

Thesis Outline

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Abstract

Over the last 20 years, there has been an increase in computational fluid dynamic codes that have made numerical analysis more and more readily available, allowing turbomachine designers to create more novel designs. However, as airport noise limitations become more restrictive over time, reducing aircraft takeoff and landing noise remains a prominent issue in the aviation community. One popular method to reduce aircraft noise is using acoustic liners placed on the walls of the engine inlet and exhaust ducts. These liners are designed to reduce the amplitude of acoustic modes emanating from the bypass fan as they propagate through the engine. The SWIRL code is a frequency-domain linearized Euler equation solver that is designed to predict the effect of acoustic liners on acoustic modes propagating in realistic sheared and swirling mean flows, guiding the design of more efficient liner configurations. The purpose of this study is to validate SWIRL using the Method Of Manufactured Solutions (MMS). This study also investigated the effect of the integration and spatial differencing methods on the convergence for a given Manufactured Solution. In addition, the effect of boundary condition implementation was tested. The improved MMS convergence rates shown for these tests suggest that the revised SWIRL code provides more accurate solutions with less computational effort than the original formulation.

1. Introduction:

1-A. Overview: Briefly explain why the study is being undertaken and what main questions of foreshadowed problems will be addressed. Do this in a general manner, because it will be done more specifically in the following sections. This study is being conducted to make progress towards further understanding the numerical results for test cases that we do not have analytical solutions for.

1-B. Statement of the Problem: Discuss the problem to be addressed in the research - the gaps, perplexities, or inadequacies in existing theory, empirical knowledge, practice, or policy that prompted the study. The problem may be a theory that appears inadequate to explain known phenomena, the lack of empirical data on a potentially interesting relationship between X and Y, or a common practice that appears ineffective. First state the problem generally, and then state the specifics that your research will address. In quantitative research, the specifics will include the constructs studied.

A turbomachine's general flow condition includes a series of axial, tangential, and radial velocity components that vary depending on the location of concern. The swirling flow between fan stages has been an area of interest due to the potential for acoustic treatment in a location previously avoided for its flow complexity, among other reasons. It is common in CAA codes to assume a uniform flow with superimposed wakes as a means of "exciting" certain acoustic modes within a duct. This is different than SWIRL the user can provide an axial and tangential component as a function of radius which does not need to be uniform in either direction.

1-C. The purpose of research is to acquire knowledge to address the problem or certain aspects of it. Quantitative research tries to fulfill that purpose by answering questions and/or testing hypotheses. Qualitative research tries to fulfill that purpose by starting with foreshadowed problems, conjectures, or exploratory questions. Mixed-methods research may use both approaches. The purpose of this study is to verify and validate SWIRL using the Method Of Manufactured solutions. The study will investigate the effect of the integration and spatial differencing methods on the convergence for a series of Manufactured Solutions. Guidelines for setting unique *auxiliary* conditions will be provided and its impact will be discussed. The use of fairing functions to set specific values at the boundaries will also be provided.

1-D.1 Research Questions or Hypotheses Research questions address problems of the study. Each research question seeks answers to a specific problem situation described in your study. The type of the data and its availability determine the research questions. *For instance, research questions should relate to the conceptual framework.* Each question should address and target a separate problem situation.

A good hypothesis clearly states the expected relationship (or difference) between two variables and defines those variables in operational, measurable terms. The hypothesis (or hypotheses) logically follows the review of related literature and is based on the implications of previous research.

A well-developed hypothesis is testable, that is, can be confirmed or disconfirmed.

