

Extra Credit Project

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Cointegration Analysis of Different Financial Markets
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Topic:

Using major stock indices for the United States (S&P500), United Kingdom (FTSE100), Germany (DAX), and France (CAC40) show that US and European financial markets cointegrate.

Method:

Reviewing cointegration condition:

Consider 2 time series variables Y and X . We have the regression equation as follow:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t \quad (1)$$

The cointegration is such that if X and Y are both non-stationary variables AND ϵ is a stationary variable, then X and Y cointegrate, i.e. they “move together” in the long run.

Thus, our primary methodology is as follow:

1. Test indices for stationarity:

Test representative stock indices in the United States (S&P 500), the United Kingdom (FTSE), Germany (DAX) and France (CAC40) for stationarity.

To execute this task, for each indices, we run the Augmented Dickey-Fuller Test (A)DF, which hypothesizes that a unit root is present in an autoregressive model. The intuition is such that if a variable is non-stationary, it tends to a constant mean—i.e. the values oscillates/ alternates from large to small. As a result, the process is not a random walk, i.e. non-stationary.

2. Test error term for stationarity:

Should the regressed result confirm non-stationarity, we check whether the error term of the regression ϵ is a stationary variable. If they are, then the indices cointegrate.

First we check for the common issue with time series data: positive autocorrelation by running Durbin-Watson Test. If autocorrelation exists, we add the first order autoregressive term $AR(1)$ into the model and subsequently $AR(2)$ as necessary.

Then, we run (A)DF test as above to test the error term for stationarity—not having a unit root in the (A)DF test.

Data Analysis

For this model, we use 20 years of data from March 2001 to March 2021.

Downloading data

```
library(quantmod)

# Set start date and end date of data
start_date <- "2001-01-01"
end_date <- "2021-03-18"

# Get data
getSymbols("^GSPC", src = "yahoo", , from = start_date, to = end_date) # S&P 500

## [1] "^GSPC"

getSymbols("^FTSE", src = "yahoo", , from = start_date, to = end_date) # S&P 500

## [1] "^FTSE"

getSymbols("^GDAXI", src = "yahoo", , from = start_date, to = end_date) # S&P 500

## [1] "^GDAXI"

getSymbols("^FCHI", src = "yahoo", , from = start_date, to = end_date) # S&P 500

## [1] "^FCHI"

# Adjusted Prices
adjGSPC_mo <- to.monthly(GSPC)$GSPC.Adjusted
adjFTSE_mo <- to.monthly(FTSE)$FTSE.Adjusted
adjGDAXI_mo <- to.monthly(GDAXI)$GDAXI.Adjusted
adjFCHI_mo <- to.monthly(FCHI)$FCHI.Adjusted

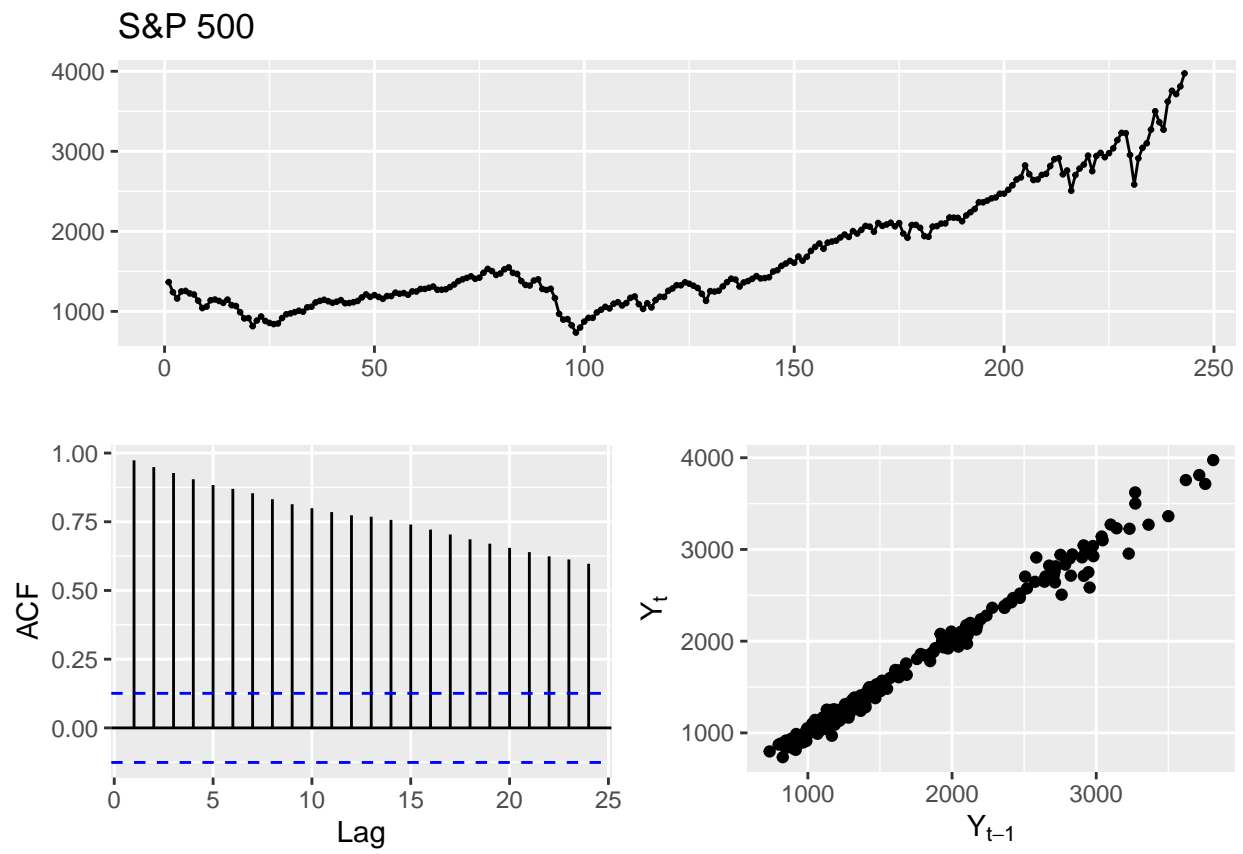
# Merge xts object
globalIndices <- merge.xts(adjGSPC_mo)
```

