Updating the SCADS Computational Model of Children's Early Arithmetic Learning and Strategy Change to Incorporate Advances in Cognitive and Developmental Neuroscience, and a Detailed Investigation of the MicroStructure of Early Addition Strategies

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This aim of this proposal is to revise the SCADS computational model of early arithmetic learning and strategy change to incorporate advances in cognitive and developmental neuroscience, and a detailed investigation of the micro-structure of early addition strategies.

From: Hansen McKensie and McClelland: " When learning addition, children appear to perform a remarkable feat: as they practice counting out sums on their fingers, they discover more efficient strategies while avoiding conceptually flawed procedures. Existing models that seek to explain how children discover good strategies while avoiding bad ones postulate metacognitive filters that reject faulty strategies. However, this leaves unexplained how the domain-specific knowledge required to evaluate a strategy might be acquired prior to addition being mastered. We introduce a biased exploration model, which demonstrates that new addition strategies can be discovered without invoking metacognitive filtering. This model provides a fit to data comparable to previous models, with the considerable advantage of avoiding an appeal to knowledge whose source is not itself explained. Specifically, we fit the pattern of changes in strategy use over time as children learn addition, as well as the overall error rate and error types reported empirically. The model suggests that the critical element allowing strategy discovery may be prior learning, rather than metacognitive strategy evaluation. We close by offering several empirical predictions and propose that what others have called strategies might often be decomposable into elements that can be assembled on the fly as problem solving unfolds in real time.

Siegler and colleagues built several computational models to explain their data, culminating in SCADS (Strategy Choice And Discovery Simulation), which posits initial knowledge of the „sum‟ strategy, a retrieval system for recalling answers based on associative learning, a module that proposes new strategies and another module that filters out proposals that do not meet criteria assuring their adequacy. SCADS captures some aspects of the successive emergence of strategies shown in the behavioral data. However, the transitions in learning are far more rapid than in the empirical data, and no account is given for how children would acquire the posited metacognitive filtering mechanism. It is this gap that we attempt to address. The Biased Exploration Model Our model approaches the problem of strategy evolution through the use of a standard reinforcement learning system. It attempts to do away with the domain specific strategy proposal and filtering modules of SCADS. It avoids incorrect strategies because action is biased towards related, previously learned, procedures. The key insight arises by breaking down the two main strategy discoveries („shortcut sum‟ and „min‟) into the component steps needed to allow a new policy to arise from a predecessor"

"... it remains unclear how children could acquire the complex knowledge required to judge the appropriateness of their own strategy proposals." [JS replies: But they DO know that they are being asked to add, as opposed to, say, count. So there is clearly some framing knowledge someplace.]

" We accomplish this by modifying a standard trial and error, reinforcement-learningbased paradigm to be biased towards previously learnt actions."

" We note that children learning the addition task have already learnt to count out numbers of objects, count on their fingers, and perform addition using a finger counting strategy (Siegler & Jenkins, 1989). As we shall demonstrate, instantiating a model with biases towards these actions obviates the need for a metacognitive filter. We also expand the notion of retrieval – a „strategy‟ that circumvents the need to engage in a structured sequence of behaviors by simply recalling the correct final answer to a problem – by suggesting that retrieval might also occur for appropriate subparts of a larger problem. Our model makes several novel predictions about the discovery process and questions the notion that selection and discovery processes necessarily take place at the level of complete strategies."

" In our view, it is important to frame the discovery process against the backdrop of relevant previously learnt procedures. The most important to our theory is what we will call the count-list procedure whereby the child learns to step through a stable ordering of the number words, sometimes while counting out fingers or other physical tokens. The count list is a prerequisite for learning addition and is known by all children in the study."

"We also assume (following Davidson, Eng & Barner, 2012) children can perform the how many task, in which the child verbally goes through his count list in order, simultaneously pointing to the next in a set of physical tokens, then responding with the number reached when the items in the set have been exhausted. It is generally accepted that this behavior is well learnt by the time children are taught their first addition procedure. Finally, we assume children have mastered the give-a-number or give-n task, which involves providing a supply of tokens and asking the child to give the experimenter (or other target) a certain number of them. Children who can perform this task for numbers larger than 4 do so by stepping through the count list as they remove them one by one from the supply, stopping when they reach the requested number."

