Assignment 04, Question 5&6

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April 1, 2021

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FBE 543 Forecasting and Risk Analysis

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Question 5

a. Do a three-period ahead forecasting using the given initial values and statistics. Write 95% confidence interval for each forecast.

a)

```
alpha_0 <- 6
alpha_1 <- .7
alpha_2 <- .12
y_0 <- 5
y_1 <- 6
sigmaSq <- 1.21
f <- function(y_tMinus1, y_tMinus2) {alpha_0 + alpha_1*y_tMinus1 + alpha_2*y_tMinus2}
y_2 <- f(y_1, y_0)
y_3 <- f(y_2, y_1)
y_4 <- f(y_3, y_2)
```

$$
\begin{aligned}
Y_t =& 6 + 0.7Y_{t-1} + 0.12Y_{t-2} + \epsilon_t \\
\text{given:} & \\
y_0 =& 5 \\
y_1 =& 6 \sigma^2 = & 1.21
\end{aligned} \tag{1}
$$

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_2^F = 6 + 0.7Y_1 + 0.12Y_0$	10.8	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[8.644, 12.956]
$Y_3^F = 6 + 0.7Y_2 + 0.12Y_1$	14.28	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2)}$	[11.6482666,13.4317334]

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_4^F = 6 + 0.7Y_3 + 0.12Y_2$	17.292	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_2^2+\alpha_1^4+2\alpha_1^2\alpha_2)}$	[14.3499486,20.2340514]

b)

```
alpha_0 <- 2.5
alpha_1 <- .3
alpha_2 <- -.28
y_0 <- 1
y_1 <- 2
sigmaSq <- 6.25
f <- function(y_tMinus1, y_tMinus2) {alpha_0 + alpha_1*y_tMinus1 + alpha_2*y_tMinus2}
y_2 <- f(y_1, y_0)
y_3 <- f(y_2, y_1)
y_4 <- f(y_3, y_2)
```

$$
\begin{aligned}
Y_t =& 2.5 + 0.3Y_{t-1} + -0.28Y_{t-2} + \epsilon_t \\
\text{given:}& \\
y_0 =& 1 \\
y_1 =& 2 \sigma^2 = & 6.25
\end{aligned} \tag{2}
$$

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_2^F = 2.5 + 0.3Y_1 + -0.28Y_0$	2.82	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[-2.08, 7.72]
$Y_3^F = 2.5 + 0.3Y_2 + -0.28Y_1$	2.786	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2)}$	[-2.3297502,7.9357502]
$Y_4^F = 2.5 + 0.3Y_3 + -0.28Y_2$	2.5462	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_2^2+\alpha_1^4+2\alpha_1^2\alpha_2)}$	[-2.6535751,7.7459751]

c)

```
alpha_0 <- 1.2
alpha_1 <- -.2
alpha_2 <- -.35
y_0 <- 1.5
y_1 <- 2
sigmaSq <- .49
f <- function(y_tMinus1, y_tMinus2) {alpha_0 + alpha_1*y_tMinus1 + alpha_2*y_tMinus2}
```

```r
y_2 <- f(y_1, y_0)
y_3 <- f(y_2, y_1)
y_4 <- f(y_3, y_2)
```

$$\begin{aligned} Y_t =& 1.2 + -0.2Y_{t-1} + -0.35Y_{t-2} + \epsilon_t \\ \text{given:} & \\ y_0 =& 1.5 \\ y_1 =& 2\sigma^2 = \quad 0.49 \end{aligned} \tag{3}$$

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_2^F = 1.2 + -0.2Y_1 + -0.35Y_0$	0.275	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[-1.097, 1.647]
$Y_3^F = 1.2 + -0.2Y_2 + -0.35Y_1$	0.445	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2)}$	[-0.954171,1.674171]
$Y_4^F = 1.2 + -0.2Y_3 + -0.35Y_2$	1.01475	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_2^2+\alpha_1^4+2\alpha_1^2\alpha_2)}$	[-0.4476372,2.4771372]

d)

```r
alpha_0 <- 2.5
alpha_1 <- -.07
alpha_2 <- .06
y_0 <- 6
y_1 <- 5
sigmaSq <- 3.69
f <- function(y_tMinus1, y_tMinus2) {alpha_0 + alpha_1*y_tMinus1 + alpha_2*y_tMinus2}
y_2 <- f(y_1, y_0)
y_3 <- f(y_2, y_1)
y_4 <- f(y_3, y_2)
```

$$\begin{aligned} Y_t =& 2.5 + -0.07Y_{t-1} + 0.06Y_{t-2} + \epsilon_t \\ \text{given:} & \\ y_0 =& 6 \\ y_1 =& 5\sigma^2 = \quad 3.69 \end{aligned} \tag{4}$$

