NE 180 Exam CheatSheet

Mathematics

Curl (Gradient Cross Product)

$$\vec{\bigtriangledown} \times \vec{B} = \begin{vmatrix} i & j & k \\ \vec{\bigtriangledown}_x & \vec{\bigtriangledown}_y & \vec{\bigtriangledown}_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Integration by parts

$$\int u dv = uv - \int v du$$

Units & Physical Constants

$$1\,[\mathrm{T}] = 10\,[\mathrm{kG}] = 1\left[\frac{\mathrm{kg}}{\mathrm{A}\cdot\mathrm{s}^2}\right] \qquad \hbar c \equiv \frac{hc}{2\pi} = 1974\,[\mathrm{eV\AA}]$$

$$\epsilon_0 = 8.854\cdot 10^{-12}\left[\frac{\mathrm{A}^2\cdot\mathrm{s}^4}{\mathrm{kg}\cdot\mathrm{m}^3}\right] \qquad \mu_0 = 4\pi\cdot 10^{-7}\left[\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{A}^2\cdot\mathrm{s}^2}\right]$$

$$c \equiv \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3\cdot 10^8\left[\frac{\mathrm{m}}{\mathrm{s}}\right] \qquad e^2 = 1.44\,[\mathrm{MeV}\cdot\mathrm{fm}]$$

$$\sigma = 5.67\cdot 10^{-8}\left[\frac{\mathrm{W}}{\mathrm{m}^2\cdot\mathrm{K}^4}\right] \qquad N_a = 6.022\cdot 10^{23}\left[\frac{\#}{\mathrm{mol}}\right]$$

$$11608\left[\frac{\mathrm{K}}{\mathrm{eV}}\right] \qquad 17.6\left[\frac{\mathrm{MeV}}{\mathrm{DT}\cdot\mathrm{fusion}}\right] \qquad 1.602\cdot 10^{-19}\left[\frac{\mathrm{J}}{\mathrm{eV}}\right]$$

$$1.66\cdot 10^{-27}\left[\frac{\mathrm{kg}}{\mathrm{amu}}\right] \qquad 1836\left[\frac{\mathrm{m_e}}{\mathrm{m_p}}\right] \qquad 931.502\left[\frac{\mathrm{MeV}}{\mathrm{c^2 amu}}\right]$$

$$3.5\left[\frac{\$}{\mathrm{Ci}\cdot{}^3_1\mathrm{H}}\right] \qquad 10\left[\frac{\mathrm{kCi}}{\mathrm{g}\cdot{}^3_1\mathrm{H}}\right] \qquad 4.18\cdot 10^{12}\left[\frac{\mathrm{J}}{\mathrm{kT}\cdot\mathrm{TNT}}\right]$$

$$t_{1/2}^{{}^3_1\mathrm{H}} = 12.3\,[\mathrm{y}] \qquad 3.7\cdot 10^{10}\left[\frac{\mathrm{Bq}}{\mathrm{Ci}}\right] \qquad 58\left[\frac{\mathrm{R}}{\mathrm{Ci}}\right](10d, 70kg, {}^3_1\mathrm{H})$$

Electricity & Magnetism

Maxwell's Equations & Simple E and M

$$\vec{\bigtriangledown}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \qquad \int\vec{B}\cdot\mathrm{d}\ell = \mu_0\int\vec{J}\cdot\mathrm{ds}$$

$$\vec{J} = \frac{\vec{B}\times\vec{\bigtriangledown}\vec{P}}{B^2} \qquad \vec{\bigtriangledown}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$V = IR \qquad P = IV = I^2R = \frac{V^2}{R}$$

Forces

$$\vec{F} = q\vec{v}\times\vec{B} = \vec{J}\times\vec{B} = \vec{\bigtriangledown}P \qquad v_{drift} = \frac{\vec{F}\times\vec{B}}{qB^2} \qquad \vec{F} = q\vec{E}$$

Waves

$$\omega = 2\pi f \qquad c = f\lambda \qquad E_{photon} = hf = \frac{hc}{\lambda}$$

Ionization

$$U = \frac{z_1 z_2 e^2}{r} = 13.6\cdot Z^2\,[\mathrm{eV}] \rightarrow \text{ Fully stripped } (E_{ion} < 3T_e)$$