" the researchers focused their analysis on when new strategies were discovered, how often they were used thereafter, and whether or not any incorrect strategies were ever used. The results of the study partially supported the idea that strategy change occurred through an exploration-based, incremental learning process. Children were not always able to describe or explain their new strategies to the experimenter. However, the authors also found no evidence that incorrect strategies were ever used and they argued that exploration of the space of possible strategies should lead to such errors. Though children did sometimes answer problems incorrectly, the authors argued that these errors did not represent the sort of conceptual mistakes one would assume children would make if they were randomly exploring the space of possible strategies."

" For the „shortcut sum‟ this means making two critical exploratory steps away from the existing „sum‟ strategy policy. The first is to continue going through the count list after reaching the end of the first addend, rather than starting the count over at one for the second addend. The second is to stop going through the count list once the correct numeral is uttered. This second step can be seen as relying on problem specific knowledge, but avoiding reliance on recall by replacing it with an easier recognition problem whereby the child merely has to stop counting when the value reached feels like it is correct"

" Exploration of this shortcut sum strategy can take place without assuming there is uniform exploration across all possible states and actions. We propose that the previously learnt counting procedure gives children a tendency to continue counting even when the first addend is reached. Thus, whilst the majority of the time the model chooses the sum strategy, occasionally a latent tendency to perform the related counting task takes over and an „exploratory‟ step is made. Critically, this exploratory step speeds up task performance but does not lead to an error."

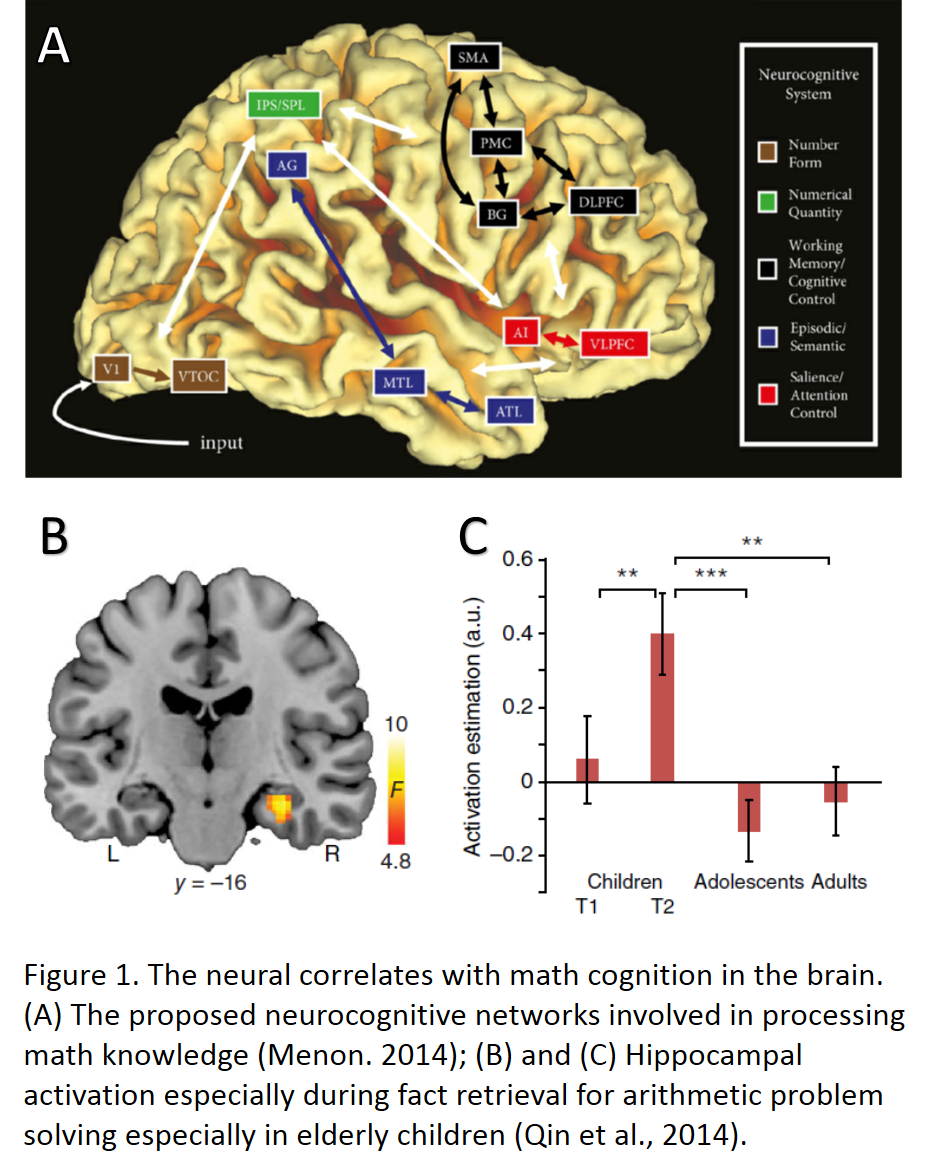
"We should stress here that the discovery process proposed above is very different from prior proposals, which focus on realizing the redundancy in having to recount both addends (Shrager & Siegler, 1998; Neches, 1987). Which of these two conceptualizations is the better account of human behavior is an empirical question, and the two mechanisms are not necessarily mutually exclusive. The size of the second addend is positively correlated with error rate when the shortcut-sum strategy is used, and this has been seen as evidence that the strategy was discovered to eliminate redundancy. It has been hypothesized that the increased error rate comes from the child having to hold two numbers in mind (one for the total count, and another for the count within the second addend). However, our recognition account is also compatible with this increased error rate, as larger second addends allow more chances to terminate the counting process, as well as providing more time to forget the problem, thus lowering the chance of recognizing the answer."

