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This edition first published 2006

Registered office
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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Library of Congress Cataloging-in-Publication Data

Names: Wystup, Uwe, author.
Title: FX options and structured products / Uwe Wystup.
Description: Second edition. | Chichester, West Sussex, United Kingdom : John Wiley & Sons, [2017] | Includes index. |
Identifiers: LCCN 2017015264 (print) | LCCN 2017023711 (ebook) | ISBN 9781118471111 (pdf) | ISBN 9781118471135 (epub) | ISBN 9781118471067 (cloth)
Subjects: LCSH: Foreign exchange options. | Structured notes (Securities) | Derivative securities.
Classification: LCC HG3853 (ebook) | LCC HG3853 .W88 2017 (print) | DDC 332.4/5—dc23
LC record available at https://lccn.loc.gov/2017015264

Cover Design: Wiley
Cover Images: Pen image: © archerix/iStockphoto;
Gears image: © sbayram/iStockphoto

Set in 10/12pt SabonLTStd by SPi Global, Chennai, India

Printed in Great Britain by TJ International Ltd, Padstow, Cornwall, UK

10 9 8 7 6 5 4 3 2 1

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Preface

SCOPE OF THIS BOOK

Treasury management of international corporates involves dealing with cash flows in different currencies. Therefore the natural service of an investment bank consists of a variety of money market and foreign exchange products. This book explains the most popular products and strategies with a focus on everything beyond vanilla options.

It explains all the FX derivatives including options, common structures and tailor-made solutions in examples, with a special focus on the application including views from traders and sales as well as from a corporate treasurer's perspective.

It contains actually traded deals with corresponding motivations explaining why the structures were traded. This way the reader gets a feeling for how to build new structures to suit clients' needs. We will also cover some examples of "bad deals," deals that traded and led to dramatic losses.

Several sections deal with some basic quantitative aspect of FX options, such as quanto adjustment, deferred delivery, vanna-volga pricing, settlement issues.

One entire chapter is devoted to hedge accounting, where after the foundations a typical structured FX forward is examined in a case study.

The exercises are meant to practice the material. Several of them are actually difficult to solve and can serve as incentives to further research and testing. Solutions to the exercises are not part of this book; however they may eventually be published on the book's web page, fxoptions.mathfinance.com.

Why I Decided to Write a Second Edition

There are numerous books on quantitative finance, and I am myself originally a quant. However, very few of these illustrate why certain products trade. There are also many books on options or derivatives in general. However, most of the options books are written in an equity options context. In my opinion, the key to really understanding options is the foreign exchange market. No other asset class makes the symmetries so obvious, and no other asset class has underlyings as liquid as the major currency pairs. With this book I am taking the effort to go beyond common literature on options, and also pure textbook material on options. Anybody can write a book on options after spending a few days on an internet search engine. Any student can learn about options doing the same thing (and save a lot on tuition at business schools). This book on FX options enables experts in the field to become more credible. My motivation to write this book was to share what I have learned in the many decades of dealing with FX derivatives in my various roles as a quant coding pricing libraries and handling market data, a structurer dealing with products from the trading and sales perspective, a risk manager running an options position, a consultant dealing with special topics in FX markets, an expert resolving legal conflicts in the area of derivatives, an adviser to the public sector and politicians on how to deal with currency risk, and last but not least as a trainer, teaching FX options to a second generation, during which time I have received so much valuable feedback that many sections of the first edition need to be updated or extended. Since the first edition, new products have been trading and new standards have been set. So it is about time. I really couldn't leave the first edition as it is. Moreover, many have asked me over the years to make solutions to the exercises available. This book now contains about 75 exercises, which I believe are very good practice material and support further learning and reflection, and all of the exercises come with solutions in a separate book. It is now possible for trainers to use this book for teaching and exam preparation. Supplementary material will be published on the web page of the book, fxoptions.mathfinance.com.

What is not Contained in this Book

This book is not a valuation of financial engineering from a programmer's or quant's point of view. I will explain the relevance and cover some basics on vanilla options. For the quantitative matters I refer to my book *Modeling Foreign Exchange Options* [142], which you may consider a second volume to this book. This does not mean that this book is not suitable for quants. On the contrary, for a quant (front-office or market risk) it may help to learn the trader's view, the buy-side view and get an overview of where all the programming may lead.

THE READERSHIP

A prerequisite for reading this book is some basic knowledge of FX markets – see, for example, *A Foreign Exchange Primer* by Shani Shamah [118]. For quantitative sections some knowledge of stochastic calculus is useful, as in Steven E. Shreve's volumes on *Stochastic Calculus for Finance* [120] are useful, but it is not essential for most of this book. The target readers are:

- Graduate students and faculty of financial engineering programs, who can use this book as a textbook for a course named *structured products* or *exotic currency options*.
- Traders, trainee structurers, product developers, sales and quants with interest in the FX product line. For them it can serve as a source of ideas as well as a reference guide.
- Treasurers of corporates interested in managing their books. With this book at hand they can structure their solutions themselves.

Those readers more interested in the quantitative and modeling aspects are recommended to read *Foreign Exchange Risk* [65]. This book explains several exotic FX options with a special focus on the underlying models and mathematics, but does not contain any structures or corporate clients' or investors' views.

About the Author

Uwe Wystup is the founder and managing director of MathFinance AG, a consulting and software company specializing in quantitative finance, implementation of derivatives models, valuation and validation services. Previously he was a financial engineer and structurer on the FX options trading desk at Commerzbank. Before that he worked for Deutsche Bank, Citibank, UBS and Sal. Oppenheim jr. & Cie. Uwe holds a PhD in mathematical finance from Carnegie Mellon University and is professor of financial option price modeling and foreign exchange derivatives at University of Antwerp and honorary professor of quantitative finance at Frankfurt School of Finance & Management, and lecturer at National University of Singapore. He has given several seminars on exotic options, numerical methods in finance and volatility modeling. His areas of specialization are the quantitative aspects and the design of structured products of foreign exchange markets. As well as co-authoring *Foreign Exchange Risk* (2002) he has written articles for journals including *Finance and Stochastics*, *Review of Derivatives Research*, *European Actuarial Journal*, *Journal of Risk*, *Quantitative Finance*, *Applied Mathematical Finance*, *Wilmott*, *Annals of Finance*, and the *Journal of Derivatives*. He also edited the section on foreign exchange derivatives in Wiley's *Encyclopedia of Quantitative Finance* (2010). Uwe has given many presentations at both universities and banks around the world. Further information and a detailed publication list are available at www.mathfinance.com.

Acknowledgments

I would like to thank Frankfurt School of Finance & Management for supporting the first edition of this book by allocating the necessary time.

I would like to thank my former colleagues on the trading floor, most of all Michael Braun, Jürgen Hakala, Tamás Korchmáros, Behnouch Mostachfi, Bereshad Nonas, Gustave Rieunier, Ingo Schneider, Jan Schrader, Noel Speake, Roman Stauss, Andreas Weber, and all my colleagues and co-authors, specially Christoph Becker, Susanne Griebsch, Christoph Kühn, Sebastian Krug, Marion Linck, Wolfgang Schmidt, and Robert Tompkins.

Special thanks to Tino Senge for his many talks on long dated FX, parts of which have become part of this book.

I would like to thank Steve Shreve for his training in stochastic calculus and for continuously supporting my academic activities.

Chris Swain, Rachael Wilkie, and many others at Wiley publications deserve respect as they were dealing with my rather slow speed in completing the first edition of this book. Similar respect applies to Werner Coetzee, Jennie Kitchin, Lori Laker, Thomas Hyrkiel, Jeremy Chia, Viv Church (the copyeditor), Abirami Srikandan (the production editor), and their colleagues for the second edition.

Many readers sent me valuable feedback, suggestions for improvement, error reports, and questions; they include but are not limited to Anupam Banerji, David Bannister, Lluis Blanc, Charles Brown, Harold Cataquet, Sven Foulon, Steffen Gregersen, Federico Han, Rupesh Mishra, Daniele Moroni, Allan Mortensen, Josua Müller, Alexander Stromilo, Yanhong Zhao. Thank you all.

Nicole van de Locht and Choon Peng Toh deserve a medal for serious detailed proof reading of the first edition, and similarly Lars Helfenstein, Archita Mishra, Armin Wendel, Miroslav Svoboda, and again Choon Peng Toh for the second edition.

CHAPTER 1

Foreign Exchange Derivatives

The FX derivatives market consists of FX swaps, FX forwards, FX or currency options, and other more general derivatives. FX structured products are either standardized or tailor-made linear combinations of simple FX derivatives including both vanilla and exotic options, or more general structured derivatives that cannot be decomposed into simple building blocks. The market for structured products is restricted to the market of the necessary ingredients. Hence, typically there are mostly structured products traded in the currency pairs that can be formed between USD, JPY, EUR, CHF, GBP, CAD and AUD. In this chapter we start with a brief history of options, followed by a technical section on vanilla options and volatility, and deal with commonly used linear combinations of vanilla options. Then we will illustrate the most important ingredients for FX structured products: the first and second generation exotics.

1.1 LITERATURE REVIEW

While there are tons of books on options and derivatives in general, very few are dedicated specifically to FX options. After the 2008 financial crisis, more such books appeared. Shamah [118] is a good source to learn about FX markets with a focus on market conventions, spot, forward, and swap contracts, and vanilla options. For pricing and modeling of exotic FX options I (obviously) suggest Hakala and Wystup's *Foreign Exchange Risk* [65] or its translation into Mandarin [68] as useful companions to this book. One of the first books dedicated to *Mathematical Models for Foreign Exchange* is by Lipton [92]. In 2010, Iain Clark published *Foreign Exchange Option Pricing* [28], and Antonio Castagna one on *FX Options and Smile Risk* [25], which both make a valuable contribution to the FX derivatives literature. A classic is Alan Hicks's *Managing Currency Risk Using Foreign Exchange Options* [76]. It provides a good overview of FX options mainly from the corporate's point of view. An introductory book on *Options on Foreign Exchange* is by DeRosa [38]. The *Handbook of Exchange Rates* [82] provides a comprehensive compilation of articles on the FX market structure, products, policies, and economic models.

1.2 A JOURNEY THROUGH THE HISTORY OF OPTIONS

The very first options and futures were traded in ancient Greece, when olives were sold before they had reached ripeness. Thereafter the market evolved in the following way.

FX Options and Structured Products, Uwe Wystup

16th century Ever since the 15th century, tulips, which were desired for their exotic appearance, were grown in Turkey. The head of the royal medical gardens in Vienna, Austria, was the first to cultivate those Turkish tulips successfully in Europe. When he fled to Holland because of religious persecution, he took the bulbs along. As the new head of the botanical gardens of Leiden, Netherlands, he cultivated several new strains. It was from these gardens that avaricious traders stole the bulbs to commercialize them, because tulips were a great status symbol.

17th century The first futures on tulips were traded in 1630. As of 1634, people could buy special tulip strains by the weight of their bulbs – the bulbs had the same value as gold. Along with the regular trading, speculators entered the market and the prices skyrocketed. A bulb of the strain, "Semper Octavian," was worth two wagonloads of wheat, four loads of rye, four fat oxen, eight fat swine, twelve fat sheep, two hogsheads of wine, four barrels of beer, two barrels of butter, 1,000 pounds of cheese, one marriage bed with linen, and one sizable wagon. People left their families, sold all their belongings, and even borrowed money to become tulip traders. When in 1637 this supposedly risk-free market crashed, traders as well as private individuals went bankrupt. The Dutch government prohibited speculative trading; the period became famous as Tulipmania.

18th century In 1728, the West India and Guinea Company, the monopolist in trading with the Caribbean Islands and the African coast, issued the first stock options. These were options on the purchase of the French island of Sainte-Croix, on which sugar plantings were planned. The project was realized in 1733 and paper stocks were issued in 1734. Along with the stock, people purchased a relative share of the island and the valuables, as well as the privileges and the rights of the company.

19th century In 1848, 82 businessmen founded the Chicago Board of Trade (CBOT). Today it is the biggest and oldest futures market in the entire world. Most written documents were lost in the great fire of 1871; however, it is commonly believed that the first standardized futures were traded as of 1860. CBOT now trades several futures and forwards, not only treasury bonds but also options and gold.

In 1870, the New York Cotton Exchange was founded. In 1880, the gold standard was introduced.

20th century

- In 1914, the gold standard was abandoned because of the First World War.
- In 1919, the Chicago Produce Exchange, in charge of trading agricultural products, was renamed the Chicago Mercantile Exchange. Today it is the most important futures market for the Eurodollar, foreign exchange, and livestock.
- In 1944, the Bretton Woods System was implemented in an attempt to stabilize the currency system.
- In 1970, the Bretton Woods System was abandoned for several reasons.

- In 1971, the Smithsonian Agreement on fixed exchange rates was introduced.
- In 1972, the International Monetary Market (IMM) traded futures on coins, currencies and precious metal.
- In 1973, the CBOE (Chicago Board of Exchange) firstly traded call options; four years later it added put options. The Smithsonian Agreement was abandoned; the currencies followed managed floating.
- In 1975, the CBOT sold the first interest rate future, the first future with no "real" underlying asset.
- In 1978, the Dutch stock market traded the first standardized financial derivatives.
- In 1979, the European Currency System was implemented, and the European Currency Unit (ECU) was introduced.
- In 1991, the Maastricht Treaty on a common currency and economic policy in Europe was signed.
- In 1999, the Euro was introduced, but the countries still used cash of their old currencies, while the exchange rates were kept fixed.

21st century In 2002, the Euro was introduced as new money in the form of cash.

FX forwards and options originate from the need of corporate treasury to hedge currency risk. This is the key to understanding FX options. Originally, FX options were not speculative products but hedging products. This is why they trade over the counter (OTC). They are tailored, i.e. cash flow matching currency risk hedging instruments for corporates. The way to think about an option is that a corporate treasurer in the EUR zone has income in USD and needs a hedge to sell the USD and to buy EUR for these USD. He would go long a forward or a EUR call option. At maturity he would exercise the option if it is in-the-money and receive EUR and pay USD. FX options are by default delivery settled. While FX derivatives were used later also as investment products or speculative instruments, the key to understanding FX options is corporate treasury.

1.3 CURRENCY OPTIONS

Let us start with a definition of a currency option:

Definition 1.3.1 *A* Currency Option Transaction *means a transaction entitling the Buyer, upon Exercise, to purchase from the Seller at the Strike Price a specified quantity of Call Currency and to sell to the Seller at the Strike Price a specified quantity of Put Currency.*

This is the definition taken from the *1998 FX and Currency Option Definitions* published by the International Swaps and Derivatives Association (ISDA) in 1998 [77]. This definition was the result of a process of standardization of currency options in the industry and is now widely accepted. Note that the key feature of an option is that the holder has a right to exercise. The definition also demonstrates clearly that calls and puts are equivalent, i.e. a call on one currency is always a put on the other currency. The definition is designed for a treasurer, where an actual cash flow of two currencies is triggered upon exercise. The definition also shows that the terms *derivative* and *option* are not synonyms. Derivative is a much wider term for financial transactions that depend on an underlying traded instrument. Derivatives include forwards, swaps, options, and exotic options. But not any derivative is also an option. For a currency option there is always a holder, the buyer after buying the option, equipped with the right to exercise, and upon exercise a cash flow of two pre-specified currencies is triggered. Anything outside this definition does not constitute a currency option. I highly recommend reading the 1998 ISDA definitions. The text uses legal language, but it does make all the terms around FX and currency options very clear and it is the benchmark in the industry. It covers only put and call options, options that are typically referred to as *vanilla* options, because they are the most common and simple products. The definition allows for different exercise styles: European for exercise permitted only at maturity, American for exercise permitted at any time between inception and maturity, as well as Bermudan for exercise permitted as finitely many pre-specified points in time. Usually, FX options are European. If you don't mention anything, they are understood to be of European exercise style. Features like cash settlement are possible: in this case one would have to make the call currency amount the net payoff and the put currency amount equal to zero. There are a number of exotic options, which we will cover later in this book, that still fit into this framework: in particular, barrier options. While they have special features not covered by the 1998 ISDA definitions, they still can be considered currency option transactions. However, variance swaps, volatility swaps, correlation swaps, combination of options, structured products, target forwards, just to mention a few obvious transactions, do not constitute currency option transactions.

1.4 TECHNICAL ISSUES FOR VANILLA OPTIONS

It is a standard in the FX options market to quote prices for FX options in terms of their implied volatility. The one-to-one correspondence between volatilities and options values rests on the convex payoff function of both call and put options. The conversion firmly rests on the Black-Scholes model. It is well known in the financial industry and academia that the Black-Scholes model has many weaknesses in modeling the underlying market properly. Strictly speaking, it is inappropriate. And there are in fact many other models, such as local volatility or stochastic volatility models or their hybrids, which reflect the dynamics much better than the Black-Scholes model. Nevertheless, as a basic tool to convert volatilities into values and values into volatilities, it is the market standard for dealers, brokers, and basically all risk management systems. This means: good news for those who have already learned it – it was not a waste of time and effort – and bad news for the quant-averse – you need to deal with it to a certain extent, as otherwise the FX volatility surface and the FX smile construction will not be accessible to you. Therefore, I do want to get the basic math done, even in this book, which I don't intend to be a quant book. However, I don't want to scare away much of my potential readership. If you don't like the math, you can still read most of this book.

We consider the model *geometric Brownian motion*

$$dS_t = (r_d - r_f)S_t\, dt + \sigma S_t\, dW_t \tag{1}$$

for the underlying exchange rate quoted in FOR-DOM (foreign-domestic), which means that one unit of the foreign currency costs FOR-DOM units of the domestic currency. In the case of EUR-USD with a spot of 1.2000, this means that the price of one EUR is 1.2000 USD. The notion of *foreign* and *domestic* does not refer to the location of the trading entity but only to this quotation convention. There are other terms used for FOR, which are *underlying*, *CCY1*, *base*; there are also other terms for DOM, which are *base*, *CCY2*, *counter* or *term*, respectively. For the quants, DOM is also considered the *numeraire* currency. I leave it to you to decide which one you wish to use. I find "base" a bit confusing, because it refers sometimes to FOR and sometimes to DOM. I also find "CCY1" and "CCY2" not very conclusive. The term "numeraire" does not have an established counterpart for FOR. So I prefer FOR and DOM. You may also stick to the most liquid currency pair EUR/USD, and think of FOR as EUR and DOM as USD.

We denote the (continuous) foreign interest rate by r_f and the (continuous) domestic interest rate by r_d. In an equity scenario, r_f would represent a continuous dividend rate. Note that r_f is *not* the interest rate that is typically used to discount cash flows in foreign currency, but is the (artificial) foreign interest rate that ensures that the forward price calculated in Equation (9) matches the market forward price. The volatility is denoted by σ, and W_t is a standard Brownian motion. The sample paths are displayed in Figure 1.1. We consider this standard model not because it reflects the statistical properties of the exchange rate (in fact, it doesn't) but because it is widely used in practice and front-office systems and mainly serves as a tool to

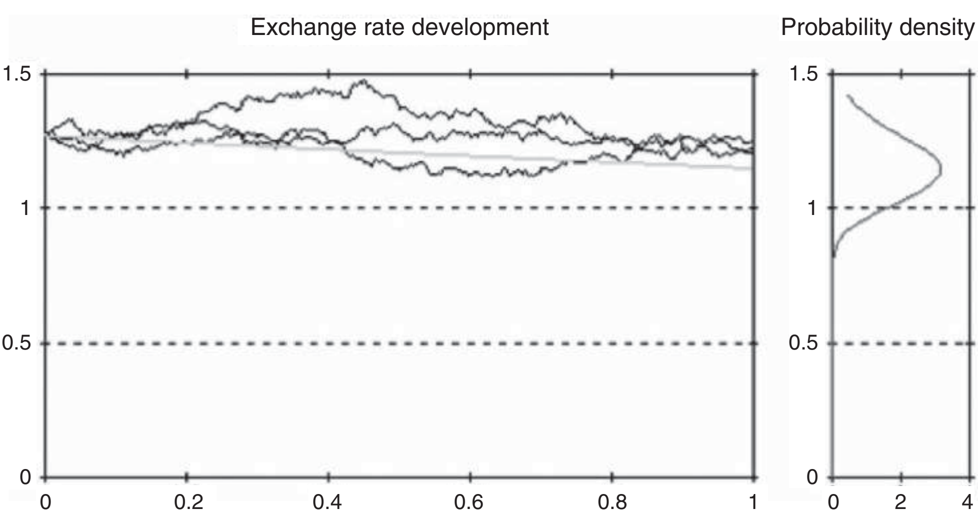


FIGURE 1.1 Simulated paths of a geometric Brownian motion. The distribution of the spot S_T at time T is log-normal. The light gray line reflects the average spot movement.

communicate prices of vanilla call and put options and switch between quotations in price and in terms of implied volatility. Currency option prices are commonly quoted in terms of volatility in the sense of this model. Model (1) is sometimes referred to as the Garman-Kohlhagen model [54]. However, all that happened there was adding the foreign interest rate r_f to the Black-Scholes model [15]. For this reason Model (1) is generally and in this book referred to as the Black-Scholes model.

Applying Itô's rule to $\ln S_t$ yields the following solution for the process S_t

$$S_t = S_0 \exp\left\{\left(r_d - r_f - \tfrac{1}{2}\sigma^2\right)t + \sigma W_t\right\}, \tag{2}$$

which shows that S_t is log-normally distributed, more precisely, $\ln S_t$ is normal with mean $\ln S_0 + (r_d - r_f - \frac{1}{2}\sigma^2)t$ and variance $\sigma^2 t$. Further model assumptions are:

1. There is no arbitrage.
2. Trading is frictionless, no transaction costs.
3. Any position can be taken at any time, short, long, arbitrary fraction, no liquidity constraints.

The payoff for a vanilla option (European put or call) is given by

$$F = [\phi(S_T - K)]^+, \tag{3}$$

where the contractual parameters are the strike K, the expiration time T and the type ϕ, a binary variable which takes the value $+1$ in the case of a call and -1 in the case of a put. The symbol x^+ denotes the positive part of x, i.e., $x^+ \stackrel{\Delta}{=} \max(0, x) \stackrel{\Delta}{=} 0 \vee x$. We generally use the symbol $\stackrel{\Delta}{=}$ to *define* a quantity. Most commonly, vanilla options on foreign exchange are of *European style*, i.e. the holder can only exercise the option at time T. *American style options*, where the holder can exercise any time, or *Bermudan style options*, where the holder can exercise at selected times, are not used very often except for *time options*, see Section 2.1.19.

1.4.1 Valuation in the Black-Scholes Model

In the Black-Scholes model the value of the payoff F at time t if the spot is at x is denoted by $v(t, x)$ and can be computed either as the solution of the *Black-Scholes partial differential equation* (see [15])

$$v_t - r_d v + (r_d - r_f)x v_x + \tfrac{1}{2}\sigma^2 x^2 v_{xx} = 0, \tag{4}$$

$$v(T, x) = F \tag{5}$$

or equivalently (*Feynman-Kac Theorem*) as the discounted expected value of the payoff-function

$$v(x, K, T, t, \sigma, r_d, r_f, \phi) = e^{-r_d \tau} I\!E[F]. \tag{6}$$

This is the reason why basic financial engineering is mostly concerned with solving partial differential equations or computing expectations (numerical integration). The result is the *Black-Scholes formula*

$$v(x, K, T, t, \sigma, r_d, r_f, \phi) = \phi e^{-r_d \tau}[f\mathcal{N}(\phi d_+) - K\mathcal{N}(\phi d_-)]. \tag{7}$$

The result of this formula is the value of a vanilla option in USD for one unit of EUR nominal. We abbreviate

$$x : \text{ current price of the underlying,}$$

$$\tau \stackrel{\Delta}{=} T - t\text{: time to maturity,} \tag{8}$$

$$f \stackrel{\Delta}{=} \mathbb{E}[S_T | S_t = x] = xe^{(r_d - r_f)\tau}\text{: forward price of the underlying,} \tag{9}$$

$$\theta_\pm \stackrel{\Delta}{=} \frac{r_d - r_f}{\sigma} \pm \frac{\sigma}{2}, \tag{10}$$

$$d_\pm \stackrel{\Delta}{=} \frac{\ln \frac{x}{K} + \sigma\theta_\pm \tau}{\sigma\sqrt{\tau}} = \frac{\ln \frac{f}{K} \pm \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}, \tag{11}$$

$$n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} = n(-t) \text{ normal density,} \tag{12}$$

$$\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt = 1 - \mathcal{N}(-x) \text{ normal distribution function.} \tag{13}$$

We observe that some authors use d_1 for d_+ and d_2 for d_-, which requires extra memory and completely ruins the beautiful symmetry of the formula.

The Black-Scholes formula can be derived using the integral representation of Equation (6)

$$\begin{aligned} v &= e^{-r_d \tau}\mathbb{E}[F] \\ &= e^{-r_d \tau}\mathbb{E}[[\phi(S_T - K)]^+] \\ &= e^{-r_d \tau} \int_{-\infty}^{+\infty} \left[\phi\left(xe^{(r_d - r_f - \frac{1}{2}\sigma^2)\tau + \sigma\sqrt{\tau}y} - K\right)\right]^+ n(y)\, dy. \end{aligned} \tag{14}$$

Next one has to deal with the positive part and then complete the square to get the Black-Scholes formula. A derivation based on the partial differential equation can be done using results about the well-studied *heat equation*. For valuation of options it is very important to ensure that the interest rates are chosen such that the forward price (9) matches the market, as otherwise the options may not satisfy the put-call parity (41).

1.4.2 A Note on the Forward

The *forward price* f is the pre-agreed exchange rate which makes the time zero value of the *forward contract* with payoff

$$F = S_T - f \tag{15}$$

equal to zero. It follows that $f = \mathbb{E}[S_T] = xe^{(r_d - r_f)T}$, i.e. the forward price is the expected price of the underlying at time T in a risk-neutral measure (drift of the geometric Brownian motion is equal to cost of carry $r_d - r_f$). The situation $r_d > r_f$ is called *contango*, and the situation $r_d < r_f$ is called *backwardation*. Note that in the Black-Scholes model the class of forward price curves is quite restricted. For example, no seasonal effects can be included. Note that the post-trade value of the forward contract after time zero is usually different from zero, and since one of the counterparties is always short, there may be settlement risk of the short party. A *futures contract* prevents this dangerous affair: it is basically a forward contract, but the counterparties have to maintain a *margin account* to ensure the amount of cash or commodity owed does not exceed a specified limit.

1.4.3 Vanilla Greeks in the Black-Scholes Model

Greeks are derivatives of the value function with respect to model and contract parameters. They are important information for traders and have become standard information provided by front-office systems. More details on Greeks and the relations among Greeks are presented in Hakala and Wystup [65] or Reiss and Wystup [107]. Initially there was a desire to use Greek letters for all these mathematical derivatives. However, it turned out that since the early days of risk management many higher order Greeks have been added whose terms no longer reflect Greek letters. Even vega is not a Greek letter but we needed a Greek sounding term that starts with a "v" to reflect volatility and Greek doesn't have such a letter. For vanilla options we list some of them now.

(Spot) Delta.

$$\frac{\partial v}{\partial x} = \phi e^{-r_f \tau} \mathcal{N}(\phi d_+) \tag{16}$$

This spot delta ranges between 0% and a *discounted* ±100%. The interpretation of this quantity is the amount of FOR the trader needs to buy to delta hedge a short option. So for instance, if you sell a call on 1 M EUR, that has a 25% delta, you need to buy 250,000 EUR to delta hedge the option. The corresponding forward delta ranges between 0% and ±100% and is symmetric in the sense that a 60-delta call is a 40-delta put, a 75-delta put is a 25-delta call, etc. I had wrongly called it "driftless delta" in the first edition of this book.

Forward Delta.

$$\phi \mathcal{N}(\phi d_+) \tag{17}$$

The interpretation of forward delta is the number of units of FOR of forward contracts a trader needs to buy to delta hedge a short option. See Section 1.4.7 for a justification.

Future Delta.

$$\phi e^{-r_d \tau} \mathcal{N}(\phi d_+) \tag{18}$$

Gamma.

$$\frac{\partial^2 v}{\partial x^2} = e^{-r_f \tau} \frac{n(d_+)}{x\sigma\sqrt{\tau}} \tag{19}$$

The interpretation of gamma is the change of delta as spot changes. A high gamma means that the delta hedge must be adapted very frequently and will hence cause transaction costs. Gamma is typically high when the spot is near a strike of a barrier, generally whenever the payoff has a kink or more dramatically a jump. Trading systems usually quote gamma as a *traders' gamma*, using a 1% *relative* change in the spot price. For example, if gamma is quoted as 10,000 EUR, then delta will increase by that amount if the spot rises from 1.3000 to 1.3130 = 1.3000 · (1 + 1%). This can be approximated by $\frac{\partial^2 v}{\partial x^2} \cdot \frac{x}{100}$.

Speed.

$$\frac{\partial^3 v}{\partial x^3} = -e^{-r_f \tau} \frac{n(d_+)}{x^2\sigma\sqrt{\tau}} \left(\frac{d_+}{\sigma\sqrt{\tau}} + 1 \right) \tag{20}$$

The interpretation of speed is the change of gamma as spot changes.

Theta.

$$\frac{\partial v}{\partial t} = -e^{-r_f \tau} \frac{n(d_+)x\sigma}{2\sqrt{\tau}} + \phi[r_f x e^{-r_f \tau} \mathcal{N}(\phi d_+) - r_d K e^{-r_d \tau} \mathcal{N}(\phi d_-)] \tag{21}$$

Theta reflects the change of the option value as the clock ticks. The *traders' theta* that you spot in a risk management system usually refers to a change of the option value in one day, i.e. the traders' theta can be approximated by $365\frac{\partial v}{\partial t}$.

Charm.

$$\frac{\partial^2 v}{\partial x \partial \tau} = -\phi r_f e^{-r_f \tau} \mathcal{N}(\phi d_+) + \phi e^{-r_f \tau} n(d_+) \frac{2(r_d - r_f)\tau - d_- \sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}} \tag{22}$$

Color.

$$\frac{\partial^3 v}{\partial x^2 \partial \tau} = -e^{-r_f \tau} \frac{n(d_+)}{2x\tau\sigma\sqrt{\tau}} \left[2r_f \tau + 1 + \frac{2(r_d - r_f)\tau - d_- \sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}} d_+ \right] \tag{23}$$

Vega.

$$\frac{\partial v}{\partial \sigma} = xe^{-r_f \tau} \sqrt{\tau} n(d_+) \tag{24}$$

Trading and risk management systems usually quote vega as a *traders' vega*, using a 1% *absolute* change in the volatility. For example, if vega is quoted as 4,000 EUR, then the option value will increase by that amount if the volatility rises from 10% to 11% = 10% + 1%. This can be approximated by $\frac{\partial v}{\partial \sigma} \cdot 100$.

Volga.

$$\frac{\partial^2 v}{\partial \sigma^2} = xe^{-r_f \tau} \sqrt{\tau} n(d_+) \frac{d_+ d_-}{\sigma} \tag{25}$$

Volga is also sometimes called *vomma* or *volgamma* or *dvega/dvol*. Volga reflects the change of vega as volatility changes. Traders' volga assumes again a 1% absolute change in volatility.

Vanna.

$$\frac{\partial^2 v}{\partial \sigma \partial x} = -e^{-r_f \tau} n(d_+) \frac{d_-}{\sigma} \tag{26}$$

Vanna is also sometimes called *dvega/dspot*. It reflects the change of vega as spot changes. Traders' vanna assumes again a 1% relative change in spot. The origin of the term vanna is not clear. I suspect it goes back to an article in *Risk* magazine by Tim Owens in the 1990s, where he asked "Wanna lose a lot of money?" and then explained how a loss may occur if second order Greeks such as vanna and volga are not hedged.

Volunga.

$$\frac{\partial^3 v}{\partial \sigma^3} = \frac{\text{vega}}{\sigma^2} \left((d_+ d_-)^2 - d_+^2 - d_+ d_- - d_-^2 \right) \tag{27}$$

This is actually not a joke.

Vanunga.

$$\frac{\partial^3 v}{\partial x \partial \sigma^2} = \frac{\text{vega}}{\sigma^2 x \sqrt{\tau}} \left(d_+ + d_+ d_- - d_+ d_-^2 \right) \tag{28}$$

This one isn't a joke either.

Rho.

$$\frac{\partial v}{\partial r_d} = \phi K \tau e^{-r_d \tau} \mathcal{N}(\phi d_-) \tag{29}$$

$$\frac{\partial v}{\partial r_f} = -\phi x \tau e^{-r_f \tau} \mathcal{N}(\phi d_+) \tag{30}$$

Trading and risk management systems usually quote rho as a *traders' rho*, using a 1% *absolute* change in the interest rate. For example, if rho is quoted as 4,000 EUR, then the option value will increase by that amount if the interest rate rises from 2% to 3% = 2% + 1%. This can be approximated by $\frac{\partial v}{\partial \rho} \cdot 100$. **Warning:** FX options always involve two currencies. Therefore, there will be two interest rates, a domestic interest rate r_d, and a foreign interest rate r_f. The value of the option can be represented in both DOM and FOR units. This means that you can have a change of the option value in FOR as the FOR rate changes, a change of the value of the option in FOR as the DOM rate changes, a change of the value of the option in DOM as the FOR rate changes, and a change of the value of the option in DOM as the DOM rate changes. Some systems add to the confusion as they list one rho, which refers to the change of the option value as the *difference of the interest rates* changes, and again possibly in both DOM and FOR terms.

Dual Delta.

$$\frac{\partial v}{\partial K} = -\phi e^{-r_d \tau} \mathcal{N}(\phi d_-) \tag{31}$$

The non-discounted version of the dual delta, also referred to as the forward dual delta, also represents the risk-neutral exercise probability of the option.

Dual Gamma.

$$\frac{\partial^2 v}{\partial K^2} = e^{-r_d \tau} \frac{n(d_-)}{K\sigma\sqrt{\tau}} \tag{32}$$

Dual Theta.

$$\frac{\partial v}{\partial T} = -v_t \tag{33}$$

Dual Greeks refer to changes of the option value as contractual parameters change. This has no application in market risk management, because the contractual parameters are fixed between counterparts and cannot be changed on the way. However, the dual Greeks contribute a lot to understanding of derivatives. The dual gamma (on the strike space) for example – up to a discount factor – is identical to the probability density of the underlying exchange rate.

1.4.4 Reoccurring Identities

$$\frac{\partial d_\pm}{\partial \sigma} = -\frac{d_\mp}{\sigma} \tag{34}$$

$$\frac{\partial d_\pm}{\partial r_d} = \frac{\sqrt{\tau}}{\sigma} \tag{35}$$

$$\frac{\partial d_\pm}{\partial r_f} = -\frac{\sqrt{\tau}}{\sigma} \tag{36}$$

$$xe^{-r_f\tau}n(d_+) = Ke^{-r_d\tau}n(d_-) \tag{37}$$

$$\mathcal{N}(\phi d_-) = I\!P[\phi S_T \geq \phi K] \tag{38}$$

$$\mathcal{N}(\phi d_+) = I\!P\left[\phi S_T \leq \phi\frac{f^2}{K}\right] \tag{39}$$

The *put-call parity* is a way to express the trivial equation $x = x^+ - x^-$ in financial terms and is the relationship on the payoff level

$$\text{call} - \text{put} = \text{forward}$$
$$(S_T - K)^+ - (K - S_T)^+ = S_T - K, \tag{40}$$

which translates to the value functions of these products via

$$v(x, K, T, t, \sigma, r_d, r_f, +1) - v(x, K, T, t, \sigma, r_d, r_f, -1) = xe^{-r_f\tau} - Ke^{-r_d\tau}. \tag{41}$$

A forward contract that is constructed using a long call and a short put option is called a *synthetic forward*.

The *put-call delta parity* is

$$\frac{\partial v(x, K, T, t, \sigma, r_d, r_f, +1)}{\partial x} - \frac{\partial v(x, K, T, t, \sigma, r_d, r_f, -1)}{\partial x} = e^{-r_f\tau}. \tag{42}$$

In particular, we learn that the absolute values of a spot put delta and a spot call delta are not exactly adding up to 100%, but only to a positive number $e^{-r_f\tau}$. They add up to one approximately if either the time to expiration τ is short or if the foreign interest rate r_f is close to zero. The corresponding forward deltas do add up to 100%.

Whereas the choice $K = f$ produces identical values for call and put, we seek the *delta-symmetric strike* or *delta-neutral strike* K_+ which produces absolutely identical deltas (spot, forward or future). This condition implies $d_+ = 0$ and thus

$$K_+ = fe^{+\frac{\sigma^2}{2}\tau}, \tag{43}$$

in which case the absolute spot delta is $e^{-r_f\tau}/2$. In particular, we learn that always $K_+ > f$, i.e., there can't be a put and a call with identical values *and* deltas. Note that the strike K_+ is usually chosen as the middle strike when trading a straddle or a butterfly. Similarly the dual-delta-symmetric strike $K_- = fe^{-\frac{\sigma^2}{2}T}$ can be derived from the condition $d_- = 0$. Note that the delta-symmetric strike K_+ also maximizes gamma and vega of a vanilla option and is thus often considered a center of symmetry.

1.4.5 Homogeneity based Relationships

We may wish to measure the value of the underlying in a different unit. This will obviously affect the option pricing formula as follows:

$$av(x,K,T,t,\sigma,r_d,r_f,\phi) = v(ax,aK,T,t,\sigma,r_d,r_f,\phi) \text{ for all } a > 0. \tag{44}$$

Differentiating both sides with respect to a and then setting $a = 1$ yields

$$v = xv_x + Kv_K. \tag{45}$$

Comparing the coefficients of x and K in Equations (7) and (45) leads to suggestive results for the delta v_x and dual delta v_K. This *space-homogeneity* is the reason behind the simplicity of the delta formulas, whose tedious computation can be saved this way.

Time Homogeneity We can perform a similar computation for the time-affected parameters and obtain the obvious equation

$$v(x,K,T,t,\sigma,r_d,r_f,\phi) = v\left(x,K,\frac{T}{a},\frac{t}{a},\sqrt{a}\sigma,ar_d,ar_f,\phi\right) \text{ for all } a > 0. \tag{46}$$

Differentiating both sides with respect to a and then setting $a = 1$ yields

$$0 = \tau v_t + \tfrac{1}{2}\sigma v_\sigma + r_d v_{r_d} + r_f v_{r_f}. \tag{47}$$

Of course, this can also be verified by direct computation. The overall use of such equations is to generate double checking benchmarks when computing Greeks. These homogeneity methods can easily be extended to other more complex options.

Put-Call Symmetry By *put-call symmetry* we understand the relationship (see [9, 10, 19] and [24])

$$v(x,K,T,t,\sigma,r_d,r_f,+1) = \frac{K}{f}v\left(x,\frac{f^2}{K},T,t,\sigma,r_d,r_f,-1\right). \tag{48}$$

The strike of the put and the strike of the call result in a geometric mean equal to the forward f. The forward can be interpreted as a *geometric mirror* reflecting a call into a certain number of puts. Note that for at-the-money options ($K = f$) the put-call symmetry coincides with the special case of the put-call parity where the call and the put have the same value.

Rates Symmetry Direct computation shows that the *rates symmetry*

$$\frac{\partial v}{\partial r_d} + \frac{\partial v}{\partial r_f} = -\tau v \tag{49}$$

holds for vanilla options. In fact, this relationship holds for all European options and a wide class of path-dependent options as shown in [107].

Foreign-Domestic Symmetry One can directly verify the *foreign-domestic symmetry* as relationship

$$\frac{1}{x}v(x, K, T, t, \sigma, r_d, r_f, \phi) = Kv\left(\frac{1}{x}, \frac{1}{K}, T, t, \sigma, r_f, r_d, -\phi\right). \tag{50}$$

This equality can be viewed as one of the faces of put-call symmetry. The reason is that the value of an option can be computed in units of domestic currency as well as in units of foreign currency. We consider the example of S_t modeling the exchange rate of EUR/USD. In New York, the call option $(S_T - K)^+$ costs $v(x, K, T, t, \sigma, r_{usd}, r_{eur}, 1)$ USD and hence $v(x, K, T, t, \sigma, r_{usd}, r_{eur}, 1)/x$ EUR. This EUR-call option can also be viewed as a USD-put option with payoff $K\left(\frac{1}{K} - \frac{1}{S_T}\right)^+$. This option costs $Kv(\frac{1}{x}, \frac{1}{K}, T, t, \sigma, r_{eur}, r_{usd}, -1)$ EUR in Frankfurt, because S_t and $\frac{1}{S_t}$ have the same volatility. Of course, the New York value and the Frankfurt value must agree, which leads to (50). We will also learn later that this symmetry is just one possible result based on *change of numeraire*.

1.4.6 Quotation Conventions

Quotation of the Underlying Exchange Rate Equation (1) is a model for the exchange rate. The quotation is a permanently confusing issue, so let us clarify this here. The exchange rate means how many units of the *domestic* currency are needed to buy one unit of *foreign* currency. For example, if we take EUR/USD as an exchange rate, then the default quotation is EUR-USD, where USD is the domestic currency and EUR is the foreign currency. The term *domestic* is in no way related to the location of the trader or any country. It merely means the *numeraire* currency. The terms *domestic*, *numeraire*, *currency two* or *base currency* are synonyms, as are *foreign*, *currency one* and *underlying*. Some market participants even refer to the foreign currency as the base currency, one of the reasons why I prefer to avoid the term base currency altogether. Throughout this book we denote with the slash (/) the currency pair and with a dash (-) the quotation. The slash (/) does *not* mean a division. For instance, EUR/USD can also be quoted in either EUR-USD, which then means how many USD are needed to buy one EUR, or in USD-EUR, which then means how many EUR are needed to buy one USD. There are certain market standard quotations listed in Table 1.1.

Trading Floor Language We call one million a *buck*, one billion a *yard*. This is because a billion is called “milliarde” in French, German and other languages. For the British pound one million is also often called a *quid*.

TABLE 1.1 Standard market quotation of major currency pairs with sample spot prices.

Currency pair	Default quotation	Sample quote
GBP/USD	GBP-USD	1.6000
GBP/CHF	GBP-CHF	2.2500
EUR/USD	EUR-USD	1.3000
EUR/GBP	EUR-GBP	0.8000
EUR/JPY	EUR-JPY	135.00
EUR/CHF	EUR-CHF	1.2000
USD/JPY	USD-JPY	108.00
USD/CHF	USD-CHF	1.0100

Certain currencies also have names, e.g. the New Zealand dollar NZD is called a *Kiwi*, the Australian dollar AUD is called *Aussie*, the Canadian dollar CAD is called *Loonie*, the Scandinavian currencies DKK, NOK (*Nokkies*) and SEK (*Stockies*) are collectively called *Scandies*.

Exchange rates are generally quoted up to five relevant figures, e.g. in EUR-USD we could observe a quote of 1.2375. The last digit "5" is called the *pip*, the middle digit "3" is called the *big figure*, as exchange rates are often displayed in trading floors and the big figure, which is displayed in bigger size, is the most relevant information. The digits left of the big figure are known anyway. If a trader doesn't know these when getting to the office in the morning, he may most likely not have the right job. The pips right of the big figure are often negligible for general market participants of other asset classes and are highly relevant only for currency spot traders. To make it clear, a rise of USD-JPY 108.25 by 20 pips will be 108.45 and a rise by 2 big figures will be 110.25.

Cable Currency pairs are often referred to by nicknames. The price of one pound sterling in US dollars, denoted by GBP/USD, is known by traders as the *cable*, which originates from the time when a communications cable under the Atlantic Ocean synchronized the GBP/USD quote between the London and New York markets. So where is the cable?

I stumbled upon a small town called Porthcurno near Land's End on the southwestern Cornish coast and by mere accident spotted a small hut called the "cable house" admittedly a strange object to find on a beautiful sandy beach. Trying to find Cornish cream tea I ended up at a telegraphic museum, which had all I ever wanted to know about the cable (see the photographs in Figure 1.2). Telegraphic news transmission was introduced in 1837, typically along the railway lines. Iron was rapidly replaced by copper. A new insulating material, gutta-percha, which is similar to rubber, allowed cables to function under the sea, and as Britain neared the height of its international power, submarine cables started to be laid, gradually creating a global network of cables, which included the first long-term successful trans-Atlantic cable of 1865 laid by the Great Eastern ship.

FIGURE 1.2 The Cable at Porthcurno, in the telegraphic museum and on the beach near the cable house.

The entrepreneur of that age was John Pender, founder of the Eastern Telegraph Company. He had started as a cotton trader and needed to communicate quickly with various ends of the world. In the 1860s telegraphic messaging was the new and only way to do this. Pender quickly discovered the value of fast communication. In the 1870s, an annual traffic of around 200,000 words went through Porthcurno. By 1900, cables connected Porthcurno with India (via Gibraltar and Malta), Australia and New Zealand. The cable network charts of the late 1800s reflect the financial trading centers of today very closely: Tokyo, Sydney, Singapore, Mumbai, London, New York.

Fast communication is ever so important for the financial industry. You can still go to Porthcurno and touch the cables. They have been in the sea for more than 100 years, but they still work. However, they have been replaced by fiber glass cables, and communications have been extended by radio and satellites. Algorithmic trading relies on getting all the market information within milliseconds.

The word "cable" itself is still used as the GBP/USD rate, reflecting the importance of fast information.

Crosses Currency pairs not involving the USD such as EUR/JPY are called a *cross* because it is the cross rate of the more liquidly traded USD/JPY and EUR/USD. If the cross is illiquid, such as ILS/MYR, it is called an illiquid cross. Spot transactions would then happen in two steps via USD. Options on an illiquid cross are rare or traded at very high bid-offer spreads.

Quotation of Option Prices Values and prices of vanilla options may be quoted in the six ways explained in Table 1.2.

TABLE 1.2 Standard market quotation types for option values. In the example we take FOR = EUR, DOM = USD, $S_0 = 1.2000$, $r_d = 3.0\%$, $r_f = 2.5\%$, $\sigma = 10\%$, $K = 1.2500$, $T = 1$ year, $\phi = +1$ (call), notional = 1,000,000 EUR = 1,250,000 USD. For the pips, the quotation 291.48 USD pips per EUR is also sometimes stated as 2.9148% USD per 1 EUR. Similarly, the 194.32 EUR pips per USD can also be quoted as 1.9432% EUR per 1 USD.

Name	Symbol	Value in units of	Example
domestic cash	**d**	DOM	29,148 USD
foreign cash	**f**	FOR	24,290 EUR
% domestic	**% d**	DOM per unit of DOM	2.3318% USD
% foreign	**% f**	FOR per unit of FOR	2.4290% EUR
domestic pips	**d pips**	DOM per unit of FOR	291.48 USD pips per EUR
foreign pips	**f pips**	FOR per unit of DOM	194.32 EUR pips per USD

The Black-Scholes formula quotes **d pips**. The others can be computed using the following instruction.

$$\textbf{d pips} \overset{\times\frac{1}{S_0}}{\longrightarrow} \%f \overset{\times\frac{S_0}{K}}{\longrightarrow} \%d \overset{\times\frac{1}{S_0}}{\longrightarrow} \textbf{f pips} \overset{\times S_0 K}{\longrightarrow} \textbf{d pips} \tag{51}$$

Delta and Premium Convention The spot delta of a European option assuming the premium is paid in DOM is well known. It will be called *raw spot delta* δ_{raw} now. It can be quoted in either of the two currencies involved. The relationship is

$$\delta_{raw}^{reverse} = -\delta_{raw}\frac{S}{K}. \tag{52}$$

The delta is used to buy or sell spot in the corresponding amount in order to hedge the option up to first order. The raw spot delta, multiplied by the FOR nominal amount, represents the amount of FOR currency the trader needs to buy in order to delta hedge a short option. How do we get to the reverse delta? It rests firmly on the symmetry of currency options. A FOR call is a DOM put. Hence, buying FOR amount in the delta hedge is equivalent to selling DOM amount multiplied by the spot S. The negative sign reflects the change from buying to selling. This explains the negative sign and the spot factor. A right to buy 1 FOR (and pay for this K DOM) is equivalent to the right to sell K DOM and receive for that 1 DOM. Therefore, viewing the FOR call as a DOM put and applying the delta hedge to one unit of DOM (instead of K units of DOM) requires a division by K. Now read this paragraph again and again and again, until it clicks. Sorry.

For consistency the premium needs to be incorporated into the delta hedge, since a premium in foreign currency will already hedge part of the option's delta risk. In a stock options context such a question never comes up, as an option on a stock is always paid in cash, rather than paid in shares of stock. In foreign exchange, both currencies are cash, and it is perfectly reasonable to pay for a currency option in either DOM or FOR currency. To make this clear, let us consider EUR-USD. In any financial markets model, $v(x)$ denotes the value or premium in USD of an option with 1 EUR notional, if the spot is at x, and the raw delta v_x denotes the number of EUR to buy to delta hedge a short position of this option. If this raw delta is negative, then EUR have to be sold (silly but hopefully helpful remark for the non-math freak). Therefore, xv_x is the number of USD to sell. If now the premium is paid in EUR rather than in USD, then we already have $\frac{v}{x}$ EUR, and the number of EUR to buy has to be reduced by this amount, i.e. if EUR is the premium currency, we need to buy $v_x - \frac{v}{x}$ EUR for the delta hedge or equivalently sell $xv_x - v$ USD. This is called a *premium-adjusted delta* or delta with premium included.

The same result can be derived by looking at the risk management of a portfolio whose accounting currency is EUR and risky currency is USD. In this case spot is $\frac{1}{x}$ rather than x. The value of the option – or in fact more generally of a portfolio of derivatives – is then $v\left(\frac{1}{x}\right)$ in USD, and $v\left(\frac{1}{x}\right)\frac{1}{x}$ in EUR, and the change of the portfolio value in EUR as the price of the USD measured in EUR is

$$\begin{aligned}
\frac{\partial}{\partial \frac{1}{x}} \frac{v\left(\frac{1}{x}\right)}{x} &= \frac{\partial}{\partial x} \frac{v\left(\frac{1}{x}\right)}{x} \frac{\partial x}{\partial \frac{1}{x}} \\
&= \frac{v_x\left(\frac{1}{x}\right) x - v\left(\frac{1}{x}\right)}{x^2} \left(\frac{\partial \frac{1}{x}}{\partial x}\right)^{-1} \\
&= \frac{xv_x - v}{x^2}\left(-\frac{1}{x^2}\right)^{-1} \\
&= -[xv_x - v]. \qquad (53)
\end{aligned}$$

We observe that both the trader's approach deriving delta from the premium and the risk manager's approach deriving delta from the portfolio risk arrive at the same number. Not really a surprise, is it?

The premium-adjusted delta for a vanilla option in the Black-Scholes model becomes

$$\begin{aligned}
-[xv_x - v] &= -\left[\phi x e^{-r_f \tau} \mathcal{N}(\phi d_+) - \phi[x e^{-r_f \tau} \mathcal{N}(\phi d_+) - K e^{-r_d \tau} \mathcal{N}(\phi d_-)]\right] \\
&= -\phi K e^{-r_d \tau} \mathcal{N}(\phi d_-) \qquad (54)
\end{aligned}$$

in USD, or $-\phi e^{-r_d \tau} \frac{K}{x} \mathcal{N}(\phi d_-)$ in EUR. If we sell USD instead of buying EUR, and if we assume a notional of 1 USD rather than 1 EUR ($= K$ USD) for the option, the premium-adjusted delta becomes just

$$\phi e^{-r_d \tau} \mathcal{N}(\phi d_-). \qquad (55)$$

If you ever wondered why delta uses $\mathcal{N}(d_+)$ and not $\mathcal{N}(d_-)$, which is really not fair, you now have an answer: both these terms are deltas, and only the FX market can really explain what's going on:

- $\phi e^{-r_f \tau} \mathcal{N}(\phi d_+)$ is the delta if the premium is paid in USD,
- $\phi e^{-r_d \tau} \mathcal{N}(\phi d_-)$ is the delta if the premium is paid in EUR.

In FX options markets there is no preference between the two, as a premium can always (well, normally always) be paid in either currency. The premium-adjusted delta is therefore also related to the dual delta (31).

Default Premium Currency Quotations in FX require some patience because we need to first sort out which currency is domestic, which is foreign, what is the notional currency of the option, and what is the premium currency. Unfortunately this is not symmetric, since the counterpart might have another notion of domestic currency for a given currency pair. Hence in the professional inter bank market there is a generic notion of delta per currency pair. Table 1.3 provides a short overview. Details on all currency pairs can be found in your risk management system (if you have a good one). Essentially there are only four currency pairs with a premium paid in domestic currency by default. All other pairs use premium adjustment.

Example of Delta Quotations We consider two examples in Table 1.4 and Table 1.5 to compare the various versions of deltas that are used in practice.

TABLE 1.3 Default premium currency for a small selection of currency pairs. LHS currency pairs assume premium paid in USD (domestic currency), RHS assume premium paid in foreign currency.

Premium-unadjusted	Premium-adjusted
EUR/USD	USD/CAD
GBP/USD	EUR/GBP
AUD/USD	USD/JPY
NZD/USD	EUR/JPY
	USD/BRL
	USD/CHF
	EUR/CHF
	USD/ILS
	USD/SGD
	EUR/TRY

TABLE 1.4 1y EUR call USD put strike $K = 0.9090$ for a EUR-based bank. Market data: spot $S = 0.9090$, volatility $\sigma = 12\%$, EUR rate $r_f = 3.96\%$, USD rate $r_d = 3.57\%$. The raw delta is 49.15% EUR and the value is 4.427% EUR.

Delta ccy	Prem ccy	FENICS	Formula	Delta
% EUR	EUR	lhs	$\delta_{raw} - P$	44.72
% EUR	USD	rhs	δ_{raw}	49.15
% USD	EUR	rhs [flip F4]	$-(\delta_{raw} - P)S/K$	−44.72
% USD	USD	lhs [flip F4]	$-(\delta_{raw})S/K$	−49.15

TABLE 1.5 1y EUR call USD put strike $K = 0.7000$ for a EUR-based bank. Market data: spot $S = 0.9090$, volatility $\sigma = 12\%$, EUR rate $r_f = 3.96\%$, USD rate $r_d = 3.57\%$. The raw delta is 94.82% EUR and the value is 21.88% EUR.

Delta ccy	Prem ccy	FENICS	Formula	Delta
% EUR	EUR	lhs	$\delta_{raw} - P$	72.94
% EUR	USD	rhs	δ_{raw}	94.82
% USD	EUR	rhs [flip F4]	$-(\delta_{raw} - P)S/K$	−94.72
% USD	USD	lhs [flip F4]	$-\delta_{raw}S/K$	−123.13

1.4.7 Strike in Terms of Delta

Since $v_x = \Delta = \phi e^{-r_f\tau}\mathcal{N}(\phi d_+)$ we can retrieve the strike as

$$K = x \exp\left\{-\phi\mathcal{N}^{-1}(\phi\Delta e^{r_f\tau})\sigma\sqrt{\tau} + \sigma\theta_+\tau\right\}. \tag{56}$$

Forward Delta I had labeled $\phi\mathcal{N}(\phi d_+)$ as forward delta in (17) and interpreted it as the number of units of FOR of forward contracts a trader needs to buy to delta hedge a short option. Here is justification: translating the above hedge ratio into calculus means that we need to compute (for a call)

$$\frac{\partial v_{\text{option}}}{\partial v_{\text{forward}}}. \tag{57}$$

The value of a forward contract v_{forward} for an agreed forward exchange rate K is obviously

$$\phi[xe^{-r_f\tau} - Ke^{-r_d\tau}], \tag{58}$$

(the Black-Scholes formula with the normal probabilities set equal to one, because a forward is always transacted at maturity), and its derivative with respect to spot x is

$$\frac{\partial v_{\text{forward}}}{\partial x} = \phi e^{-r_f\tau}. \tag{59}$$

Using the chain rule and the derivative of the inverse function, we obtain for the forward delta

$$\begin{aligned}\frac{\partial v_{\text{option}}}{\partial v_{\text{forward}}} &= \frac{\partial v_{\text{option}}}{\partial x}\frac{\partial x}{\partial v_{\text{forward}}}\\ &= \phi e^{-r_f\tau}\mathcal{N}(\phi d_+)\left(\frac{\partial v_{\text{forward}}}{\partial x}\right)^{-1}\\ &= \phi e^{-r_f\tau}\mathcal{N}(\phi d_+)\left(e^{-r_f\tau}\right)^{-1}\\ &= \phi\mathcal{N}(\phi d_+),\end{aligned} \tag{60}$$

and Bob's your uncle. High-school calculus – no university degree needed. Needless to say, there can also be a premium-adjusted forward delta which is (for the sake of completeness)

$$\phi \frac{K}{f} \mathcal{N}(\phi d_-). \tag{61}$$

By now you are hopefully aware that in FX options markets the notion of delta is highly important. It is imperative to develop a habit to ask which notion of delta is applied when traders or quants talk about it: with or without premium adjustment, spot or forward, nominal currency and the quotation of the respective currency pair. And all of this will expand even further once other models beyond Black-Scholes are applied and smile is included.

1.4.8 Volatility in Terms of Delta

The mapping $\sigma \mapsto \Delta = \phi e^{-r_f \tau} \mathcal{N}(\phi d_+)$ is not one-to-one. The two solutions are given by

$$\sigma_\pm = \frac{1}{\sqrt{\tau}} \left\{ \phi \mathcal{N}^{-1}(\phi \Delta e^{r_f \tau}) \pm \sqrt{(\mathcal{N}^{-1}(\phi \Delta e^{r_f \tau}))^2 - \sigma \sqrt{\tau}(d_+ + d_-)} \right\}. \tag{62}$$

Thus using just the delta to retrieve the volatility of an option is not advisable.

1.4.9 Volatility and Delta for a Given Strike

The determination of the volatility and the delta for a given strike is an iterative process involving the determination of the delta for the option using at-the-money volatilities in a first step and then using the determined volatility to re-determine the delta and to continuously iterate the delta and volatility until the volatility does not change more than $\epsilon = 0.001\%$ between iterations. More precisely, one can perform the following algorithm. Let the given strike be K.

1. Choose σ_0 = at-the-money volatility from the volatility matrix.
2. Calculate $\Delta_{n+1} = \Delta(\text{call}(K, \sigma_n))$.
3. Take $\sigma_{n+1} = \sigma(\Delta_{n+1})$ from the volatility matrix, possibly via a suitable interpolation.
4. If $|\sigma_{n+1} - \sigma_n| < \epsilon$, then quit, otherwise continue with step 2.

In order to prove the convergence of this algorithm we need to establish convergence of the recursion

$$\begin{aligned} \Delta_{n+1} &= e^{-r_f \tau} \mathcal{N}(d_+(\Delta_n)) \\ &= e^{-r_f \tau} \mathcal{N}\left(\frac{\ln(S/K) + (r_d - r_f + \frac{1}{2}\sigma^2(\Delta_n))\tau}{\sigma(\Delta_n)\sqrt{\tau}} \right) \end{aligned} \tag{63}$$

for sufficiently large $\sigma(\Delta_n)$ and a sufficiently smooth volatility smile surface. We must show that the sequence of these Δ_n converges to a fixed point $\Delta^* \in [0, 1]$ with a fixed volatility $\sigma^* = \sigma(\Delta^*)$.