Data Observation:

Observing each indices:

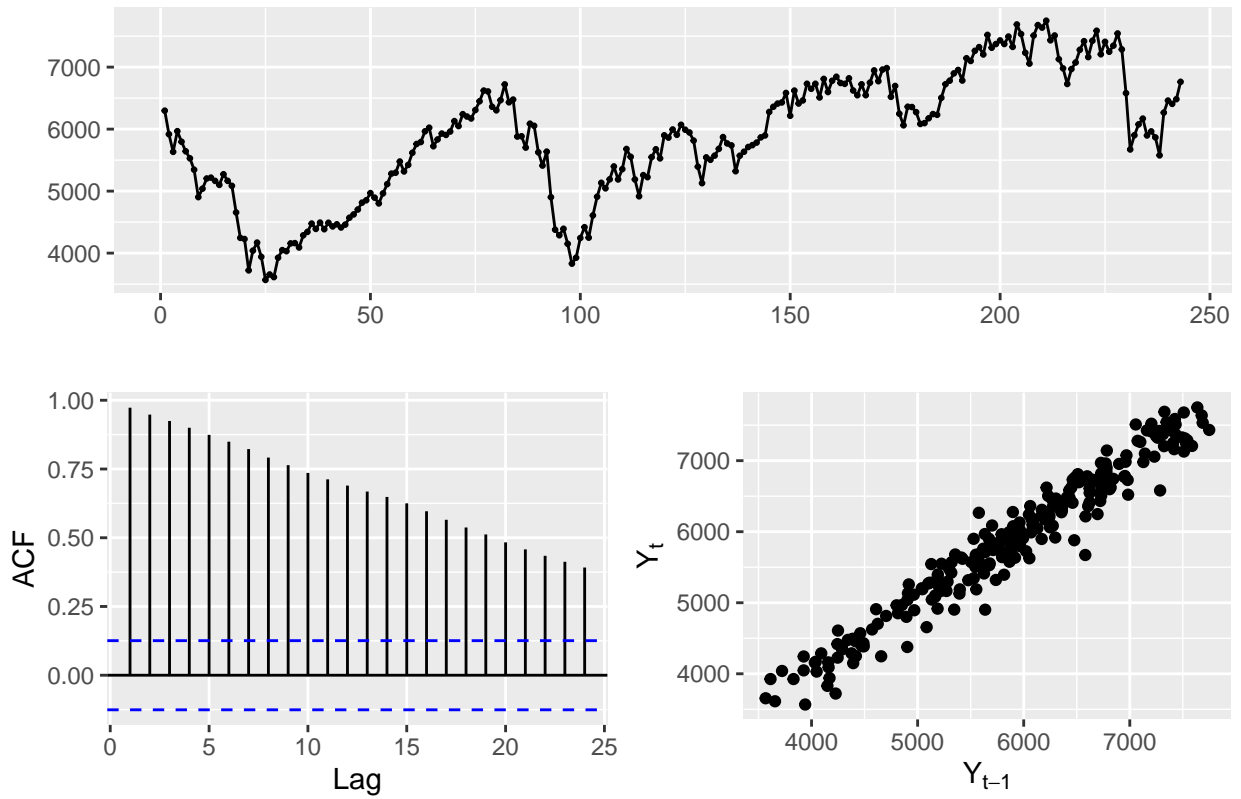
```
library(forecast)

ggtsdisplay(adjGSPC_mo, main="S&P 500", plot.type="scatter")
```

