02	0.104557021514E+04	-0.238606522403E+00	0.149367173591E+02	-0.106920547464E-02	-0.669320344334E-01
35	-0.212710709991E+02	-0.102328071558E+04	-0.303872442845E+00	-0.146182959368E+02	-0.142137999173E-02	0.683778797551E-01
36	-0.212710709991E+02	0.102328071558E+04	-0.303872442845E+00	0.146182959368E+02	-0.142137999173E-02	-0.683778797551E-01
37	-0.228054873142E+02	-0.998324712386E+03	-0.325792675918E+00	-0.142617816055E+02	-0.160091097336E-02	0.700808961023E-01
38	-0.212774834152E+02	-0.976894964284E+03	-0.303964048789E+00	-0.139556423469E+02	-0.155997119231E-02	0.716216280121E-01
39	-0.228054873142E+02	0.998324712386E+03	-0.325792675918E+00	0.142617816055E+02	-0.160091097336E-02	-0.700808961023E-01
40	-0.208993998168E+02	-0.953074686225E+03	-0.298562854526E+00	-0.136153526604E+02	-0.160978991568E-02	0.734111999492E-01
41	-0.212774834152E+02	0.976894964284E+03	-0.303964048789E+00	0.139556423469E+02	-0.155997119231E-02	-0.716216280121E-01
42	-0.209953926758E+02	-0.930873564586E+03	-0.299934181083E+00	-0.132981937798E+02	-0.169519543977E-02	0.751599479970E-01
43	-0.208993998168E+02	0.953074686225E+03	-0.298562854526E+00	0.136153526604E+02	-0.160978991568E-02	-0.734111999492E-01
44	-0.214307849008E+02	-0.907211471256E+03	-0.306154070012E+00	-0.129601638751E+02	-0.182169962394E-02	0.771164847049E-01
45	-0.750743340143E+02	-0.867636273988E+03	-0.107249048592E+01	-0.123948039141E+02	-0.692906567927E-02	0.800794147177E-01
46	-0.211032442940E+02	-0.885049284498E+03	-0.301474918485E+00	-0.126435612071E+02	-0.188480103133E-02	0.790466992164E-01
47	-0.209953926758E+02	0.930873564586E+03	-0.299934181083E+00	0.132981937798E+02	-0.169519543977E-02	-0.751599479970E-01
48	-0.214307849008E+02	0.907211471256E+03	-0.306154070012E+00	0.129601638751E+02	-0.182169962394E-02	-0.771164847049E-01
49	-0.750743340143E+02	0.867636273988E+03	-0.107249048592E+01	0.123948039141E+02	-0.692906567927E-02	0.800794147177E-01
50	-0.211032442940E+02	0.885049284498E+03	-0.301474918485E+00	0.126435612071E+02	-0.188480103133E-02	-0.790466992164E-01
51	-0.208598209490E+02	-0.861876916702E+03	-0.297997442128E+00	-0.123125273815E+02	-0.196455319739E-02	0.811705458359E-01
52	-0.208598209490E+02	0.861876916702E+03	-0.297997442128E+00	0.123125273815E+02	-0.196455319739E-02	-0.811705458359E-01
53	-0.210016771115E+02	-0.839223423030E+03	-0.300023958735E+00	-0.119889060433E+02	-0.208605103367E-02	0.833582422869E-01
54	-0.210016771115E+02	0.839223423030E+03	-0.300023958735E+00	0.119889060433E+02	-0.208605103367E-02	-0.833582422869E-01
55	-0.208255231249E+02	-0.816057187878E+03	-0.297507473213E+00	-0.116579598268E+02	-0.218761063733E-02	0.857224749726E-01
56	-0.208255231249E+02	0.816057187878E+03	-0.297507473213E+00	0.116579598268E+02	-0.218761063733E-02	-0.857224749726E-01

57 -0.210021156194E+02 -0.793422863730E+03 -0.300030223134E+00 -0.113346123390E+02 -0.233371313108E-02 0.881635635733E-01  
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130 -0.115005890022E+02 0.000000000000E+00 0.164294128603E+00 0.000000000000E+00 0.608664477849E+01 0.000000000000E+00  
131 0.401187152504E+02 0.000000000000E+00 0.573124503578E+00 0.000000000000E+00 0.174482157674E+01 0.000000000000E+00  
132 0.526993221422E+02 0.000000000000E+00 0.752847459174E+00 0.000000000000E+00 0.132829033002E+01 0.000000000000E+00  
133 0.567458536332E+02 0.000000000000E+00 0.810655051903E+00 0.000000000000E+00 0.123357030546E+01 0.000000000000E+00

