Extra Credit Project

Marc Luiz, Nelly Shieh, Jeff Nguyen

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Coitegration Analysis of Different Financial Markets
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Directed by Professor Mohammad Safarzadeh

Topic:

Using major stock indices for the United States (S&P500), Unite Kingdom (FTSE100), Germany (DAX), and France (CAC40) show that US and European financial markets cointegrate.

Method:

Reviewing cointegration condition:

Consider 2 time series variables Y and X. We have the regression equation as follow:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t \tag{1}$$

The cointegration is such that if X and Y are both non-stationary variables AND ϵ is a stationary variable, then X and Y cointegrate, i.e. they "move together" in the long run.

Thus, our primary methodology is as follow:

1. Test indices for stationarity:

Test representative stock indices in the United States (S&P 500), the United Kingdom (FTSE), Germany (DAX) and France (CAC40) for stationarity.
To execute this task, for each indices, we run the Augmented Dickey-Fuller Test (A)DF, which hypothesizes that a unit root is present in an autoregressive model. The intuition is such that if a variable is non-stationary, it tends to a constant mean–i.e. the values oscillates/ alternates from large to small. As a result, the process is not a random walk, i.e. non-stationary.

2. Test error term for stationarity:

Should the regressed result confirm non-stationarity, we check whether the error term of the regression ϵ is a stationary variable. If they are, then the indices cointegrate.

First we check for the common issue with time series data: positive autocorrelation by running Durbin-Watson Test. If autocorrelation exists, we add the first order autoregressive term $AR(1)$ into the model and subsequently $AR(2)$ as necessary.

Then, we run (A)DF test as above to test the error term for stationarity–not having a unit root in the (A)DF test.

Data Analysis

For this model, we use 20 years of data from March 2001 to March 2021.

Downloading data

```
library(quantmod)

# Set start date and end date of data
start_date <- "2001-01-01"
end_date <- "2021-03-18"

# Get data
getSymbols("^GSPC", src = "yahoo", , from = start_date, to = end_date) # S&P 500
```

```
## [1] "^GSPC"
```

```
getSymbols("^FTSE", src = "yahoo", , from = start_date, to = end_date) # S&P 500
```

```
## [1] "^FTSE"
```

```
getSymbols("^GDAXI", src = "yahoo", , from = start_date, to = end_date) # S&P 500
```

```
## [1] "^GDAXI"
```

```
getSymbols("^FCHI", src = "yahoo", , from = start_date, to = end_date) # S&P 500
```