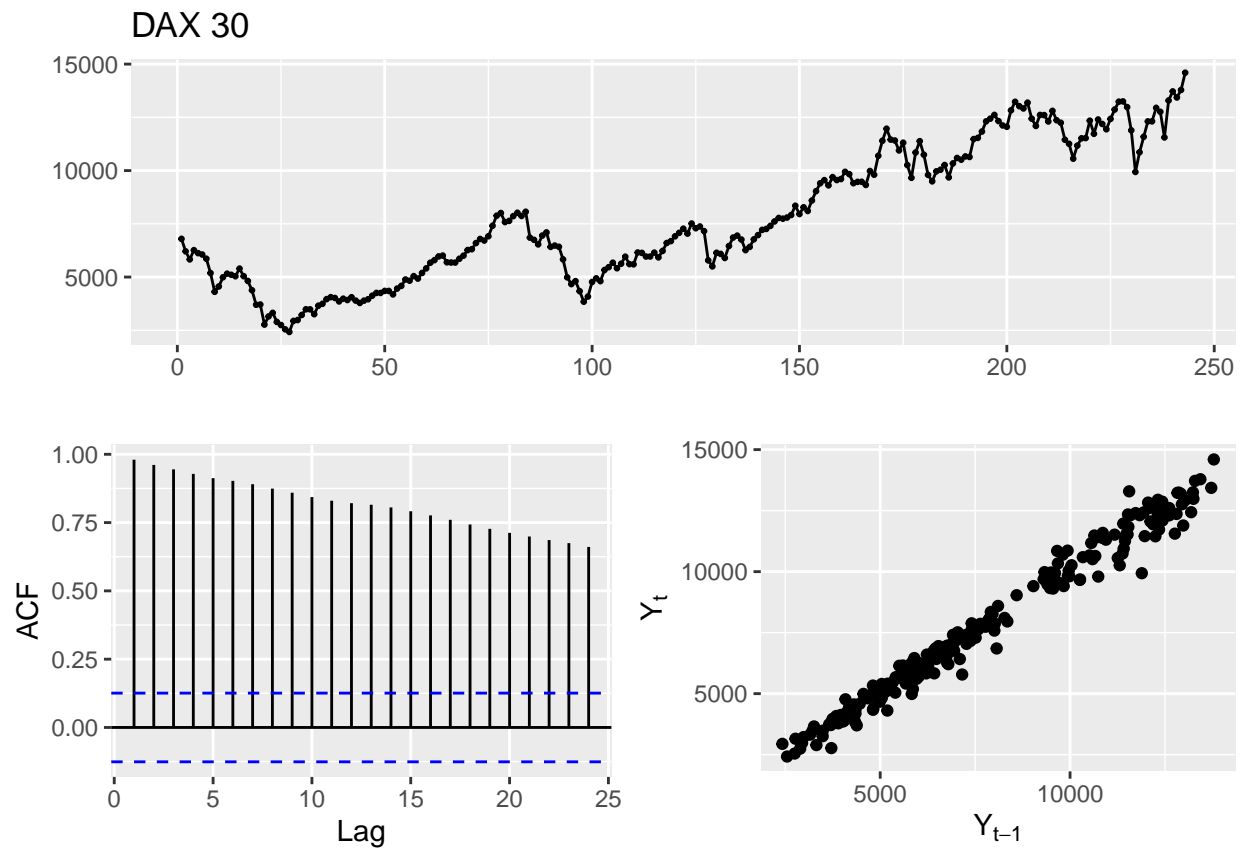


```
ggtsdisplay(adjFTSE_mo, main="FTSE 100", plot.type="scatter")
```