This proof has been carried out in the thesis by Borowski [18] and works like this. We consider the derivative

$$\frac{\partial \Delta_{n+1}}{\partial \Delta_n} = -e^{-r_f \tau} n(d_+(\Delta_n)) \frac{d_-(\Delta_n)}{\sigma(\Delta_n)} \cdot \frac{\partial}{\partial \Delta_n} \sigma(\Delta_n). \tag{64}$$

The term

$$-e^{-r_f \tau} n(d_+(\Delta_n)) \frac{d_-(\Delta_n)}{\sigma(\Delta_n)}$$

converges rapidly to zero for very small and very large spots, being an argument of the standard normal density n. For sufficiently large $\sigma(\Delta_n)$ and a sufficiently smooth volatility surface in the sense that $\frac{\partial}{\partial \Delta_n}\sigma(\Delta_n)$ is sufficiently small, we obtain

$$\left| \frac{\partial}{\partial \Delta_n} \sigma(\Delta_n) \right| \stackrel{\Delta}{=} q < 1. \tag{65}$$

Thus for any two values $\Delta_{n+1}^{(1)}$, $\Delta_{n+1}^{(2)}$, and continuously differentiable smile surface we obtain

$$|\Delta_{n+1}^{(1)} - \Delta_{n+1}^{(2)}| < q|\Delta_n^{(1)} - \Delta_n^{(2)}|, \tag{66}$$

due to the mean value theorem. Hence the sequence Δ_n is a contraction in the sense of the fixed point theorem of Banach. This implies that the sequence converges to a unique fixed point in $[0, 1]$, which is given by $\sigma^* = \sigma(\Delta^*)$.

1.4.10 Greeks in Terms of Deltas

In foreign exchange markets the moneyness of vanilla options is always expressed in terms of deltas and prices are quoted in terms of volatility. This makes a ten-delta call a financial object as such independent of spot and strike. This method and the quotation in volatility makes objects and prices transparent in a very intelligent and user-friendly way. At this point we list the Greeks in terms of deltas instead of spot and strike. Let us introduce the quantities

$$\Delta_+ \stackrel{\Delta}{=} \phi e^{-r_f \tau} \mathcal{N}(\phi d_+) \text{ spot delta}, \tag{67}$$

$$\Delta_- \stackrel{\Delta}{=} -\phi e^{-r_d \tau} \mathcal{N}(\phi d_-) \text{ dual delta}, \tag{68}$$

which we assume to be given. From these we can retrieve

$$d_+ = \phi \mathcal{N}^{-1}(\phi e^{r_f \tau} \Delta_+), \tag{69}$$

$$d_- = \phi \mathcal{N}^{-1}(-\phi e^{r_d \tau} \Delta_-). \tag{70}$$

Interpretation of Dual Delta The dual delta introduced in (31) as the sensitivity with respect to strike has another – more practical – interpretation in a foreign exchange setup. We have seen in Section 1.4.5 that the domestic value

$$v(x, K, \tau, \sigma, r_d, r_f, \phi) \tag{71}$$

corresponds to a foreign value

$$v\left(\frac{1}{x}, \frac{1}{K}, \tau, \sigma, r_f, r_d, -\phi\right) \tag{72}$$

up to an adjustment of the nominal amount by the factor xK. From a foreign viewpoint the delta is thus given by

$$\begin{aligned}
&-\phi e^{-r_d\tau}\mathcal{N}\left(-\phi\frac{\ln\left(\frac{K}{x}\right)+\left(r_f-r_d+\frac{1}{2}\sigma^2\tau\right)}{\sigma\sqrt{\tau}}\right)\\
&=-\phi e^{-r_d\tau}\mathcal{N}\left(\phi\frac{\ln\left(\frac{x}{K}\right)+\left(r_d-r_f-\frac{1}{2}\sigma^2\tau\right)}{\sigma\sqrt{\tau}}\right)\\
&=\Delta_-,
\end{aligned} \tag{73}$$

which means the dual delta is the delta from the foreign viewpoint. This is again in line with its interpretation of a premium-adjusted delta as in Equation (55). We will see below that foreign rho, vega and gamma do not require knowing the dual delta. We will now state the Greeks in terms of $x, \Delta_+, \Delta_-, r_d, r_f, \tau, \phi$.

Value.

$$v(x, \Delta_+, \Delta_-, r_d, r_f, \tau, \phi) = x\Delta_+ + K\Delta_-\frac{e^{-r_f\tau}n(d_+)}{e^{-r_d\tau}n(d_-)} \tag{74}$$

(Spot) Delta.

$$\frac{\partial v}{\partial x} = \Delta_+ \tag{75}$$

Forward Delta.

$$\frac{\partial v}{\partial v_f} = e^{r_f\tau}\Delta_+ \tag{76}$$

Gamma.

$$\frac{\partial^2 v}{\partial x^2} = e^{-r_f\tau}\frac{n(d_+)}{x(d_+ - d_-)} \tag{77}$$

Taking a trader's gamma (change of delta if spot moves by 1%) additionally removes the spot dependence, because

$$\Gamma_{trader} = \frac{x}{100}\frac{\partial^2 v}{\partial x^2} = e^{-r_f \tau}\frac{n(d_+)}{100(d_+ - d_-)} \tag{78}$$

Speed.

$$\frac{\partial^3 v}{\partial x^3} = -e^{-r_f \tau}\frac{n(d_+)}{x^2(d_+ - d_-)^2}\left(2d_+ - d_-\right) \tag{79}$$

Theta.

$$\begin{aligned}\frac{1}{x}\frac{\partial v}{\partial t} &= -e^{-r_f \tau}\frac{n(d_+)(d_+ - d_-)}{2\tau} \\ &\quad + \left[r_f \Delta_+ + r_d \Delta_- \frac{e^{-r_f \tau} n(d_+)}{e^{-r_d \tau} n(d_-)}\right]\end{aligned} \tag{80}$$

Charm.

$$\frac{\partial^2 v}{\partial x \partial \tau} = -\phi r_f e^{-r_f \tau}\mathcal{N}(\phi d_+) + \phi e^{-r_f \tau} n(d_+)\frac{2(r_d - r_f)\tau - d_-(d_+ - d_-)}{2\tau(d_+ - d_-)} \tag{81}$$

Color.

$$\frac{\partial^3 v}{\partial x^2 \partial \tau} = -\frac{e^{-r_f \tau} n(d_+)}{2x\tau(d_+ - d_-)}\left[2r_f \tau + 1 + \frac{2(r_d - r_f)\tau - d_-(d_+ - d_-)}{2\tau(d_+ - d_-)} d_+\right] \tag{82}$$

Vega.

$$\frac{\partial v}{\partial \sigma} = x e^{-r_f \tau}\sqrt{\tau} n(d_+) \tag{83}$$

Volga.

$$\frac{\partial^2 v}{\partial \sigma^2} = x e^{-r_f \tau}\tau n(d_+)\frac{d_+ d_-}{d_+ - d_-} \tag{84}$$

Vanna.

$$\frac{\partial^2 v}{\partial \sigma \partial x} = -e^{-r_f \tau} n(d_+)\frac{\sqrt{\tau} d_-}{d_+ - d_-} \tag{85}$$

Rho.

$$\frac{\partial v}{\partial r_d} = -x\tau\Delta_- \frac{e^{-r_f \tau} n(d_+)}{e^{-r_d \tau} n(d_-)} \tag{86}$$

$$\frac{\partial v}{\partial r_f} = -x\tau\Delta_+ \tag{87}$$

Dual Delta.

$$\frac{\partial v}{\partial K} = \Delta_- \tag{88}$$

Dual Gamma.

$$K^2 \frac{\partial^2 v}{\partial K^2} = x^2 \frac{\partial^2 v}{\partial x^2} \tag{89}$$

Dual Theta.

$$\frac{\partial v}{\partial T} = -v_t \tag{90}$$

As an important example we consider vega.

Vega in Terms of Delta The mapping $\Delta \mapsto v_\sigma = xe^{-r_f \tau}\sqrt{\tau}n(\mathcal{N}^{-1}(e^{r_f \tau}\Delta))$ is important for trading vanilla options. Observe that this function does not depend on r_d or σ, just on r_f. Quoting vega in % foreign will additionally remove the spot dependence. This means that for a moderately stable foreign term structure curve, traders will be able to use a moderately stable vega matrix. For $r_f = 3\%$ the vega matrix is presented in Table 1.6. There are traders who know this table by heart. Just as motivation in case you don't have any plans for tonight. Usually, a vega hedge uses an option with the same maturity, so the table is in fact also independent of r_f. The vega hedging default product is an at-the-money straddle with the same maturity. However, note that vega is mostly hedged on a portfolio level, rather than on individual transactions. Individual vega hedging would be performed on big size tickets.

TABLE 1.6 Vega in terms of delta for the standard maturity labels and various deltas. It shows that one can vega hedge a long 9M 35 delta call with 4 short 1M 20 delta puts.

Mat/Δ	50%	45%	40%	35%	30%	25%	20%	15%	10%	5%
1D	2	2	2	2	2	2	1	1	1	1
1W	6	5	5	5	5	4	4	3	2	1
1W	8	8	8	7	7	6	5	5	3	2
1M	11	11	11	11	10	9	8	7	5	3
2M	16	16	16	15	14	13	11	9	7	4
3M	20	20	19	18	17	16	14	12	9	5
6M	28	28	27	26	24	22	20	16	12	7
9M	34	34	33	32	30	27	24	20	15	9
1Y	39	39	38	36	34	31	28	23	17	10
2Y	53	53	52	50	48	44	39	32	24	14
3Y	63	63	62	60	57	53	47	39	30	18

1.4.11 Settlement

Standard textbooks dealing with the valuation of options deal with only two times, one for the beginning and one for the end of the deal. In practice, the story is a bit more advanced and deals with the dates listed in Figure 1.3.

We are going to use the following notation:

T_h *horizon date*, represents the date on which the derivative is evaluated. In many cases it represents today.

T_{hs} *horizon spot date*, two business days after the horizon date.

T_e *expiry date*. For path independent options the payoff depends on the quoted spot or forward on this date.

T_{es} *expiry spot date*, two business days after the expiry date.

T_d *delivery date*, represents the date on which cash flows implied by the option payoff will be settled.

For corridors, faders, target forwards and other derivatives transactions whose payoff depends on a fixing schedule, additional fixing dates are introduced:

T_f *fixing date*, on this date it is decided whether the underlying is inside a specified range.

T_{fs} *fixing spot date*, two business days after the fixing date.

In general we have

$$T_h \overset{2bd}{\leq} T_{hs} \leq T_f \overset{2bd}{\leq} T_{fs} \leq T_e \overset{2bd}{\leq} T_{es} \leq T_d. \tag{91}$$

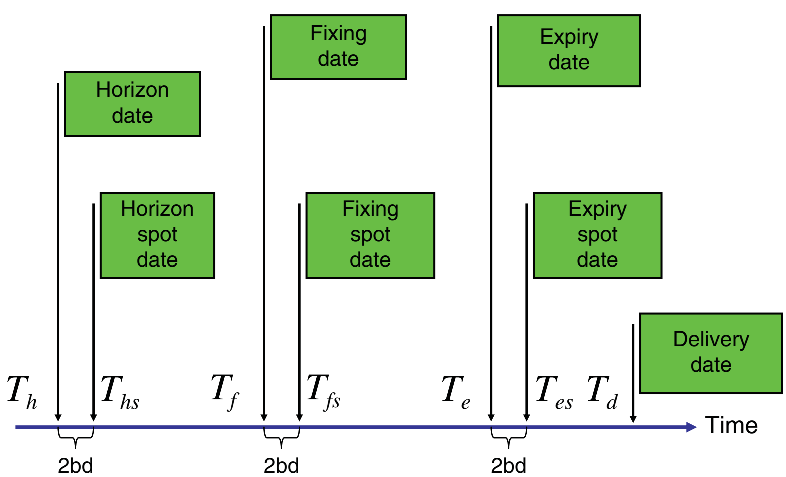


FIGURE 1.3 Relevant dates for trading options. The spot dates are usually two business days (2bd) after the horizon, fixing or expiry date.

The payoff of FX options depends either on the quoted (2bd) spot S_{T_e} at the expiry date T_e, or the forward F_{T_{es},T_d} from that date to a specific delivery date T_d. In many cases the delivery date T_d corresponds to the expiry spot date T_{es}, where we have $F_{T_{es},T_d} = S_{T_e}$.

The constant volatility used in the model (see Equation (1)) corresponds to the time period $T_h = 0$ to $T_e = T$. This is the time interval that is relevant for the exchange rate risk. The risk starts right after the transaction and ends as soon as the option is exercised or expires.

All interest rates r_d and r_f are assumed to correspond to the respective period in their context, for example: the term $e^{-r_d(T_d-T_{hs})}$ represents the factor used for discounting the payoff, using the domestic (forward) interest rate from the spot date T_{hs} to the delivery date T_d.

Remark 1.4.1 *In rare cases the delivery date can be* before *the expiry spot date, that is $T_{es} > T_d$. However, it can never be before the expiry date T_e. For instance, in Canada, the delivery date is usually only one business day after the expiry date. One can have a delivery on the same day upon special request.*

The Black-Scholes Model for the Actual Spot The standard approach is to assume a Black-Scholes model as in Equation (1)

$$\frac{d\hat{S}_t}{\hat{S}_t} = (r_d - r_f)\,dt + \sigma\,dW_t \tag{92}$$

for the underlying, where $\hat{S}_t$ means the price of the underlying at time t.

Instead of modeling this zero-day spot $\hat{S}$ we need to model the *2bd spot* S, which is usually quoted in FX markets.

Assuming no arbitrage opportunities this leads to the relationships

$$\begin{aligned} \hat{S}_t &= S_t e^{-(r_d-r_f)(T_{hs}-t)}, \\ \hat{S}_{T_e} &= S_{T_e} e^{-(r_d-r_f)(T_{es}-T_e)}. \end{aligned} \tag{93}$$

Assuming the exponents in (93) are deterministic, the quoted spot at some future time T_e satisfies

$$S_{T_e} = S_t \exp\left[(r_d - r_f)(T_{es} - T_{hs}) - \tfrac{1}{2}\sigma^2(T_e - t) + \sigma W_{T_e-t}\right], \tag{94}$$

which follows directly from (93) and (2).

Cash Settlement In case of *cash settlement* the seller of the option pays a cash amount depending on the payoff formula and the quoted spot S_{T_e} to the holder. By default, the cash arrives in the holder's account on the expiry spot date T_{es}, but in general on the delivery date T_d. For example, in the case of a vanilla quoted in FOR-DOM, the DOM cash amount

$$\left(\phi\left(S_{T_e} - K\right)\right)^+ \tag{95}$$

is paid, where as usual $\phi = \pm 1$ for calls and puts respectively. The option will usually be exercised if $\phi S_{T_e} > \phi K$.

The value of the vanilla option at time $t = T_{hs}$ is then given by

$$\begin{aligned} v_t^C &= \mathbb{E}\left[e^{-r_d(T_d - T_{hs})}\left(\phi\left(S_{T_e} - K\right)\right)^+ \middle| S_t\right] \\ &= \mathbb{E}\left[e^{-r_d(T_d - T_{es})}e^{-r_d(T_{es} - T_{hs})}\left(\phi\left(S_{T_e} - K\right)\right)^+ S_t\right] \\ &= e^{-r_d(T_d - T_{es})}v_t^{PV}, \end{aligned} \tag{96}$$

where v_t^{PV} denotes the value of a vanilla in the Black-Scholes model published in textbooks – compare with Equation (7). In particular, we have

$$d_\pm^C = \frac{\ln\frac{S_t}{K} + (r_d - r_f)(T_{es} - T_{hs}) \pm \frac{1}{2}\sigma^2(T_e - t)}{\sigma\sqrt{T_e - t}}, \tag{97}$$

and

$$\begin{aligned} v_t^C &= \phi e^{-r_d(T_d - T_{hs})}\left(e^{(r_d - r_f)(T_{es} - T_{hs})}S_t\mathcal{N}(\phi d_+^C) - K\mathcal{N}(\phi d_-^C)\right) \\ &= \phi e^{-r_d(T_d - T_{es})}\left(e^{-r_f(T_{es} - T_{hs})}S_t\mathcal{N}(\phi d_+^C) - e^{-r_d(T_{es} - T_{hs})}K\mathcal{N}(\phi d_-^C)\right). \end{aligned} \tag{98}$$

For other options this works similarly.

Delivery Settlement In the case of *delivery settlement* the cash amounts of the two involved currencies FOR and DOM are physically exchanged on the delivery date T_d. Therefore the intrinsic value is given by

$$\left(\phi\left(F_{T_{es},T_d} - K\right)\right)^+ \tag{99}$$

$$= \left(\phi\left(S_{T_e}e^{(r_d - r_f)\cdot(T_d - T_{es})} - K\right)\right)^+, \tag{100}$$

where the second equality is meant to hold at expiry date T_e, when the rates are known. The option will typically be exercised if

$$\phi F_{T_{es},T_d} > \phi K, \tag{101}$$

which is equivalent to

$$\phi S_{T_e} > \phi K e^{-(r_d - r_f)(T_d - T_{es})}. \tag{102}$$

The value of a vanilla option at time $t = T_{hs}$ is then given by

$$v_t^D = \mathbb{E}\left[e^{-r_d(T_d-T_{hs})}\left(\phi\left(F_{T_{es},T_d} - K\right)\right)^+ \middle| S_t\right] \tag{103}$$

$$= \phi e^{-r_d(T_d-T_{hs})}\left(e^{(r_d-r_f)(T_d-T_{hs})}S_t\mathcal{N}(\phi d_+^D) - K\mathcal{N}(\phi d_-^D)\right)$$

$$= \phi e^{-r_f(T_d-T_{hs})}S_t\mathcal{N}(\phi d_+^D) - \phi e^{-r_d(T_d-T_{hs})}K\mathcal{N}(\phi d_-^D),$$

$$d_\pm^D = \frac{\ln\frac{S_t}{K} + (r_d - r_f)(T_d - T_{hs}) \pm \frac{1}{2}\sigma^2(T_e - t)}{\sigma\sqrt{T_e - t}}.$$

Options with Deferred Delivery Options in FX OTC markets are typically delivery-settled, as corporates do have cash to exchange. Options are bought for the purpose of investment, or speculation as well, and in that case a cash settlement is more suitable. In this section we deal with delivery-settled options, where the delivery date is deferred.

In the case of an option with delivery settlement often it becomes important to the corporate treasurer to have a settlement date significantly after the expiration date of the contract. The default is two business days, but corporates sometimes wish to delay the settlement up to one year or even further. The reason is that a decision to exchange amounts in different currencies in the future is often taken much earlier than the actual payment time, just like in the case of *compound* or *installment* options.

This means that the option under consideration is not an option on the FX *spot* but rather an option on the FX *forward*, which is sometimes called *compound on forward*.

To be concrete, let us derive the formula for the deferred delivery-settled vanilla call. We let the current time be T_h, the expiration time be T_e, and the delivery time be T_d. The buyer of a deferred-delivery call with strike K has the right to enter into a forward contract with strike K at time T_e, which is then delivered at time T_d. In a Black-Scholes model framework with constant interest rates, the forward price at time t maturing at time T_d is given by the random variable

$$f_t(T_d) = S_t e^{(r_d-r_f)(T_d-t)}, \tag{104}$$

in particular, at time zero,

$$f_0(T_d) = S_0 e^{(r_d-r_f)T_d}, \tag{105}$$

is the current forward price. We let T_d be fixed and view t as the variable. Using Itô's formula,[1] we see that the forward price satisfies

$$df_t(T_d) = \sigma f_t(T_d)dW_t, \tag{106}$$

hence it is a martingale. In a risk-neutral valuation approach, the value of a call on $f_t(T_d)$ is given by

$$v(0) = e^{-r_dT_d}\mathbb{E}[(f_t(T_d) - K)^+]. \tag{107}$$

[1] $df(S_t) = f'(S_t)\, dS_t + \frac{1}{2}f''(S_t)\,(dS_t)^2$

In order to compute this, we notice that we can use the existing Black-Scholes Equation (7) for the special case $r_d = r_f$ (due to Equation (106)) and $S_0 = f_0(T_d)$, which is

$$\begin{aligned} v(0) &= e^{-r_d T_d}\left[f_0(T_d)\mathcal{N}(d_+) - K\mathcal{N}(d_-)\right] \\ &= S_0 e^{-r_f T_d}\mathcal{N}(d_+) - Ke^{-r_d T_d}\mathcal{N}(d_-), \end{aligned} \tag{108}$$

$$\begin{aligned} d_\pm &= \frac{\ln\frac{f_0(T_d)}{K} \pm \frac{1}{2}\sigma^2 T_e}{\sigma\sqrt{T_e}} \\ &= \frac{\ln\frac{S_0}{K} + (r_d - r_f)T_d \pm \frac{1}{2}\sigma^2 T_e}{\sigma\sqrt{T_e}}. \end{aligned} \tag{109}$$

This calculation works similarly for all European style path-independent options. The basic procedure is to reuse existing formulas for options as a function of the spot for a zero drift $r_d = r_f$ and replace the spot variable by the forward.

1.4.12 Exercises

Vega Maximizing Strike Show that the delta-symmetric strike K_+ maximizes vega.

Gamma Maximizing Strike Show that the delta-symmetric strike K_+ maximizes gamma.

Forward Delta Normally, when selling an option, the trader buys delta times the notional of the option contract of the underlying in the spot market. However, in many situations, such as trading long-term contracts in-the-money, it turns out to be more suitable to replace this spot hedge by a forward hedge. This means that instead of buying the underlying, the trader buys a forward contract on the underlying with the same maturity as the option.

(i) The *forward delta* is defined to be the number of units of forward contracts to buy so the total delta of the short option and the long forward contract is zero. Show that the forward delta is equal to

$$\phi\mathcal{N}(\phi d_+). \tag{110}$$

(ii) Let us fix the initial notional of the forward traded to hedge and call it

$$\phi\tilde{\mathcal{N}}(\phi d_+). \tag{111}$$

Keeping this quantity fixed, write down the value of the portfolio P of the short vanilla option and the long forward contract. Verify that the total initial spot delta is zero.

(iii) Argue why replacing the spot hedge with a forward hedge does not change the vega and gamma positions.

(iv) Show that the portfolio P has a foreign rho $\frac{\partial P}{\partial r_f}$ of zero.

(v) Compute the domestic rho of the portfolio $\frac{\partial P}{\partial r_d}$ and argue why this is often considerably smaller than the domestic rho of a vanilla contract.

Overall these considerations show that forward hedging also takes care of most of the interest rate risk, which is why, in principle, it is the better way to hedge. However, spot markets are normally more liquid than forward markets, and bid-ask spreads are smaller, which prevents traders from doing forward hedges most of the time. A common practice is to always do a spot hedge at inception of the trade and replace it by a forward hedge if the underlying contract has a high sensitivity to the carry (difference of interest rates). This can be done by an FX swap.

Vega-Delta Implied volatility is sometimes quoted in terms of the spot delta in the Black-Scholes model, rather than in terms of the strike, particularly in foreign exchange markets. The reason is that vega

$$\frac{\partial v}{\partial \sigma} = xe^{-r_f\tau}\sqrt{\tau}n(d_+) \tag{112}$$

does not depend on the domestic rate r_d and not on the volatility σ, when quoted in terms of delta. Prove these two statements.

Premium-Adjusted Delta If the underlying security is an exchange rate, then the option premium can be paid in both the domestic and the foreign currency. Therefore, we must re-think our notion of delta, which represents the number of units of foreign currency to buy when selling a vanilla option. This works fine if the premium is paid in domestic currency. However, if the premium is paid in foreign currency, applying a standard delta hedge would be wrong, because the trader would end up having too many units of foreign currency in his book. We define the *premium-adjusted spot delta* as

$$x \cdot \frac{\partial}{\partial x}\left(\frac{v}{x}\right). \tag{113}$$

(i) Show that the premium-adjusted spot delta can be written as

$$\Delta_S - \frac{v}{x}, \tag{114}$$

where Δ_S is the standard vanilla delta, which is not premium-adjusted. Argue intuitively, why this is the correct quantity.

(ii) Similarly, we define the *premium-adjusted forward delta* as

$$\Delta_{Fpa} \stackrel{\Delta}{=} \frac{x \cdot \frac{\partial}{\partial x}\left(\frac{v}{x}\right)}{\frac{\partial v_f}{\partial x}}, \tag{115}$$

which is the number of forward contracts to buy when the premium is paid in foreign currency. Show that the premium-adjusted forward delta is given by

$$\phi \frac{K}{f} \mathcal{N}(\phi d_-). \tag{116}$$

(iii) Show that the strike K_- that generates a premium-adjusted forward delta position of zero of a straddle (call and put with strike K_-) is given by

$$K_- = f e^{-\frac{1}{2}\sigma^2\tau}. \tag{117}$$

Such a straddle is used to hedge vega without affecting the delta position.

(iv) Show that in the case of spot deltas without premium adjustment the relationship of strike and delta is monotone and show how to retrieve the strike from a given spot delta.

(v) Show that in the case of premium-adjusted forward deltas the relationship of strike to delta is no longer guaranteed to be monotone. In particular, show that there is another strike that generates the same delta as $\check{K}$ if

$$\sigma > \sqrt{\frac{2}{\pi\tau}}. \tag{118}$$

This ambiguity would pop up for 30-year options for volatilities above 14.6%, which is not unrealistic, even in FX markets.

Deferred Delivery Driven by Forward Discuss the implications of a deferred delivery on the value of a vanilla call with a fixed strike. What happens to the value if the forward curve is downward sloping? What are the main risk factors involved if the deferral period is very long? How about correlation between the spot and the interest rates?

Greeks in a Binomial Tree Model Implement an N-step binomial tree model that allows the value and delta of European style call and put options to be computed.

1. Compute the values and deltas of put and call options for the following sample contract data: strike $K = 1.4500$, maturity $\tau = 1$ year. Use $N = 120$ time steps, so a single time step will be $\Delta_t = \frac{\tau}{N}$. Let the market data be given by $S_0 = 1.5000$, $r = 5\%$ p.a., $r_f = 4\%$ p.a., volatility $\sigma = 10\%$. For the implementation use the formulas

$$u = e^{+\sigma\sqrt{\Delta_t}}, \tag{119}$$

$$d = e^{-\sigma\sqrt{\Delta_t}}, \tag{120}$$

$$\tilde{p} = \frac{e^{(r-r_f)\Delta_t} - d}{u - d}. \tag{121}$$

For the discounting per time step you need to use $e^{-r\Delta_t}$.

2. Examine the convergence behavior of the call option value and delta for a strike of $K = 1.4500$ and maturity of $T = 1$ year empirically. This means computing the sequences of results for values of N ranging from 1 to 200. Again, a single time step will be $\Delta_t = \frac{T}{N}$. Let the market data be given by $S_0 = 1.5000$, $r = 5\%$ p.a., $r_f = 0$, volatility $\sigma = 10\%$. Plot your results in a graph taking the values for N on the x-axis and the value and delta on the y-axis. Describe the convergence behavior you see, explain, why it possibly looks the way it looks, and suggest and try methods to improve the convergence.

1.5 VOLATILITY

Volatility is the *annualized standard deviation of the log-returns*. It is *the* crucial input parameter in determining the value of an option. Hence, the crucial question is where to derive the volatility from. If no active option market is present, the only source of information is estimating the historic volatility. This would give some clue about the *past*. In liquid currency pairs volatility is often a traded quantity on its own, quoted by traders, brokers, and real-time data pages. These quotes reflect market participants' views about the *future*.

Since volatility normally does not stay constant, option traders are highly concerned with hedging their volatility exposure. Hedging vanilla options' vega is comparatively easy because vanilla options have convex payoffs, whence vega is always positive, i.e. the higher the volatility, the higher the price. Let us take for example a EUR-USD market with spot 1.2000, USD- and EUR rate at 2.5%. A three-month at-the-money call with 1 M EUR notional would cost 29,000 USD at a volatility of 12%. If the volatility now drops to a value of 8%, then the value of the call would be only 19,000 USD. This monotone dependence is not guaranteed for non-convex payoffs, as we illustrate in Figure 1.4.

1.5.1 Historic Volatility

We briefly describe how to compute the historic volatility of a time series

$$S_0, S_1, \ldots, S_N \tag{122}$$

of daily data. First, we create the sequence of log-returns

$$r_i = \ln \frac{S_i}{S_{i-1}}, \quad i = 1, \ldots, N. \tag{123}$$

Then, we compute the average log-return

$$\bar{r} = \frac{1}{N} \sum_{i=1}^{N} r_i, \tag{124}$$

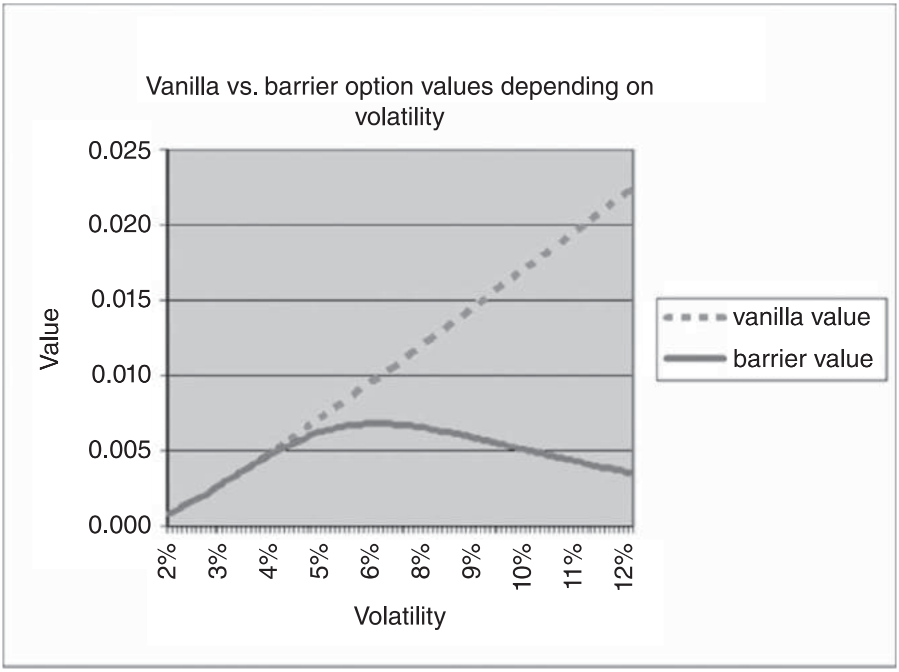


FIGURE 1.4 Dependence of the value of a vanilla call and a reverse knock-out call on volatility. The vanilla value is monotone in the volatility, whereas the barrier value is not. The reason is that as the spot gets closer to the upper knock-out barrier, an increasing volatility would increase the chance of knock-out and hence decrease the value.

their variance

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2, \tag{125}$$

and their standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2}. \tag{126}$$

The annualized standard deviation, which is the volatility, is then given by

$$\hat{\sigma}_a = \sqrt{\frac{B}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2}, \tag{127}$$

where the *annualization factor B* is given by

$$B = \frac{N}{k}d, \tag{128}$$

and k denotes the number of calendar days within the time series and d denotes the number of calendar days per year.

Assuming normally distributed log-returns, we know that $\hat{\sigma}^2$ is χ^2-distributed. Therefore, given a confidence level of p and a corresponding error probability $\alpha = 1 - p$, the p-confidence interval is given by

$$\left[\hat{\sigma}_a \sqrt{\frac{N-1}{\chi^2_{N-1;1-\frac{\alpha}{2}}}}, \hat{\sigma}_a \sqrt{\frac{N-1}{\chi^2_{N-1;\frac{\alpha}{2}}}}\right], \tag{129}$$

where $\chi^2_{n;p}$ denotes the p-quantile of a χ^2-distribution[2] with n degrees of freedom.

As an example let us take the 256 ECB fixings of EUR-USD from 4 March 2003 to 3 March 2004 displayed in Figure 1.5. We get $N = 255$ log-returns. Taking $k = d = 365$, we obtain

$$\bar{r} = \frac{1}{N} \sum_{i=1}^{N} r_i = 0.0004166,$$

$$\hat{\sigma}_a = \sqrt{\frac{B}{N-1} \sum_{i=1}^{N} (r_i - \bar{r})^2} = 10.85\%,$$

and a 95% confidence interval of [9.99%, 11.89%].

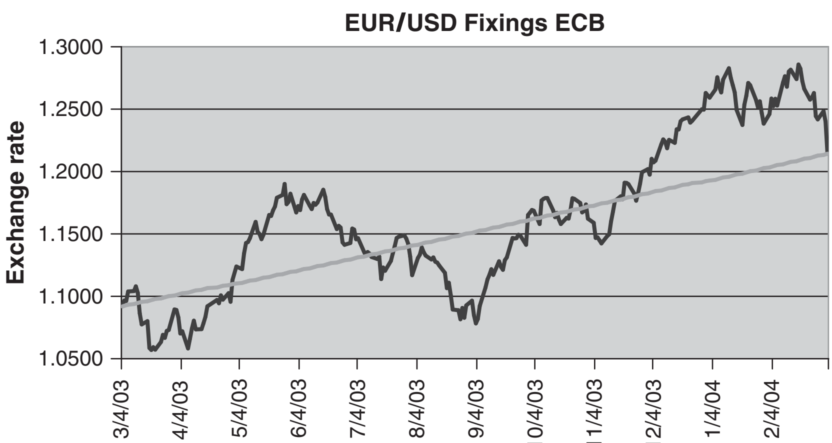


FIGURE 1.5 ECB fixings of EUR-USD from 4 March 2003 to 3 March 2004 and the line of average growth.

[2]Values and quantiles of the χ^2-distribution and other distributions can be computed on the internet, e.g. at http://eswf.uni-koeln.de/allg/surfstat/tables.htm or via EXCEL function CHIINV.

Notice that the quality of the return estimate is horrendously bad. All that matters is the slope of the line connecting the start point and the end point. Therefore, the return estimate is completely determined by the choice of these two points and can hence be considered arbitrary. This is good news for asset managers as they will always be able to find a time interval in a historic data series with positive return. Furthermore, the fact that returns are impossible to estimate justifies more than 95% of all jobs in asset management. It is also why Markowitz doesn't work but Black-Scholes does.

Quality of the volatility estimate is substantially better than the return estimate. In risk-neutral pricing (assuming it works), returns can be inferred from money market rates and do not need to be estimated from historic time series. The remaining uncertainty is volatility, which in turn has become a liquid tradable quantity in many currency pairs. If there are no options in a currency pair, then volatility can only be estimated from historic data.

1.5.2 Historic Correlation

As in the preceding section we briefly describe how to compute the historic correlation of two time series,

$$x_0, x_1, \dots, x_N,$$
$$y_0, y_1, \dots, y_N,$$

of daily data. First, we create the sequences of log-returns

$$\begin{aligned} X_i &= \ln \frac{x_i}{x_{i-1}}, \quad i = 1, \dots, N, \\ Y_i &= \ln \frac{y_i}{y_{i-1}}, \quad i = 1, \dots, N. \end{aligned} \tag{130}$$

Then, we compute the average log-returns

$$\begin{aligned} \overline{X} &= \frac{1}{N} \sum_{i=1}^{N} X_i, \\ \overline{Y} &= \frac{1}{N} \sum_{i=1}^{N} Y_i, \end{aligned} \tag{131}$$

their variances and covariance

$$\hat{\sigma}_X^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \overline{X})^2, \tag{132}$$

$$\hat{\sigma}_Y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \overline{Y})^2, \tag{133}$$

$$\hat{\sigma}_{XY} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})(Y_i - \overline{Y}), \tag{134}$$

and their standard deviations

$$\hat{\sigma}_X = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2}, \tag{135}$$

$$\hat{\sigma}_Y = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2}. \tag{136}$$

The estimate for the correlation of the log-returns is given by

$$\hat{\rho} = \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X\hat{\sigma}_Y}. \tag{137}$$

This correlation estimate is often not very stable, yet often it is the only available information. More recent work by Jäkel [51] treats robust estimation of correlation. We will revisit FX correlation risk in Section 1.9.2.

1.5.3 Volatility Smile

The Black-Scholes model assumes constant volatility throughout. However, market prices of traded options imply different volatilities for different maturities and different deltas. We start with some technical issues on how to imply the volatility from vanilla options.

Retrieving the Volatility from Vanilla Options Given the value of an option. Recall the Black-Scholes formula in Equation (7). We now look at the function $v(\sigma)$, whose derivative (vega) is

$$v'(\sigma) = xe^{-r_f\tau}\sqrt{\tau}n(d_+). \tag{138}$$

The function $\sigma \mapsto v(\sigma)$ is

1. strictly increasing,
2. concave up for $\sigma \in [0, \sqrt{2|\ln f - \ln K|/\tau})$,
3. concave down for $\sigma \in (\sqrt{2|\ln f - \ln K|/\tau}, \infty)$

and also satisfies

$$v(0) = [\phi(xe^{-r_f\tau} - fe^{-r_d\tau})]^+, \tag{139}$$

$$v(\infty, \phi = 1) = xe^{-r_f\tau}, \tag{140}$$

$$v(\sigma = \infty, \phi = -1) = Ke^{-r_d\tau}, \tag{141}$$

$$v'(0) = xe^{-r_f\tau}\sqrt{\tau}/\sqrt{2\pi}I\!I_{\{f=K\}}. \tag{142}$$

In particular the mapping $\sigma \mapsto v(\sigma)$ is invertible. However, the starting guess for employing Newton's method should be chosen with care, because the mapping $\sigma \mapsto v(\sigma)$ has a saddle point at

$$\left(\sqrt{\frac{2}{\tau}\left|\ln\frac{f}{K}\right|}, \phi e^{-r_d\tau}\left\{f\mathcal{N}\left(\phi\sqrt{2\tau\left[\ln\frac{f}{K}\right]^+}\right) - K\mathcal{N}\left(\phi\sqrt{2\tau\left[\ln\frac{K}{f}\right]^+}\right)\right\}\right), \tag{143}$$

as illustrated in Figure 1.6.

To ensure convergence of Newton's method, we are advised to use initial guesses for σ on the same side of the saddle point as the desired implied volatility. The danger is that a large initial guess could lead to a negative successive guess for σ. Therefore one should start with small initial guesses at or below the saddle point. For at-the-money options, the saddle point is degenerate for a zero volatility and small volatilities serve as good initial guesses.

The problem is well known and has been studied in detail by Jäckel [80]. He uses a clever rearrangement of the variables in combination with Newton's and Halley's method along with a good initial guess. Li and Lee show that the calculation speed can be substantially improved by *Successive Over-Relaxation (SOR)* [90, 91].

Market Data Now that we know how to imply the volatility from a given value, we can take a look at the market. We take EUR/GBP at the beginning of April 2005. The at-the-money volatilities for various maturities are listed in Table 1.7. We observe that implied volatilities are not constant but depend on the time to maturity of the option

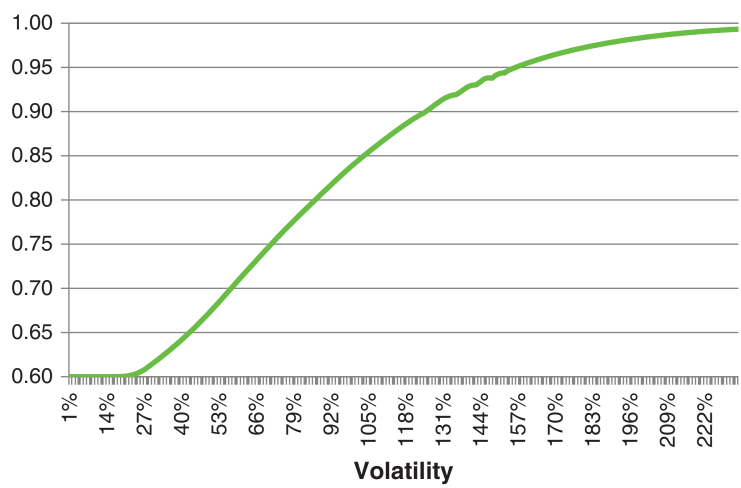


FIGURE 1.6 Value of a European call in terms of volatility with parameters $x = 1$, $K = 1$, $\tau = 5$, $r_d = 20\%$, $r_f = 0\%$. The saddle point is at $\sigma = 60\%$. The value starts at the value of the forward contract 0.5981 USD per EUR and converges to 1.0000 EUR, which is the (foreign discounted) value of the call currency amount expressed in USD.

TABLE 1.7 EUR/GBP implied volatilities in % for at-the-money vanilla options. Source: BBA (British Bankers' Association), http://www.bba.org.uk.

Date	Spot	1 week	1 month	3 month	6 month	1 year	2 year
1-Apr-05	0.6864	4.69	4.83	5.42	5.79	6.02	6.09
4-Apr-05	0.6851	4.51	4.88	5.34	5.72	5.99	6.07
5-Apr-05	0.6840	4.66	4.95	5.34	5.70	5.97	6.03
6-Apr-05	0.6847	4.65	4.91	5.39	5.79	6.05	6.12
7-Apr-05	0.6875	4.78	4.97	5.39	5.79	6.01	6.10
8-Apr-05	0.6858	4.76	5.00	5.41	5.78	6.00	6.09

as well as on the current time. This shows that the Black-Scholes assumption of a constant volatility is not fully justified looking at market data. We have a *term structure of volatility* as well as a stochastic nature of the term structure curve as time passes.

Besides the dependence on the time to maturity (term structure) we also observe different implied volatilities for different degrees of moneyness. This effect is called the *volatility smile*. The term structure and smile together are called a *volatility matrix* or *volatility surface*, if it is graphically displayed. Various possible reasons for this empirical phenomenon are discussed by Bates, among others, for example in [10].

In foreign exchange options markets implied volatilities are generally quoted and plotted against the deltas of out-of-the-money call and put options. This allows market participants to ask various partners for quotes on a 25-delta call, which is spot independent. The actual strike will be set depending on the spot once the trade is close to being finalized. The at-the-money option has a specific strike. In foreign exchange options markets, there are essentially three variants. For retail products, strike is commonly understood to be at the current spot. In the interbank market, strike is equal to the forward rate for options with tenors above one year or for emerging markets currency pairs. Setting the strike equal to the outright forward rate is equivalent to the *value* of the call and the put being equal. For tenors up to one year in standard currency pairs, strike is chosen to make the *delta* of the call and put equal. Other types of *at-the-money* are discussed in Section 1.5.6. The delta of a vanilla option in the case of strike equal to the outright forward rate is

$$\frac{\partial v}{\partial x} = \phi e^{-r_f \tau} \mathcal{N}\left(\phi \tfrac{1}{2}\sigma\sqrt{\tau}\right), \tag{144}$$

for a small volatility σ and short time to maturity τ, a number near $\phi 50\%$. This is why an at-the-money option is often called a 50-delta option. This is acceptable as a rough approximation; however, it is incorrect. In particular, it is incorrect for long-term vanilla options. Further market information is the implied volatilities for puts and calls with a delta of $\phi 25\%$. Other or additional implied volatilities for $\phi 10\%$ deltas are also quoted. Volatility matrices for more delta pillars are usually interpolated. Which at-the-money version is applied is convention-based and may change over time. A first source of information can be your risk management system, provided it is good and current.

Symmetric Decomposition Generally in Foreign Exchange, volatilities are decomposed into a *symmetric* part of the smile reflecting the *convexity* and a *skew-symmetric* part of the smile reflecting the *skew*. The way this works is that the market quotes *risk reversals (RR)* and *butterflies (BF)* or strangles – see Sections 1.6.2 and 1.6.4 for the description of the products and Figure 1.7 for the payoffs. Note that the terms risk reversal and butterfly/strangle are used both as names for trading strategies and numbers indicating the symmetric decomposition of the volatility smile. Not easy for newcomers. Here we are talking about the respective *volatilities* and *volatility differences* and about how to use these to construct the volatility smile and eventually price all vanilla options across the range of deltas. Sample quotes are listed in Tables 1.8 and 1.9. The relationship between risk reversal and strangle/butterfly quotes and the volatility smile are graphically explained in Figure 1.8.

FIGURE 1.7 The risk reversal (upper payoff) is a skew-symmetric product, the butterfly (lower payoff) is a symmetric product.

TABLE 1.8 EUR/GBP 25 delta risk reversal in %. Source: BBA (British Bankers' Association). This means that for example on 4 April 2005, the 1-month 25-delta EUR call was priced with a volatility of 0.15% higher than the 25-delta EUR put. At that moment the market apparently favored calls indicating a market sentiment in an upward movement.

Date	Spot	1 month	3 month	1 year
1-Apr-05	0.6864	0.18	0.23	0.30
4-Apr-05	0.6851	0.15	0.20	0.29
5-Apr-05	0.6840	0.11	0.19	0.28
6-Apr-05	0.6847	0.08	0.19	0.28
7-Apr-05	0.6875	0.13	0.19	0.28
8-Apr-05	0.6858	0.13	0.19	0.28

TABLE 1.9 EUR/GBP 25 delta butterfly in %. Source: BBA. This means that for example on 4 April 2005, the 1-month 25-delta EUR call and the 1-month 25-delta EUR put are on average quoted with a volatility of 0.15% higher than the 1-month at-the-money calls and puts. Using the quotes in Table 1.7 and Table 1.8, the result is that the 1-month 25-delta EUR call is quoted with a volatility of 4.88% + 0.15% + 0.075% and the 1-month 25-delta EUR put is quoted with a volatility of 4.88% + 0.15% – 0.075%. Note that some market participants (including BBA) also use the term "strangle" as a synonym for the butterfly quote. The vocabulary is not consistent, but the meaning has to be inferred from the context.

Date	Spot	1 month	3 month	1 year
1-Apr-05	0.6864	0.15	0.16	0.16
4-Apr-05	0.6851	0.15	0.16	0.16
5-Apr-05	0.6840	0.15	0.16	0.16
6-Apr-05	0.6847	0.15	0.16	0.16
7-Apr-05	0.6875	0.15	0.16	0.16
8-Apr-05	0.6858	0.15	0.16	0.16

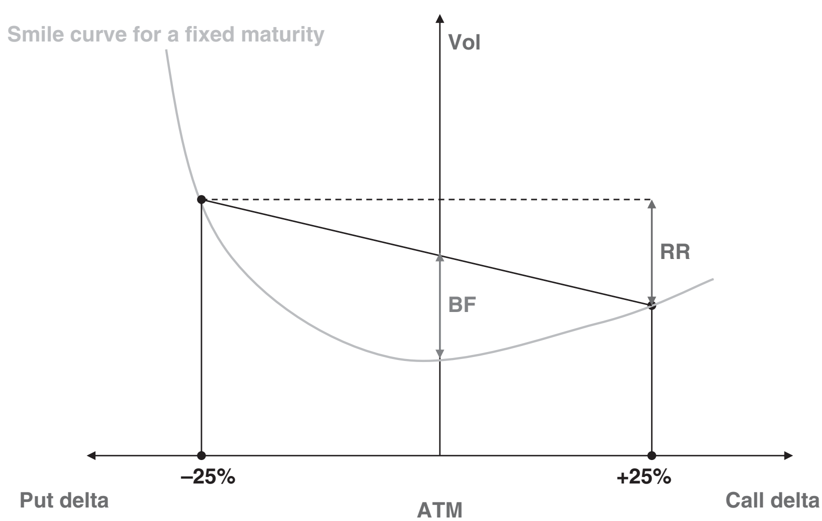


FIGURE 1.8 Risk reversal and butterfly in terms of volatility for a given FX vanilla option smile.

Generally, there are several ways the FX volatility smile can be constructed from quotes for at-the-money, risk reversals and butterflies/strangles. In the next two paragraphs I outline the basic two versions, one being the smile version, which works purely on the level of implied volatilities, the other one being the brokers' version, which requires a detour via values of strangles, in which the construction is harder than just solving a set of linear equations in volatility.

Smile Version In the smile version, the relationships between risk reversal quoted in terms of volatility (RR) and butterfly/strangle (BF) quoted in terms of volatility are defined by

$$\mathrm{RR} = \sigma_+ - \sigma_-, \tag{145}$$

$$\mathrm{BF} = \frac{\sigma_+ + \sigma_-}{2} - \mathrm{ATM}, \tag{146}$$

and therefore, equivalently, the volatilities of 25-delta calls and puts are

$$\sigma_+ = \mathrm{ATM} + \mathrm{BF} + \tfrac{1}{2}\mathrm{RR}, \tag{147}$$

$$\sigma_- = \mathrm{ATM} + \mathrm{BF} - \tfrac{1}{2}\mathrm{RR}, \tag{148}$$

where ATM $= \sigma_0$ denotes the at-the-money volatility of both put and call, see Table 1.7, σ_+ the volatility of an out-of-the-money call (usually $25\,\Delta$) and σ_- the volatility of an out-of-the-money put (usually $25\,\Delta$). Our sample market data is given in terms of RR and BF. Translated into implied volatilities of vanilla options we obtain the data listed in Table 1.10 and illustrated in Figure 1.9.

Brokers' Version In the more common brokers' version of the smile, (146) is replaced by

$$\sigma_{Str} + \sigma_{Str} \stackrel{\Delta}{=} 2(\mathrm{ATM} + \mathrm{BF}) = \sigma_+ + \sigma_- \tag{149}$$

$$\mathrm{Call}(K_{C25}, \sigma_{Str}) + \mathrm{Put}(K_{P25}, \sigma_{Str}) = \mathrm{Call}(K_{C25}, \sigma_+) + \mathrm{Put}(K_{P25}, \sigma_-), \tag{150}$$

where the 25-delta call strike K_{C25} and 25-delta put strike K_{P25} are calculated with the – yet to be found – 25-delta call volatility σ_+ and 25-delta put volatility σ_-. The non-linear (150) means that the LHS value of the 25-delta strangle is calculated as the sum of a call and a put value, both of which are wrong, because they are valued with the at-the-money volatility rather than with the volatilities from the actual FX smile. The RHS value adds up to the correct value, where also both summands are correct values. The smile version is much easier to solve because it is linear, and can

TABLE 1.10 EUR/GBP implied volatilities in % of 4 April 2005. Source: BBA. They are computed based on the market data displayed in Tables 1.7, 1.8 and 1.9 using Equations (147) and (148).

Maturity	25 delta put	at-the-money	25 delta call
1M	4.955	4.880	5.105
3M	5.400	5.340	5.600
1Y	6.030	5.990	6.295

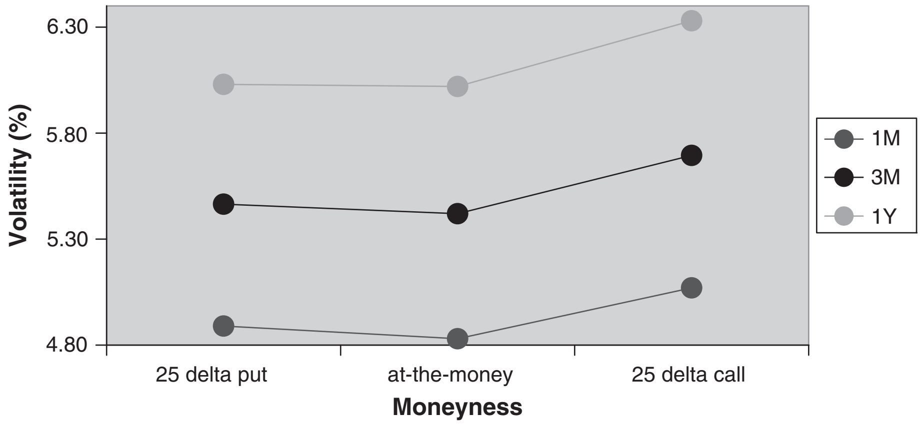


FIGURE 1.9 Implied volatilities for EUR-GBP vanilla options as of 4 April 2005. Source: BBA. Note that the dots are the only market input. Their interpolation and extrapolation are part of the volatility smile construction.

be considered a good approximation, especially if risk reversals are small. Note that the risk reversal (145) stays the same, and is not translated to the level of values of the building blocks. The strikes of the market strangle can also be determined using the at-the-money volatility σ_0. This is easier, but the delta of the strangle will no longer be zero after it is booked.

You might – legitimately – ask the question: why is this made so complicated? As a matter of legacy, brokers liked to quote just one number, which is the market strangle volatility σ_{Str}, to be interpreted approximately as the average volatility for 25-delta calls and puts. Figure 1.10 illustrates this.

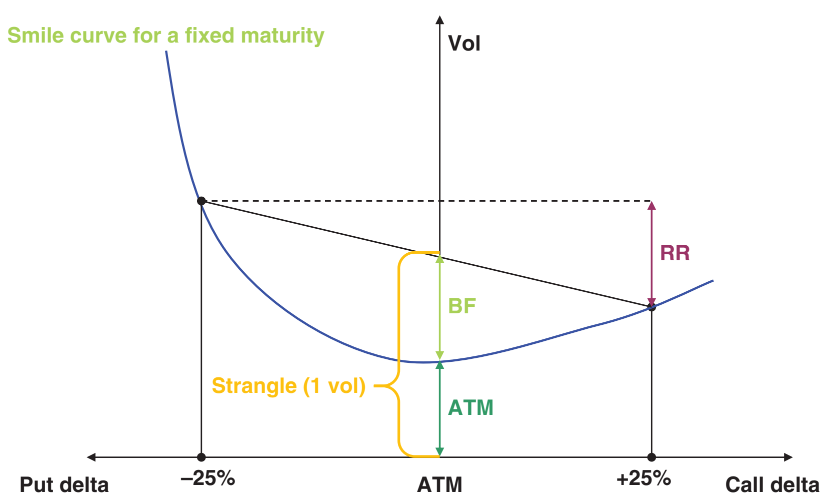


FIGURE 1.10 Relationship of risk reversal, butterfly, and market strangle volatility (the 1-vol strangle).

FX smile construction has been studied in detail in a number of papers by Dimitri Reiswich and myself, see [108, 109, 110, 111], and we will revisit the topic in [141] for quantitative matters. Bloomberg's method has been documented in [62]. An advanced parametric construction of the FX implied volatility surface is treated by Damghani in [35].

1.5.4 At-The-Money Volatility Interpolation

We denote by σ the volatility, which is the *spot volatility* for the time interval from horizon to expiry. The corresponding spot variance is denoted by $\sigma^2\tau$, where τ is the time difference between expiry and horizon. Corresponding forward volatilities σ_f and forward variances apply to a time interval starting later than horizon. The interpolation of at-the-money volatilities takes into account the effect of reduced volatility on weekends and on days closed in the global main trading centers London or New York and the local market, e.g. Tokyo for JPY trades. The change is done for the one-day forward volatility. You may apply a reduction in the one-day forward variance of 25% for each London and New York closed day. For local market holidays you may use a reduction of 25%, where local holidays for EUR are ignored. Weekends can be accounted for by a reduction to 15% variance. The variance on trading days is adjusted to match the volatility on the pillars of the ATM–volatility curve exactly. Obviously, the reduction percentages are arbitrary and different traders may have different opinions about these.

The procedure starts from the two pillars t_1, t_2 surrounding the date t_r in question. The ATM forward volatility for the period is calculated based on the consistency condition

$$\sigma^2(t_1)(t_1 - t_0) + \sigma_f^2(t_1, t_2)(t_2 - t_1) = \sigma^2(t_2)(t_2 - t_0), \tag{151}$$

which means that total variance over a time interval from t_0 to t_2 is the sum of all the variances of the sub-intervals, in this case the spot variance and the forward variance. Therefore,

$$\sigma_f(t_1, t_2) = \sqrt{\frac{\sigma^2(t_2)(t_2 - t_0) - \sigma^2(t_1)(t_1 - t_0)}{t_2 - t_1}}. \tag{152}$$

For each day the factor is determined, and from the constraint that the sum of one-day forward variances matches exactly the total variance, the factor for the enlarged one-day business variances $\alpha(t)$ with t business day is determined.

$$\sigma^2(t_1, t_2)(t_2 - t_1) = \sum_{t=t_1}^{t_r} \alpha(t)\sigma_f^2(t, t+1) \tag{153}$$

The variance for the period is the sum of variances to the start and sum of variances to the required date.

$$\sigma^2(t_r) = \sqrt{\frac{\sigma^2(t_1)(t_1 - t_0) + \sum_{t=t_1}^{t_r} \alpha(t)\sigma_f^2(t, t+1)}{t_r - t_0}} \tag{154}$$

1.5.5 Volatility Smile Conventions

The volatility smile is quoted in terms of delta and one at-the-money pillar. We recall that there are several notions of delta, namely

- spot delta $e^{-r_f \tau} N(d_+)$,
- forward delta $N(d_+)$,

and there is the premium which might be included in either delta. It is important to specify the notion that is used to quote the smile. There are four different deltas concerning plain vanilla options.

1.5.6 At-The-Money Definition

There is one specific at-the-money pillar in the middle. There are numerous notions for the meaning of *at-the-money (ATM)*.

Value parity: choose the strike such that call value = put value

Delta parity: choose the strike such that delta call = − delta put

Fifty delta: call delta = 50% and put delta = 50%

Maximum vega: choose the strike that maximizes vega of a call (or put)

Maximum gamma: choose the strike that maximizes gamma of a call (or put)

ATM spot: set strike equal to spot

Maximum time value: choose the strike that maximizes the time value of the call (or put)

Moreover, the notions involving delta use different versions of delta, namely either spot, forward, and premium included or excluded.

Obviously, put–call parity implies that *value parity* is equivalent to setting the strike equal to the outright forward rate. Therefore, *value parity* is also referred to as ATM forward. The *50-delta* notion is flawed, as for a fixed strike we cannot have both the call *and* the put have a spot delta of 50% (except in the case where $r_f = 0$). However, this flawed notion that an ATM option has a 50-delta is surprisingly widespread. Probably caused by herds of backyard academics writing books and papers about options ignoring interest and dividend rates. The 50-delta notion is only well defined when we use forward deltas and in that case becomes equivalent to *delta parity*. This notion is also referred to as *ATM delta-neutral*. I leave it to you to determine the details of which notions of ATM are equivalent. For risk management systems and common FX pricing tools there are two essential notions: *ATM forward* ($K = f$) and *ATM delta-neutral* ($K = K_\pm$) as derived in Equation (43) in Section 1.4.4. Entering “ATM” into the box for the strike in a pricing tool for EUR/USD options up to one-year tenor is expected to produce K_+ (K_- respectively for USD/JPY).

1.5.7 Interpolation of the Volatility on Fixed Maturity Pillars

Now let me provide a short introduction into the handling of FX implied volatility market data – especially their inter- and extrapolation across delta space and time.

We discuss a low-dimensional Gaussian kernel approach as an example showing several advantages over usual smile interpolation methods, for example cubic splines.

Before the discussion of specific interpolation methods it is recommended to take a step backwards and remember Rebonato's well-known statement of implied volatility as the wrong number in the wrong formula to obtain the right price [105]. So the explanatory power of implied volatilities for the dynamics of a stochastic process remains limited. Implied volatilities give a lattice on which marginal distributions can be constructed. However, even using many data points to generate marginal distributions, forward distributions, and extremal distributions – determining the prices of compound and barrier products, for example – cannot be uniquely defined by implied volatilities – see Tistaert *et al.* [127] for a discussion of this.