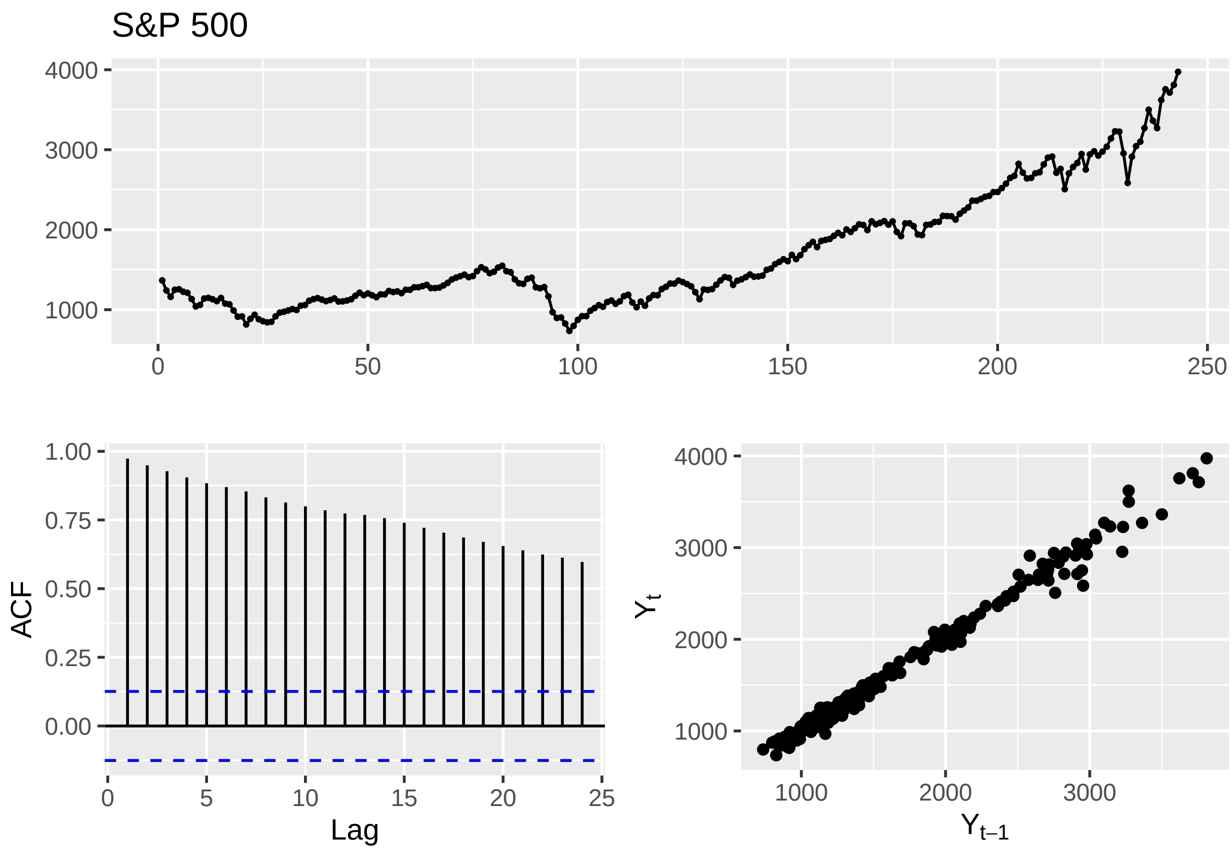
```
## [1] "^FCHI"
```

```
# Adjusted Prices
adjGSPC_mo <- to.monthly(GSPC)$GSPC.Adjusted
adjFTSE_mo <- to.monthly(FTSE)$FTSE.Adjusted
adjGDAXI_mo <- to.monthly(GDAXI)$GDAXI.Adjusted
adjFCHI_mo <- to.monthly(FCHI)$FCHI.Adjusted

# Merge xts object
globalIndices <- merge.xts(adjGSPC_mo)
```

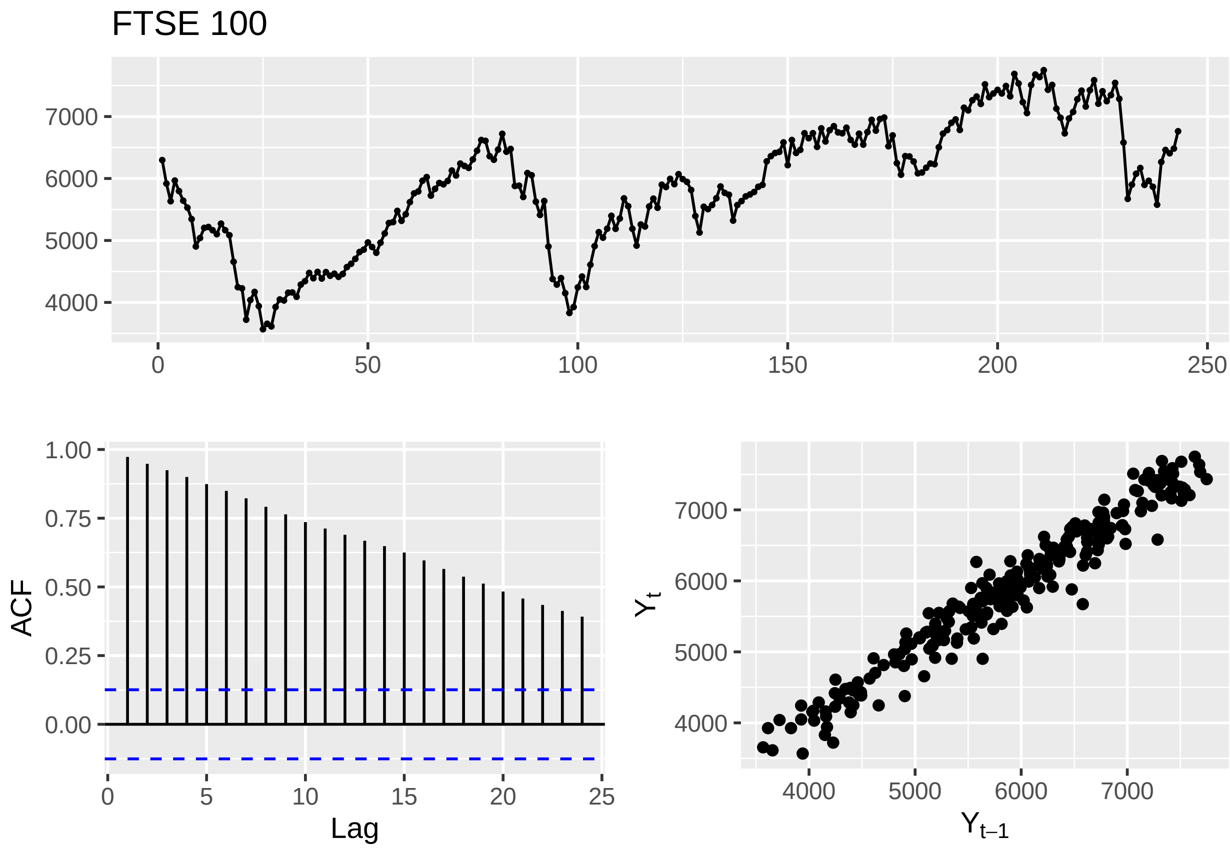
Data Observation:

