Assignment 04, Question 3&4

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FBE 543 Forecasting and Risk Analysis

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Question 3

a.

$$Y_t = 2.5 + .35Y_{t-1} + \epsilon_t \tag{1}$$

- **ARIMA model:** $ARIMA(1, 0, 0)$.
- **Stationarity:** Stationary as $\sum_{i=1}^{n} |\alpha_i| = .35 < 1$.
- **Invertibility:** N/A.

b.

$$Y_t = 4.5 - 1.5Y_{t-1} + \epsilon_t - .5\epsilon_{t-1} \tag{2}$$

- **ARIMA model:** $ARIMA(1, 0, 1)$.
- **Stationarity:** Non-Stationary as $\sum_{i=1}^{n} |\alpha_i| = |1.5| > 1$.
- **Invertibility:** Invertible as $\sum_{i=1}^{n} \beta_i| = .5 < 1$.

c.

$$Y_t = 1.2 - .75Y_{t-1} + .3Y_{t-2} + \epsilon_t \tag{3}$$

- **ARIMA model:** $ARIMA(2, 0, 0)$.
- **Stationarity:** Non-Stationary as $\sum_{i=1}^{n} |\alpha_i| = | - .75| + |.3| = 1.05 > 1$.
- **Invertibility:** N/A.

d.

$$Y_t = 2.5 - .95Y_{t-1} + \epsilon_t - .5\epsilon_{t-1} - .2\epsilon t - 2 \tag{4}$$

- **ARIMA model:** $ARIMA(1, 0, 2)$.
- **Stationarity:** Stationary as $\sum_{i=1}^{n} |\alpha_i| = 0.95 < 1$.
- **Invertibility:** Invertible as $\sum_{i=1}^{n} |\beta_i| = |-.5| + |-.2| = 0.7 < 1$.

e.

$$Y_t = .52 - 1.2Y_{t-1} + \epsilon_t + .2\epsilon_{t-1} \tag{5}$$

- **ARIMA model:** $ARIMA(1, 0, 1)$.
- **Stationarity:** Non-Stationary as $\sum_{i=1}^{n} |\alpha_i| = 1.2 > 1$.
- **Invertibility:** Invertible as $\sum_{i=1}^{n} |\beta_i| = |.2| = 0.2 < 1$.

f.

$$DY_t = 1.2DY_{t-1} + \epsilon_t \tag{6}$$

- **ARIMA model:** $ARIMA(1, 1, 0)$.
- **Stationarity:** Non-Stationary as $\sum_{i=1}^{n} |\alpha_i| = 1.2 > 1$.
- **Invertibility:** N/A.

g.

$$DY_t = .42DY_{t-1} + \epsilon_t - .6\epsilon_{t-1} \tag{7}$$

- **ARIMA model:** $ARIMA(1, 1, 1)$.
- **Stationarity:** Stationary as $\sum_{i=1}^{n} |\alpha_i| = 0.42 < 1$.
- **Invertibility:** Invertible as $\sum_{i=1}^{n} |\beta_i| = |-.6| = 0.6 < 1$.

h.

$$DY_t = .62Y_{t-1} + \epsilon_t - .6\epsilon_{t-1} \tag{8}$$

- **ARIMA model:** Incorrect model as LHS and RHS does not have similar order difference term.
- **Stationarity:** N/A.
- **Invertibility:** N/A.

i.

$$Y_t = 2.5 + .95DY_{t-1} + \epsilon_t \tag{9}$$

- **ARIMA model:** Incorrect model as LHS and RHS does not have similar order difference term.
- **Stationarity:** N/A.
- **Invertibility:** N/A.

j.

$$Y_t = 1.6 + \epsilon_t - .6\epsilon_{t-1} \tag{10}$$

- **ARIMA model:** $ARIMA(0,0,1)$.
- **Stationarity:** Stationary as $\sum_{i=1}^{n} |\beta_i| = |-.6| = 0.6 < 1$.
- **Invertibility:** Invertible as $\sum_{i=1}^{n} |\beta_i| = |-.6| = 0.6 < 1$.

Question 4

a. Do a three-period ahead forecasting using the given initial values and statistics. Write 95% confidence interval for each forecast.

a)

$$\begin{aligned} Y_t =& 1.6 + .75Y_{t-1} + \epsilon_t \\ \text{given:}& \\ y_0 =& 2 \\ \sigma^2 =& 1.21 \end{aligned} \tag{11}$$

```
y_0 <- 2
sigmaSq <- 1.21

f <- function(y_t) {1.6 + .75*y_t}
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_1^F = 1.6 + .75Y_0$	3.1	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[0.944, 5.256]
$Y_2^F = 1.6 + .75Y_1$	3.925	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha^2)}$	[1.23,6.62]
$Y_3^F = 1.6 + .75Y_2$	4.54375	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_1^4)}$	[1.5884518,7.4990482]

b)

$$
\begin{aligned}
Y_t =& 2.5 + .3Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& 10 \\
\sigma^2 =& 6.25
\end{aligned}
\tag{12}
$$