The attempt to capture FX smile features can lead into two different general approaches.

Parameterization One possibility for expressing smile or skew patterns is just to capture them as the calibration parameter set of an arbitrary stochastic volatility or jump diffusion model which generates the observed market implied volatilities. However, as spreads are rather narrow in liquid FX options markets, it is preferable to exactly fit the given input volatilities. This automatically leads to an interpolation approach.

Pure Interpolation As an introduction we would like to pose four requirements for an acceptable volatility surface interpolation:

1. Smoothness in the sense of continuous differentiability. Especially with respect to the possible application of Dupire-style local volatility models it is crucial to construct an interpolation which is at least C^2 (twice continuously differentiable). This becomes obvious when looking at the expression for the local volatility in this context:

$$\sigma_t^{local}(S(t)) = \left(2 \frac{\frac{\partial Call(S,t;K,T)}{\partial T} + \frac{\partial Call(S,t;K,T)}{\partial K}}{K^2 \frac{\partial^2 Call(S,t;K,T)}{\partial K^2}} \right)^{\frac{1}{2}}.$$

   Note in addition that local volatilities can directly be extracted from delta-based FX volatility surfaces, i.e. the Dupire formula can alternatively be expressed in terms of delta. See Hakala and Wystup [66] for details.
2. Absence of oscillations, which is guaranteed if the sign of the curvature of the surface does not change over different strike or delta levels.
3. Absence of arbitrage possibilities on single smiles of the surface as well as absence of calendar arbitrage.
4. A reasonable extrapolation available for the interpolation method.

A classical interpolation method widely spread are cubic splines. They attempt to fit surfaces by fitting piecewise cubic polynomials to given data points. They are specified by matching their second derivatives at each intersection. While this ensures the required smoothness by construction, it does not prevent oscillations – which directly leads to the danger of arbitrage possibilities – nor does it define how to extrapolate the smile. We therefore introduce the concept of a slice kernel volatility surface – as described in Hakala and Wystup [66] – as an alternative:

Definition 1.5.1 (Slice Kernel) *Let $(x_1, y_1), (x_2, y_2) \cdots, (x_n, y_n)$ be n given points and $g : I\!R \mapsto I\!R$ a smooth function which fulfills*

$$g(x_n) = y_n, \forall n = 1, \ldots, n. \tag{155}$$

A smooth interpolation is then given by

$$g(x) \triangleq \frac{1}{\Phi_\lambda(x)} \sum_{i=1}^{N} \alpha_i K_\lambda(\| x - x_i \|), \tag{156}$$

where

$$\Phi_\lambda(x) \triangleq \sum_{i=1}^{N} K_\lambda(\| x - x_i \|) \tag{157}$$

and

$$K_\lambda(u) \triangleq \exp\left\{-\frac{u^2}{2\lambda^2}\right\}. \tag{158}$$

The described kernel is also called a Gaussian kernel. The interpolation reduces to the determination of the α_i which is straightforward via solving a linear equation system. Note that λ remains a free smoothing parameter which also impacts the condition of the equation system. At the same time it can be used to fine-tune the extrapolation behavior of the kernel.

The idea behind this approach is as follows. The parameters which solve the interpolation conditions hyperref[interpolbed] (156) are $\alpha_1, \ldots, \alpha_n$. The parameter λ determines the "smoothness" of the resulting interpolation g and should be fixed according to the nature of the points (x_n, y_n). If these points yield a smooth surface, a "large" λ might yield a good fit, whereas in the opposite case when for neighboring points x_k, x_n the appropriate values y_k, y_n vary significantly, only a small λ, that means $\lambda << \min_{n,k} \| X_k - X_n \|$, can provide the needed flexibility.

For the set of delta pillars of $10\%, 25\%, ATM, -25\%, -10\%$ one can use $\lambda = 25\%$ for a smooth interpolation.

Normally the slice kernel produces reasonable output smiles based on a maximum of seven delta-volatility points. Then it fulfills all above mentioned requirements: it is C^∞, does not create oscillations, passes typical no-arbitrage conditions as they are posed by Gatheral [55], for example, and finally has an inherent extrapolation method.

In time direction one might connect different slice kernels by linear interpolation of the variances for same deltas. This also normally ensures the absence of calendar arbitrage, for which a necessary condition is a non-decreasing variance for constant moneyness F/K (see also Gatheral [55] for a discussion of this).

Figure 1.11 displays the shape of a slice kernel applied to a typical FX vol surface constructed from 10- and 25-delta volatilities, and the ATM volatility.

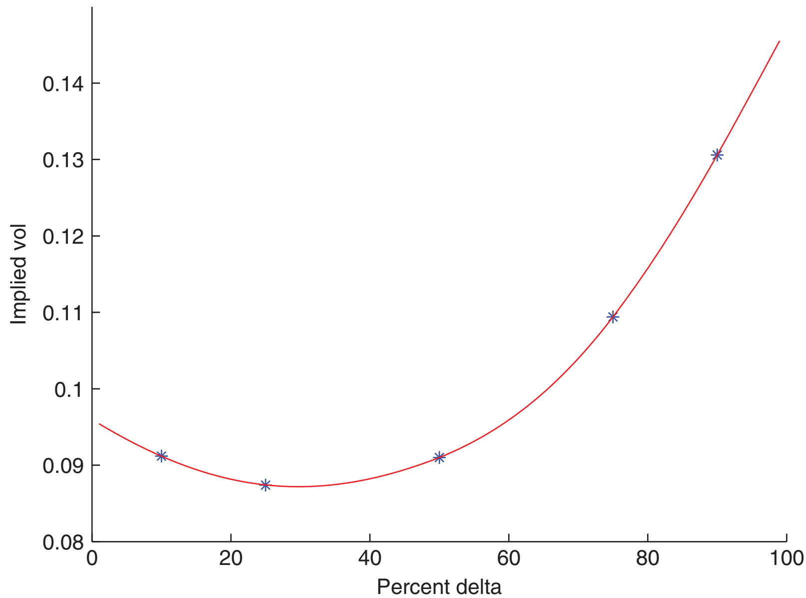


FIGURE 1.11 Kernel interpolation to generate an FX volatility smile.

Critical Judgment Kernels have been used in the industry going back to the 1990s. I had discussed slice kernels in [66] and [137]. However, kernels are by no means the only way to interpolate. Other methods include splines, stochastic volatility-based approaches (Heston, SABR), vanna-volga-based approaches. Malz' parabolic approach is discussed in the exercises in Section 1.5.12. The so-called stochastic volatility-inspired (SVI) approach reported by Gatheral [55] appears to be one of the most promising. A clever way of illustrating its practical implementation can be found in Zeliade [144]. We come back to the issue of interpolation and extrapolation on the smile in more detail in [141].

1.5.8 Interpolation of the Volatility Spread between Maturity Pillars

Interpolation of smile curves between maturity pillars is another issue. One can come up with numerous ideas, which we revisit in [141]. A simple method to interpolate the volatility spread to ATM uses the interpolation of the spread on the two surrounding maturity pillars for the initial Black–Scholes delta of the option. The spread is interpolated using square root of time where $\tilde{\sigma}$ is the volatility spread,

$$\tilde{\sigma}(t) = \tilde{\sigma}_1 + \frac{\sqrt{t} - \sqrt{t_1}}{\sqrt{t_2} - \sqrt{t_1}}(\tilde{\sigma}_2 - \tilde{\sigma}_1). \tag{159}$$

The spread is added to the interpolated ATM volatility as calculated above.

An example of a complete volatility surface with interpolation is shown in Figure 1.12.

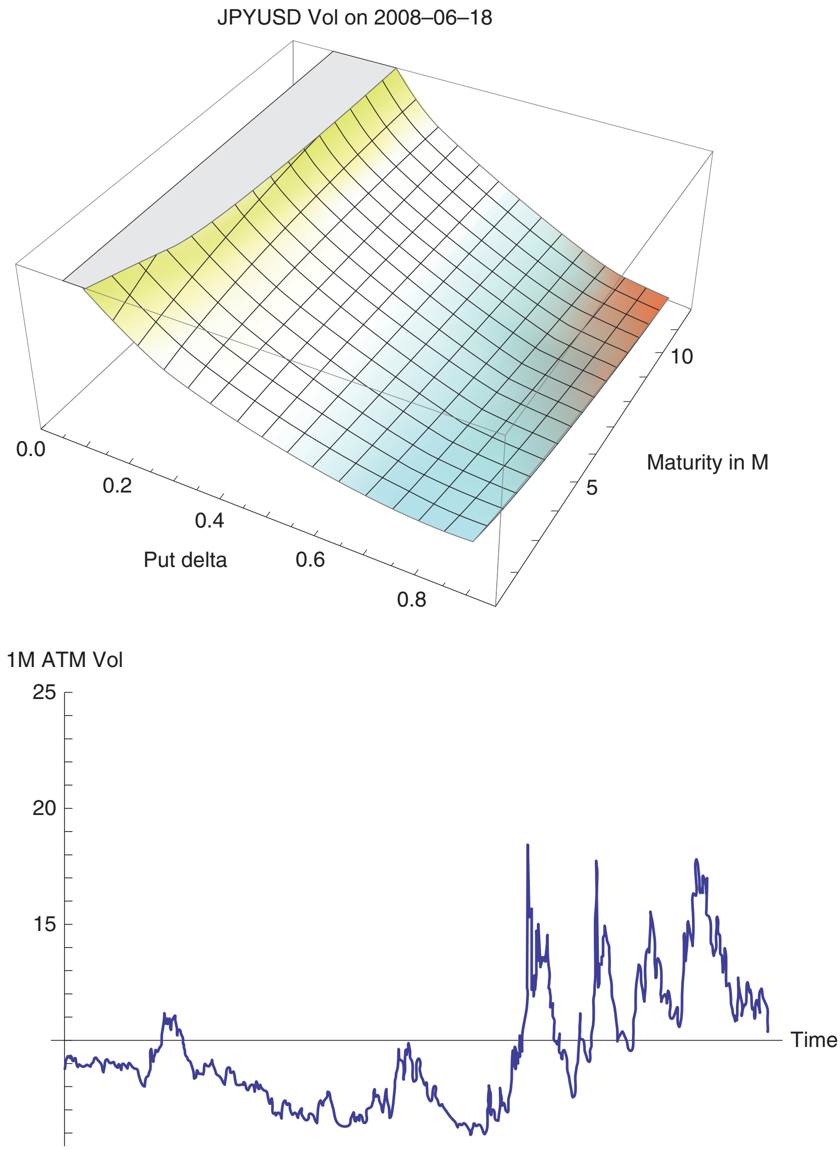


FIGURE 1.12 USD/JPY volatility surface on the delta space up to one-year tenor and historic ATM volatilities.

1.5.9 Volatility Sources

1. BBA, the British Bankers' Association, used to provide historic smile data for all major currency pairs in spreadsheet format at www.bba.org.uk.
2. Olsen Data (www.olsendata.com) can provide tic data of historic spot rates, from which the historic volatilities can be computed.

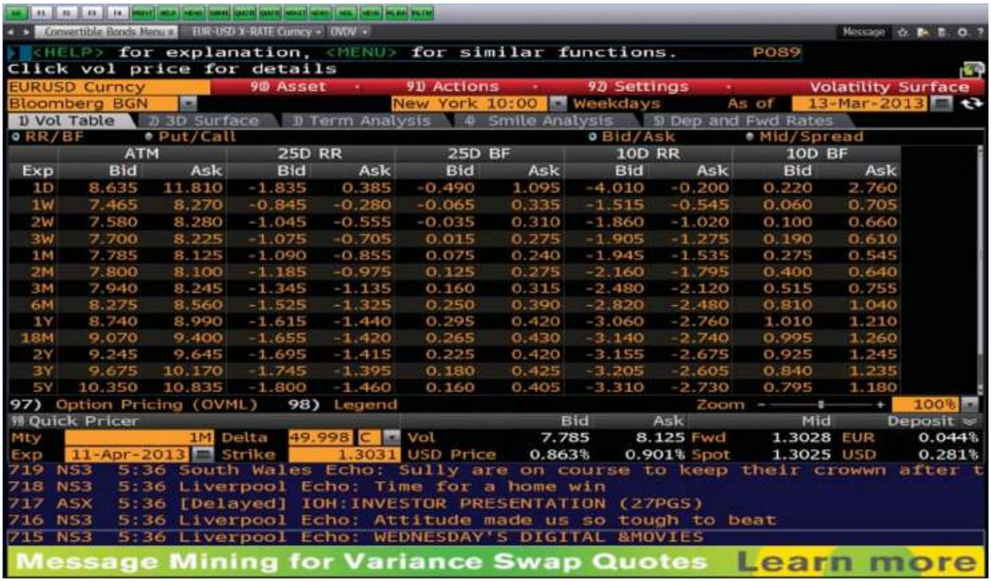
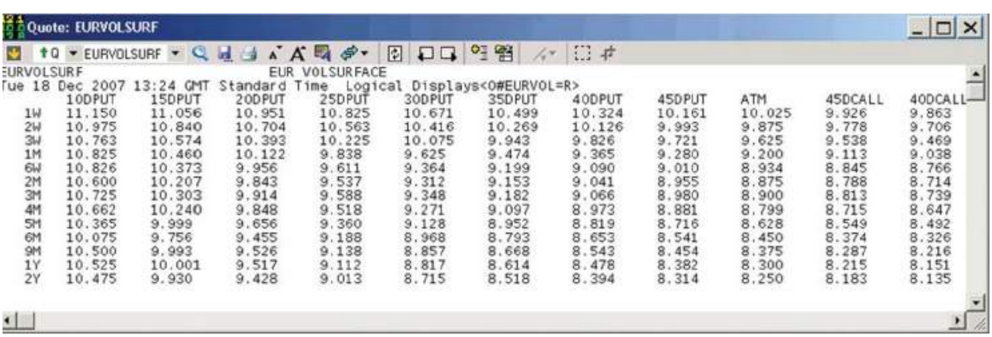


FIGURE 1.13 Bloomberg page OVDV quoting currency option volatilities.

3. Bloomberg provides both implied volatilities (OVDV) and historic volatilities, see Figure 1.13.
4. SuperDerivatives (www.superderivatives.com) besides being an internet pricing platform also provides FX volatility surfaces. An example of such a surface is presented in Figure 1.14.
5. Reuters pages such as FXMOX, SGFXVOL01, and others are commonly used and contain mostly implied volatilities. JYSKEOPT is a common reference for volatilities of Scandinavia (scandie-vols). NMRC has some implied volatilities for precious metals. An example of a EUR/USD volatility surface is presented in Figure 1.15. The common RICs (Reuters Instrument Codes) used for EUR-USD are EUR= for the spot, EUR6M= for the 6-month outright forward, EUR6MD= and USD6MD= for the 6-month money market deposit rates in EUR and USD, EUR6MO= for the 6-month ATM volatility, EUR6MRR=, EUR6MR10=, EUR6MBF=, EUR6MB10= for 6-month 25-delta and 10-delta risk reversals and butterflies. The entries of the EURVOLSURF page are available as RIC via EUR20P3M=R for the 6-month 20-delta EUR put volatility; the different contributing brokers can be addressed directly by stating EUR6MRR=FI for GFI (FENICS), =ICAP, =TPI for Tullett, =R for a recalculation performed by Reuters. Good luck!
6. Among the common brokers we find ICAP, GFI, or Tullett Prebon – see Figure 1.16 for an example of USD-JPY volatilities.
7. Telerate pages such as 4720 deliver implied volatilities.
8. Cantorspeed 90 also provides implied volatilities.

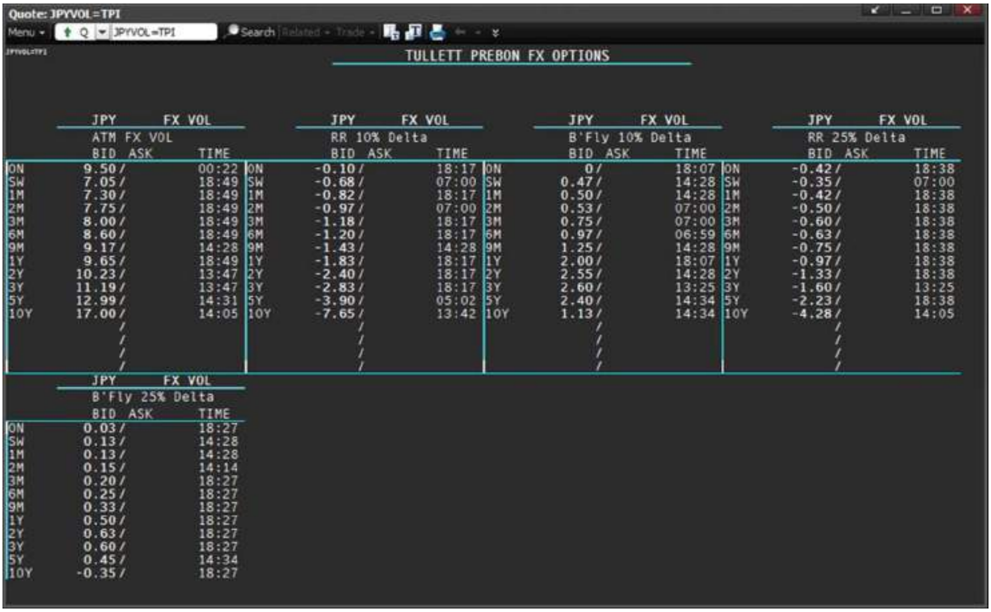
	25ΔRR	25ΔFly		5 Δ	10 Δ	15 Δ	20 Δ	25 Δ	30 Δ	ATM	30 Δ	25 Δ	20 Δ	15 Δ	10 Δ	5 Δ
1W	0.520	0.275	Vol	11.58	11.26	11.10	11.00	10.92	10.87	10.910	11.28	11.45	11.61	11.82	12.12	12.68
			Strike	68.238	68.675	68.952	69.166	69.346	69.505	70.049	70.632	70.810	71.013	71.259	71.585	72.117
1M	0.785	0.273	Vol	10.77	10.57	10.48	10.42	10.39	10.36	10.500	10.98	11.17	11.37	11.62	11.96	12.59
			Strike	66.718	67.554	68.098	68.523	68.883	69.204	70.312	71.571	71.962	72.410	72.957	73.688	74.899
2M	0.884	0.308	Vol	10.89	10.69	10.59	10.53	10.50	10.47	10.625	11.16	11.38	11.61	11.88	12.27	12.96
			Strike	65.730	66.861	67.600	68.177	68.668	69.106	70.614	72.384	72.938	73.577	74.360	75.415	77.181
3M	0.998	0.349	Vol	11.28	11.06	10.96	10.90	10.86	10.83	11.000	11.61	11.85	12.11	12.42	12.84	13.62
			Strike	64.739	66.176	67.115	67.851	68.476	69.035	70.951	73.277	74.011	74.861	75.908	77.328	79.733
6M	1.239	0.409	Vol	11.72	11.45	11.32	11.24	11.19	11.17	11.400	12.14	12.43	12.73	13.09	13.60	14.53
			Strike	63.192	65.268	66.621	67.681	68.581	69.382	72.123	75.693	76.841	78.177	79.839	82.127	86.100
9M	1.439	0.451	Vol	12.04	11.68	11.51	11.41	11.35	11.33	11.619	12.45	12.79	13.11	13.50	14.05	15.08
			Strike	62.429	65.055	66.754	68.078	69.197	70.187	73.565	78.217	79.737	81.508	83.723	86.801	92.253
1Y	1.635	0.493	Vol	12.35	11.88	11.65	11.51	11.42	11.41	11.750	12.68	13.06	13.39	13.79	14.34	15.40
			Strike	61.528	64.676	66.690	68.248	69.556	70.701	74.591	80.219	82.088	84.258	86.976	90.767	97.563
2Y	2.038	0.575	Vol	12.48	11.95	11.70	11.55	11.46	11.46	11.924	13.05	13.50	13.90	14.38	15.05	16.33
			Strike	60.684	65.157	68.038	70.272	72.151	73.792	79.280	88.400	91.555	95.270	100.012	106.813	119.568
				5 Δ	10 Δ	15 Δ	20 Δ	25 Δ	30 Δ	ATM	30 Δ	25 Δ	20 Δ	15 Δ	10 Δ	5 Δ

FIGURE 1.14 SuperDerivatives displaying EUR/INR option volatilities. The figures in boxes are meant to be read as market input; all other figures are calculated by SuperDerivatives' proprietary interpolation and extrapolation methods.



	10DPUT	15DPUT	20DPUT	25DPUT	30DPUT	35DPUT	40DPUT	45DPUT	ATM	45DCALL	40DCALL
1W	11.150	11.056	10.951	10.825	10.671	10.499	10.324	10.161	10.025	9.926	9.863
2W	10.975	10.840	10.704	10.563	10.416	10.269	10.126	9.993	9.875	9.778	9.706
3W	10.763	10.574	10.393	10.225	10.075	9.943	9.826	9.721	9.625	9.538	9.469
1M	10.825	10.460	10.122	9.838	9.625	9.474	9.365	9.280	9.200	9.113	9.038
6W	10.826	10.373	9.956	9.611	9.364	9.199	9.090	9.010	8.934	8.845	8.766
2M	10.600	10.207	9.843	9.537	9.312	9.153	9.041	8.955	8.875	8.788	8.714
3M	10.725	10.303	9.914	9.588	9.348	9.182	9.066	8.980	8.900	8.813	8.739
4M	10.662	10.240	9.848	9.518	9.271	9.097	8.973	8.881	8.799	8.715	8.647
5M	10.365	9.999	9.656	9.360	9.128	8.952	8.819	8.716	8.628	8.549	8.492
6M	10.075	9.756	9.455	9.188	8.968	8.793	8.653	8.541	8.450	8.374	8.326
9M	10.500	9.993	9.526	9.138	8.857	8.668	8.543	8.454	8.375	8.287	8.216
1Y	10.525	10.001	9.517	9.112	8.817	8.614	8.478	8.382	8.300	8.215	8.151
2Y	10.475	9.930	9.428	9.013	8.715	8.518	8.394	8.314	8.250	8.183	8.135

FIGURE 1.15 Reuters displaying EUR/USD option volatilities. It is not obvious to me (but hopefully to Reuters and its subscribers) which figures are used as market input and how the rest of the figures are interpolated or extrapolated.



Quote: JPYVOL=TPI

TULLETT PREBON FX OPTIONS

	JPY FX VOL ATM FX VOL BID ASK	TIME	JPY FX VOL RR 10% Delta BID ASK	TIME	JPY FX VOL B'Fly 10% Delta BID ASK	TIME	JPY FX VOL RR 25% Delta BID ASK	TIME	JPY FX VOL B'Fly 25% Delta BID ASK	TIME
ON	9.50 /	00:22	-0.10 /	18:17	0 /	18:07	-0.42 /	18:38	0.03 /	18:27
SW	7.05 /	18:49	-0.68 /	07:00	0.47 /	14:28	-0.35 /	07:00	0.13 /	14:28
1M	7.30 /	18:49	-0.82 /	18:17	0.50 /	14:28	-0.42 /	18:38	0.13 /	14:28
2M	7.75 /	18:49	-0.97 /	07:00	0.53 /	07:00	-0.50 /	18:38	0.15 /	14:14
3M	8.00 /	18:49	-1.18 /	18:17	0.75 /	07:00	-0.60 /	18:38	0.20 /	18:27
6M	8.60 /	18:49	-1.20 /	18:17	0.97 /	06:59	-0.63 /	18:38	0.25 /	18:27
9M	9.17 /	14:28	-1.43 /	14:28	1.25 /	14:28	-0.75 /	18:38	0.33 /	18:27
1Y	9.65 /	18:49	-1.83 /	18:17	2.00 /	18:07	-0.97 /	18:38	0.50 /	18:27
2Y	10.23 /	13:47	-2.40 /	18:17	2.55 /	14:28	-1.33 /	18:38	0.63 /	18:27
3Y	11.19 /	13:47	-2.83 /	18:17	2.60 /	13:25	-1.60 /	13:25	0.60 /	18:27
5Y	12.99 /	14:31	-3.90 /	05:02	2.40 /	14:34	-2.23 /	18:38	0.45 /	14:34
10Y	17.00 /	14:05	-7.65 /	13:42	1.13 /	14:34	-4.28 /	14:05	-0.35 /	18:27

FIGURE 1.16 Tullett Prebon quoting USD-JPY volatilities of 14 April 2014 in terms of at-the-money, risk reversals and butterflies.

The way market data is organized can be best seen in *Volmaster*, as shown in Figure 1.17.

1.5.10 Volatility Cones

Volatility cones visualize whether current at-the-money volatility levels for various maturities are high or low compared with a recent history of these implied volatilities. This may provide information on whether it is currently advisable to buy volatility or sell volatility, i.e. to buy vanilla options or to sell vanilla options. We fix a time horizon of historic observations of mid-market at-the-money implied volatility and look at the maximum, the minimum that traded over this time horizon, and compare this with the current volatility level. Since long-term volatilities tend to fluctuate less than short-term volatility levels, the chart of the minimum and the maximum typically looks like a part of a cone. We illustrate this in Figure 1.18 based on the data provided in Table 1.11.

1.5.11 Stochastic Volatility

Stochastic volatility models are very popular in FX options, whereas *jump diffusion models* can be considered as the cherry on the cake and are left to currency pairs with high jump risk. The most prominent reason for the popularity is very simple: FX volatility *appears* stochastic, as *is* shown for instance in Figure 1.19. Treating stochastic volatility in detail here is way beyond the scope of this book. A more recent overview

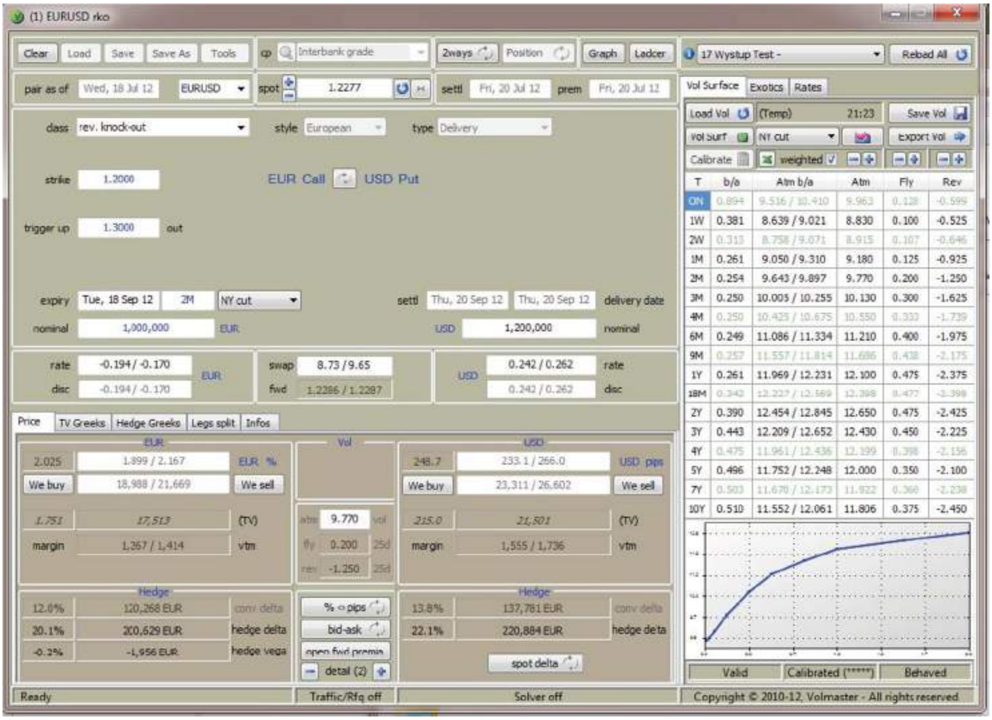


FIGURE 1.17 Volmaster single leg pricing screen with market input data on the right. "Fly" and "Rev" represent butterfly and risk reversal respectively.

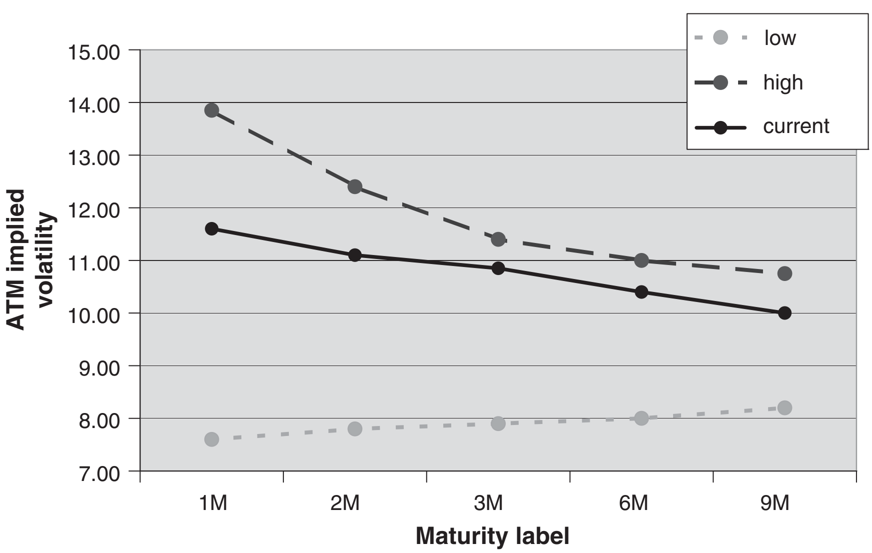


FIGURE 1.18 Example of a volatility cone in USD-JPY for a 6-month time horizon from 6 September 2003 to 24 February 2005.

TABLE 1.11 Sample data of a volatility cone in USD-JPY for a 6-month time horizon from 6 September 2003 to 24 February 2005.

Maturity	Low	High	Current
1M	7.60	13.85	11.60
2M	7.80	12.40	11.10
3M	7.90	11.40	10.85
6M	8.00	11.00	10.40
12M	8.20	10.75	10.00

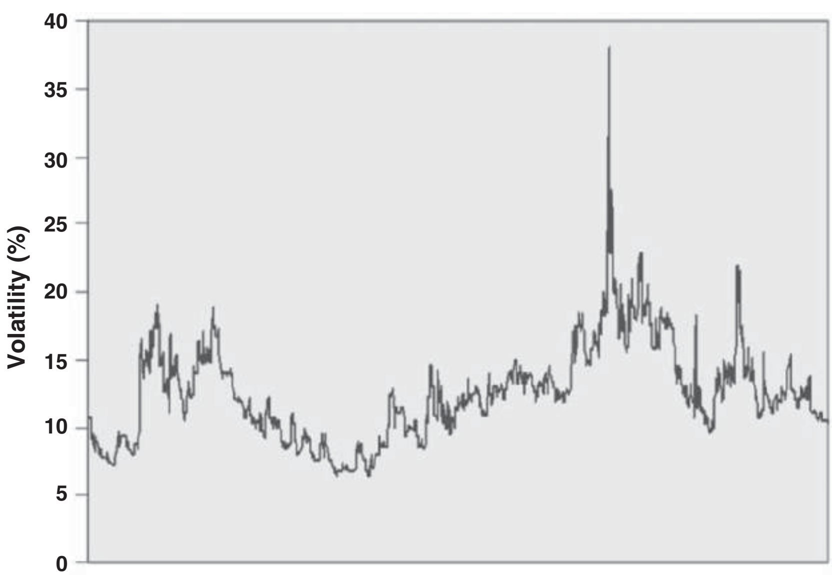


FIGURE 1.19 Historic implied volatilities for USD-JPY 1-month vanilla at-the-money options for the period 1994–2000.

can be found in the article *The Heston Model and the Smile* in Weron and Wystup [132] and subsequently by Janek *et al*. [81].

1.5.12 Exercises

Converting Risk Reversals and Strangles into Volatilities For the market data in Tables 1.7, 1.8 and 1.9 determine a smile matrix for at-the-money and the 25-deltas. Also compute the corresponding strikes for the three pillars or moneyness. We assume the continuous EUR interest rate to be $r_f = 3\%$ and the continuous GBP interest rate to be $r_d = 5\%$. The at-the-money notion here is spot delta without premium adjustment for the delta-neutral straddle.

Malz Parabolic Interpolation One way of generating a quick (and semi-dirty) volatility smile from ATM, risk reversals, and butterflies is to fit the three pieces of information to a parabola on the delta space. Assume you want to generate a parabolic function

$$\sigma(\Delta_f) = A + B(\Delta_f - 50\%) + C(\Delta_f - 50\%)^2, \tag{160}$$

where the variable is the forward delta Δ_f, the market input parameters are ATM, RR, and BF (in %), and σ is the desired parabolic interpolation function. Determine A, B, and C as functions of ATM, RR, and BF quoted in the smile version (not the brokers' version), and hereby derive the formula by Malz [95].

Slope of the Malz-Smile on the Strike Space Using the Malz parabola [95]

$$\sigma(\Delta_f) = A + B(\Delta_f - 50\%) + C(\Delta_f - 50\%)^2 \tag{161}$$

of the previous exercise, calculate the slope of the smile on the strike space in the Black-Scholes model, i.e. determine the function $\sigma'(K)$. Show that for the ATM delta-neutral strike, the windmill-adjustment (which is vanilla vega times $\sigma'(K)$) converges to

$$-\frac{\mathrm{RR}}{\pi\sigma_{\mathrm{ATM}}}, \tag{162}$$

as time to maturity turns to zero. As usual RR denotes the risk reversal. Assume that RR and the ATM volatility σ_{ATM} are constant over time.

Volatility Cones Using the historic data, generate a volatility cone for USD-JPY.

At-the-money Delta for Long-Term Options It is often believed that an at-the-money (in the sense that the strike is set equal to the forward) vanilla call has a delta near 50%. What can you say about the delta of a 15-year at-the-money USD-JPY call if USD rates are at 5%, JPY rates are at 1%, and the volatility is at 11%?

Vega Hedging ATM Calls Suppose you have just sold a 2Y ATM call. How many 1Y ATM calls do you need to buy to be vega neutral? You may assume the ATM convention to be delta-neutral.

Which Forward Rate Does the Smile Surface Use? Suppose you are provided with a EUR/USD volatility smile matrix as in Reuters VOLSURF, and for 6 months (182 days) you see a 25-delta put volatility 12.191% for strike 1.0273, 10-delta put volatility 14.147% for strike 0.9591: which forward rate and EUR money market rate are used? You may assume that delta is spot delta, premium excluded.

1.6 BASIC STRATEGIES CONTAINING VANILLA OPTIONS

Linear combinations of vanillas are quite well known and have been explained in several textbooks, including the one by Spies [121]. Therefore, we will restrict our attention in this section to the most basic strategies.

1.6.1 Call and Put Spread

A call spread is a combination of a long and a short call option. It is also called *capped call*. The motivation to do this is the fact that buying a simple call may be too expensive and the buyer wishes to lower the premium. At the same time he does not expect the underlying exchange rate to appreciate above the strike of the short call option.

The call spread entitles the holder to buy an agreed amount of a currency (say EUR) on a specified date (maturity) at a pre-determined rate (long strike) as long as the exchange rate is above the long strike at maturity. However, if the exchange rate is above the short strike at this time, the holder's profit is limited to the spread as defined by the short and long strikes (see example below). Buying a call spread provides protection against a rising EUR with full participation in a falling EUR. The holder has to pay a premium for this protection. The holder will typically exercise the option at maturity if the spot is above the long strike.

Advantages

- Protection against stronger EUR/weaker USD
- Low-cost product
- Maximum loss is the premium paid

Disadvantages

- Protection is limited when the exchange rate is above the short strike at maturity

The buyer has the chance of full participation in a weaker EUR/stronger USD. However, in case of very high EUR at maturity the protection works only up to the higher strike.

For example, a company wants to buy 1 M EUR. At maturity:

1. If $S_T < K_1$, it will not exercise the option. The overall loss will be the option's premium. But instead the company can buy EUR at a lower spot in the market.
2. If $K_1 < S_T < K_2$, it will exercise the option and buy EUR at strike K_1.
3. If $S_T > K_2$, it will buy the 1 M EUR at a rate $K_2 - K_1$ below S_T.

Example A company in the EUR-zone wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be able to buy EUR at a lower spot rate if the EUR weakens on the one hand, but on the other hand be protected against a stronger EUR. The vanilla call is too expensive, but the company does not expect a large upward movement of the EUR.

In this case a possible form of protection that the company can use is to buy a call spread as for example listed in Table 1.12. The payoff and effective final exchange rate are exhibited in Figure 1.20.

TABLE 1.12 Example of a call spread.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put with lower strike
Company sells	EUR call USD put with higher strike
Maturity	1 year
Notional of both call options	EUR 1,000,000
Strike of the long call option	1.1400 EUR-USD
Strike of the short call option	1.1800 EUR-USD
Premium	USD 20,000.00
Premium of the long EUR call only	USD 63,000.00

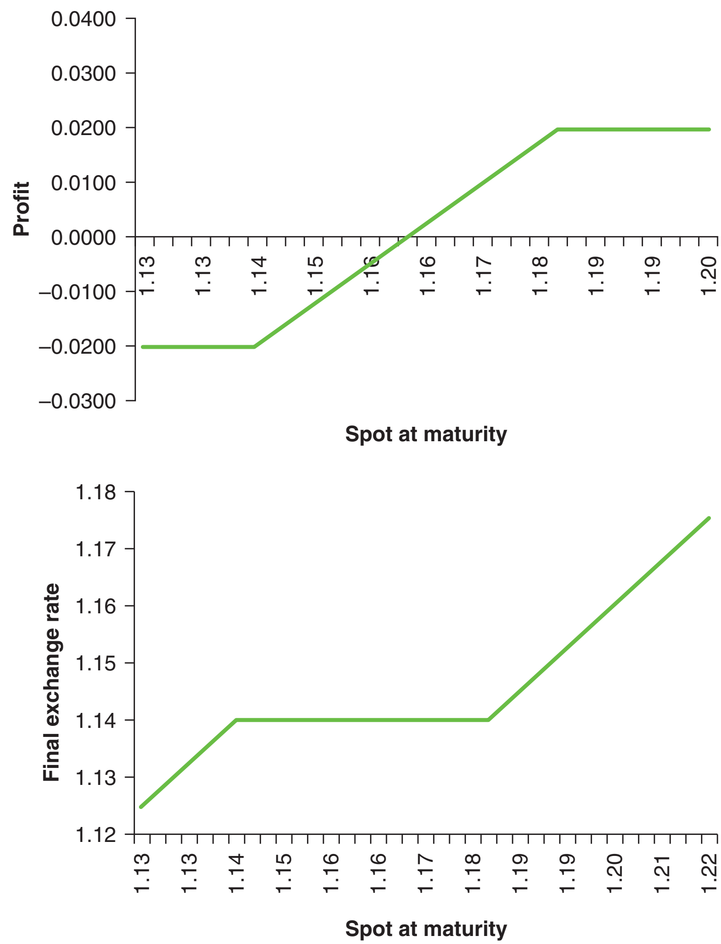


FIGURE 1.20 Profit & loss and final exchange rate of a call spread.

- If the company's market expectation is correct, it can buy EUR at maturity at the strike of 1.1400.
- If the EUR-USD exchange rate is below the strike at maturity, the option expires worthless. However, the company would benefit from a lower spot when buying EUR.
- If the EUR-USD exchange rate is above the short strike of 1.1800 at maturity, the company can buy the EUR amount 400 pips below the spot. Its risk is that the spot at maturity is very high.

The EUR seller can buy a EUR put spread in a similar fashion.

Critical Assessment The Call Spread lowers the cost of the protection against a rising EUR for the treasurer, but fails to protect the extreme risk. Viewed as an insurance it covers small losses but fails to cover potential big losses. One can use it, but we would want to be sure the treasurer understands the consequences. The situation is different for a different client type: the investor, i.e. the market participant *without* the underlying cash flow. The investor buying a Call Spread merely waives participation in an extreme rise of the underlying market. This is why Call Spreads are commonly used in private banking and retail banking.

Ratio Call Spread A variation of a Call Spread is a *Ratio Call Spread*. The treasurer/investor sells more than one Call with the higher strike, e.g. two Calls. The Call Spread becomes *leveraged* with a leverage of 2. The incentive is to generate a strategy that is even cheaper than the Call Spread, ideally in fact zero cost. One can achieve this by lowering the higher strike or by increasing the leverage. The problem will then be that the position of a long Ratio Call Spread can become negative if spot increases substantially. Let me tell you the story of a Turkish trader who set up a speculative position in USD-TRY in 2008. The trader used a highly leveraged ratio call spread on a margined account following a trade idea that the Turkish Lira would depreciate over the coming months, but not become weaker than 1.6000 USD-TRY, a level the trader had chosen for the higher strike, as illustrated in Figure 1.21.

It turned out that the trader had underestimated the Gamma/Vega exposure with rising spot, which consequently led to a high VaR. The trader received repeated margin calls, could eventually not meet them, and the bank had force-closed his position. The trader filed a claim against the bank stating that the close-out was unfair, and that eventually the Lira did what he had predicted.

The claimant being an experienced SuperDerivatives user alleged he had never heard of smile. The effect of smile on the valuation is illustrated in Figure 1.22. What went wrong? At the trade's inception, the value of the position was close to zero, because the leverage of the Ratio Call Spread was designed like that. Furthermore, since the vega and gamma of the long Call and the vega and gamma of the short Call neutralized each other, the trader was deceived by spotting a zero-cost and zero-risk strategy in his portfolio. Then, ignoring the smile effect and underestimating the change of the vega/gamma position with a rising USD-TRY spot led to unexpected continued margin calls. It was impossible to navigate this position through the October 2008 financial crisis with a limited amount of cash.

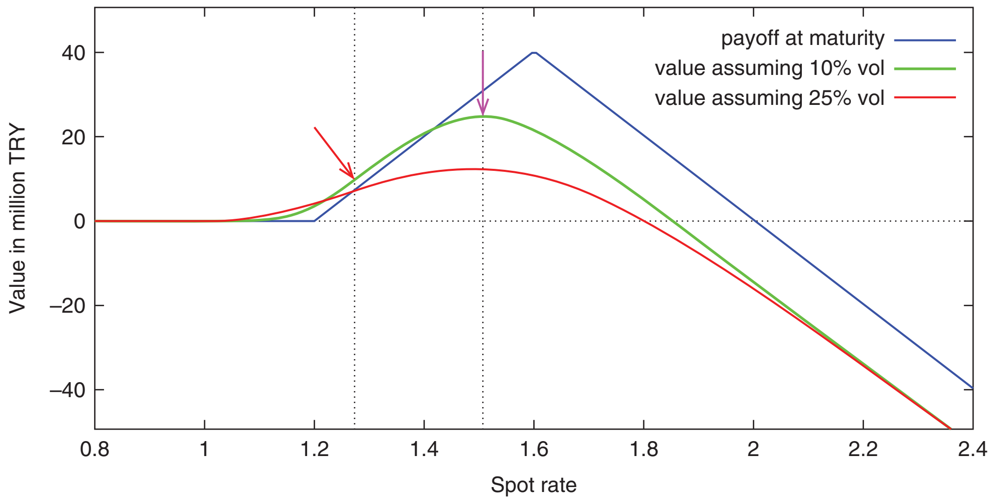


FIGURE 1.21 Position of a Ratio Call Spread reflecting the view of a rise and sharp landing of USD-TRY at about 1.6000 in 2008.

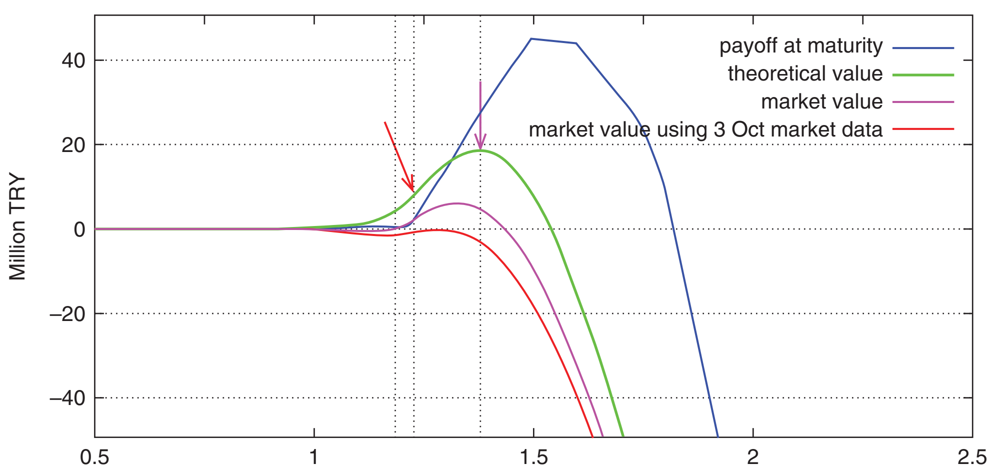


FIGURE 1.22 Smile effect in a ratio call spread in USD-TRY.

Loss Calculation In order to calculate the loss of the position, which was delta-hedged on top of everything for a reason I failed to understand, the claimant's dodgy expert calculated the loss amount by constructing a "zero gamma – long theta" portfolio.

- Consider the change of value of the option book V:

$$V(S + \delta S, t + \delta t) = V(S, t) + \Delta\ \delta S + \frac{1}{2}\Gamma\ \delta S^2 + \Theta\ \delta t + o(\delta t, \delta S^2)$$

- Then the change of the delta-hedged option portfolio $P(S,t) \stackrel{\Delta}{:=} V(S,t) + h\ S$ becomes

$$P(S+\delta S, t+\delta t) = P(S,t) + (h+\Delta)\delta S + \tfrac{1}{2}\Gamma\ \delta S^2 + \Theta\ \delta t + o(\delta t, \delta S^2).$$

- The P&L at time $t+\delta t$ is

$$\begin{aligned} P(S+\delta S, t+\delta t) - P(S,t) \\ = (h_t + \Delta)\delta S + \tfrac{1}{2}\Gamma\ \delta S^2 + \Theta\ \delta t + o(\delta t, \delta S^2) \end{aligned} \tag{163}$$

- The standard delta hedge $h_t := -\Delta$ yields

$$\text{P\&L at time } (t+\delta t) = \tfrac{1}{2}\Gamma\ \delta S^2 + \Theta\ \delta t + o(\delta t, \delta S^2)$$

The dodgy expert had argued that the spot reference for the delta hedge was unknown because the delta hedge had been executed during the day, and that therefore one must use the average of the previous day's end of day spot and the current day's end of day spot as a proxy for the spot reference for the intra-day delta hedge. While this may sound reasonable to an outsider, it is obvious that this average approach uses the information of the future spot, and is therefore a crystal ball approach. Some basic calculations make this very clear. Using the dodgy expert's "average hedge"

$$h_t = -\frac{\Delta_{t+\delta t} + \Delta_t}{2} = -\frac{V'(S+\delta S, t+\delta t) + V'(S,t)}{2},$$

the fact that $\Delta_t = V'(S,t)$, and Equation (163) we obtain the P&L at time $t+\delta t$ as

$$\begin{aligned} &\delta S(h_t + \Delta_t) + \tfrac{1}{2}\delta S^2\ \Gamma + \Theta\ \delta t + o(\delta t, \delta S^2) \\ &= \delta S \frac{-\Delta_{t+\delta t} - \Delta_t + 2\Delta_t}{2} + \tfrac{1}{2}\delta S^2\ \Gamma + \Theta\ \delta t + o(\delta t, \delta S^2) \\ &= -\delta S \frac{\Delta_{t+\delta t} - \Delta_t}{2} + \tfrac{1}{2}\delta S^2\ \Gamma + \Theta\ \delta t + o(\delta t, \delta S^2) \\ &= -\delta S \tfrac{1}{2} V''(S,t)\delta S + \tfrac{1}{2}\delta S^2\ \Gamma + \Theta\ \delta t + o(\delta t, \delta S^2), \\ &= \Theta\ \delta t + o(\delta t, \delta S^2). \end{aligned}$$

The dodgy expert introduced a systematic error in the P&L by artificially removing the entire gamma risk. The claim was based on artificially generated money. The mistakes were spotted by the court rather quickly and the Turkish trader withdrew his claim. As a conclusion I would like to reiterate that even simple vanilla structures can cause surprises and losses, which are unpleasant for all parties involved. Issues like leverage and short options should always be carefully discussed with the buy-side to avoid such surprises and losses.

1.6.2 Risk Reversal

Very often corporates seek so-called zero-cost strategies to hedge their international cash flows. Since buying a call requires a premium, the buyer can sell another option to finance the purchase of the call. A popular liquid product in FX markets is the risk reversal or collar or range forward. The term *cylinder* is also used as a synonym for the Risk Reversal, or more often actually refers to a more general form of a Risk Reversal to distinguish it from the standard case. A risk reversal is a combination of a long call and a short put. It entitles the holder to buy an agreed amount of a currency (say EUR) on a specified date (maturity) at a pre-determined rate (long strike) assuming the exchange rate is above the long strike at maturity. However, if the exchange rate is below the strike of the short put at maturity, the holder is obliged to buy the amount of EUR determined by the short strike. Therefore, buying a risk reversal provides full protection against rising EUR. The holder will typically exercise the option only if the spot is above the long strike at maturity. The risk on the upside is financed by a risk on the downside. Since the risk is reversed, the strategy is named Risk Reversal.

Advantages

- Full protection against stronger EUR/weaker USD
- Can be structured as a zero-cost strategy

Disadvantages

- Participation in weaker EUR/stronger USD is limited to the strike of the sold put

For example, a company wants to sell 1 M USD. At maturity T:

1. If $S_T < K_1$, it will be obliged to sell USD at K_1. Compared with the market spot the loss can be large. However, compared with the outright forward rate at inception of the trade, K_1 is usually only marginally worse.
2. If $K_1 < S_T < K_2$, it will not exercise the call option. The company can trade at the prevailing spot level.
3. If $S_T > K_2$, it will exercise the option and sell USD at strike K_2.

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be fully protected against a stronger EUR. But it finds that the corresponding plain vanilla EUR call is too expensive and would prefer a zero-cost strategy by financing the call with the sale of a put.

In this case a possible form of protection that the company can use is to buy a risk reversal, for example as indicated in Table 1.13 and exhibited in Figure 1.23.

If the company's market expectation is correct, it can buy EUR at maturity at the strike of $K_2 = 1.2250$. The risk is when the EUR-USD exchange rate is below the strike of $K_1 = 1.0775$ at maturity, the company is obliged to buy 1 M EUR at the rate of 1.0775. $K_2 = 1.2250$ is the guaranteed worst case, which can be used as a budget rate.

TABLE 1.13 Example of a Risk Reversal.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put with higher strike
Company sells	EUR put USD call with lower strike
Maturity	1 year
Notional of both options	EUR 1,000,000
Strike of the long Call option	1.2250 EUR-USD
Strike of the short Put option	1.0775 EUR-USD
Premium	EUR 0.00

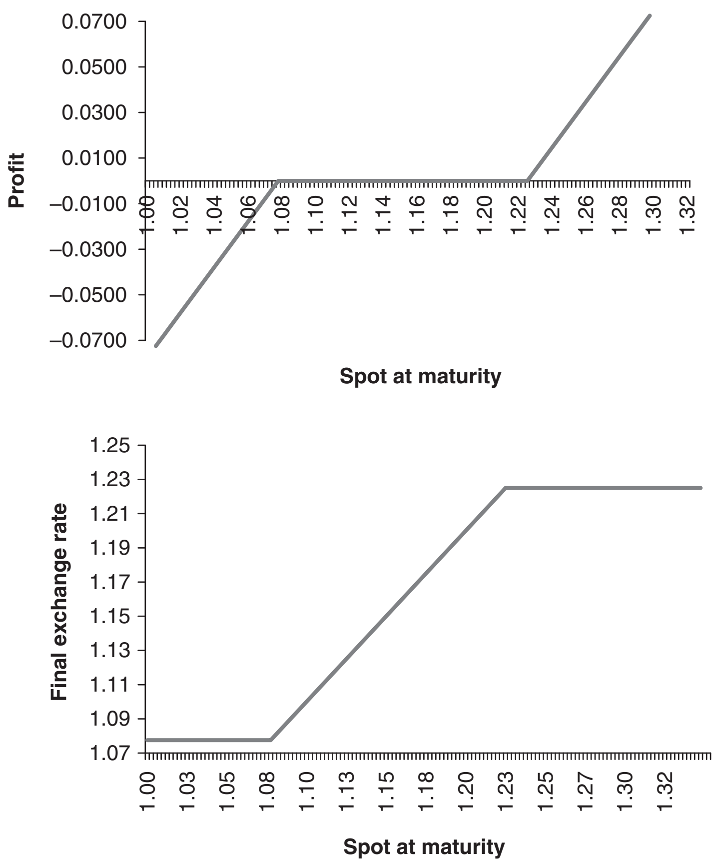


FIGURE 1.23 Payoff and final exchange rate of a risk reversal.

Critical Assessment The risk of the spot ending below the put strike is a risk only if the strategy was traded by an investor *without* the underlying cash flow. For a treasurer using the Risk Reversal as a hedge for an existing cash flow, the situation is not risky at all; in fact the treasurer can normally still buy the EUR at a rate lower than the outright forward rate he could have agreed on at inception of the trade. A common misconception is to judge the quality of a strategy based on the prevailing spot reference and compare the strategy with the "do nothing; wait for a better spot" strategy. This is not fair because the decision about how to hedge has to be taken at inception of the trade when the future spot is still unknown. The Risk Reversal is one of the most popular hedging strategies for corporate treasurers, which is why the brokers' market quote risk reversals directly, rather than stand-alone out-of-the-money option volatilities – see Section 1.5.3 on the symmetric decomposition of the smile, where risk reversal is a market quote for the skew in the smile curve. The risk-warehousing sell-side uses risk reversals as a vanna hedge, i.e. a hedge of the skew of the volatility surface – see the exercises in Section 1.6.9.

Risk Reversal Flip As a variation of the standard risk reversal, we consider the following trade on EUR/USD spot reference 1.2400 with a tenor of two months.

1. Long 1.2500/1.1900 risk reversal (long 1.2500 EUR call, short 1.1900 EUR put)
2. If 1.3000 trades before expiry, it flips into a 1.2900/1.3100 risk reversal (long 1.2900 EUR put, short 1.3100 EUR call)
3. Zero premium

The corresponding view is that EUR/USD looks bullish and may break on the upside of a recent trading range. However, a runaway higher EUR/USD setting a new all-time high within two months looks unlikely. However, if EUR/USD overshoots to 1.30, then it will likely retrace thereafter.

The main thrust is to long EUR/USD for zero cost, with a safe cap at 1.30. So the initial risk is EUR/USD below 1.19. If 1.30 is breached, then all accrued profit from the 1.25/1.19 risk reversal is lost, and the maximum risk becomes spot levels above 1.31. Therefore, this trade is not suitable for EUR bulls who feel there is scope above 1.30 within two months. On the other hand, this trade is suitable for those who feel that if spot overshoots to 1.30, then it will retrace down quickly. For early profit taking: with two weeks to go and spot at 1.28, this trade should be worth approximately 0.84 % EUR. However, with that spot the maximum profit occurs at the trade's maturity.

Composition Clearly, this risk reversal flip is rather a proprietary trading strategy than a corporate hedging structure.

The composition is presented in Table 1.14. The options used are standard barrier options, see Section 1.7.3.

TABLE 1.14 Example of a Risk Reversal flip.

client buys	1.2500 EUR call up-and-out at 1.3000
client sells	1.1900 EUR put up-and-out at 1.3000
client buys	1.2900 EUR put up-and-in at 1.3000
client sells	1.3100 EUR call up-and-in at 1.3000

1.6.3 Straddle

A straddle is a combination of a put and a call option with the same strike. At inception of the trade the strike is usually the delta-neutral at-the-money strike $K_{\pm}$ as in Equation (43). It entitles the holder to buy an agreed amount of a currency (say EUR) on a specified date (maturity) at a pre-determined rate (strike) if the exchange rate is above the strike at maturity. Alternatively, if the exchange rate is below the strike at maturity, the holder is entitled to sell the amount at this strike. Buying a straddle provides participation in both an upward and a downward movement where the direction of the rate is unclear. The buyer has to pay a premium for this product.

Advantages

- Full participation in or protection against market movement or increasing volatility
- Maximum loss is the premium paid

Disadvantages

- Expensive product
- Not suitable as directional hedge and consequently not suitable for hedge accounting of spot positions as it should be clear if the client wants to sell or buy EUR

Potential profits of a long straddle arise from movements in the spot and also from increases in implied volatility. If the spot moves, the call or the put can be sold before maturity with profit. Conversely, if a quiet market phase persists the strategy is unlikely to generate much revenue.

Figure 1.24 shows the payoff of a long straddle. The payoff of a short straddle looks like the straddle below a seesaw on a children's playground, which is where the name straddle originated.

For example, a company buys a Straddle with a nominal of 1 M EUR. At maturity T:

1. If $S_T < K$, it would sell 1 M EUR at strike K.
2. If $S_T > K$, it would buy 1 M EUR at strike K.

Example A company wants to benefit from its view that the EUR-USD exchange rate will move far from a specified strike (Straddle's strike). In this case a possible product to use is a Straddle as presented in Table 1.15 for example.

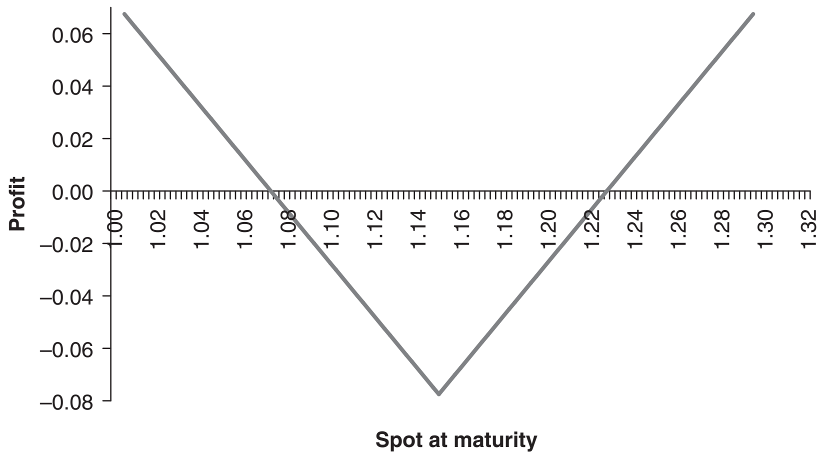


FIGURE 1.24 Profit and loss of a long Straddle.

TABLE 1.15 Example of a Straddle, valued at a volatility of about 7.8%.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put
Company buys	EUR put USD call
Maturity	1 year
Notional of both the options	EUR 1,000,000
Strike of both options	1.1500 EUR-USD
Premium	USD 77,500.00

- If the spot rate is above the strike at maturity, the company can buy 1 M EUR at the strike of 1.1500.
- If the spot rate is below the strike at maturity, the company can sell 1 M EUR at the strike of 1.1500.

The break even points are 1.0726 for the put and 1.2274 for the call. If the spot is between the break even points at maturity, then the company will make an overall loss.

