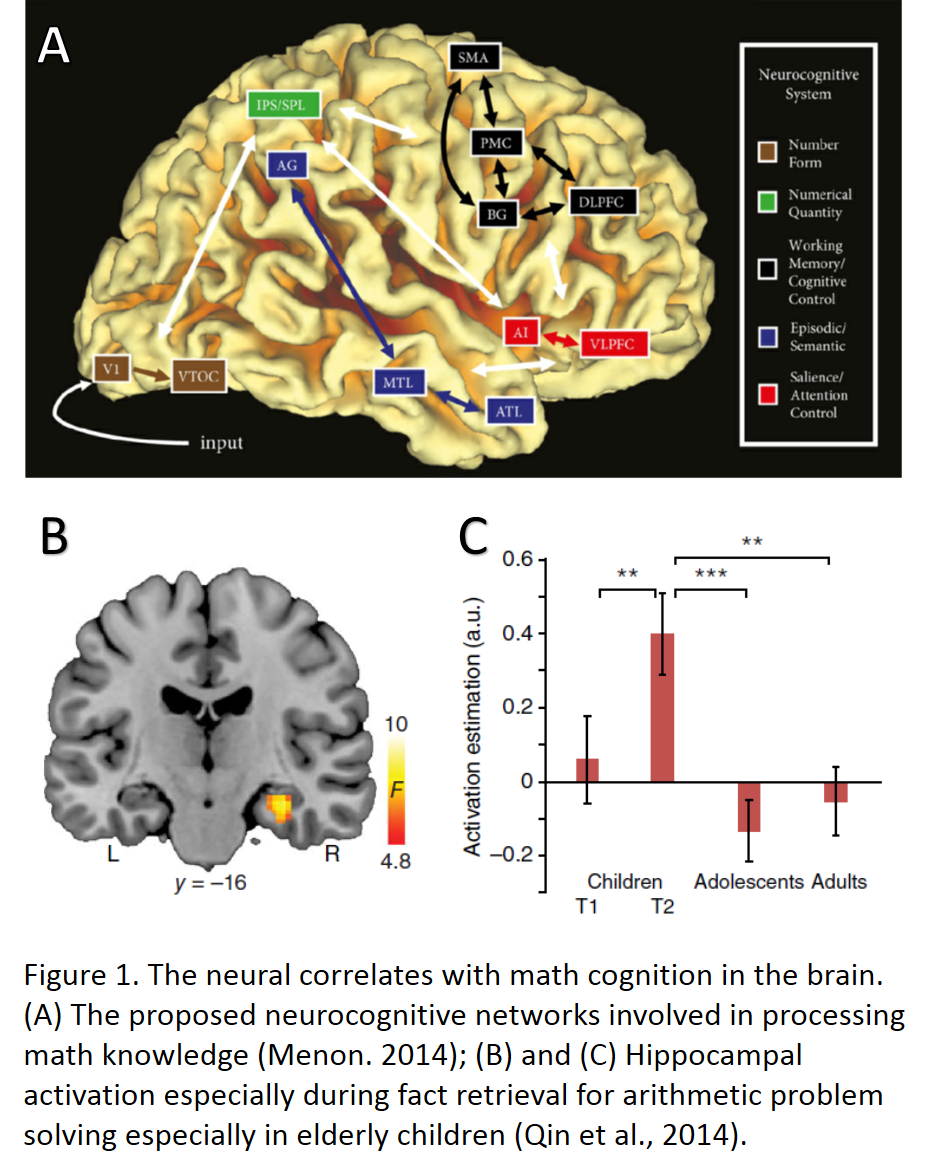
**BACKGROUND**

Cognitive neuroscience is entering an era in which the aim is to understand how complex behavior arises from the dynamic information exchange and control process across the network of brain regions, and how these change with learning and development. Children’s arithmetic, more specifically children's small number addition (hereafter “addition”) provides a unique opportunity to investigate these questions, as the phenomena in this domain are complex and foundational, yet still tractable. Addition involves a range of cognitive, often physical actions[1](#_ENREF_1), [2](#_ENREF_2), yet it has well-defined structure with formally correct answers, so that the space of possible correct (or plausibly incorrect) algorithms can be clearly mapped out[3](#_ENREF_3). Moreover, how children carry out addition, what mistakes they make, and how they transition from pre-addition (only knowing counting sequences) through adult-type “pure retrieval”[4-8](#_ENREF_4), occurs in a relatively consistent cognitive- and neuro-developmental time-window[4](#_ENREF_4), [9](#_ENREF_9).

As a result of its great educational importance[10-14](#_ENREF_10), children's arithmetic has been well-studied by psychologists, cognitive neuroscientists, and computational modelers. Several core brain regions have been delineated as contributing to math competence and processing[15-19](#_ENREF_15), including ventral visual stream (e.g., posterior fusiform gyrus; pFG) for decoding number forms, parietal circuits (majorly around inferior parietal sulcus; IPS) for anchoring the visuospatial numerical representations, prefrontal-parietal cortices for manipulating quantity representations in working memory, and medial temporal lobe (MTL), and especially hippocampus for associative memory processing only in children[20-22](#_ENREF_20) (see Figure 1). Difficulties in math processing in some children (e.g., developmental dyscalculia; DD) are associated with abnormality, either hypo-activation in these brain regions[23](#_ENREF_23) or increased functional connectivity among these regions[24](#_ENREF_24). Studies further suggest that that MTL, left prefrontal and bilateral posterior parietal cortices may relate to uses of different strategies[25](#_ENREF_25), and hippocampus seems crucial for the transition from overt to more implicit strategies[22](#_ENREF_22). Even given all this detailed knowledge regarding children's arithmetic, we still do not understand how the large-scale brain system (the dynamic network of widely-distributed brain regions) support the organized execution of arithmetic reasoning, nor how these systems and especially their interactions and change through learning and development.

