**Background**

Cognitive neuroscience is entering an era in which the aim is to understand how complex behavior arises from the dynamic information exchange and control process across the network of brain regions, and how these change with learning and development. Children’s arithmetic, more specifically children's small number addition (hereafter simply called "addition"), provides a unique opportunity to investigate these questions, as the phenomena in this domain are complex and foundational, yet still tractable. Unlike many domains, addition involves a range of cognitive (and often physical) action (Geary et al., 1992; Siegler, 1996), yet it has well-defined structure with formally correct answers, so that the space of possible correct (or plausibly incorrect) algorithms can be clearly mapped out (van Lehn's thesis). Moreover, how children carry out addition, what mistakes they make, and how they transition from pre-addition (usually only knowing counting sequences) through adult-type "pure retrieval" (Ascraft, 1982; Geary et al., 2007; Groen & Parkman, 1972; Siegler & Shrager, 1984; Siegler, 1986), occurs in a relatively observable cognitive- and neuro-developmental time-window (Svenson & Sjoberg, 1983; Siegler & Shrager, 1984).

As a result of its great educational importance (Geary et al., 1992; Pazza et al., 2010; Price et al., 2007), children's arithmetic has been well-studied via psychological experiments, cognitive neuroscience, computational modeling. On the cognitive neuroscience side, several core brain regions have been delineated as contributing to math competence and processing (Ansari, 2008; Dehaene, et al., 2003; Menon, 2014), including ventral visual stream (e.g., posterior fusiform gyrus; pFG) for decoding number forms, parietal circuits (majorly around inferior parietal sulcus; IPS) for anchoring the visual numerical representations, prefrontal-parietal cortices for manipulating quantity representations in working memory, and medial temporal lobe (MTL), and especially hippocampus for associative memory processing only in children (Cho et al., 2012; Qin et al., 2014; Supekar et al., 2013). Difficulties in math processing in some children (e.g., developmental dyscalculia; DD) are associated with abnormality, either hypo-activation in these brain regions (Ashkenazi et al., 2012) or increased functional connectivity among these regions (Rosenberg-Lee et al., 2014). Studies further suggest that that MTL, left prefrontal and bilateral posterior parietal cortices may relate to uses of different strategies (Cho et al., 2011), and hippocampus seems crucial for the transition from overt to more implicit strategies (Qin et al., 2014). Even given all this detailed knowledge regarding children's arithmetic, we still do not understand how the whole brain system -- the dynamic network of widely-distributed brain regions -- support the organized execution of arithmetic reasoning, nor how these systems, and especially their interactions, change through learning and development.

Computational studies, which have been undertaken for decades (Siegler & Shrager, 1984; Siegler & Shipley, 1995; Shrager & Siegler, 1998), have generally been rendered in purely symbolic, purely connectionist, and in hybrid symbolic/connectionist paradigms. However, due to unavailability of cognitive neuroscientific data at that time, these models are not rich enough to adequately constrain our theorizing about how the brain develops arithmetic skill.

Recently, my colleagues and I have demonstrated that computational neural networks can account for neural activities in human and animal studies, as well as provide mechanistic explanations for dynamic changes and evolvement of brain network functions (Chen & Rogers, 2015; Plaut & Buhrmann, 2011; Stoianov & Zorzi, 2012). Furthermore, theorists have argued the role of prefrontal cortex in system control of cognitive tasks (Miller & Cohen, 2001). Therefore, the primary goal of the proposed research is build from these findings and employ a novel systems-control/connectionist framework to understand the interactive dynamics and evolution of arithmetic skill and number sense. By modeling the neurocognitive networks in human brain, I thereby bridge the rich infrastructure of cognitive theory, data, and computational experimentation with recent findings from systems neuroscience. Specifically, I propose to build a new computational model, which is a hybrid of classical connectionist models and classical control system models of children's arithmetic and its development, focusing especially on strategy use.

I will be guided in this effort by several leading thinkers with a wide range of expertise very relevant to this specific domain: Vinod Menon is one of the leading systems neuroscientists, and is among the only ones specifically capturing data and theorizing about the development of the brain as a multi-scale dynamical system, especially in math cognition; Jay McClelland is one of the world's leading connectionist modelers and cognitive scientists, and runs a lab at Stanford focused specifically on arithmetic learning; and Jeff Shrager, consulting professor in the Symbolic Systems program here at Stanford, is one of the founders (with Bob Siegler, of CMU) of the field of the computational modeling of arithmetic development. Dr. Shrager wrote three prior, highly cited, computer models of children's arithmetic development, and is also a leading expert on children's and adult's complex cognitive behavior.

**Goals and Hypotheses**

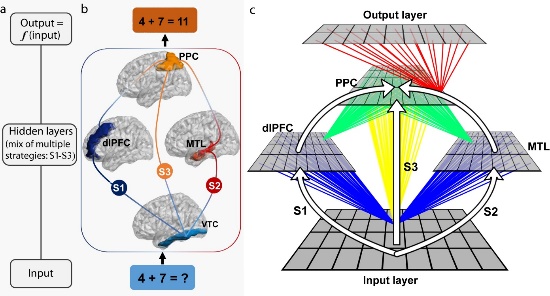
**Goal 1**: **Develop a neuro-computational model for both behavioral and neurobiological outcomes of arithmetic development in children.** Using connectionist neural network models (NN models), I will establish a NN model to demonstrate face validity of this neuro-computational approach to account for behavioral and neurobiological findings on children’s development of arithmetic skills. ***Hypothesis 1***: The efficiency of neural pathways for different strategy use leads to observed behavioral and neurobiological developments of arithmetic problem solving in children.