134	0.555434885843E+02	0.000000000000E+00	0.793478408347E+00	0.000000000000E+00	0.126027373837E+01	0.000000000000E+00
135	0.402641558115E+03	0.000000000000E+00	0.575202225878E+01	0.000000000000E+00	0.173851900255E+00	0.000000000000E+00
136	0.396121698821E+03	0.000000000000E+00	0.565888141173E+01	0.000000000000E+00	0.176713369170E+00	0.000000000000E+00
137	0.373309777565E+03	0.000000000000E+00	0.533299682236E+01	0.000000000000E+00	0.187511831210E+00	0.000000000000E+00
138	0.368150631881E+03	0.000000000000E+00	0.525929474116E+01	0.000000000000E+00	0.190139562283E+00	0.000000000000E+00
139	0.352330933134E+03	0.000000000000E+00	0.503329904478E+01	0.000000000000E+00	0.198676850134E+00	0.000000000000E+00
140	0.347895533404E+03	0.000000000000E+00	0.496993619148E+01	0.000000000000E+00	0.201209826741E+00	0.000000000000E+00
141	0.336356851415E+03	0.000000000000E+00	0.480509787735E+01	0.000000000000E+00	0.20811236039E+00	0.000000000000E+00
142	0.332545345539E+03	0.000000000000E+00	0.475064779342E+01	0.000000000000E+00	0.210497608639E+00	0.000000000000E+00
143	0.323605489636E+03	0.000000000000E+00	0.462293556623E+01	0.000000000000E+00	0.216312770462E+00	0.000000000000E+00
144	0.320212435720E+03	0.000000000000E+00	0.457446336742E+01	0.000000000000E+00	0.218604876612E+00	0.000000000000E+00
145	0.3131118185803E+03	0.000000000000E+00	0.447311694005E+01	0.000000000000E+00	0.223557759254E+00	0.000000000000E+00
146	0.310085459170E+03	0.000000000000E+00	0.442979227386E+01	0.000000000000E+00	0.225744219633E+00	0.000000000000E+00
147	0.304281414191E+03	0.000000000000E+00	0.434687734559E+01	0.000000000000E+00	0.230050199372E+00	0.000000000000E+00
148	0.301526331610E+03	0.000000000000E+00	0.430751902300E+01	0.000000000000E+00	0.232152195883E+00	0.000000000000E+00
149	0.296704614015E+03	0.000000000000E+00	0.423863734307E+01	0.000000000000E+00	0.235924878460E+00	0.000000000000E+00
150	0.294188782745E+03	0.000000000000E+00	0.420269689635E+01	0.000000000000E+00	0.237942450922E+00	0.000000000000E+00
151	0.290112734356E+03	0.000000000000E+00	0.414446763365E+01	0.000000000000E+00	0.241285513218E+00	0.000000000000E+00
152	0.287795224939E+03	0.000000000000E+00	0.411136035628E+01	0.000000000000E+00	0.243228496980E+00	0.000000000000E+00
153	0.284310943755E+03	0.000000000000E+00	0.406158491078E+01	0.000000000000E+00	0.246209305472E+00	0.000000000000E+00
154	0.282165506001E+03	0.000000000000E+00	0.403093580002E+01	0.000000000000E+00	0.248081351233E+00	0.000000000000E+00
155	0.279153483577E+03	0.000000000000E+00	0.398790690824E+01	0.000000000000E+00	0.250758110209E+00	0.000000000000E+00
156	0.277155900145E+03	0.000000000000E+00	0.395937000208E+01	0.000000000000E+00	0.252565433257E+00	0.000000000000E+00
157	0.274530082035E+03	0.000000000000E+00	0.392185831479E+01	0.000000000000E+00	0.254981164472E+00	0.000000000000E+00
158	0.272661978839E+03	0.000000000000E+00	0.389517112628E+01	0.000000000000E+00	0.256728130185E+00	0.000000000000E+00
159	0.270354802085E+03	0.000000000000E+00	0.386221145835E+01	0.000000000000E+00	0.258919018491E+00	0.000000000000E+00
160	0.268600082685E+03	0.000000000000E+00	0.383714403836E+01	0.000000000000E+00	0.260610493117E+00	0.000000000000E+00
161	0.266559862753E+03	0.000000000000E+00	0.380799803933E+01	0.000000000000E+00	0.262605177227E+00	0.000000000000E+00
162	0.264905307952E+03	0.000000000000E+00	0.378436154217E+01	0.000000000000E+00	0.264245365792E+00	0.000000000000E+00
163	0.263090838979E+03	0.000000000000E+00	0.375844055684E+01	0.000000000000E+00	0.266067797236E+00	0.000000000000E+00
164	0.261524965584E+03	0.000000000000E+00	0.373607093692E+01	0.000000000000E+00	0.267660870707E+00	0.000000000000E+00
165	0.259903504222E+03	0.000000000000E+00	0.371290786317E+01	0.000000000000E+00	0.269330680117E+00	0.000000000000E+00
166	0.258416541210E+03	0.000000000000E+00	0.369166487443E+01	0.000000000000E+00	0.270880492681E+00	0.000000000000E+00
167	0.256961692367E+03	0.000000000000E+00	0.367088131953E+01	0.000000000000E+00	0.272414146075E+00	0.000000000000E+00
168	0.255544939220E+03	0.000000000000E+00	0.365064198885E+01	0.000000000000E+00	0.273924422897E+00	0.000000000000E+00
169	0.234425495389E+03	0.000000000000E+00	0.334893564842E+01	0.000000000000E+00	0.298602333692E+00	0.000000000000E+00
170	0.234637004147E+03	0.000000000000E+00	0.335195720211E+01	0.000000000000E+00	0.298333164687E+00	0.000000000000E+00
171	0.235363438154E+03	0.000000000000E+00	0.336233483077E+01	0.000000000000E+00	0.297412378698E+00	0.000000000000E+00
172	0.254235150367E+03	0.000000000000E+00	0.363193071953E+01	0.000000000000E+00	0.275335648508E+00	0.000000000000E+00
173	0.252881099538E+03	0.000000000000E+00	0.361258713626E+01	0.000000000000E+00	0.276809932130E+00	0.000000000000E+00
174	0.236219246406E+03	0.000000000000E+00	0.337456066294E+01	0.000000000000E+00	0.296334871375E+00	0.000000000000E+00
175	0.236798796630E+03	0.000000000000E+00	0.338283995185E+01	0.000000000000E+00	0.295609610337E+00	0.000000000000E+00
176	0.251698717418E+03	0.000000000000E+00	0.359569596311E+01	0.000000000000E+00	0.278110276914E+00	0.000000000000E+00
177	0.237809858589E+03	0.000000000000E+00	0.339728369413E+01	0.000000000000E+00	0.294352809489E+00	0.000000000000E+00
178	0.238401550533E+03	0.000000000000E+00	0.340573643618E+01	0.000000000000E+00	0.293622251380E+00	0.000000000000E+00
179	0.250400611875E+03	0.000000000000E+00	0.357715159822E+01	0.000000000000E+00	0.279552032544E+00	0.000000000000E+00
180	0.239473359952E+03	0.000000000000E+00	0.342104799931E+01	0.000000000000E+00	0.292308088107E+00	0.000000000000E+00
181	0.240124014968E+03	0.000000000000E+00	0.343034307097E+01	0.000000000000E+00	0.291516031870E+00	0.000000000000E+00
182	0.249331116082E+03	0.000000000000E+00	0.356187308689E+01	0.000000000000E+00	0.280751159742E+00	0.000000000000E+00
183	0.248082903115E+03	0.000000000000E+00	0.354404147308E+01	0.000000000000E+00	0.282163740915E+00	0.000000000000E+00
184	0.241942301496E+03	0.000000000000E+00	0.345631859279E+01	0.000000000000E+00	0.289325180290E+00	0.000000000000E+00
185	0.241223340883E+03	0.000000000000E+00	0.344604772690E+01	0.000000000000E+00	0.290187507327E+00	0.000000000000E+00
186	0.247114276078E+03	0.000000000000E+00	0.353020394397E+01	0.000000000000E+00	0.283269753213E+00	0.000000000000E+00
187	0.245910310431E+03	0.000000000000E+00	0.351300443474E+01	0.000000000000E+00	0.284656628985E+00	0.000000000000E+00
188	0.245032674911E+03	0.000000000000E+00	0.350046678444E+01	0.000000000000E+00	0.285676185943E+00	0.000000000000E+00
189	0.243073018772E+03	0.000000000000E+00	0.347247169675E+01	0.000000000000E+00	0.287979309072E+00	0.000000000000E+00
190	0.243867966407E+03	0.000000000000E+00	0.348382809153E+01	0.000000000000E+00	0.287040569663E+00	0.000000000000E+00
191	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
192	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
193	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00
194	0.567911472111E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
195	0.466240772809E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
196	0.415767284045E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
197	0.383621352506E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
198	0.360695293338E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
199	0.343240039104E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
200	0.329368527703E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
201	0.318004530586E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650408E+00	0.000000000000E+00
202	0.308479382536E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
203	0.300350954098E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
204	0.293312800074E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
205	0.287144565956E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
206	0.281683072401E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
207	0.276804538102E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886023735E+00	0.000000000000E+00
208	0.272413165137E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
209	0.268433540210E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
210	0.264805420345E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00