Observing each indices:

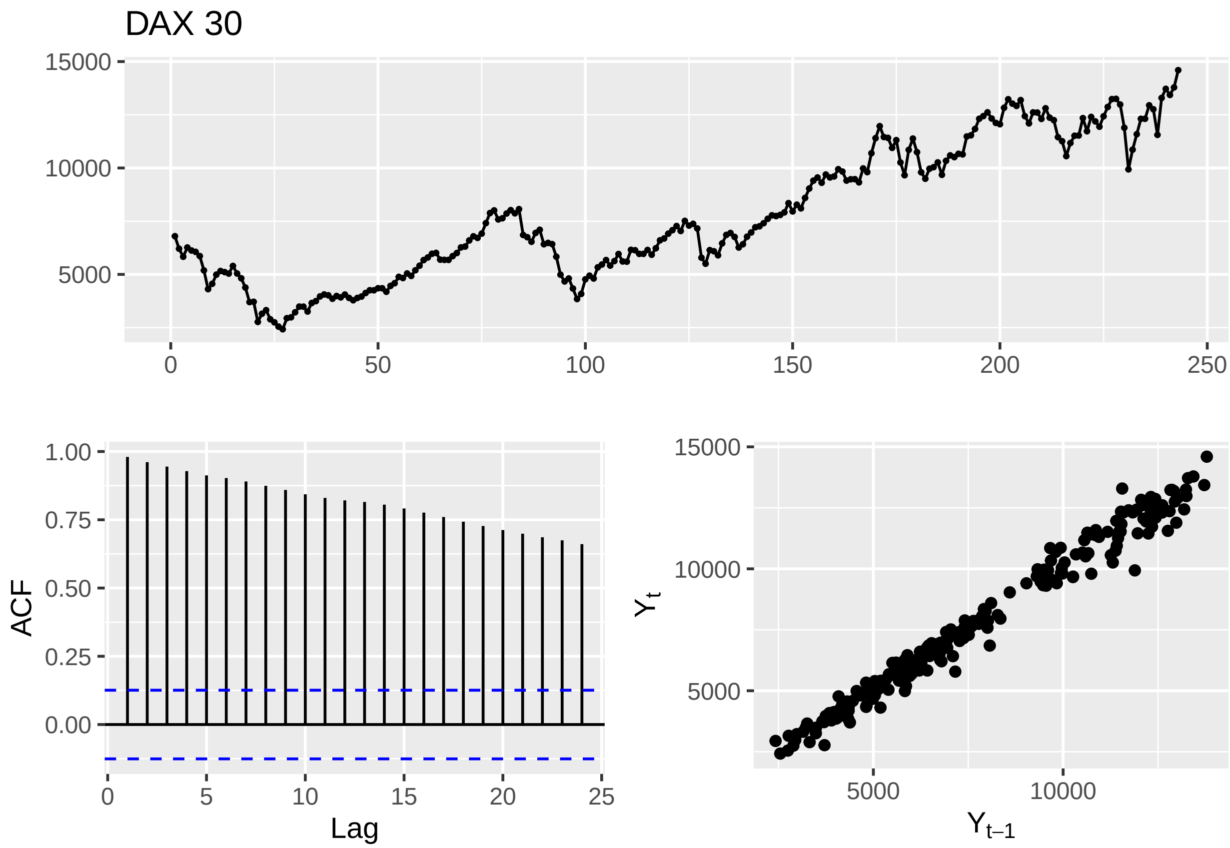
```
library(forecast)

ggtsdisplay(adjGSPC_mo, main="S&P 500", plot.type="scatter")
```



