LSM Reloaded – Differentiate xVA on your iPad Mini

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Erratum LSM Reloaded (Huge and Savine, 2017) section 2.2.5

We address an error pointed out by Stephane Crepey in our paper 'LSM Reloaded' from 2017. The main result remains that a collateralized CVA is computed as the expected sum of the cash-flows in the netting set, discounted by an adapted process:

$$CVA = E\left[\sum_k \eta_{T_k} CF_k\right]$$

However, the formula for the discounting process in the paper is wrong:

$$\begin{aligned}\eta_T &= \sum_{T_p<T}\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\}}-\left(1-{}^{B}R_{T_p}\right)1_{\{T_{p-1}\le{}^{B}\tau<T_p\}}1_{\{{}^{B}\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\}}\\ &\ne \sum_{T_p<T}\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}\left(1_{\{\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\}}-1_{\{{}^{B}\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\}}\right)\end{aligned}$$

Let us recall the formula for a fully collateralized CVA with MPR theta:

$$\begin{aligned}CVA &= E\left[\sum_p\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}\max\left(0,V_{T_p}-V_{T_p-\vartheta}\right)\right]\\ &= E\left[\sum_p\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{V_{T_p}>V_{T_p-\vartheta}\}}V_{T_p}\right]-E\left[\sum_p\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{V_{T_p}>V_{T_p-\vartheta}\}}V_{T_p-\vartheta}\right]\\ &= LHS-RHS\end{aligned}$$

Starting with the LHS, as in the article:

$$\begin{aligned}LHS &= E\left[\sum_p\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{V_{T_p}>V_{T_p-\vartheta}\}}V_{T_p}\right]\\ &= E\left[\sum_p\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{V_{T_p}>V_{T_p-\vartheta}\}}E_{T_p}\left(\sum_{T_k>T_p}CF_k\right)\right]\\ &= E\left[\sum_p\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{V_{T_p}>V_{T_p-\vartheta}\}}\sum_{T_k>T_p}CF_k\right]\\ &= E\left[\sum_k CF_k\sum_{T_p<T_k}\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{V_{T_p}>V_{T_p-\vartheta}\}}\right]\\ &\approx E\left[\sum_k\left(\sum_{T_p<T_k}\left(1-R_{T_p}\right)1_{\{T_{p-1}\le\tau<T_p\}}1_{\{\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\}}\right)CF_k\right]\end{aligned}$$

In line 2, we replaced the future value by its definition. Line 3 applies the boxed expectation formula, knowing everything outside the conditional expectation is measurable on the exposure date. Note that the filtration includes everything market and everything credit up to the exposure date. Line 4 reverses the order of the sums as in the document. Line 5 applies proxies in indicators.

Now, focusing on the RHS:

$$RHS = E\left[\sum_p \left(1-R_{T_p}\right) 1_{\left\{T_{p-1}\le\tau<T_p\right\}} 1_{\left\{V_{T_p}>V_{T_p-\vartheta}\right\}} V_{T_p-\vartheta}\right]$$
$$= E\left[\sum_p \left(1-R_{T_p}\right) 1_{\left\{T_{p-1}\le\tau<T_p\right\}} 1_{\left\{V_{T_p}>V_{T_p-\vartheta}\right\}} E_{T_p-\vartheta}\left(\sum_{T_k>T_p} CF_k\right)\right]$$

(assuming no cash-flow during the MRP)

$$RHS \approx E\left[\sum_p \left(1-R_{T_p}\right) 1_{\left\{T_{p-1}\le\tau<T_p\right\}} 1_{\left\{\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}} E_{T_p-\vartheta}\left(\sum_{T_k>T_p} CF_k\right)\right]$$

(applying POI, using equality in place of approx. from there on)

$$RHS = E\left[\sum_p E_{T_p-\vartheta}\left[\left(1-R_{T_p}\right) 1_{\left\{T_{p-1}\le\tau<T_p\right\}} 1_{\left\{\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}}\right] E_{T_p-\vartheta}\left(\sum_{T_k>T_p} CF_k\right)\right]$$

Now, we are branching out everything market and everything credit on the margin date: conditionally to the (complete) filtration on the margin date, variables read on the main branch and the secondary branch are independent and identically distributed (including joint distributions) by construction.

$$RHS = E\left[\sum_p E_{T_p-\vartheta}\left[\left(1-{}^B R_{T_p}\right) 1_{\left\{T_{p-1}\le{}^B\tau<T_p\right\}} 1_{\left\{{}^B\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}}\right] E_{T_p-\vartheta}\left(\sum_{T_k>T_p} CF_k\right)\right]$$

(because of identical conditional distributions)

$$RHS = E\left[\sum_p \left(1-{}^B R_{T_p}\right) 1_{\left\{T_{p-1}\le{}^B\tau<T_p\right\}} 1_{\left\{{}^B\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}} \sum_{T_k>T_p} CF_k\right]$$

(because of conditional independence) and reversing the order of the sums:

$$RHS = E\left[\sum_k \left(\sum_{T_p<T_k} \left(1-{}^B R_{T_p}\right) 1_{\left\{T_{p-1}\le{}^B\tau<T_p\right\}} 1_{\left\{{}^B\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}}\right) CF_k\right]$$

Putting it all together:

$$\begin{aligned}
CVA &= LHS - RHS \\
&= E\left[\sum_k \left(\sum_{T_p<T_k} \left(1-R_{T_p}\right) 1_{\left\{T_{p-1}\le\tau<T_p\right\}} 1_{\left\{\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}} - \left(1-{}^B R_{T_p}\right) 1_{\left\{T_{p-1}\le{}^B\tau<T_p\right\}} 1_{\left\{{}^B\tilde{V}_{T_p}>\tilde{V}_{T_p-\vartheta}\right\}}\right) CF_k\right] \\
&= E\left[\sum_k \eta_{T_k} CF_k\right]
\end{aligned}$$

where $\eta_T \equiv \sum_{T_p < T_k} \left(1 - R_{T_p}\right) 1_{\left\{T_{p-1} \leq \tau < T_p\right\}} 1_{\left\{\tilde{V}_{T_p} > \tilde{V}_{T_p - \vartheta}\right\}} - \left(1 - {}^B R_{T_p}\right) 1_{\left\{T_{p-1} \leq {}^B \tau < T_p\right\}} 1_{\left\{{}^B \tilde{V}_{T_p} > \tilde{V}_{T_p - \vartheta}\right\}}$

We note that the fundamental result of the document remains: the CVA is the expected sum of the cash-flows in the netting set, discounted by an adapted process. However, the formula in the document is wrong: the credit components must be read on the secondary branch along with the proxies in indicators.

We thank Stephane Crepey for pointing the mistake. Any remaining error is our own responsibility.

Antoine Savine and Brian Huge, January 2019