"We implemented the model within the actor-critic reinforcement learning architecture. While this architecture has traditionally been chosen for its relative biological plausibility, here its utility comes from the fact that an actor-critic mdel learns by modifying the current policy. This feature limits how drastically a single learning step can affect the behavior of the agent (Sutton & Barto, 1998). This is important in part because it prevents the large number of errors typically associated with reinforcement learning. Since the initial policy (the explicitly taught sum strategy) is accurate, the model avoids a big change unless it consistently outperforms this existing solution."

"In addition to these actions, the agent tries to retrieve the sum at the start of each problem, and the action selection process described above only occurs when this initial retrieval fails. The assumptions made for retrieving this sum (as well as the subtask end states) are taken straight from SCADS. This memory mechanism, the „distributions of associations‟ model, learns by accumulating association strengths between the task at hand and the various possible end states, with the idea being that it will converge to the correct answer as this is the most common end state for an agent competent at the task. When called upon to make a retrieval, a threshold is stochastically set and compared to each association strength. If no associations are higher than this threshold, then the retrieval fails to return an answer. If multiple associations are above the threshold, the retrieved association is randomly chosen from this set (Siegler & Shrager, 1984)."

"Distinguishing our account from that of the SCADS model and investigating the extent to which metacognition plays a role in the discovery process will be another focus of our future work. One area where the models make distinct predictions is in the rationale behind the use of the shortcut sum strategy. The SCADS model claims that children track the total while also counting out the second addend, while our account relies on habitually counting on until the sum is recognized, avoiding the need to keep track of the second addend. This may be amenable to empirical exploration. Self-reports might also be used to differentiate these accounts, but we stress that we do not claim children do not eventually discover a rationale for their actions. Our claim is only that they need not do so before the actions themselves emerge. Another area for future work will be to address the problem-specific representations of our current model and to explore the consequences of this for the model‟s predictions. Sharing information between problems might simply accelerate the learning process, but more fundamental changes are also possible. For example, sharing could increase certain errors due to confusion of one problem with another, which would change the pressures that lead to strategy discovery. Another approach we are exploring is to let a neural network control the policy across all of the problems (in this case, the problem state would be represented as an input feature vector), as this could allow a more nuanced sharing of discovery information to emerge (it is possible to see at least some versions of a table-driven model as an alternative implementation of this neural-network based approach). Going forward, we plan to extend our model to account for another stream of evidence that has previously been used to support the notion of metacognition: the recognition of never-before seen strategies. Children that have been shown the min strategy before discovering it still rate it as better than an incorrect strategy (Siegler & Crowley, 1994). While this has previously been taken as support for the proposed metacognitive filter, we suggest that the biased exploration model can account for this data as well by using the agent‟s value function to evaluate novel strategies. Such an extension is indicative of our overall goal with this model: to set up a new foundation for selfguided learning that will allow a rethinking of the role of metacognition in strategy discovery." ========================================

