Formula Sheet

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Gauss-Markov Theorem

6th Assumptions: Stationarity

A variable if overtime, the mean μ_X, variance σ_X and covariance $Cov(Y_t, Y_{t-5})$ stays the same.
- If the mean changes: it's a "mean non-stationary" variable.
- If the variance changes: it's a "variance non-stationary" variable.

Review the regression equation:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t \tag{1}$$

Y	X	ϵ	Regression OK?	Note
Stationary	Stationary	Stationary	Yes	
Non-Stationary	Non-Stationary	Stationary	Yes	Cointegration, needs more data
Non-Stationary	Non-Stationary	Non-Stationary	No	Spurious Regression

Time Trend

Every time series variable can be decomposed to the following trend:
- Time
- Seasonal
- Cyclical
- Autocorrelation
- Randomness

Note: Recession is usually an economic variable.

How to measure time trend?

Step 1:

Run Regression:

$$
\begin{aligned}
Y =& \beta_0 + \beta_1 T + \beta_2 T^2 + \beta T^3 + \epsilon \\
& \text{Note: Nothing beyond } T^3
\end{aligned} \tag{2}
$$

If T^3 is insignificant, drop it and so on. The trend estimated equation is:

$$
\begin{aligned}
\hat{Y} =& \hat{\beta_0} + \hat{\beta_1} T + \hat{\beta_2} T^2 + \hat{\beta_3} T^3 \\
Y =& \hat{Y} + \epsilon
\end{aligned} \tag{3}
$$

$\hat{\epsilon}$ represents seasonality, cyclicality, autocorrelation, and randomness.

Step 2:

Use $\hat{\epsilon_t}$ to estimate autocorrelation:

$$
\hat{\epsilon_t} = \alpha_0 + \alpha_1 \hat{\epsilon_{t-1}} \tag{4}
$$

What to do when you have spurious regression, i.e. ϵ is non-stationary?

In this case, ϵ shows as not scattered on a scatterplot. We have to de-trend the variable.

Two type of trend:
- Deterministic: when variable explicitly driven by trend (saving account balance).
- Stochastic: random variable (S&P 500 price movement).

Important
- If the variable is Tr

De-trend Deterministic Trend Variable:

- Run a regression on time: $Y = \beta_0 + \beta_1 T + \beta_2 T^2 + \beta T^3 + \epsilon$.
- The estimated $\hat{Y}$ is

$$
\hat{Y} = \hat{\beta_0} + \hat{\beta_1} T + \hat{\beta_2} T^2 + \hat{\beta_3} T^3 \tag{5}
$$

- The equation is then $Y = \hat{Y} + \hat{\epsilon}$.
- Take the error of regression $\hat{\epsilon}$ as “de-trended” Y.

De-trend Stochastic Trend Variable:

- Differencing Y:

$$\Delta Y = Y_t - Y_{t-1} \tag{6}$$

For example, stock price model:

$$\begin{aligned}
Y_t &= Y_{t-1} + \epsilon_t \\
&\text{via inspection:} \\
Y_t &= Y_0 + \sum_{i=0}^{t} \epsilon_i \\
&\text{thus:} \\
E[Y_t] &= Y_0 \quad \text{The model is mean-stationary.} \\
Var[Y_t] &= t\sigma_\epsilon^2 \quad \text{Variance non-stationary}
\end{aligned} \tag{7}$$

Note that the random-walk hypothesis is variance-non-stationary.

Removing "wrong" Nonstationarity:

- If the variable is Trend Stationary (TS) and was treated as Difference Stationary (DS), the estimated $\hat{\beta}$ will be unbiased, inefficient and consistent. Use more data to fix the problem.
- If the variable is DS and was treated as TS, $\hat{\beta}$ will be biased, inefficient and inconsistent.