Applications and Assessment The straddle is not a product for corporate treasury. Buyers are typically investors. It is a vehicle to go long or short vega, i.e. the main application on the sell-side is volatility risk management of the at-the-money volatility or levels of the volatility surface. In the exercises in Section 1.5.12 you can verify that the delta-neutral straddle does in fact maximize the vega position.

1.6.4 Strangle

A strangle is a combination of an out-of-the-money put and call option with two different strikes. At inception, the strikes are typically 25-delta. It entitles the holder to buy an agreed amount of currency (say EUR) on a specified date (maturity) at a pre-determined rate (call strike), if the exchange rate is above the call strike at maturity. Alternatively, if the exchange rate is below the put strike at maturity, the holder is entitled to sell the amount at this strike. Buying a strangle provides full participation in a strongly moving market, where the direction is not clear. The buyer has to pay a premium for this product.

Advantages

- Full participation in or protection against a highly volatile exchange rate or increasing volatility
- Maximum loss is the premium paid
- Cheaper than the straddle

Disadvantages

- Expensive product
- Not suitable for directional hedging and consequently not suitable for hedge accounting as it should be clear if the client wants to sell or buy EUR

As in the straddle the chance of the strangle lies in spot movements. If the spot moves significantly, the call or the put can be sold before maturity with profit. Conversely, if a quiet market phase persists, the strategy is unlikely to generate much revenue. Figure 1.25 shows the profit and loss diagram of a long strangle.

For example, a company buys a strangle with a nominal of 1 M EUR. At maturity T:

1. If $S_T < K_1$, it would sell 1 M EUR at strike K_1.
2. If $K_1 < S_T < K_2$, it would not exercise either of the two options. The overall loss will be the strategy's premium.
3. If $S_T > K_2$, it would buy 1 M EUR at strike K_2.

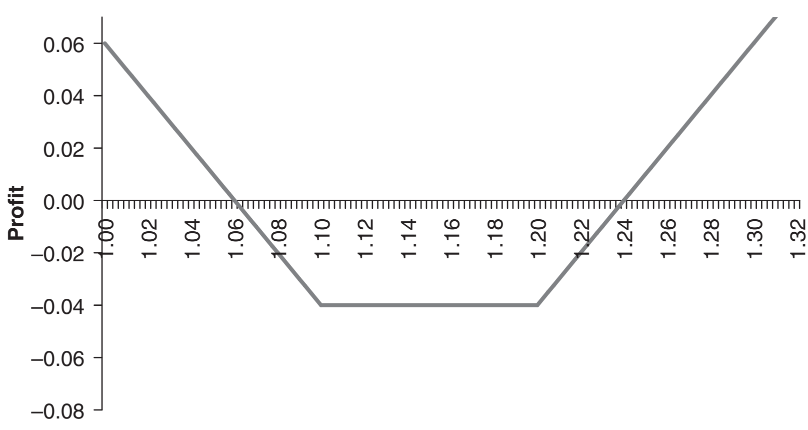


FIGURE 1.25 Profit and loss of a Strangle.

TABLE 1.16 Example of a Strangle, valued at a volatility of about 8%.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put
Company buys	EUR put USD call
Maturity	1 year
Notional of both the options	EUR 1,000,000
Put strike	1.1000 EUR-USD
Call strike	1.2000 EUR-USD
Premium	USD 40,000.00

Example A company wants to benefit from its view that the EUR-USD exchange rate will move far from two specified strikes (strangle's strikes). In this case a possible product to use is a strangle as listed in Table 1.16, for example.

- If the spot rate is above the call strike at maturity, the company can buy 1 M EUR at the strike of 1.2000.
- If the spot rate is below the put strike at maturity, the company can sell 1 M EUR at the strike of 1.1000.

However, the risk is that if the spot rate is between the put strike and the call strike at maturity, both options expire worthless.

The break even points are 1.0600 for the put and 1.2400 for the call. If the spot is between these points at maturity, then the company makes an overall loss.

Applications and Assessment The strangle is not a product for corporate treasury. Buyers are typically investors. It is a vehicle to go long or short volga, i.e. the main application on the sell-side is volatility risk management of the convexity of the volatility surface. In the exercises in Section 1.6.9 you can verify that a delta-neutral straddle does not have much vanna, but a strangle does. This explains why a risk-warehousing sell-side needs strangles as a hedging instrument for second order risk on the volatility surface. In fact, the importance of the strangle can be observed by the brokers' market quoting a *one-vol-strangle* or a *market strangle* as a strategy rather than the building blocks. For details on the market strangle revisit Section 1.5.3.

1.6.5 Butterfly

A long butterfly is a combination of a long strangle and a short straddle. Buying a long butterfly provides participation where a highly volatile exchange rate condition exists. The buyer typically *receives* a premium for this product.

Advantages of a Short Butterfly

- Limited participation in or protection against market movement or increasing volatility
- Maximum loss is the premium paid
- Cheaper than the straddle

Disadvantages of a Short Butterfly

- Limited profit
- Not suitable for directional hedging and consequently not suitable for hedge accounting as it should be clear if the client wants to sell or buy EUR

If the spot will remain volatile, a short butterfly can be bought back before maturity with profit. Conversely, if a quiet market phase persists, a short butterfly strategy is unlikely to be bought back early. Figure 1.26 shows the profit and loss diagram of a long and a short butterfly.

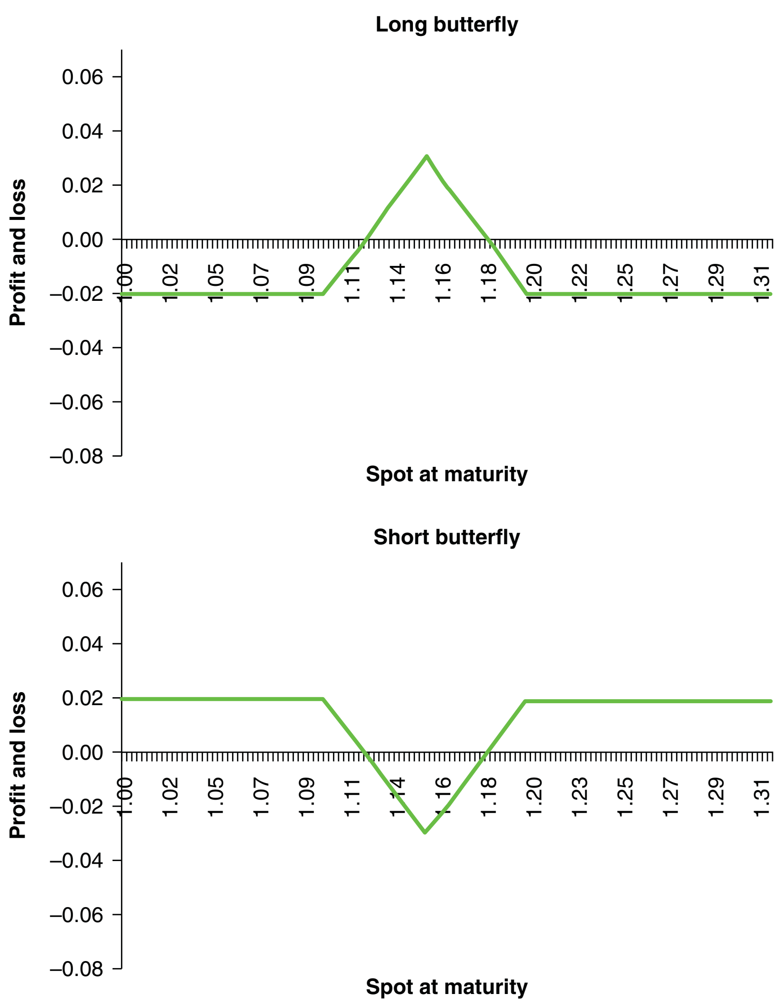


FIGURE 1.26 Profit and loss of a long and short butterfly.

For example, a company trades a short butterfly with a nominal of 1 M EUR and strikes $K_L < K_0 < K_U$. At maturity T:

1. If $S_T < K_L$, it would sell 1 M EUR at a rate $K_0 - K_L$ higher than the market.
2. If $K_L < S_T < K_0$, it would sell 1 M EUR at strike K_0.
3. If $K_0 < S_T < K_U$, it would buy 1 M EUR at strike K_0.
4. If $S_T > K_U$, it would buy 1 M EUR at a rate $K_U - K_0$ less than the market.

Example A company wants to benefit from its view that the EUR-USD exchange rate will remain volatile from a specified strike (the middle strike K_0).

In this case a possible product to use is a short butterfly as listed in Table 1.17 for example.

- If the spot rate is between the lower and the middle strike at maturity, the company can sell 1 M EUR at the strike of 1.1500.
- If the spot rate is between the middle and the higher strike at maturity, the company can buy 1 M EUR at the strike of 1.1500.
- If the spot rate is above the higher strike at maturity, the company will buy EUR 500 points below the spot.
- If the spot rate is below the lower strike at maturity, the company will sell EUR 500 points above the spot.

Applications and Assessment In principle, one can argue what is a long butterfly and what is a short butterfly. The version presented above is the common market standard, although it appears unusual to receive a premium when going long a strategy. The idea behind it is to go long convexity on the smile curve by taking a long position in a butterfly. Just like a strangle, a butterfly serves as a volga hedging strategy for the risk-warehousing sell-side. The butterfly is often preferred to the strangle because the premium is lower, delta is close to zero, vega is partially netted, and the volga of a butterfly and a strangle is identical, since at-the-money straddles do not have (much) volga. One can even go one step further and increase the notional of the strangle to generate a vega-neutral butterfly. This construction will then become a pure volga hedge without delta, gamma, or vega and is referred to as a *vega-weighted butterfly*.

TABLE 1.17 Example of a short butterfly.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional of all options	EUR 1,000,000
Lower strike K_L	1.1000 EUR-USD
Middle strike K_0	1.1500 EUR-USD
Upper strike K_U	1.2000 EUR-USD
Premium	USD 30,000.00

Alternative Construction of a Butterfly A long butterfly can also be constructed by going

- long a call struck at the lower strike,
- short two calls struck at the middle strike,
- long a call struck at the upper strike.

This construction shows that a butterfly is a market equivalent of the second derivative,

$$\text{call}(K_L) - 2\text{call}(K_0) + \text{call}(K_U), \tag{164}$$

which again underlines that a long butterfly reflects a position of long convexity. As a limiting case as difference of the strikes goes to zero, the butterfly can be viewed as an *Arrew-Debreu security*, or the probability density of the terminal spot price. Therefore, the long butterfly should have a non-negative value. If the long butterfly price is negative, it means that we have a negative probability density of the final spot distribution, which is referred to as *butterfly arbitrage*.

Note that the premium of a long butterfly constructed via call options is different from the premium of a long butterfly constructed via strangle and straddle. However, the shape of the payoff as well as the Greeks gamma, vega, vanna, and volga are the same for both constructions of the butterfly.

1.6.6 Condor

A long condor is a combination of a long strangle with far away strikes (e.g. 10-delta) and a short strangle with closer strikes (e.g. 25-delta). Buying a short condor provides participation in a large market movement up or down. A long condor provides participation in a low-volatility market. The buyer of a long condor typically *receives* a premium for this product.

Advantages of a Short Condor

- Limited participation in or protection against larger market movement or increasing volatility
- Maximum loss is the premium paid
- Cheaper than the short butterfly

Disadvantages of a Short Condor

- Limited profit
- Not suitable for directional hedging and consequently not suitable for hedge accounting as it should be clear if the client wants to sell or buy EUR

If the spot becomes more volatile, a short condor can be bought back before maturity with profit. Conversely, if a quiet market phase persists, a short condor strategy is unlikely to be bought back early. Figure 1.27 shows the profit and loss diagram of a

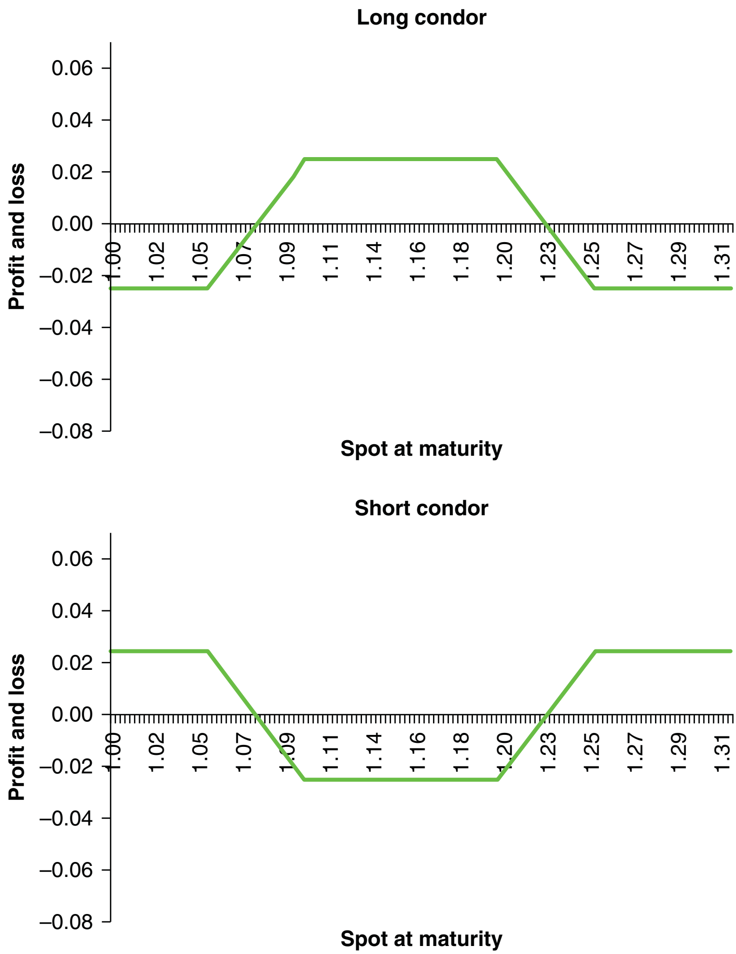


FIGURE 1.27 Profit and loss of a long and short condor, valued at a volatility of about 10%.

long and a short condor. The wide wing span of the south American condor bird is the origin of the name of the trading strategy.

For example, a company trades a short condor with a nominal of 1 M EUR and strikes $K_1 < K_2 < K_3 < K_4$. At maturity T:

1. If $S_T < K_1$, it would sell 1 M EUR at a rate $K_2 - K_1$ higher than the market.
2. If $K_1 < S_T < K_2$, it would sell 1 M EUR at strike K_2.
3. If $K_2 < S_T < K_3$, all options are out-of-the-money and no actions would follow from the strategy.
4. If $K_3 < S_T < K_4$, it would buy 1 M EUR at strike K_3.
5. If $S_T > K_4$, it would buy 1 M EUR at a rate $K_4 - K_3$ less than the market.

TABLE 1.18 Example of a short condor.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional of all options	EUR 1,000,000
Lower down strike K_1	1.0500 EUR-USD
Higher down strike K_2	1.1000 EUR-USD
Lower up strike K_3	1.2000 EUR-USD
Higher up strike K_4	1.2500 EUR-USD
Premium	USD 25,000.00

Example A company wants to benefit from its view that the EUR-USD exchange rate will become very volatile.

In this case a possible product to use is a short condor as listed in Table 1.18 for example.

- If the spot rate is between the lower and the higher down strike at maturity, the company can sell 1 M EUR at the strike of 1.1000.
- If the spot rate is between the lower and the higher up strike at maturity, the company can buy 1 M EUR at the strike of 1.2000.
- If the spot rate is above the higher up strike at maturity, the company will buy EUR 500 points below the spot.
- If the spot rate is below the lower down strike at maturity, the company will sell EUR 500 points above the spot.

Applications and Assessment As in the butterfly, one can argue what is a long condor and what is a short condor. The version presented above is the common market standard, although it appears unusual to receive a premium when going long a strategy. The idea behind it is to go long outer wing convexity on the smile curve by taking a long position in a butterfly. Obviously, this is a rather specialized market view.

Alternative Construction of a Condor A short condor can also be constructed by going

- long a call spread with lower and higher up strike,
- long a put spread with lower and higher down strike.

There is also an alternative construction of a long condor, where the buyer goes long a call at the lowest strike, short two calls at two different medium strikes, and long a call at the upper far strike, which can be remembered by going *long the wings short the belly*.

1.6.7 Seagull

A long seagull call strategy is a combination of a long call with a center strike, a short call with a higher strike, and a short put with a lower strike. It is similar to a risk reversal and typically trades at zero cost at inception. It entitles its holder to purchase an agreed amount of a currency (say EUR) on a specified date (maturity) at a pre-determined long call strike if the exchange rate at maturity is between the long call strike and the short call strike (see below for more information). If the exchange rate is below the short put strike at maturity, the holder must buy this amount in EUR at the short put strike. Buying a seagull call strategy provides good protection against a rising EUR.

Advantages

- Good protection against stronger EUR/weaker USD
- Better strikes than in a risk reversal
- Zero-cost product

Disadvantages

- Maximum loss depends on spot rate at maturity and can be arbitrarily large

As in a call spread, the protection against a rising EUR is limited to the interval from the long call strike and the short call strike. The biggest risk is a large upward movement of EUR.

Figure 1.28 shows the payoff and final exchange rate diagram of a seagull call. Rotating the payoff clockwise by about 45 degrees shows the shape of a flying seagull.

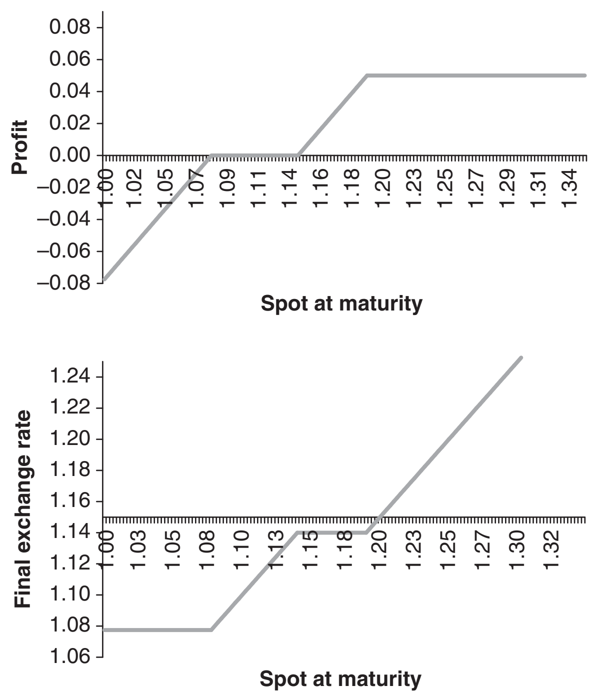


FIGURE 1.28 Profit (equivalent to the payoff for a zero-value structure) and final exchange rate of a seagull call.

Note that the diagram shows linear dependencies. This is correct only if the notional is in foreign currency (EUR). If the principal amount to be sold is in domestic currency (USD), then the relationships are no longer exactly linear.

For example, a company wants to sell 1 M USD and buy EUR. At maturity T:

1. If $S_T < K_1$, the company must sell 1 M USD at rate K_1.
2. If $K_1 < S_T < K_2$, all involved options expire worthless and the company can sell USD in the spot market.
3. If $K_2 < S_T < K_3$, the company would buy EUR at strike K_2.
4. If $S_T > K_3$, the company would sell 1 M USD at a rate $K_2 - K_1$ less than the market.

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD but not a large upward movement of the EUR. The company wishes to be protected against a stronger EUR and finds that the corresponding plain vanilla is too expensive and would prefer a zero-cost strategy and is willing to limit protection on the upside.

In this case a possible form of protection that the company can use is a seagull call as presented in Table 1.19 for example.

- If the company's market expectation is correct, it can buy EUR at maturity at the strike of 1.1400.
- If the EUR-USD exchange rate will be above the short call strike of 1.1900 at maturity, the company will sell USD at 500 points less than the spot.
- If the EUR-USD exchange rate is below the strike of 1.0775 at maturity, it will have to sell 1 M USD at the strike of 1.0775.

Critical Assessment Which part of the seagull is risky depends on the client type. For a treasurer *with* the underlying cash flow, the risky part is the lack of protection against large spot moves up. The same problem as in the case of a call spread applies. For an investor *without* the underlying cash flow, the short put is the driver of risk. One can remove the risk for both if we alter the seagull structure: the EUR buyer USD seller could buy a EUR call and finance it by selling a EUR put spread. This construction would remove the risk on the downside for the investor because the potential loss is limited. It would also fix the lack of missing upside protection for the treasurer. However, this version of a seagull generates less attractive levels.

TABLE 1.19 Example of a seagull call.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional	USD 1,000,000
Company buys	EUR call USD put strike 1.1400
Company sells	EUR call USD put strike 1.1900
Company sells	EUR put USD call strike 1.0775
Premium	USD 0.00

1.6.8 Calendar Spread

A calendar spread is a strategy comprised of a long option with a longer maturity and a short option with a shorter maturity. Typically, both options are at-the-money. For at-the-money options the longer dated option is more expensive than the shorter dated option, whence the buyer has to pay a premium for the calendar spread. The trader of a calendar spread reflects his view of the slope of the volatility on the tenor space. Negative calendar spread prices indicate a *calendar arbitrage*. A calendar spread of options with different moneyness is called a *diagonal spread*.

More generally, calendar spreads can be differences of derivatives or structured products with two different maturity dates. For example, one can trade a rolling strategy of calendar spreads of variance swaps as a crash protection.

1.6.9 Exercises

Seagull For EUR/GBP spot ref 0.7000, volatility 8%, EUR rate 2.5%, GBP rate 4%, and flat smile, find the strike of the short EUR put for a 6-month zero-cost seagull put, where the strike of the long EUR put is 0.7150, the strike of the short call is 0.7300, and the desired sales margin is 0.1% of the GBP notional. What is the value of the seagull put after three months if the spot is at 0.6900 and the volatility is at 7.8%?

Straddle Vanna At-the-money straddles are commonly used as vega hedge instruments. Since vega of a vanilla option is concentrated around ATM, explain why vanna is not also concentrated around ATM. To clarify: vega attains its maximum if the strike is chosen as the one that makes a straddle delta-neutral. Vega is the first derivative of the option's value as volatility changes. Vanna is the second derivative of the option's value as spot and volatility change. As a mixed second derivative it has two interpretations:

Interpretation 1: Vanna = ∂ vega $/\partial$ spot

Interpretation 2: Vanna = ∂ delta $/\partial$ vol

1. To understand why vega peaks ATM and vanna OTM, plot vanilla vega on the spot space.
2. Calculate vanna for an at-the-money straddle as a formula in the Black-Scholes model.
3. Following interpretation 2, plot a vanilla (call) delta for a small volatility and a big volatility.

Straddle Volga Calculate volga for an at-the-money straddle.

Butterfly Premium Difference By how much does the premium of a long butterfly constructed via call options as in Equation (164) differ from the premium of a long butterfly constructed via strangle and straddle?

Short Gamma Long Vega Find a strategy (in the sense of a linear combination) of vanilla options that is short gamma and long vega (in the Black-Scholes model). Explain why this is not possible for single vanilla option. Hint: revisit Equation (19) for gamma and Equation (24) for vega of a vanilla option in the Black-Scholes model.

1.7 FIRST GENERATION EXOTICS

For the sake of example we consider EUR/USD – the most liquidly traded currency pair in the foreign exchange market. Internationally active market participants are always subject to changing foreign exchange rates. To hedge this exposure an immense variety of derivatives transactions is traded worldwide. Besides vanilla (European style put and call) options, the so-called first generation exotics have become standard derivative instruments.

1.7.1 Classification

The term *first generation exotic* does not refer to a clearly defined set of derivatives contracts, especially not in a legal sense. However, it is universally agreed that Foreign Exchange transactions (spot and forward contracts) and vanilla options are not in the set. It is also universally agreed that flip-flop-kiko-tarns and correlation swaps are not in the set either. We can then classify first generation exotics by:

Time of Introduction: Here we consider the history and the time when certain contracts first traded.

Existence of Standardized Deal Confirmations: We would classify a transaction as first generation exotic if there exists a standardized deal confirmation template, such as the ones provided by ISDA.

Replicability: We would classify a transaction as first generation exotic if it can be statically or semi-statically replicated or approximated by spot, forward, and vanilla option contracts.

Trading Volume: We would classify a transaction as first generation exotic if its trading volume is sufficiently high (and the transaction is not a spot, forward, or vanilla option).

There can also be other approaches to classify first generation exotics. I would like to point out that a first generation exotic does not necessarily need to be a currency option. For example, a flexi forward can be considered a first generation exotic in terms of both timing and standardization, but is clearly not an option. A variance swap can be considered a first generation exotic in terms of both standardization and replicability, but is clearly not an option because there is no right to exercise. Classification by trading volume would change the set of first generation exotics over time and is consequently not suitable for classification purposes. The various classifications would generate overlaps as well as differences. One could certainly argue to label barrier options as first generation exotic because they would satisfy all of the above: timing, standardization, replicability, and volume. For Asian options, the timing criterion would make them first generation as they started trading in Tokyo in 1987, but there was – even in 2016 – no standardized deal confirmation provided by ISDA. Power options satisfy timing and replicability, but not standardization or trading volume. This leads to the effect that the transition between the generations is not strict and can depend on the person you ask and classification they have in mind. A clean approach to classification could be sticking to the standardization, which would classify barrier options and touch products, as well as variance and volatility swaps as first generation exotic, based on the existing ISDA definitions and their supplements. The question of which transaction is standardized can then be viewed in light of ISDA's *Barrier Option Supplement* [78], which appeared in 2005. ISDA has extended the 1998 *FX and Currency Option Definitions* [77] to the range of touch products and single and double barrier options, including time windows for barriers. These are (a) options that knock in or out if the underlying hits a barrier (or one of two barriers) and (b) all kind of touch products: a one-touch [no-touch] pays a fixed amount of either USD or EUR if the spot ever [never] trades at or beyond the touch level and zero otherwise. Double one-touch and no-touch contracts work the same way but have two barriers. More on barrier options can be found in Section 1.7.3. The ISDA *Barrier Option Supplement* contains all the relevant definitions required to confirm these transactions by standardized short templates. It is clearly defined what a *barrier event* or a *determination agent* is. However, for purposes of classification, the product range covered by this ISDA supplement is not necessarily viewed as equivalent by all market participants. Moreover, the set of first generation exotics would then change each time ISDA publishes a new supplement. My personal preference is to classify the set of first generation exotics by the time of introduction in the market. This is reflected in this section.

1.7.2 European Digitals and the Windmill Effect

In this section we discuss the digital options along with the questions

1. How can we price digital options with smile?
2. Is the implied volatility of a digital option the same as the implied volatility of the corresponding vanilla option?
3. What is the windmill effect?

Digital Options (European) digital options pay off

$$v(T;S_T) = I\!I_{\{\phi S_T \geq \phi K\}} \text{ domestic paying,} \tag{165}$$

$$w(T;S_T) = S_T I\!I_{\{\phi S_T \geq \phi K\}} \text{ foreign paying.} \tag{166}$$

In the domestic paying case the payment of the fixed amount is in domestic currency, whereas in the foreign paying case the payment is in foreign currency. We obtain for the theoretical value functions

$$v(t;x) = e^{-r_d \tau} \mathcal{N}(\phi d_-), \tag{167}$$

$$w(t;x) = x e^{-r_f \tau} \mathcal{N}(\phi d_+), \tag{168}$$

of the digital options paying one unit of domestic and paying one unit of foreign currency respectively.

The question is how we can use the existing smile for vanilla options to read off a suitable volatility that we can plug into (167) to get a smile-adjusted value for the digital. In particular, can we take the same volatility as for the vanilla with strike K? The answer can be found by looking at the static replication and its associated cost.

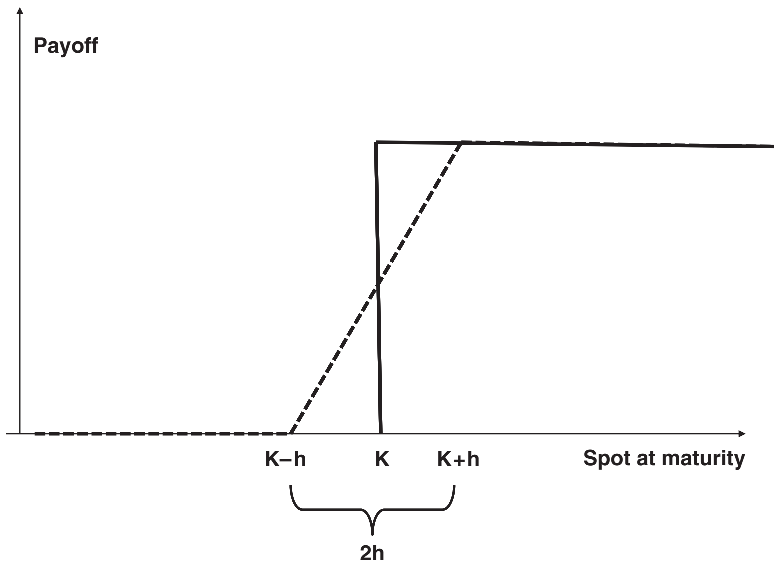


FIGURE 1.29 Replicating a digital call with a vanilla call spread.

Replication of Digital Options An obvious attempt to replicate a domestic digital is the call spread illustrated in Figure 1.29.

In the limit we have

$$\text{digital}(K) = \lim_{h \to 0} \frac{\text{vanilla}(K - h) - \text{vanilla}(K + h)}{2h} \tag{169}$$

However, since the number $\frac{1}{2h}$ corresponds to the (foreign!) notional of the vanilla options to trade, there are practical limitations we need to approximate the digital with a call spread with finite notional. To be on the safe side, the replication can be built as a super-replication with the upper strike chosen as the strike of the digital. This is too expensive to be used for pricing. For pricing we go for the symmetric compromise and choose one strike to be lower than the strike of the digital and the other one higher. The practical limitation for the difference of the two strikes is two pips. This is equivalent to a factor of 5000 to compute the notional of the vanilla options. In practice one would mostly take a larger difference or equivalently a smaller notional, say a factor of 50, for the notional multiplier. In this case the two strikes are two big figures apart. Consequently, we need to think very carefully about which volatilities to choose for the pricing. Taking the same volatility for the digital as for the corresponding vanilla would mean that we would price the options in the replicating portfolio with a flat volatility. Since the smile is not constant, this could produce a significant error. We should take the market volatilities for the replication to find a good price for the digital with smile. The mismatch is caused by the windmill effect and is illustrated in Figure 1.30.

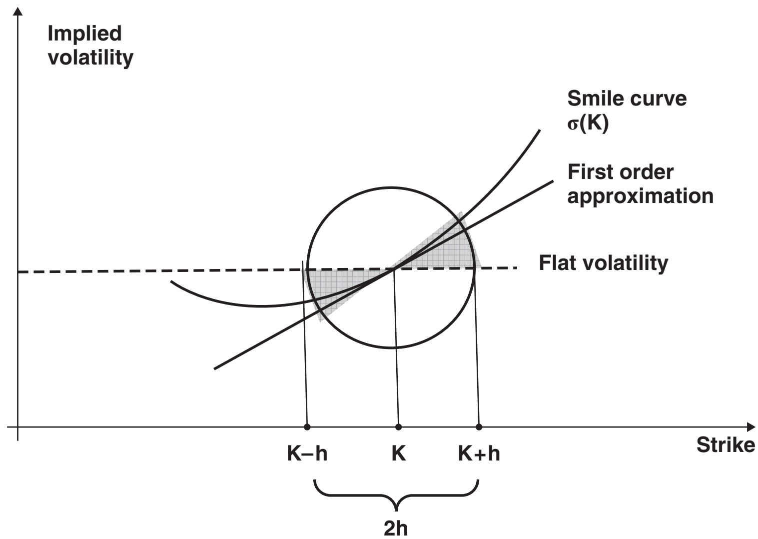


FIGURE 1.30 Windmill effect. The shaded gray areas show the mis-pricing. Working with a flat volatility is not sufficient. A first order approximation is attained using the windmill adjustment and will be considerably better. Working on the smile curve exactly is achieved by the call spread replication.

We conclude from Equation (169) that the value of the domestic-paying digital call is generally, and independent of any model, given by

$$\text{digital}(K) = -\frac{\partial}{\partial K}\text{vanilla}(K). \tag{170}$$

This helps us to get a much better approximation of the smile-adjusted value of the digital call. Let us abbreviate the vanilla call value function by $v(K,\sigma)$ and notice that, in a valuation with smile, the volatility σ is itself a function of K. The rest is a consequent application of calculus 101:

$$\begin{aligned}\text{digital}(K) &= -\frac{\partial}{\partial K}v(K,\sigma(K))\\ &= -v_K(K,\sigma(K)) - v_\sigma(K,\sigma(K))\sigma'(K).\end{aligned} \tag{171}$$

The first term is the dual delta of the vanilla, i.e. $e^{-r_d\tau}\mathcal{N}(\phi d_-)$ as in (167). The second term is the windmill adjustment. It depends on the vanilla vega at the given strike and the slope of the smile on the strike space. The vega impact is maximal for the ATM (delta-neutral) strike and the contribution decreases from that ATM strike in both directions. Vanilla vega is always positive. The slope of the smile on the strike space means that we need to consider the implied volatility as a function of strike, i.e. strike is on the x-axis and implied volatility on the y-axis. This slope can be positive or negative. For a strongly down-skewed market, the slope is negative, whence the windmill adjustment is positive, and the digital call (put) will be valued higher (lower) when smile is taken into consideration. In short: down-skew leads to marking the digital calls up and the digital puts down; up-skew leads to marking the digital calls down and the digital puts up. This effect is highly important, as the market price of the digital translates to the market price of European knock-out, the one-touch, and hence the market price of the reverse knock-out and therefore into all structured products that contain these first generation exotics as building blocks. Note that the slope of the implied volatility curve is the local slope around the strike under consideration. Its sign is not necessarily equal to the sign of the 25-delta-risk reversal. We provide an example in Table 1.20. The windmill-adjustment is analytically tractable for a parametric smile interpolation – see Formula (162) in Section 1.5.12.

Volatility Implied by Digital Options With the windmill-adjustment it is obvious that digital options cannot be priced with the same volatility as the corresponding vanilla. Technically, it is possible to retrieve the volatility from a digital option's price. This is equivalent to retrieving the volatility from a given delta. It boils down to a quadratic equation with two solutions, similar to Equation (62). The volatility implied by the digital call price listed in Table 1.20 is based on this result. Note that the implied volatility of about 22% is very different from the smile volatility 15%, so in terms of volatility, the windmill effect is all the more visible.

Drift Sensitivity of Digital Options Looking at the payoff of digital options again shows that their value is highly drift sensitive. This means that we need to be very careful about which values for the rates and forward points to use for the valuation. For instance, selling a digital call would require buying foreign currency in the delta hedge. If we hedge delta with a forward contract (or even if we just pretend to do this), then we need to use the offer side of the forward points.

Using the same market data as in Table 1.20, we see that the forward points come out to be −597 mid market, implying a value of 30.89% DOM of a three-year digital call. The offer side of the forward points will be around 558 and the digital call value 31.22% DOM. On a notional of 1 M domestic currency, this difference accounts for 3,325 units of domestic currency. The trader selling the digital call would lose this amount if he sold the digital call priced with mid market forward points. Hence, always make sure you use the quote of the forward points consistent with your hedge: offer side for selling digital calls, bid side for selling digital puts.

TABLE 1.20 Windmill-adjustment for a digital call paying one unit of domestic currency. Contract data: Time to maturity $\tau = 186/365$, strike $K = 1.4500$, market data spot $S = 1.4000$, $r_d = 2.5\%$, $r_f = 4.0\%$, volatilities $\sigma(K) = 15.0000\%$, $\sigma(1.4499) = 15.0010\%$, $\sigma(1.4501) = 14.9990\%$, $h = 0.0001$.

Digital value without smile	$-v_K(K, \sigma(K)) = e^{-r_d\tau}\mathcal{N}(\phi d_-)$	0.322134
Value of the replication	$\frac{1}{2h}[v(K-h, \sigma(K-h)) - v(K+h, \sigma(K+h))]$	0.358975
Windmill-adjustment	$-v_\sigma(K, \sigma(K))\sigma'(K)$	0.036845
Digital value with smile	$-v_K(K, \sigma(K)) - v_\sigma(K, \sigma(K))\sigma'(K)$	0.358978
Implied volatility		22.005%

Domestic and Foreign Paying Digitals We leave it as an exercise for you to replicate a foreign-paying digital statically with vanillas and domestic-paying digitals. This perfect static replication will tell you all about the theoretical value and the smile-adjusted value of the foreign-paying digital.

Applications of Digital Options European digitals as a stand-alone transaction is mainly a vehicle of speculators. Trading desks and institutional investors could use digitals to hedge themselves against unexpected market moves, such as the EUR-CHF plunge on 15 January 2015. As a building block it comes up frequently in structured forwards (e.g. in Section 2.1.10) and structured FX-linked swaps (e.g. in Section 2.5.3). Conceptually, with the digital call one trades the probability distribution function directly.

European Knock-Out Barrier Options (EKO) A further application of digital options is the construction of European barrier options with payoff

$$\mathrm{EKO} = [\phi(S_T - K)]^+ \mathbb{I}_{\{\eta S_T > \eta B\}}, \tag{172}$$

where K denotes the strike, ϕ the put-call indicator, $\eta = +1$ for a lower barrier B and $\eta = -1$ for an upper barrier B. Note that for European knock-outs, the barrier is always in-the-money, i.e. $B > K$ for the call and $B < K$ for the put. This payoff can be replicated statically with vanillas and digitals (exercise!), and therefore all we need to know about the market price follows from the vanilla smile and the windmill-adjustment. Since EKOs are building blocks of a dynamic replication of target forwards (see Section 2.2), most of what you need to know about target forwards goes back to understanding the EKO, which in turn goes back to the digital and the windmill effect. I would like to stress at this point that understanding the features of the volatility smile for vanilla options can help you understand the smile-adjusted price of many exotics and structured products.

1.7.3 Barrier Options

Barrier Options clearly belong to the first generation exotic options. The definitions of their terms have been standardized by the 2005 ISDA *Barrier Option Supplement* [78]. An entire book devoted to *FX Barrier Options* has been written by Dadachanji [34].

Knock-out Call Option (American Style Barrier) A knock-out call option entitles the holder to purchase an agreed amount of a currency (say EUR) on a specified expiration date at a pre-determined rate called the strike K provided the exchange rate never hits or crosses a pre-determined barrier level B. However, there is no obligation to do so. Buying a EUR knock-out call provides protection against a rising EUR if no knock-out event occurs between the trade date and expiration date while enabling full participation in a falling EUR. The holder has to pay a premium for this protection. The holder will typically exercise the option only if at expiration time the spot is above the strike and can exercise the option only if the spot has failed to touch the barrier between the trade date and expiration date (American style barrier) or if the spot on the expiry date does not touch or cross the barrier (European style barrier) – see Figure 1.31. We display the profit and the final exchange rate of an up-and-out call in Figure 1.32.

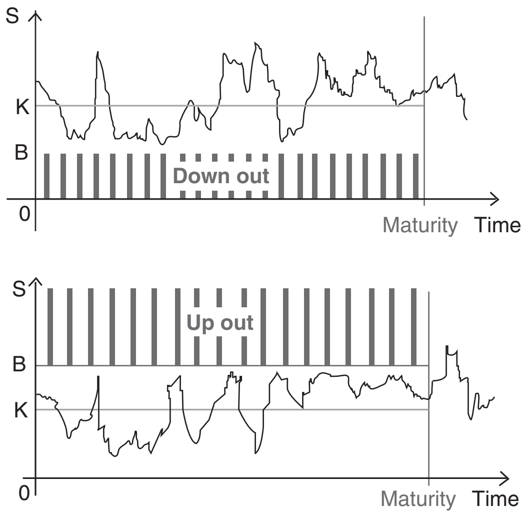


FIGURE 1.31 *Down-and-out* American barrier: if the exchange rate is never at or below *B* between the trade date and maturity, the option can be exercised. *Up-and-out* American barrier: if the exchange rate is never at or above *B* between the trade date and maturity, the option can be exercised.

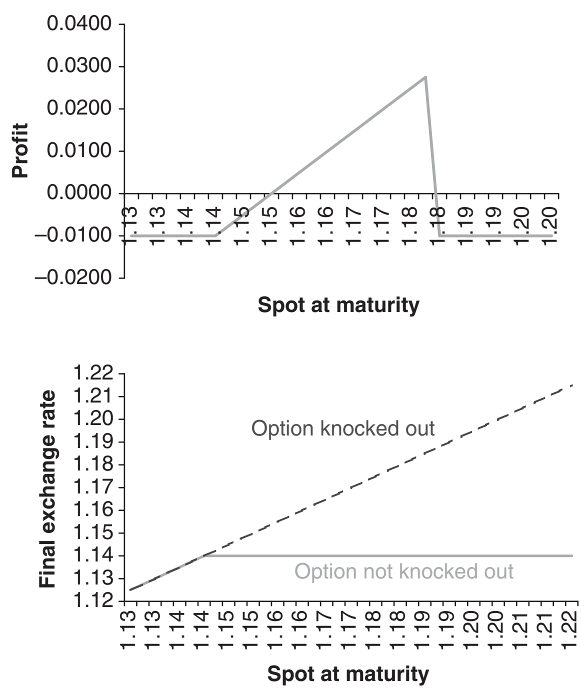


FIGURE 1.32 *Up-and-out* American barrier option payoff (above) and final exchange rate (below).

Advantages

- Cheaper than a plain vanilla
- Conditional protection against stronger EUR/weaker USD
- Full participation in a weaker EUR/stronger USD

Disadvantages

- Option may knock out, i.e. the hedge is lost if the option is used to hedge FX risk
- Premium has to be paid

For example, a company wants to sell 1 M USD and buy EUR at time T. If as usual S_t denotes the exchange rate of EUR-USD at time t then at maturity

1. if $S_T < K$, the company would not exercise the option,
2. if $S_T > K$ and if S has respected the conditions pre-determined by the barrier, the company would exercise the option and sell 1 M USD at strike K.

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be able to buy EUR at a lower spot rate if the EUR becomes weaker on the one hand, but on the other hand be protected against a stronger EUR, and finds that the corresponding vanilla EUR call USD put is too expensive and is prepared to take more risk.

In this case a possible form of protection that the company can use is to buy a EUR knock-out call option as listed in Table 1.21 for example.

If the company's market expectation is correct, then it can buy EUR at maturity at the strike of 1.1500.

If the EUR-USD exchange rate touches the barrier at least once between the trade date and maturity the option will expire worthless.

Types of Barrier Options Generally the payoff of a standard knock-out option can be stated as

$$[\phi(S_T - K)]^+ \mathbb{I}_{\{\eta S_t > \eta B,\, 0 \leq t \leq T\}}, \tag{173}$$

where $\phi \in \{+1, -1\}$ is the usual put/call indicator and $\eta \in \{+1, -1\}$ takes the value $+1$ for a lower barrier (down-and-out) or -1 for an upper barrier (up-and-out). Compare

TABLE 1.21 Example of an up-and-out call.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional	EUR 1,000,000
Company buys	EUR call USD put
Strike	1.1500 EUR-USD
Up-and-out American barrier	1.3000 EUR-USD
Premium	EUR 12,553.00

this path-dependent payoff with the payoff (172) of the European barrier option. The corresponding *knock-in* options become alive only if the spot ever trades at or beyond the barrier between trade date and expiration date. Naturally,

$$\text{knock-out} + \text{knock-in} = \text{vanilla}, \tag{174}$$

which means that a portfolio containing both a knock-out and a knock-in option with otherwise identical contractual parameters can be considered economically equivalent to a vanilla option. Furthermore, we distinguish (see Figure 1.33):

(Regular) knock-out (KO): the barrier is out-of-the-money.

Reverse knock-out (RKO): the barrier is in-the-money. This version of a knock-out/knock-in is also referred to as *kick-out/kick-in*.

Strike out: the barrier is at the strike.

Losing a regular barrier option due to the spot hitting the barrier is not critical when used as a hedge for FX risk management. If the spot drops and a down-and-out EUR call USD put knocks out, then the hedge is lost, but at hitting time EUR is cheaper than at inception of the trade. The treasurer can trade another FX forward at a lower exchange rate at hitting time and lock in a better FX rate at no extra cost. Losing a reverse barrier option due to the spot hitting the barrier is more painful since the treasurer already has accumulated a positive intrinsic value, loses the hedge and is faced with a very expensive

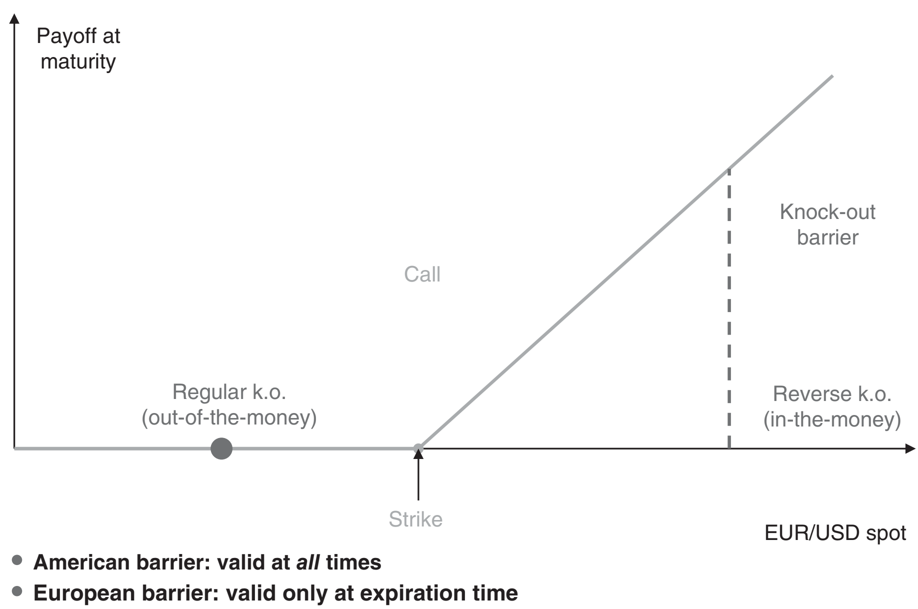


FIGURE 1.33 Barrier option terminology: regular barriers are out-of-the-money, reverse barriers are in-the-money.

EUR at hitting time. As a treasurer I would think twice before using a reverse knock-out as a stand-alone element of a currency hedge. In fact, reverse barrier options are used more commonly as part of structured FX forwards rather than as stand-alone elements.

This means that there are in total 16 different types of barrier options, call or put, in or out, up or down, regular or reverse (knock or kick).

Theoretical Value of Barrier Options For the standard type of FX barrier options a detailed derivation of values and Greeks can be found in [65].

Foreign–Domestic Symmetry for Barrier Options For a standard knock-out barrier option we let the value function be

$$v(S, r_d, r_f, \sigma, K, B, T, t, \phi, \eta), \tag{175}$$

where B denotes the barrier and the variable η takes the value $+1$ for a lower barrier and -1 for an upper barrier. With this notation at hand, we can state our FOR-DOM symmetry as

$$v(S, r_d, r_f, \sigma, K, B, T, t, \phi, \eta) = v(1/S, r_f, r_d, \sigma, 1/K, 1/B, T, t, -\phi, -\eta) \cdot S \cdot K. \tag{176}$$

Note that the rates r_d and r_f have been purposely interchanged. This implies that if we know how to price barrier contracts with upper barriers, we can derive the formulas for lower barriers. This symmetry is contractual and hence model-independent.

Barrier Option Terminology This paragraph is based on Hakala and Wystup [64].

American vs. European – Traditionally barrier options are of American style, which means that the barrier level is active during the entire duration of the option: any time between today and maturity the spot hits the barrier, the option becomes worthless. If the barrier level is active only at maturity, the barrier option is of European style and can in fact be replicated by a vertical spread and a digital option – see Section 1.7.2.

Single, double and outside barriers – Instead of taking just a lower or an upper barrier one could have both if one feels confident that the spot will remain in a range for a while. In this case besides vanillas, constant payoffs at maturity are popular – they are called range binaries. If the barrier and strike are in different exchange rates, the contract is called an outside barrier option or double currency barrier option. Such options traded a few years ago with the strike in USD/DEM and the barrier in USD/FRF taking advantage of the imbalance between implied and historic correlation between the two currency pairs.

Rebates – For knock-in options an amount R is paid at expiration by the seller of the option to the holder of the option if the option failed to kick in during its lifetime. For knock-out options an amount R is paid by the seller of the option to the holder of the option if the option knocks out. The payment of the rebate is either at maturity or at the first time the barrier is hit. Including such rebate features makes hedging easier for reverse barrier options and serves as a consolation for the holder's disappointment in case of a knock-out or a missing knock-in. The rebate part of a barrier option can be completely separated from the barrier contract and can in fact be traded separately, in which case it is called a one-touch (digital) or hit binary (in the knock-out case) and no-touch (in the knock-in case). We treat the touch contracts in detail in Section 1.7.2.

Determination of knock-out event – *Barrier events* are best described in ISDA's *Barrier Option Supplement* [78]. We highlight some of the relevant issues in Section 1.7.4.

How the Barrier is Monitored (Continuous vs. Discrete) and how this Influences the Value How often and when exactly do you check whether an option has knocked out or kicked in? This question is not trivial and should be clearly stated in the deal. The intensity of monitoring can create any price between a standard barrier and a vanilla contract. The standard for barrier options is continuous monitoring. Any time the exchange rate hits the barrier the option is knocked out. An alternative is to consider just daily/weekly/monthly currency fixings which makes the knock-out option more expensive because chances of knocking out are smaller (see Figure 1.34). A detailed discussion of the valuation of discrete barriers can be found in [57]. Discretely monitored barrier options are not very popular in FX markets, mainly because a trader is exposed to a fixing risk and you do know that all fixings are manipulated, don't you?

The Popularity of Barrier Options

- They are less expensive than vanilla contracts: in fact, the closer the spot is to the barrier, the cheaper the knock-out option. Any price between zero and the vanilla premium can be obtained by taking an appropriate barrier level, as we see

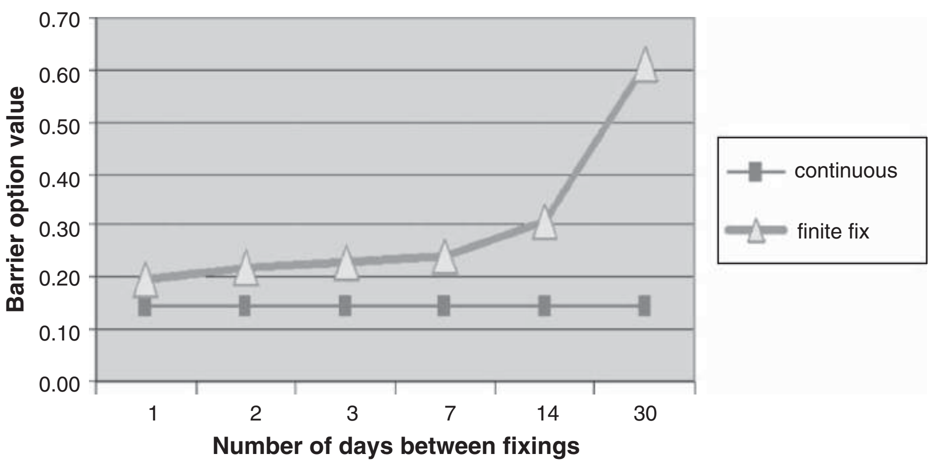


FIGURE 1.34 Comparison of a discretely and a continuously monitored knock-out barrier option.

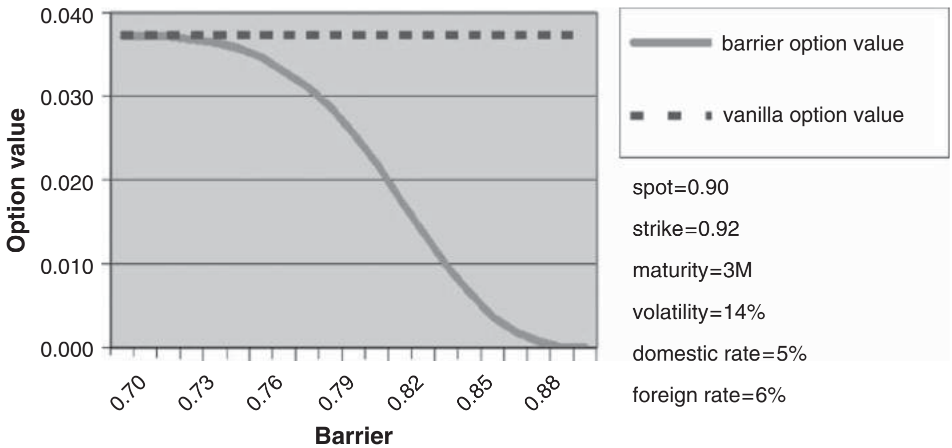


FIGURE 1.35 Comparison of a vanilla put and a down-and-out put. As the barrier moves far away from the current spot, the barrier option behaves like a vanilla option. As the barrier moves close to the current spot, the barrier option becomes worthless.

in Figure 1.35. One must be aware, however, that the lower the value of a barrier option, the more likely it will knock out.

- They allow foreign exchange risk exposure to be designed to customers' special needs. Instead of lowering the premium one can increase the nominal coverage of the vanilla contract by adding a barrier. Several customers feel sure about exchange rate levels not being hit during the next month which could be exploited to lower the premium. Others really want to cover their exchange rate exposure only if the market moves drastically which can be reflected by a knock-in option.
- The reduced cost allows another hedge of foreign exchange risk exposure if the first barrier option happens to knock out.
- The contract is easy to understand if one knows about vanillas.
- Many pricing and trading systems include barrier options in their standard.
- Pricing and hedging barriers in the Black-Scholes model are well understood and most pricing tools use closed-form solutions for the theoretical value, which allow fast and stable implementation; some even closed-form solutions in more advanced systems. Most of the big players have a stochastic-local volatility model in place to calculate values and Greeks of barrier options.
- Barrier options are standard building blocks in structured FX forwards – see for example the *shark forward* in Section 2.1.6.

Barrier Option Crisis 1994–1996 In the currency market barrier options became popular in 1994. The exchange rate between USD and DEM was then between 1.50 and 1.70. Since the all time low before 1995 was 1.3870 at September 2 1992 there were a lot of down and out barrier contracts written with a lower knock-out barrier of 1.3800. The sudden fall of the US dollar at the beginning of 1995 was unexpected and the 1.3800 barrier was hit at 10:30 am on March 29 1995 and fell even more to its all time low of

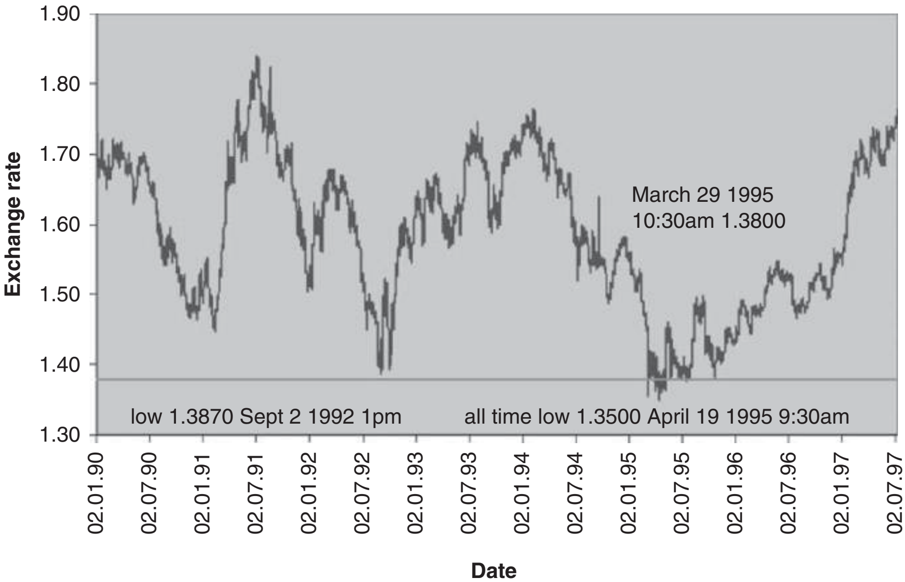


FIGURE 1.36 Barrier had lost popularity in 1994–1996 when USD-DEM had dropped below its historic low.

1.3500 at 9:30 am on April 19 1995. Numerous barrier option holders were shocked that a knock-out event was something that can really happen (see Figure 1.36). The shock lasted for more than a year and barrier options were unpopular for a while until many market participants had forgotten the event. Events like this often lead traders to question using exotics. Complicated products can in fact lead to unpleasant surprises. However, to cover foreign exchange risk reflecting individual market views at minimal cost requires exotic options. Often they appear as an integral part of an investment portfolio. The number of market participants who understand the advantages and pitfalls is growing steadily.

Risk Management of Barrier Options There are many ways to risk manage barrier options for a trading desk. Most typical is *hedging* the spot risk in the underlying and the vega and gamma risk using vanilla options. In order to understand the driving risk factors and the impact of the vanilla volatility surface on the price of barrier options, it is helpful to also consider *replication* of barrier options with vanilla options, either as a *static (buy-and-hold) replication* of the payoff or a *semi-static replication*, where the replicating portfolio must be unwound if and when the spot hits the barrier. We consider both hedging and replication. Since payoffs with discontinuities as we find them in touch contracts and reverse- and double-knock-out barrier options generate exploding Greeks, smoothing or face-lifting of payoffs is applied in practice before Greeks are used for hedging.

Hedging For barrier contracts a delta and vega hedge is most common. Obviously, both delta and vega hedges must be readjusted during the lifetime; however, this is normally conducted on a portfolio level. A vega hedge can be done using two vanilla options or more. In the example we consider a three-month up-and-out put with strike 1.0100 and barrier 0.9800. The vega minimizing hedge consists of 0.9 short three-month 50-delta calls and 0.8 long two-month 25-delta calls. Spot reference for EUR/USD is 0.9400 with rates 3.05% and 6.50% and volatility 11.9% – see Figure 1.37. This up-and-out put knocks out in-the-money, so is generally referred to as a reverse knock-out (RKO). Typically, the standard RKO put would be with a barrier below strike and the spot above the barrier. This is a less common example of a RKO put with a barrier above initial spot.