211	0.261480061434E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
212	0.258417575385E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
213	0.255584990968E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
214	0.252954806970E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
215	0.250503896645E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
216	0.248212667187E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
217	0.246064407246E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
218	0.244044775008E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
219	0.242141392684E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
220	0.240343522485E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
221	0.238641805624E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
222	0.237028050595E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
223	0.235495060273E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
224	0.234036489934E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
225	0.234036489934E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
226	0.235495060273E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
227	0.237028050595E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
228	0.238641805624E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
229	0.240343522485E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
230	0.242141392684E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
231	0.244044775008E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
232	0.246064407246E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
233	0.248212667187E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
234	0.250503896645E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
235	0.252954806970E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
236	0.255584990968E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
237	0.258417575385E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
238	0.261480061434E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
239	0.264805420345E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
240	0.268433540210E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
241	0.272413165137E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
242	0.276804538102E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
243	0.281683072401E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
244	0.287144565956E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
245	0.293312800074E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
246	0.300350954098E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
247	0.308479382536E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
248	0.318004530586E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00
249	0.329368527703E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
250	0.343240039104E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
251	0.360695293338E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
252	0.383621352506E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
253	0.415767284045E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
254	0.466240772809E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
255	0.567911472111E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
256	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00

The comparison with the original data is given in the table. Note that, in order to make sure of a valid comparison, 64 radial grid points were run.

In the data, note the propagating mode at index 133. This needs to be investigated further.

## 7.4 Lined Annulus, Shear Flow (Table 4.6)

This test case is given in Ref. 10 by Shankar. In order to get the correct input data,

$$\begin{aligned}\tilde{r}_i &= 2.0 \\ \frac{r_i}{b} &= 2.0 \\ \frac{r_i}{r_o - r_i} &= 2.0\end{aligned}$$

$\gamma_n^\pm$	Shankar $\beta_n$	Kousen report	$\frac{\gamma_n}{k}$ (current)	index
$\gamma_0^-$	0.79293	0.79353	0.793478	134
$\gamma_1^-$	0.75075	0.75292	0.752847	132
$\gamma_2^-$	0.57143	0.57320	0.573124	131
$\gamma_3^-$	-0.00969	0.16437	0.164294	130
$\gamma_4^-$	$-0.28733 + 0.73219i$	$-0.28357 + 0.73425i$	$-0.283637 + 0.734325i$	127
$\gamma_5^-$	$-0.29118 + 1.21721i$	$-0.28622 + 1.2198i$	$-0.286225 + 1.219841i$	125
$\gamma_6^-$	$-0.29248 + 1.62569i$	$-0.28766 + 1.6281i$	$-0.287553 + 1.628282i$	122
$\gamma_7^-$	$-0.29519 + 2.00221i$	$-0.28947 + 2.0055i$	$-0.289320 + 2.005620i$	120
$\gamma_8^-$	$-0.29567 + 2.36511i$	$-0.29035 + 2.3683i$	$-0.289988 + 2.368514i$	114
$\gamma_9^-$	$-0.29768 + 2.71665i$	$-0.29167 + 2.7209i$	$-0.291333 + 2.720846i$	112
$\gamma_{10}^-$	$-0.29776 + 3.06414i$	$-0.29243 + 3.0679i$	$-0.291720 + 3.068215i$	110

Table 3: Table 4.5 data

$$\begin{aligned}
r_i &= 2r_o - 2r_i \\
3r_i &= 2r_o \\
\frac{r_i}{r_o} &= \frac{2}{3} \\
\sigma &= \frac{2}{3} \\
b &= \frac{1}{3} \\
kb &= 10 \\
k &= 30
\end{aligned}$$

The axial Mach number profile is again given by:

$$\begin{aligned}
M\left(\frac{r}{b}\right) &= M_0 \left(1 - 2 \left| \frac{r_i - r}{b} + 0.5 \right| \right)^{\frac{1}{7}} \\
M_0 &= 0.3
\end{aligned}$$

The admittance is given as:

$$\eta = 0.3 + 0.1i$$

It is not clear from Shankar whether both walls are lined or only one. Running the code for each case, it appears that the outer wall is lined and the inner wall is hard.