Computational studies, which have been undertaken for decades[4](#_ENREF_4), [5](#_ENREF_5), [26-28](#_ENREF_26), have generally been rendered in purely symbolic, purely connectionist, and in hybrid symbolic/connectionist paradigms. However, these models suffer from several critical drawbacks: (a) implementation of both procedural memory and executive control systems were not theory-driven; (b) memory knowledge was not represented in distributed manner; and (c) the model structure was not informed by current understanding of the brain. Therefore, their value to inform the brain dynamics for arithmetic problem solving was clearly undermined. Recently, Along with other researchers, I have demonstrated that computational neural networks can account for neural activities in human and animal studies, as well as provide mechanistic explanations for dynamic changes and evolvement of brain network functions [29-31](#_ENREF_29). Furthermore, theorists have argued the role of prefrontal cortex in system control of cognitive tasks[32](#_ENREF_32), and neural networks are shown to be nicely combined with system-control models for adaptive learning and optimal control[33-36](#_ENREF_33).

Therefore, the primary goal of the proposed research is to build from these findings and employ a novel systems-control/connectionist framework to understand the interactive dynamics and evolution of arithmetic skill and number sense. To be specific, I will first use a connectionist model to bridge both behavioral and neuroimaging data to provide an unprecedented neuro-computational modeling framework for mental arithmetic. I will then employ a hybrid model of strategy representation and execution that is much more aligned with our modern knowledge of the cognitive functions of brain regions, how these are interconnected, and how the execution of complex procedures are initiated and controlled. By modeling the neurocognitive networks in human brain, I thereby bridge the rich infrastructure of cognitive theory, data, and computational experimentation with recent findings from systems neuroscience.