- 1-D.2 Significance of the Study: Discuss the potential significance of the research. Significance comes from the uses that might be made of your results-how they might be of benefit to theory, knowledge, practice, policy, and future research. The potential significance should be based upon your literature review in Chapter 2. The significance of this study is that it will offer guidance when verifying a frequency domain CAA code which has little literature on. The goal is to couple code verification and validation to the sound propagation in a duct problem and highlight the issues that arise when using couple numerical techniques. The impact of the numerical scheme and number of grid points will be discussed/
- 1-E. Conceptual Framework: Briefly summarize the theoretical foundation or conceptual framework(s)
 - (a) Research Questions:
2. Literature Review:
 - a] The problem to be addressed and its significance
 - b] The theoretical foundation or conceptual framework
 - c] The research questions, hypotheses, foreshadowed problems, or conjectures
 - d] The research paradigm and the methodology
 - (a) Intro - Topics, Purposes, and Methods of the Literature Review.
 - i. Following the discovery of the acoustic impact of swirling flow in 1977, there has been a concerted effort to model sound propagation under such conditions (Cooper, A.J. and Peake N., Kousen, Posson).
 - (b) Description and Critique of Scholarly Literature
 - i. Sound propagation in ducted flows can be modeled with the Linearized Euler Equations (LEE) though an initial value problem and normal mode analysis. The two normal mode analysis' have been studied by Golubev and Kousen where the flow is assumed to be isentropic, however Tam & Auriault used a non isentropic mean flow with constant density. For isentropic flows, the normal mode analysis requires the perturbation variables to take the form of a linear equation. The components of the perturbation variables has been split into its mean and fluctuating parts in Kousen's study while Golubev , and Atassi splits the perturbation into a vortical and potential part. This study will use Kousen's definition for the perturbation variables
 - A. While validation is apart of existing literature, there is currently no literature where the code verification technique of the method of manufactured solutions (MMS) is used for the

problem of sound propagation is a ducted flow. Although various code verification methods exist, the MMS is considered to be a gold-standard by experts in computational modeling and simulation (Oberkampf, W. L., and Roy, C. J., 2010. Verification and validation in scientific computing.) (Roache, P. J., 2009. Fundamentals of verification and validation). The MMS provides a mathematical technique to verify that the LEE simulation solves the equations correctly and that the spatial grid refinement reduces the numerical error at an expected rate based on the numerical scheme used (Roache, P. J., 2002. “Code verification by the method of manufactured solutions”.)

(c) Inferences for Forthcoming study

- i. The problem to be addressed in your research and its significance
 - A. The code verification presented in Maldando’s work will be improved by providing an observed order of accuracy
 - B. One key question that remains, is how well do unsteady linearized equations capture the mechanisms of noise generation within realistic turbomachinery flow? The numerical simulations for more complex (axial shear + swirl) flows do not have analytical solutions and contain different categories for the axial wavenumbers.
- ii. possible research questions, hypotheses, foreshadowed problems, or conjectures
 - A. What combination of spatial grid resolution and numerical scheme aid the final result? The use of dissipation is apparent in literature, but the amount is not reported.
 - B. Can the use of a desired damping rate help aid the sorting of axial wavenumbers
- iii. possible theoretical or conceptual framework to be used and possible research paradigms and methodologies used
 - A. MMS study
 - Additional MMS framework will be explored (tanhSummation) and utilized to find MS’ that meet the guidelines in Roaches work
 - The use of MMS to test matrix construction for eigenvalue problem
 - B. MES study
 - the analytical solution is known for uniform flow and is used for MES
 - C. Literature test case studies (some are shared with Maldonado)
 - Kousen’s Test Cases

Cylinder, Uniform Flow with Liner (Table 4.3)

$$m = 2$$

$$k = \frac{\omega r_T}{A_T} = -1$$

$$M_x = 0.5$$

$$\eta_T = 0.72 + 0.42i$$

Confirm if 32 grid points is enough

Cylinder, Shear Flow without Liner (Table 4.4)

$$m = 0$$

$$kb = \left(\frac{\omega r_T}{A_T} \right) b = 20$$

$$b = r_{max} - r_{min}$$

$$\tilde{r} = \frac{r}{b}$$

$$M_x = 0.3(1 - \tilde{r})^{\frac{1}{7}}$$

$$\eta_T = 0$$

Confirm if 32 grid points is enough

Annulus, Shear Flow without Liner (Table 4.5)