Replication of the Regular Knock-Out We start by semi-statically hedging regular barriers with a risk reversal as indicated in Figure 1.38. We choose the put strike to be zero for spot at the barrier and then recalculate the price of the risk reversal under current spot. The problem is, of course, that the value of the risk reversal at knock-out time does not need to be zero, and we do not know the market value of the risk reversal at hitting time. In fact, considering hedging a down-and-out call with a spot approaching the barrier and assuming a down trend in the spot triggers a down trend in the risk reversals, the calls will tend to be even cheaper than the

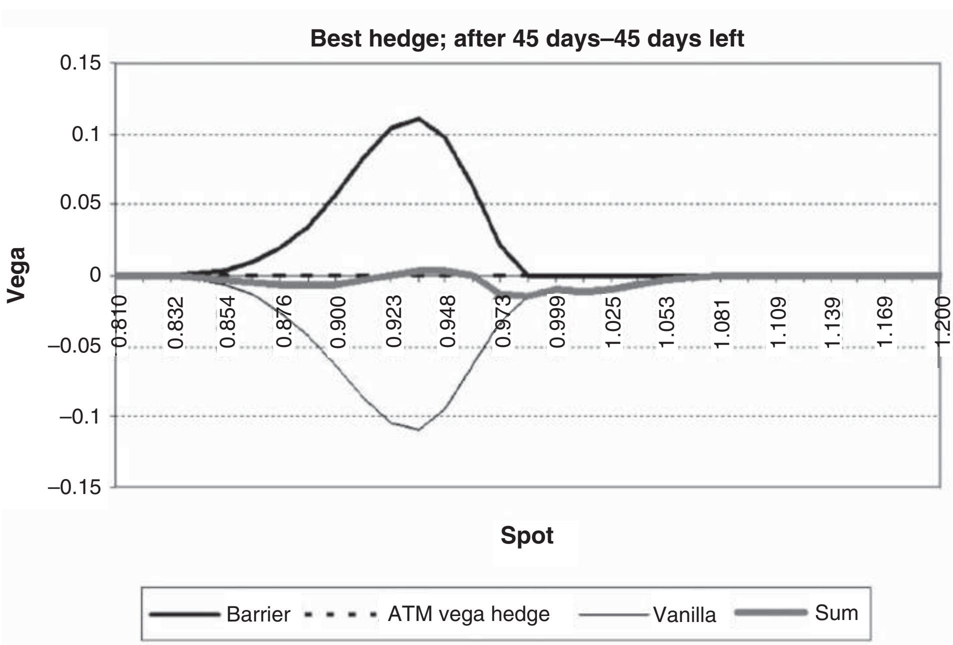


FIGURE 1.37 Vega depending on spot of an up-and-out put and a vega hedge consisting of two vanilla options.

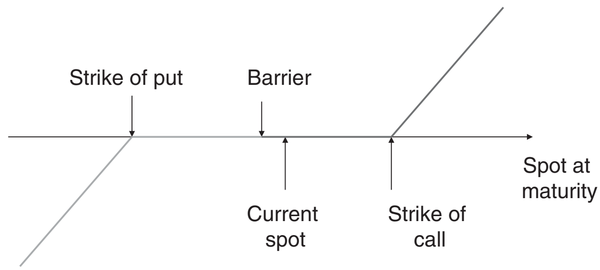


FIGURE 1.38 Semi-static replication of the regular knock-out with a risk reversal. A short down-and-out call is hedged by a long call with the same strike and a short put with a strike chosen in such a way that the value of the call and put portfolio is zero if the spot is at the barrier.

puts, so to unwind the hedge one would get less for the call one could sell and pay more than expected for the put to be bought back. Quantitatively, one can estimate the probability distribution of the first hitting time or its expected value under a pre-specified model. In the Black-Scholes model, it is in fact known in closed form. Furthermore, one can come up with an educated guess of the future risk reversal at hitting time, for example by running a regression analysis on historic relative returns versus relative risk reversals (relative meaning in relation to the at-the-money volatility). And if you believe in your model for the underlying and also believe in any sort of predictive power of historic financial data, then you can determine a put strike, that generates an unwind cost closer to zero. However, at the end of the day, the future risk reversal is unknown and the trading desk has to take a view on it. Future skew is the only risk that a risk reversal cannot replicate and this feature justifies the existence of the regular knock-out option. In fact, treasurers hardly ever use regular knock-out options to hedge their FX (directional) risk, but would much rather use a risk reversal. A view on the future skew at hitting time is more likely to be taken by a hedge fund. Therefore, many traders and researchers like to think of stochastic skew models taking exactly this effect into account. In summary, the regular knock-out can be seen essentially as a risk reversal, can be semi-statically replicated with a risk reversal. This replication is not used much in practice though: the treasurer would not replicate but replace the regular knock-out with the risk reversal, and the risk managing trading desk would hedge using the Greeks. The semi-static replication helps us understand the regular knock-out in full detail and can be used for valuation.

Semi-Static Replication of Reverse Knock-Out Options The perfect static replication of the reverse knock-out uses a portfolio of regular knock-out and single touch contracts (exercise). The regular knock-out in turn is essentially a risk reversal plus future skew risk, and the touch is approximately two European digitals. The European digital in turn can be viewed as a vanilla call spread with high notional amounts. Therefore, the risk drivers of the reverse knock-out are well understood. Several authors claim that reverse knock-out barrier options can be replicated semi-statically with a portfolio of vanilla options. These approaches are problematic if the hedging portfolio has to be unwound at hitting time, since volatilities for the vanillas may have changed between the time the replicating portfolio is composed and the first hitting time. Moreover, the occasionally high nominals and low deltas can cause a high price for the replicating portfolio. The approach by Maruhn and Sachs in [96] appears most promising. To understand the key problem and risk driver, note that a first order approximation of the reverse knock-out call is a ratio call spread with infinite leverage – see Section 1.6.1, and remember the disaster of the USD-TRY trader. This tells it all: high notionals that do not work in practice and the infinitely many at-the-money calls to buy back at first hitting time are driven by the future at-the-money volatility level. This exposure is very high because of the large notional of the short call.

Hedging the Reverse Knock-Out Reverse barrier options have extremely high values for delta, gamma, and theta when the spot is near the barrier and the time is close to expiry – see for example the delta in Figure 1.39. This is because the intrinsic value of the option jumps from a positive value to zero when the barrier is hit. In such a situation a simple delta hedge is impractical. However, there are ways to tackle this undesirable state of affairs by moving the barrier or more systematically applying valuation subject to portfolio constraints such as limited leverage – see Schmock *et al.* in [117]. The idea is to keep the *contractual barrier* for the contract and to determine the barrier event but

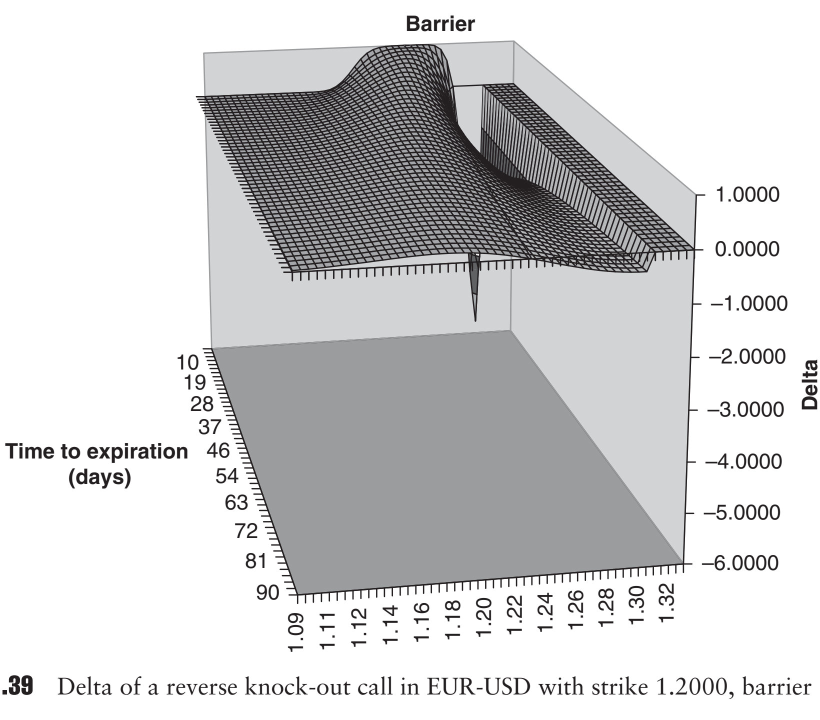


FIGURE 1.39 Delta of a reverse knock-out call in EUR-USD with strike 1.2000, barrier 1.3000.

use a *shadow barrier* to derive the value and the Greeks. A more advanced approach is called *barrier bending*, where the shadow barrier's distance from the contractual barrier is a function of time to maturity [126].

How Large Barrier Contracts Affect the Market Let's take the example of a reverse up-and-out call in EUR/USD with strike 1.2000 and barrier 1.3000. A trading desk delta-hedging a short position with nominal EUR 10 M has to buy 10 million times delta EUR, which is negative in this example. As the spot goes up to the barrier, delta reduces in size, requiring the hedging trading desk to sell more and more EUR. This can influence the market since steadily offering EUR slows down the spot movement towards the barrier and can in extreme cases prevent the spot from crossing the barrier. This is illustrated in Figure 1.40.

On the other hand, if the hedging trading desk runs out of breath or the upward market movement cannot be stopped by the delta-hedging institution, then the option knocks out and the hedge is unwound. Then suddenly more EUR are in demand whence the upward movement of an exchange rate can be accelerated once a large barrier contract in the market has knocked out. Situations like this happened to the USD-DEM spot in the early 1990s (see Figure 1.36), where many reverse knock-out puts have been written by banks, as traders are telling.

The reverse situation occurs when the bank hedges a long position, in which case EUR has to be bought when the spot approaches the barrier. This can cause an accelerated movement of the exchange rate towards the barrier and a sudden halt once the barrier is breached.

In modern compliance-dominated markets, regulators have actually started asking banks to ensure that by delta-hedging their own options positions they need to have a policy in place that prevents them from moving the market. While I know what to do

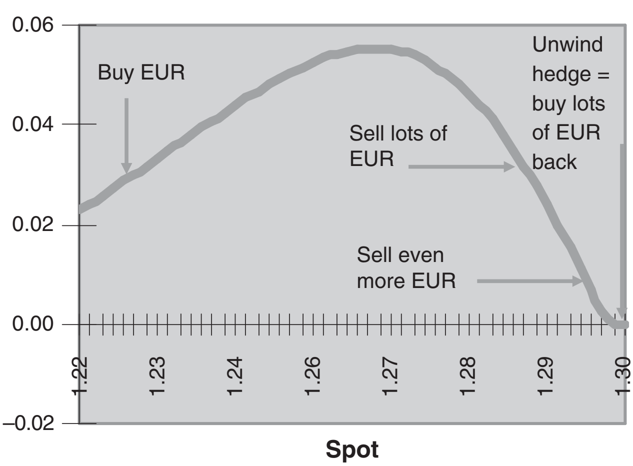


FIGURE 1.40 Delta hedging a short reverse knock-out call. The y-axis shows the USD value per EUR notional. Recall that the slope of the value function of a long contract represents the delta hedge to employ.

to reduce exploding Greeks, I fail to understand how one can ensure that markets are not affected by trading. Regulation should be regulated!

1.7.4 Touch Contracts

Now we take a more detailed look at the pricing of one-touch contracts – often also called (American style) binary or digital options, hit options, or rebate options. They trade as listed and over-the-counter products.

The touch-time is the first time the underlying trades at or beyond the touch level. The barrier determination agent, who is specified in the contract, determines the touch- or barrier event. The *Foreign Exchange Committee* recommends to the foreign exchange community a set of best practices for the barrier options and touch market. In the next stage of this project, the Committee is planning on publishing a revision of the *International Currency Options Master Agreement (ICOM) User Guide* to reflect the new recommendations.[3] Some key features are

- Determination whether the spot has breached the barrier must be due to actual transactions in the foreign exchange markets.
- Transactions must occur between 5:00 a.m. Sydney time on Monday and 5:00 p.m. New York time on Friday.
- Transactions must be of commercial size. In liquid markets, dealers generally accept that commercial size transactions are a minimum of 3 M USD.
- The barrier event determination agent may use cross-currency rates to determine whether a barrier has been breached in respect of a currency pair that is not commonly quoted.

The barrier or touch-level is usually monitored continuously over time. A further contractual issue to specify is the time of the payment of the one-touch. Typically, the notional is paid at the delivery date, which is two business days after the maturity. Another common practice is two business days after the barrier event (first hitting time, as we say in probability theory). In FX markets the former is the default.

Classification of One-Touch Contracts I was once asked whether the one-touch is an option. At first glance my reaction was that it is an option, because it is a derivative with a non-negative payoff. However, when going back to ISDA's 1998 **FX and Currency Option Definitions**, it appeared that there are two ways to view the one-touch:

Right to Receive: One can squeeze the one-touch into ISDA's 1998 FX and Currency Option Definitions and set the call currency amount equal to the one-touch notional, the put currency amount equal to zero, furthermore American style. Considering the option as a right to exercise would not work here because the holder would always exercise immediately and claim his call currency amount. Therefore, one must additionally agree on cash settlement and on a spot reference to define clearly in which case the option will be

[3]For details see http://www.ny.frb.org/fxc/fxann000217.html

automatically exercised. Once you think about the legal terms, it is no longer that obvious that the one-touch is actually a currency option.

Obligation to Pay: Indeed, an alternative way to consider the one-touch as a legal contract is to confirm it as an obligation of the seller to pay a fixed amount in case of a barrier event. Such deal confirmations have been used and clearly take the one-touch outside the class of currency options.

This doesn't really matter for the financial engineer, trader, or structurer, and if you find this subtle distinction irrelevant, then fair enough. However, it matters for clients who have agreed to trade only currency options and forwards. A payment obligation is clearly not in this set.

Applications of One-Touch Contracts Market participants of a rather speculative nature like to use one-touch contracts to reflect a view on a rising or falling exchange rate. In fact, nowadays, even individuals can trade them over the internet on a whole bunch of binary trading platforms. Hedging-focused clients often buy one-touch contracts as a rebate, so they receive a payment as a consolation if the strategy they believe in does not work. One-touch contracts also often serve as parts of structured products designed to enhance a forward rate or an interest rate.

Theoretical Value of the One-Touch In the standard Black-Scholes model for the underlying exchange rate of EUR/USD,

$$dS_t = S_t[(r_d - r_f)dt + \sigma dW_t], \tag{177}$$

where t denotes the running time in years, r_d the USD interest rate, r_f the EUR interest rate, σ the volatility, W_t a standard Brownian motion under the risk-neutral measure, the payoff is given by

$$R\mathbb{I}_{\{\tau_B \leq T\}}, \tag{178}$$

$$\tau_B \stackrel{\Delta}{=} \inf\{t \geq 0 : \eta S_t \leq \eta B\}. \tag{179}$$

This type of contract pays a domestic cash amount R USD if a barrier B is hit any time before the expiration time. We use the binary variable η to describe whether B is a lower barrier ($\eta = 1$) or an upper barrier ($\eta = -1$). The stopping time τ_B is called the first hitting time. The contract can be viewed either as the rebate portion of a knock-out barrier option or as an American cash-or-nothing digital. In FX markets it is usually called a *(single) one-touch (option)*, *one-touch-digital* or *hit binary*. The modified payoff of a *(single) no-touch (option)*, $R\mathbb{I}_{\{\tau_B \geq T\}}$, describes a rebate which is being paid if a knock-in option has not knocked in by the time it expires and can be valued similarly simply by exploiting the identity

$$R\mathbb{I}_{\{\tau_B \leq T\}} + R\mathbb{I}_{\{\tau_B > T\}} = R. \tag{180}$$

We will further distinguish the cases

$$\omega = \mathbf{0}, \text{ rebate paid at hit,}$$
$$\omega = \mathbf{1}, \text{ rebate paid at end.}$$

It is important to mention that the payoff is one unit of the base currency. For a payment in the underlying currency EUR, one needs to exchange r_d and r_f, replace S and B by their reciprocal values, and change the sign of η.

For the one-touch we will use the abbreviations

- T: expiration time (in years)
- t: running time (in years)
- $\tau \stackrel{\Delta}{=} T - t$: time to expiration (in years)
- $\theta_\pm \stackrel{\Delta}{=} \frac{r_d - r_f}{\sigma} \pm \frac{\sigma}{2}$
- $S_t = S_0 e^{\sigma W_t + \sigma\theta_- t}$: price of the underlying at time t
- $n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$
- $\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\,dt$
- $\vartheta_- \stackrel{\Delta}{=} \sqrt{\theta_-^2 + 2(1-\omega) r_d}$
- $e_\pm \stackrel{\Delta}{=} \frac{\pm \ln \frac{x}{B} - \sigma\vartheta_-\tau}{\sigma\sqrt{\tau}}$

We can describe the value function of the one-touch as a solution to a partial differential equation setup. Let $v(t,x)$ denote the value of the option at time t when the underlying is at x. Then $v(t,x)$ is the solution of

$$v_t + (r_d - r_f) x v_x + \frac{1}{2}\sigma^2 x^2 v_{xx} - r_d v = 0, t \in [0,T], \eta x \geq \eta B, \tag{181}$$

$$v(T,x) = 0, \eta x \geq \eta B, \tag{182}$$

$$v(t,B) = Re^{-\omega r_d \tau}, t \in [0,T]. \tag{183}$$

The theoretical value of the one-touch turns out to be

$$v(t,x) = Re^{-\omega r_d \tau}\left[\left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}} \mathcal{N}(-\eta e_+) + \left(\frac{B}{x}\right)^{\frac{\theta_- - \vartheta_-}{\sigma}} \mathcal{N}(\eta e_-)\right] \tag{184}$$

Note that $\vartheta_- = |\theta_-|$ for rebates paid at end ($\omega = 1$).

Greeks We list some of the sensitivity parameters of the one-touch here, as they seem hard to find in the existing literature, but many people have asked me for them, so here we go.

Delta

$$v_x(t,x) = -\frac{Re^{-\omega r_d \tau}}{\sigma x}\left\{\left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}}\left[(\theta_- + \vartheta_-)\mathcal{N}(-\eta e_+) + \frac{\eta}{\sqrt{\tau}} n(e_+)\right]\right.$$

$$\left. + \left(\frac{B}{x}\right)^{\frac{\theta_- - \vartheta_-}{\sigma}}\left[(\theta_- - \vartheta_-)\mathcal{N}(\eta e_-) + \frac{\eta}{\sqrt{\tau}} n(e_-)\right]\right\} \tag{185}$$

Gamma can be obtained using $v_{xx} = \frac{2}{\sigma^2 x^2}[r_d v - v_t - (r_d - r_f)x v_x]$ and turns out to be

$$v_{xx}(t,x) = \frac{2Re^{-\omega r_d \tau}}{\sigma^2 x^2} \cdot \tag{186}$$

$$\left\{\left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}} \mathcal{N}(-\eta e_+)\left[r_d(1-\omega) + (r_d - r_f)\frac{\theta_- + \vartheta_-}{\sigma}\right]\right.$$

$$+ \left(\frac{B}{x}\right)^{\frac{\theta_- - \vartheta_-}{\sigma}} \mathcal{N}(\eta e_-)\left[r_d(1-\omega) + (r_d - r_f)\frac{\theta_- - \vartheta_-}{\sigma}\right]$$

$$+ \eta\left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}} n(e_+)\left[-\frac{e_-}{2\tau} + \frac{r_d - r_f}{\sigma\sqrt{\tau}}\right]$$

$$\left. + \eta\left(\frac{B}{x}\right)^{\frac{\theta_- - \vartheta_-}{\sigma}} n(e_-)\left[\frac{e_+}{2\tau} + \frac{r_d - r_f}{\sigma\sqrt{\tau}}\right]\right\}.$$

Theta

$$v_t(t,x) = \omega r_d v(t,x) + \frac{\eta Re^{-\omega r_d \tau}}{2\tau}\left[\left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}} n(e_+)e_- - \left(\frac{B}{x}\right)^{\frac{\theta_- - \vartheta_-}{\sigma}} n(e_-)e_+\right]$$

$$= \omega r_d v(t,x) + \frac{\eta Re^{-\omega r_d \tau}}{\sigma \tau^{(3/2)}}\left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}} n(e_+)\ln\left(\frac{B}{x}\right). \tag{187}$$

The computation exploits the identities (209), (210), and (211) derived below.

Vega requires the identities

$$\frac{\partial \theta_-}{\partial \sigma} = -\frac{\theta_+}{\sigma} \tag{188}$$

$$\frac{\partial \vartheta_-}{\partial \sigma} = -\frac{\theta_- \theta_+}{\sigma \vartheta_-} \tag{189}$$

$$\frac{\partial e_\pm}{\partial \sigma} = \pm\frac{\ln \frac{B}{x}}{\sigma^2\sqrt{\tau}} + \frac{\theta_- \theta_+}{\sigma \vartheta_-}\sqrt{\tau} \tag{190}$$

$$A_\pm \stackrel{\Delta}{=} \frac{\partial}{\partial\sigma}\frac{\theta_- \pm \vartheta_-}{\sigma} = -\frac{1}{\sigma^2}\left[\theta_+ + \theta_- \pm \left(\frac{\theta_-\theta_+}{\vartheta_-} + \vartheta_-\right)\right] \tag{191}$$

and turns out to be

$$\begin{aligned} v_\sigma(t,x) = Re^{-\omega r_d \tau} \cdot & \qquad (192)\\ & \left\{\left(\frac{B}{x}\right)^{\frac{\theta_-+\vartheta_-}{\sigma}}\left[\mathcal{N}(-\eta e_+)A_+ \ln\left(\frac{B}{x}\right) - \eta n(e_+)\frac{\partial e_+}{\partial\sigma}\right]\right.\\ & \left. + \left(\frac{B}{x}\right)^{\frac{\theta_--\vartheta_-}{\sigma}}\left[\mathcal{N}(\eta e_-)A_- \ln\left(\frac{B}{x}\right) + \eta n(e_-)\frac{\partial e_-}{\partial\sigma}\right]\right\}. \end{aligned}$$

Vanna uses the identity

$$d_- = \frac{\ln\frac{B}{x} - \sigma\theta_-\tau}{\sigma\sqrt{\tau}} \tag{193}$$

and turns out to be

$$\begin{aligned} v_{\sigma x}(t,x) = \frac{Re^{-\omega r_d \tau}}{\sigma x} \cdot & \qquad (194)\\ & \left\{\left(\frac{B}{x}\right)^{\frac{\theta_-+\vartheta_-}{\sigma}}\left[\mathcal{N}(-\eta e_+)A_+\left(-\sigma - (\theta_- + \vartheta_-)\ln\left(\frac{B}{x}\right)\right)\right.\right.\\ & \left. -\frac{\eta n(e_+)}{\sqrt{\tau}}\left(d_-\frac{\partial e_+}{\partial\sigma} + A_+\ln\left(\frac{B}{x}\right) - \frac{1}{\sigma}\right)\right]\\ & + \left(\frac{B}{x}\right)^{\frac{\theta_--\vartheta_-}{\sigma}}\left[\mathcal{N}(\eta e_-)A_-\left(-\sigma - (\theta_- - \vartheta_-)\ln\left(\frac{B}{x}\right)\right)\right.\\ & \left.\left. + \frac{\eta n(e_-)}{\sqrt{\tau}}\left(d_-\frac{\partial e_-}{\partial\sigma} - A_-\ln\left(\frac{B}{x}\right) + \frac{1}{\sigma}\right)\right]\right\} \end{aligned}$$

Volga uses the identities

$$g = \frac{1}{\sigma^2\vartheta_-}\left[-\theta_+^2 - \theta_-^2 - \theta_+\theta_- + \frac{\theta_-^2\theta_+^2}{\vartheta_-^2}\right] \tag{195}$$

$$\frac{\partial^2 e_\pm}{\partial\sigma^2} = \mp\frac{2\ln\left(\frac{B}{x}\right)}{\sigma^3\sqrt{\tau}} + g\sqrt{\tau} \tag{196}$$

$$\frac{\partial A_\pm}{\partial\sigma} = \frac{\theta_+ + \theta_-}{\sigma^3} - \frac{2A_\pm \pm g}{\sigma} \tag{197}$$

and turns out to be

$$
\begin{aligned}
v_{\sigma\sigma}(t,x) = {} & Re^{-\omega r_d \tau} \cdot \qquad (198)\\
& \left\{ \left(\frac{B}{x}\right)^{\frac{\theta_- + \vartheta_-}{\sigma}} \left[\mathcal{N}(-\eta e_+) \ln\left(\frac{B}{x}\right) \left(A_+^2 \ln\left(\frac{B}{x}\right) + \frac{\partial A_+}{\partial \sigma} \right) \right.\right.\\
& \qquad \left. - \eta n(e_+) \left(2 \ln\left(\frac{B}{x}\right) A_+ \frac{\partial e_+}{\partial \sigma} - e_+ \left(\frac{\partial e_+}{\partial \sigma}\right)^2 + \frac{\partial^2 e_+}{\partial \sigma^2} \right) \right]\\
& + \left(\frac{B}{x}\right)^{\frac{\theta_- - \vartheta_-}{\sigma}} \left[\mathcal{N}(\eta e_-) \ln\left(\frac{B}{x}\right) \left(A_-^2 \ln\left(\frac{B}{x}\right) + \frac{\partial A_-}{\partial \sigma} \right) \right.\\
& \qquad \left.\left. + \eta n(e_-) \left(2 \ln\left(\frac{B}{x}\right) A_- \frac{\partial e_-}{\partial \sigma} - e_- \left(\frac{\partial e_-}{\partial \sigma}\right)^2 + \frac{\partial^2 e_-}{\partial \sigma^2} \right) \right] \right\}
\end{aligned}
$$

Touch Probability The risk-neutral probability of knocking out is given by

$$
\begin{aligned}
\mathbb{P}[\tau_B \le T] &= \mathbb{E}\left[\mathbb{I}_{\{\tau_B \le T\}}\right]\\
&= \frac{1}{R} e^{r_d T} v(0, S_0). \qquad (199)
\end{aligned}
$$

The touch probability is a non-discounted one-touch unit price for a one-touch paying domestic currency. This relationship is generic, i.e. independent of the model. However, it is not independent of the measure. The market price of a one-touch is derived under a risk-neutral measure. This must be taken into consideration when deriving touch probabilities from market prices of the one-touch.

Properties of the First Hitting Time τ_B As derived in [119], for example, the first hitting time

$$
\tilde{\tau} \stackrel{\Delta}{=} \inf\{t \ge 0 : \theta t + W(t) = x\} \qquad (200)
$$

of a Brownian motion with drift θ and hit level $x > 0$ has the density

$$
\mathbb{P}[\tilde{\tau} \in dt] = \frac{x}{t\sqrt{2\pi t}} \exp\left\{ -\frac{(x - \theta t)^2}{2t} \right\} dt, \quad t > 0, \qquad (201)
$$

the cumulative distribution function

$$
\mathbb{P}[\tilde{\tau} \le t] = \mathcal{N}\left(\frac{\theta t - x}{\sqrt{t}}\right) + e^{2\theta x} \mathcal{N}\left(\frac{-\theta t - x}{\sqrt{t}}\right), \quad t > 0, \qquad (202)
$$

the Laplace-transform

$$
\mathbb{E} e^{-\alpha \tilde{\tau}} = \exp\left\{ x\theta - x\sqrt{2\alpha + \theta^2} \right\}, \quad \alpha > 0, \; x > 0, \qquad (203)
$$

and the property

$$\mathbb{P}[\tilde{\tau} < \infty] = \begin{Bmatrix} 1 & \text{if } \theta \geq 0 \\ e^{2\theta x} & \text{if } \theta < 0 \end{Bmatrix} \tag{204}$$

For upper barriers $B > S_0$ we can now rewrite the first passage time τ_B as

$$\begin{aligned} \tau_B &= \inf\{t \geq 0 : S_t = B\} \\ &= \inf\left\{t \geq 0 : W_t + \theta\_t = \frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right)\right\}. \end{aligned} \tag{205}$$

The density of τ_B is hence

$$\mathbb{P}[\tilde{\tau_B} \in dt] = \frac{\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right)}{t\sqrt{2\pi t}} \exp\left\{-\frac{\left(\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right) - \theta\_t\right)^2}{2t}\right\}, \quad t > 0. \tag{206}$$

Derivation of the Value Function Using the density (206) the value of the paid-at-end ($\omega = 1$) upper rebate ($\eta = -1$) option can be written as the following integral:

$$\begin{aligned} v(T, S_0) &= Re^{-r_d T}\mathbb{E}\left[\mathbb{I}_{\{\tau_B \leq T\}}\right] \\ &= Re^{-r_d T}\int_0^T \frac{\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right)}{t\sqrt{2\pi t}} \exp\left\{-\frac{\left(\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right) - \theta\_t\right)^2}{2t}\right\} dt. \end{aligned} \tag{207}$$

To evaluate this integral, we introduce the notation

$$e_{\pm}(t) \stackrel{\Delta}{=} \frac{\pm\ln\frac{S_0}{B} - \sigma\theta\_t}{\sigma\sqrt{t}} \tag{208}$$

and list the properties

$$e\_(t) - e_+(t) = \frac{2}{\sqrt{t}}\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right), \tag{209}$$

$$n(e_+(t)) = \left(\frac{B}{S_0}\right)^{-\frac{2\theta\_}{\sigma}} n(e\_(t)), \tag{210}$$

$$\frac{\partial e_{\pm}(t)}{\partial t} = \frac{e_{\mp}(t)}{2t}. \tag{211}$$

We evaluate the integral in (207) by rewriting the integrand in such a way that the coefficients of the exponentials are the inner derivatives of the exponentials using properties (209), (210), and (211).

$$
\begin{aligned}
&\int_0^T \frac{\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right)}{t\sqrt{2\pi t}} \exp\left\{-\frac{\left(\frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right)-\theta_{-}t\right)^2}{2t}\right\} dt \\
&= \frac{1}{\sigma}\ln\left(\frac{B}{S_0}\right)\int_0^T \frac{1}{t^{(3/2)}} n(e_{-}(t))\, dt \\
&= \int_0^T \frac{1}{2t} n(e_{-}(t))[e_{-}(t)-e_{+}(t)]\, dt \\
&= -\int_0^T n(e_{-}(t))\frac{e_{+}(t)}{2t} + \left(\frac{B}{S_0}\right)^{\frac{2\theta_{-}}{\sigma}} n(e_{+}(t))\frac{e_{-}(t)}{2t}\, dt \\
&= \left(\frac{B}{S_0}\right)^{\frac{2\theta_{-}}{\sigma}} \mathcal{N}(e_{+}(T)) + \mathcal{N}(-e_{-}(T)).
\end{aligned}
\tag{212}
$$

The computation for lower barriers ($\eta = 1$) is similar.

Semi-Static Replication and Smile Effect The one-touch can be valued in many models, including stochastic-local volatility models and jump diffusion models. The purpose of this is to include the smile effect and the dynamics of the underlying consistently into the valuation of the one-touch. In order to see the driving risk and really understand it, we may again turn to a semi-static replication. We start by analyzing the touch probability.

Touch Probability and Moneyness Probability The touch probability for a touch level B above initial spot S_0 can be rewritten as

$$\text{touch-probability} = I\!P[M_T \geq B], \tag{213}$$

$$M_T \stackrel{\Delta}{=} \max_{0\leq t\leq T} S_t. \tag{214}$$

The event $\{M_T \geq B\}$ can be split into a case where the final spot S_T is also at or above B and another case where it is below B. We continue as

$$
\begin{aligned}
I\!P[M_T \geq B] &= I\!P[M_T \geq B, S_T \geq B] + I\!P[M_T \geq B, S_T < B] \\
&= I\!P[S_T \geq B] + I\!P[M_T \geq B, S_T < B] \\
&= I\!P[S_T \geq B] + I\!P[S_T \geq B].
\end{aligned}
\tag{215}
$$

In the first summand we have dropped the event $\{M_T \geq B\}$ because it is implied by the event $\{S_T \geq B\}$. The second summand collapses to $I\!P[S_T \geq B]$ because of the *reflection principle*. It states that if a spot path starts at a level below B and ends at a level above B, then the first hitting time must occur before T and from the first hitting time, there is a spot path reflected at the barrier level that ends above B, and now comes the crucial part: and that the reflected path is equally likely. Now the reflection principle as we know it from probability theory can be proved under the assumption of a *symmetric* Brownian motion. As we introduce an up or down drift, this will clearly fail. For practical matters, we may work with an assumption of a symmetric FX spot model and view the result of the reflection principle as a very handy rule of thumb:

$$\text{one-touch} \approx 2 \cdot \text{digitals}. \tag{216}$$

The closer the spot price to a perfect symmetry, the better the approximation. So for a market with high swap points as in emerging markets, this approximation is not very good, but still provides a first orientation. I always check the European digital price when I see or need to quote a one-touch price. As for the intuition: if the spot is on the barrier at inception, then the one-touch price is 100%, and the digital price is around 50%, because there is a 50/50 chance for the spot to end up at or above the barrier at maturity. In this extreme case, we see how important the assumption about symmetry is.

Semi-Static Replication To semi-statically replicate the one-touch we follow the recipe and buy two digitals, obviously with the same barrier, same maturity, same pay currency, and same notional. If the spot doesn't hit the barrier, everything is zero, so our replication is perfect. If the spot hits the barrier, we sell the two digitals at first hitting time. Both European digitals are then at-the-money spot, and their value is consequently approximately 50%. Since we have two, the total value adds up to 100%, and Bob's your uncle. Again, we see that the symmetry is crucial. However, we can get a very good smile-adjusted value of the European digital using the windmill effect. This shows us that the slope of the smile on the strike space at the ATM point drives the risk of the one-touch. It is the term structure of this future slope that matters, and which is unknown at inception. Now, would you actually do this replication?

Quotation Conventions and Bid-Ask Spreads If the payoff is at maturity, the undiscounted value of the one-touch is the touch probability under the risk-neutral measure. The market standard is to quote the price of a one-touch in percent of the payoff, a number between 0 and 100%. The market value of a one-touch depends on the theoretical value (TV) of the above formula, the smile adjustment (either derived by our semi-static replication thought experiment or from quantifying other hedge cost such as vanna and volga explained in Section 4.1), and the bid-ask spread. The spread in turn depends on the currency pair and the client. For interbank trading, spreads are usually between 2% and 4% for liquid currency pair – see Section 4.2 for details. Bid and offer quotes are usually rounded to the next quarter.

Two-Touch A two-touch pays one unit of currency (either foreign or domestic) if the underlying exchange rate hits both an upper and a lower barrier during its lifetime. This can be structured using basic touch contracts in the following way. The long two-touch with barriers L and H is equivalent to

1. a long single one-touch with lower barrier L,
2. a long single one-touch with upper barrier H,
3. a short double one-touch with barriers L and H.

This is easily verified by looking at the possible cases.

If the order of touching L and H matters, then the above hedge no longer works, but we have a new product, which can be valued, for example, using a finite-difference grid or Monte Carlo Simulation.

Double-No-Touch The payoff

$$\mathbb{I}_{\{L \leq \min_{[0,T]} S_t < \max_{[0,T]} S_t \leq H\}} \tag{217}$$

of a double-no-touch (DNT) is in units of domestic currency and is paid at maturity T. The lower barrier is denoted by L, the higher barrier by H.

Derivation of the Value Function To compute the expectation, let us introduce the stopping time

$$\tau \stackrel{\Delta}{=} \min\{\, \inf\{t \in [0,T] | S_t = L \;\text{ or }\; S_t = H\}, T\} \tag{218}$$

and the notation

$$\tilde{\theta}_{\pm} \stackrel{\Delta}{=} \frac{r_d - r_f \pm \frac{1}{2}\sigma^2}{\sigma} \tag{219}$$

$$\tilde{h} \stackrel{\Delta}{=} \frac{1}{\sigma} \ln \frac{H}{S_t} \tag{220}$$

$$\tilde{l} \stackrel{\Delta}{=} \frac{1}{\sigma} \ln \frac{L}{S_t} \tag{221}$$

$$\theta_{\pm} \stackrel{\Delta}{=} \tilde{\theta}_{\pm} \sqrt{T-t} \tag{222}$$

$$h \stackrel{\Delta}{=} \tilde{h} / \sqrt{T-t} \tag{223}$$

$$l \stackrel{\Delta}{=} \tilde{l} / \sqrt{T-t} \tag{224}$$

$$y_{\pm} \stackrel{\Delta}{=} y_{\pm}(j) = 2j(h-l) - \theta_{\pm} \tag{225}$$

$$n_T(x) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi T}} \exp\left(-\frac{x^2}{2T}\right) \tag{226}$$

$$n(x) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \tag{227}$$

$$\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt. \tag{228}$$

On $[t, \tau]$, the value of the double-no-touch is

$$v(t) = I\!E^t \left[e^{-r_d(T-t)} I\!I_{\{L \le \min_{[0,T]} S_t < \max_{[0,T]} S_t \le H\}} \right], \tag{229}$$

on $[\tau, T]$,

$$v(t) = e^{-r_d(T-t)} I\!I_{\{L \le \min_{[0,T]} S_t < \max_{[0,T]} S_t \le H\}}. \tag{230}$$

The joint distribution of the maximum and the minimum of a Brownian motion can be taken from [113] and is given by

$$I\!P \left[\tilde{l} \le \min_{[0,T]} W_t < \max_{[0,T]} W_t \le \tilde{h} \right] = \int_{\tilde{l}}^{\tilde{h}} k_T(x)\, dx \tag{231}$$

with

$$k_T(x) = \sum_{j=-\infty}^{\infty} \left[n_T(x + 2j(\tilde{h} - \tilde{l})) - n_T(x - 2\tilde{h} + 2j(\tilde{h} - \tilde{l})) \right]. \tag{232}$$

Hence the joint density of the maximum and the minimum of a Brownian motion with drift $\tilde{\theta}$, $W_t^{\tilde{\theta}} \stackrel{\Delta}{=} W_t + \tilde{\theta} t$, is given by

$$k_T^{\tilde{\theta}}(x) = k_T(x) \exp\left\{ \tilde{\theta} x - \tfrac{1}{2} \tilde{\theta}^2 T \right\}. \tag{233}$$

We obtain for the value of the double-no-touch on $[t, \tau]$

$$\begin{aligned}
v(t) &= e^{-r_d(T-t)} I\!E I\!I_{\{L \le \min_{[0,T]} S_t < \max_{[0,T]} S_t \le H\}} \\
&= e^{-r_d(T-t)} I\!E I\!I_{\{\tilde{l} \le \min_{[0,T]} W_t^{\tilde{\theta}_-} < \max_{[0,T]} W_t^{\tilde{\theta}_-} \le \tilde{h}\}} \\
&= e^{-r_d(T-t)} \int_{\tilde{l}}^{\tilde{h}} k_{(T-t)}^{\tilde{\theta}_-}(x) dx \qquad (234) \\
&= e^{-r_d(T-t)} \qquad (235) \\
&\quad \cdot \sum_{j=-\infty}^{\infty} \left[e^{-2j\theta_-(h-l)} \left\{ \mathcal{N}\left(h + y_-\right) - \mathcal{N}\left(l + y_-\right) \right\} \right. \\
&\quad \left. - e^{-2j\theta_-(h-l)+2\theta_- h} \left\{ \mathcal{N}\left(h - 2h + y_-\right) - \mathcal{N}\left(l - 2h + y_-\right) \right\} \right]
\end{aligned}$$

and on $[\tau, T]$

$$v(t) = e^{-r_d(T-t)} I\!I_{\{L \le \min_{[0,T]} S_t < \max_{[0,T]} S_t \le H\}}. \tag{236}$$

Of course, the value of the double-one-touch (DOT) on $[t, \tau]$ is given by

$$e^{-r_d(T-t)} - v(t), \tag{237}$$

following the fact that a DNT and a DOT in combination are economically equivalent to the zero-coupon bond.

Greeks We take the space to list some of the sensitivity parameters I have been asked about frequently.

Vega

$$\begin{aligned} v_\sigma(t) = \frac{e^{-r_d(T-t)}}{\sigma} \cdot \sum_{j=-\infty}^{\infty} & \\ \Big\{ e^{-2j\theta_-(h-l)} & \big[2j(h-l)(\theta_+ + \theta_-)\left\{\mathcal{N}\left(h+y_-\right) - \mathcal{N}\left(l+y_-\right)\right\} \\ & + n(h+y_-)(-h-y_+) - n(l+y_-)(-l-y_+)\big] \\ - e^{-2j\theta_-(h-l)+2\theta_- h} & \big[2(\theta_+ + \theta_-)(j(h-l)-h)\left\{\mathcal{N}\left(-h+y_-\right) - \mathcal{N}\left(l-2h+y_-\right)\right\} \\ & + n(-h+y_-)(h-y_+) - n(l-2h+y_-)(-l+2h-y_+)\big]\Big\} \end{aligned} \tag{238}$$

Vanna

$$v_{\sigma S_t}(t) = \frac{e^{-r_d(T-t)}}{S_t \sigma^2 \sqrt{T-t}} \cdot \sum_{j=-\infty}^{\infty} \left\{ e^{-2j\theta_-(h-l)}(T_1 - T_2) - e^{-2j\theta_-(h-l)+2\theta_- h}(T_3 + T_4 - T_5) \right\} \tag{239}$$

$$T_1 = n(h+y_-)\left\{1 - 2j(h-l)(\theta_+ + \theta_-) - (h+y_-)(h+y_+)\right\} \tag{240}$$

$$T_2 = n(l+y_-)\left\{1 - 2j(h-l)(\theta_+ + \theta_-) - (l+y_-)(l+y_+)\right\} \tag{241}$$

$$\begin{aligned} T_3 = {} & 2(\theta_+ + \theta_-)\left[-2\theta_- j(h-l) + 2\theta_- h + 1\right] \\ & \cdot \left\{\mathcal{N}(-h+y_-) - \mathcal{N}(l-2h+y-)\right\} \end{aligned} \tag{242}$$

$$\begin{aligned} T_4 = {} & n(-h+y_-)\left\{-2\theta_-(h-y_+) + 2(\theta_+ + \theta_-)(j(h-l)-h) \right. \\ & \left. + (h-y_-)(h-y_+) - 1\right\} \end{aligned} \tag{243}$$

$$\begin{aligned} T_5 = {} & n(l-2h+y_-)\left\{-2\theta_-(-l+2h-y_+) + 2(\theta_+ + \theta_-)(j(h-l)-h) \right. \\ & \left. + (-l+2h-y_-)(-l+2h-y_+) - 1\right\} \end{aligned} \tag{244}$$

Volga

$$v_{\sigma\sigma}(t) = \frac{e^{-r_d(T-t)}}{\sigma^2} \cdot \sum_{j=-\infty}^{\infty} \left\{ e^{-2j\theta_-(h-l)}(T_1 + T_2) - e^{-2j\theta_-(h-l)+2\theta_- h}(T_3 + T_4) \right\} \tag{245}$$

$$T_1 = (2j(\theta_+ + \theta_-)(h-l) - 3)\left\{2j(h-l)(\theta_+ + \theta_-)\left[\mathcal{N}(h+y_-) - \mathcal{N}(l+y_-)\right]\right\}$$
$$+ (4j(\theta_+ + \theta_-)(h-l) - 1)\left[n(h+y_-)(-h-y_+) - n(l+y_-)(-l-y_+)\right] \quad (246)$$

$$T_2 = n(h+y_-)(h+y_-)\left[1-(h+y_+)^2\right] - n(l+y_-)(l+y_-)\left[1-(l+y_+)^2\right] \quad (247)$$

$$T_3 = (2(\theta_+ + \theta_-)(j(h-l)-h) - 3)\left\{2(\theta_+ + \theta_-)(j(h-l)-h)\right.$$
$$\left.\cdot\left[\mathcal{N}(-h+y_-) - \mathcal{N}(l-2h+y_-)\right]\right\} + (4(\theta_+ + \theta_-)(j(h-l)-h) - 1)$$
$$\cdot\left[n(-h+y_-)(h-y_+) - n(l-2h+y_-)(-l+2h-y_+)\right] \quad (248)$$

$$T_4 = n(-h+y_-)(h-y_-)\left[(h-y_+)^2 - 1\right]$$
$$- n(l-2h+y_-)(-l+2h-y_-)\left[(-l+2h-y_+)^2 - 1\right] \quad (249)$$

Applications of Double-Touch Contracts A double-no-touch is a very liquid instrument. Going long a DNT reflects a view of spot staying quiet over the lifetime of the contract. Long a DNT means short vega, so it is a contract one can buy and be vega short. DNT and DOT serve as rebates to double-knock-out and double-knock-in options. They also appear as building blocks in structured FX forwards and yield enhancing deposits like wedding cakes, towers, and onions – see Section 2.4.6.

1.7.5 Compound and Installment

Compound Options A compound call (put) option gives its holder the right to exercise and upon exercise buy (sell) a vanilla option called the daughter option for a pre-specified price called the strike of the mother option. It works in a similar way to a vanilla call, but allows the holder to pay the premium of the call option spread over time. A first payment is made on inception of the trade. On the following payment day the holder of the compound call can decide to turn it into a plain vanilla call, in which case he has to pay the second part of the premium, or terminate the contract by simply not paying any more.

Advantages

- Full protection against stronger EUR/weaker USD
- Maximum loss is the premium paid
- Initial premium required is less than in the vanilla call
- Easy termination process, especially useful if future cash flows are uncertain

Disadvantages

- Premium required (not a zero-cost structure)
- More expensive than the vanilla call

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be able to buy EUR at a lower spot rate if the EUR becomes weaker on the one hand, but

TABLE 1.22 Example of a compound call option.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional	USD 1,000,000
Company buys	EUR call USD put strike 1.1500
Premium per half year of the compound	USD 23,000.00
Premium of the vanilla call	USD 40,000.00

on the other hand be fully protected against a stronger EUR. The future income in USD is uncertain but will be under review at the end of the next half year.

In this case a possible form of protection that the company can use is to buy a EUR compound call option with two equal semi-annual premium payments as illustrated in Table 1.22 for example.

The company pays 23,000 USD on the trade date. After a half year, the company has the right to buy a plain vanilla call. To do this the company must pay another 23,000 USD.

Of course, besides not paying the premium, another way to terminate the contract is always to sell it in the market or to the seller. So if the option is not needed but deep in-the-money, the company can take profit from paying the premium to turn the compound into a plain vanilla call and then sell it.

If the EUR-USD exchange rate is above the strike at maturity, then the company can buy EUR at maturity at a rate of 1.1500.

If the EUR-USD exchange rate is below the strike at maturity, the option expires worthless. However, the company would benefit from being able to buy EUR at a lower rate in the market.

Variations of Compound Options

Distribution of payments The payments do not have to be equal. However, the rule is that the more premium is paid later, the higher the total premium. The cheapest distribution of payments is to pay the entire premium in the beginning, which corresponds to a plain vanilla call.

Exercise style Both the mother and the daughter of the compound option can be European and American style. The market default is European style.

Compound strategies One can think of a compound option on any structure, as for instance a compound put on a knock-out call or a compound call on a structured forward.

Forward Volatility The daughter option of the compound requires knowing the volatility for its lifetime, which starts on the exercise date T_1 of the mother option and ends on the maturity date T_2 of the daughter option. This volatility is not known at inception of the trade, so the only proxy traders can take is the forward volatility $\sigma(T_1, T_2)$ for this time interval. In the Black-Scholes model the consistency equation for the forward volatility is given by Equation (152).

The more realistic way to look at this unknown forward volatility is that the fairly liquid market of forward volatility-sensitive derivatives could be taken to back out the forward volatilities since this is the only unknown. These derivatives include forward start options and forward volatility agreements.

In a market with smile, the payoff of the compound option can be approximated by a linear combination of vanillas, whose market prices are known. For the payoff of the compound option itself we can take the forward volatility as in Equation (152) for the at-the-money value and the smile of today as a proxy. More details on this can be found in Schilling [116] for example. The actual forward volatility, however, is a trader's view and with any luck can be taken from other market prices.

Installment Options This section is based on Griebsch *et al.* – see [61].

An installment call option generalizes the idea of a compound call: it allows the holder to pay the premium of the call option in more than two installments spread over time. A first payment is made at inception of the trade. On the following payment days the holder of the installment call can decide to prolong the contract, in which case he has to pay the second installment of the premium, or to terminate the contract by simply not paying any more. After the last installment payment the contract turns into a plain vanilla call. We illustrate two scenarios in Figure 1.41.

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be able to buy EUR at a lower spot rate if the EUR becomes weaker on the one hand, but on the other hand be fully protected against a stronger EUR. The future income in USD is uncertain but will be under review at the end of each quarter.

In this case a possible form of protection that the company can use is to buy a EUR installment call option with four equal quarterly premium payments as illustrated in Table 1.23 for example.

The company pays 12,500 USD on the trade date. After one quarter, the company has the right to prolong the installment contract. To do this the company must pay another 12,500 USD. After six months, the company has the right to prolong the contract and must pay 12,500 USD in order to do so. After nine months the same decision has to be taken. If on one of these three decision days the company does not pay, then the contract terminates. If all premium payments are made, then the contract turns into a plain vanilla EUR call.

Of course, besides not paying the premium, another way to terminate the contract is always to resell it in the market. So if the option is not needed but deep in-the-money,

TABLE 1.23 Example of an installment call.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional	USD 1,000,000
Company buys	EUR call USD put strike 1.1500
Premium per quarter of the installment	USD 12,500.00
Premium of the vanilla call	USD 40,000.00

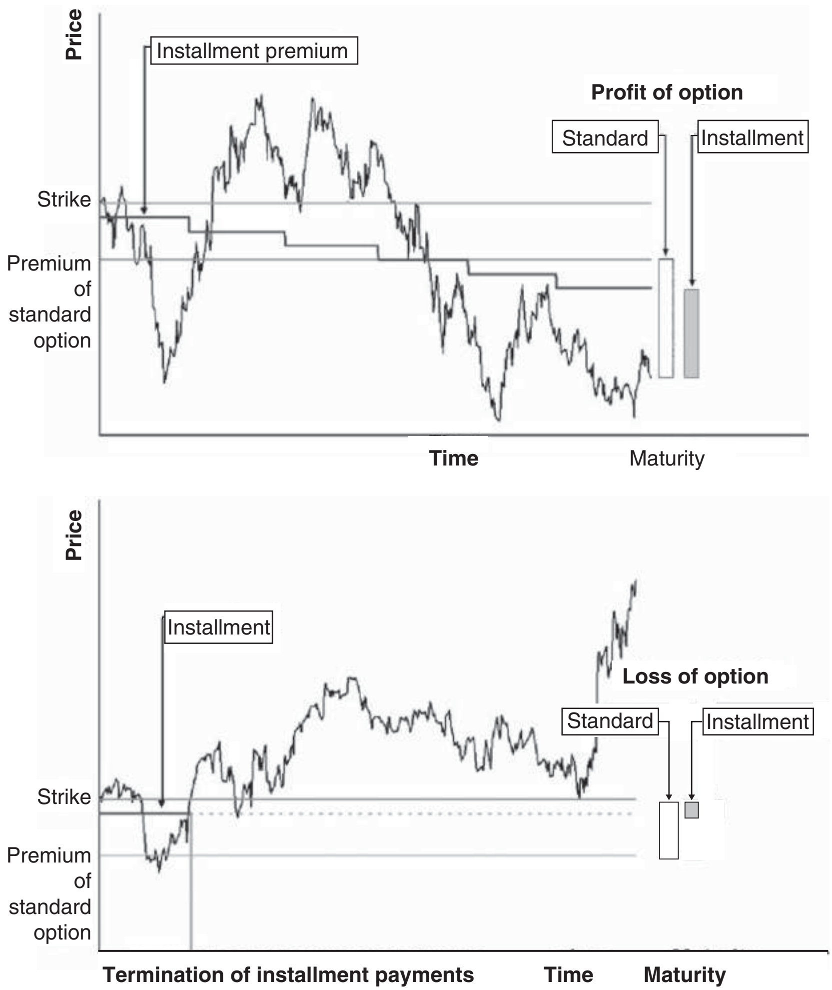


FIGURE 1.41 Comparison of two scenarios of an installment option. The top graph shows a continuation of all installment payments until expiration. The graph below shows a scenario where the installment option is terminated after the first decision date.

the company can take profit from paying the premium to prolong the contract and then sell it.

If the EUR-USD exchange rate is above the strike at maturity, then the company can buy EUR at maturity with a rate of 1.1500.

If the EUR-USD exchange rate is below the strike at maturity, the option expires worthless. However, the company would benefit from being able to buy EUR at a lower rate in the market.

Compound options can be viewed as a special case of installment options, and the possible variations of compound options apply analogously to installment options.

Reasons for Trading Compound and Installment Options We observe that in the buy-and-hold-to-maturity scenario, compound and installment options are always more expensive than buying a vanilla, sometimes substantially more expensive. So why are people buying them? The number one reason is an *uncertainty* about a future cash flow in a foreign currency. If the cash flow is certain, then buying a vanilla is in principle the better deal. An exception may be the situation that a treasurer has a budget constraint, i.e. limited funds to spend for foreign exchange risk. With an installment he can then split the premium over time. The main issue is, however, if a treasurer has to deal with an uncertain cash flow, and buys a vanilla instead of an installment, and then is faced with a far out-of-the-money vanilla at time T_1, then selling the vanilla does not give him as much as the savings between the vanilla and the sum of the installment payments.

The Theory of Installment Options This book is not concerned primarily with valuation of options. However, we do want to give some insight into selected topics that come up very often and are of particular relevance to foreign exchange options and have not been published in books so far. We will now take a look at the valuation, the implementation of installment options, and the limiting case of a continuous flow of premium payments.

Valuation in the Black-Scholes Model The intention of this section is to obtain a closed-form formula for the n-variate installment option in the Black-Scholes model. For the cases $n = 1$ and $n = 2$, the Black-Scholes formula and Geske's compound option formula (see [56]) are already well known.

We consider an exchange rate process S_t modeled by a geometric Brownian motion,

$$S_{t_2} = S_{t_1} \exp((r_d - r_f - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta t}Z) \quad \text{for } 0 \le t_1 \le t_2 \le T, \tag{250}$$

where $\Delta t = t_2 - t_1$ and Z is a standard normal random variable independent of the past of S_t up to time t_1.

Let $t_0 = 0$ be the installment option inception date and $t_1, t_2, \ldots, t_n = T$ a schedule of decision dates in the contract on which the option holder has to pay the premiums $k_1, k_2, \ldots, k_{n-1}$ to keep the option alive. To compute the price of the installment option, which is the upfront payment V_0 at t_0 to enter the contract, we begin with the option payoff at maturity T

$$V_n(s) \stackrel{\Delta}{=} [\phi_n(s - k_n)]^+ \stackrel{\Delta}{=} \max[\phi_n(s - k_n), 0],$$

where $s = S_T$ is the price of the underlying asset at T and as usual $\phi_n = +1$ for a call option, $\phi_n = -1$ for a put option.

At time t_i the option holder can either terminate the contract or pay k_i to continue. Therefore by the risk-neutral pricing theory, the holding value is

$$e^{-r_d(t_{i+1}-t_i)}\mathbb{E}[V_{i+1}(S_{t_{i+1}}) \mid S_{t_i} = s], \quad \text{for } i = 0, \ldots, n-1, \tag{251}$$

where

$$V_i(s) = \begin{cases} \left[e^{-r_d(t_{i+1}-t_i)}\mathbb{E}[V_{i+1}(S_{t_{i+1}}) \mid S_{t_i} = s] - k_i\right]^+ & \text{for } i = 1, \ldots, n-1 \\ V_n(s) & \text{for } i = n \end{cases}. \tag{252}$$

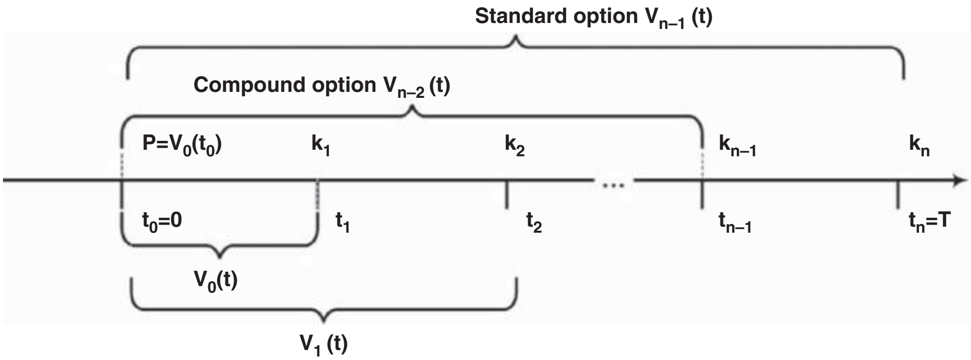


FIGURE 1.42 Lifetime structure of the options with value V_i for the i-th Option.

Then the unique arbitrage-free value of the initial premium is

$$P \stackrel{\Delta}{=} V_0(s) = e^{-r_d(t_1-t_0)} I\!E[V_1(S_{t_1}) \mid S_{t_0} = s]. \tag{253}$$

Figure 1.42 illustrates this context.

One way of pricing this installment option is to evaluate the nested expectations through multiple numerical integration of the payoff functions via backward iteration. Alternatively, one can derive a solution in closed form in terms of the n-variate cumulative normal.

The Curnow and Dunnett Integral Reduction Technique Denote the n dimensional multivariate normal integral with upper limits $h_1, \dots, h_n$ and correlation matrix $R_n \stackrel{\Delta}{=} (\rho_{ij})_{i,j=1,\dots,n}$ by $\mathcal{N}_n(h_1, \dots, h_n; R_n)$, and the univariate normal density function by $n(\cdot)$. Let the correlation matrix be non-singular and $\rho_{11} = 1$.

Under these conditions Curnow and Dunnett [33] derived the following reduction formula for multivariate normal integrals:

$$\begin{aligned}
\mathcal{N}_n(h_1, \dots, h_n; R_n) &= \int_{-\infty}^{h_1} \mathcal{N}_{n-1}\left(\frac{h_2 - \rho_{21}y}{(1-\rho_{21}^2)^{1/2}}, \dots, \frac{h_n - \rho_{n1}y}{(1-\rho_{n1}^2)^{1/2}}; R_{n-1}^*\right) n(y)dy, \\
R_{n-1}^* &\stackrel{\Delta}{=} (\rho_{ij}^*)_{i,j=2,\dots,n}, \\
\rho_{ij}^* &\stackrel{\Delta}{=} \frac{\rho_{ij} - \rho_{i1}\rho_{j1}}{(1-\rho_{i1}^2)^{1/2}(1-\rho_{j1}^2)^{1/2}}.
\end{aligned} \tag{254}$$

A Closed-Form Solution for the Value of an Installment Option Heuristically, the formula which is given in the theorem below has the structure of the Black-Scholes formula in higher dimensions, namely $S_0\mathcal{N}_n(\cdot) - k_n\mathcal{N}_n(\cdot)$ minus the later premium payments $k_i\mathcal{N}_i(\cdot)$ $(i = 1, \dots, n-1)$. This structure is a result of the integration of the vanilla option payoff, which is again integrated minus the next installment, which in turn is integrated with the following installment and so forth. By this iteration the vanilla payoff is integrated with respect to the normal density function n times and the i-payment is integrated i times for $i = 1, \ldots, n-1$.

The correlation coefficients ρ_{ij} of these normal distribution functions contained in the formula arise from the overlapping increments of the Brownian motion, which models the price process of the underlying S_t at the particular exercise dates t_i and t_j.