The input data for SWIRL is then:

```
&INPUTS
MM      = 0,
NPTS    = 64,
SIG      = 0.666666666667,
AKRE     = 30.,
AKIM     = 0.,
IX       = 0,
NX       = 0,
IR       = 5,
RMAX     = 0.30,
SLOPE    = 0.00,
IS       = 0,
ANGOM    = 0.00,
GAM      = 0.00,
IREPEAT  = 0,
IFD      = 0,
ITEST    = 0,
ETAHR    = 0.30,
ETAHI    = 0.10,
ETADR    = 0.00,
ETADI    = 0.00,
ED2      = 0.0,
ED4      = 0.0,
&END
```

The output in 'gam.acc' for this test case is:

```
#      j      Re{gam}      Im{gam}      Re{gam/ak}      Im{gam/ak}      nz
1      0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
2      -0.494841896977E+03  0.150986662444E+04  -0.164947298992E+02  0.503288874813E+02  -0.588032102032E-02  -0.179420952506E-01
3      -0.497294064275E+03  0.149993467498E+04  -0.165764688092E+02  0.499978224995E+02  -0.597444618683E-02  -0.180200803573E-01
4      -0.505958390575E+03  -0.149630330504E+04  -0.168652796858E+02  -0.498767768346E+02  -0.608386925721E-02  0.179922180293E-01
5      -0.498555082515E+03  -0.150141804284E+04  -0.166185027505E+02  -0.500472680945E+02  -0.597593612666E-02  0.179967643257E-01
6      0.142037265777E+03  0.133362305854E+04  0.473457552591E+01  0.444541019515E+02  0.236896571828E-02  -0.222428057138E-01
7      0.139416488582E+03  0.133894446939E+04  0.464721628607E+01  0.446314823132E+02  0.230795348761E-02  -0.221653951357E-01
8      0.141036909142E+03  -0.134302692906E+04  0.470123030475E+01  -0.447675643021E+02  0.232017860864E-02  0.220939495242E-01
9      0.139746687507E+03  -0.133783150023E+04  0.465822291691E+01  -0.445943833410E+02  0.231711106343E-02  0.221823087580E-01
10     0.116993225396E+04  0.533196962418E+01  0.389977417986E+02  0.177732320806E+00  0.256419778038E-01  -0.116863387851E-03
11     0.115495640560E+04  0.118684119891E+02  0.384985468533E+02  0.395613732970E+00  0.259722637678E-01  -0.266892780707E-03
12     -0.115674962011E+03  0.847926389266E+03  -0.385583206704E+01  0.282642129755E+02  -0.473844766381E-02  -0.347340059460E-01
13     -0.117275874013E+03  0.841217168836E+03  -0.390919580044E+01  0.280405722945E+02  -0.487701197132E-02  -0.349826956090E-01
14     -0.120493264847E+03  -0.850335234923E+03  -0.401644216155E+01  -0.283445078308E+02  -0.490083192189E-02  0.345857510702E-01
15     -0.120697214617E+03  -0.843119145332E+03  -0.402324048723E+01  -0.281039715111E+02  -0.499149163909E-02  0.348675997042E-01
16     0.128310479661E+02  0.698601126039E+03  0.427701598869E+00  0.232867042013E+02  0.788457595244E-03  -0.429284782761E-01
17     0.136141333173E+02  0.693634333204E+03  0.453804443909E+00  0.231211144401E+02  0.848562717863E-03  -0.432338554033E-01
18     0.127707775857E+02  -0.700149880393E+03  0.425692528594E+00  -0.233383293464E+02  0.781289582772E-03  0.428337177020E-01
19     0.128219566282E+02  -0.694953062925E+03  0.427398554272E+00  -0.231651020975E+02  0.796190149259E-03  0.431536932267E-01
20     -0.302122088218E+02  0.585577036313E+03  -0.100707362739E+01  0.195192345438E+02  -0.263621685045E-02  -0.510955044522E-01
21     -0.312804390770E+02  0.577898124652E+03  -0.104268130257E+01  0.192632708217E+02  -0.280169576928E-02  -0.517606139390E-01
22     -0.311266178279E+02  -0.586777849899E+03  -0.103755392760E+01  -0.195592616633E+02  -0.270449011480E-02  0.509832100427E-01
23     -0.321611308713E+02  -0.579101925202E+03  -0.107203769571E+01  -0.193033975067E+02  -0.286817157804E-02  0.516450646371E-01
24     -0.207101933357E+01  -0.523423941320E+03  -0.690339777857E-01  -0.174474647107E+02  -0.226773050599E-03  0.573140202051E-01
25     -0.222421602840E+01  -0.516360663993E+03  -0.741405342799E-01  -0.172120221331E+02  -0.250255632895E-03  0.580978480146E-01
26     -0.191317221512E+01  0.522790944339E+03  -0.637724071708E-01  0.174263648113E+02  -0.209997145388E-03  -0.573835460697E-01
27     -0.189508674844E+01  0.515786010045E+03  -0.631695582813E-01  0.171928670015E+02  -0.213700417223E-03  -0.581628707156E-01
28     -0.141002297852E+02  -0.483633277818E+03  -0.470007659506E+00  -0.161211092606E+02  -0.180694984536E-02  0.619777897154E-01
29     -0.148117655806E+02  -0.473728789630E+03  -0.493725519352E+00  -0.157909596543E+02  -0.197808147393E-02  0.632655261343E-01
30     -0.138069448192E+02  0.483207309186E+03  -0.460231493974E+00  0.161069103062E+02  -0.177254561420E-02  -0.620345056680E-01
31     -0.144935585303E+02  0.473271385981E+03  -0.483118617676E+00  0.157757128660E+02  -0.193940575147E-02  -0.633291848970E-01
32     -0.752876863706E+01  -0.459004952518E+03  -0.250958954569E+00  -0.153001650839E+02  -0.107175035780E-02  0.653411926712E-01
33     -0.743135075327E+01  0.458704726677E+03  -0.247711691776E+00  0.152901575559E+02  -0.105927473193E-02  -0.653843887226E-01
34     -0.744149683653E+01  -0.448246417354E+03  -0.248049894551E+00  -0.149415472451E+02  -0.111078044309E-02  0.669090325537E-01
35     -0.920576576135E+01  -0.438615627382E+03  -0.306858858712E+00  -0.146205209127E+02  -0.143490023161E-02  0.683668997924E-01
36     -0.729169063155E+01  0.447958336767E+03  -0.243056354385E+00  0.149319445589E+02  -0.108983107347E-02  -0.669527740134E-01
37     -0.905199945737E+01  0.438343085580E+03  -0.301733315246E+00  0.146114361860E+02  -0.141270763053E-02  0.684103688589E-01
38     -0.982781908426E+01  -0.427924858523E+03  -0.327593969475E+00  -0.142641619508E+02  -0.160921610079E-02  0.700688083856E-01
39     -0.964650345945E+01  0.427678777629E+03  -0.321550115315E+00  0.142559592098E+02  -0.158137507886E-02  -0.701104354994E-01
40     -0.920768274853E+01  -0.418744951766E+03  -0.306922758284E+00  -0.139581650589E+02  -0.157457191233E-02  0.716080318458E-01
41     -0.905697339145E+01  0.418467949883E+03  -0.301899113048E+00  0.139489316628E+02  -0.155087414673E-02  -0.716565122430E-01
42     -0.909752712317E+01  -0.408529278134E+03  -0.303250904106E+00  -0.136176426045E+02  -0.163449247002E-02  0.733977508232E-01
43     -0.891499448382E+01  0.408296879301E+03  -0.297166482794E+00  0.136098959767E+02  -0.160355260706E-02  -0.734409344218E-01
44     -0.910026665053E+01  -0.399027175525E+03  -0.303342221684E+00  -0.133009058508E+02  -0.171373867613E-02  0.751437655383E-01
```