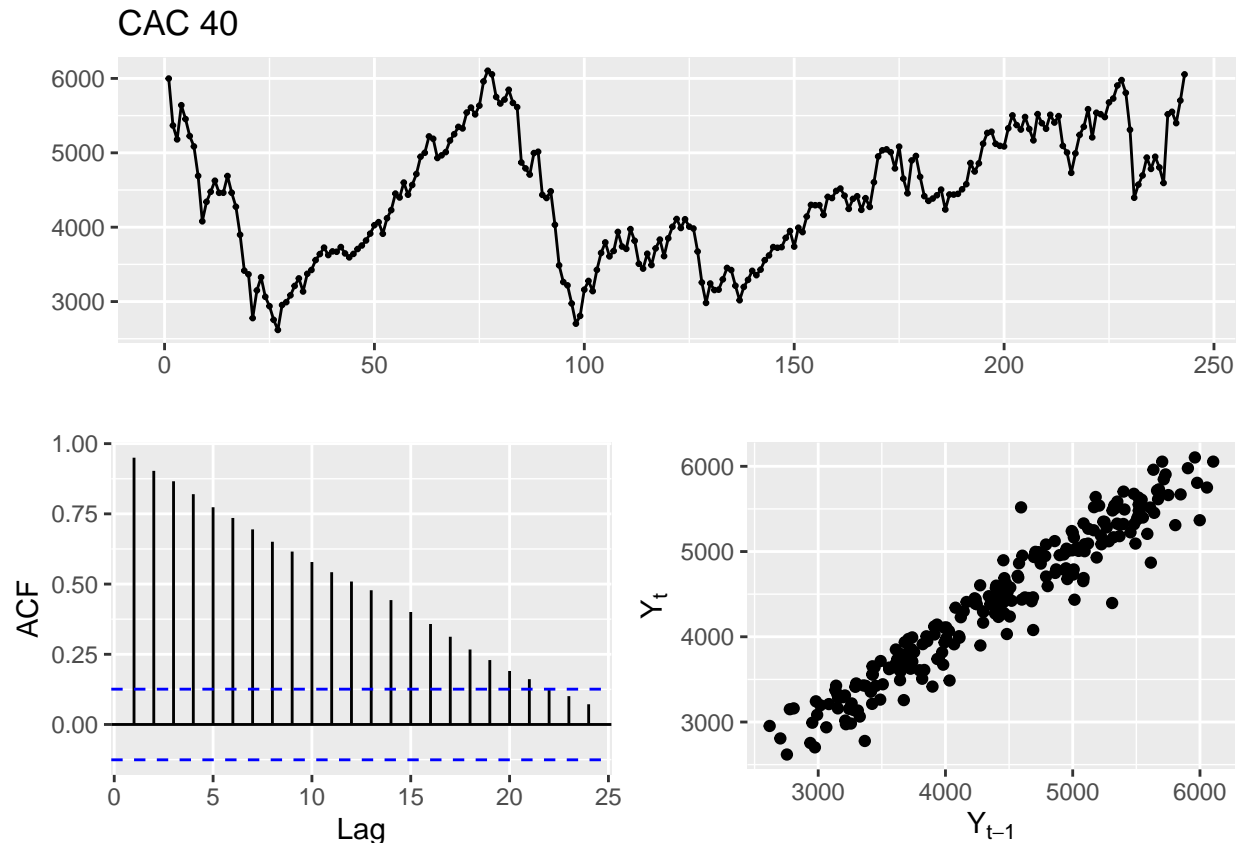
FTSE 100



```
ggtsdisplay(adjGDAXI_mo, main="DAX 30", plot.type="scatter")
```



```
ggtsdisplay(adjFCHI_mo, main="CAC 40", plot.type="scatter")
```



Remarks

We can also see all indices' lag plots exhibit a linear pattern, implying that the data are strongly non-random and thus, a first-order autoregressive model might be appropriate.

