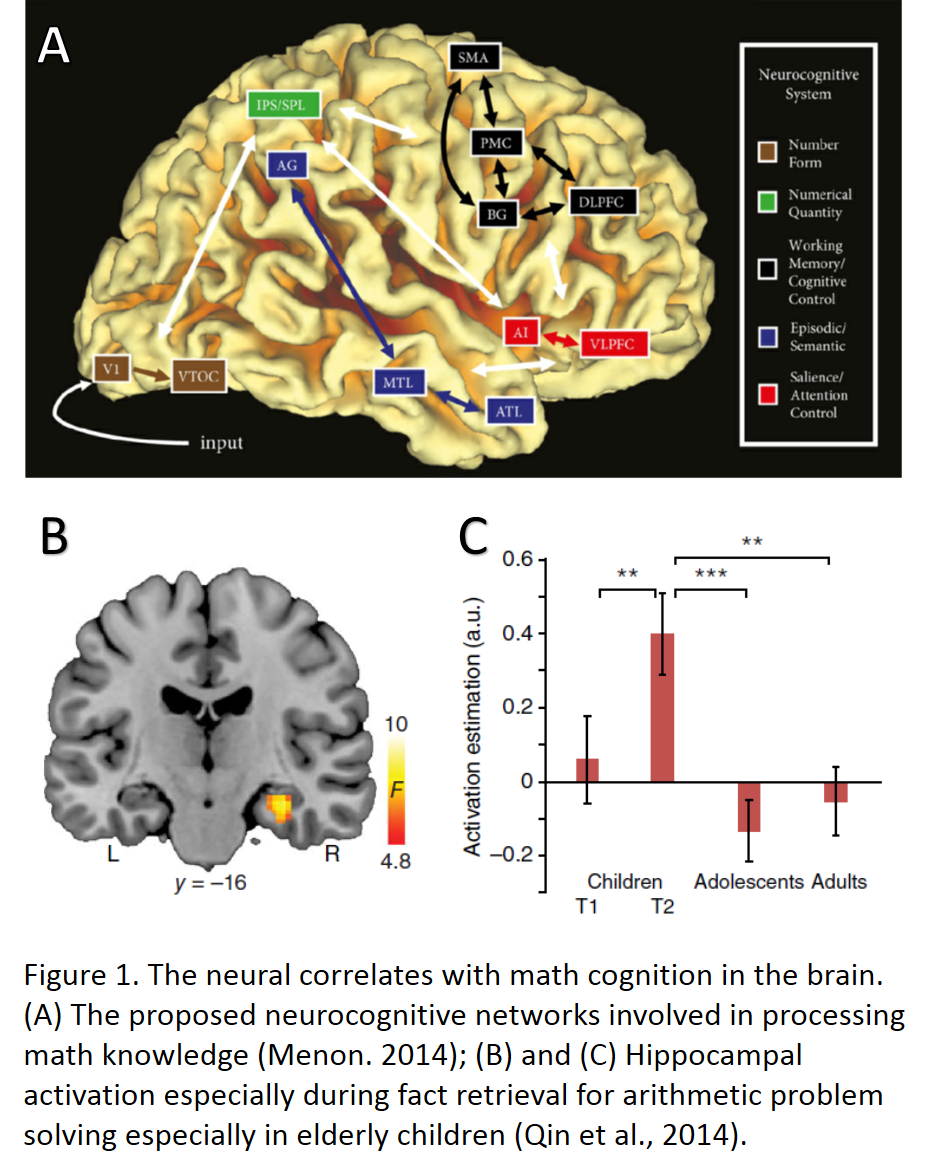
**BACKGROUND**

Modern systems neuroscience seeks to understand how complex behavior arises from the dynamic information exchange and control process across the network of brain regions, and how these change with learning and development. Children’s arithmetic, more specifically children's small number addition (hereafter “addition”) provides a unique opportunity to investigate these questions, as the phenomena in this domain are complex and foundational, yet still tractable. Addition involves a range of cognitive, often physical actions[1](#_ENREF_1), [2](#_ENREF_2), yet it has well-defined structure with formally correct answers, so that the space of possible correct (or plausibly incorrect) algorithms can be clearly mapped out[3](#_ENREF_3). Moreover, how children carry out addition, what mistakes they make, and how they transition from pre-addition (only knowing counting sequences) through adult-type “pure retrieval”[4-8](#_ENREF_4), occurs in a relatively consistent cognitive- and neuro-developmental time-window[4](#_ENREF_4), [9](#_ENREF_9).