**BACKGROUND**

Modern systems neuroscience seeks to understand how complex behavior arises from the dynamic information exchange and control process across the network of brain regions, and how these change with learning and development. Children’s arithmetic, more specifically children's small number addition (hereafter just “arithmetic” or “addition”) provides a unique opportunity to investigate these questions, as the phenomena in this domain are complex and foundational, yet still tractable. Addition involves a range of cognitive, often physical actions[1](#_ENREF_1), [2](#_ENREF_2), yet it has well-defined structure with formally correct answers, so that the space of possible correct (or plausibly incorrect) algorithms can be clearly mapped out[3](#_ENREF_3). Moreover, how children carry out addition, what mistakes they make, and how they transition from pre-addition (only knowing counting sequences) through adult-type “pure retrieval”[4-8](#_ENREF_4), occurs in a relatively consistent cognitive- and neuro-developmental time-window[4](#_ENREF_4), [9](#_ENREF_9). As a result of its great educational importance[10-14](#_ENREF_10), children's arithmetic has been well-studied by psychologists, cognitive neuroscientists, and computational modelers. Several core brain regions contribute to math competence and processing[15-19](#_ENREF_15), including the ventral visual stream (e.g., posterior fusiform gyrus; pFG) for decoding number forms, parietal circuits (majorly around inferior parietal sulcus; IPS) for anchoring the visuospatial numerical representations, prefrontal-parietal cortices for manipulating quantity representations in working memory, and medial temporal lobe (MTL) and especially hippocampus for associative memory processing in children[20-22](#_ENREF_20) (Figure 1). Difficulties in math processing in some children (e.g., developmental dyscalculia; DD) are associated with abnormalities such as hypo-activation in these brain regions[23](#_ENREF_23), or increased functional connectivity among these regions[24](#_ENREF_24). Studies further suggest that that MTL, left prefrontal, and bilateral posterior parietal cortices correlate with the use of different strategies[25](#_ENREF_25), and hippocampus seems crucial for the transition from overt to implicit strategies[22](#_ENREF_22). Even given all this detailed knowledge regarding children's arithmetic, we still do not understand how the large-scale dynamic network of widely-distributed brain regions supports the organized execution of arithmetic reasoning, nor, especially, how their interactions and change through learning and development. Computational studies of arithmetic development, which have been undertaken for decades[4](#_ENREF_4), [5](#_ENREF_5), [26-28](#_ENREF_26), have generally been rendered either in purely symbolic, purely connectionist, or hybrid symbolic/connectionist paradigms. These paradigms suffer from several critical drawbacks: (a) implementation of procedural memory and executive control systems were not theory-driven (not relevant theory was available at that time); (b) memory and knowledge was not represented in distributed manner (as is not believed to be the case); and (c) model structure was not informed by current understanding of the brain. For example, theorists have elicidated the role of prefrontal cortex in system control of cognitive tasks[32](#_ENREF_32), and neural networks have be shown to naturally combined with system-control models for adaptive learning and optimal control[33-36](#_ENREF_33), but these advances have not made their way into computational models of arithmetic.

Modelers have recently demonstrated that systems-oriented computational neural networks can account well for neural dynamics in human and animal studies, as well as provide good mechanistic explanations for dynamic changes and developmental evolution of brain network functions [29-31](#_ENREF_29).Therefore, the primary goal of the presently proposed research is to apply this methodology to develop a novel systems-control/connectionist model of the interactive dynamics and developmental evolution of arithmetic skill and number sense. By modeling the neurocognitive networks in human brain, this new model will be able to provide explanations that encompass the existing rich body of cognitive theory, data, and computational experimentation in arithmetic within a framework of modern systems neuroscience.