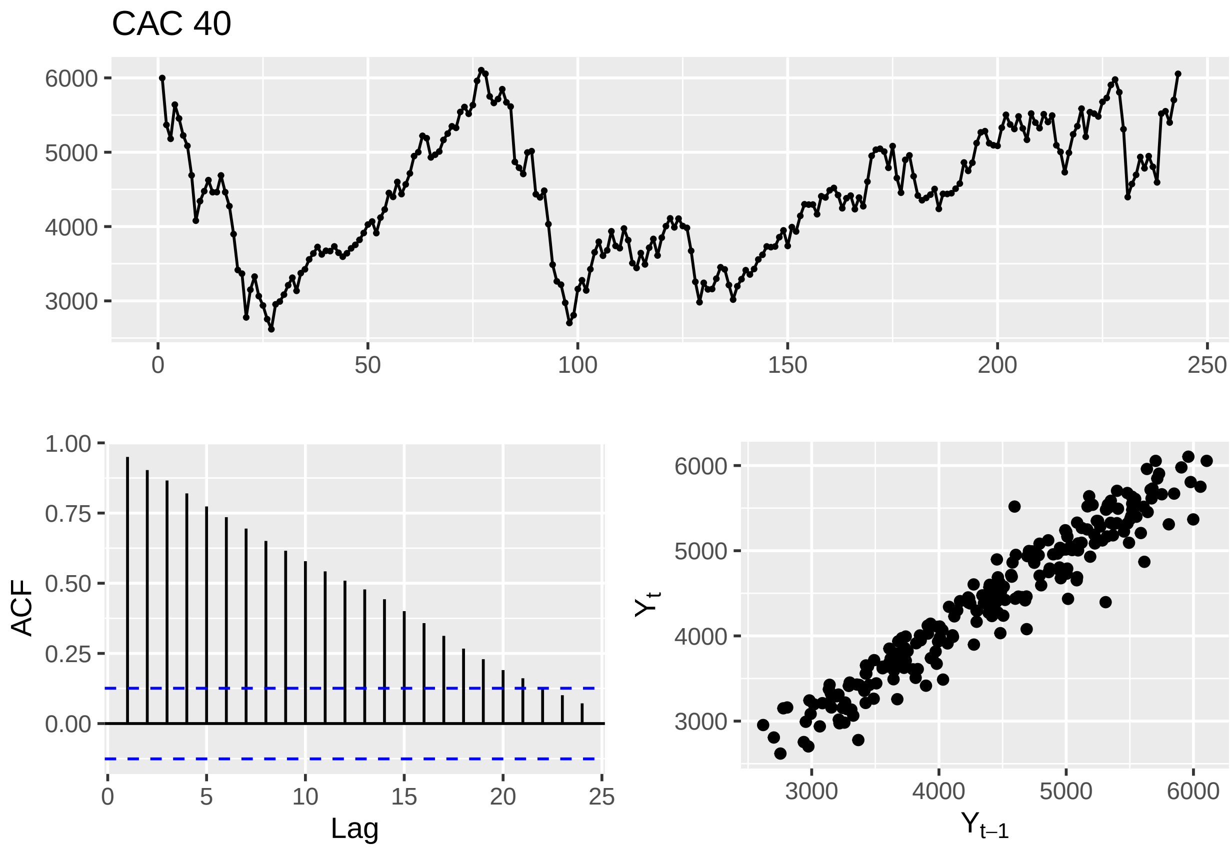

```
ggtsdisplay(adjFTSE_mo, main="FTSE 100", plot.type="scatter")
```




```
ggtsdisplay(adjGDAXI_mo, main="DAX 30", plot.type="scatter")
```




```
ggtsdisplay(adjFCHI_mo, main="CAC 40", plot.type="scatter")
```



Remarks
We can also see all indices' lag plots exhibit a linear pattern, implying that the data are strongly non-random and thus, a first-order autoregressive model might be appropriate.

