PERTURBATION THEORY FORMULAS FOR THE EFFECT OF THE DIMENSIONS ON THE CRITICAL MASS IN A FAST REACTOR

S. B. Shikhov

This paper deals with a method for calculating the critical mass of a reactor provided with a sufficiently thick reflector as a function of the composition of the core and its dimensions.

The working formula is obtained by an application of the theory of similarity to the usual first-approximation scheme of perturbation theory. For the calculation of the coefficients of the formula it is necessary to have the spatial and energy spectra of the neutron fluxes and values calculated numerically for some fixed volume of the reactor with a sufficiently thick reflector. By means of the coefficients so obtained for the formula it is possible to predict the critical mass over a wide range of variation of the dimensions of the core (over a change by about a factor two).

If the size of the core goes beyond the limits of the interval thus accessible, one has to calculate the coefficients for a new range of the dimensions, and this requires a new numerical computation of the spectra.

In this paper we give a formula containing coefficients calculated for a certain typical spectrum of a fast reactor for a number of isotopes contained in the core. The formula is checked by a nine-group calculation for volumes of the core ranging from 200 to 1000 cubic decimeters. The constants for the nine-group calculation were obtained from Soviet and foreign material published up to 1955.

Introduction

The calculation of the critical mass of a fast reactor involves cumbersome computations, even in relatively simple cases in which the reactor consists only of the core and a single-layer reflector. The calculation involves the determination of the spatial distribution of the neutrons of the fission spectrum, as softened by the effects of inelastic and elastic scattering by the heavy nuclei contained in the core and the screen.

If the dimensions of the reactor exceed several mean free paths of the fast neutrons, the calculation reduces to the solution of a system of multigroup diffusion equations with boundary conditions. In the case of design calculations for a small reactor one has to solve a multigroup system of gas-kinetic equations with the corresponding boundary conditions.

The calculations are complicated by the presence of inelastic transitions from each energy group into all the lower groups, and by the necessity of applying iteration methods for the determination of the k_{eff} of the system. The critical charge is determined by "sending" k_{eff} to unity. All of these computations can be carried out only by numerical methods by the use of high-speed computing machines. Under such conditions it is extremely useful to possess a formula providing a sufficiently reliable estimate of the critical charge without the solution of spatial-energetic boundary-value problems.

Theory

In its general form, useful for the computation of any reactor, the formula can be derived from the exact satationary gas-kinetic equation for the balance of additions and removals from the number of neutrons possessing lethargy U and having their velocities directed along the unit vector $\vec{\Omega}$:*

$$0 = -\vec{\Omega}\nabla\varphi(\vec{r}, u, \vec{\Omega}) - \Sigma(\vec{r}, u)\varphi(\vec{r}, u, \vec{\Omega}) + \int\int_{-\infty}^{u} \varphi(\vec{r}, u', \vec{\Omega}')\, du'\, d\Omega'\, \Sigma_{si}(\vec{r}, u', U, \mu_0) + \frac{\chi(u)}{4\pi}\cdot\frac{Q(\vec{r})}{k_{\text{eff}}}. \quad (1)$$

* Cf. e. g., reference 1, pp. 280, 281.

Here the notations are those adopted in [4], and

$$\Sigma_{si}(\vec{r}, u', U, \mu_0) = \Sigma_s(\vec{r}, u', U, \mu_0) + \Sigma_i(\vec{r}, u', U, \mu_0)$$

is the macroscopic differential cross section for elastic and inelastic scattering with transition of a neutron from the state $(u', \vec{n}')$ into the state $(u, \vec{n})$, and $U = u - u'$ characterizes the change of the lethargy in the transition.

In accordance with the definition of the total cross section in terms of the differential cross section we have

$$\int_0^{\infty}\int \Sigma_{si}(\vec{r}, u', U, \mu_0)\, dU\, d\Omega = \Sigma_{si}(\vec{r}, u') = \Sigma_s(\vec{r}, u') + \Sigma_i(\vec{r}, u').$$

The quantity

$$Q(\vec{r}) = \int_{-\infty}^{\infty}\int \nu_f(\vec{r}, u)\, \Sigma_f(\vec{r}, u)\, \varphi(\vec{r}, u, \vec{n})\, du\, d\Omega \quad (2)$$

gives the total number of fission neutrons produced in 1 cm^3 in 1 sec at the point with the coordinates $\vec{r}$.