45 -0.929621464180E+01 -0.388858401426E+03 -0.309873821393E+00 -0.129619467142E+02 -0.184330112669E-02 0.771048385920E-01  
46 -0.319851132466E+02 -0.372057084071E+03 -0.106617044155E+01 -0.124019028024E+02 -0.688100890280E-02 0.800412394387E-01  
47 -0.914406205284E+01 -0.379388111387E+03 -0.304802068428E+00 -0.126462703796E+02 -0.190476222746E-02 0.790287882938E-01  
48 -0.894190061775E+01 0.398755079185E+03 -0.298063353925E+00 -0.132918359728E+02 -0.168624355589E-02 -0.751963381607E-01  
49 -0.911104481931E+01 0.388661811995E+03 -0.303701493977E+00 -0.129553937332E+02 -0.180845248488E-02 -0.771455341971E-01  
50 -0.318473298411E+02 0.371657633048E+03 -0.106157766137E+01 -0.123885877683E+02 -0.686642836164E-02 -0.801310667209E-01  
51 -0.898186122908E+01 0.379117826555E+03 -0.299395374303E+00 -0.126372608852E+02 -0.187368029162E-02 -0.790866816686E-01  
52 -0.891763543528E+01 0.369209586090E+03 -0.297254514509E+00 -0.123069862030E+02 -0.196142517728E-02 -0.812072867415E-01  
53 -0.903194288961E+01 -0.369394909142E+03 -0.301064762987E+00 -0.123131636381E+02 -0.198454553156E-02 0.811653733066E-01  
54 -0.893940291858E+01 0.359477445153E+03 -0.297980097286E+00 -0.119825815051E+02 -0.207404410167E-02 -0.834028940851E-01  
55 -0.910218986587E+01 -0.359753714580E+03 -0.303406328862E+00 -0.119917904860E+02 -0.210852447751E-02 0.833370347403E-01  
56 -0.889225304768E+01 0.349558391338E+03 -0.296408434923E+00 0.116519463779E+02 -0.218178852980E-02 -0.857670699008E-01  
57 -0.899450067985E+01 -0.349811161187E+03 -0.299816689328E+00 -0.116603720396E+02 -0.220365680681E-02 0.857038955120E-01  
58 -0.893126655765E+01 0.339847097800E+03 -0.297708885255E+00 0.113282365933E+02 -0.231828770710E-02 -0.882140672926E-01  
59 -0.910464716525E+01 -0.340131445635E+03 -0.303488238842E+00 -0.113377148545E+02 -0.235928133903E-02 0.881380418087E-01  
60 -0.890093534012E+01 0.329998891847E+03 -0.296697844671E+00 0.109999630616E+02 -0.245028213633E-02 -0.908433056529E-01  
61 -0.903322163435E+01 -0.330282612282E+03 -0.30110738712E+00 -0.110094204094E+02 -0.248237711543E-02 0.907634099483E-01  
62 -0.889958958471E+01 0.320210998080E+03 -0.296652986157E+00 0.106736999936E+02 -0.260185686715E-02 -0.936159112574E-01  
63 -0.908471779200E+01 -0.320499735910E+03 -0.302823926400E+00 -0.106833245303E+02 -0.265111485297E-02 0.935286741645E-01  
64 -0.887526605486E+01 0.310364676562E+03 -0.295842201829E+00 0.103454892187E+02 -0.276186892691E-02 -0.965815053778E-01  
65 -0.903311156364E+01 -0.310658567968E+03 -0.301103718788E+00 -0.103552855989E+02 -0.280559462379E-02 0.964874619322E-01  
66 -0.888274470653E+01 0.300570433265E+03 -0.296091490218E+00 0.100190144422E+02 -0.294711296615E-02 -0.997231205422E-01  
67 -0.908038034459E+01 -0.300863070781E+03 -0.302679344820E+00 -0.100287690260E+02 -0.300671392218E-02 0.996223899069E-01  
68 -0.886297830793E+01 0.290732733321E+03 -0.295432610264E+00 0.969109111071E+01 -0.314274874427E-02 -0.103091748712E+00  
69 -0.904203678693E+01 -0.291032569029E+03 -0.301401226231E+00 -0.970108563428E+01 -0.319952367891E-02 0.102981841136E+00  
70 -0.885701100393E+01 0.280917084543E+03 -0.295233700131E+00 0.936390281810E+01 -0.336372612447E-02 -0.106687022932E+00  
71 -0.906811637152E+01 -0.281213652143E+03 -0.302270545717E+00 -0.937378840476E+01 -0.343648285443E-02 0.106569639650E+00  
72 -0.884016079083E+01 0.271077478227E+03 -0.294672026361E+00 0.903591594091E+01 -0.360522955685E-02 -0.110551896037E+00  
73 -0.903898154434E+01 -0.271381807093E+03 -0.301299384811E+00 -0.904606023642E+01 -0.367788179221E-02 0.104422861486E+00  
74 -0.883679750612E+01 0.261250410280E+03 -0.294559916871E+00 0.870834700935E+01 -0.387976650971E-02 -0.114701122409E+00  
75 -0.906314714190E+01 -0.261551324261E+03 -0.302104904730E+00 -0.871837747538E+01 -0.396976942885E-02 0.114562682793E+00  
76 -0.881985480292E+01 0.251407374339E+03 -0.293995160097E+00 0.838024581130E+01 -0.418111863130E-02 -0.119181560284E+00  
77 -0.881191261802E+01 0.241563896357E+03 -0.293730420610E+00 0.805212987858E+01 -0.452428399316E-02 -0.124025704406E+00  
78 -0.903881993856E+01 -0.251716344723E+03 -0.301293997952E+00 -0.839054482409E+01 -0.427415749664E-02 0.119028292314E+00  
79 -0.879552096851E+01 0.231713064885E+03 -0.293184032284E+00 0.772376882951E+01 -0.490745665081E-02 -0.129284191968E+00  
80 -0.905578809533E+01 -0.241869622526E+03 -0.301859603178E+00 -0.806232075085E+01 -0.463742052199E-02 0.123860136670E+00  
81 -0.878788292286E+01 0.221853292690E+03 -0.292929430762E+00 0.739510975633E+01 -0.534801804324E-02 -0.135012655798E+00  
82 -0.903634292032E+01 -0.232026944498E+03 -0.301211430677E+00 -0.773423148326E+01 -0.502781058034E-02 0.129099519214E+00  
83 -0.876950546294E+01 0.211990594959E+03 -0.292316848765E+00 0.706635316531E+01 -0.584414002315E-02 -0.141273955044E+00  
84 -0.905240047945E+01 -0.222164468182E+03 -0.301746682648E+00 -0.740548227275E+00 -0.549307290878E-02 0.134811271799E+00  
85 -0.875952082791E+01 0.202109640725E+03 -0.291984027597E+00 0.673698802415E+01 -0.642114583187E-02 -0.148155989650E+00  
86 -0.903511935829E+01 -0.212310101908E+03 -0.301170645276E+00 -0.707700339695E+01 -0.600244267121E-02 0.141047302718E+00  