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t \tag{2}$$

Testing for Stationarity for indices

Per methodology, we run Augmented Dickey Fuller Test for each indices. Recall that the null hypothesis for Dickey-Fuller Test is that a unit root is present in our autoregressive model, meaning the variable is a non-stationary variable.

S&P 500

```
library(aTSA)

adf.test(adjGSPC_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
```

```
## [1,]   0 3.47    0.99
## [2,]   1 3.47    0.99
## [3,]   2 4.06    0.99
## [4,]   3 3.83    0.99
## [5,]   4 3.83    0.99
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 3.80    0.99
## [2,]   1 3.73    0.99
## [3,]   2 4.32    0.99
## [4,]   3 4.25    0.99
## [5,]   4 4.33    0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 2.71    0.99
## [2,]   1 2.95    0.99
## [3,]   2 3.46    0.99
## [4,]   3 3.35    0.99
## [5,]   4 3.45    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p - value = .99 > .05$. Thus, we fail to reject the null hypothesis. In other words, S&P 500 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

FTSE 100

```
adf.test(adjFTSE_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag   ADF p.value
## [1,]   0 0.440   0.770
## [2,]   1 0.551   0.802
## [3,]   2 0.640   0.828
## [4,]   3 0.538   0.799
## [5,]   4 0.572   0.808
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 1.84    0.99
## [2,]   1 1.87    0.99
## [3,]   2 1.92    0.99
## [4,]   3 1.86    0.99
## [5,]   4 1.79    0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 2.08    0.99
## [2,]   1 2.36    0.99
## [3,]   2 2.62    0.99
## [4,]   3 2.39    0.99
```

```
## [5,]   4 2.44    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p - value > .05$. Thus, we fail to reject the null hypothesis. In other words, FTSE100 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

DAX 30

```
adf.test(adjGDAXI_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag  ADF p.value
## [1,]   0 1.86   0.984
## [2,]   1 1.83   0.983
## [3,]   2 2.05   0.990
## [4,]   3 2.00   0.989
## [5,]   4 2.04   0.990
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 2.16    0.99
## [2,]   1 2.01    0.99
## [3,]   2 2.19    0.99
## [4,]   3 2.23    0.99
## [5,]   4 2.23    0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 2.03    0.99
## [2,]   1 2.19    0.99
## [3,]   2 2.56    0.99
## [4,]   3 2.44    0.99
## [5,]   4 2.60    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p - value = .99 > .05$. Thus, we fail to reject the null hypothesis. In other words, DAX 30 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

CAC 40

```
adf.test(adjFCHI_mo)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
```

```
##      lag   ADF p.value
## [1,]   0 0.413   0.763
## [2,]   1 0.611   0.820
## [3,]   2 0.673   0.837
## [4,]   3 0.493   0.786
## [5,]   4 0.563   0.806
## Type 2: with drift no trend
##      lag  ADF p.value
## [1,]   0 2.12    0.99
## [2,]   1 2.20    0.99
## [3,]   2 2.42    0.99
## [4,]   3 2.09    0.99
## [5,]   4 2.15    0.99
## Type 3: with drift and trend
##      lag  ADF p.value
## [1,]   0 1.61    0.99
## [2,]   1 1.87    0.99
## [3,]   2 2.08    0.99
## [4,]   3 1.66    0.99
## [5,]   4 1.78    0.99
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe $p-value > .05$. Thus, we fail to reject the null hypothesis. In other words, CAC 40 monthly adjusted closing price has a unit root and therefore, is a non-stationary variable.

Remarks

Through (A)DF tests, we can observe that the indices adjusted monthly closing prices are non-stationary variables.