```
y_0 <- 10
sigmaSq <- 6.25

f <- function(y_t) {2.5 + .3*y_t}
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_1^F = 2.5 + .3Y_0$	5.5	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[0.6, 10.4]
$Y_2^F = 2.5 + .3Y_1$	4.15	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha^2)}$	[-1.975,10.275]
$Y_3^F = 2.5 + .3Y_2$	3.745	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_1^4)}$	[-2.9715869,10.4615869]

c)

$$
\begin{aligned}
Y_t =& 1.2 - .2Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& 1.5 \\
\sigma^2 =& .49
\end{aligned}
\tag{13}
$$

```
y_0 <- 1.5
sigmaSq <- .49

f <- function(y_t) {1.2 - .2*y_t}
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_1^F = 1.2 - .2Y_0$	0.9	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[-0.472, 2.272]
$Y_2^F = 1.2 - .2Y_1$	1.02	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha^2)}$	[-0.695,2.735]
$Y_3^F = 1.2 - .2Y_2$	0.996	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_1^4)}$	[-0.8846443,2.8766443]

d)

$$\begin{aligned} Y_t =& 2.5 - .8Y_{t-1} + \epsilon_t \\ \text{given:}& \\ y_0 =& 6 \\ \sigma^2 =& 3.69 \end{aligned} \tag{14}$$

```
y_0 <- 6
sigmaSq <- 3.69

f <- function(y_t) {2.5 - .8*y_t}
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_1^F = 2.5 - .8Y_0$	-2.3	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[-6.0650371, 1.4650371]
$Y_2^F = 2.5 - .8Y_1$	4.34	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha^2)}$	[-0.3662963,9.0462963]
$Y_3^F = 2.5 - .8Y_2$	-0.972	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_1^4)}$	[-6.1328568,4.1888568]

e)

$$\begin{aligned} Y_t =& -.5Y_{t-1} + \epsilon_t \\ \text{given:}& \\ y_0 =& -1.6 \\ \sigma^2 =& 1.44 \end{aligned} \tag{15}$$

```
y_0 <- -1.6
sigmaSq <- 1.44

f <- function(y_t) {-.5*y_t}
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Variable	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_1^F = -.5Y_0$	0.8	$Y_{t+1}^F \pm 1.96\sqrt{\sigma^2}$	[-1.552, 3.152]
$Y_2^F = -.5Y_1$	-0.4	$Y_{t+2}^F \pm 1.96\sqrt{\sigma^2(1+\alpha^2)}$	[-3.34,2.54]
$Y_3^F = -.5Y_2$	0.2	$Y_{t+3}^F \pm 1.96\sqrt{\sigma^2(1+\alpha_1^2+\alpha_1^4)}$	[-3.0239617,3.4239617]

Do a long-run (unconditional) forecasting and write 95% confidence interval.

a)

$$
\begin{aligned}
Y_t =& 1.6 + .75Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& 2 \\
\sigma^2 =& 1.21
\end{aligned} \tag{16}
$$

```r
alpha_0 <- 1.6
alpha_1 <- .75
y_0 <- 2
sigmaSq <- 1.21

yLR <- alpha_0 / (1 - alpha_1)
sigmaSq_yLR <- sigmaSq / (1 - alpha_1^2)
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{1.6}{0.75}$	6.4	$Y_{LR} \pm 1.96\sqrt{\frac{\sigma^2}{1-\alpha_1^2}}$	[3.1404344, 9.6595656]

b)

$$
\begin{aligned}
Y_t =& 2.5 + .3Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& 10 \\
\sigma^2 =& 6.25
\end{aligned} \tag{17}
$$

```r
alpha_0 <- 2.5
alpha_1 <- .3
y_0 <- 10
sigmaSq <- 6.25

yLR <- alpha_0 / (1 - alpha_1)
sigmaSq_yLR <- sigmaSq / (1 - alpha_1^2)
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{2.5}{0.3}$	3.5714286	$Y_{LR} \pm 1.96\sqrt{\frac{\sigma^2}{1-\alpha_1^2}}$	[-1.5651671, 8.7080243]

c)

$$
\begin{aligned}
Y_t =& 1.2 - .2Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& 1.5 \\
\sigma^2 =& .49
\end{aligned} \tag{18}
$$

```r
alpha_0 <- 1.2
alpha_1 <- -.2
y_0 <- 1.5
sigmaSq <- .49

yLR <- alpha_0 / (1 - alpha_1)
sigmaSq_yLR <- sigmaSq / (1 - alpha_1^2)
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{1.2}{-0.2}$	1	$Y_{LR} \pm 1.96\sqrt{\frac{\sigma^2}{1-\alpha_1^2}}$	[-0.4002916, 2.4002916]

d)

$$
\begin{aligned}
Y_t =& 2.5 - .8Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& 6 \\
\sigma^2 =& 3.69
\end{aligned} \tag{19}
$$

```r
alpha_0 <- 2.5
alpha_1 <- -.8
y_0 <- 6
sigmaSq <- 3.69

yLR <- alpha_0 / (1 - alpha_1)
sigmaSq_yLR <- sigmaSq / (1 - alpha_1^2)
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{2.5}{-0.8}$	1.3888889	$Y_{LR} \pm 1.96\sqrt{\frac{\sigma^2}{1-\alpha_1^2}}$	[-4.8861729, 7.6639506]

e)

$$
\begin{aligned}
Y_t =& -.5Y_{t-1} + \epsilon_t \\
\text{given:}& \\
y_0 =& -1.6 \\
\sigma^2 =& 1.44
\end{aligned}
\tag{20}
$$

```
alpha_0 <- 0
alpha_1 <- -.5
y_0 <- -1.6
sigmaSq <- 1.44

yLR <- alpha_0 / (1 - alpha_1)
sigmaSq_yLR <- sigmaSq / (1 - alpha_1^2)
y_1 <- f(y_0)
y_2 <- f(y_1)
y_3 <- f(y_2)
```

Forecast Long Run	Forecasted Value	95% Confidence Interval Equation	95% Confidence Interval
$Y_{LR} = \frac{0}{-0.5}$	0	$Y_{LR} \pm 1.96\sqrt{\frac{\sigma^2}{1-\alpha_1^2}}$	[-2.7158557, 2.7158557]