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t \quad (2)$$

Testing for Stationarity for indices

Per methodology, we run Augmented Dickey Fuller Test for each indices. Recall that the null hypothesis for Dickey-Fuller Test is that a unit root is present in our autoregressive model, meaning the variable is a non-stationary variable.

S&P 500

```
library(aTSA)

adf.test(adjGSPC_mo)

## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
```

```
## [1,] 0 3.47 0.99
## [2,] 1 3.47 0.99
## [3,] 2 4.06 0.99
## [4,] 3 3.83 0.99
## [5,] 4 3.83 0.99
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,] 0 3.80 0.99
## [2,] 1 3.73 0.99
## [3,] 2 4.32 0.99
## [4,] 3 4.25 0.99
## [5,] 4 4.33 0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,] 0 2.71 0.99
## [2,] 1 2.95 0.99
## [3,] 2 3.46 0.99
## [4,] 3 3.35 0.99
## [5,] 4 3.45 0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p\text{-value} = .99 > .05$. Thus, we fail to reject the null hypothesis. In other words, S&P 500 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

FTSE 100

```
adf.test(adjFTSE_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,] 0 0.440 0.770
## [2,] 1 0.551 0.802
## [3,] 2 0.640 0.828
## [4,] 3 0.538 0.799
## [5,] 4 0.572 0.808
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,] 0 1.84 0.99
## [2,] 1 1.87 0.99
## [3,] 2 1.92 0.99
## [4,] 3 1.86 0.99
## [5,] 4 1.79 0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,] 0 2.08 0.99
## [2,] 1 2.36 0.99
## [3,] 2 2.62 0.99
## [4,] 3 2.39 0.99
```

```
## [5,]    4 2.44    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p - value > .05$. Thus, we fail to reject the null hypothesis. In other words, FTSE100 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

DAX 30

```
adf.test(adjGDAXI_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,]   0 1.86   0.984
## [2,]   1 1.83   0.983
## [3,]   2 2.05   0.990
## [4,]   3 2.00   0.989
## [5,]   4 2.04   0.990
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 2.16   0.99
## [2,]   1 2.01   0.99
## [3,]   2 2.19   0.99
## [4,]   3 2.23   0.99
## [5,]   4 2.23   0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 2.03   0.99
## [2,]   1 2.19   0.99
## [3,]   2 2.56   0.99
## [4,]   3 2.44   0.99
## [5,]   4 2.60   0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p - value = .99 > .05$. Thus, we fail to reject the null hypothesis. In other words, DAX 30 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

CAC 40

```
adf.test(adjFCHI_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
```



```
##      lag   ADF p.value
## [1,]   0 0.413  0.763
## [2,]   1 0.611  0.820
## [3,]   2 0.673  0.837
## [4,]   3 0.493  0.786
## [5,]   4 0.563  0.806
## Type 2: with drift no trend
##      lag   ADF p.value
## [1,]   0 2.12  0.99
## [2,]   1 2.20  0.99
## [3,]   2 2.42  0.99
## [4,]   3 2.09  0.99
## [5,]   4 2.15  0.99
## Type 3: with drift and trend
##      lag   ADF p.value
## [1,]   0 1.61  0.99
## [2,]   1 1.87  0.99
## [3,]   2 2.08  0.99
## [4,]   3 1.66  0.99
## [5,]   4 1.78  0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p\text{-value} > .05$. Thus, we fail to reject the null hypothesis. In other words, CAC 40 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

Remarks

Through (A)DF tests, we can observe that the indices adjusted monthly closing prices are non-stationary variables.