**Goal 2: Establish the control/connectionist hybrid model to explain arithmetic development through learning.** Combining the connectionist and system-control models, I will establish an integrated NN model to account for the joint efforts of sensory, motor, memory and prefrontal control brain systems in children’s learning of skills for arithmetic problem solving. ***Hypothesis 2A***: The prefrontal control system maintains the control of task-specific information exchange amongst sensory, motor and memory systems and it facilitates the developmental changes in the network dynamics amongst those brain systems. ***Hypothesis 2B***: Learning experience plays a critical role in achieving desirable outcomes of mental arithmetic skills in children.

**Goal 3: Explore sources of individual differences in neural basis to explain typical and atypical development**. In the aforementioned NN models, I plan to explain possible neural mechanisms of math difficulties in children with developmental dyscalculia (DD). ***Hypothesis***: The behavioral and neurobiological abnormality of DD can be explained by different learning parameters analogous to biological dysfunctions in human brain.

**Experimental methods**

**Part 1**. The development of arithmetic skills from overt strategies to retrieval strategies is accompanied by processing efficiency observed in behavioral data, and a general tendency of prefrontal-to-parietal (anterior-to-posterior) in brain activations. ***Modeling study proposal***. I will use feed-forward connectionist model (example depicted in Figure X) to establish three distinct neurocognitive pathways hypothesized for distinct strategy uses (3a and 3b), including effortful counting strategies mediated by prefrontal working memory system (S1; dlPFC, dorsolateral prefrontal cortex), hippocampal-dependent episodic-like memory retrieval (S2; MTL), hippocampal-independent semantic-like memory retrieval (S3, PPC, posterior parietal cortex). By manipulating the additional potentiation, I will manipulate the processing efficiency of each neuro-pathway for strategy use and test the performance and internal activations of child-like (i.e., recruiting more inefficient pathways) and adult-like models (i.e., recruiting more efficient pathways) after training to learn simple addition tasks. ***Behavior and neuroimaging study proposal***. In order to validate the model behaviors, I will re-analyze *behavioral* data (e.g., accuracy, response latency, inter-problem variability, etc.) and *neuroimaging* data (e.g., functional activation, representational stability, etc.) from Qin et al. (2014) to show the developmental changes in behavioral efficiency and the brain involvement from prefrontal to parietal cortices from children to adults.



**Figure X. Proposed model architecture for Goal 1**

**Part 2.** In order to explain how children learn to use different strategies and what drives the shift of strategy uses, we need to extend the NN models to be a more ecological and neurally-faithful way by including other brain systems for sensory, motor, memory and control processes. ***Modeling study proposal***. Importantly, I will integrate the connectionist and system-control models by adding information control system, analogous to the proposed function of prefrontal cortex (Miller & Cohen, 2001), in maintaining and controlling task-relevant information exchange in distributed brain networks (Figure X). This model can specifically examine (a) the interaction between different neuro-pathways/systems underlying learning of different strategies; and (b) how learning of overt problem solving skills may bolster the development of retrieval strategy use. ***Behavior and neuroimaging study proposal***. I will conduct a large-scale *behavioral* data re-analysis from previous studies to demonstrate the developmental changes in strategy use for addition problems in children from age 5-10 years old. Specifically, I examine whether the model can account for (a) the response distribution across correct and incorrect answers for different addition problems; and (b) a U-shaped retrieval usage that retrieval strategy is heavily used early in the learning process but rapidly gives away to extended period of over strategy use, and eventually comes back again as an adult-like behavior (Hypothesis 2A). I will also propose a dynamic causal modeling (DCM) analysis on *neuroimaging* data from Qin et al. (2014) to investigate the role of prefrontal cortex in information control over the development (Hypothesis 2A). I also plan to conduct a *behavioral* treatment-control study to whether teaching retrieval strategy use can be unprecedented by overt strategy learning (Hypothesis 2B).

**Part 3.** I will manipulate several model parameters related to neurobiological functions and learning in the NN models to account for behavioral and neurobiological dysfunctions in DD. ***Modeling study proposal***. I will manipulate the following parameters during model training to see their effects on model performance and internal representations for arithmetic and number knowledge: (a) learning rate(LR) and weight decay (WD): as synaptic plasticity for learning; (b) number of processing units (NPU): as a number of cortical minicolumns or neural pathway capacity for learning; and (c) internal noise (IN): as signal-to-noise in neural processing (i.e., high baseline excitation). ***Behavior and neuroimaging study proposal***. I propose to conduct a meta-analysis on behavioral data of previous studies on DD population to examine whether the modeling simulation can explain the behavioral performance of DD on arithmetic problem solving. I also propose to conduct a meta-analysis on functional and structural neuroimaging data on DD to investigate: (a) whether focal disruptions (LR, WD and/or IN) to separate systems in the model predict the abnormality in functional activation in DD; and (b) structural lesion (NPU) in model as observed in structural abnormality in DD can produce similar behavior performance on arithmetic tasks.