**GOALS AND HYPOTHESES**

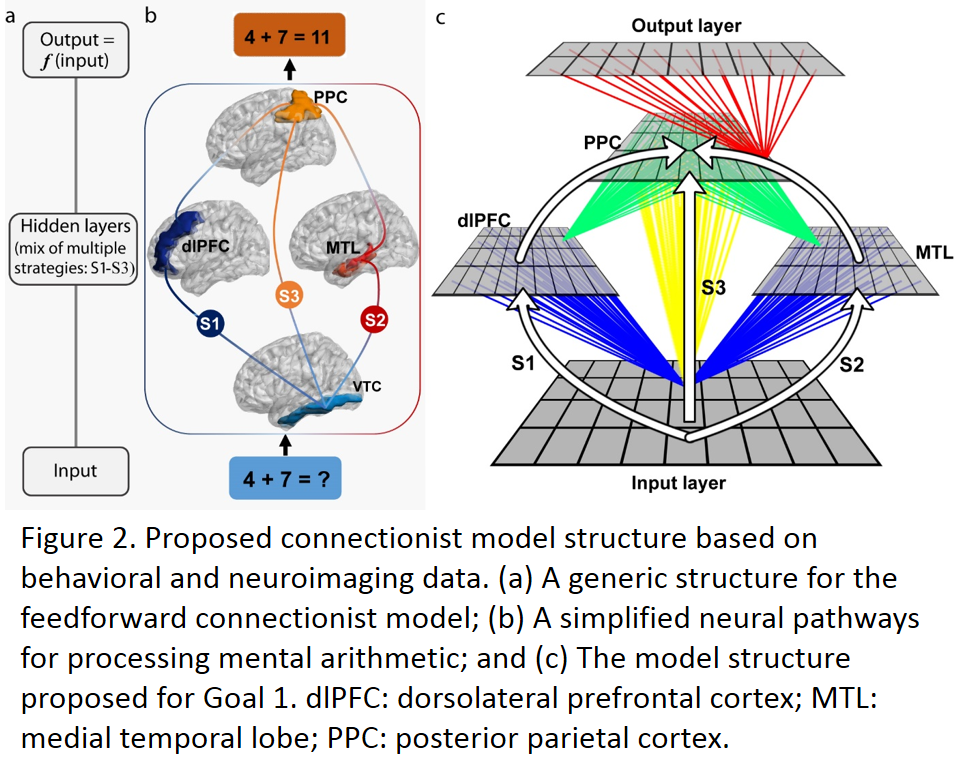
**Goal 1**: **Develop a neuro-computational model for both behavioral and neurobiological outcomes of arithmetic development in children.** I will establish a connectionist model to demonstrate the face validity and feasibility of this neuro-computational approach to account for behavioral and neurobiological findings on children’s development of arithmetic skills. ***Hypothesis 1***: The efficiency of neural pathways for different strategy use leads to observed behavioral and neurobiological developments of arithmetic problem solving in children.