As a result of its great educational importance[10-14](#_ENREF_10), children's arithmetic has been well-studied by psychologists, cognitive neuroscientists, and computational modelers. Several core brain regions contribute to math competence and processing[15-19](#_ENREF_15), including the ventral visual stream (e.g., posterior fusiform gyrus; pFG) for decoding number forms, parietal circuits (majorly around inferior parietal sulcus; IPS) for anchoring the visuospatial numerical representations, prefrontal-parietal cortices for manipulating quantity representations in working memory, and medial temporal lobe (MTL) and especially hippocampus for associative memory processing in children[20-22](#_ENREF_20) (Figure 1). Difficulties in math processing in some children (e.g., developmental dyscalculia; DD) are associated with abnormalities such as hypo-activation in these brain regions[23](#_ENREF_23), or increased functional connectivity among these regions[24](#_ENREF_24). Studies further suggest that that MTL, left prefrontal, and bilateral posterior parietal cortices correlate with the use of different strategies[25](#_ENREF_25), and hippocampus seems crucial for the transition from overt to implicit strategies[22](#_ENREF_22). Even given all this detailed knowledge regarding children's arithmetic, we still do not understand how the large-scale dynamic network of widely-distributed brain regions supports the organized execution of arithmetic reasoning, nor, especially, how their interactions and change through learning and development.

Computational studies of arithmetic development, which have been undertaken for decades[4](#_ENREF_4), [5](#_ENREF_5), [26-28](#_ENREF_26), have generally been rendered either in purely symbolic, purely connectionist, or hybrid symbolic/connectionist paradigms. These paradigms suffer from several critical drawbacks: (a) implementation of both procedural memory and executive control systems were not theory-driven; (b) memory and knowledge was not represented in distributed manner; and (c) model structure was not informed by current understanding of the brain, for example, theorists have argued the role of prefrontal cortex in system control of cognitive tasks[32](#_ENREF_32" \o "Miller, 2001 #1323), and neural networks are shown to be nicely combined with system-control models for adaptive learning and optimal control[33-36](#_ENREF_33" \o "Suykens, 2012 #1327), and these advances have not made their way into computational models of arithmetic.

Modelers have recently demonstrated that systems-oriented computational neural networks that account well for neural dynamics in human and animal studies, as well as provide good mechanistic explanations for dynamic changes and evolution of brain network functions [29-31](#_ENREF_29" \o "Chen, 2015 #1284).The primary goal of the presently proposed research is to apply this methodology to develop a novel systems-control/connectionist model of the interactive dynamics and evolution of arithmetic skill and number sense. In the first part of the project (goal 1) I will develop an updated connectionist model that bridges both behavioral and neuroimaging data in arithmetic model. Next, (goal 2) I will extend this model into a hybrid model of strategy representation and execution that is aligned with our modern knowledge of the cognitive functions of brain regions, how these are interconnected, and how the execution of complex procedures are initiated and controlled. Finally (goal 3) I will employ the new model to try to explain aspects of typical and atypical arithmetic development by trying to model individual differences in neural architecture. By modeling the neurocognitive networks in human brain, this new model will be able to provide explanations that encompass that existing rich bank of cognitive theory, data, and computational experimentation in arithmetic within a framework of modern systems neuroscience.