**GOALS AND HYPOTHESES**

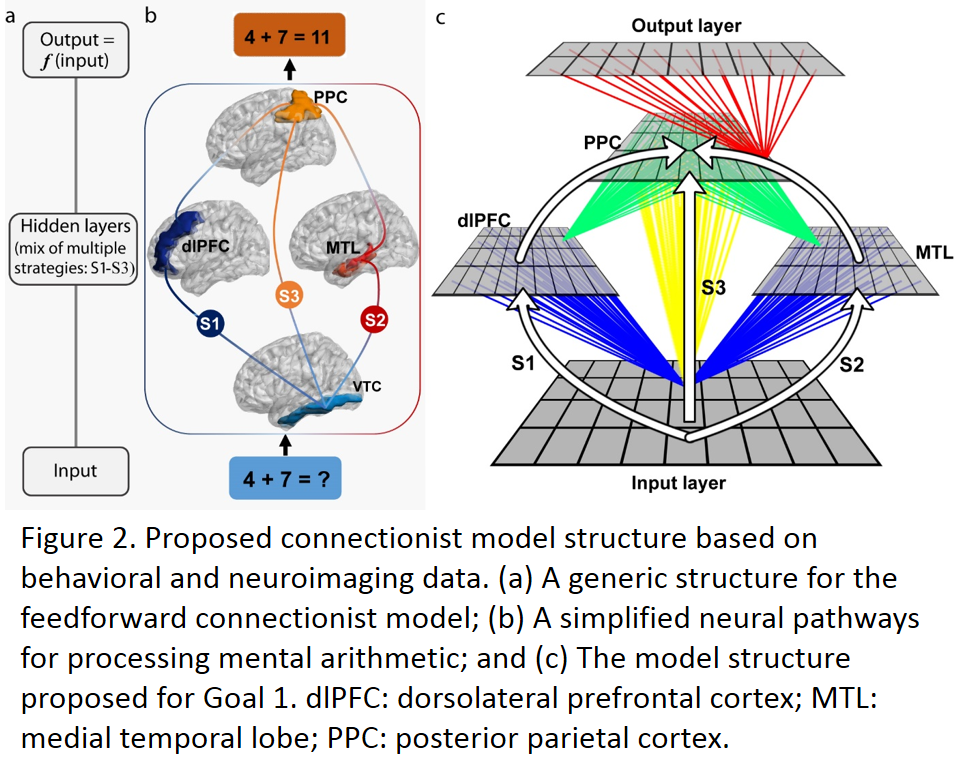
**Goal 1**: **Develop a neuro-computational model accounting for both behavioral and neurobiological outcomes in arithmetic development.** I will establish a connectionist model to demonstrate the face validity and feasibility of this neuro-computational approach that bridges both behavioral and neuroimaging data in arithmetic model. ***Hypothesis 1***: The development of arithmetic problem solving in children results in behavioral neurobiological changes over development, which can be partly explained by the differences in efficiency of neural pathways in a connectionist model.

**Goal 2: Extend the connectionist model into a hybrid model to explain arithmetic development through learning.** The development of the prefrontal control systems is facilitated by the learning process of arithmetic problem solving. To explore this feature, I will combine the connectionist model above with a system-control model, thus creating a more comprehensive model that is a hybrid of strategy representation and execution aligned with our current knowledge of the cognitive functions of brain regions, how these regions are interconnected, and how the execution of complex procedures are initiated and controlled, in the context of development of arithmetic problem solving. ***Hypothesis 2A***: The revised hybrid model, wherein the activation status of different neural pathways for strategy use is monitored by prefrontal control units and changes their controlling parameters, can more accurately explain the developmental evolution of strategy application and change in arithmetic and number knowledge. ***Hypothesis 2B***: The model will exhibit and thereby serve as an account for well establish phenomena, such as that learning overt and inefficient strategies can scaffold the development of more efficient, retrieval-like strategies. I will specifically demonstrate that manipulating the training environment in the model will lead to different behavioral outcomes in arithmetic development that are similar to those observed in the data.