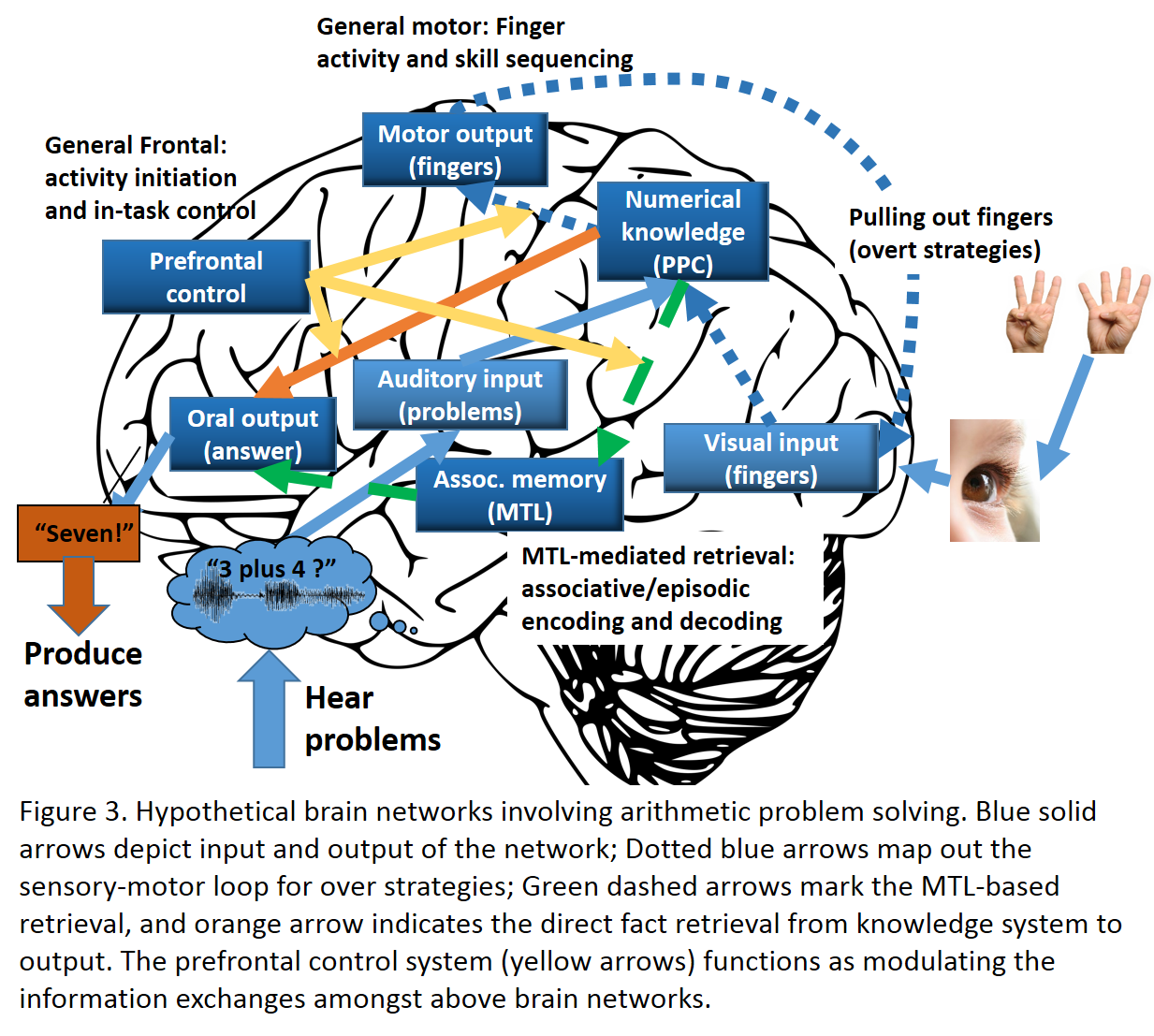
**Goal 2: Establish the control/connectionist hybrid model to explain arithmetic development through learning.** Combining the connectionist and system-control models, I will establish an integrated neural network (NN) model to account for the joint efforts of sensory, motor, memory and prefrontal control systems in children’s development of arithmetic problem solving. ***Hypothesis 2A***: The prefrontal system maintains the control of task-specific information exchange amongst sensory, motor and memory systems and it facilitates the developmental changes in the network dynamics amongst those brain systems. ***Hypothesis 2B***: Learning experience plays a critical role in achieving desirable outcomes of mental arithmetic skills.

**Goal 3: Explore sources of individual differences in neural basis to explain typical and atypical development**. In the aforementioned NN models, I plan to explain possible neural mechanisms of math difficulties in children with developmental dyscalculia (DD). ***Hypothesis***: The behavioral and neurobiological abnormality of DD can be explained by different learning parameters analogous to biological dysfunctions in human brain.

**EXPERIMENTAL METHODS**

**Part 1**. The development of arithmetic skills from overt strategies to retrieval strategies is accompanied by processing efficiency observed in behavioral data, and a general tendency of prefrontal-to-parietal (anterior-to-posterior) in brain activations.