The quantity $\chi(u)$ gives the spectrum of the fission neutrons normalized to unit lethargy range; for simplicity it is assumed that the angular distribution of the fission neutrons is isotropic and that the energy spectrum of the neutrons does not depend on the energy of the incident neutron.

It is convenient to write Eq. 1 in the abbreviated form

$$\hat{L}\varphi = \hat{\omega}\varphi - \Sigma_t\varphi + \frac{\chi(u)}{4\pi}\cdot\frac{Q(\vec{r})}{k_{\vartheta\Phi\Phi}} = 0, \quad (1')$$

where the linear integro-differential operator $\hat{\omega}$ is defined by the expression

$$\hat{\omega}\varphi = -\vec{\Omega}\nabla\psi(\vec{r}, u, \vec{\Omega}) + \int\int_{-\infty}^{u} \varphi(\vec{r}, u', \vec{\Omega}')\, du'\, d\Omega' \cdot \Sigma_{si}(\vec{r}, u', U, \mu_0).$$

The general perturbation theory is developed by the use of the adjoint equation

$$\hat{L}^+\varphi^+ = \hat{\omega}^+\varphi^+ + \frac{\nu_f(\vec{r}, u)}{k_{\text{eff}}}\Sigma_f(\vec{r}, u)\,Q^+(\vec{r}). \tag{3}$$

Here the operator $\hat{\omega}^+$ adjoint to the operator $\hat{\omega}$ is defined by the expression

$$\hat{\omega}^+\varphi^+(\vec{r}, u', \vec{\Omega}') = \vec{\Omega}'\nabla\varphi^+(\vec{r}, u', \vec{\Omega}') + \int_{u'}^{\infty}\int \Sigma_{si}(\vec{r}, u', U, \vec{\Omega}_0)\,du\,d\Omega\,\varphi^+(\vec{r}, u, -\vec{\Omega}).$$

The adjoint function $\varphi^+ = \varphi^+(\vec{r}, u, \vec{\Omega})$ has the physical significance of the importance function for a neutron having the coordinates r, the lethargy U, and the direction of motion given by the unit vector $\vec{\Omega}$*.

Then $Q^+(\vec{r})$ has the physical meaning of the importance of a fission neutron, and is given by

$$Q^+(\vec{r}) = \int_{-\infty}^{\infty}\int \frac{\chi_i(u)}{4\pi}\varphi^+(\vec{r}, u, -\vec{\Omega})\,du\,d\Omega. \tag{4}$$

The fact that Eq. 3 is adjoint to Eq. 1 is obvious. As is well known, the Eqs. 1 and 3 determine the same value of k_{eff}. Let us rewrite these equations in dimensionless form. For this purpose we introduce the dimensionless coordinates.

$$\vec{r}_0 = \frac{\vec{r}}{l},$$

where l is some characteristic dimension of the system. For this dimension we take the largest linear dimension of the core. The gradient operator is now expressed in the form

$$\nabla = \frac{\partial}{\partial \vec{r}} = \frac{1}{l}\cdot\frac{\partial}{\partial \vec{r}_0} = \frac{1}{l}\nabla_0.$$

After substituting this expression in Eqs. 1 and 3 we multiply these two equations by l. Then all of the macroscopic cross sections take the dimensionless form

$$l\Sigma = \Sigma_0 \tag{5}$$

(hereafter we shall use the index zero for all operators expressed in terms of dimensionless coordinates and cross sections).

The balance equations for the density and the importance function now take the form

$$0 = \hat{\omega}_0\varphi - \Sigma_{t_0}\varphi + \frac{\chi(u)}{4\pi}\cdot\frac{Q_0}{k_{\text{eff}}} = \hat{L}_0\varphi, \tag{6}$$

$$0 = \hat{\omega}_0^+\varphi^+ - \Sigma_{t_0}\varphi^+ + \frac{\nu_f\Sigma_{f_0}}{k_{\text{eff}}}Q^+ = \hat{L}_0^+\varphi^+. \tag{7}$$

Obviously, corresponding to similar forms and identical values of the dimensionless macroscopic cross sections Σ_0 one has identical energy and spatial distributions of the neutrons and importance functions, and also identical values of k_{eff}.

The values of the macroscopic cross sections will be preserved if the nuclear densities ρ of all isotopes occuring in the composition of the reactor are changed in inverse proportion to the change of the linear dimensions of the system, i.e., if we set

$$\rho \sim \frac{1}{l}. \tag{8}$$

* We here use the definition of the importance of a neutron given in reference 2.