Testing for stationarity for error term

Regression model

```r
# Converting xts to numeric because DW doesn't play nice with xts
adjGSPC_mo_num <- as.numeric(adjGSPC_mo)
adjFTSE_mo_num <- as.numeric(adjFTSE_mo)
adjGDAXI_mo_num <- as.numeric(adjGDAXI_mo)
adjFCHI_mo_num <- as.numeric(adjFCHI_mo)

model1 <- lm(adjGSPC_mo_num ~ adjFTSE_mo_num + adjGDAXI_mo_num + adjFCHI_mo_num, data=globalIndices)
model1
```

```
##
## Call:
## lm(formula = adjGSPC_mo_num ~ adjFTSE_mo_num + adjGDAXI_mo_num +
##     adjFCHI_mo_num, data = globalIndices)
##
## Coefficients:
##     (Intercept)   adjFTSE_mo_num  adjGDAXI_mo_num   adjFCHI_mo_num
##        928.0168          -0.3887           0.3024           0.1580
```

Checking for positive autocorrelation

We first run Durbin-Watson Test to check for positive autocorrelation.

```
library(car)

durbinWatsonTest(model1, max.lag=4)
```

```
##  lag Autocorrelation D-W Statistic p-value
##    1       0.9062800     0.1760415       0
##    2       0.8296681     0.3159679       0
##    3       0.7679952     0.4225422       0
##    4       0.7256038     0.4945158       0
##  Alternative hypothesis: rho[lag] != 0
```

We can observe $p - value < .05$ from the DW Test, implying autocorrelation of the residuals in this model. We will attempt to remedy by apply an ARIMA model.

Fitting ARIMA Model

```
model2 <- auto.arima(adjGSPC_mo, xreg=cbind(adjFTSE_mo, adjGDAXI_mo, adjFCHI_mo))
summary(model2)
```

```
## Series: adjGSPC_mo
## Regression with ARIMA(0,1,1) errors
##
## Coefficients:
##           ma1   drift  FTSE.Adjusted  GDAXI.Adjusted  FCHI.Adjusted
##       -0.1345  7.3917         0.0796          0.0997         0.0241
## s.e.   0.0631  2.5589         0.0248          0.0147         0.0341
##
## sigma^2 estimated as 2088:  log likelihood=-1265.76
## AIC=2543.52   AICc=2543.88   BIC=2564.46
##
## Training set error measures:
##                       ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -0.01669197 45.12256 30.55832 -0.2428897 1.751505 0.1420251
##                       ACF1
## Training set -0.0003775466
```

We can see that with $ARIMA(0, 1, 1)$, we have great $ACF1$ statistics, implying a good fit for forecasting.

Re-running Durbin Watson Test with ARIMA(0,1,1)

We rerun the Durbin Watson Test to verify if autocorrelation has been fixed. Since $durbinWatsonTest$ requires a linear model object, we calculate the statistics using the following equation:

$$\begin{aligned} d = &\frac{\sum_{t=2}^{T}(\epsilon_t - \epsilon_{t-1})^2}{\sum_{t=1}^{T}\epsilon_t^2} \\ =&1.9957965 \approx 2 \end{aligned} \tag{3}$$

Since the new Durbin-Watson Test is $d \approx 2$, we addressed the autocorrelation issue with our model.

Test for stationarity of error term

We run the (A)DF test to check for stationarity of the error terms:

```
adf.test(model2$residuals)
```

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##      lag    ADF p.value
## [1,]   0 -15.49    0.01
## [2,]   1 -10.98    0.01
## [3,]   2  -8.26    0.01
## [4,]   3  -6.88    0.01
## [5,]   4  -6.10    0.01
## Type 2: with drift no trend
##      lag    ADF p.value
## [1,]   0 -15.46    0.01
## [2,]   1 -10.95    0.01
## [3,]   2  -8.24    0.01
## [4,]   3  -6.86    0.01
## [5,]   4  -6.09    0.01
## Type 3: with drift and trend
##      lag    ADF p.value
## [1,]   0 -16.21    0.01
## [2,]   1 -11.78    0.01
## [3,]   2  -9.06    0.01
## [4,]   3  -7.72    0.01
## [5,]   4  -7.08    0.01
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

We can observe that $p - value = .01 < .05$, thus we reject the null hypothesis that the variable–the error term in this case–has a unit root. Thus the error term is a stationary variable.

Conclusion

In conclusion, through our analysis, we note that all indices are non-stationary variables. However the error term is a stationary variable. This means that the global indices cointegrate.