One of the most interesting features of children's arithmetic is how strategic is it, both in the sense that children use many different strategies to solve arithmetic problems (adults do too, but on larger problems), and in the sense that learners learn to strategically choose which strategy to use on a particular problem. In order to avoid confusion, we will reserve the term "strategy" to mean the way (i.e., method) in which a problem is solved, and will use "judicious" to mean how one choses a particular strategy for a given problem. One might be tempted to use "algorithm" to refer to what we are calling "strategy", but this is too "in the head", whereas many, perhaps most, strategies that children use in arithmetic problems involve external action, such as raising and counting one's fingers, or making and counting piles of various things. We restrict ourselves here to finger counting strategies, but as has been well demonstrated in the literature, there children employ a wide range of finger counting strategies, and do so (somewhat) strategically. A striking feature of any even slightly complex finger counting strategy is the extent to which it engages many part of the brain. Indeed, one might say "choreographs" rather than merely "engages", because in order to get the correct answer (or even anything close to the correct answer, allowing for various small errors), the child must correctly execute a relatively complex strategy that lasts several seconds and employs numerous memories, motor, and perceptual subsystems. Take, for example, the simple addition problem: 3+4=? (we generally imagine these to be spoken, as: "What is three plus four?", even though for simplicity we write them as "3+4=?") Most adult, and many children by around 5 or 6 years old will know the answer to this question, but before that a child might employ a finger counting strategy. Surprisingly, there are many possible correct finger counting strateies for small addition problems (we leave the reader to prove this to themselves); here we consider just the one called the "shortcut sum" strategy. What you would hear if all you did was listen to what the child says while executing this strategy is him or her counting up to 7: "one, two, three, four, five, six, seven," and sometimes repeating the last number as the answer: "seven". On the face of it this seems simple enough (although one might wonder that if the child knew to stop at seven, why did she bother counting up to it)? However, if one watched what the child is doing with her body, specifically her eyes, hands, and fingers, the strategy looks quite complex indeed: In one version of the shortcut sum strategy the child puts up three fingers on one hand, let's say her left hand, while saying "one, two, three", and then starts "four" on the other (right) hand, putting up four fingers while saying: "four, five, six, seven". Let us look in more detail at just the second part -- the right hand. On the one hand (pun only slightly intended!) putting up fingers resolves the problem noted just above of how the child knows the answer but is still counting: she did the first addend (3) on one hand, and the second addend (4) on the other, and when she got up to four, the answer to be reported is just the last number stated ("seven"). But this raises more problems than it resolves; in order to stop at 7, that is 4 fingers raised on the second hand, she had to be somehow either counting down, or up, or doing some sort of subitizing match between her fingers and the second addend (4) *without actually saying "one, two, three, four"* (because, of course, she was busy saying "four, five, six, seven"). This very small example brings out several important features of the present proposal. First, it demonstrates how (small number) arithmetic is at the same time extremely simple and extremely complex -- we posit that there is plenty of richness in such phenomena, especially in looking at their developmental and learning aspects, while the behaviors are not so diffuse as to require philosophical inquiry. Second, arithmetic strategy execution demonstrates the coordination of many brain and physical actions, at least including: supervisory control (to choose a strategy, initiate it, and ensure that it operates in correct sequence), verbalization, number identification (both initial for the problem), object identity (number) comparison (for counting termination), counting sequencing, semi-automatic action sequences, memories (for addends, for counting sequence, etc.), perception (subitizing), visual/motor attention, motor action (finger raising, moving eyes)). It is clear that the child, by operating this strategy over and over, i.e., through practice, eventually ends up simply being able to retrieve that sum of 3 and 4 (i.e., 7) reliably from memory. This (behaviorally) 'adult' behavior -- just giving the sum -- has been termed the "retrieval" strategy (or, in our terms "strategy"), but it is almost certainly the case that retrieval is not a strategy in the same sense as the shortcut sum strategy. Indeed, one can clearly see that retrieval processes are required at many points throughout the execution of strategies like shortcut sum; even if one