Theorem 1.7.1 *Let $\vec{k} = (k_1, \ldots, k_n)$ be the strike price vector, $\vec{t} = (t_1, \ldots, t_n)$ the vector of the exercise dates of an n-variate installment option, and $\vec{\phi} = (\phi_1, \ldots, \phi_n)$ the vector of the put/call indicators of these n options.*

The value function of an n-variate installment option is given by

$$
\begin{aligned}
V_n(S_0, M, \vec{k}, \vec{t}, \vec{\phi}) = {} & e^{-r_f t_n} S_0 \phi_1 \cdot \cdots \cdot \phi_n \\
& \times \mathcal{N}_n \left[\frac{\ln \frac{S_0}{S_1^*} + \mu^{(+)} t_1}{\sigma \sqrt{t_1}}, \frac{\ln \frac{S_0}{S_2^*} + \mu^{(+)} t_2}{\sigma \sqrt{t_2}}, \ldots, \frac{\ln \frac{S_0}{S_n^*} + \mu^{(+)} t_n}{\sigma \sqrt{t_n}}; R_n \right] \\
& - e^{-r_d t_n} k_n \phi_1 \cdot \cdots \cdot \phi_n \\
& \times \mathcal{N}_n \left[\frac{\ln \frac{S_0}{S_1^*} + \mu^{(-)} t_1}{\sigma \sqrt{t_1}}, \frac{\ln \frac{S_0}{S_2^*} + \mu^{(-)} t_2}{\sigma \sqrt{t_2}}, \ldots, \frac{\ln \frac{S_0}{S_n^*} + \mu^{(-)} t_n}{\sigma \sqrt{t_n}}; R_n \right] \\
& - e^{-r_d t_{n-1}} k_{n-1} \phi_1 \cdot \cdots \cdot \phi_{n-1} \\
& \times \mathcal{N}_{n-1} \left[\frac{\ln \frac{S_0}{S_1^*} + \mu^{(-)} t_1}{\sigma \sqrt{t_1}}, \frac{\ln \frac{S_0}{S_2^*} + \mu^{(-)} t_2}{\sigma \sqrt{t_2}}, \ldots, \frac{\ln \frac{S_0}{S_{n-1}^*} + \mu^{(-)} t_{n-1}}{\sigma \sqrt{t_{n-1}}}; R_{n-1} \right] \\
& \vdots \\
& - e^{-r_d t_2} k_2 \phi_1 \phi_2 \mathcal{N}_2 \left[\frac{\ln \frac{S_0}{S_1^*} + \mu^{(-)} t_1}{\sigma \sqrt{t_1}}, \frac{\ln \frac{S_0}{S_2^*} + \mu^{(-)} t_2}{\sigma \sqrt{t_2}}; \rho_{12} \right] \\
& - e^{-r_d t_1} k_1 \phi_1 \mathcal{N} \left[\frac{\ln \frac{S_0}{S_1^*} + \mu^{(-)} t_1}{\sigma \sqrt{t_1}} \right],
\end{aligned} \tag{255}
$$

where S_i^* $(i = 1, \ldots, n)$ is to be determined as the spot price S_t for which the payoff of the corresponding i-installment option $(i = 1, \ldots, n)$ is equal to zero and $\mu^{(\pm)}$ is defined as $r_d - r_f \pm \frac{1}{2}\sigma^2$.

The correlation coefficients in R_i of the i-variate normal distribution function can be expressed through the exercise dates t_i,

$$\rho_{ij} = \sqrt{t_i/t_j} \text{ for } i, j = 1, \ldots, n \text{ and } i < j. \tag{256}$$

The proof is established with Equation (254). Formula (255) has been independently derived by Thomassen and van Wouwe in [125].

Valuation of Installment Options with the Algorithm of Ben-Hameur, Breton, and François The value of an installment option at time t is given by the snell envelope of the discounted payoff processes, which is calculated with the dynamic programming method used by the algorithm of Ben-Hameur, Breton, and François below. Their original work in [14] deals with installment options with an additional exercise right at each installment date. This means that at each decision date the holder can either exercise, terminate, or continue.

We examine this algorithm now for the special case of zero value in case it is exercised at $t_1, \ldots, t_{n-1}$. The difference between the above mentioned types of installment options consists in the (non-)existence of an exercise right at the installment dates, but this does not change the algorithm in principle.

Model Description The algorithm developed by Ben-Hameur, Breton, and François approximates the value of the installment option in the Black-Scholes model, which is the premium P paid at time t_0 to enter the contract.

The exercise value of an installment option at maturity t_n is given by $V_n(s) \stackrel{\Delta}{=} \max[0, \phi_n(s - k_n)]$ and zero at earlier times. The value of a vanilla option at time t_{n-1} is denoted by $V_{n-1}(s) = e^{-r_d \Delta t} \mathbb{E}[V_n(s) \mid S_{t_{n-1}} = s]$. At an arbitrary time t_i the holding value is determined as

$$V_i^h(s) = e^{-r_d \Delta t} \mathbb{E}[V_{i+1}(S_{t_{i+1}}) \mid S_{t_i} = s] \text{ for } i = 0, \ldots, n-1, \tag{257}$$

where

$$V_i(s) = \begin{cases} V_0^h(s) & \text{for } i = 0, \\ \max[0, V_i^h(s) - k_i] & \text{for } i = 1, \ldots, n-1, \text{ (DP)}. \\ V_n^e(s) & \text{for } i = n. \end{cases} \tag{258}$$

The function $V_i^h(s) - k_i$ is called net holding value at t_i, for $i = 1, \ldots, n-1$, which is shown in Figure 1.43.

The option value is the holding value or the exercise value, whichever is greater. The value function V_i, for $i = 0, \ldots, n-1$, is unknown and has to be approximated. Ben-Hameur, Breton, and François propose an approximation method, which solves the above dynamic programming (DP)-equation (258) in a closed form for all s and i.

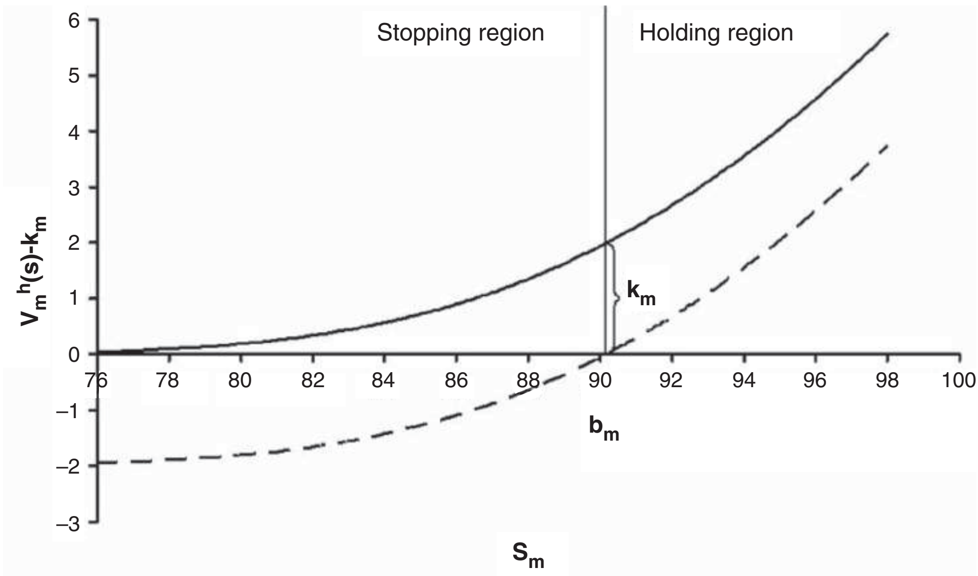


FIGURE 1.43 The holding value shortly before t_3 for an installment option with 4 rates is shown by the solid line. The positive slope of this function is less than 1 and the function is continuous and convex. The net holding value of an installment call option $V_m^h(s) - k_m$ for $(s > 0)$ and a decision time m is presented by the dashed line. This curve intersects the x-axis in the point, where it divides the stopping region and the holding region. The value function is zero in the stopping region $(0, b_i)$ and equal to the net holding value in the holding region $[b_i, \infty)$, where b_i is a threshold for every time t_i, which depends on the parameters of the installment option.

Valuation of Installment Options with Stochastic Dynamic Programming The idea of the above mentioned authors is to partition the positive real axis into intervals and approximate the option value through piecewise linear interpolation. Let $a_0 = 0 < a_1 < \cdots < a_p < a_{p+1} = +\infty$ be points in $I\!R_0^+ \cup \{\infty\}$ and $(a_j, a_{j+1}]$for $j = 0, \ldots, p$ a partition of $I\!R_0^+$ in (p+1) intervals.

Given approximations $\tilde{V}_i$ of option values V_i at supporting points a_j at the i-th step (at the beginning of the algorithm, at T, this is provided through the input values), this function is piecewise linearly interpolated by

$$\hat{V}_i(s) = \sum_{i=0}^{p} (\alpha_j^i + \beta_j^i s) I\!I_{\{a_j < s \le a_{j+1}\}}, \tag{259}$$

where $I\!I$ is the indicator function. The local coefficients of this interpolation in step i, the y-axis intercepts α_j^i and the slopes β_j^i, are obtained by solving the following linear equations,

$$\tilde{V}_i(a_j) = \hat{V}_i(a_j) \text{ for } j = 0, \ldots, p-1. \tag{260}$$

For $j = p$, one chooses

$$\alpha_p^i = \alpha_{p-1}^i \text{ and } \beta_p^i = \beta_{p-1}^i. \tag{261}$$

Now it is assumed that $\hat{V}_{i+1}$ is known. This is a valid assumption in this context because the values $\hat{V}_{i+1}$ are known from the previous step. The mean value (257) is calculated in step i through

$$\begin{aligned}\tilde{V}_i^h(a_k) &= e^{-r_d\Delta t}I\!E[\hat{V}_{i+1}(S_{t_{i+1}})|S_{t_i} = a_k] \\ &\overset{(259)}{=} e^{-r_d\Delta t}\sum_{j=0}^{p}\alpha_j^{i+1} I\!E\left[I\!I_{\left\{\frac{a_j}{a_k} < e^{\mu\Delta t+\sigma\sqrt{\Delta t}z} \leq \frac{a_{j+1}}{a_k}\right\}}\right] \\ &\quad + \beta_j^{i+1} a_k I\!E\left[e^{\mu\Delta t+\sigma\sqrt{\Delta t}z} I\!I_{\left\{\frac{a_j}{a_k} < e^{\mu\Delta t+\sigma\sqrt{\Delta t}z} \leq \frac{a_{j+1}}{a_k}\right\}}\right],\end{aligned} \tag{262}$$

where $\mu \stackrel{\Delta}{=} r_d - r_f - \sigma^2/2$ and $\tilde{V}_i^h$ denotes the approximated holding value of the installment option. Define

$$x_{k,j} \stackrel{\Delta}{=} \frac{\ln\left(\frac{a_j}{a_k}\right) - \mu\Delta t}{\sigma\sqrt{\Delta t}}, \tag{263}$$

so for $k = 1, \ldots, p$ and $j = 0, \ldots, p$ the first mean values in Equation (262), namely

$$A_{k,j} \stackrel{\Delta}{=} I\!E\left[I\!I_{\left\{\frac{a_j}{a_k} < e^{\mu\Delta t+\sigma\sqrt{\Delta t}z} \leq \frac{a_{j+1}}{a_k}\right\}}\right] \tag{264}$$

can be expressed as

$$(264) = \begin{cases} \mathcal{N}(x_{k,1}) & \text{for } j = 0, \\ \mathcal{N}(x_{k,j+1}) - \mathcal{N}(x_{k,j}) & \text{for } 1 \leq j \leq p-1, \\ 1 - \mathcal{N}(x_{k,p}) & \text{for } j = p, \end{cases} \tag{265}$$

and similarly

$$B_{k,j} \stackrel{\Delta}{=} I\!E\left[a_k e^{\mu\Delta t+\sigma\sqrt{\Delta t}z} I\!I_{\left\{\frac{a_j}{a_k} < e^{\mu\Delta t+\sigma\sqrt{\Delta t}z} \leq \frac{a_{j+1}}{a_k}\right\}}\right] \tag{266}$$

can be expressed in the following way

$$(266) = \begin{cases} a_k\mathcal{N}(x_{k,1} - \sigma\sqrt{\Delta t})e^{(r_d-r_f)\Delta t} & \text{for } j = 0, \\ a_k[\mathcal{N}(x_{k,j+1} - \sigma\sqrt{\Delta t}) - \mathcal{N}(x_{k,j} - \sigma\sqrt{\Delta t})]e^{(r_d-r_f)\Delta t} & \text{for } 1 \leq j \leq p-1, \\ a_k[1 - \mathcal{N}(x_{k,p} - \sigma\sqrt{\Delta t})]e^{(r_d-r_f)\Delta t} & \text{for } j = p, \end{cases} \tag{267}$$

where $n(z) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$ and $\mathcal{N}$ denotes the cumulative normal distribution function.

In the simplifying notation (265) and (267) the points a_i ($i = 1, \dots, p$) can be understood as the quantiles of the log-normal distribution. These are not chosen directly but are calculated as the quantiles of (e.g. equidistant) probabilities of the log-normal distribution. Thereby the supporting points lie closer together, in areas where the modification rate of the distribution function is great. The number a_k in Equation (263) is the given exchange rate at time t_i and therefore constant. In the implementation it requires an efficient method to calculate the inverse normal distribution function. One possibility is to use the Cody-Algorithm taken from [21].

An Algorithm in Pseudo Code For a better understanding let us sketch the procedure described in Section 1.7.5 in the form of an algorithm. The algorithm works according to the dynamic programming principle backwards in time, based on the values of the exercise function of the installment option at maturity T at pre-determined supporting points a_j. Through linear connection of these points an approximation of the exercise function can be obtained. The exercise function at maturity is the payoff function of the vanilla option, which is constant up to the strike price K and in the region behind (i.e. $\geq K$) it is linear. The linear approximation at maturity T is therefore exact, except on the interval $K \in (a_l, a_{l+1})$, in case K does not correspond to one of these supporting points. For this reason the holding value of this linear approximation is calculated by the means of $A_{k,j}$ and $B_{k,j}$ from above. The transition parameters $A_{k,i}$ and $B_{k,j}$ can be calculated before the first iteration, because only values which are known in the beginning are required. The advantage of this approach is that the holding value needs to be calculated only at the supporting points a_j and because of linearity, the function values for all s are obtained. The values of the holding value at a_j are used again as approximations of the exercise values at time t_{n-1} and it proceeds as in the beginning. The output of the algorithm is the value of the installment option at time t_0.

A Description in Pseudo Code First the a_k are generated as quantiles of the distribution of the price at maturity of the exchange rate S_T and can be approximated by the Cody-Algorithm, for example.

1. Calculate $q_1, \dots, q_p$-quantiles of the standard normal distribution via the inverse distribution function.
2. Calculate $a_1, \dots, a_p$-quantiles of the log-normal distribution with mean $\log S_0 - \mu T$ and variance $\sigma\sqrt{T}$ by

$$\exp(q_i \sigma \sqrt{T} + \log S_0 + \mu T) = a_i.$$

In pseudo code the implementation of the theoretical consideration of Section 1.7.5 can be worked out in the following way. The principle of the backward induction is realized as a for-loop that counts backwards from $n - 1$ to 0.

1. Calculate $\hat{V}_n(s)$ for all s, using (259), i.e. calculate all α_i^n, β_i^n for $i = 0, \dots, p$.
2. For $j = n$ to 1
   a. Calculate $\tilde{V}_{j-1}^h(a_k)$ for a_k ($k = 1, \dots, p$) in closed form using (262).
   b. Calculate $\tilde{V}_{j-1}(a_k)$ for $k = 1, \dots, p$ using (DP) with $\tilde{V}_{j-1}^h(a_k)$ for $V_{j-1}^h(a_k)$.

c. Calculate $\hat{V}_{j-1}(s)$ for all $s > 0$ using (259), i.e. calculate all α_i^{j-1}, β_i^{j-1} for $i = 1, \ldots, k$. Unless $j-1$ is already equal to zero, calculate $\hat{V}_{j-1}(s)$ for $s = S_0$ and break the algorithm.

d. Substitute $j \leftarrow j-1$.

Repeat these steps until $\hat{V}_0(S_0)$ is calculated, which is the value of the installment option at time 0.

This algorithm works with equidistant installment dates, constant volatility, and constant interest rates. Constant volatility and interest rates are assumptions of the applied Black-Scholes Model, but the algorithm would be extendable for piecewise constant volatility and interest rate as functions of time, with jumps at the installment dates. The interval length Δt in the calculation can be replaced in every period by arbitrary $t_{i+1} - t_i$. Furthermore the computational time could be decreased by omitting smaller supporting points in the calculation as soon as one of them generates a zero value in the maximum function.

Installment Options with a Continuous Payment Plan Let $g = (g_t)_{t\in[0,T]}$ be the stochastic process describing the discounted net payoff of an installment option expressed as multiples of the domestic currency. If the holder stops paying the premium at time t, the difference between the option payoff and premium payments (all discounted to time 0) amounts to

$$g(t) = \begin{cases} e^{-r_d T}(S_T - K)^+ \mathbf{1}_{(t=T)} - \dfrac{p}{r_d}(1 - e^{-r_d t}) & \text{if } r_d \neq 0 \\ (S_T - K)^+ \mathbf{1}_{(t=T)} - pt & \text{if } r_d = 0 \end{cases}, \tag{268}$$

where K is the strike. Given the premium rate p, there is a unique no-arbitrage premium P_0 to be paid at time 0 (supplementary to the rate p) given by

$$P_0 = \sup_{\tau \in \mathcal{T}_{0,T}} \mathbb{E}_Q(g_\tau). \tag{269}$$

Ideally, p is chosen as the *minimal* rate such that

$$P_0 = 0. \tag{270}$$

Note that P_0 from (269) can never become negative as it is always possible to stop payments immediately. Thus, besides (270), we need a minimality assumption to obtain a unique rate. We want to compare the installment option with the American contingent claim $f = (f_t)_{t\in[0,T]}$ given by

$$f_t = e^{-r_d t}(K_t - C_E(T-t, S_t))^+, t \in [0, T], \tag{271}$$

where $K_t = \frac{p}{r_d}\left(1 - e^{-r_d(T-t)}\right)$ for $r_d \neq 0$ and $K_t = p(T-t)$ when $r_d = 0$. C_E is the value of a standard European call. Equation (271) represents the payoff of an American put on a European call where the variable strike K_t of the put equals the part of the installments *not* to be paid if the holder decides to terminate the contract at time t. Define by $\tilde{f} = (\tilde{f}_t)_{t\in[0,T]}$ a similar American contingent claim with

$$\tilde{f}(t) = e^{-r_d t}\left[(K_t - C_E(T-t,S_t))^+ + C_E(T-t,S_t)\right], \quad t \in [0,T]. \tag{272}$$

As the process $t \mapsto e^{-r_d t} C_E(T-t,S_t)$ is a Q-martingale we obtain that

$$\sup_{\tau\in\mathcal{T}_{0,T}} \mathbb{E}_Q(\tilde{f}_\tau) = C_E(T,s_0) + \sup_{\tau\in\mathcal{T}_{0,T}} \mathbb{E}_Q(f_\tau). \tag{273}$$

The following theorem has been proved in [60] using earlier results of El Karoui *et al.* in [47].

Theorem 1.7.2 *An installment option is the sum of a European call plus an American put on this European call, i.e.*

$$\underbrace{P_0 + p\int_0^T e^{-r_d s}\,ds}_{\textit{total premium payments}} = C_E(T,s_0) + \sup_{\tau\in\mathcal{T}_{0,T}} \mathbb{E}_Q(f_\tau)$$

1.7.6 Asian Options

This section is joint work with Silvia Baumann, Marion Linck, Michael Mohr, and Michael Seeberg.

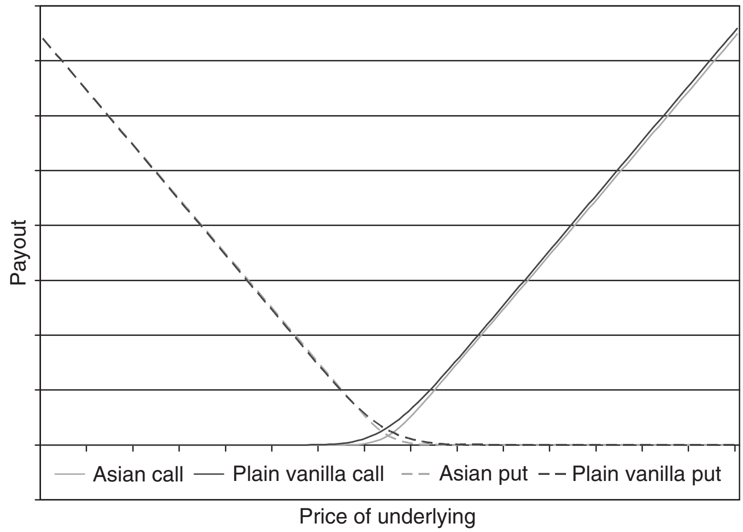
Asian Options are options on the average usually of spot fixings and are very popular and common hedging instruments for corporates. Average options belong to the class of path dependent options. The Term *Asian Options* comes from their origin in the Tokyo office of Bankers Trust in 1987.[4] The payoff of an Asian Option is determined by the path taken by the underlying exchange rate over a fixed period of time. We distinguish the four cases listed in Table 1.24 and compare values of average price options with vanilla options in Figure 1.44.

Variations of Asian options refer particularly to the way the average is calculated.

TABLE 1.24 Types of Asian options for $T_0 \le t \le T$, where $[T_0, T]$ denotes the time interval over which the average is taken, K denotes the strike, S_T the spot price at expiration time and A_T the average.

Product name	Payoff	Product name	Payoff
Average price call	$(A_T - K)^+$	Average strike call	$(S_T - A_T)^+$
Average price put	$(K - A_T)^+$	Average strike put	$(A_T - S_T)^+$

[4]see https://en.wikipedia.org/wiki/Asian_option



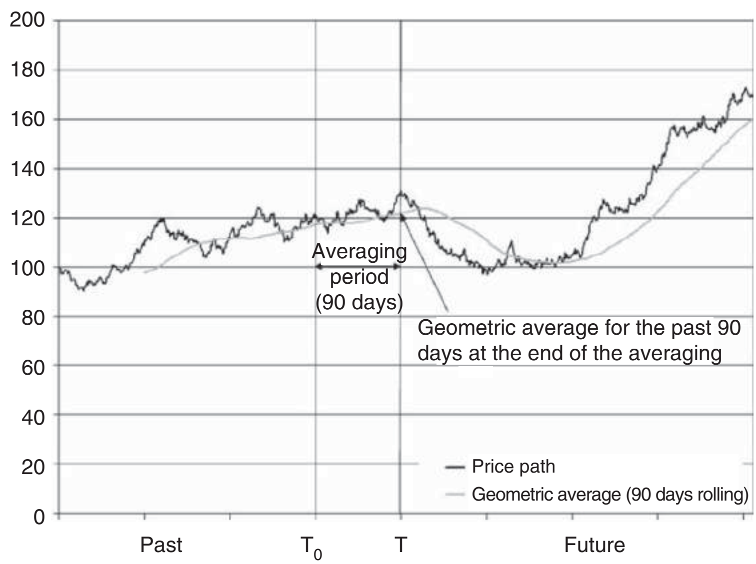


FIGURE 1.44 Above: comparing the value of average price options with vanillas, we see that average price options are cheaper. The reason is that averages are less volatile and hence less risky. Below: ingredients for average options: a price path, 90-days rolling price average (here: geometric), and an averaging period for an option with 90-days maturity.

Kind of average We find geometric, arithmetic, or harmonic average of prices. Harmonic averaging originates from a payoff in *domestic* currency and will be treated in Section 1.8.7.

Time interval We need to specify the period over which the prices are taken. The end of the averaging interval can be shorter than or equal to the option's expiration date, the starting value can be any time before. In particular, after an average option is traded, the beginning of the averaging period typically lies in the past, so that parts of the values contributing to the average are already known.

Sampling style The market generally uses discrete sampling, like daily fixings. In the literature we often find continuous sampling.

Weighting Different weights may be assigned to the prices to account for a non-linear, i.e. skewed, price distribution – see Hansen and Jorgensen [69], pp. 1116–1117, and the example below under 3.

Variations The wide range of variations covers also the possible right for early exercise, Asian options with barriers.

Asian Options are applied in risk management, especially for currencies for the following reasons.

1. Protection against rapid price movements or manipulation in thinly traded underlyings at maturity, i.e. reduction of significance of reference prices through averaging.
2. Reduction of hedging cost through
   - the lower fair value compared with regular options since an average is less volatile than single prices, and
   - to achieve a similar hedging effect as with vanilla options, buying a chain of such options – obviously a more expensive strategy.
3. Adjustment of option payoff to payment structure of the firm
   - Average price options can be used to hedge a stream of (received) payments (e.g. a USD average call can be bought to hedge the ongoing EUR revenues of a US-based company). Different amounts of the payments can be reflected in flexible weights, i.e. the prices related to higher payments are assigned a higher weight than those related to smaller cash flows when calculating the average.
   - With average strike options the strike price can be set at the average of the underlying price – a helpful structure in volatile or hardly predictable markets.

Valuation The valuation approaches developed differ depending on the specific characteristics considered, for example averaging method, option style, etc. In the following, we present the value formula for a *European geometric average price call*. In the sequel two common approaches to evaluating arithmetic average price options are introduced. Henderson and Wojakowski prove the symmetry between average price options and average strike options in [72], allowing the use of the more established fixed-strike valuation methods to price *floating strike Asian options*. Asian options were the first ones traded in history, where *pre-trade valuation* and *post-trade valuation* are different.

Geometric Average Options Kemna and Vorst [86] derive a closed form solution for geometric average price options in a geometric Brownian motion model

$$dS_t = S_t[(r_d - r_f)dt + \sigma dW_t]. \tag{274}$$

A geometric average price call pays $(A_T - K)^+$, where A_T denotes the geometric average of the foreign exchange rate. In the discrete case, A_T is calculated as

$$A_T \triangleq \sqrt[n+1]{\prod_{i=0}^{n} S_{t_i}}, \tag{275}$$

in the continuous case as

$$A_T \triangleq \exp\left\{\frac{1}{T - T_0}\int_{T_0}^{T} \log S_t \, dt\right\}. \tag{276}$$

The random variable $\int_t^T W(u)\, du$ is normally distributed with mean zero variance

$$\Sigma^2 \triangleq \frac{T^3}{3} + \frac{2t^3}{3} - t^2 T \tag{277}$$

for any $t \in [T_0, T]$. This can be calculated following the instructions in Shreve's lecture notes [119]. Therefore, the geometric average of a log-normally distributed random variable is log-normally distributed. In the continuous case, the distribution parameters can be derived as

$$\log A_T \sim \mathcal{N}\left[\frac{1}{2}\left(r_d - r_f - \frac{1}{2}\sigma^2\right)(T - T_0) + \log S_0; \frac{1}{3}\sigma^2(T - T_0)\right]. \tag{278}$$

The interesting feature of these terms is that the average has half the drift and one third of the variance of the spot price. In the Black-Scholes model the value of the option can be computed as the expected payoff under the risk-neutral probability measure. Using the money market account $e^{-r_d(T-T_0)}$ as numeraire leads to the value of the continuously sampled geometric Asian fixed strike call,

$$C_{G-Asian} = \mathbb{E}\left[e^{-r_d(T-T_0)}(A_T - K)\mathbb{I}_{\{A_T > K\}}\right], \tag{279}$$

where we observe that the remaining computation works just like a vanilla. In order to derive a useful general result we need to generalize the payoff of the continuously sampled geometric Asian fixed strike option to

$$[\phi(A(-s, T) - K)]^+, \tag{280}$$

$$A(-s, T) \triangleq \exp\left\{\frac{1}{T + s}\int_{-s}^{T} \log S(u)\, du\right\}, s \geq 0. \tag{281}$$

This definition includes the case where parts of the average are already known, which is important to value the option after it has been written (post-trade valuation).

With the abbreviations

- T for the expiration time (in years),
- s for the time before valuation date (in years), for which the values and average of the underlying are known,
- K for the strike of the option,
- ϕ taking the values $+1$ or -1 if the option is a call or a put respectively,
- $\alpha \stackrel{\Delta}{=} \frac{T}{T+s} \in [0,1]$,
- $\theta_\pm \stackrel{\Delta}{=} \frac{r_d - r_f}{\sigma} \pm \frac{\sigma}{2}$,
- $S_t = S_0 e^{\sigma W_t + \sigma\theta_- t}$ for the price of the underlying at time t,
- $d_\pm \stackrel{\Delta}{=} \frac{\ln \frac{S_0}{K} + \sigma\theta_\pm T}{\sigma\sqrt{T}}$,
- $n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$,
- $\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt$,
- $\text{Vanilla}(S_0, K, \sigma, r_d, r_f, T, \phi) = \phi\left(S_0 e^{-r_f T}\mathcal{N}(\phi d_+) - K e^{-r_d T}\mathcal{N}(\phi d_-)\right)$,
- $H \stackrel{\Delta}{=} \exp\left\{-\frac{\alpha T}{2}\left(r_d - r_f + \frac{\sigma^2}{2}[1 - \frac{2\alpha}{3}]\right)\right\}$,

the value of the continuously sampled geometric Asian fixed strike call or put is then given by

$$\text{Asiangeo}(S_0, K, T, s, \sigma, r_d, r_f, \phi) = e^{(\alpha-1)r_d T} H \left(\frac{S_0}{A(-s,0)}\right)^{\alpha-1}$$

$$\text{Vanilla}\left(S_0, \frac{K}{H}\left(\frac{S_0}{A(-s,0)}\right)^{1-\alpha}, \frac{\alpha\sigma}{\sqrt{3}}, \alpha r_d, \alpha r_f, T, \phi\right). \tag{282}$$

This way a geometric Asian option with fixed strike can be viewed as a version of a vanilla option with the same spot and time to maturity but different parameters such as notional, strike, volatility, and interest rates. We observe in particular that as time to maturity becomes smaller, the known part of the average becomes more prominent, α tends to zero, and hence the volatility of the auxiliary vanilla option tends to zero. Moreover, the properties known for the function vanilla carry over to the function Asiangeo. Greeks can also be derived from this relation.

Let us now consider the case where averaging starts after T_0, i.e. the payoff is changed to

$$[\phi(A(t,T) - K)]^+, \tag{283}$$

$$A(t,T) \stackrel{\Delta}{=} \exp\left\{\frac{1}{T-t}\int_t^T \log S(u)\, du\right\}, t \in [0,T]. \tag{284}$$

Then the value becomes

$$\begin{aligned}
&\text{Asiangeowindow}(S_0, K, T, t, \sigma, r_d, r_f, \phi) \\
&= H \text{ Vanilla}\left(S_0, \frac{K}{H}, \frac{\Sigma\sigma}{(T-t)\sqrt{T}}, r_d, r_f, T, \phi\right), \qquad (285) \\
&H \stackrel{\Delta}{=} \exp\left\{-\frac{\sigma\theta_-}{2}(T-t) - \frac{\sigma^2}{2}(t - \frac{\Sigma}{T-t})\right\}. \qquad (286)
\end{aligned}$$

Derivation of the Value Function First we consider the call without history ($s = 0$). We rewrite the geometric average as

$$\begin{aligned}
A(0,T) &= \exp\left\{\frac{1}{T}\int_0^T \log S(u)\, du\right\} \\
&= S_0 \exp\left\{\frac{\sigma}{2}\theta_- T + \frac{\sigma}{T}\int_0^T W(u)\, du\right\} \qquad (287)
\end{aligned}$$

and compute the value function as

$$\begin{aligned}
&\text{Asiangeo}(S_0, K, T, 0, \sigma, r_d, r_f, \phi) \\
&= e^{-r_d T}\mathbb{E}[(A(0,T) - K)^+] \\
&= e^{-r_d T}\int_{-\infty}^{+\infty}\left(S_0 \exp\left\{\frac{\sigma}{2}\theta_- T + \sigma\sqrt{\frac{T}{3}}x\right\} - K\right)^+ n(x)\, dx \\
&= S_0 e^{-r_f T} e^{-\frac{T}{2}(r_d - r_f + \frac{\sigma^2}{6})}\mathcal{N}\left(\frac{\ln\frac{S_0}{K} + \frac{\sigma}{2}\theta_- T}{\sigma\sqrt{\frac{T}{3}}} + \sigma\sqrt{\frac{T}{3}}\right) \\
&\quad - K e^{-r_d T}\mathcal{N}\left(\frac{\ln\frac{S_0}{K} + \frac{\sigma}{2}\theta_- T}{\sigma\sqrt{\frac{T}{3}}}\right), \qquad (288)
\end{aligned}$$

which leads to the desired result. The analysis for the put option is similar. We obtain

$$\text{Asiangeo}(S_0, K, \sigma) = \sqrt{\frac{S_0}{K_+}}\text{vanilla}\left(S_0, K\sqrt{\frac{K_+}{S_0}}, \frac{\sigma}{\sqrt{3}}\right), \qquad (289)$$

where K_+ is the delta-neutral straddle strike as in Equation (43); however, in the case of the Asian option calculated with $\frac{\sigma}{\sqrt{3}}$ instead of σ. For $s > 0$ (real history) note that

$$A(-s, T) = A(-s, 0)^{1-\alpha}A(0,T)^{\alpha}. \qquad (290)$$

The first factor of this product is non-random at time 0, hence the value of a call with history is given by

$$
\begin{aligned}
&\text{Asiangeo}(S_0, K, T, s, \sigma, r_d, r_f, \phi) \qquad (291)\\
&= e^{-r_d T} I\!E[(A(-s,T) - K)^+]\\
&= e^{-r_d T} A(-s,0)^{1-\alpha} I\!E\left[\left(A(0,T)^\alpha - \frac{K}{A(-s,0)^{1-\alpha}}\right)^+\right]\\
&= e^{-r_d T}\int_{-\infty}^{+\infty}\left(S_0^\alpha \exp\left\{\frac{\alpha\sigma}{2}\theta\_T + \alpha\sigma\sqrt{\frac{T}{3}}x\right\} - \frac{K}{A(-s,0)^{1-\alpha}}\right)^+ n(x)\,dx.
\end{aligned}
$$

It is now an easy exercise to complete this calculation.

Arithmetic Average Options Since the distribution of the arithmetic average of log-normally distributed random variables is not normal, a closed form solution for the frequently used arithmetic average price options is not immediately available. Some of the approaches to solve this valuation task are

1. Numerical approaches, e.g. Monte Carlo simulations work well, as one can take the geometric Asian option as a highly correlated control variate. Taking a PDE approach is equally fast as Večeř has shown how to reduce the valuation problem to a PDE in one dimension in [133].
2. Modifications of the geometric average approach.
3. Approximations of the density function for the arithmetic average – see [88] on p. 430.

For instance, Turnbull and Wakeman (see [131]) develop an approximation of the density function by defining an alternative distribution for the arithmetic average with moments that match the moments of the true distribution, similar to that in Section 1.9.2. One can also match the cumulants up to fourth order: mean, variance, skew, and kurtosis. The adjusted mean and variance are finally plugged into the general Black-Scholes formula. Lévy states in [88] that considering only the first two moments delivers acceptable results for typical ranges of volatility and simultaneously reduces the complexity of the Turnbull and Wakeman approach. Hakala and Perissé show in [65] how to include higher moments. We apply a Monte Carlo simulation of price paths to value arithmetic average price options. To improve the quality of the results, we take geometric average options with similar specifications as control variate – see [86], p. 124. For variance reduction techniques see [58], pp. 414–418. For further suggestions on the implementation of pricing models see, for example, [29], pp. 118–123. We show in Table 1.25 that the results are close to the analytical approximations provided by Turnbull and Wakeman as well as Lévy.

Sensitivity Analysis We analyze now the sensitivities of the values with respect to various input parameters and compare them with vanilla options. Throughout we will use the parameters $K = 1.2000$, $S_0 = 1.2000$, $r_d = 3\%$, $r_f = 2.5\%$, $\sigma = 10\%$, $T - T_0 = 3$ months

TABLE 1.25 Values of average options. Input parameters are $K = 1.2000$, $S_0 = 1.2000$, $\sigma = 20\%$, $r_d = 3\%$, $r_f = 2.5\%$, $T - T_0 = 90$ days $= 90/365$ years, 90 observations (implying a time step of 0.002739726 years), 10,000 price paths in the Monte Carlo simulation. The arithmetic average options are average price options. All values are in domestic pips.

Method	Ar. call	Ar. put	Geo. price call	Geo. price put	Geo. strike call	Geo. strike put
Analytical	—	—	271.19	273.63	295.21	248.19
Monte Carlo	295.92	251.95	290.53	256.44	295.38	244.62
With control variate	276.57	269.14	—	—	—	—
Turnbull/Wakeman	276.36	269.02	—	—	—	—
Lévy	276.36	269.02	—	—	—	—

(91 days). The similarity of vanilla and average options, and the effects from averaging prices, which already dominated the derivation of the value formula, are reflected in the *Greeks* as well. Both option types react in the same direction to parameter changes and differ only in the quantity of the option value change. This holds especially for delta, gamma, and vega. These sensitivities, which are related to the underlying, represent best the properties of average options, i.e. initially, the option is very sensitive to price changes in the underlying. Delta, gamma, and vega have accordingly high values. With decreasing time to maturity, the impact of single prices on the final payoff diminishes, delta stabilizes, and gamma approaches zero, see [101], pp. 63–64. Figure 1.45 illustrates the similarity between vanilla and average price options with respect to delta and gamma.

For the same level of volatility in the underlying, average options have a lower vega compared with vanilla options because fluctuations of the underlying price are smoothed by the average. Note that the lower the volatility, the smaller the value difference between average and vanilla options, see Figure 1.46.

Since single prices – especially at maturity – influence the payoff of average options less significantly than for vanilla options, time, i.e. the chance of a finally favorable performance, plays a less important role in determining the value of average options, leading to a lower theta. The interest rate sensitivity rho of average options is smaller than for vanilla options.

Risk Management With the sensitivity analysis in mind, the question arises as to how the writer of an average option should deal with the risks of a short position.

Dynamic Hedging For a call position, for instance, one way is hedging with an investment in the underlying that is funded by borrowing. The delta of the option suggests how many units of the foreign currency have to be bought. Since delta changes over time, as in a vanilla option, the amount invested in the underlying has to be adjusted frequently. From the risk analysis it can be inferred that average options are easier to hedge than vanilla options, in particular the delta of average options stabilizes over time. Accordingly, the scope of required re-balancing of the hedge and the related transaction cost decrease over time. The costs of the hedge include interest payments as well as commissions and bid-ask spreads due at every re-balancing transaction. See [128] and

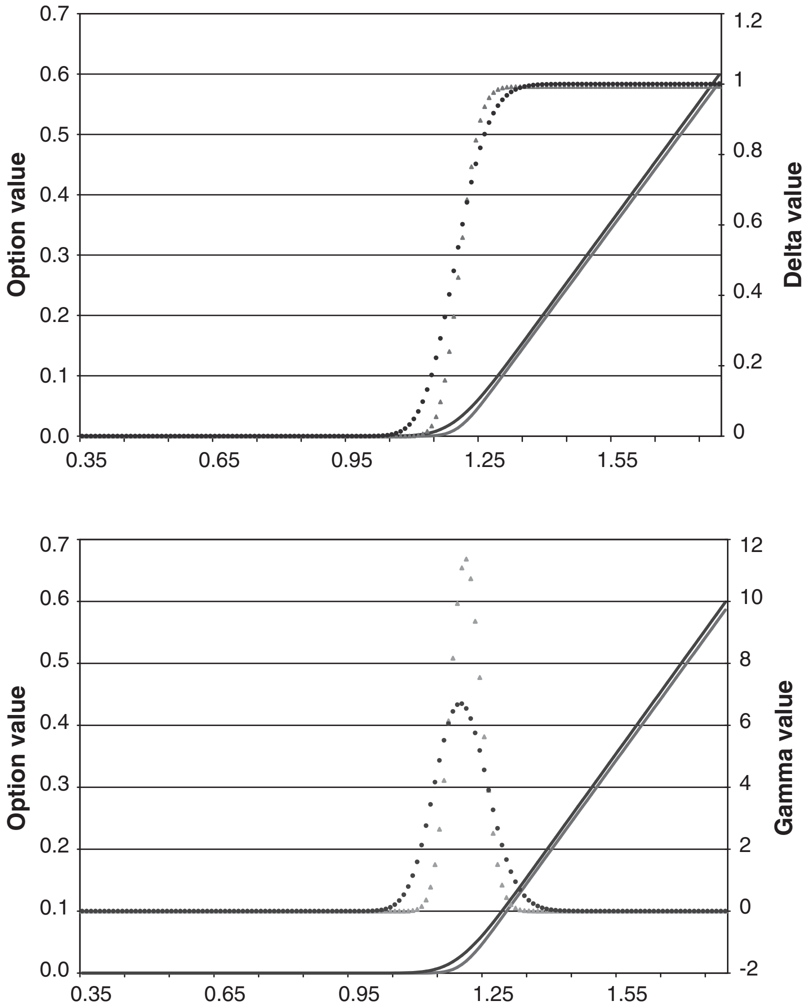


FIGURE 1.45 Above: option values and delta depending on the underlying price; Asian value is lower than vanilla value, Asian delta is smoother than vanilla delta; below: option values and gamma depending on the underlying price; Asian value is lower than vanilla value, Asian gamma is lower than vanilla gamma.

[101] for empirical analysis on the cost of dynamic and static hedging. Dynamic hedging neutralizes the delta exposure inherent in the option position. The volatility exposure can be hedged with vanilla contracts, typically ATM straddles.

Static Replication Alternatively, a static replication involving vanilla options can be set up. The position remains generally unchanged until maturity of the average option. Vanilla options are traded in liquid markets at relatively small bid-ask spreads. Furthermore, not only the delta risk but also the gamma and volatility exposure can be reduced

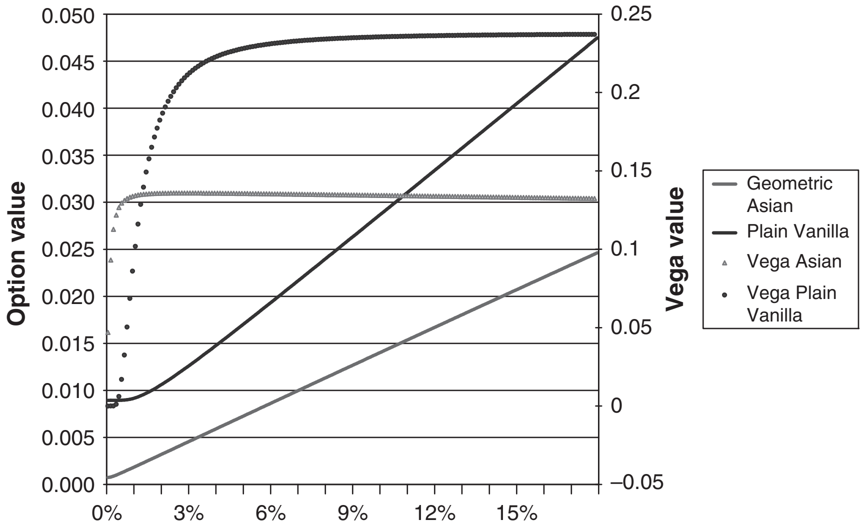


FIGURE 1.46 Option values and vega depending on volatility for at-the-money options.

with an option based replication. Static replication with vanilla options has therefore become common market practice, see [89]. For instance, Lévy suggests in [89] as a rule of thumb choosing a vanilla call with a similar strike as a short average price call and an expiration that is one-third of the averaging period of the exotic, based on the appearance of the factor $\frac{T}{3}$ in Equation (288). As the graph below of Figure 1.47 shows, the sensitivities of the short average price call are at their highest levels in the first third of the averaging period. Hedging with options only during this most critical time period already significantly reduces the sensitivity of the position to underlying price changes. Simultaneously, choosing vanilla calls with shorter maturity saves hedging costs. Nevertheless, this approach leaves the option writer with an open position for the remaining time to maturity unless she decides to build up a new replicating portfolio (semi-static replication). Since the stabilized delta in the later lifetime of the average option reduces the re-balancing effort, a dynamic hedge could be an alternative to a renewed replication with vanilla options.

1.7.7 Lookback Options

This section is joint work with Silvia Baumann, Marion Linck, Michael Mohr, and Michael Seeberg.

Lookback options are, like Asian options, path dependent. At expiration the holder of the option can "look back" over the lifetime of the option and exercise based upon the optimal underlying value (extremum) achieved during that period. Thus, lookback options (like Asians) avoid the problem of European options that the underlying performed favorably throughout most of the option's lifetime but

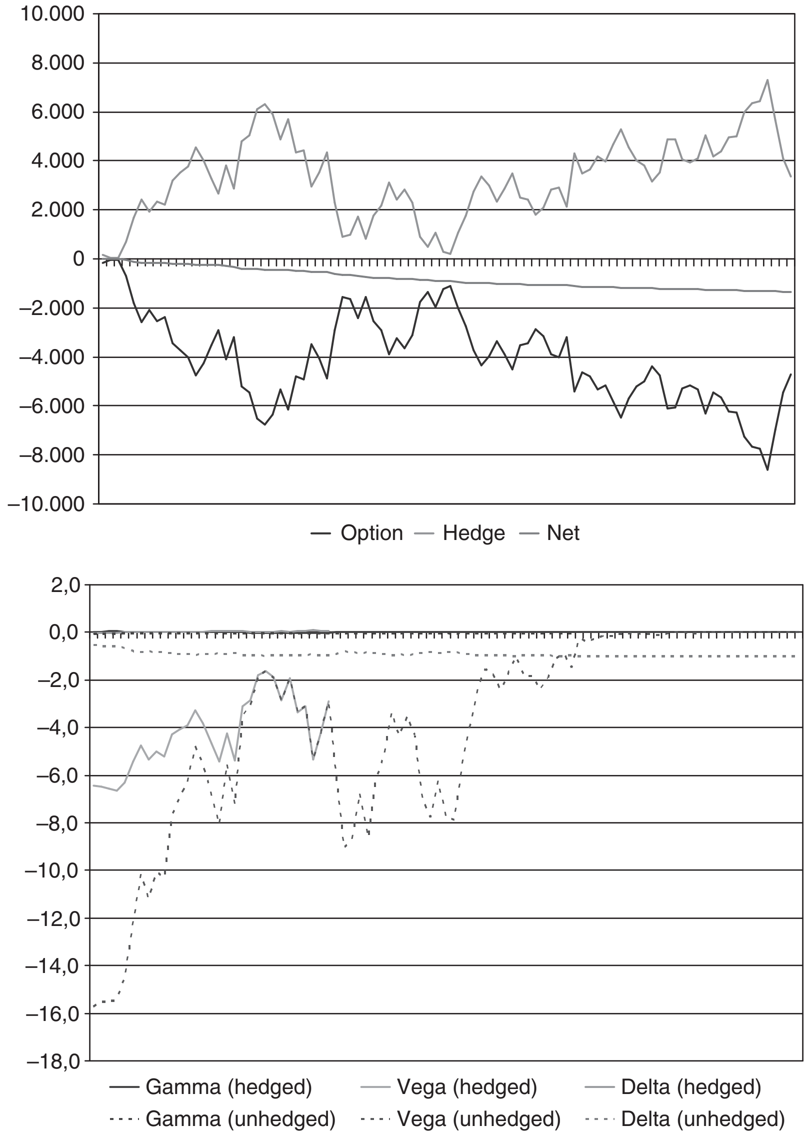


FIGURE 1.47 Graph above: dynamic hedging: performance of option position and hedge portfolio; graph below: static hedging: comparison of hedged and unhedged "Greek" exposure. For both, sample prices were generated randomly.

moves into a non-favorable direction towards maturity. Moreover (unlike American options), lookback options optimize the market timing because the investor gets – by definition – the most favorable underlying price. As summarized in Table 1.26 lookback options can be structured in two different types with the extremum representing either

TABLE 1.26 Types of lookback options. The contract parameters T and X are the time to maturity and the strike price respectively, and S_T denotes the spot price at expiration time. Fixed strike lookback options are also called hindsight options.

Payoff	Lookback type	Parameter
$M_T - S_T$	floating strike put	$\phi = -1, \overline{\eta} = -1$
$S_T - m_T$	floating strike call	$\phi = +1, \overline{\eta} = +1$
$(M_T - X)^+$	fixed strike call	$\phi = +1, \overline{\eta} = -1$
$(X - m_T)^+$	fixed strike put	$\phi = -1, \overline{\eta} = +1$

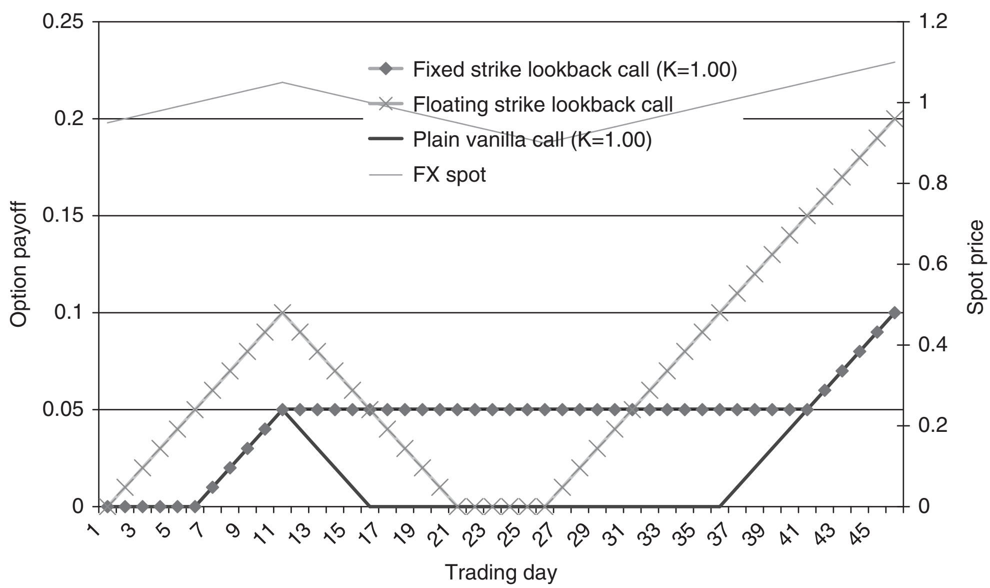


FIGURE 1.48 Payoff profile of lookback calls (sample underlying price path, 20 trading days).

the strike price or the underlying value. Figure 1.48 shows the development of the payoff of lookback options depending on a sample price path. In detail we define

$$M_{t,T} \stackrel{\Delta}{=} \max_{t \le u \le T} S(u), \tag{292}$$

$$M_T \stackrel{\Delta}{=} M_{0,T}, \tag{293}$$

$$m_{t,T} \stackrel{\Delta}{=} \min_{t \le u \le T} S(u), \tag{294}$$

$$m_T \stackrel{\Delta}{=} m_{0,T}. \tag{295}$$

Variations of lookback options include *partial lookback options*, where the monitoring period for the underlying is shorter than the lifetime of the option. Conze and Viswanathan [31] present further variations, such as *limited risk* and *American lookback options*. Lookback options are not traded much with corporate treasurers, as they are too expensive; they are mainly used by speculators, see [27]. An often cited strategy is building *lookback straddles* paying

$$M_{t,T} - m_{t,T}, \tag{296}$$

(also called *range* or *hi-lo option*), a combination of lookback put(s) and call(s) that guarantees a payoff equal to the observed range of the underlying asset. In theory, Garman pointed out in [53] that lookback options can also add value for risk managers because floating (fixed) strike lookback options are good means to solve the timing problem of market entries (exits), see [75]. For instance, a minimum strike call is suitable to avoid missing the best exchange rate in currency linked security issues. However, this right is very expensive. Since one buys a guarantee for the best possible exchange rate observed in a time interval, lookback options are generally way too expensive and hardly ever trade. Exceptions are performance notes, where lookback and average features are mixed, e.g. performance notes paying say 50% of the best of 36 monthly average gold price returns.

Valuation As in the case of Asian options, closed form solutions exist only for specific products – in this case basically for any lookback option with continuously monitored underlying value. We consider the example of the floating strike lookback call. Again, the value of the option is given by

$$\begin{aligned} v(0, S_0) &= \mathbb{E}\left[e^{-r_d T}(S_T - m_T)\right] \\ &= S_0 e^{-r_f T} - e^{-r_d T}\mathbb{E}\left[m_T\right]. \end{aligned} \tag{297}$$

In the standard Black-Scholes model (1), the value can be derived using the reflection principle and results in

$$\begin{aligned} v(t,x) = \phi \Bigg\{ & x e^{-r_f \tau}\mathcal{N}(\phi b_1) - K e^{-r_d \tau}\mathcal{N}(\phi b_2) + \frac{1-\eta}{2}\phi e^{-r_d \tau}[\phi(R - X)]^+ \\ & + \eta x e^{-r_d \tau}\frac{1}{h}\left[\left(\frac{x}{K}\right)^{-h}\mathcal{N}(-\eta\phi(b_1 - h\sigma\sqrt{\tau})) - e^{(r_d - r_f)\tau}\mathcal{N}(-\eta\phi b_1)\right]\Bigg\}. \end{aligned} \tag{298}$$

This value function has a removable discontinuity at $h = 0$ where it turns out to be

$$\begin{aligned} v(t,x) = \phi \Bigg\{ & x e^{-r_f \tau}\mathcal{N}(\phi b_1) - K e^{-r_d \tau}\mathcal{N}(\phi b_2) + \frac{1-\eta}{2}\phi e^{-r_d \tau}[\phi(R - X)]^+ \\ & + \eta x e^{-r_d \tau}\sigma\sqrt{\tau}\left[-b_1\mathcal{N}(-\eta\phi b_1) + \eta\phi n(b_1)\right]\Bigg\}. \end{aligned} \tag{299}$$

The abbreviations we use are

$$t\text{: running time (in years),} \tag{300}$$

$$x \stackrel{\Delta}{=} S_t\text{: known spot at time of evaluation,} \tag{301}$$

$$\tau \stackrel{\Delta}{=} T - t\text{: time to expiration (in years),} \tag{302}$$

$$n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}, \tag{303}$$

$$\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt, \tag{304}$$

$$h \stackrel{\Delta}{=} \frac{2(r_d - r_f)}{\sigma^2}, \tag{305}$$

$$K \stackrel{\Delta}{=} \begin{cases} R & \text{floating strike lookback} \quad (X \leq 0) \\ \overline{\eta}\min(\overline{\eta}X, \overline{\eta}R) & \text{fixed strike lookback} \quad (X > 0) \end{cases}, \tag{306}$$

$$\eta \stackrel{\Delta}{=} \begin{cases} +1 & \text{floating strike lookback} \quad (X \leq 0) \\ -1 & \text{fixed strike lookback} \quad (X > 0) \end{cases}, \tag{307}$$

$$b_1 \stackrel{\Delta}{=} \frac{\ln \frac{x}{K} + (r_d - r_f + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \tag{308}$$

$$b_2 \stackrel{\Delta}{=} b_1 - \sigma\sqrt{\tau}. \tag{309}$$

Note that this formula basically consists of that for a vanilla call (first two terms) plus another term. Conze and Viswanathan also show closed form solutions for fixed strike lookback options and the variations mentioned above in [31]. Heynen and Kat develop equations for *partial fixed and floating strike lookback options* in [74]. For those preferring the PDE approach of deriving formulas, we refer to [59]. For most practical matters, where we have to deal with fixings and lookback features in combination with averaging, the only reasonable valuation technique is Monte Carlo simulation.

Example We list some sample results in Table 1.27.

Sensitivity Analysis

delta.

$$\begin{aligned} v_x(t,x) = \phi \Bigg\{ & e^{-r_f\tau}\mathcal{N}(\phi b_1) + \eta e^{-r_d\tau}\frac{1}{h} \\ & \cdot \left[\left(\frac{x}{K}\right)^{-h} \mathcal{N}(-\eta\phi(b_1 - h\sigma\sqrt{\tau}))(1-h) - e^{(r_d-r_f)\tau}\mathcal{N}(-\eta\phi b_1) \right] \Bigg\} \end{aligned} \tag{310}$$

TABLE 1.27 Sample valuation results for lookback options. For the input data we used spot $S_0 = 0.8900$, $r_d = 3\%$, $r_f = 6\%$, $\sigma = 10\%$, $\tau = \frac{1}{12}$, running min = 0.9500, running max = 0.9900, $m = 22$. We find the analytic results in the continuous case in agreement with the ones published in Haug [71]. We can also reproduce the numerical results for the discretely sampled floating strike lookback put contained in Nahum [98].

Payoff	Analytic model	Continuous
$M_T - S_T$	0.0231	0.0255
$S_T - m_T$	0.0310	0.0320
$(M_T - 0.99)^+$	0.0107	0.0131
$(0.97 - m_T)^+$	0.0235	0.0246

At $h = 0$ this simplifies to

$$\begin{aligned} v_x(t,x) = \phi \{ & e^{-r_f\tau}\mathcal{N}(\phi b_1) \\ & + \eta e^{-r_d\tau}\left[\sigma\sqrt{\tau}\left[-b_1\mathcal{N}(-\eta\phi b_1) + \eta\phi n(b_1)\right] - \mathcal{N}(-\eta\phi b_1)\right]\} \end{aligned} \tag{311}$$

gamma.

$$v_{xx}(t,x) = \frac{2e^{-r_f\tau}}{x\sigma\sqrt{\tau}} n(b_1) - \phi\eta e^{-r_d\tau}\frac{1-h}{x}\mathcal{N}(-\phi\eta(b_1 - h\sigma\sqrt{\tau})) \tag{312}$$

theta. We can use the Black-Scholes partial differential equation to obtain theta from value, delta, and gamma.

vega.

$$\begin{aligned} v_\sigma(t,x) = \phi\eta x e^{-r_d\tau}\frac{2}{\sigma}\Bigg[& \left(\frac{x}{K}\right)^{-h}\mathcal{N}(-\eta\phi(b_1 - h\sigma\sqrt{\tau}))\left(\frac{1}{h} + \ln\frac{x}{K}\right) \\ & - e^{(r_d - r_f)\tau}\frac{1}{h}\mathcal{N}(-\eta\phi b_1)\Bigg] \end{aligned} \tag{313}$$

At $h = 0$ this simplifies to

$$v(t,x) = \phi\eta x e^{-r_d\tau}\sqrt{\tau}\left[-\sigma\sqrt{\tau}b_1\mathcal{N}(-\eta\phi b_1) + 2\eta\phi n(b_1)\right] \tag{314}$$

Discrete Sampling In practice, one cannot take the average over a continuum of exchange rates. The standard is to specify a *fixing calendar* and take only a finite number of fixings into account. Suppose there are m equidistant sample points left until expiration at which we evaluate the extremum. In this case the value can be determined by an approximation described by Broadie *et al.* [20]. We set

$$\beta_1 = 0.5826 = -\zeta(1/2)/\sqrt{2\pi}, \tag{315}$$

$$a = e^{\phi\beta_1\sigma\sqrt{\tau/m}}, \tag{316}$$

and obtain for fixed strike lookback options

$$v(t, x, r_d, r_f, \sigma, R, X, \phi, \overline{\eta}, m) = v(t, x, r_d, r_f, \sigma, aR, aX, \phi, \overline{\eta})/a, \tag{317}$$

and for floating strike lookback options

$$v(t, x, r_d, r_f, \sigma, R, X, \phi, \overline{\eta}, m) = av(t, x, r_d, r_f, \sigma, R/a, X, \phi, \overline{\eta}) - \phi(a-1)xe^{-r_f\tau}. \tag{318}$$

One interesting observation is that when the options move deep in-the-money and have the same strike price, lookback options and vanilla options have the same value, except for extreme risk parameter inputs. This can be explained recalling that a floating strike lookback option has an exercise probability of 1 and buys (sells) at the minimum (maximum). When the strike price of a vanilla option equals the extremum of the exotic and is deep in-the-money, the holder of the option will also buy (sell) at the extremum with a probability very close to 1. Moreover, recall that the floating strike lookback option consists of a vanilla option and an additional term. Garman names this term a *strike-bonus option*, see [53]. It can be considered as an option that has an increased payoff whenever a new extremum is reached. When the underlying price moves very far away from the current extremum, the strike-bonus option has almost zero value.