Since the critical mass P of the reactor is proportional to the product of the nuclear density by the volume V_c of the core, from the condition (8) we get the following dependence of the critical mass P on the linear dimension:

$$P \sim V\rho \sim \frac{l^3}{l} = l^2. \tag{9}$$

The relation (9) has reasonably good accuracy for the evaluation of the critical mass over a certain limited range of variation of the dimensions. An improvement of the relation (9) can be obtained on the basis of the first approximation of perturbation theory. In developing the theory we shall use the scheme proposed by L. N. Usachev. Suppose that there is some change of the macroscopic dimensionless cross sections for the system:

$$\Sigma_0 \to \Sigma_0'.$$

Then there are also changes of the operators $\hat{L}$ and $\hat{\omega}$. Since these operators are linear functions of the macroscopic cross sections, we have

$$\hat{L}_0 \to \hat{L}_0', \quad \hat{\omega}_0 \to \hat{\omega}_0'.$$

Let us denote the variations of these operators by the symbol δ

$$\delta\Sigma_0 = \Sigma_0' - \Sigma_0, \quad \delta\hat{L}_0 = \hat{L}_0' - \hat{L}_0, \quad \delta\hat{\omega}_0 = \hat{\omega}_0' - \hat{\omega}_0.$$

Obviously, $\delta\hat{L}_0$ and $\delta\hat{\omega}_0$ are linear functions of $\delta\Sigma_0$. The change of the macroscopic cross sections is accompanied by a change of the spatial and energy distribution of the neutrons. The perturbed neutron fluxes now satisfy the equation

$$\hat{L}_0'\varphi' = 0. \tag{10}$$

We form the scalar product of Eq. 7 by φ', and that of Eq. 10 by φ^+, and subtract one result from the other. Then we get the identity

$$(\varphi^+, \hat{L}_0'\varphi') - (\hat{L}_0^+\varphi^+, \varphi') = 0,$$

where the brackets denote the triple integral over the entire volume of the reactor, over the lethargy, and over the solid angle. By means of a property of adjoint operators this identity can be brought into the following form

$$(\varphi^+, \hat{L}_0'\varphi') - (\varphi^+, \hat{L}_0\varphi') \equiv 0.$$

From this it follows that

$$\begin{gathered}(\varphi^+, (\hat{L}_0' - \hat{L}_0)\varphi') \equiv 0,\\ (\varphi^+, \delta L_0\varphi') \equiv 0.\end{gathered} \tag{11}$$

If in the identity (11) we replace the perturbed fluxes by their unperturbed values, we get the first approximation of perturbation theory

$$(\varphi^+, \delta\hat{L}_0\varphi) = 0. \tag{12}$$

The symbolic product in this equation is linear in the quantities

$$\delta\Sigma_0, \quad \frac{1}{k_{\text{eff}}}, \quad \frac{1}{k_{\text{eff}}},$$

and from it one can directly calculate the difference

$$\delta\left(\frac{1}{k_{\text{eff}}}\right) = \frac{1'}{k_{\text{eff}}} - \frac{1}{k_{\text{eff}}} \cong \frac{\delta k}{k_{\text{eff}}},$$

which gives the change of the reactivity caused by the perturbation of the dimensionless cross sections.

If we require that the perturbed system be critical, then we must set $k'_{\text{eff}} = 1$ in Eq. 12 and calculate the corresponding change of the critical mass. The calculations will be accurate enough if the replacement of φ' by φ in the passage from the identity (11) to Eq. 12 does not bring with it much change in the value of the symbolic product, i.e., if the spatial and energy perturbations of the neutron fluxes are small.

If in Eq. 12 we substitute the expression (1') for $\hat{L}\varphi$ and if we then use the definition (4) of the importance function of a fission neutron, we get

$$(\varphi^+, \delta\hat{L}_0\varphi) = -(\varphi^+, \delta\Sigma_{t_0}\varphi) + (\varphi^+, \delta\hat{\omega}_0\varphi) + \frac{1}{k_{\text{eff}}}\int Q^+Q'_0\,dV - \frac{1}{k_{\text{eff}}}\int Q^+Q_0\,dV = 0, \tag{13}$$

where

$$Q'_0 = \int\int_{-\infty}^{\infty} \nu_f\Sigma'_{f_0}\varphi\,du\,d\Omega, \tag{14}$$

$$Q_0 = \int\int_{-\infty}^{\infty} \nu_f\Sigma_{f_0}\varphi\,du\,d\Omega. \tag{15}$$