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_2^F = 2.5 + -0.07Y_1 + 0.06Y_0$	2.51	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[-1.2550371, 6.2750371]

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_3^F = 2.5 + -0.07Y_2 + 0.06Y_1$	2.6243	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2)}$	[-1.1499501,6.2842501]
$Y_4^F = 2.5 + -0.07Y_3 + 0.06Y_2$	2.466899	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_2^2+\alpha_1^4+2\alpha_1^2\alpha_2)}$	[-1.3152527,6.2490507]

Do a long-run (unconditional) forecasting and write 95% confidence interval.

a)

```
alpha_0 <- 6
alpha_1 <- .7
alpha_2 <- .12
y_0 <- 5
y_1 <- 6
sigmaSq <- 1.21
yLR <- alpha_0 / (1 - alpha_1 - alpha_2)
sigmaSq_yLR <- (1 - alpha_2)*sigmaSq / ((1 + alpha_2)*((1 - alpha_2)^2 - alpha_1^2))
```

$$\begin{aligned} Y_t =& 6 + 0.7Y_{t-1} + 0.12Y_{t-2} + \epsilon_t \\ \text{given:}& \\ y_0 =& 5 \\ y_1 =& 6\sigma^2 = \quad 1.21 \end{aligned} \tag{5}$$

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{6}{1-0.7-0.12}$	33.3333333	$Y_{LR} \pm 1.96\sqrt{\frac{(1-\alpha_2)*\sigma^2}{(1+\alpha_2)[(1-\alpha_2)^2-\alpha_1]}}$	[29.7497601, 36.9169066]

b)

```
alpha_0 <- 2.5
alpha_1 <- .3
alpha_2 <- -.28
y_0 <- 1
y_1 <- 2
sigmaSq <- 6.25
yLR <- alpha_0 / (1 - alpha_1 - alpha_2)
sigmaSq_yLR <- (1 - alpha_2)*sigmaSq / ((1 + alpha_2)*((1 - alpha_2)^2 - alpha_1^2))
```

$$
\begin{aligned}
Y_t =& 2.5 + 0.3Y_{t-1} + -0.28Y_{t-2} + \epsilon_t \\
\text{given:}& \\
y_0 =& 1 \\
y_1 =& 2\sigma^2 = \qquad 6.25
\end{aligned} \tag{6}
$$

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{2.5}{1-0.3--0.28}$	2.5510204	$Y_{LR} \pm 1.96\sqrt{\frac{(1-\alpha_2)*\sigma^2}{(1+\alpha_2)[(1-\alpha_2)^2-\alpha_1]}}$	[-2.6993898, 7.8014306]

c)

```
alpha_0 <- 1.2
alpha_1 <- -.2
alpha_2 <- -.35
y_0 <- 1.5
y_1 <- 2
sigmaSq <- .49
yLR <- alpha_0 / (1 - alpha_1 - alpha_2)
sigmaSq_yLR <- (1 - alpha_2)*sigmaSq / ((1 + alpha_2)*((1 - alpha_2)^2 - alpha_1^2))
```

$$
\begin{aligned}
Y_t =& 1.2 + -0.2Y_{t-1} + -0.35Y_{t-2} + \epsilon_t \\
\text{given:}& \\
y_0 =& 1.5 \\
y_1 =& 2\sigma^2 = \qquad 0.49
\end{aligned} \tag{7}
$$

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{1.2}{1--0.2--0.35}$	0.7741935	$Y_{LR} \pm 1.96\sqrt{\frac{(1-\alpha_2)*\sigma^2}{(1+\alpha_2)[(1-\alpha_2)^2-\alpha_1]}}$	[-0.7067877, 2.2551748]

d)

```
alpha_0 <- 2.5
alpha_1 <- -.07
alpha_2 <- .06
y_0 <- 6
y_1 <- 5
sigmaSq <- 3.69
yLR <- alpha_0 / (1 - alpha_1 - alpha_2)
sigmaSq_yLR <- (1 - alpha_2)*sigmaSq / ((1 + alpha_2)*((1 - alpha_2)^2 - alpha_1^2))
```

$$
\begin{aligned}
Y_t =& 2.5 + -0.07Y_{t-1} + 0.06Y_{t-2} + \epsilon_t \\
\text{given:}& \\
y_0 =& 6 \\
y_1 =& 5\sigma^2 = \qquad 3.69
\end{aligned} \tag{8}
$$

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{2.5}{1--0.07-0.06}$	2.4752475	$Y_{LR} \pm 1.96\sqrt{\frac{(1-\alpha_2)*\sigma^2}{(1+\alpha_2)[(1-\alpha_2)^2-\alpha_1]}}$	[-1.307087, 6.257582]