Magnetic Confinement Fusion

Toroid geometry

$$R_0 \equiv \text{Major radius} \quad 2a \equiv \text{D width} \quad 2b \equiv \text{D height}$$

$$A \equiv \frac{R_0}{a} \quad \mathcal{S}_{ellipse} = (\pi ab) = \frac{\pi R_0^2\epsilon}{A^2}$$

$$\epsilon \equiv \frac{b}{a} \quad \mathcal{V}_{toroid} = (2\pi\bar{R}\mathcal{S}_{ellipse}) = \frac{2\pi^2 R_0^3\epsilon}{A^2}$$

Safety Factor & Plasma Beta

$$q(r) \equiv \frac{r}{R}\frac{B_\phi}{B_\theta} \qquad \beta_{pol} \equiv \frac{2\mu_0 P_{kinetic}}{B_{pol}} \qquad \beta_{tor} \equiv \frac{\beta_{pol}}{q^2A^2}$$

Bennett Z-Pinch (straight B-fields)

$$p_{kin} + \frac{B^2}{2\mu_0} = const \qquad \beta \equiv \frac{p_{kin}}{p_{mag}} \qquad 0 \le \beta \le 1$$

Particles

Maxwell-Boltzmann distributions

$$\langle v_\alpha\rangle = \sqrt{\frac{8}{\pi}\frac{T_\alpha}{m_\alpha}} \qquad v_{\alpha rms} = \sqrt{\frac{3T_\alpha}{m_\alpha}} \qquad \left(\frac{v_\alpha}{c}\right)^2 = \frac{T_\alpha}{m_\alpha c^2}$$

Kinetic pressure

$$p = n_eT_e + \sum n_zT_z \rightarrow 2nT(n_e = n_i \& T_e = T_i) \qquad n_e = \sum n_zZ$$

Fusion Power

Fusion reaction rate

$$RR_{ab} = \frac{n_a n_b}{1+\delta_{ab}}\langle\sigma v\rangle \rightarrow \frac{n^2}{4}\langle\sigma v\rangle\,(50/50)$$

Plant Efficiency

$$\eta_0 = \eta_{th}\left(1+\frac{1}{Q}\right) - \frac{1}{Q\eta_{inj}}$$

Confinement

$$n\tau = \frac{12T}{\langle\sigma v\rangle\,(f_\alpha(1-\chi_R)+\frac{1}{Q})E_{fusion}}$$

$$\chi_R = \frac{P_{rad}}{P_\alpha} = 0.055\sqrt{\frac{T}{15\,[\mathrm{keV}]}}Z_{eff}$$

$$f_\alpha = \frac{P_\alpha}{P_{fusion}} = \frac{1}{5} \qquad Z_{eff} = \frac{\sum_i n_iZ_i^2}{n_e}$$

Transport Coefficients

Timesteps & Coulomb logarithms

$$n\tau_e = 3.44\cdot 10^5\frac{T_{e,eV}^{3/2}}{\log\Lambda_e}\left[\frac{\mathrm{s}}{\mathrm{cm}^3}\right] \qquad \log\Lambda_e = 17.5$$

$$n\tau_i = 2.09\cdot 10^7\frac{T_{i,eV}^{3/2}}{\log\Lambda_i}\frac{m}{m_p}^{1/2}\left[\frac{\mathrm{s}}{\mathrm{cm}^3}\right] \qquad \log\Lambda_i = 15$$

Electrical Conductivities & Inter-species heating

$$\sigma_\perp = ne^2\frac{\tau_e}{m_{ei}} \qquad \sigma_\parallel = 1.96\sigma_\perp \qquad \dot{q}'''_{e\rightarrow i} = 3\left(\frac{m_e}{m_i}\right)\frac{(T_e-T_i)\,n_i}{\tau_e}$$

Dimensionless collisionality & Neoclassical heating

$$\nu_i^* = \frac{qR}{v_{th,i}^{2D}\tau_i} \qquad \dot{q}''_{neo} = \dot{q}''_{class}\times X$$

$$X = \begin{pmatrix} 2q^2A^{3/2} & : \nu^* < A^{-3/2} \text{ (banana)} \\ 1 & : A^{-3/2} < \nu^* < 1.0 \text{ (plateau)} \\ 2q^2 & : 1.0 < \nu^* \text{ (Pfirsch-Schlüter)} \end{pmatrix}$$