If we set

$$\delta Q_0 = \int\int_{-\infty}^{\infty} \nu_f\delta\Sigma_{f_0}\varphi\,du\,d\Omega,$$

$$Q'_0 = \delta Q_0 + Q_0,$$

$$\frac{1}{k_{\text{eff}}} = \delta\left(\frac{1}{k_{\text{eff}}}\right) + \frac{1}{k_{\text{eff}}}$$

and neglect the product

$$\delta\left(\frac{1}{k_{\text{eff}}}\right)\delta Q_0$$

as a higher-order quantity, we get as the result the expression

$$-(\varphi^+, \delta\Sigma_{t_0}\varphi) + (N^*, \delta\hat{\omega}_0\varphi) + \frac{1}{k_{\text{eff}}}\int Q^+\delta Q_0\,dV + \delta\left(\frac{1}{k_{\text{eff}}}\right)\int Q^+Q_0\,dV, \tag{16}$$

which is convenient for the determination of $\delta\left(\frac{1}{k_{\text{eff}}}\right)$.

Equation 13 gives in explicit form the relation between the dimensions of the system and the volume fractions of the isotopes of which it is composed. Let the reactor be divided into separate zones, the nth zone having the volume V_n. Then any dimensionless macroscopic cross section of the nth zone can be represented in the form

$$\Sigma_0^{(n)} = l\sum_m \varepsilon_m^{(n)}\sigma_m\rho_m,$$

where ρ_m is the nuclear density of the mth isotope, which has microscopic cross section σ_m and volume fraction ε_m. The left member of Eq. 12 can now be written as a linear function of the volume fractions

$$\sum_{n,\,m}(l'\varepsilon_m^{(n)\prime} - l\varepsilon_m^{(n)})A_m^{(n)} + \sum_{n,\,m}\left(\frac{l'\varepsilon_m^{(n)\prime}}{k_{\text{eff}}} - \frac{l\varepsilon_m^{(n)}}{k_{\text{eff}}}\right)B_m^{(n)} = 0. \tag{17}$$

It is not hard to verify that the coefficients $A_m^{(n)}$ and $B_m^{(n)}$ can be expressed as follows:

$$A_m^{(n)} = \rho_m\int_{V_n} dV\int d\Omega\int_{-\infty}^{\infty} du\left[-\sigma_m N + \int d\Omega'\int_{-\infty}^{u}\varphi(\vec{r}, u', \vec{n}')\,\sigma_{si,\,m}(u', U, \mu_0)\,du'\right]; \tag{18}$$

$$B_m^{(n)} = \rho_m\int_{V_n} dV\,Q^+\int d\Omega\times\int_{-\infty}^{\infty}\nu_{fm}(u)\,\sigma_{fm}(u)\,\varphi(\vec{r}, u, \vec{n}). \tag{19}$$

Equation 17 gives the desired relation between the reactivity of the system, its linear dimensions, and the volume fractions of the elements contained in it.

If one uses the age-diffusion approximation, the expressions for the coefficients $A_m^{(n)}$ and $B_m^{(n)}$ take the following forms:

$$A_m^{(n)} = \rho_m\int_{V_n} dV\int_{-\infty}^{\infty} du\left\{\Phi^+(\vec{r}, u)\times\left[-\sigma_{cfi,\,m}\Phi(\vec{r}, u) + \int_{-\infty}^{u}\Phi(\vec{r}, u')\,du'\,\sigma_{in,\,m}\times(u', v) - \xi_m(u)\frac{\partial\sigma_{s,\,m}(u)\,\Phi(\vec{r}, u)}{\partial u}\right] + 3\sigma_{tr}(\vec{r}, u)\,\vec{i}^+(\vec{r}, u)\,\vec{i}(\vec{r}, u)\right\}; \tag{20}$$

$$B_m^{(n)} = \rho_m\int_{V_n} dV\,Q^+(\vec{r})\times\int_{-\infty}^{\infty}\nu_{fm}(u)\,\sigma_{fm}(u)\,\Phi(\vec{r}, u). \tag{21}$$