$$m = 0$$

$$kb = \left(\frac{\omega r_T}{A_T} \right) b = 10$$

$$b = r_{max} - r_{min} = \frac{1}{7}$$

$$k = 70$$

$$\tilde{r} = \frac{r}{b} = 6.0$$

$$M_x = 0.3 \left(1 - 2 \left| \frac{r_{max} - r}{b} + 0.5 \right| \right)^{\frac{1}{7}}$$

$$\eta_T = 0$$

Confirm if 32 grid points is enough

Annulus, Shear Flow with Liner (Table 4.6)

$$m = 0$$

$$kb = \left(\frac{\omega r_T}{A_T} \right) b = 10$$

$$b = r_{max} - r_{min} = \frac{1}{3}$$

$$k = 30$$

$$\tilde{r} = \frac{r}{b} = 2.0$$

$$M_x = 0.3 \left(1 - 2 \left| \frac{r_{max} - r}{b} + 0.5 \right| \right)^{\frac{1}{7}}$$

$$\eta_T = 0.3 + 0.1i$$

Confirm if 32 grid points is enough

- Kousen's test cases which were compared to P.N. Shankar's work and then expanded for cases where data is needed . Maldonado has compared the same tests

- (d) Theoretical/Conceptual Framework for Forthcoming Study (May appear in methods chapter)

3. Research Methodology

- (a) Conjectures, or Exploratory Questions:

- What is the observed order of accuracy for SWIRL's second order and fourth order solutions (MMS/MES)? How does the number of grid points and manufactured solution effect the solution accuracy.

- (b) Research Procedures - Describe in detail how the inquiry was undertaken. Generally the description should be thorough enough that other skilled researchers could approximately replicate your study from the description.

- i. Introduce the epistemology that will guide the inquiry/ Indicate the methodology used and why it was selected.

- A. Aerodynamic Theoretical Background: The steady and unsteady aerodynamic models are presented here.

- B. The MMS and tanh summation method will be used to define a sufficient test case for the second/fourth order differencing schemes for the radial derivatives and for the approximated speed of sound obtained through numerical integration. To account for acoustic liners as boundary conditons, fairing functions were used to impose boundary values for the radial velocity perturbation and for the radial derivative of pressure.

- ii. Explain the theoretical perspective that will drive the research, and why it was selected.

- A. The need for a code verification study prompted the use of MMS and MES to investigate if SWIRL was coded correctly.

In addition, the findings from these studies inform the user how the numerical scheme is effecting the final solution which will offer insight for the more complicated flows.

- iii. Indicate the specific methods used and the justification for them. How were sites, cases, and informants selected? Why? What access did you unsuccessfully seek? Which people perhaps tried to minimize contact with you and which repeatedly sought it out? How did you collect your data? Why? The manufactured solutions were chosen such in a way that allows testing for each term in SWIRL. Setting up the manufactured solutions this way ensures that there is no variable unused.
- iv. Indicate how you analyzed and interpreted your data, making sure the analysis was consistent with the selected methodology. If you inferred themes, explain how. If you coded the transcripts, explain the coding system and checks for coding reliability and validity. How did you analyze the data from the coding? How did you triangulate or otherwise verify findings? How did you interpret the full set of data?
The

4. Results of the Study

- (a) statement of the received results and their analysis
 - i. Results for MMS/MES/set of test cases will be presented. The MMS and MES will have errors and convergence rates associated with the schemes that are available in SWIRL.
- (b) Comparison of the obtained results and the initial goals/questions

5. Summary

- (a) Discussion of the research results;
 - i. Discuss the convergence rates and make the case for fourth order schemes.
 - ii. Discuss the effect on axial wavenumber categorization.
- (b) Comparison of the obtained results with the findings of prior researchers
 - i. Discuss the appearance of nearly convected wavenumbers as the tests get more complex.
- (c) Suggestions regarding the use of the obtained findings for the further development of the topic and future investigation
 - i. The use of GCI could give a grid which is closer to convergence
 - ii. Limitations of the eigensolver
 - iii. The use of a desired damping rate can be correlated to the length of a Nacelle
 - iv. Comparison against experimental data

6. Appendices