Thermal Conduction

$$\hat{\kappa}^e = n\tau_e\left(\frac{T_e}{m_e}\right) \quad \kappa_\parallel^e = 3.2\hat{\kappa}^e \quad \kappa_\perp^e = \frac{4.7\hat{\kappa}^e}{(\omega_{ce}\tau_e)^2} \quad \kappa_\wedge^e = \frac{2.5\hat{\kappa}^e}{(\omega_{ce}\tau_e)}$$

$$\hat{\kappa}^i = n\tau_i\left(\frac{T_i}{m_i}\right) \quad \kappa_\parallel^i = 3.9\hat{\kappa}^i \quad \kappa_\perp^i = \frac{2.0\hat{\kappa}^i}{(\omega_{ci}\tau_i)^2} \quad \kappa_\wedge^i = \frac{2.5\hat{\kappa}^i}{(\omega_{ci}\tau_i)}$$

$$\frac{\kappa_\parallel^e}{\kappa_\parallel^i} \sim 60 \quad \frac{\kappa_\perp^e}{\kappa_\perp^i} \sim \frac{1}{60} \rightarrow \frac{4.7m_e\tau_e}{2m_i\tau_i}(n_e = n_i \& T_e = T_i)$$

$$\dot{q}'' \equiv -\kappa\vec{\bigtriangledown}T \qquad \dot{q}_{th}^e = \dot{q}_\parallel^e + \dot{q}_\perp^e + \dot{q}_\wedge^e \qquad \dot{q}_{th}^i = \dot{q}_\parallel^i + \dot{q}_\perp^i - \dot{q}_\wedge^i$$

Engineering Equations

$$P_{fusion} = 1.86\cdot n_{20}^2\left[\frac{\mathrm{MW}}{\mathrm{m}^3}\right]\ (DT@15keV)$$

$$f_{pe} = 89\cdot\sqrt{n_{20}}\,[\mathrm{GHz}] \qquad f_{ce} = 2.8\cdot B_{kG}[GHz]$$

$$\langle\sigma v\rangle = 2.71\cdot 10^{-22}\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]\ (DT@15keV)$$

$$\eta_\parallel = \frac{1}{\sigma_\parallel} = 2.8\cdot 10^{-8}Z_{eff}T_{keV}^{-3/2}\,[\Omega\cdot\mathrm{m}]$$

$$P_{brem} = 5.35\cdot 10^{-37}T_{keV}^{1/2}Z_{eff}n_en_iVol\,[\mathrm{W}]$$

Plasma Parameters

Gyrofrequency & thermal velocity & Debye length

$$\omega_{c\alpha} = \frac{qB}{m_\alpha} \qquad v_{th}^{XD} = \sqrt{\frac{XkT}{2m}} \qquad \lambda_D = \frac{v_{th}^{1D}}{\omega_{pe}} = \sqrt{\frac{\epsilon_0kT}{n_\alpha e^2}}$$

Gyroradius & Alfven speed

$$\rho_e = \frac{v_{th}^{2D}}{\omega_{ce}} \qquad v_A = \frac{B}{\sqrt{\mu_0\rho}}$$

Plasma frequency & plasma dispersion & Phase velocity

$$\omega_{pe}^2 = \frac{n_ee^2}{\epsilon_0m_e} \qquad \omega^2 = \omega_{pe}^2k^2c^2 \qquad v_\phi = \frac{\omega}{k}$$

Impurities in Plasmas

$$P_{brem} \sim n_e\sum n_zZ^2 \quad \frac{P_{dirt}}{P_{clean}} = (1+fZ)\left(1+fZ^2\right)$$

$$p \sim \sum n_iT_i \quad \frac{p_{dirt}}{p_{clean}} = (1-fZ)^2$$

$$P_{fusion} \sim n_{DT}^2 \quad \frac{P_{dirt}}{P_{clean}} = \frac{1}{(1+fZ)^2}$$

Inertial Confinement Fusion

$$Yield = f_B\mathcal{Y}\mathcal{M} \quad \mathcal{Y} = 3.39\cdot 10^{11}\left[\frac{\mathrm{J}}{\mathrm{g}}\right] \quad f_B = \frac{\rho R}{6+\rho R}$$

$$\mathcal{M} = \frac{\frac{4}{3}\pi(\rho R)^3}{\rho^2} \qquad c_S = \sqrt{\frac{\gamma p}{\rho}} \qquad \gamma = \frac{5}{3}$$

Fusion Technology

$$\frac{\mathcal{U}_{TH}}{\tau_E} = P_{f\alpha} + P_{inj} - P_{rad} \quad NI = 5RB$$

$$q'_{ohmic} = \frac{I^2\rho_{stabilizer}}{A_{stabilizer}} \quad \dot{q}''_{ohmic} \le \dot{q}''_{CHF}$$