In the diffusion approximation the relations between the neutron flux $\Phi(\vec{r}, u)$ and importance function $\Phi^+(\vec{r}, u)$ and the spatial, angular, and energy distributions of the neutron flux and importance is given by the well-known relations:*

$$\varphi(\vec{r}, u, \vec{n}) \cong \frac{\Phi(\vec{r}, u)}{4\pi} + \frac{3}{4\pi}\vec{i}(\vec{r}, u)\cdot\vec{\Omega},$$

$$\varphi^+(\vec{r}, u, -\vec{n}) \cong \frac{\Phi^+(\vec{r}, u)}{4\pi} - \frac{3}{4\pi}\vec{i}^+(\vec{r}, u)\cdot\vec{\Omega},$$

which contain the first two terms of the expansions of φ and φ^+ in series of spherical harmonics. Furthermore

$$\sigma_{tr}(u) = \sigma_t(u) - \bar{\mu}_0(u)\,\sigma_s(u),$$

$$\vec{i}(\vec{r}, u) = -\frac{1}{3\Sigma_{tr}(\vec{r}, u)}\nabla\Phi(\vec{r}, u), \tag{22}$$

$$\vec{i}^+(\vec{r}, u) = -\frac{1}{3\Sigma_{tr}(\vec{r}, u)}\nabla\Phi^+(\vec{r}, u), \tag{23}$$

$$Q^+(\vec{r}) = \int_{-\infty}^{\infty}\chi(u)\,\Phi^+\,du, \quad Q(\vec{r}) = \int_{-\infty}^{\infty}\nu_j\Sigma_j\Phi\,du. \tag{24}$$

The quantity $\xi_m(u)$, appearing in the expression (20) is the mean logarithmic energy loss in an elastic collision; if there is anisotropy in the center-of-mass system this quantity depends on the energy.

*Cf. e. g., reference 3.

In the multigroup approximation the integrals over the lethargy that appear in the expressions (22) and (21) are represented as finite sums over all the p-groups

$$A_m^{(n)} = \rho_m \int_{V_n} dV \sum_{k=1}^{p} \left\{ \Phi_k^+(\vec{r}) \cdot \left[-\sigma_{cf,\,\text{yB},\,m}^{(k)} \Phi_k(\vec{r}) + \right.\right.$$

$$\left.\left. + \sum_{j=1}^{k-1} \sigma_{\text{re},\,m}^{kj} \Phi_j(\vec{r}) \right] + 3\sigma_{tr,\,m}^{(k)} \vec{i}_k^+(\vec{r})\, \vec{i}_k(\vec{r}) \right\}; \qquad (25)$$

$$B_m^{(n)} = \rho_m \int_{V_n} dV \cdot Q^+(\vec{r}) \sum_{k=1}^{p} (\nu_f \sigma_f)_k \Phi_k(\vec{r}). \qquad (26)$$

The multigroup microscopic cross sections appearing in these expressions are averaged over the neutron spectrum of the given reactor, as is recommended, for example, in reference 4, or over the neutron spectrum and the importance spectrum (reference 1, Section 62), which latter procedure seems better justified.

The removal cross section σ_{re}^{kj} takes account of both elastic and inelastic scattering with passage of a neutron from group j to group k. The main practical importance of the formula (17) is in the solution of the problem of two zones, i.e., the problem of a reactor consisting only of the core (zone $n = 1$) and an infinitely thick reflector (zone $n = 2$). If, however, the porosity of a uranium screen does not exceed 30 percent, then if the thickness is larger than 20 cm it can be regarded as practically infinite in the calculation of the reactivity.

In such cases one can confidently use the formula (17) with the coefficients A_m^2 and B_m^2 calculated for an infinite screen.

Test of the Formula and Discussion of Results

The formula (17) has been tested by means of nine-group calculations in the age-diffusion approximation for spherical geometry. Calculations were carried out for two spherical reactors of volumes 205 and 930 cubic decimeters. We shall call them "system I" and "system II", respectively. The thickness of the reflector in terms of the extrapolated boundary was taken the same for both systems—50 cm. The volume fractions of U^{238}(ε'_8), iron (ε_{Fe}) and sodium (ε_{Na}) making up the reflector were also taken to be the same in both systems, with the values $\varepsilon'_8 = 0.7$; $\varepsilon_{Fe} = 0.1$; $\varepsilon_{Na} = 0.2$.

It was assumed that the core consists of fuel alloy with volume fraction ε_{fuel}, iron with volume fraction ε_{Fe}, and sodium with volume fraction ε_{Na}. The values of the volume fractions of the core materials are not the same for systems I and II; they are given in Table 1. The table also gives the values of R, the radius of the spherical core. The fuel alloy assumed was plutonium diluted with some other substance. The calculations were made for each system with five types of alloys, containing U^{238}, Ba, Zr, Fe, and Mo with the plutonium. For all the elements making up the active zone a nine-group system of constants was constructed, based on the experimental data published up to 1955.