87 -0.873921254405E+01 0.192229591713E+03 -0.291307084802E+00 0.640765305709E+01 -0.708037702703E-02 -0.155741490233E+00  
88 -0.904875768037E+01 -0.202427026509E+03 -0.301625856012E+00 -0.674756755031E+01 -0.661160861229E-02 0.147906006194E+00  
89 -0.872800653443E+01 0.182322226120E+03 -0.290933551148E+00 0.607740753734E+01 -0.785892183806E-02 -0.16416727369E+00  
90 0.233541341856E+03 -0.415769538715E-01 0.778471139520E+01 -0.138589846238E-02 0.12845690945E+00 0.228689574374E-04  
91 0.227775772988E+03 -0.248422186077E-01 0.759252576626E+01 -0.828073953588E-03 0.131708474741E+00 0.143647003326E-04  
92 -0.30398126098E+03 -0.192555515388E+03 -0.301132706899E+00 -0.641851717961E+01 -0.729346028465E-02 0.155457042903E+00  
93 -0.904757920794E+01 -0.182646798584E+03 -0.301585973598E+00 -0.608822661946E+01 -0.811643191268E-02 0.163849386748E+00  
94 -0.870348120172E+01 0.172416734835E+03 -0.290116040057E+00 0.574722449451E+01 -0.876092923237E-02 -0.173554785420E+00  
95 -0.903366807175E+01 -0.172750232910E+03 -0.301122269058E+00 -0.575834109700E+01 -0.905653578557E-02 0.173187530678E+00  
96 -0.868947465762E+01 0.162473437250E+03 -0.289649155254E+00 0.541578124166E+01 -0.984712767682E-02 -0.184118919006E+00  
97 -0.904711042114E+01 -0.162806484361E+03 -0.301570347371E+00 -0.542688281203E+01 -0.102081893478E-01 0.183700578642E+00  
98 -0.865952770748E+01 0.152530586203E+03 -0.288650923583E+00 0.508435287342E+01 -0.111302278811E-01 -0.196049974158E+00  
99 -0.903422219812E+01 -0.152873108479E+03 -0.301140739937E+00 -0.509577028263E+01 -0.115567510521E-01 0.195558224992E+00  
100 -0.864182197255E+01 0.142536668707E+03 -0.288060732418E+00 0.475122229023E+01 -0.127139321060E-01 -0.209701326215E+00  
101 -0.904875383972E+01 -0.142879926638E+03 -0.301625127991E+00 -0.476266422125E+01 -0.132443055246E-01 0.209127735734E+00  
102 -0.860341958591E+01 0.132536722769E+03 -0.286780652864E+00 0.441789075898E+01 -0.146316645432E-01 -0.225402567881E+00  
103 -0.903626497240E+01 -0.132890361232E+03 -0.301208832413E+00 -0.442967870772E+01 -0.152798757526E-01 0.224711007760E+00  
104 -0.857943083451E+01 0.122466913713E+03 -0.285981027817E+00 0.408223045709E+01 -0.170771740456E-01 -0.243767779079E+00  
105 -0.905280119253E+01 -0.122822927562E+03 -0.301760039751E+00 -0.409409758540E+01 -0.179057447361E-01 0.242934307502E+00  
106 -0.852885156158E+01 0.112375755408E+03 -0.284295052053E+00 0.374585851361E+01 -0.201452251859E-01 -0.265432559330E+00  
107 -0.904153907991E+01 -0.112743531948E+03 -0.301384635997E+00 -0.375811773159E+01 -0.212029453779E-01 0.264390269009E+00  
108 -0.849424202667E+01 0.102183778772E+03 -0.283141400899E+00 0.340612595906E+01 -0.242376920890E-01 -0.291573863634E+00  
109 -0.906229141429E+01 -0.102556395545E+03 -0.302076380476E+00 -0.341854651818E+01 -0.256481412158E-01 0.290255609236E+00  
110 0.193287502557E+03 0.486753676173E-02 0.644291675191E+01 0.162251122539E-03 0.155209206928E+00 -0.390861545876E-05  
111 0.189759766976E+03 0.138129334324E-01 0.632532556587E+01 0.460431114414E-03 0.158094628377E+00 -0.115079745965E-04  
112 -0.842434935075E+01 0.919353541009E+02 -0.280811645025E+00 0.306451180336E+01 -0.296524859915E-01 -0.323599091882E+00  
113 -0.905454087297E+01 -0.923221185214E+02 -0.301818029099E+00 -0.307740395071E+01 -0.315659403585E-01 0.321853369254E+00  
114 -0.836955207578E+01 0.815256520043E+02 -0.278985069193E+00 0.271752173348E+01 -0.373836462408E-01 -0.364144473502E+00  
115 -0.908436302123E+01 -0.819214231273E+02 -0.30281200708E+00 -0.273071410424E+01 -0.401155612140E-01 0.3617566113942E+00  
116 -0.826549491684E+01 0.709726958254E+02 -0.275516497228E+00 0.236575652751E+01 -0.485687330302E-01 -0.417041441639E+00  
117 -0.908860924559E+01 -0.713869169742E+02 -0.302953641520E+00 -0.237956389914E+01 -0.526499999363E-01 0.413541948232E+00  
118 -0.81647528673E+01 0.601070472972E+02 -0.272158542958E+00 0.200356824324E+01 -0.665691892863E-01 -0.490066974135E+00  
119 -0.914261469033E+01 -0.605381107644E+02 -0.304753823011E+00 -0.201793702548E+01 -0.731711565606E-01 0.484505114857E+00  
120 -0.409294296057E+02 -0.102275549246E+00 -0.136431432019E+01 -0.340918497480E-02 0.732964347809E+00 0.183155084179E-02  
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129 -0.771927763371E+01 0.366406367650E+02 -0.257309254457E+00 0.122135455883E+01 -0.165162547469E+00 -0.783967386126E+00  
130 -0.706875252877E+01 0.223114268841E+02 -0.235625084292E+00 0.743714229471E+00 -0.387140086033E+00 -0.127149795875E+01  
131 0.429293559785E+01 0.218566893343E+01 0.143097853262E+00 0.728556311144E-01 0.554966818108E+01 -0.282551113516E+01  
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133 0.170155153961E+03 -0.150075958345E-02 0.567183846537E+01 -0.500253194484E-04 0.176309675607E+00 0.155504214338E-05  
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135 0.220780843504E+02 0.789988134835E+00 0.735936145015E+00 0.263329378278E-01 0.135707605121E+01 -0.48582880068E-01  
136 0.236168578482E+02 0.112727201685E+00 0.787228594938E+00 0.375757338951E-02 0.127025012027E+01 -0.606311569553E-02  
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198 0.154583697145E+03 0.000000000000E+00 0.515278990483E+01 0.000000000000E+00 0.194069624120E+00 0.000000000000E+00