does not retrieve the sum initially, one must retrieve the addends (esp. the second addend) in order to terminate the finger counting, and one must retrieve the next number from each previous during counting, moreover, one must retrieve the strategy itself in order to execute it to begin with. These are almost certainly different sorts of retrieval (short term or working memory for addends (perhaps auditory), long term memory for counting sequence, and procedural memory for the strategy and its component actions). Siegler and Shrager's 1984 work on strategy choice demonstrates results learning, but not strategy change; that would have been impossible in that model because strategies were not explicitly represented. Shrager and Sielger's SCADs model (1999) extends their 1984 model by explicitly representing and reasoning about strategies, which enables SCADS to model strategy change. Note that both the 1984 and the 1999 (SCADS) models are able to model learning both results (i.e., the association between problems and both correct and incorrect answers), and what we have termed "judicious" strategy choice (i.e., choosing a strategy that is appropriate to the stated problem), although judicious strategy choice is better handled by Siegler and Shipley's (1995) adaptive strategy choice model. The 1995 model did choose strategies adaptively, but was not able to discover NEW strategies; all the possible strategies were build into that model. Only the 1999 SCADS model accomplished all three: results memory, adaptive strategy choice, and discovery of new strategies. Even though the 1999 SCADS model succeeds in roughly demonstrating the desired phenomena, it may be criticized for the way in which it represented and executed arithmetic strategies as being neuropsychologically implausible, thus calling into question the whole model. SCADS had a set of plausible mico-operations that were intended to roughly represent the core operations required to execute an arithmetic strategy, for example, it had separate operations that would recover an addend from the stated problem, place a number in a putative echoic buffer, say the next number from the one in the echoic buffer, select a hand, raise a number of fingers on the selected hand, and so on. Strategies consisted of sets of these operations executed in order. It was necessary to have these operations explicitly represented in SCADS because discovering new strategies resulted from the existing strategies being modified in various ways, usually by dropping out operations that the supervisory system deemed redundant. (This is an oversimplification of the SCADS discovery algorithm, but it is adequate to the present purpose.) There are several problems with the way that SCADS represented and executed strategies. First, there was no theory of procedural memory; strategies were simply called out of machine memory all at once, as a whole, as needed. Second, there was no theory of strategy execution; the operations that comprised a given strategy were simply executed one after the other until an operation was reached to say the answer and stop. Third, the theory of results memory was implausible as compared to modern theories of memory, which in the last decade would more likely have been modeled through distributed connectionist representation. Aside from being implausible with regard to current understanding of the brain, these three problems (and other related sub problems) limited SCADS' ability to model certain subtleties in children's strategic activity, most especially as regards observed deficits in the development of arithmetic skill and number sense. The present proposal seeks to addresses this problem by building a new model of arithmetic learning and development as regards small number addition strategies, that is based in our modern, much better understanding of how the brain represents number and procedures, how these are used in the (somewhat) complex operations that are observed in children's small number arithmetic. The models that we will develop will be more correct in the ways implied above: First, we will use a connectionist/distributed model of results (associative) memory, and the same for counting memory. Moreover, we will employ a number of such memories as needed to represent arithmetic results and the separate knowledge of counting sequence (which is more like a song than like a set of number facts). Second, we will employ a model of strategy representation and execution that is much more aligned with our modern knowledge of how the brain is organized and how it works, especially as regards the functions of brain regions, and how these are interconnected, and how the execution of complex procedures are initiated and controlled. Third, we will use a more accurate representation of the interaction between procedural execution (as just described) and perceptual systems, especially as regards phenomena that clearly participate in arithmetic, such as subitizing. In this regard we are especially interested in getting the phonological interplay correctly situated because of the apparent important relationship between arithmetic development and phonological development, especially as regards comorbididies.