Testing for stationarity for error term

Regression model

```
# Converting xts to numeric because DW doesn't play nice with xts
adjGSPC_mo_num <- as.numeric(adjGSPC_mo)
adjFTSE_mo_num <- as.numeric(adjFTSE_mo)
adjGDAXI_mo_num <- as.numeric(adjGDAXI_mo)
adjFCHI_mo_num <- as.numeric(adjFCHI_mo)

model1 <- lm(adjGSPC_mo_num ~ adjFTSE_mo_num + adjGDAXI_mo_num + adjFCHI_mo_num, data=globalIndices)
model1

##
## Call:
## lm(formula = adjGSPC_mo_num ~ adjFTSE_mo_num + adjGDAXI_mo_num +
##      adjFCHI_mo_num, data = globalIndices)
##
## Coefficients:
##      (Intercept)      adjFTSE_mo_num      adjGDAXI_mo_num      adjFCHI_mo_num
##          928.0168             -0.3887              0.3024              0.1580
```

Checking for positive autocorrelation

We first run Durbin-Watson Test to check for positive autocorrelation.

```
library(car)

durbinWatsonTest(model1, max.lag=4)

## lag Autocorrelation D-W Statistic p-value
## 1 0.9062800 0.1760415 0
## 2 0.8296681 0.3159679 0
## 3 0.7679952 0.4225422 0
## 4 0.7256038 0.4945158 0
## Alternative hypothesis: rho[lag] != 0
```

We can observe $p - value < .05$ from the DW Test, implying autocorrelation of the residuals in this model. We will attempt to remedy by apply an ARIMA model.

Fitting ARIMA Model

```
model2 <- auto.arima(adjGSPC_mo)
summary(model2)

## Series: adjGSPC_mo
## ARIMA(2,2,2)
##
## Coefficients:
##          ar1          ar2          ma1          ma2
##      -0.1066  -0.1791  -0.8580  -0.1083
## s.e.   0.3806   0.0662   0.3862   0.3789
##
## sigma^2 estimated as 6091:  log likelihood=-1391.63
## AIC=2793.25  AICc=2793.51  BIC=2810.67
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 9.05527 77.07304 53.04823 0.5072379 3.288932 0.2465509
##              ACF1
## Training set -0.01047505
```

We can see that with $ARIMA(2,2,2)$, we have great $ACF1$ statistics, implying a good fit for forecasting.

Re-running Durbin Watson Test with ARIMA(2,2,2)

We rerun the Durbin Watson Test to verify if autocorrelation has been fixed. Since *durbinWatsonTest* requires a linear model object, we calculate the statistics using the following equation:

$$d = \frac{\sum_{t=2}^T (\epsilon_t - \epsilon_{t-1})^2}{\sum_{t=1}^T \epsilon_t^2} = 1.9851751 \approx 2 \quad (3)$$

Since the new Durbin-Watson Test is $d \approx 2$, we addressed the autocorrelation issue with our model.

Test for stationarity of error term

We run the (A)DF test to check for stationarity of the error terms:

```
adf.test(model2$residuals)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag      ADF p.value
## [1,]  0 -15.41    0.01
## [2,]  1 -10.78    0.01
## [3,]  2  -8.89    0.01
## [4,]  3  -8.03    0.01
## [5,]  4  -7.16    0.01
## Type 2: with drift no trend
##      lag      ADF p.value
## [1,]  0 -15.59    0.01
## [2,]  1 -10.98    0.01
## [3,]  2  -9.13    0.01
## [4,]  3  -8.28    0.01
## [5,]  4  -7.43    0.01
## Type 3: with drift and trend
##      lag      ADF p.value
## [1,]  0 -15.57    0.01
## [2,]  1 -10.96    0.01
## [3,]  2  -9.11    0.01
## [4,]  3  -8.26    0.01
## [5,]  4  -7.41    0.01
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe that $p - value = .01 < .05$, thus we reject the null hypothesis that the variable—the error term in this case—has a unit root. Thus the error term is a stationary variable.

Conclusion

In conclusion, through our analysis, we note that all indices are non-stationary variables. However the error term is a stationary variable. This means that the global indices cointegrate.