**GOALS AND HYPOTHESES**

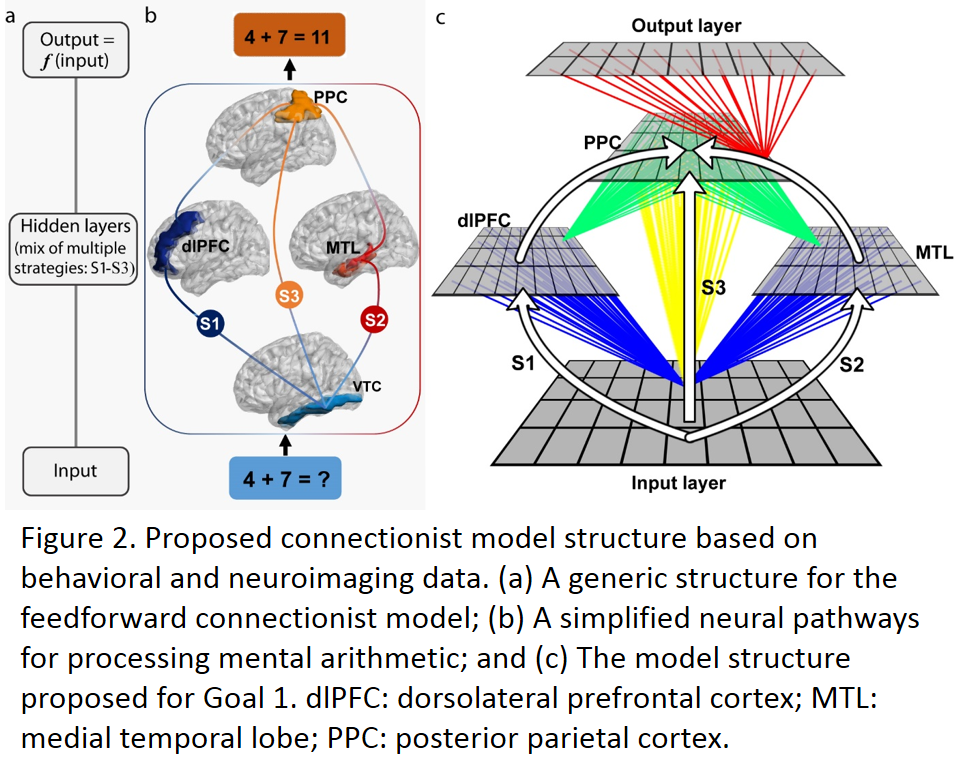
**Goal 1**: **Develop a neuro-computational model for both behavioral and neurobiological outcomes of arithmetic development in children.** I will establish a connectionist model to demonstrate the face validity and feasibility of this neuro-computational approach to account for behavioral and neurobiological findings on children’s development of arithmetic skills. ***Hypothesis 1***: The efficiency of neural pathways for different strategy use leads to observed behavioral and neurobiological developments of arithmetic problem solving in children.