***Modeling study proposal***. I will use ******feed-forward connectionist models (example depicted in Figure 2) to establish three distinct neurocognitive pathways hypothesized for distinct strategy uses (2b), including effortful counting strategies (S1; dlPFC), hippocampal-dependent associative memory retrieval (S2; MTL), and hippocampal-independent fact memory retrieval (S3, PPC). By manipulating the additional potentiation from controlling units, the processing efficiency of each neuro-pathway for strategy use is manipulated to test the performance and neural activities of child-like (i.e., recruiting more inefficient pathways) and adult-like models (i.e., recruiting more efficient pathways) after training to solve simple addition problems. ***Behavior and neuroimaging study proposal***. In order to validate the model behaviors, I will re-analyze *behavioral* data (e.g., accuracy, response latency, inter-problem variability, etc.) and *neuroimaging* data (e.g., functional activation, representational stability, etc.) from Qin et al. (2014) to show the developmental changes in behavioral efficiency and the brain involvement from prefrontal to parietal cortices over development.

**Part 2.** In order to explain the development and the causes of strategy uses and shifts, I will extend the NN models to a more neurally-faithful way by including other brain systems for sensory, motor, memory and control processes.

***Modeling study proposal***. I will integrate the connectionist and system-control models by adding information control system, analogous to the proposed function of prefrontal cortex32, in maintaining and controlling task-relevant information exchange in distributed brain networks (Figure 3). This model can specifically examine (a) the interaction between different neuro-pathways/systems underlying learning of different strategies; and (b) how learning of overt problem solving skills may bolster the development of retrieval strategy use. ***Behavior and neuroimaging study proposal***. I will conduct a large-scale *behavioral* data re-analysis from previous studies to demonstrate the developmental changes in strategy use for addition problems in children from age 5-10 years old. Specifically, I examine whether the model can account for (a) the response distribution across correct and incorrect answers for different addition problems; and (b) a U-shaped retrieval usage that retrieval strategy is heavily used early in the learning process but rapidly gives away to extended period of over strategy use, and eventually comes back again as an adult-like behavior (Hypothesis 2A). I also plan to conduct a *behavioral* treatment-control study with videotaping to examine whether teaching retrieval strategy use can be unprecedented by overt strategy learning (Hypothesis 2B). For the *neuroimaging* part, I propose: (a) a dynamic causal modeling (DCM) analysis[20](#_ENREF_20), [37](#_ENREF_37), [38](#_ENREF_38) on data from Qin et al. (2014) and (b) functional connectivity analysis on resting-state data and DTI analysis from the same dataset to investigate the role of prefrontal cortex in information control and the dynamic evolvement of this brain network over the development, both functionally and structurally (Hypothesis 2A).

**Part 3. *Modeling study proposal***. I will manipulate the following parameters during model training to see their effects on model performance and internal representations for arithmetic and number knowledge to account for behavioral and neurobiological dysfunctions in DD: (a) learning rate(LR) and weight decay (WD)[39-41](#_ENREF_39): as synaptic plasticity for learning; (b) number of processing units (NPU)[40](#_ENREF_40), [42](#_ENREF_42): as neural pathway capacity (e.g., number of cortical minicolumns) for learning; and (c) internal noise (IN) [41](#_ENREF_41), [42](#_ENREF_42): as signal-to-noise in neural processing (i.e., high baseline excitation). ***Behavior and neuroimaging study proposal***. I propose to conduct a meta-analysis on behavioral data on DD population to examine whether the model simulation can explain the behaviors of DD on arithmetic problem solving. I also propose to conduct data analyses on functional and structural neuroimaging data on DD to investigate: (a) whether focal disruptions (LR, WD and/or IN) to separate systems in the model predict the abnormality in functional activation in DD; and (b) structural lesion (NPU) in model as observed in structural abnormality in DD can produce similar behavior performance on arithmetic tasks.

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