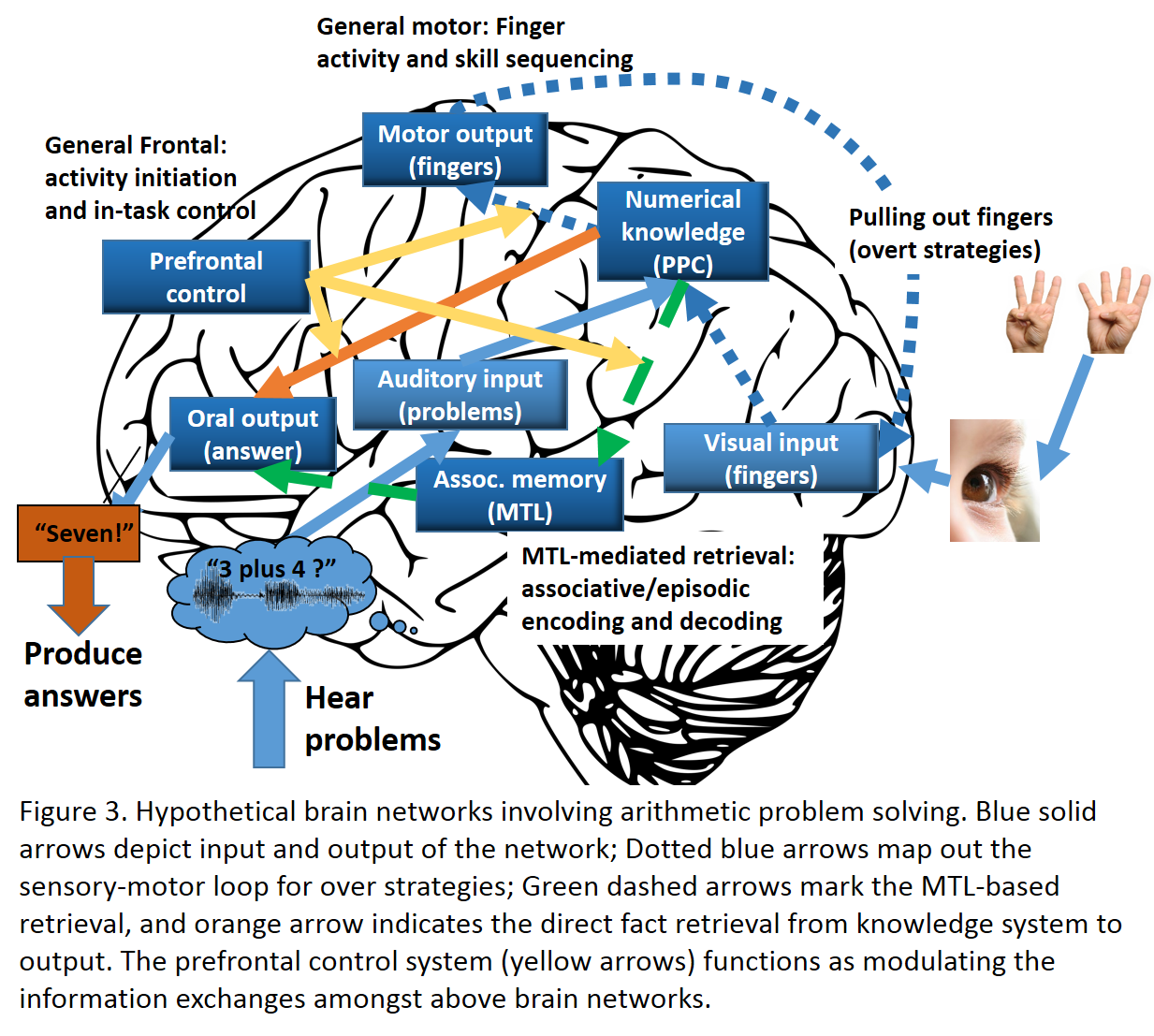
**Goal 3: Explore sources of individual differences in neural basis to explain typical and atypical development**. I will employ the new model above to try to explain aspects of typical and atypical (DD population) arithmetic development as individual differences in the models neural architecture and parameterization. ***Hypothesis 3***: The behavioral and neurobiological abnormality of DD can be explained by different settings of global learning parameters analogous to biological dysfunctions in human brain. To the extent that this is not possible, we will document phenomena that must be explained by by systems-level architectural changes in the model.