Table 1. Comparative Characteristics of Systems I and II

	ϵ_{fuel}	ε_{Na}	ε_{Fe}	R, cm
System I	0.33	0.50	0.17	36.5
System II	0.36	0.53	0.11	60.3

The determinations of the critical mass and the calculations of the neutron spectra and of the spatial distributions of the neutron fluxes and importance functions were made by solving a system of multigroup diffusion equations for the neutron density and importance function. The quantity k_{eff} was determined by the method of iteration of the sources. The computations were carried out by the method of finite-difference factorization explained in the book by G.I. Marchuk [4], and were done under his direction. A study of the spectra obtained with different alloying materials in systems I and II showed that in general they have the same character. The results of the solution of the critical problem for systems I and II are presented in Table 2, where ε_9 denotes the volume fraction of the main fissionable isotope, plutonium. The spectra obtained for each modification of system I were used to construct the coefficients of the formula (17) according to Eqs. 25, 26. System I was regarded as an unperturbed system. The quantities k_{eff} and k'_{eff} for each modification were taken in accordance with Table 2.

Table 2. Results of the Calculations for Systems I and II

Composition of fuel alloy	System I		System II	
	ε_9	k_{eff}	ε_9	k_{eff}
Pu+U^{238}	0.050	0.97	0.034	1.02
Pu+Zr	0.052	1.05	0.023	1.01
Pu+Mo	0.050	0.99	0.033	1.02
Pu+Fe	0.052	0.99	0.031	1.07
Pu+Ba	0.055	0.98	0.032	1.07

For each modification with the different alloying elements the value of ε'_9 for system II was found from the formula (17), with the coefficients $A_m^{(n)}$ and $B_m^{(n)}$ computed from the calculated spectrum for the same modification of system I. The error in the calculation of ε'_9 did not exceed 10 percent. This made it possible to take as a basis a certain typical spectrum for a fast reactor and calculate from it a set of universal coefficients for the formula (17). The spectrum adopted for this purpose was that of system I with U^{238} as the alloying element.

Table 3 shows the results of calculations of the quantity ε'_9 by Eq. 17 with the coefficients computed from this spectrum. The table also shows the comparison with the computed value of ε'_9. Table 4 gives the coefficients $A_m^{(n)}$ and $B_m^{(n)}$ for the types of materials for which the results are listed in Table 3.* The nuclear densities are taken under normal conditions. The coefficients relating to the screen are given only for U^{238}, Na, and Fe.

* The calculations of coefficients and the check of the formula (17) were carried out by Engineers A. P. Karbaeva and V. A. Kuz'micheva.

Table 3

Comparison of the Results of Calculations of ϵ'_9 by the Formula (17) With Those of Diffusion Calculations

Composition of fuel alloy	System I			System II		
	Eq. (17)	computed values	error, percent	Eq. (17)	computed values	error, percent
Pu+U^{238}	—	0.050	—	0.030	0.0340	10
Pu+Zr	0.051	0.052	2	0.024	0.0230	3
Pu+Mo	0.052	0.050	4	0.032	0.0330	3
Pu+Fe	0.051	0.052	2	0.032	0.0312	2
Pu+Ba	0.049	0.055	10	0.031	0.032	3

Table 4. Coefficients for Formula (17)

Element	$A_m^{(1)}$	$B_m^{(1)}$	$A_m^{(2)}$	$B_m^{(2)}$
U^{238}	−0.2651	0.2524	−0.0006	0.0235
Pu^{239}	−1.9925	6.3167	—	—
Zr	0.0665	0	—	0
Mo	−0.0297	0	—	0
Na	0.0187	0	0.0097	0
Fe	−0.0025	0	0.0264	0

Conclusions

The check of the formula (17) by the solution of nine-group age-diffusion equations has shown the applicability of the formula for an increase of the dimensions of a system by a factor of two, or possibly more than two. The errors in the calculations of the constants shown in Table 3 for the different alloying elements and for spherical volumes from 200 to 1000 cubic decimeters do not exceed 10 percent. The possibility of application to other fissionable isotopes and alloying elements and to other ranges of volumes of the core, on the basis of the neutron spectra and importance spectra adopted here, still needs to be specially checked by calculations of the spectra in the new ranges of sizes.

For the alloying elements considered here and in the range of dimensions indicated, the coefficients given in Table 4 can be accepted, subject to the condition that the screen can be regarded as infinite.

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