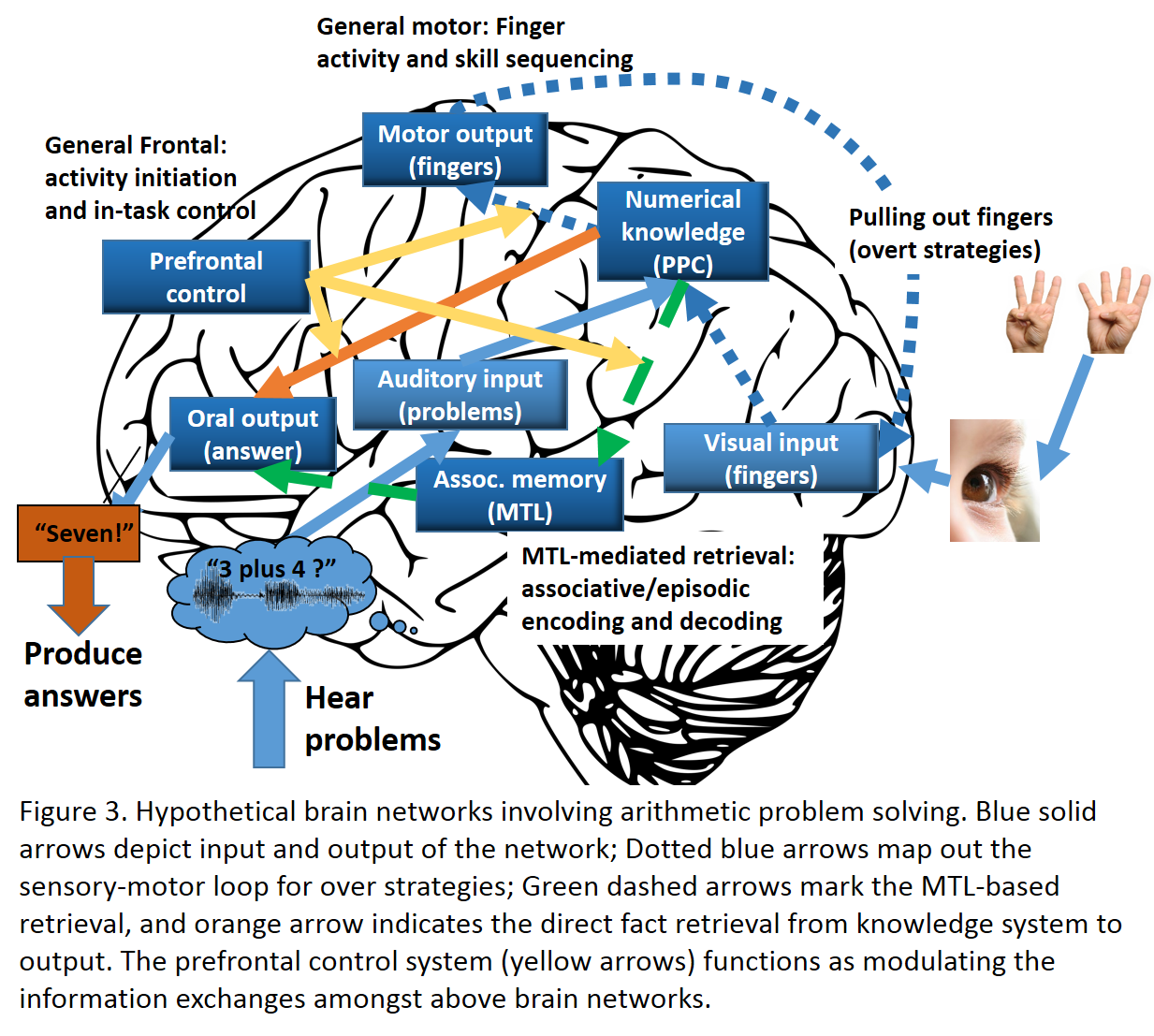
**Goal 2: Establish the control/connectionist hybrid model to explain arithmetic development through learning.** Combining the connectionist and system-control models, I will establish an integrated neural network (NN) model to account for the joint efforts of sensory, motor, memory and prefrontal control systems in children’s development of arithmetic problem solving. ***Hypothesis 2A***: The prefrontal system maintains the control of task-specific information exchange amongst sensory, motor and memory systems and it facilitates the developmental changes in the network dynamics amongst those brain systems. ***Hypothesis 2B***: Learning experience plays a critical role in achieving desirable outcomes of mental arithmetic skills.

**Goal 3: Explore sources of individual differences in neural basis to explain typical and atypical development**. In the aforementioned NN models, I plan to explain possible neural mechanisms of math difficulties in children with developmental dyscalculia (DD). ***Hypothesis***: The behavioral and neurobiological abnormality of DD can be explained by different learning parameters analogous to biological dysfunctions in human brain.

**EXPERIMENTAL METHODS**

**Part 1**. The development of arithmetic skills from overt strategies to retrieval strategies is accompanied by processing efficiency observed in behavioral data, and a general tendency of prefrontal-to-parietal (anterior-to-posterior) in brain activations.