The structure of the Greeks delta, rho, theta, and vega is similar for lookback and vanilla calls. Nonetheless, the intensity of these sensitivities against changes differs, see Figure 1.49.

Close to or at the money, lookback calls have a significantly lower *delta* than their vanilla counterparts. The reason is that the strike-bonus option in the lookback call has a negative delta when the underlying value is close to the current extremum and a delta next to zero when it is far in-the-money. Intuitively, the lower lookback delta is explained by the fact that the closer the current exchange rate is to the extremum, the more likely it is that the payoff of the lookback option remains unchanged, which is different for vanilla options where the payoff changes with every spot movement. Note that whenever a new extremum is achieved, the payoff for a lookback option equals zero and remains unchanged until the underlying value moves into the adverse direction.

Floating strike lookback options have a lower *rho* than vanilla options (with equal strikes at the time of observation), which can be explained by the fact that the option holder needs to pay more up-front and thus has a lower principal profiting from favorable interest rate movements. As a rule of thumb, a floating strike lookback option is worth twice as much as a vanilla option. The longer the time to maturity, the more intensively floating strike lookback options react compared with vanilla options.

The higher *theta* for lookback options reflects the fact that the optimal value achieved to date is “locked in” and the longer the time to maturity, the higher the chance to lock in an even better extremum.

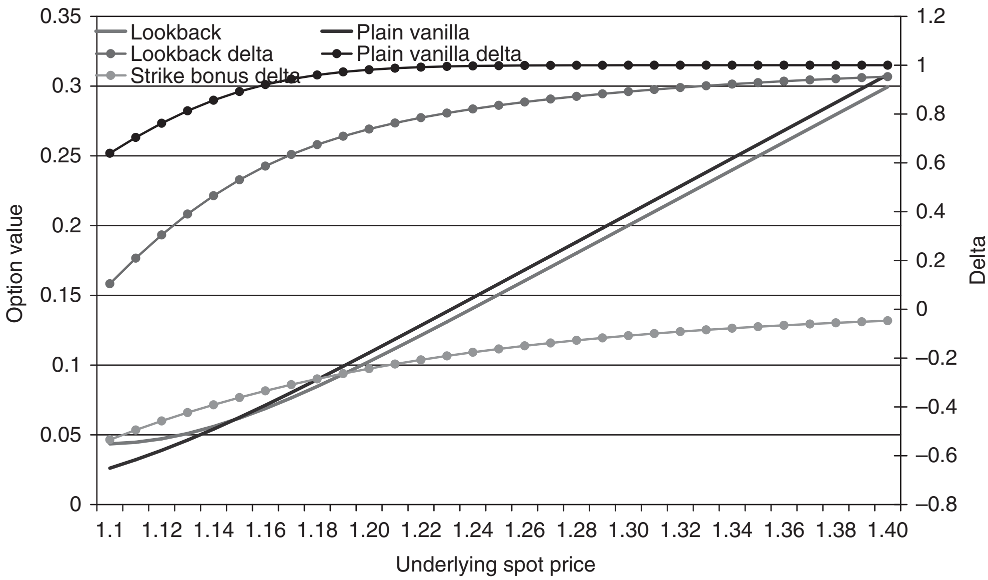


FIGURE 1.49 Vanilla and lookback call (left-hand scale) with deltas (right-hand scale) using $\min_t S_t = 1.00$. The lookback delta equals the sum of the delta of a vanilla option plus the delta of a strike bonus option.

Regarding *vega*, lookback options show a stronger reaction than regular options. The higher the volatility of the underlying, the higher the probability of reaching a new extremum. Moreover, having "locked in" this new extremum the option value can benefit even more from the higher chance of adverse price movements.

As pointed out by Taleb in [123], one particularly interesting risk parameter is the lookback *gamma* since it is *one-sided*, while the vanilla gamma changes symmetrically for up and down movements of the underlying, see Figure 1.50. A lookback option always has its maximum gamma at the extremum which can move over time. Vanilla options, however, have their maximum gamma at the strike price. The *lookback gamma asymmetry* indicates that gamma risk cannot be consistently (statically) hedged with vanilla options. The fact that gamma is considerably higher for lookback options implies that a frequent re-balancing of the hedging portfolio and hence high transaction costs are likely, see [32].

Semi-Static Replication Due to the maximum (minimum) function that allows the strike price to change there exists no buy-and-hold static replication for floating strike lookback options. Instead, a *semi-static rollover strategy* can be applied, see [53]. As can be read from the value in Equation (298), we can replicate parts of a lookback with a vanilla option. Whenever the maximum (minimum) changes, the writer of the option buys a new put (call) struck at the current market price and sells the old put (call). However, this does not work without costs. While the new put (call) is at-the-money, the old put (call) is out-of-the-money at the time of the sale and hence returns less money

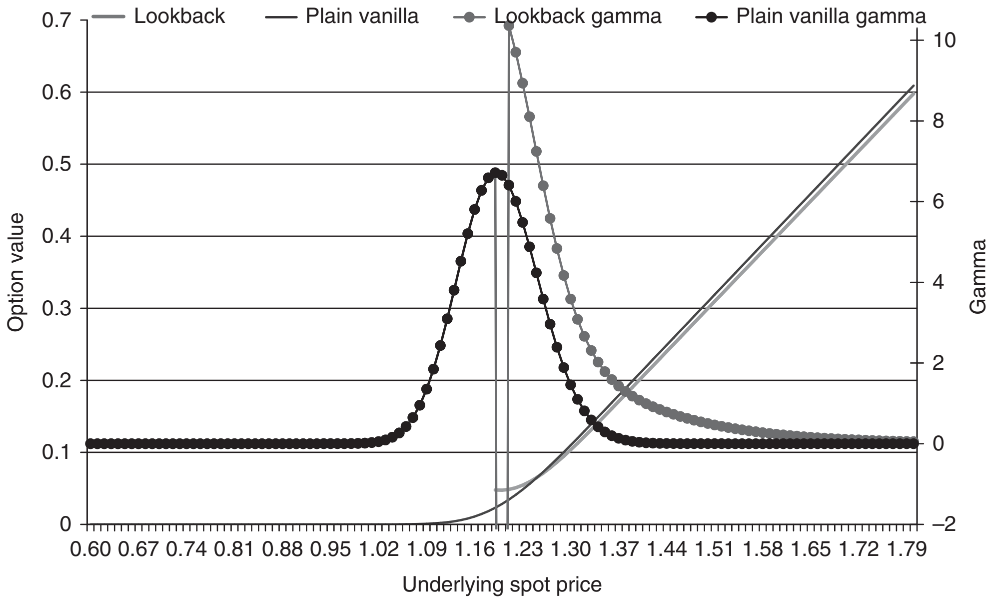
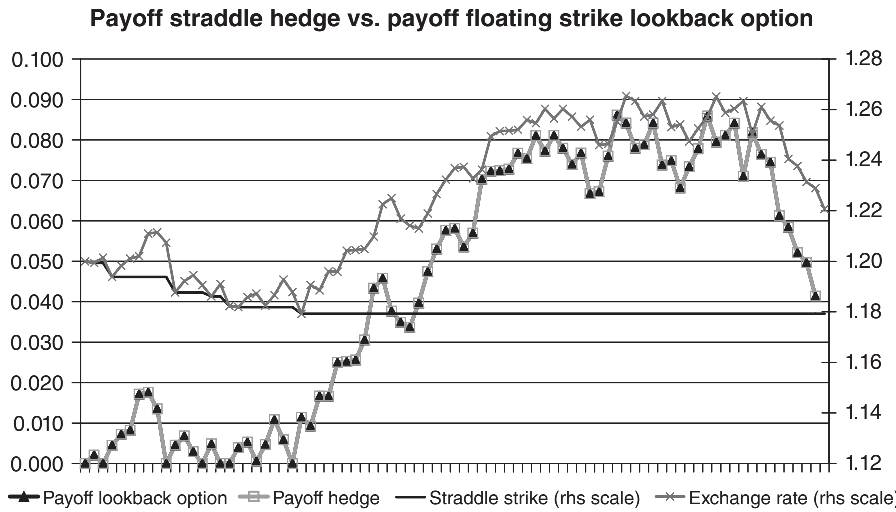


FIGURE 1.50 Value (left-hand scale) and gamma (right-hand scale) of an at-the-money floating strike lookback and a vanilla call.

than the amount necessary to purchase the new option. We encounter vanilla option bid-ask spreads, and smile risk. The strike-bonus option returns exactly the money that is needed for the rollover (in theory, and assuming no bid-offer spreads). This approach, however, is rather theoretical, since strike-bonus options are hardly available in the market.

In practice, floating strike lookback options are usually approximated by a straddle, see Section 1.6.3. Cunningham and Karumanchi also explain a dynamic replication strategy for fixed strike lookback options in [32]. The straddle to use is a combination of a vanilla put and a vanilla call, which have a term to maturity equal to that of the lookback option to be replicated, and a strike equal to the maximum (minimum) achieved by the underlying. At maturity T, the call (put) of this straddle becomes worthless since the strike is below (above) the terminal stock price S_T. The remaining put (call) exactly satisfies the obligation of the lookback option (see Figure 1.51). Over the lifetime of the option, the strike price of the straddle needs to be adapted if the current exchange rate S_t rises above (falls below) the current maximum (minimum). Regarding the intrinsic value, the holder of the hedging portfolio will not lose money since for instance the intrinsic value lost by the put will be exactly gained by the call. However, the deltas of the two options differ, not only in their sign. In addition, attempting to create a replicating portfolio with zero delta, the trader has to buy a certain number of puts per one call. Figure 1.51 shows that for the latter two reasons this is not a self-financing replicating strategy. Note that the strategy would not be self-financing even if the straddle was not adapted for a zero delta of the position. Specifying a *re-hedge threshold* and a maximum number of trades per period can help to balance the risk taken with



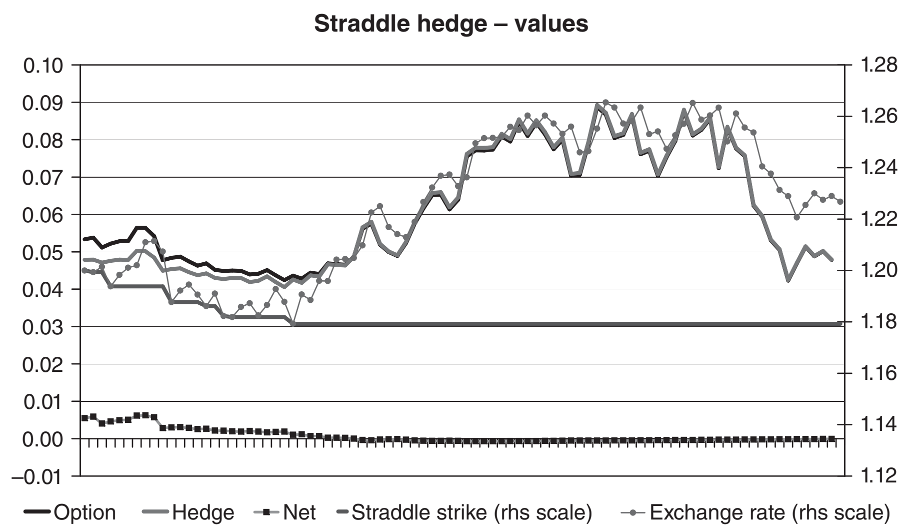


FIGURE 1.51 Comparison of the payoffs of a floating strike lookback option and a vanilla straddle (graph above) and the values of the positions (graph below) for a random time path (exchange rate and straddle strike on the rhs scale).

transaction and administrative costs. We refer to this replication as semi-static since the basic idea of the replication is that of a static one: initialize the hedge and wait until maturity. However, due to the changes of the extrema, the static replication has to be adapted – a characteristic which is usually associated with dynamic hedging. Apart from this *semi-static* replication, a *dynamic hedge* using spot and money market is also possible and more commonly applied. Due to the risk parameters, especially gamma and vega, which are difficult to hedge, the hedge appears to deviate considerably in value relative to the option over time.

1.7.8 Forward Start, Ratchet, and Cliquet Options

A forward start vanilla option is just like a vanilla option, except that the strike is set on a future date t. It pays off

$$[\phi(S_T - K)]^+, \tag{319}$$

where K denotes the strike and ϕ takes the values +1 for a call and −1 for a put. The strike K is set as αS_t at time $t \in [0, T]$. Very commonly α is set to 1.

Advantages

- Protection against spot market movement and against increasing volatility
- Buyer can lock in current volatility level
- Spot risk easy to hedge

Disadvantages

- Protection level not known in advance

The Value of Forward Start Options Using the abbreviations

- x for the current spot price of the underlying,
- $\tau \stackrel{\Delta}{=} T - t$,
- $F_s \stackrel{\Delta}{=} \mathbb{E}[S_s|S_0] = S_0 e^{(r_d - r_f)s}$ for the outright forward of the underlying,
- $\theta_\pm \stackrel{\Delta}{=} \frac{r_d - r_f}{\sigma} \pm \frac{\sigma}{2}$,
- $d_\pm \stackrel{\Delta}{=} \frac{\ln \frac{x}{K} + \sigma\theta_\pm \tau}{\sigma\sqrt{\tau}} = \frac{\ln \frac{f}{K} \pm \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$,
- $d_\pm^\alpha \stackrel{\Delta}{=} \frac{-\ln \alpha + \sigma\theta_\pm \tau}{\sigma\sqrt{\tau}}$,
- $n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} = n(-t)$,
- $\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt = 1 - \mathcal{N}(-x)$,

we recall the value of a vanilla option in Equation (7),

$$v(x, K, T, t, \sigma, r_d, r_f, \phi) = \phi e^{-r_d \tau}[f\mathcal{N}(\phi d_+) - K\mathcal{N}(\phi d_-)]. \tag{320}$$

For the value of a forward start vanilla option in a constant-coefficient geometric Brownian motion model we obtain

$$\begin{aligned} v &= e^{-r_d t}\mathbb{E} v(S_t, K = \alpha S_t, T, t, \sigma, r_d, r_f, \phi) \\ &= \phi e^{-r_d T}[F_T \mathcal{N}(\phi d_+^\alpha) - \alpha F_t \mathcal{N}(\phi d_-^\alpha)]. \end{aligned} \tag{321}$$

Noticeably, the value computation is easy here because the strike K is set as a *multiple* of the future spot. If we were to choose to set the strike as a constant *difference* of the future spot, the integration would not work in closed form, and we would have to use numerical integration.

The crucial pricing issue here is that one needs to know the volatility, which is the *forward volatility*, i.e. the volatility that will materialize at the future time t for a maturity $T - t$. It is not obvious from the market which proxy to take for this forward volatility. The standard is to use Equation (152).

Greeks The Greeks are the same as for vanilla options after time t, when the strike has been set. Before time t they are given by

(Spot) Delta.

$$\frac{\partial v}{\partial S_0} = \frac{v}{S_0} \tag{322}$$

Gamma.

$$\frac{\partial^2 v}{\partial x^2} = 0 \tag{323}$$

Theta.

$$\frac{\partial v}{\partial t} = r_f v \tag{324}$$

Vega.

$$\frac{\partial v}{\partial \sigma} = -\frac{e^{-r_d T}}{\sigma}[F_T n(d_+^\alpha)d_-^\alpha - \alpha F_t n(d_-^\alpha)d_+^\alpha] \tag{325}$$

Rho.

$$\frac{\partial v}{\partial r_d} = \phi e^{-r_d T}\alpha F_t(T-t)\mathcal{N}(\phi d_-^\alpha) \tag{326}$$

$$\frac{\partial v}{\partial r_f} = -Tv - \phi e^{-r_d T}\alpha F_t(T-t)\mathcal{N}(\phi d_-^\alpha) \tag{327}$$

Example We consider an example in Table 1.28.

Reasons for Trading Forward Start Options The key reason for trading a forward start is trading the forward volatility without any spot exposure. In quiet market phases with low volatility, buying a forward start is cheap. Keeping a long position will allow participation in rising volatility, independent of the spot level. In recent years, forward start options have become more popular, especially among institutional clients who hedge their volatility exposure. They use *forward volatility agreements (FVAs)*, see

TABLE 1.28 Value and Greeks of a forward start vanilla in USD on EUR/USD – spot of 0.9000, $\alpha = 99\%$, $\sigma = 12\%$, $r_d = 2\%$, $r_f = 3\%$, maturity $T = 186$ days, strike set at $t = 90$ days.

	Call	Put		Call	Put
Value	0.0251	0.0185			
Delta	0.0279	0.0206	vega	0.1793	0.1793
Gamma	0.0000	0.0000	rho_d	0.1217	−0.1052
Theta	0.0007	0.0005	rho_f	−0.1329	0.0950

Section 1.8.9, which is pretty much a re-branding of the good old forward start option or a combination of them as a forward start straddle. The key reason for sales is that for notes there is typically a subscription phase, where a product such as a dual currency investment is announced and investors can subscribe for it during the *subscription phase*. This phase can be for example four weeks. During this time notional is collected and the investor subscribes to a product whose strike will be fixed at the end of the subscription phase.

Variations Forward start options can be altered in all kind of ways: they can be of American style, they can come with a deferred delivery or deferred premium, they can have barriers or appear as a strip.

A strip of forward start options is generally called a *cliquet*.

A *ratchet* consists of a series of forward start options, where the strike for the next forward start option is set equal to the spot at maturity of the previous.

1.7.9 Power Options

This section is joint work with Silvia Baumann, Marion Linck, Michael Mohr, and Michael Seeberg.

For power options, the vanilla option payoff function $[\phi(S_T - K)]^+$ is adjusted by raising the entire function or parts of the function to the n-th power, see e.g. Zhang [146]. The result is a non-linear profile with the potential of a higher payoff at maturity with a greater leverage than standard options. Interestingly, many people don't know that numbers smaller than 1 tend to get smaller when squared. If the exponent n is exactly 1, the option is equal to a vanilla option. We distinguish between *asymmetric* and *symmetric* power options. Their payoffs in comparison with vanilla options are illustrated in Figure 1.52 and Figure 1.53.

Asymmetric Power Options With an asymmetric power option, the underlying S_T and strike K of a standard option payoff function are individually raised to the n-th power,

$$[\phi(S_T^n - K^n)]^+. \tag{328}$$

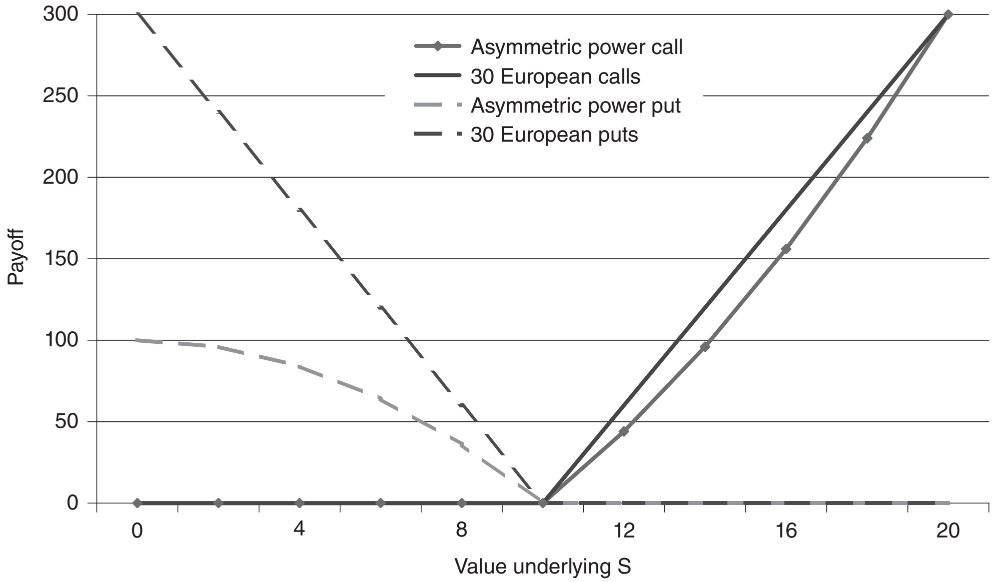


FIGURE 1.52 Payoff of asymmetric power options vs. vanilla options, using $K = 10$, $n = 2$.

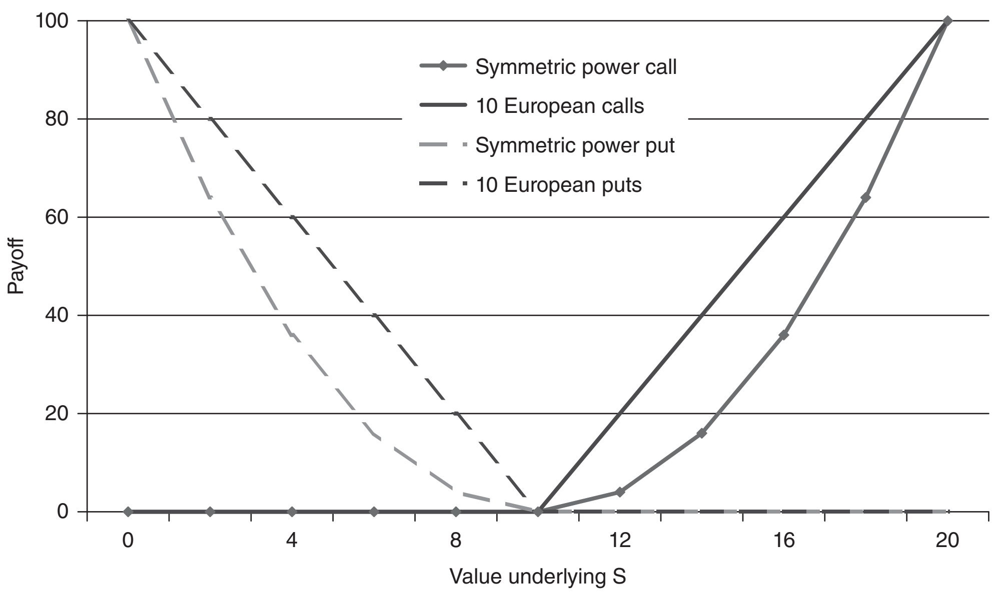


FIGURE 1.53 Payoff of symmetric power options vs. vanilla options, using $K = 10$, $n = 2$.

Figure 1.52 illustrates why this option type is called *asymmetric*. With increasing S_T, the convex call payoff grows exponentially. Given the limited and fixed profit potential of $K^2 = 10^2$, the concave put payoff decreases exponentially. It requires 30 vanilla options to replicate the call payoff if the underlying S_T moves to 20.

Symmetric Power Options In the symmetric type, the entire vanilla option payoff is raised to the n-th power,

$$[[\phi(S_T - K)]^+]^n, \tag{329}$$

see [129]. Figure 1.53 insinuates naming this option type *symmetric*, since put and call display the same payoff shape. Here, 10 vanilla options suffice to replicate the symmetric power option if the underlying S_T moves to 20.

Combining a symmetric power call and put as in Figure 1.53 leads to a symmetric power straddle, which pays

$$|S_T - K|^n. \tag{330}$$

Reasons for Trading Power Options Power options are often equipped with a payoff cap C to limit the short position risk as well as the option premium for the buyer. For example, the payments of the short position at $n = 3$ for $K = 10$ shoots to 2375(125) for the asymmetric (symmetric) power call if S_T moves to 15. Even with cap, the highly leveraged payoff motivates speculators to invest in the product that demands a considerably higher option premium than a vanilla option. Power options are mostly popular in the listed derivatives and retail market, due to their high leverage and due to their mere name. There exist more advanced power-like derivatives, see for example the multiplicity power option in Section 1.8.1. Furthermore, a *self-quanto* option comes up in retail markets if the vanilla payoff $\phi(S_T - K)^+$ is meant to pay in foreign currency, which means that it must pay

$$\phi(S_T - K)^+ S_T \tag{331}$$

in domestic currency, such that a division of the final payoff by the final spot price S_T will yield the vanilla payoff in foreign currency. This is a version of a symmetric power option, which is why some market participants refer to self-quanto options as power options. Quanto options are treated in Section 1.7.10. Besides the obvious reasons one can think of the following additional motives.

1. Hedging future levels of implied volatility. Vega, which is volatility risk, is extremely difficult to hedge as there is no directly observable measure available, see [106]. A power straddle is an effective instrument to do so as it preserves the volatility exposure better than a vanilla straddle when the price of the underlying moves significantly as shown in Section 1.7.9 on sensitivities to risk parameters.
2. Through their exponential, non-linear payoff, power options can hedge non-linear price risks. An example is an importer earning profits merely through a percentage mark-up on imported products. An exchange rate change will lead to a price change, which in turn may affect demand volumes. The importer faces a risk of non-linearly decreasing earnings, see [70].

3. With very large short positions in vanilla options, a re-balancing of a dynamic delta hedge of this short gamma position may require such massive buying (selling) of the underlying that this impacts the price of the underlying, which in turn requires further hedge adjustments and may "pin" the underlying to the strike price, see p. 37 in [129]. To *smooth this pin risk*, option sellers propose a *soft strike option* with a similarly smooth and continuous payoff curvature as power options. As we will show in the hedging analysis of this section, this payoff curvature can be effectively replicated using vanilla options with different strike prices. The *diversified* range of strikes then softens any effect of a move in the underlying price. For details on *soft strike options* see [129], p. 37 and [70], p. 51.

Valuation of the Asymmetric Power Option The value can be written as the discounted expected value of the payoff under the risk-neutral measure. Using the domestic discount factor $e^{-r_d T}$ yields

$$\text{asymmetric power option value } v_{aPC} = e^{-r_d T} I\!E \left[\phi(S_T^n - K^n) I\!I_{\{\phi S_T > \phi K\}} \right]. \tag{332}$$

As K is a constant, S_T is the only random variable which simplifies the equation to

$$v_{aPC} = \phi e^{-r_d T} I\!E \left[S_T^n I\!I_{\{\phi S_T > \phi K\}} \right] - \phi e^{-r_d T} K^n I\!E \left[I\!I_{\{\phi S_T > \phi K\}} \right]. \tag{333}$$

The expectation of an indicator function is just the probability that the event $\{S_T > K\}$ occurs. In the Black-Scholes model, S_T is log-normally distributed and evolves according to a geometric Brownian motion (1). Itô's Lemma implies that S_T^n is also a geometric Brownian motion following

$$dS_t^n = \left[n(r_d - r_f) + \frac{1}{2} n(n-1)\sigma^2 \right] S_t^n \, dt + n\sigma S_t^n \, dW_t. \tag{334}$$

Solving the differential equation and calculating the expected value in Equation (333) leads to the desired closed form solution

$$v_{aPC} = \phi e^{-r_d T} \left[f^n e^{\frac{1}{2} n(n-1)\sigma^2 T} \mathcal{N}(\phi d_+^n) - K^n \mathcal{N}(\phi d_-) \right], \tag{335}$$

$$f \stackrel{\Delta}{=} S_0 e^{(r_d - r_f)T},$$

$$d_- \stackrel{\Delta}{=} \frac{\ln \frac{f}{K} - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}},$$

$$d_+^n \stackrel{\Delta}{=} \frac{\ln \frac{f}{K} + \left(n - \frac{1}{2}\sigma^2 \right) T}{\sigma\sqrt{T}}.$$

Valuation of the Symmetric Power Option Due to the binomial term $(S_T - K)^n$ the general value formula derivation for the symmetric version looks more complicated. That is why a more intuitive approach is taken and the valuation logic is shown based on the asymmetric option discussed above. Taking the example of $n = 2$ the difference between asymmetric and symmetric call is

$$[S_T^2 - K^2] - [S_T^2 - 2S_T K + K^2] = 2K(S_T - K). \tag{336}$$

The symmetric version for $n = 2$ is thus exactly equal to the asymmetric power option minus $2K$ vanilla options. This way pricing and hedging the symmetric power option becomes a structuring exercise, see Figure 1.54. Tompkins and Zhang both discuss the more complicated derivation of the general formula for symmetric power options in [129] and [146]. Tompkins also presents a formula for symmetric power straddles for $n = 2$.

Sensitivity Analysis Looking at the *Greeks* of asymmetric power options compared to vanilla options, the exponential elements of power options are well reflected in the exposures. This is especially true for delta and gamma, as can be seen in Figure 1.55, but is also valid for theta and vega. The power option rhos are very similar to the vanilla version.

Contrary to the asymmetric power option, the symmetric power option sensitivities exhibit new features that cannot be found with vanilla options, namely extreme delta values and a gamma that resembles the plain vanilla delta. When combined in a straddle this creates a *constant gamma exposure*, see Figure 1.56.

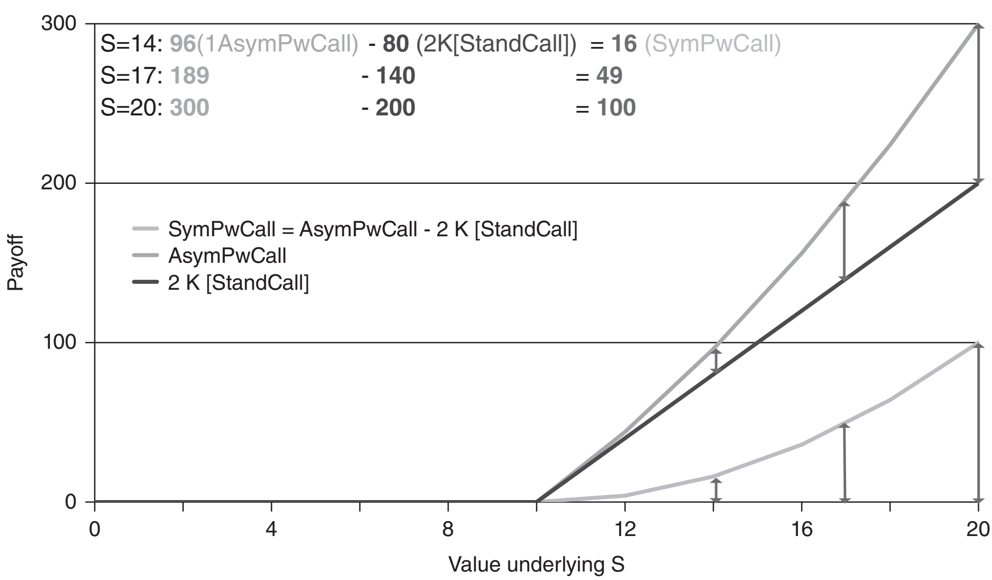


FIGURE 1.54 Symmetric power call replicated with asymmetric power and vanilla calls, using $K = 10$, $n = 2$.

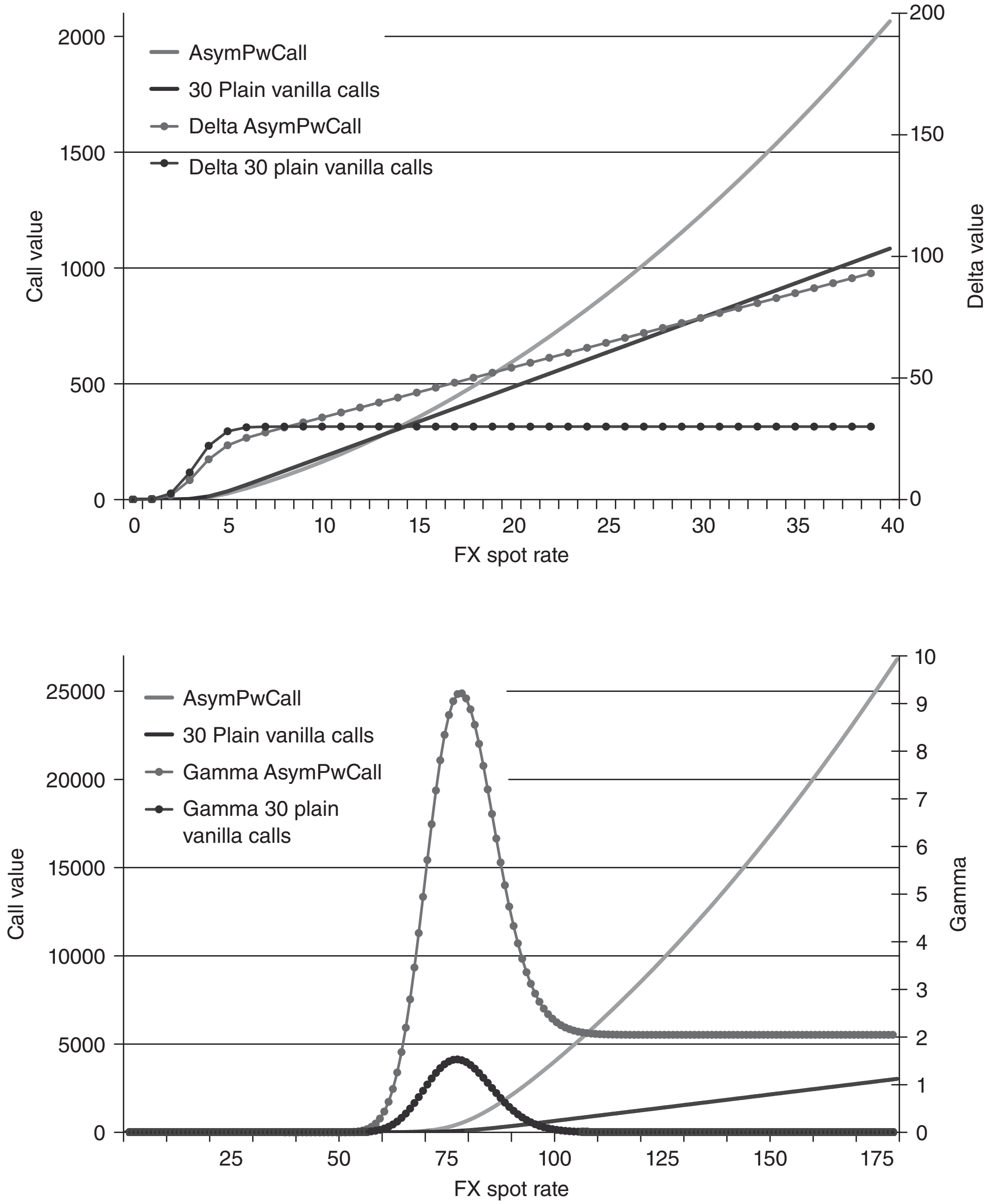


FIGURE 1.55 Asymmetric power call and vanilla call value, delta (lhs) and gamma (rhs) on the spot space, using $K = 10$, $n = 2$, $\sigma = 20\%$, $r_d = 5\%$, $r_f = 0\%$, $T = 90$ days.

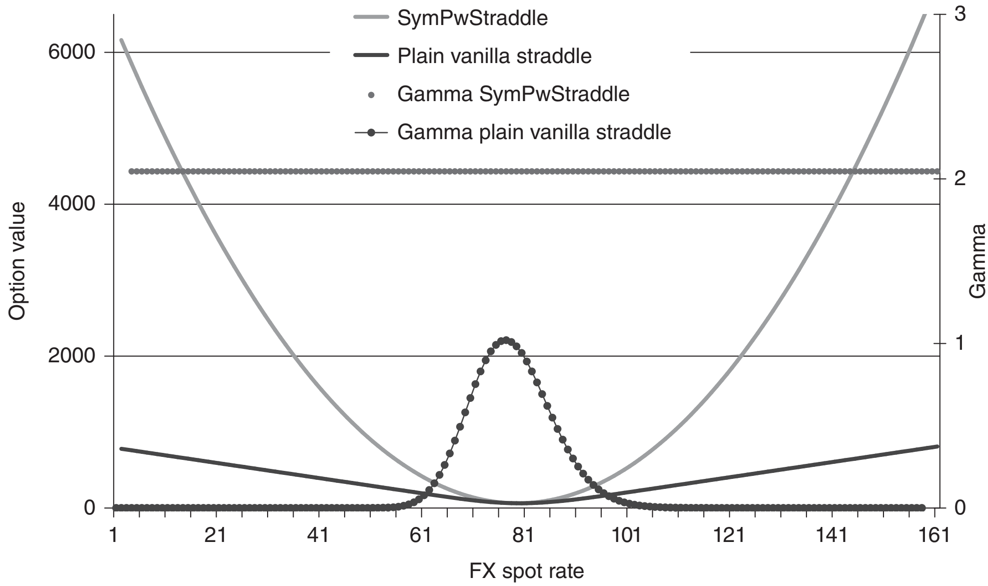


FIGURE 1.56 Gamma exposure of a symmetric power versus vanilla straddle, using $K = 80$ (at-the-money), $n = 2$, $\sigma = 20\%$, $r_d = 5\%$, $r_f = 0\%$, $T = 90$ days.

At the same time, if the underlying increases significantly, the symmetric power straddle preserves the exposure to volatility, whereas the vanilla straddle value becomes more and more invariant to the volatility input. Therefore, the power straddle is useful to hedge implied volatility, see Figure 1.57. This feature is similar to a variance swap explained in Section 1.8.8.

Replication with Vanilla Options The insights from the option payoffs, valuation, and the sensitivity analysis provide an effective static replication strategy for both asymmetric and symmetric power options. The respective call values are considered as an example.

Approximate Static Replication The continuous curvature of a power option can be approximated piecewise, adding up linear payoffs of vanilla options with different strike prices, see [129].

The symmetric power call for $n = 2$ can be replicated statically, as explained in the pricing section, just an asymmetric power call less $2K$ vanilla options, see Table 1.29.

The piecewise linear approximation with vanilla options is a super-replication, whence the value of the super-replicating portfolio is a natural upper boundary for the value of the symmetric power option as it overestimates the option value, see Table 1.30. The complexity of a static replication increases enormously with higher values of n. For the above example, a package of 25499 (3439) vanilla options is required to replicate one asymmetric (symmetric) power call. Overall, the static replication works very well, as can be seen in Figure 1.58. The static super-replication of power options with vanilla options takes into account the smile correctly, whence the value of the static

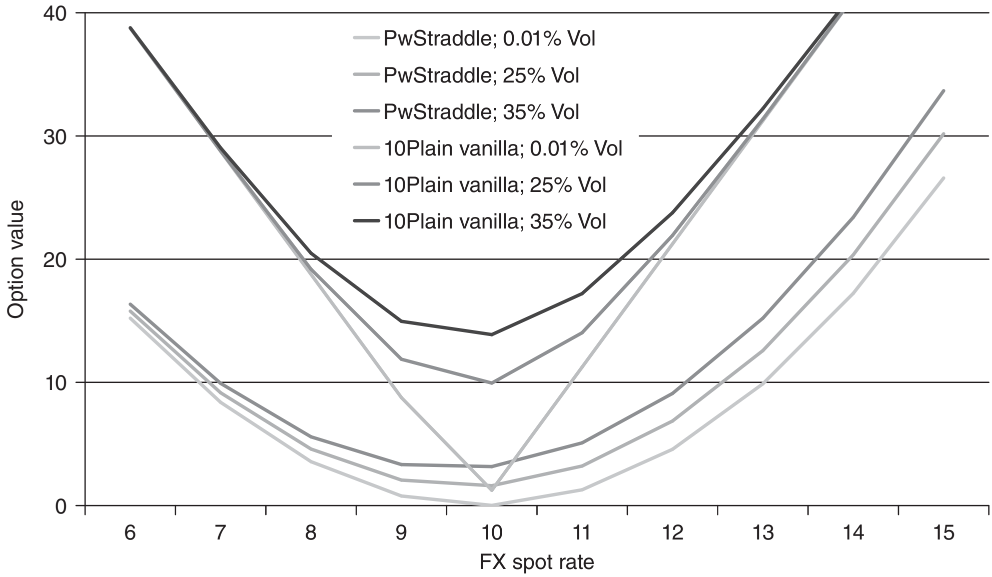


FIGURE 1.57 Vega exposure of a symmetric power versus vanilla straddle, using $K = 10$ (at-the-money), $n = 2$, $\sigma = 20\%$, $r_d = 5\%$, $r_f = 0\%$, $T = 90$ days.

TABLE 1.29 Static replication for the asymmetric power call, using $K = 20$, $n = 2$. For the symmetric power call the $2K$ standard calls need to be removed.

Underlying price		10	11	12	13	14	15	16	17	18	19	20
Asym. power call		0	21	44	69	96	125	156	189	224	261	300
Vanilla call package	Sum	0	21	44	69	96	125	156	189	224	261	300
Package components	Strike											
$2K$ standard calls	10		20	40	60	80	100	120	140	160	180	200
One standard call	10		1	2	3	4	5	6	7	8	9	10
Two standard calls	11			2	4	6	8	10	12	14	16	18
Two standard calls	12				2	4	6	8	10	12	14	16
Two standard calls	13					2	4	6	8	10	12	14
Two standard calls	14						2	4	6	8	10	12
Two standard calls	15							2	4	6	8	10
Two standard calls	16								2	4	6	8
Two standard calls	17									2	4	6
Two standard calls	18										2	4
Two standard calls	19											2

super-replicating portfolio can serve as a market value of the power option. All the sensitivities can be correctly derived from the sensitivities of the super-replicating portfolio. Since even far-out strikes need to be taken into consideration, the precision of the smile on the wings (extrapolation) is crucial. The power option shares this feature with the variance swap.

TABLE 1.30 Asymmetric power call replication versus formula value, using $K = 10$ (at-the-money), $n = 2$, $\sigma = 15\%$, $r_d = 5\%$, $r_f = 0\%$, $T = 90$ days.

Asym. power call	Formula value	11.91
Vanilla call package	Sum	12
Package components	Strike	
$2K$ standard calls	10	11.0354
One standard call	10	0.55177
Two standard calls	11	0.33257
Two standard calls	12	0.06928
Two standard calls	13	0.01034
Two standard calls	14	0.00116
Two standard calls	15	0.00010
Two standard calls	16	7.55E-06
Two standard calls	17	4.74E-07
Two standard calls	18	2.64E-08
Two standard calls	19	1.33E-09

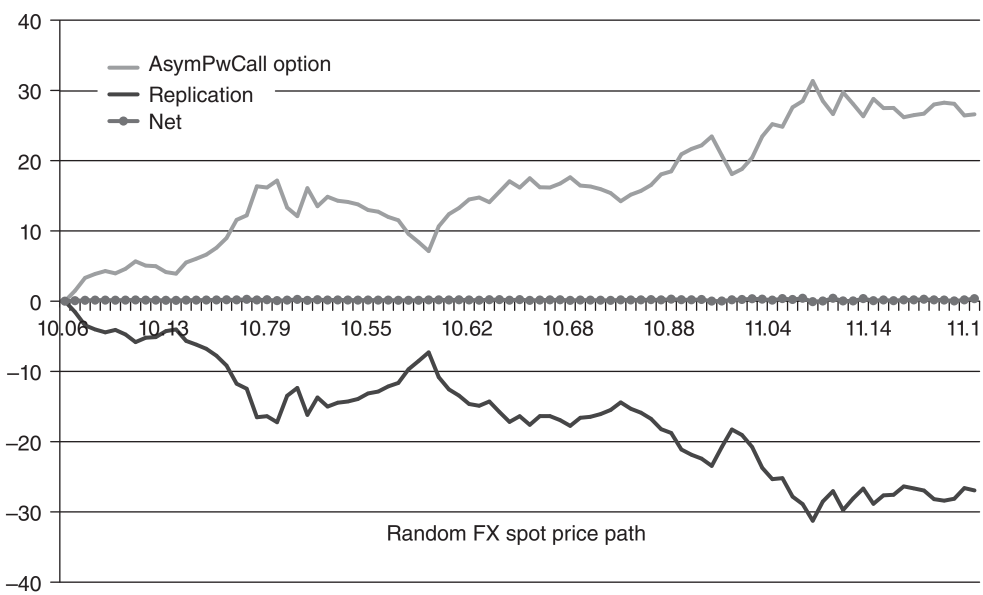


FIGURE 1.58 Static replication performance of an asymmetric power call, using $K = 10$, $n = 2$, $\sigma = 150\%$, $r_d = 5\%$, $r_f = 0\%$, $T = 90$ days.

Dynamic Hedging A dynamic hedge involves setting up and managing a position in the underlying currencies that offsets any value change in the option position. Usually the difficulty of dynamic hedging lies in second order, that is in gamma, and in vega risk. As the symmetric power straddle has a constant gamma, this simplifies delta hedging activities. In practice, however, it is common for the risk-warehousing desk to first set up a static replication and then hedge the Greeks of the residual position dynamically.

1.7.10 Quanto Options

A quanto option can be any cash-settled option, whose payoff is converted into a third currency at maturity at a pre-specified rate, called the *quanto factor*. There can be quanto plain vanilla, quanto barriers, quanto forward starts, quanto corridors, etc. The valuation theory is covered for example in [120] and [65]. This section is based on [138].

FX Quanto Drift Adjustment We take the example of a gold contract with underlying XAU/USD in XAU-USD quotation that is quantoed into EUR. Since the payoff is in EUR, we let EUR be the numeraire or domestic or base currency and consider a Black-Scholes model

$$\text{XAU-EUR: } dS_t^{(3)} = (r_{EUR} - r_{XAU})S_t^{(3)}\,dt + \sigma_3 S_t^{(3)}\,dW_t^{(3)}, \tag{337}$$

$$\text{USD-EUR: } dS_t^{(2)} = (r_{EUR} - r_{USD})S_t^{(2)}\,dt + \sigma_2 S_t^{(2)}\,dW_t^{(2)}, \tag{338}$$

$$dW_t^{(3)}dW_t^{(2)} = \rho_{23}\,dt, \tag{339}$$

where we use an (implicit) plus sign in front of the correlation because both $S^{(3)}$ and $S^{(2)}$ have the same base currency (DOM), which is EUR in this case. The scenario is displayed in Figure 1.59. The actual underlying is then

$$\text{XAU-USD: } S_t^{(1)} = \frac{S_t^{(3)}}{S_t^{(2)}}. \tag{340}$$

Using Itô's formula, we first obtain

$$\begin{aligned} d\frac{1}{S_t^{(2)}} &= -\frac{1}{(S_t^{(2)})^2}\,dS_t^{(2)} + \tfrac{1}{2}\cdot 2\cdot \tfrac{1}{(S_t^{(2)})^3}\,(dS_t^{(2)})^2 \\ &= (r_{USD} - r_{EUR} + \sigma_2^2)\frac{1}{S_t^{(2)}}\,dt - \sigma_2\frac{1}{S_t^{(2)}}\,dW_t^{(2)}, \end{aligned} \tag{341}$$

and hence

$$\begin{aligned} dS_t^{(1)} &= \frac{1}{S_t^{(2)}}\,dS_t^{(3)} + S_t^{(3)}\,d\frac{1}{S_t^{(2)}} + dS_t^{(3)}\,d\frac{1}{S_t^{(2)}} \\ &= \frac{S_t^{(3)}}{S_t^{(2)}}(r_{EUR} - r_{XAU})\,dt + \frac{S_t^{(3)}}{S_t^{(2)}}\sigma_3\,dW_t^{(3)} \\ &\quad + \frac{S_t^{(3)}}{S_t^{(2)}}(r_{USD} - r_{EUR} + \sigma_2^2)\,dt - \frac{S_t^{(3)}}{S_t^{(2)}}\sigma_2\,dW_t^{(2)} - \frac{S_t^{(3)}}{S_t^{(2)}}\rho_{23}\sigma_2\sigma_3\,dt \\ &= (r_{USD} - r_{XAU} + \sigma_2^2 - \rho_{23}\sigma_2\sigma_3)S_t^{(1)}\,dt + S_t^{(1)}(\sigma_3\,dW_t^{(3)} - \sigma_2\,dW_t^{(2)}). \end{aligned}$$

Since $S_t^{(1)}$ is a geometric Brownian motion with volatility σ_1, we introduce a new Brownian motion $W_t^{(1)}$ and find

$$dS_t^{(1)} = (r_{USD} - r_{XAU} + \sigma_2^2 - \rho_{23}\sigma_2\sigma_3)S_t^{(1)}\,dt + \sigma_1 S_t^{(1)}\,dW_t^{(1)}. \tag{342}$$

Now Figure 1.59 and the *law of cosine* imply

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2, \tag{343}$$

$$\sigma_1^2 = \sigma_2^2 + \sigma_3^2 - 2\rho_{23}\sigma_2\sigma_3, \tag{344}$$

which yields

$$\sigma_2^2 - \rho_{23}\sigma_2\sigma_3 = \rho_{12}\sigma_1\sigma_2. \tag{345}$$

As explained in the *currency triangle* in Figure 1.59, ρ_{12} is the correlation between XAU-USD and EUR-USD, whence $\rho \stackrel{\Delta}{=} -\rho_{12}$ is the correlation between XAU-USD and USD-EUR (i.e. the correlation between the currency pairs FOR-DOM and DOM-QUANTO). Inserting this into Equation (342), we obtain the usual formula for the drift adjustment

$$dS_t^{(1)} = (r_{USD} - r_{XAU} - \rho\sigma_1\sigma_2)S_t^{(1)}\,dt + \sigma_1 S_t^{(1)}\,dW_t^{(1)}. \tag{346}$$

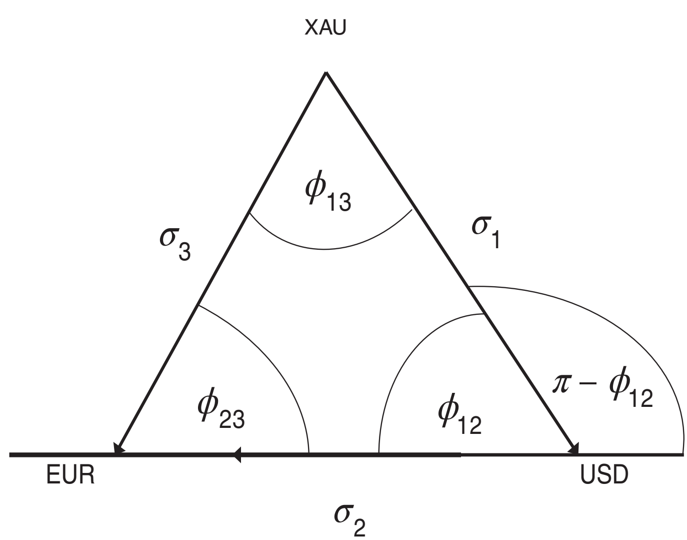


FIGURE 1.59 XAU-USD-EUR FX quanto triangle. The arrows point in the direction of the respective base currencies. The length of the edges represents the volatility. The cosine of the angles $\cos\phi_{ij} = \rho_{ij}$ represents the correlation of the currency pairs $S^{(i)}$ and $S^{(j)}$, *if* both $S^{(i)}$ and $S^{(j)}$ have the same base currency (DOM). If the base currency (DOM) of $S^{(i)}$ is the underlying currency (FOR) of $S^{(j)}$, then the correlation is denoted by $-\rho_{ij} = \cos(\pi - \phi_{ij})$.

This is the risk-neutral pricing process that can be used for the valuation of any derivative depending on $S_t^{(1)}$ which is quantoed into EUR.

Extensions to Other Models The previous derivation can be extended to the case of term structure of volatility and correlation. However, introduction of volatility smile would distort the relationships. Nevertheless, accounting for smile effects is important in real market scenarios. To do this, one could, for example, capture the smile for a multi-currency model with a *weighted Monte Carlo technique* as described by Avellaneda *et al.* in [5]. This would still allow the previous result to be used.

Quanto Vanilla Common among foreign exchange options is a quanto plain vanilla paying

$$Q[\phi(S_T - K)]^+, \tag{347}$$

where K denotes the strike, T the expiration time, ϕ the usual put-call indicator taking the value +1 for a call and −1 for a put, S the underlying in FOR-DOM quotation, and Q the quanto factor from the domestic currency into the quanto currency. We let

$$\tilde{\mu} \stackrel{\Delta}{=} r_d - r_f - \rho\sigma\tilde{\sigma}, \tag{348}$$

be the *adjusted drift*, where r_d and r_f denote the risk-free interest rates of the domestic and foreign underlying currency pair respectively, $\sigma = \sigma_1$ the volatility of this currency pair, $\tilde{\sigma} = \sigma_2$ the volatility of the currency pair DOM-QUANTO, and

$$\rho = \frac{\sigma_3^2 - \sigma^2 - \tilde{\sigma}^2}{2\sigma\tilde{\sigma}} \tag{349}$$

the correlation between the currency pairs FOR-DOM and DOM-QUANTO in this quotation. Furthermore we let r_Q be the risk-free interest rate of the quanto currency. With the same principles as in [139] we can derive the formula for the value as

$$v = Qe^{-r_Q T}\phi[S_0 e^{\tilde{\mu}T}\mathcal{N}(\phi d_+) - K\mathcal{N}(\phi d_-)], \tag{350}$$

$$d_\pm = \frac{\ln\frac{S_0}{K} + \left(\tilde{\mu} \pm \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{351}$$

where $\mathcal{N}$ denotes the cumulative standard normal distribution function and n its density.

Quanto Forward Similarly, we can easily determine the value of a quanto forward paying

$$Q[\phi(S_T - K)], \tag{352}$$

where K denotes the strike, T the expiration time, ϕ the usual long-short indicator, S the underlying in FOR-DOM quotation, and Q the quanto factor from the domestic currency into the quanto currency. Then the formula for the value can be written as

$$v = Qe^{-r_Q T}\phi[S_0 e^{\tilde{\mu} T} - K]. \tag{353}$$

This follows from the vanilla quanto value formula by taking both the normal probabilities to be one. These normal probabilities are exercise probabilities under some measure. Since a forward contract is always exercised, both these probabilities must be equal to 1.

Quanto Digital A European style quanto digital pays

$$Q\mathbb{I}_{\{\phi S_T \geq \phi K\}}, \tag{354}$$

where K denotes the strike, S_T the spot of the currency pair FOR-DOM at maturity T, ϕ takes the values +1 for a digital call and −1 for a digital put, and Q is the pre-specified conversion rate from the domestic to the quanto currency. The valuation of European style quanto digitals follows the same principle as in the quanto vanilla option case. The value is

$$v = Qe^{-r_Q T}\mathcal{N}(\phi d_-). \tag{355}$$

We provide an example of a European style digital put in USD/JPY quanto into EUR in Table 1.31.

Hedging of Quanto Options Dynamic hedging of quanto options can be done by running a multi-currency options book. All the usual Greeks can be hedged. Delta hedging is done by trading in the underlying spot market. An exception is the *correlation risk*, which can only be hedged with other derivatives depending on the same correlation. This is

TABLE 1.31 Example of a quanto digital put. The buyer receives 100,000 EUR if at maturity the ECB fixing for USD-JPY (computed via EUR-JPY and EUR-USD) is below 108.65. Terms were created on January 12 2004 with the following market data: USD-JPY spot ref 106.60, USD-JPY ATM vol 8.55%, EUR-JPY ATM vol 6.69%, EUR-USD ATM vol 10.99% (corresponding to a correlation of −27.89% for USD-JPY against JPY-EUR), USD rate 2.5%, JPY rate 0.1%, EUR rate 4%.

Notional	100,000 EUR
Maturity	3 months (92 days)
European style barrier	108.65 USD-JPY
Theoretical value	71,555 EUR
Fixing source	ECB

generally not possible in other asset classes. In FX the correlation risk can be translated into a vega position as shown in [135] or in Section 1.9.2 on foreign exchange basket options. We illustrate this approach for quanto plain vanilla options now.

Vega Positions of Quanto Plain Vanilla Options Starting from Equation (350), we obtain the sensitivities

$$\begin{aligned}
\frac{\partial v}{\partial \sigma} &= QS_0 e^{(\tilde{\mu}-r_Q)T}\left[n(d_+)\sqrt{T} - \phi\mathcal{N}(\phi d_+)\rho\tilde{\sigma}T\right], \\
\frac{\partial v}{\partial \tilde{\sigma}} &= -QS_0 e^{(\tilde{\mu}-r_Q)T}\phi\mathcal{N}(\phi d_+)\rho\sigma T, \\
\frac{\partial v}{\partial \rho} &= -QS_0 e^{(\tilde{\mu}-r_Q)T}\phi\mathcal{N}(\phi d_+)\sigma\tilde{\sigma}T, \\
\frac{\partial v}{\partial \sigma_3} &= \frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial \sigma_3} \\
&= \frac{\partial v}{\partial \rho}\frac{\sigma_3}{\sigma\tilde{\sigma}} \\
&= -QS_0 e^{(\tilde{\mu}-r_Q)T}\phi\mathcal{N}(\phi d_+)\sigma\tilde{\sigma}T\frac{\sigma_3}{\sigma\tilde{\sigma}} \\
&= -QS_0 e^{(\tilde{\mu}-r_Q)T}\phi\mathcal{N}(\phi d_+)\sigma_3 T \\
&= -QS_0 e^{(\tilde{\mu}-r_Q)T}\phi\mathcal{N}(\phi d_+)\sqrt{\sigma^2 + \tilde{\sigma}^2 + 2\rho\sigma\tilde{\sigma}}T.
\end{aligned}$$

Note that the computation is standard calculus and repeatedly uses Identity (37). The understanding of these Greeks is that σ and $\tilde{\sigma}$ are both risky parameters, independent of each other. The third independent source of risk is either σ_3 or ρ, depending on what is more likely to be known. This shows exactly how the three vega positions can be hedged with plain vanilla options in all three legs, provided there is a liquid vanilla options market in all three legs. In the example with XAU-USD-EUR the currency pairs XAU-USD and EUR-USD are traded; however, there is a very illiquid vanilla market in XAU-EUR. Therefore, the correlation risk remains unhedgeable. Similar statements would apply for quantoed stocks or stock indices. However, in FX, there are situations with all legs being hedgeable, for instance EUR-USD-JPY.

The signs of the vega positions are not uniquely determined in all legs. The FOR-DOM vega is smaller than the corresponding vanilla vega in the case of a call and positive correlation or put and negative correlation, larger in the case of a put and positive correlation or call and negative correlation. The DOM-Q vega takes the sign of the correlation in the case of a call and its opposite sign in the case of a put. The FOR-Q vega takes the opposite sign of the put-call indicator ϕ.

We provide an example of pricing and vega hedging scenario in Table 1.32, where we notice, that dominating vega risk comes from the FOR-DOM pair, whence most of the risk can be hedged.

Applications Standard applications are performance linked deposits or notes explained in Section 2.4.2 or participation notes explained in Section 2.6. Any time the performance of an underlying asset needs to be converted into the notional currency invested,

TABLE 1.32 Example of a quanto plain vanilla.