**EXPERIMENTAL METHODS**

**Part 1**. The development of arithmetic skills from overt strategies to retrieval strategies is accompanied by processing efficiency observed in behavioral data, and a general tendency of prefrontal-to-parietal (anterior-to-posterior) in brain activations.

***Modeling***. I will use feed-forward connectionist models (example depicted in Figure 2) to establish three distinct neurocognitive pathways hypothesized for distinct strategy uses (2b), including effortful counting strategies (S1; dlPFC), hippocampal-dependent associative memory retrieval (S2; MTL), and hippocampal-independent fact memory retrieval (S3, PPC). By manipulating the additional potentiation from controlling units, the processing efficiency of each neuro-pathway for strategy use is manipulated to test the performance and neural activities of child-like (i.e., recruiting more inefficient pathways) and adult-like models (i.e., recruiting more efficient pathways) after practice solving simple addition problems. ***Behavior and neuroimaging***. I will re-analyze *behavioral* data (e.g., accuracy, response latency, inter-problem variability, etc.) and *neuroimaging* data (e.g., functional activation, representational stability, etc.) from Qin et al. (2014) to evaluate the model in terms of developmental changes in behavioral efficiency and the brain involvement from prefrontal to parietal cortices.

**Part 2.** I will extend the NN model to include recent understanding of the involvement of other brain systems, especially sensory, motor, memory, and control processes, and including external aspects of behavior, especially regarding phonological (hearing the problem) and visual (finger counting) involvement.

***Modeling***. I will extend the previous models by adding information and control system analogous to the proposed function of prefrontal cortex32, in maintaining and controlling task-relevant information exchange in distributed brain networks, as well as other systems (Figure 3), focusing on the interaction between different neuro-pathways/systems underlying the execution of different strategies, and how the control and other brain systems support the development of each other. ***Behavior and neuroimaging***. I will conduct a large-scale *behavioral* data re-analysis from previous studies to demonstrate the developmental changes in strategy use for addition problems in children from age 5-10 years old. I will use this analysis to examine how well the model can accounts for (a) the response distribution across correct and incorrect answers for different addition problems; and (b) the U-shaped retrieval usage that retrieval strategy is heavily used early in the learning process but rapidly gives away to extended period of over strategy use, and eventually comes back again as an adult-like behavior (Hypothesis 2A). I will also conduct a *behavioral* study, using video capture to record overt strategy use, to examine, in much detail greater than any currently available, how overt strategy learning and practice evolve, eventually into retrieval (Hypothesis 2B). I will conduct additional *neuroimaging* analyses: (a) dynamic causal modeling (DCM) analysis[20](#_ENREF_20), [37](#_ENREF_37), [38](#_ENREF_38) on data from Qin et al. (2014) and (b) functional connectivity analysis on resting-state data and DTI analysis from the same dataset to investigate the role of prefrontal cortex in information control and the evolution of this brain network across development (Hypothesis 2A).

**Part 3. *Modeling***. I will study the effect of different global learning parameters on performance and internal representations for arithmetic and number knowledge, and explore their implications for behavioral and neurobiological dysfunctions in DD. The specific manipulations will include (but not restricted to): (a) learning rate(LR) and weight decay (WD)[39-41](#_ENREF_39): as synaptic plasticity for learning; (b) number of processing units (NPU)[40](#_ENREF_40), [42](#_ENREF_42): as neural pathway capacity (e.g., number of cortical minicolumns) for learning; and (c) internal noise (IN) [41](#_ENREF_41), [42](#_ENREF_42): as signal-to-noise in neural processing (i.e., high baseline excitation). ***Behavior and neuroimaging***. I will conduct meta-analysis of behavioral data on DD on arithmetic problem solving to examine to what extent the model accounts for DD phenomena. I will also conduct analyses of functional and structural neuroimaging data from DD populations to investigate how well the different settings of global learning parameters in the model can account for functional and structural abnormalities in DD.

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