***Modeling***. I will use ******feed-forward connectionist models (example depicted in Figure 2) to establish three distinct neurocognitive pathways hypothesized for distinct strategy uses (2b), including effortful counting strategies (S1; dlPFC), hippocampal-dependent associative memory retrieval (S2; MTL), and hippocampal-independent fact memory retrieval (S3, PPC). By manipulating the additional potentiation from controlling units, the processing efficiency of each neuro-pathway for strategy use is manipulated to test the performance and neural activities of child-like (i.e., recruiting more inefficient pathways) and adult-like models (i.e., recruiting more efficient pathways) after practice solving simple addition problems. ***Behavior and neuroimaging***. I will re-analyze *behavioral* data (e.g., accuracy, response latency, inter-problem variability, etc.) and *neuroimaging* data (e.g., functional activation, representational stability, etc.) from Qin et al. (2014) to evaluate the model in terms of developmental changes in behavioral efficiency and the brain involvement from prefrontal to parietal cortices.

**Part 2.** I will extend the NN model to include recent understanding of the involvement of other brain systems, especially sensory, motor, memory, and control processes, and including external aspects of behavior, especially regarding phonological (hearing the problem) and visual (finger counting) involvment.

***Modeling***. I will extend the previous models by adding information and control system analogous to the proposed function of prefrontal cortex32, in maintaining and controlling task-relevant information exchange in distributed brain networks, as well as other systems (Figure 3), focusing on the interaction between different neuro-pathways/systems underlying the execution of different strategies, and how both strategies and arithmetic knowledge and skill change developmentally. ***Behavior and neuroimaging***. I will conduct a large-scale *behavioral* data re-analysis from previous studies to demonstrate the developmental changes in strategy use for addition problems in children from age 5-10 years old. I will use this analysis to examine how well the model can accounts for (a) the response distribution across correct and incorrect answers for different addition problems; and (b) the U-shaped retrieval usage that retrieval strategy is heavily used early in the learning process but rapidly gives away to extended period of over strategy use, and eventually comes back again as an adult-like behavior (Hypothesis 2A). I will also conduct a *behavioral* study, using video capture to record overt strategy use, to examine, in much detail greater than any currently available, how overt strategy learning and practice evolve, eventually into retrieval (Hypothesis 2B). I will conduct additional *neuroimaging* analyses: (a) dynamic causal modeling (DCM) analysis[20](#_ENREF_20), [37](#_ENREF_37), [38](#_ENREF_38) on data from Qin et al. (2014) and (b) functional connectivity analysis on resting-state data and DTI analysis from the same dataset to investigate the role of prefrontal cortex in information control and the dynamic evolution of this brain network over the development (Hypothesis 2A).

**Part 3. *Modeling***. I will study the effect of the manipulation of model parameters effect performance and internal representations for arithmetic and number knowledge, and correlate these observations with behavioral and neurobiological knowledge regarding dysfunctions in DD. The specific manipulations will include: (a) learning rate(LR) and weight decay (WD)[39-41](#_ENREF_39): as synaptic plasticity for learning; (b) number of processing units (NPU)[40](#_ENREF_40), [42](#_ENREF_42): as neural pathway capacity (e.g., number of cortical minicolumns) for learning; and (c) internal noise (IN) [41](#_ENREF_41), [42](#_ENREF_42): as signal-to-noise in neural processing (i.e., high baseline excitation). ***Behavior and neuroimaging***. I will conduct meta-analysis of behavioral data on DD on arithmetic problem solving to examine to what extent the model accounts for DD phenomena. I will also conduct analyses of functional and structural neuroimaging data from DD populations to investigate: (a) whether focal disruptions (LR, WD and/or IN) to separate systems in the model predict the abnormality in functional activation in DD; and (b) structural lesion (NPU) in model as observed in structural abnormality in DD can produce similar behavior performance on arithmetic tasks.

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