199	0.147102873902E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
200	0.141157940444E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
201	0.136287655966E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00
202	0.132205449658E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
203	0.128721837471E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
204	0.125705485746E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
205	0.123061956838E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
206	0.120721316743E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
207	0.118630516330E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
208	0.116748499344E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
209	0.115042945804E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
210	0.113488037291E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
211	0.112062883472E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
212	0.110750389451E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
213	0.109536424701E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
214	0.108409202987E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
215	0.107358812848E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
216	0.106376857366E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
217	0.105456174534E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
218	0.104590617860E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
219	0.103774882579E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
220	0.103004366779E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
221	0.102275059553E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
222	0.101583450255E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
223	0.100926454403E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
224	0.100301352829E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
225	0.100301352829E+03	0.000000000000E+00	0.334337842762E+01	0.000000000000E+00	0.299098657734E+00	0.000000000000E+00
226	0.100926454403E+03	0.000000000000E+00	0.336421514676E+01	0.000000000000E+00	0.297246149956E+00	0.000000000000E+00
227	0.101583450255E+03	0.000000000000E+00	0.338611500850E+01	0.000000000000E+00	0.295323696180E+00	0.000000000000E+00
228	0.102275059553E+03	0.000000000000E+00	0.340916865178E+01	0.000000000000E+00	0.293326644160E+00	0.000000000000E+00
229	0.103004366779E+03	0.000000000000E+00	0.343347889264E+01	0.000000000000E+00	0.291249788122E+00	0.000000000000E+00
230	0.103774882579E+03	0.000000000000E+00	0.345916275263E+01	0.000000000000E+00	0.289087294097E+00	0.000000000000E+00
231	0.104590617860E+03	0.000000000000E+00	0.348635392868E+01	0.000000000000E+00	0.286832610933E+00	0.000000000000E+00
232	0.105456174534E+03	0.000000000000E+00	0.351520581781E+01	0.000000000000E+00	0.284478363951E+00	0.000000000000E+00
233	0.106376857366E+03	0.000000000000E+00	0.354589524553E+01	0.000000000000E+00	0.282016227428E+00	0.000000000000E+00
234	0.107358812848E+03	0.000000000000E+00	0.357862709493E+01	0.000000000000E+00	0.279436770995E+00	0.000000000000E+00
235	0.108409202987E+03	0.000000000000E+00	0.361364009957E+01	0.000000000000E+00	0.276729273654E+00	0.000000000000E+00
236	0.109536424701E+03	0.000000000000E+00	0.365121415668E+01	0.000000000000E+00	0.273881497246E+00	0.000000000000E+00
237	0.110750389451E+03	0.000000000000E+00	0.369167964836E+01	0.000000000000E+00	0.270879408630E+00	0.000000000000E+00
238	0.112062883472E+03	0.000000000000E+00	0.373542944906E+01	0.000000000000E+00	0.267706836292E+00	0.000000000000E+00
239	0.113488037291E+03	0.000000000000E+00	0.378293457636E+01	0.000000000000E+00	0.264345042140E+00	0.000000000000E+00
240	0.115042945804E+03	0.000000000000E+00	0.383476486014E+01	0.000000000000E+00	0.260772181991E+00	0.000000000000E+00
241	0.116748499344E+03	0.000000000000E+00	0.389161664481E+01	0.000000000000E+00	0.256962617665E+00	0.000000000000E+00
242	0.118630516330E+03	0.000000000000E+00	0.395435054432E+01	0.000000000000E+00	0.252886027375E+00	0.000000000000E+00
243	0.120721316743E+03	0.000000000000E+00	0.402404389144E+01	0.000000000000E+00	0.248506235761E+00	0.000000000000E+00
244	0.123061956838E+03	0.000000000000E+00	0.410206522794E+01	0.000000000000E+00	0.243779643773E+00	0.000000000000E+00
245	0.125705485746E+03	0.000000000000E+00	0.419018285820E+01	0.000000000000E+00	0.238653069291E+00	0.000000000000E+00
246	0.128721837471E+03	0.000000000000E+00	0.429072791569E+01	0.000000000000E+00	0.233060687988E+00	0.000000000000E+00
247	0.132205449658E+03	0.000000000000E+00	0.440684832195E+01	0.000000000000E+00	0.226919541347E+00	0.000000000000E+00
248	0.136287655966E+03	0.000000000000E+00	0.454292186552E+01	0.000000000000E+00	0.220122650048E+00	0.000000000000E+00
249	0.141157940444E+03	0.000000000000E+00	0.470526468148E+01	0.000000000000E+00	0.212527895388E+00	0.000000000000E+00
250	0.147102873902E+03	0.000000000000E+00	0.490342913006E+01	0.000000000000E+00	0.203938911622E+00	0.000000000000E+00
251	0.154583697145E+03	0.000000000000E+00	0.515278990483E+01	0.000000000000E+00	0.194069624120E+00	0.000000000000E+00
252	0.164409151074E+03	0.000000000000E+00	0.548030503581E+01	0.000000000000E+00	0.182471594823E+00	0.000000000000E+00
253	0.178185978876E+03	0.000000000000E+00	0.593953262921E+01	0.000000000000E+00	0.168363415512E+00	0.000000000000E+00
254	0.199817474061E+03	0.000000000000E+00	0.666058246869E+01	0.000000000000E+00	0.150137019502E+00	0.000000000000E+00
255	0.243390630905E+03	0.000000000000E+00	0.811302103015E+01	0.000000000000E+00	0.123258647584E+00	0.000000000000E+00
256	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00	0.000000000000E+00

