# assignment01

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**University of Southern California**
**Marshall School of Business**
**FBE 506 Quantitative Method in Finance**
Assignment 01
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## Question 1

### a. Solve $x^2 + 2x - 8 = 0$

Solving polynomial equations in R using polyroot. The quadratic equation is expected to have 2 roots in the form of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

```
polyroot(c(-8, 2, 1))
```

```
## [1]  2-0i -4+0i
```

### b. Solve $x^2 - 9 = 0$

Solving polynomial equations in R using polyroot. The quadratic equation is expected to have 2 roots in the form of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

```
polyroot(c(-9, 0, 1))
```

```
## [1]  3+0i -3+0i
```

### c. Solve $x^3 - x^2 + 4x - 4 = 0$

Solving polynomial equations in R using polyroot. The cubic equation is expected to have 3 roots.

```r
polyroot(c(-4, 4, -1, 1))
```

```
## [1] 1-0i 0+2i 0-2i
```

### d. Solve $x^5 + 3x^2 - 8 = 0$

Solving polynomial equations in R using polyroot. The equation is expected to have 5 roots.

```r
polyroot(c(-8, 0, 3, 0, 0, 1))
```

```
## [1]  0.673841+1.598073i -1.305358+0.633886i -1.305358-0.633886i
## [4]  1.263035-0.000000i  0.673841-1.598073i
```

## Question 2

For discrete compounding (presented in a,b,c,d), we use the formula $S = A(1 + \frac{r}{n})^{nt}$ ,substitute $A = 100$, $r = 6\%$, $t = 5$ and use the correct value for n based on the number compounding period per year. Then we use the below custom function in R to solve it.

```r
# Discrete Compounding Function
# Compounding a=100 at an interest rate r=6% incurring with frequency n a year, for 5 years.
fv_discrete <- function(n) {
  100*(1+0.06/n)^(n*5)
}
```

a. Compound annually

```r
# compounded annually, so n=1
fv_discrete(1)
```

```
## [1] 133.8226
```

b. Compound quarterly

```r
# compounded quarterly, so n=4
fv_discrete(4)
```

```
## [1] 134.6855
```

c. Compound monthly

```r
# compounded monthly, so n=12
fv_discrete(12)
```

```
## [1] 134.885
```

d. Compound quarterly

```r
# compounded daily, so n=365
fv_discrete(365)
```

```
## [1] 134.9826
```

For continuous compounding, we use the formula $S = Ae^{rt}$ ,substitute $A = 100$, $r = 6\%$, $t = 5$. Thus:

```r
fv_continuous <- 100*exp(0.06*5)
fv_continuous
```

```
## [1] 134.9859
```

# Question 3

Using the two formulas: Present value of cash flow (future returns): $PV_{CF} = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + ... + \frac{R_n}{(1+r)^n}$

Present value of the investment $PV_{investment} = PV_{CF} + \frac{MV}{(1+r)^n}$
with $R_1$, $R_2$, $R_3$, $\ldots$, and $R_n$ be the return each year and $MV$ the market value at the end of the final year. Establishing the r functions

```r
# Present value of cash flow (future returns)
pv_cf <- function(R,r) {
  sum(R/((1+r)^(1:length(R))))
}

pv_investment <- function(R,r,MV) {
  pv_cf(R,r) + MV/((1+r)^length(R))
}
```

Substituting the given value of future returns:
$R_1 = 22000$, $R_2 = 28500$, $R_3 = 32000$, $R_4 = 18000$, $R_5 = 12000$ and the final year market value $MV = 52000$, we have:

Present value of future returns:

```r
R <- c(22000, 28500, 32000, 18000, 12000) # Future returns
r <- 0.06 # interest rate

pv_cf(R,r) # PV of future returns
```

```
## [1] 96212.22
```

Present value of investment:

```r
R <- c(22000, 28500, 32000, 18000, 12000) # Future returns
r <- 0.06 # interest rate
MV <- 52000 # market value of investment in the final year

pv_investment(R,r,MV) #PV of investment
```

```
## [1] 135069.6
```

# Question 4

Calculate NPV using this function $NPV = PV_{Investment} - Initial\ Investment$

Using the method and parameters in **Question 3** to calculate $PV_{Investment}$ and the initial investment valued at $120,000, we have:

```r
R <- c(22000, 28500, 32000, 18000, 12000) # Future returns
r <- 0.06 # interest rate
MV <- 52000 # market value of investment in the final year
init_investment <- 120000 # initial investment

npv <- pv_investment(R,r,MV) - init_investment #NPV
npv
```

```
## [1] 15069.64
```

# Question 5

Calculate PV of the perpetuity using the formula $PV_{CF} = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + ... + \frac{R_n}{(1+r)^n} = \frac{R}{r}$ with $R_1 = R_2 = R_3 = ... = R_n$ and $n = \infty$

Subtituing the parameters $R = 1$ and $r = 5\%$, we have

```r
perpetuity <- function(R,r) {
  R/r
}

R <- 1
r <- .05

perpetuity(1,.05)
```

```
## [1] 20
```

# Question 6

## a. Find IRR of question 4

IRR is the break-even root of the equation $I_0 = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + ... + \frac{R_n}{(1+r)^n}$
Thus the problem turns into finding the non-complex root of the polynomial equation in the form of $I_0 - \frac{R_1}{(1+r)^1} - \frac{R_2}{(1+r)^2} - \frac{R_3}{(1+r)^3} - ... - \frac{R_n}{(1+r)^n} = 0$
Note that the root will be in the form $\frac{1}{1+r}$

Using the parameters in **Question 4**

```
R <- c(22000, 28500, 32000, 18000, 12000 + 52000) # Future returns, adding the scrap value to the final

init_investment <- 120000 # initial investment
R_combine <- c(init_investment, R*(-1)) # rearrange the polynomial equation
r <- polyroot(R_combine) # find roots of the polynomial equation, the return will be 1+r

(1/(Re(r)[abs(Im(r)) < 1e-6])) - 1 # find only the non-complex root of said equation and subtract 1
```

```
## [1] 0.09894825
```

## b. Find the rate of return on a $24,000 investment with an annual return of $4,500 for five years and a scrap value of $2,500 at the end of the fifth year.

In this case $I_0 = 24000$, $R_1 = R_2 = R_3 = R_4 = R_5 = 4500$, and $MV = 2500$ Using similar method in a. with the new parameters

```
R <- c(4500, 4500, 4500, 4500, 4500 + 2500) # Future returns, adding the scrap value to the final year

init_investment <- 24000 # initial investment
R_combine <- c(init_investment, R*(-1)) # rearrange the polynomial equation
r <- polyroot(R_combine) # find roots of the polynomial equation, the return will be 1+r

(1/(Re(r)[abs(Im(r)) < 1e-6])) - 1 # find only the non-complex root of said equation and subtract 1
```

```
## [1] 0.01289476
```