The comparison with the original data is given in the table. Note that, in order to make sure of a valid comparison, 64 radial grid points were run (though the 32 point dataset looked good too).

In the data, note the (very nearly) propagating mode at index 137. This needs to be investigated further – are these modes real?

$\gamma_n^\pm$	Shankar $\beta_n$	Kousen report	$\frac{\gamma_n}{k}$ (current)	index
$\gamma_0^-$	$0.78698 + 0.00400i$	$0.78093 + 0.00913i$	$0.787228 + 0.003757i$	136
$\gamma_1^-$	$0.73438 + 0.02541i$	$0.75079 + 0.03387i$	$0.735936 + 0.026332i$	135
$\gamma_2^-$	$0.55840 + 0.03148i$	$0.57267 + 0.03246i$	$0.560121 + 0.031371i$	134
$\gamma_3^-$	$0.14308 + 0.07638i$	$0.16875 + 0.06982i$	$0.143097 + 0.072855i$	131
$\gamma_4^-$	$-0.23900 + 0.74173i$	$-0.23734 + 0.72727i$	$-0.235625 + 0.743714i$	130
$\gamma_5^-$	$-0.26149 + 1.21973i$	$-0.25993 + 1.2120i$	$-0.257309 + 1.221354i$	129
$\gamma_6^-$	$-0.26996 + 1.62627i$	$-0.26860 + 1.6207i$	$-0.265988 + 1.627490i$	124
$\gamma_7^-$	$-0.27669 + 2.00192i$	$-0.27468 + 1.9983i$	$-0.272158 + 2.003568i$	118
$\gamma_8^-$	$-0.27974 + 2.36439i$	$-0.27813 + 2.3612i$	$-0.275516 + 2.365756i$	116
$\gamma_9^-$	$-0.28359 + 2.71568i$	$-0.28147 + 2.7139i$	$-0.278985 + 2.717521i$	114
$\gamma_{10}^-$	$-0.28502 + 3.06304i$	$-0.28361 + 3.0610i$	$-0.280811 + 3.064511i$	112

Table 4: Table 4.6 data