		Data set 1	Data set 2	Data set 3
FX pair	FOR-DOM	XAU-USD	XAU-USD	XAU-USD
Spot	FOR-DOM	800.00	800.00	800.00
Strike	FOR-DOM	810.00	810.00	810.00
Quanto	DOM-Q	1.0000	1.0000	1.0000
Volatility	FOR-DOM	10.00%	10.00%	10.00%
Quanto volatility	DOM-Q	12.00%	12.00%	12.00%
Correlation	FOR-DOM – DOM-Q	25.00%	25.00%	−75.00%
Domestic interest rate	DOM	2.0000%	2.0000%	2.0000%
Foreign interest rate	FOR	0.5000%	0.5000%	0.5000%
Quanto currency rate	Q	4.0000%	4.0000%	4.0000%
Time in years	T	1	1	1
1=call −1=put	FOR	1	−1	1
Quanto vanilla option	value	30.81329	31.28625	35.90062
Quanto vanilla option	vega FOR-DOM	298.14188	321.49308	350.14600
Quanto vanilla option	vega DOM-Q	−10.07056	9.38877	33.38797
Quanto vanilla option	vega FOR-Q	−70.23447	65.47953	−35.61383
Quanto vanilla option	correlation risk	−4.83387	4.50661	−5.34207
Quanto vanilla option	vol FOR-Q	17.4356%	17.4356%	8.0000%
Vanilla option	value	32.6657	30.7635	32.6657
Vanilla option	vega	316.6994	316.6994	316.6994

and the exchange rate risk is with the seller, we need a quanto product. Naturally, an underlying like gold, which is quoted in USD, would be a default candidate for a quanto product, when the investment is in a currency other than USD. This shows that quanto features play a key role in asset management whenever the investor wants to protect his investment against foreign exchange rate risk. Quanto options are embedded in many investment strategies with underlying assets being stocks or stock indices. Quanto options in foreign exchange markets do not trade as much because FX is not viewed as an interesting asset class by many asset managers. This may change in the future. On the treasury side, quanto options do sometimes come up in currency related swaps, as explained in Section 2.5.7.

1.7.11 Exercises

Foreign Digital Value via Static Replication Derive Equation (168) using a static replication of the foreign digital by domestic digital and vanilla options.

Foreign Digital Value via Change of Measure Derive Equation (168) by calculating the expected value of the payoff (166) and a change of measure.

Compound Consider a EUR-USD market with spot at 1.2500, EUR rate at 2.5%, USD rate at 2.0%, volatility at 10.0%, and the situation of a treasurer expecting to receive 1 M USD in one year, that he wishes to change into EUR at the current spot rate of

1.2500. In six months he will know whether the company gets the definite order. Compute the price of a vanilla EUR call USD put in EUR. Alternatively compute the price of a compound with two thirds of the total premium to be paid at inception and one third to be paid in six months. Do the same computations if the sales margin for the vanilla is 1 EUR per 1,000 USD notional and for the compound is 2 EUR per 1,000 USD notional. After six months the company ends up not getting the order and can waive its hedge. How much would it get for the vanilla if the spot is at 1.1500, at 1.2500 and at 1.3500? Would it be better for the treasurer to own the compound and not pay the second premium? How would you split up the premium for the compound to persuade the treasurer to buy the compound rather than the vanilla? (After all, there is more margin to earn.)

Perpetual One-Touch Replication Find the fair price and a semi-static replication of a *perpetual one-touch*, which pays one unit of the domestic currency if the barrier $H > S_0$ is ever hit, where S_0 denotes the current exchange rate. How about payment in the foreign currency? How about a *perpetual no-touch*? These thoughts are developed further to a *vanilla-one-touch duality* by Peter Carr [22].

Perpetual Double-One-Touch Find the value of a *perpetual double-one-touch*, which pays a rebate R_H, if the spot reaches the higher level H before the lower level L, and R_L, if the spot reaches the lower level first. Consider as an example the EUR-USD market with a spot of S_0 at time zero between L and H. Let the interest rates of both EUR and USD be zero and the volatility be 10%. The specified rebates are paid in USD. There is no finite expiration time, but the rebate is paid whenever one of the levels is reached. How would you replicate a short position (semi-) statically?

Strike-Out Replication and Impact of Jumps A call (put) option is the right to buy (sell) one unit of foreign currency on a maturity date T at a pre-defined price K, called the strike price. A knock-out call with barrier B is like a call option that becomes worthless if the underlying ever touches the barrier B at any time between inception of the trade and its expiration time. Let the market parameters be spot $S_0 = 120$, all interest rates be zero, volatility $\sigma = 10\%$. In a liquid and jump-free market, find the value of a one-year *strike-out*, i.e. a down-and-out knock-out call, where $K = B = 100$.

Suppose now that the spot price movement can have downward jumps but the forward price is still constant and equal to the spot (since we assume zero interest rates). How do these possible jumps influence the value of the knock-out call?

The solution to this problem is used for the design of *turbo notes*, see Section 2.6.4.

Strike-Out Call Vega What is the vega profile as a function of spot for a strike-out call? What can you say about the sign of vega?

Double-No-Touch with Notional in Foreign Currency Given Equation (235), which represents the theoretical value (TV) of a double-no-touch in units of domestic currency, where the payoff currency is also domestic, let us denote this function by

$$v^d(S, r_d, r_f, \sigma, L, H), \tag{356}$$

where the superscript d indicates that the payoff currency is domestic. Using this formula, prove that the corresponding value in domestic currency of a double-no-touch paying one unit of *foreign* currency is given by

$$v^f(S, r_d, r_f, \sigma, L, H) = Sv^d\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right). \tag{357}$$

Assuming you know the sensitivity parameters of the function v^d, derive the following corresponding sensitivity parameters for the function v^f,

$$\begin{aligned}
\frac{\partial v^f}{\partial S} &= v^d\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right) - \frac{1}{S}\frac{\partial v^d}{\partial S}\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right), \qquad (358)\\
\frac{\partial^2 v^f}{\partial S^2} &= \frac{1}{S^3}\frac{\partial^2 v^d}{\partial S^2}\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right),\\
\frac{\partial v^f}{\partial \sigma} &= S\frac{\partial v^d}{\partial \sigma},\\
\frac{\partial^2 v^f}{\partial \sigma^2} &= S\frac{\partial^2 v^d}{\partial^2 \sigma},\\
\frac{\partial^2 v^f}{\partial S \partial \sigma} &= S\frac{\partial v^d}{\partial \sigma}\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right) - \frac{1}{S}\frac{\partial^2 v^d}{\partial S \partial \sigma}\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right),\\
\frac{\partial v^f}{\partial r_d} &= \frac{\partial v^d}{\partial r_f},\\
\frac{\partial v^f}{\partial r_f} &= \frac{\partial v^d}{\partial r_d}.
\end{aligned}$$

Static Replication of DNT with DKO Suppose your front-office application for double-no-touch contracts is out of order, but you can use double-knock-out options. Replicate statically a double-no-touch paying one unit of domestic currency using two double-knock-out options.

Static Replication of a FOR-Paying Double-No-Touch The nominal amounts of the respective double-knock-out options that statically replicate the double-no-touch depend on the currency in which the payoff is settled. In the case of a EUR-USD double-no-touch paying one unit of USD (domestic currency), the nominal amounts of the double-knock-out call and put must be chosen to be both $\frac{1}{H-L}$ (which currency?). Find a static replication of a double-no-touch paying one unit of EUR (foreign currency).

Static Replication of a FOR-Paying Double-No-Touch with One Double-Knock-Out Option Can you statically replicate a double-no-touch paying one unit of foreign currency using just *one* double-knock-out option?

Gold Price Return vs. EUR-USD Return The price of an ounce of gold is quoted in USD. If the price of Gold drops by 5%, but the price of Gold in EUR remains constant, determine the change of the EUR-USD exchange rate.

Vega Profile of Double-No-Touch Suppose you are long a double-no-touch. Draw the possible vega profiles as a function of the spot and discuss the possible scenarios.

Vanilla Vega Hedging Suppose you know the vega of a two-month at-the-money vanilla. By what factor is the vega of a four-month at-the-money vanilla bigger? How does this look for a five-year vanilla in comparison with a 10-year vanilla? You may assume a delta-neutral straddle notion of at-the-money.

One-Touch Replication with Digitals Suppose the exchange rate S follows a Brownian motion without drift and constant volatility. How can you replicate semi-statically a single-one-touch with European digital call or put options? Hint: Use the reflection principle. Which risk remains unhedged in your semi-static replication?

Static Replication of European Barrier Options with Vanilla and Digital Options Given vanillas and digitals, how can you statically replicate European style barrier options?

Self-Quanto Option Determine the Black-Scholes value of a CHF-EUR *self-quanto option* with strike K, which is cash-settled in CHF at maturity. As an example you may consider the payoff

$$N \cdot \frac{(K - S_T)^+}{S_T} \tag{359}$$

paid in CHF, where N is a CHF notional amount, K a strike in EUR-CHF, and S_T the spot price in EUR-CHF at maturity time T.

Implementing the View of a Rising USD/JPY Suppose a client believes very strongly that USD/JPY will reach a level of 120.00 or higher in three months' time. With a current spot level of 110.00, volatility of 10%, JPY rate of 0%, USD rate of 3%, find a contract reflecting the client's view and create a term sheet for the client explaining chances and risks.

Gamma of a Symmetric Power Straddle Prove that a symmetric power straddle has a constant gamma. What does this imply for delta, vega, rho (domestic and foreign), and theta?

Static Replication of RKO with KO and Touch Contracts Let $\mathrm{NT}(B)$ and $\mathrm{OT}(B)$ denote the value of a no-touch and a one-touch with barrier B respectively, both paid at the end. Let $\mathrm{KOPut}(K, B)$ and $\mathrm{KOCall}(K, B)$ denote the value of a regular knock-out put and call with strike K and barrier B respectively. Let $\mathrm{SOPut}(K)$ and $\mathrm{SOCall}(K)$ denote the value of a strike-out put and call with strike K and barrier K respectively. Finally, let

RKOPut(K, B) and RKOCall(K, B) denote the value of a reverse knock-out put and call with strike K and barrier B respectively. How can you replicate statically reverse knock-outs using touch contracts, strike-outs and regular knock-outs? Support your answer with a suitable figure. This implies in particular that the market prices for reverse knock-outs can be implied from the market prices of touch contracts and regular barrier options.

Non-Deliverable Self-Quanto Forward Consider the value function (353) of a quanto forward. Derive the value function of a (non-deliverable) *self-quanto forward*, for example in a EUR-USD market, with payoff $S_T - K$ in EUR rather than USD. If the amount he receives is negative, then the client pays.

No-Touch Value with Infinite Volatility Derive the limiting value in the Black-Scholes model of a no-touch with either an upper barrier or a lower barrier as volatility goes to infinity, and justify your answer intuitively.

1.8 SECOND GENERATION EXOTICS (SINGLE CURRENCY PAIR)

In this section we present an overview of some of the most common second generation exotics, of which some are exotic options and others are more similar to forwards and swaps. Many of the transactions are considered second generation because they are multi-currency transactions with correlation risk (and are not simple quantos). These include outside barrier options, spread and exchange options, basket options, best-of and worst-of options. Other second generation exotics are considered volatility trades, including the variance swap, volatility swap, and forward volatility agreement. The correlation swap is a correlation trade as its name indicates. Other second generation exotics comprise options or strategies with path-dependent notional amounts (corridors, faders) or some barrier options with special features. There are also path-independent second generation exotics on one underlying. As an example we consider the multiplicity power option.

1.8.1 Multiplicity Power Options

The basic idea of a power option (see Section 1.7.9) can be further extended. An extreme case of selling options in private banking was this: BNPP offered something called a "multiplicity" before the 2008 financial crisis. An investor sells a USD Put/JPY Call with strike X to the bank presumably for some huge premium expressed as a yield on some yield-enhancing note or swap. The strike X is taken as $\frac{K}{S^2}$, where S is the USD-JPY spot and K would be set somewhere around initial spot cubed, S^3, to make the initial strike below ATM. Thus as spot S falls, the strike X of the USD Put shoots up. Furthermore, X was capped at 600. Figure 1.60 illustrates the payoff of the multiplicity compared with a vanilla put. The risk the investor takes is substantial but well hidden in the nested definition of the powers. Hard to imagine that these products were very popular and traded a lot in Asia. The industry can gain back a lot of trust by explaining the risks appropriately before the product trades.

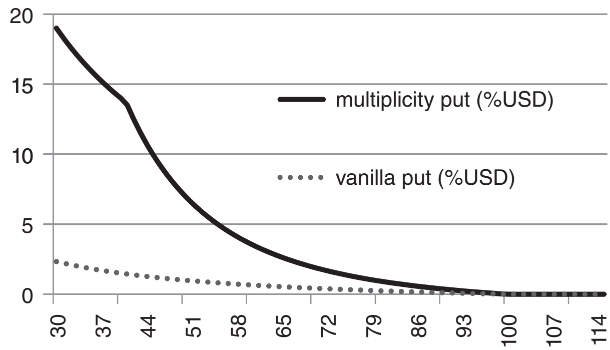


FIGURE 1.60 Payoff of a multiplicity power put compared with a vanilla put with strike $K = 100$ and cap of 600. USD-JPY final spot on the x-axis.

1.8.2 Corridors/Range Accruals

A European *corridor* or *range accrual (RAC)* entitles its holder to receive a pre-specified amount of a currency (say EUR) on a specified date (maturity) proportional to the number of fixings inside a range between the start date and maturity. The buyer has to pay a premium for this product.

Advantages

- High leverage product, high profit potential
- Can take advantage of a quiet market phase
- Easy to price and to understand

Disadvantages

- Not suitable for the long term
- Expensive product
- Price spikes and large market movements can lead to loss

Figure 1.61 shows a sample scenario for a corridor. At delivery, the holder receives $\frac{n}{N}\cdot$ notional, where n is the number of fixings inside the range and N denotes the maximum number of fixings.

Types of Corridors

European style corridor. The corridor is *resurrecting*, i.e. all fixings inside the range count for the accumulation, even if some of the fixings are outside. Given a *fixing schedule* $\{S_{t_1}, S_{t_2}, \ldots, S_{t_N}\}$ the payoff can be specified by

$$\text{notional} \cdot \frac{1}{N} \sum_{i=1}^{N} I\!I_{\{S_{t_i} \in [L,H]\}}, \tag{360}$$

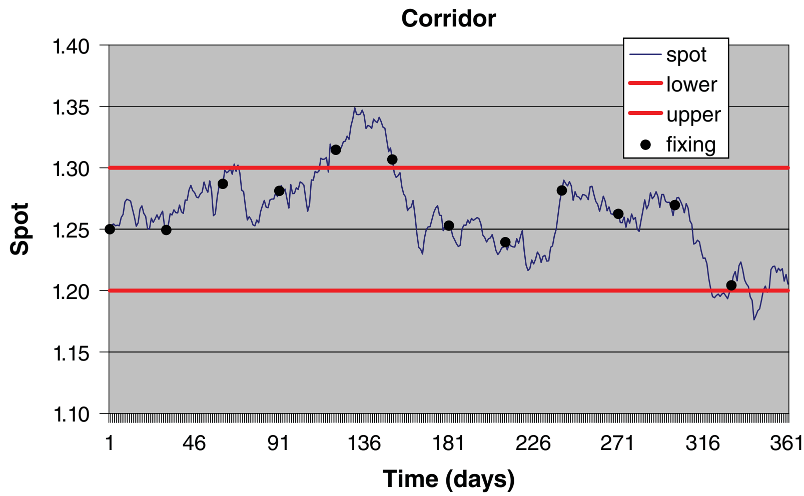


FIGURE 1.61 Example of a corridor or range accrual with spot 1.2500, domestic interest rate 3.00%, foreign interest rate 2.75%, volatility 10%, for a maturity of 1 year with 12 monthly fixings indicated by the dots. The range is 1.2000–1.3000. In a resurrecting corridor, the investor would accumulate 10 out of 12 fixings. In a non-resurrecting corridor, the investor would accumulate 4 out of 12 fixings as the fifth is outside the range.

where N denotes the total number of fixings, L the lower barrier, H the higher barrier.

American style corridor. This is *non-resurrecting*, i.e. only fixing dates count for the accumulation that occur before the first fixing is outside the range. The holder of the corridor keeps the accumulated amount. Introducing the stopping time

$$\tau \stackrel{\Delta}{=} \min\{t : S_{t_i} \notin (L, H)\}, \tag{361}$$

the payoff can be specified by

$$\text{notional} \cdot \frac{1}{N} \sum_{i=1}^{N} I\!I_{\{S_{t_i} \in [L,H]\}} I\!I_{\{t_i < \tau\}}. \tag{362}$$

American style corridor with continuously observed knock-out. This is an American style corridor, where all of the accumulated amount is lost once the exchange rate trades at or outside the range. This is equivalent to a double-no-touch. The payoff can be specified by

$$\text{notional} \cdot \frac{1}{N} \sum_{i=1}^{N} I\!I_{\{S_{t_i} \in [L,H]\}} I\!I_{\{L < \min_{0 \le t \le T} S_t \le \max_{0 \le t \le T} S_t < H\}}, \tag{363}$$

where T denotes the maturity time. This type of corridor can be generalized as the range for the fixings does not need to be identical to the range for the continuously observed knock-out condition.

American style corridor with discrete knock-out. This is like an American style corridor where the knock-out occurs when the fixing is outside the range for the first time, i.e. we use the stopping time in Equation (361) and replace the payoff by

$$\text{notional} \cdot \frac{1}{N} \sum_{i=1}^{N} I\!I_{\{S_{t_i} \in [L,H]\}} I\!I_{\{\tau > T\}}. \tag{364}$$

In this case the holder receives either the full notional or nothing, so it is very similar to a double-no-touch.

Forward start corridor. In this type, which can be European or American as before, the range will be set relative to a future spot level, see also Section 1.7.8.

Example An investor wants to benefit from his view that the EUR-USD exchange rate will be often between two levels during the next 12 months. In this case he may consider buying a European corridor as presented in Table 1.33 for example.

If the investor's market expectation is correct, then he will receive 1 M EUR at delivery, twice the initial premium.

Explanations

Fixings are *official* exchange rate sources such as from the European Central Bank, the Federal Reserve Bank or private banks, which take place on each business day. For details on the impact on pricing see Becker and Wystup [13].

Fixing source is the exact source of the fixing, for example Reuters pages ECB37, WMRSPOT11, or BFIX on Bloomberg.[5]

Fixing schedule requires a start date, an end date, and a frequency such as daily, weekly, or monthly. It can also be customized. Since there are often disputes about holidays, it is advisable to specify any fixing schedule explicitly in the deal confirmation. A common way is to agree on the open days in the TARGET system.

TABLE 1.33 Example of a European corridor. To compare, the premium for the same corridor in American style would be 100,000 EUR.

Spot reference	1.1500 EUR-USD
Notional	1,000,000 EUR
Maturity	1 year
European style corridor	1.1000–1.18000 EUR-USD
Fixing schedule	monthly
Fixing source	ECB37
Premium	500,000 EUR

[5] Also exhibited on http://www.bloomberg.com/markets/currencies/fx-fixings

Composition and Applications Obviously, a European style corridor is a sum of digital call spread options (or equivalently digital put spread options). The only modification is that the expiration times are the fixing times and the delivery time is the same for all digital options and in fact deferred. Furthermore the digital payout in a corridor is usually fixing based, whereas a stand-alone digital may be exercised based on the usual NY or Tokyo cut.

Similarly, an American style corridor is a sum of double-barrier digitals with deferred delivery. We refer the reader to the exercises to work out the details.

Corridors occur very often as part of structured products such as a *range accrual forward* explained in Section 2.1.10 or a *corridor deposit* explained in Section 2.4.4.

1.8.3 Faders

Fader options are options whose nominal is directly proportional to the number of fixings inside or outside a pre-defined range. A *fade-in option* has a progressive activation of the nominal. In a *fade-out option* the concept of a progressive activation of the nominal is changed to a progressive deactivation. The term fader is sometimes used in a more general way to describe transactions with fade-in or fade-out notional amounts. These transactions do not need to be options but can be combinations of options or structured products. We discuss as an example the fade-in put option, whose characteristics are the pre-defined range and the associated fixing schedule with the maximal number of fixing being M. For each fixing date with the fixing inside the pre-defined range, the holder of a fade-in put option holds a contract, which at maturity is economically equivalent to a vanilla put option with the total notional multiplied by

$$\frac{\text{number of fixings inside the range}}{M}. \tag{365}$$

Buying a fade-in put option provides protection against falling EUR and allows full participation in a rising EUR. The holder has to pay a premium for this protection. He will typically exercise the option only if at maturity the spot is below the strike. The seller of the option receives the premium but is exposed to market movements and would need to hedge his exposure accordingly.

Advantages

- Protection against weaker EUR/stronger USD
- Premium not as high as for a plain vanilla put option
- Full participation in a favorable spot movement

Disadvantages

- Selling amount dependent on market movements between inception and maturity
- No guaranteed worst case exchange rate for the full notional

Example for the Computation of the Notional We explain this product with a EUR Put-USD Call with strike K, which has two ranges and six fixings, in Figure 1.62.

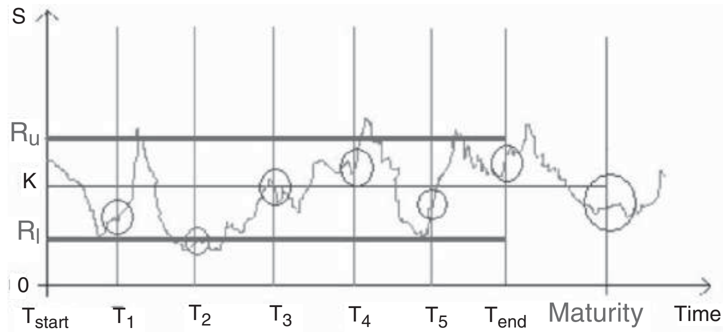


FIGURE 1.62 Notional of a fade-in put. At T_{end}, the holder would be entitled to sell $\frac{5}{6} \cdot 1$ M EUR, where 5 is the number of fixings between the lower and the upper level R_l and R_u on a resurrecting basis (here $n = 5$ because at T_2 the spot fixing is below the lower level). The total number of fixings inside the range will be known only at T_{end}. Hence, the notional of the put will be known only at T_{end}.

At maturity, the fade-in put works like a vanilla put. The holder would typically exercise the option and sell $\frac{5}{6} \cdot 1$ M EUR at the strike K if the spot is below the strike. If the spot ends up above the strike, the holder would let the option expire. For an investor client type, the maximum potential loss of the buyer is the fader's initial premium. For a treasurer client type, it can be worse in case he has used the fade-in put to protect the full notional of 1 M EUR. His worst case is that all fixings are outside the range. In this case he will have lost the initial premium and in the extreme worst case will receive nothing for 1 M EUR, assuming a limiting worst case of the EUR-USD spot going to infinity.

Example A company wants to hedge receivables from an export transaction in EUR due in 12 months' time. It expects a weaker EUR/stronger USD. The company wishes to be able to sell EUR at a higher spot rate if the EUR becomes stronger on the one hand, but on the other hand be protected against a weaker EUR. The company finds the corresponding vanilla EUR put/USD call too expensive and is prepared to take more risk. The treasurer believes that EUR/USD will not trade outside the range 1.1000–1.2000 for a significantly long time.

In this case a possible form of protection that the company can use is to buy a EUR fade-in put option, as presented in Table 1.34 for example.

- If the EUR-USD exchange rate is below the strike at maturity, then the company can sell EUR at maturity at the strike of 1.1600.
- If the EUR-USD exchange rate is above the strike at maturity, the company would let the option expire. However, the company will benefit from a higher spot when selling EUR.

The biggest risk is that all EUR-USD fixings are outside the range and the spot at maturity is low. In this case the company would need to sell EUR at the prevailing low market

TABLE 1.34 Example of a fade-in put. In comparison the corresponding vanilla put costs 50,000.00 EUR.

Spot reference	1.1500 EUR-USD	Strike	1.1600 EUR-USD
Company buys	EUR put USD call	Lower level	1.0000 EUR-USD
Fixing schedule	Monthly	Upper level	1.2000 EUR-USD
Maturity	1 year	Premium	EUR 6,000.00
Notional amount	EUR 1,000,000	Vanilla premium	EUR 50,000.00

TABLE 1.35 Example of a fade-in forward.

Spot reference	1.1500 EUR-USD	Strike	1.0000 EUR-USD
Company buys	EUR-USD forward	Lower level	1.0000 EUR-USD
Fixing schedule	Monthly	Upper level	1.1800 EUR-USD
Maturity	1 year	Premium	EUR 9,000.00
Notional amount	EUR 1,000,000	Fade-in call premium	EUR 27,000.00

spot price. Therefore, the company should have a risk policy in place that triggers an action as soon as EUR-USD drops below the lower level. Such an action could be to trade a forward contract for the rest of the unprotected notional to prevent further losses. The company should be aware that this is not a buy-and-hold strategy.

Variations Besides puts, there are fade-in calls or fade-in forwards, see Table 1.35 or the live trade in Table 2.3 in Section 2.1.4. Also more exotic types of faders can be created by taking exotic transactions and let them fade in or out.

Faders often have an additional knock-out range just like corridors, see Section 1.8.2. One then classifies faders into *resurrecting*, *non-resurrecting, keeping the accrued amount*, and *non-resurrecting losing parts or all of the accrued amount*. Faders are most popularly applied in structuring *accumulative forwards*, see Section 2.1.11.

1.8.4 Exotic Barrier Options

Digital Barrier Options Just like barrier options, which are calls or puts with knock-out or knock-in barriers, one can consider digital calls and puts with additional American style knock-out or knock-in barriers. Knowing the digitals, we can derive the knock-in digitals from the knock-out digitals. The knock-out digitals can be viewed as the limiting case of tight knock-out call spread with very high notionals and can to some extent be approximated by those – with the usual practical limitations. It is clear from this approximation that Greeks are expected to take extreme values and change their signs in different regions of spot and time. The motivation for such products is to reduce the cost in taking a view in a market event.

Window Barriers Barriers need not be active for the entire lifetime of an option. Window barrier options are extensions of (digital) call or put options with barriers where the

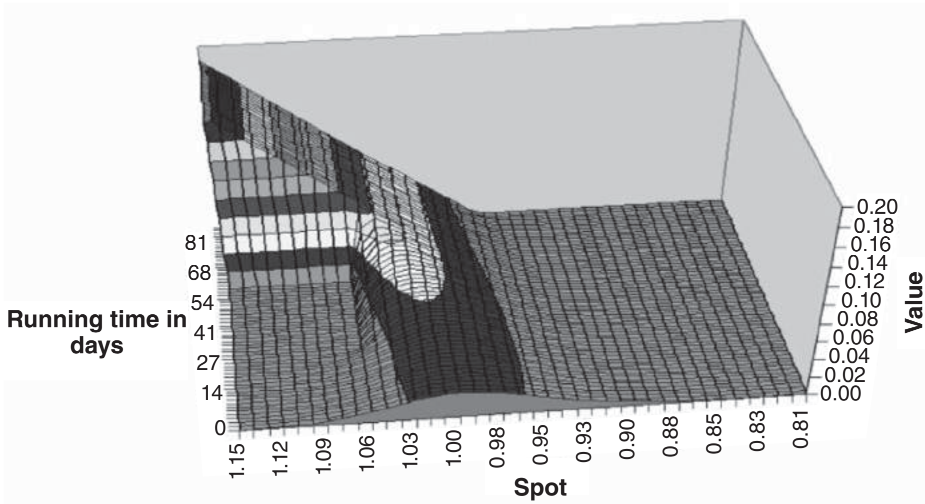


FIGURE 1.63 Value function $v(t, x)$ of an up-and-out call option with window barrier active only for the second month, with strike $K = 0.9628$, knock-out barrier $B = 1.0590$, and maturity 3 months. We used the interest rates $r_d = 6.68\%$, $r_f = 5.14\%$, volatility $\sigma = 11.6\%$, and $R = 0$.

barriers are active during a period of time which is shorter than the whole lifetime of the option – for example only the first three months from an option with six months' maturity. One can specify arbitrary time ranges with piecewise constant barrier levels or even non-constant barriers. Window barrier options are the most exotic options that still fit and can be confirmed with the definitions of the *ISDA 2005 Barrier Option Supplement* [78]. See Figure 1.63 for the value function of a window barrier option. Linear and exponential barriers are useful if there is a high drift in the exchange rate caused, for example by a high interest rate differential (high swap points).

Step and Soft Barriers In case of a knock-out event, a client might argue: "Come on, the spot only crossed the barrier for a very short moment, can't you make an exception and not let my option knock out?" This is a very common concern: how to get protection against price spikes. Such a protection is certainly possible, but surely has its price. One way is to measure the time the spot spends opposite the knock-out barrier and let the option knock out gradually. For instance, one could agree that the option's nominal decreases by 10% for each day the exchange rate fixing is opposite the barrier. Barriers can be constant, linear, or exponential functions of time. Continuously observed barrier contracts of this type are referred to as *occupation time derivatives*. There are even closed form solutions for the value of occupation time derivatives in the Black-Scholes model and some jump-diffusion models. While the occupation time of a (geometric) Brownian motion at a specific level is well understood and defined, it would be difficult, if not impossible, to specify a continuous occupation time of a traded spot. Consequently, occupation time derivatives do not trade. The discrete version of these are called faders and are explained in Section 1.8.3.

Fluffy Barriers Protection against price spikes can be achieved by having a spot spend time *sufficiently long* beyond the barrier. Another way to define a knock-out event is based on the spot going *sufficiently far* beyond the barrier. This feature can be structured by a fluffy barrier contract, where a payoff is kept constant, but the notional to which the payoff is applied depends on how far the spot goes beyond a pre-specified barrier. For instance, with a knock-out on the upside, one can specify a first and second barrier level and let the notional be proportional to the ratio of the difference of level 2 and the maximum spot and the difference of level 2 and level 1. If the spot goes beyond level 1 and also reaches level 2, then the total notional and the entire contract terminate worthless. If the spot does not hit level 1, then there is no knock-out. If the spot goes through level 1 and then to the middle of level 1 and level 2, then 50% of the notional of the contract is gone. The more common version is one with a discrete gradual knock-out: for instance one can specify a barrier range of 2.20 to 2.30 where the option loses 25% of its nominal when 2.20 is breached, 50% when 2.25 is breached, 75% when 2.275 is breached, and 100% when 2.30 is breached.

Parisian and Parasian Barriers Another way to get price spike protection is to let the option knock out only if the spot spends a certain pre-specified length of time opposite the barrier – either in total (Parasian) or in a row (Parisian). Clearly the plain barrier option is the least expensive, followed by the Parasian, then the Parisian barrier option, and finally the corresponding vanilla contract. See Figure 1.64. The name Parisian probably originates from Société Générale trading such contracts in Paris. However, there are other theories. Valuation is typically done by Monte Carlo simulation, although there are PDE methods available. The Parasian knock-out event is equivalent to a *counter*-based early termination in a target forward, see Section 2.2.3.

Resettable Barriers This is a way to give the holder of a barrier option a chance to reset the barrier during the life of the option n times at pre-specified N decision times in the future ($N \geq n$). This kind of extra protection also makes the barrier option more

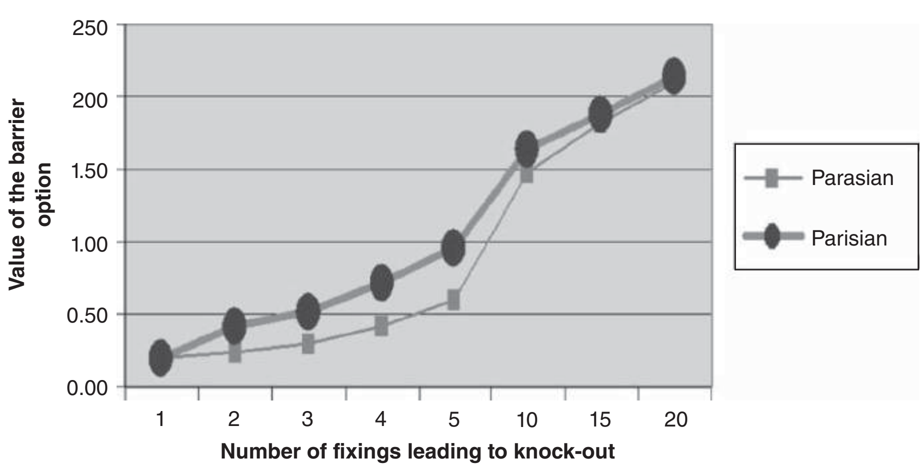


FIGURE 1.64 Comparison of Parisian and Parasian barrier option values.

expensive. Obviously, this contract requires many specifications, in particular to which new barrier level a barrier can be reset. A common way to do it is to reset the barrier to x% beyond the current spot fixing, but there can be more complex reset functions or even rights of choice for the holder. Resettable barriers occur mostly in complex retail certificates, but are also used as building blocks for corporate hedging strategies.

Transatlantic Barrier Options For transatlantic barrier options one barrier is of American style, the other one of European style. Naturally, the European style barrier is in-the-money, the American style barrier usually out-of-the-money. Therefore, there are essentially two versions,

1. a call with strike K, a European style up-and-out $H > K$, and an American style down-and-out at $L \leq K$,
2. a put with strike K, a European style down-and-out $L < K$, and an American style up-and-out $H \geq K$.

The motivation for such products is of course the lower cost in comparison with vanilla or single barrier options on the one hand and the fear of price spikes and a resulting preference for European style barriers on the other hand.

The pricing and hedging is comparatively easy provided we have regular and digital barrier options available as basic products. Then we can structure the transatlantic barrier option just like in Equation (23), with an additional out-of-the-money knock-out barrier.

Knock-In-Knock-Out Options Knock-In-Knock-Out Options (KIKOs) are barriers with both a knock-out and a knock-in barrier. However, it is not as simple, because there are three fundamentally different types:

1. The knock-out can happen *any time*.
2. The knock-out can happen only *after* the knock-in.
3. The knock-out can happen only *before* the knock-in.

The first one is the market standard, but when dealing one should always clarify which type of knock-in-knock-out is agreed upon. For example, let the lower barrier L be a knock-out barrier and the upper barrier H be a knock-out barrier. Standard type 1 KIKO can be exercised only if L is never touched *and* H has been touched at least once. This can be replicated by standard barrier options via

$$\mathrm{KIKO}(L, H) = \mathrm{KO}(L) - \mathrm{DKO}(L, H). \tag{366}$$

Therefore, pricing and hedging of this KIKO is no more complicated than pricing and hedging of knock-out options.

The second type is a special case of a *knock-in on strategy* contract. Any structure can be equipped with a *global* knock-in barrier, that has to be touched before the structure becomes alive. Knock-out events in the structure are active only *after* the structure knocks in. This is a product of its own and requires an individual valuation, pricing, and hedging approach.

In the third type of KIKO a knock-out can happen only before the knock-in. Once the option is knocked in, the knock-out barrier is no longer active. This is also a product of its own and requires an individual valuation, pricing, and hedging approach.

James Bond Range As James Bond can only live twice, the *James Bond range* is a double-no-touch type contract. Given an upper barrier H and a lower barrier L, it pays one unit of currency if the spot remains inside (L, H) at all times until expiry T, or if the spot hits L the spot thereafter remains in a new range to be set around L, or similarly if the spot hits H the spot thereafter remains in a new range to be set around H. The contract is also called *tolerant double-no-touch*.

1.8.5 Pay-Later Options

A pay-later option is a vanilla option whose premium is paid only if the option is in-the-money or is exercised at the expiration time. If the spot is not in-the-money, the holder of the option would normally not exercise the option, and will end up not having paid anything. However, if the spot is in-the-money, the holder of the option has to pay the option premium, which will then be noticeably higher than the plain vanilla. For this reason pay-later options are not traded very often. Note that the payment of the premium is conditional. The pay-later option is not to be confused with a vanilla option whose premium is (unconditionally) deferred to its maturity date.

Advantages

- Full protection against spot market movement
- Premium is paid only if the option ends up in-the-money
- Premium is paid only at maturity

Disadvantages

- More expensive than a plain vanilla
- Credit risk for the seller as payoff can be negative

The Valuation for the Pay-Later Option The payoff of a pay-later option is defined as

$$\left[\phi(S_T - K) - P\right] I\!I_{\{\phi S_T \geq \phi K\}} \tag{367}$$

and illustrated in Figure 1.65. As usual, the binary variable ϕ takes the value $+1$ for a call and -1 for a put, K the strike in units of the domestic currency, and T the expiration time in years. The *price* P of the pay-later option is paid at time T, but it is set at time zero in such a way that the time zero *value* of the above payoff is zero. Carefully notice the difference between price and value. After the option is written, the price P does not change any more.

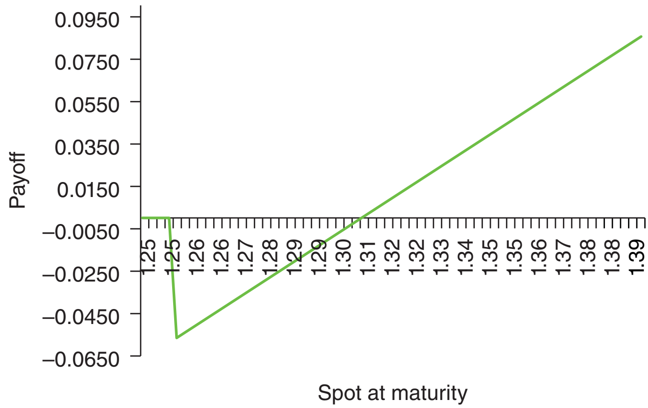


FIGURE 1.65 Payoff of a pay-later EUR call USD put. We use the market input spot $S_0 = 1.2000$, volatility $\sigma = 10\%$, EUR rate $r_f = 2\%$, USD rate $r_d = 2.5\%$, strike $K = 1.2500$, time to maturity $T = 0.5$ years. The vanilla value is 0.0158 USD, the digital value is 0.2781 USD, the resulting pay-later price is 0.0569 USD, which is substantially higher than the plain vanilla value. Consequently the break-even point is at 1.3075, which is quite far off. For this reason pay-later type structures do not trade very often.

We denote the current spot by x and the current time by t and define furthermore the abbreviations

$$n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}, \tag{368}$$

$$\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt, \tag{369}$$

$$\tau \stackrel{\Delta}{=} T - t, \tag{370}$$

$$f = xe^{(r_d - r_f)\tau}, \tag{371}$$

$$d_{\pm} \stackrel{\Delta}{=} \frac{\log \frac{f}{K} \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}, \tag{372}$$

$$\text{vanilla}(x, K, T, t, \sigma, r_d, r_f, \phi) = \phi e^{-r_d\tau}[f\mathcal{N}(\phi d_+) - K\mathcal{N}(\phi d_-)], \tag{373}$$

$$\text{digital}(x, K, T, t, \sigma, r_d, r_f, \phi) = e^{-r_d\tau}\mathcal{N}(\phi d_-). \tag{374}$$

The formulas of vanilla and digital options have been derived in Sections 1.4 and Section 1.7.2 respectively.

The payoff can be rewritten as

$$\left[\phi(S_T - K)\right]^+ - P\mathbb{I}_{\{\phi S_T \geq \phi K\}}, \tag{375}$$

whence the value of the pay-later option in the Black-Scholes model

$$dS_t = S_t \left[(r_d - r_f)dt + \sigma \, dW_t\right] \tag{376}$$

is easily read as

$$\begin{aligned} \text{paylater}(x, K, P, T, t, \sigma, r_d, r_f, \phi) = \text{vanilla}(x, K, T, t, \sigma, r_d, r_f, \phi) \\ - P \cdot \text{digital}(x, K, T, t, \sigma, r_d, r_f, \phi). \end{aligned} \tag{377}$$

In particular, this leads to a quick implementation of the value and all the Greeks having the functions vanilla and digital at hand. This relationship between the pay-later option, the vanilla, and the digital is generic, and not model dependent.

For the *pay-later price* setting

$$\text{paylater}(x, K, P, T, 0, \sigma, r_d, r_f, \phi) = 0 \tag{378}$$

yields

$$P = \frac{\text{vanilla}(x, K, T, 0, \sigma, r_d, r_f, \phi)}{\text{digital}(x, K, T, 0, \sigma, r_d, r_f, \phi)} \tag{379}$$

$$= \text{vanilla}(x, K, T, 0, \sigma, r_d, r_f, \phi)\frac{e^{r_d T}}{\mathcal{N}(\phi d_-)}. \tag{380}$$

This can be interpreted as follows. The value P is like the value of a vanilla option, except that

- we must pay interest $e^{r_d T}$, since the premium is due only at time T and
- the premium needs to be paid only if the option is in-the-money or is exercised, which is why we divide by the (risk-neutral) probability that the option is exercised $\mathcal{N}(\phi d_-)$.

We observe that the pay-later option can be viewed as a *structured product*. All we need are vanilla and digital options. The structurer will easily replicate a short pay-later with a long vanilla and a short digital. We learn that several types of options can be composed from existing ones, which is the actual job of structuring. This way it is also straightforward to determine a market price, given a vanilla market.

Variations Pay-later options are an example of the family of contingent or deferred payment options. We can also simply defer the payment of a vanilla without any conditions on the moneyness. Another variation is paying back the vanilla premium if the spot stays inside some range, see the exercises in Section 2.1.24. Naturally, the pay-later effect can be extended beyond vanilla options to all kind of options.

1.8.6 Step Up and Step Down Options

The step option is an option where the strike of the option is readjusted at predefined fixing dates, but only if the spot is more favorable than that of the previous fixing date. The step option can either be a plain vanilla option or a single barrier option. The concept of a progressive *step up* or *step down* could be changed also to a progressive *step up* or *step down* for a forward rate.

1.8.7 Options and Forwards on the Harmonic Average

Consider a schedule of observation times $t_1, \ldots, t_n$ of some underlying. Options and forwards on the arithmetic average

$$\frac{1}{n}\sum_{i=1}^{n} S(t_i) \tag{381}$$

have been analyzed and traded for some time, see Section 1.7.6. The geometric average

$$\sqrt[n]{\prod_{i=1}^{n} S(t_i)} \tag{382}$$

has often been used as control variate for pricing the arithmetic average, whose distribution in a multiplicative model like Black-Scholes is cumbersome to deal with. The *harmonic average*

$$\frac{n}{\sum_{i=1}^{n} \frac{1}{R(t_i)}} \tag{383}$$

comes up if a client wants to exchange an amount of *domestic* currency into the *foreign* currency at an average rate of the currency pair FOR-DOM, e.g. wants to exchange USD into EUR at a rate which is an average of observed EUR-USD rates. In this case USD is the domestic currency and we need to actually look at the exchange rate of $R = 1/S$ in DOM-FOR quotation in order to allow the domestic currency as a notional amount. As in the case of standard Asian contracts, there can be forwards and options on the harmonic average, both with fixed and floating strike. We treat one possible example in the next section. Options and forwards on the harmonic average are also sometimes referred to as *Australian* derivatives, possibly because Australia is on the other side of the equator. The fact that a harmonic average is used is often not explicitly shown in the systems. A domestic notional amount and cash settlement in the foreign currency are strong indications that harmonic averaging is performed behind the curtains. The valuation of harmonic average contracts can be related to arithmetic average contracts by a clever change of numeraire, see Večeř [134].

Harmonic Asian Swap We consider a EUR-USD market with spot reference 1.0070, swap points for time T_1 of −45, swap points for time $T_2 > T_1$ of −90. As a contract specification, the client buys N USD at the daily average of the period of one month before T_1, denoted by A_1. Then the client sells N USD at the daily average of a period of one month before T_2, denoted by A_2. The payoff in EUR of this structure (cash settled two business days after T_2) is

$$\frac{N}{A_2} - \frac{N}{A_1}. \tag{384}$$

To replicate this using the fixed strike Asian forward we can decompose it as follows:

1. We sell to the client the payoff $1 - \frac{1}{A_1}$ (using strike 1 by default) with notional N.
2. We buy from the client the payoff $1 - \frac{1}{A_2}$ (using strike 1 by default) with notional N.

On a notional of $N = 5$ M USD this could have a theoretical value of 23,172 EUR. This is what the sell-side should charge the client in addition to overhedge and sales margin. One problem is that the structure is very transparent for the client. If we take the forward for mid February, we have −45 swap points, for mid June −90 swap points. This means that the client would know that in a first order approximation he owes the bank 45 swap points, which is

$$5 \text{ M USD} \cdot 0.0045 = 22{,}500 \text{ EUR}.$$

If the swap ticket requires entering a strike, one can use 1.0000 in both tickets, but this value does not influence the value of the swap.

1.8.8 Variance and Volatility Swaps

A variance swap is a contract that pays the difference of a pre-determined fixed variance (squared volatility), which is usually determined in such a way that the trading price is zero, and a realized historic annualized variance, which can be computed only at maturity of the trade. Therefore, the variance swap is an ideal instrument to hedge volatility exposure, a need for funds and institutional clients. Of course, one can hedge vega with vanilla options or straddles, but this is then also subject to spot movements and time decay of the hedge instruments. The variance swap also serves as a tool to take a view on volatility. Variance and volatility swaps were standardized by ISDA in 2013 [79].

Advantages

- Insurance against changing volatility levels
- Independence of spot
- Zero-cost product
- Fixed volatility (break-even point) easy to approximate as average of smile

Disadvantages

- Difficult to understand
- Many details in the contract to be set
- Variance harder to capture than volatility
- Volatility swaps harder to price than variance swaps

Example Suppose the one-month implied volatility for EUR/USD at-the-money options is close to its one-year historic low. This can easily be noticed by looking at *volatility cones*, see Section 1.5.10. Suppose further that you are expecting a period of higher volatility during the next month. You are looking for a zero-cost strategy, where you would profit from a rising volatility, but you are ready to encounter a loss otherwise. In this case a suitable strategy to trade is a variance or volatility swap. We consider an example of a variance swap in Table 1.36.

To make this clear we consider the following two scenarios with possible fixing results listed in Table 1.37 and Figure 1.66.

- If the realized variance is 0.41% (corresponding to a volatility of 6.42%), then the market was quieter than expected and you need to pay 10 M USD · (0.85% − 0.41%) = 44,000 USD.
- If the realized variance is 1.15% (corresponding to a volatility of 10.7%), then your market expectation turned out to be correct and you will receive 10 M USD · (1.15% − 0.85%) = 30,000 USD.

A volatility swap trades

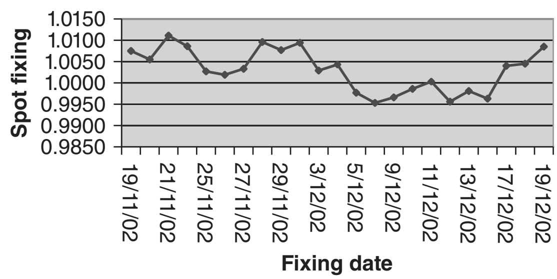
$$\sqrt{\frac{B}{N-1}\sum_{i=1}^{N}(r_i-\bar{r})^2} \tag{385}$$

TABLE 1.36 Example of a variance swap in EUR-USD. The quantity r_i is called the log-return from fixing day $i-1$ to day i and the average log-return is denoted by $\bar{r}$. The notation %% means a multiplication with 0.0001. It is also sometimes denoted as $\%^2$.

Spot reference	1.0075 EUR-USD
Notional M	USD 10,000,000
Start date	19 November 2002
Expiry date	19 December 2002
Cash settlement	23 December 2002
Fixing period	Every weekday from 19-Nov-02 to 19-Dec-02
Fixing source	ECB fixings $F_0, F_1, \dots, F_N$
Number of fixing days N	23 (32 actual days)
Annualization factor B	262.3 = 23/32·365
Fixed strike K	85.00%% corresponding to a volatility of 9.22%
Payoff	$M \cdot$ (realized variance − K)
Realized variance	$\frac{B}{N-1}\sum_{i=1}^{N}(r_i-\bar{r})^2; \bar{r}=\frac{1}{N}\sum_{i=1}^{N} r_i; r_i=\ln\frac{F_i}{F_{i-1}}$
Premium	none

TABLE 1.37 Example of two variance scenarios in EUR-USD. The left column shows a possible fixing set with a lower realized variance, the right column a scenario with a higher variance.

Date	Fixing (low vol)	Fixing (high vol)	Date	Fixing (low vol)	Fixing (high vol)
19/11/02	1.0075	1.0075	6/12/02	0.9953	1.0037
20/11/02	1.0055	1.0055	9/12/02	0.9966	0.9962
21/11/02	1.0111	1.0111	10/12/02	0.9986	0.9986
22/11/02	1.0086	1.0086	11/12/02	1.0003	0.9907
25/11/02	1.0027	1.0027	12/12/02	0.9956	1.0018
26/11/02	1.0019	1.0067	13/12/02	0.9981	1.0000
27/11/02	1.0033	0.9997	16/12/02	0.9963	0.9963
28/11/02	1.0096	1.0113	17/12/02	1.0040	1.0040
29/11/02	1.0077	1.0062	18/12/02	1.0045	1.0017
2/12/02	1.0094	1.0094	19/12/02	1.0085	1.0114
3/12/02	1.0029	0.9999	variance	0.41%	1.15%
4/12/02	1.0043	1.0043	volatility	6.42%	10.70%
5/12/02	0.9977	0.9977			



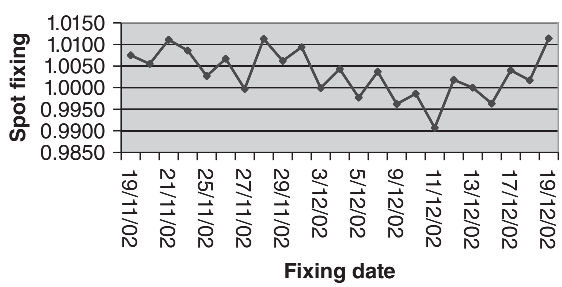


FIGURE 1.66 Comparison of scenarios for a low variance (graph above) and a higher variance (graph below).

against a fixed volatility, which is usually determined in such a way that the trading price is zero. Since the square root is not a linear function of the variance, this product is more difficult to price than a standard variance swap. For details on pricing and hedging we refer to Carr and Lee [23], as well as the paper on *more than you ever wanted to know about volatility swaps* [37]. As a rule of thumb, the fixed variance or volatility to make the contract worth zero is the average of the volatilities in the volatility smile matrix for the maturity under consideration as for the variance swap there exists a static hedging portfolio consisting of vanilla options with the same maturity. In principle, the replicating portfolio of a variance swap consists of a portfolio of vanilla options approximating the payoff $\ln S_T$. This requires options with arbitrary far away strikes. Therefore, the approximation is quite sensitive to implied volatilities for very small and very big deltas. The variance swap price can be viewed as an indicator of how an FX options desk handles the extrapolation of the volatility smile on the far wings. Aggressive market makers tend to ignore the extreme strikes. In fact, one may ask how a variance swap differs from its replication. Options in the replication with extremely far away strikes would be required if there is a jump in the underlying exchange rate. The correctly priced variance swap would quantify the jump risk, whereas the replication for all practical matters would not, simply because options with extremely far away strikes are not tradable, or market prices are not available. Note that in FX markets, other than in equity markets, variance and volatility swaps normally do not have caps.

Forward Variance Swap In a standard variance swap, the first spot fixing is at inception of the trade or two business days thereafter. However, there may be situations where a client needs to hedge a forward volatility exposure that originates from a compound, installment, forward start, cliquet, or other exotic option with a significant forward volatility dependence. We will illustrate now how to structure a forward variance swap, where the first fixing is at some time in the future, using standard variance swaps. Let there be J fixings in the initial period and M fixings in the second period. The total number of fixings is hence $M + J$. We can then split the payoff

$$\frac{B}{M-1} \sum_{i=J+1}^{J+M} (r_i - \bar{r})^2 - K \tag{386}$$

into the two parts

$$\begin{aligned} &\frac{B}{M-1} \sum_{i=1}^{J+M} (r_i - \bar{r})^2 - K - \left[\frac{B}{M-1} \sum_{i=1}^{J} (r_i - \bar{r})^2 - 0 \right] \\ &= \frac{C}{J+M-1} \sum_{i=1}^{J+M} (r_i - \bar{r})^2 - K - \left[\frac{D}{J-1} \sum_{i=1}^{J} (r_i - \bar{r})^2 - 0 \right] \end{aligned} \tag{387}$$

and find as the only solution for the numbers C and B

$$\begin{aligned} C &= \frac{(J+M-1)B}{M-1}, \\ D &= \frac{(J-1)B}{M-1}. \end{aligned} \tag{388}$$

Modifications When computing the variance of a random variable X whose mean is small, we can take the second moment $\mathbb{E}X^2$ as an approximation of the variance

$$\mathbf{var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2. \tag{389}$$

Following this idea and keeping in mind that the average of log-returns of FX fixings is indeed often close to zero, the variance swap is sometimes understood as a second moment swap rather than an actual variance swap. To clarify, traders specify in their dialogue whether the product is *mean subtracted* or not. We have presented here the variance swap with the mean subtracted.

1.8.9 Forward Volatility Agreements (FVAs)

Besides variance and volatility swaps, the forward volatility agreement is another instrument to trade volatility. In this case the forward volatility is the focus. We have already seen in Section 1.7.8 that the value of a forward start option essentially depends on the forward volatility σ_f, the one that applies between fixing time T_f in the future and the maturity time T_e. Equation (152) establishes the relationship between the forward volatility and the spot volatilities σ_{T_f} and σ_{T_e}. In a forward volatility agreement party A agrees to buy a strategy or a single option from party B. The strategy will start on the future fixing time T_f. However, the premium to pay for the strategy on the fixing value date T_{fd} is calculated with a fixed forward volatility σ_f. The prevailing spot volatility on the fixing date will most likely be different. Party A locks in the volatility using the forward volatility agreement.

Typical strategies to trade are ATM call and put vanilla options, but the most common is the ATM straddle (see Section 1.6.3). The reason to trade ATM strategies is to focus on the ATM volatility and its view on the term structure, while keeping smile effects aside. As ATM volatility, the usual FX conventions apply: delta-neutral straddle strikes $K_\pm$ as in Equation (43). Forward volatility agreements typically trade at the *par volatility* σ_f as in Equation (152). Essentially party A pays a par-vol premium for a future market spot vol product. Therefore, if the future market spot volatility turns out to be higher than the par volatility at inception, party A makes a profit; conversely, if market spot volatility turns out to be lower, then party A makes a loss. This transaction makes the forward volatility indirectly tradable.

Traded FVA Example We consider the terms of a traded forward volatility agreement in Table 1.38. Essentially, one can view a forward volatility agreement as a re-branding of the forward start option or combination of forward starting options as a straddle. In fact, it is common that forward volatility agreement is confirmed as a *forward setting currency option transaction* or *forward start straddle*. The difference is that usually, in a forward start option, the premium is paid at the horizon spot date, whereas in a forward volatility agreement the premium is paid at the fixing spot date. Generally, forward start options allow more flexibility in setting the strike different from ATM, whereas forward volatility agreements typically work with the ATM strike. Another difference is that in a forward volatility agreement, the premium will be calculated with the prevailing market data (spot, interest, and forward rates) and the pre-specified fixed volatility, whereas in a forward start option, the current forward interest rates are used.

TABLE 1.38 Example of a traded forward volatility agreement in EUR/GBP.

Trade date	29 November 2007
Strategy	European EUR Call/GBP Put and EUR Put/GBP Call
Notional amount	EUR 365,000,000.00
Business days	London & Any TARGET Settlement day
Business day Convention	Following
Expiration date	26 November 2008
Expiration time	10.00 a.m. (local time in New York)
Settlement date	28 November 2008
Strike price	delta neutral with fixed volatility of 7.30%, the prevailing EUR/GBP Spot Rate, Forward and Deposit Rate at the Fixing Time on the Fixing Date, as determined by the Calculation Agent in its sole discretion
Delta neutral straddle	the Strike Price where the Premium Currency delta of the Call and the Premium Currency delta of the Put are equal and opposite and thus sum to zero
Fixing time	10.00 a.m. (local time in New York)
Fixing date	28 May 2008
Fixing rate	Spot Rate at the Fixing Time on the Fixing Date
Premium	to be determined by using 7.30% Volatility
Premium payment date	30 May 2008

GBP/USD Forward Volatility Agreement Example We consider an example in the version of an ATM straddle. Market data (possibly on trade date T_h 7 April 2016): spot $S_0 = 1.4000$, spot date T_s 7 April 2016, 6 M ATM volatility $\sigma_{T_f} = 13.736\%$, 6 M GBP money market $r_f = 0.482\%$, 6 M USD money market $r_d = 0.667\%$, 6 M GBP-USD forward rate $F_{T_f} = 1.40136$, 6 M RR -4.377% (favoring GBP puts), 6 M BF 0.395%, 12 M ATM volatility $\sigma_{T_e} = 12.866\%$, 12 M GBP money market $r_f = 0.439\%$, 12 M USD money market $r_d = 0.731\%$, 12 M GBP-USD forward rate $F_{T_f} = 1.40421$, 12 M RR -4.055% (favoring GBP puts), 12 M BF 0.435%. Contract data: expiry T_e 5 April 2017, delivery T_d 7 April 2017, fixing data T_f 5 Oct 2016 (183 days), fixing spot date T_{fd} 7 Oct 2016 (365 days), notional GBP 1 M. The par volatility is

$$\sigma_f = \sqrt{\frac{\sigma_{T_e}^2(T_e - T_s) - \sigma_{T_f}^2(T_f - T_s)}{T_e - T_f}} = \sqrt{\frac{12.866\%^2 365 - 13.736\%^2 183}{182}} = 11.93\%. \tag{390}$$

A bid and offer par volatility could be 11.25%–12.60%, assuming a bid offer spread of 1.30%. The working spot volatility spreads are 0.533% for 6 M and 0.417% for 12 M. In this case the buyer would pay a premium calculated for a straddle with strike K_+ and the prevailing money market and spot rates at the fixing date and a volatility of 12.60%. The exact premium is not known at the trade date. The buyer then holds a straddle. With a fixed volatility of 8% (instead of the par volatility), the initial bid and offer price would be GBP 18,400–26,000. Obviously, the FVA has no delta or gamma before the fixing date. Initial vega would be GBP 5,638.

1.8.10 Exercises

European Style Corridor Starting with the value for digital options, derive exactly the value of a European style corridor in the Black-Scholes model. Discuss how to find a market price based on the market of vanilla options.

Fade-Out Call How would you structure a *fade-out call* that starts with a nominal amount of M? As the exchange rate evolves, the notional will be decreased by $\frac{M}{N}$ for each of the N fixings that is outside a pre-defined range.

Fader Payoff As for the corridors in Section 1.8.2, write down the exact payoff formulas for the various variations of faders in Section 1.8.3.

Fade-In Forward Describe a possible client view that could lead to trading a fade-in forward in Table 1.35.

Static Replication of the Tolerant Double-No-Touch As a variation of the James Bond range in Section 1.8.4, we consider barriers A, B, C, D as illustrated in Figure 1.67.

A rather *tolerant double-no-touch* knocks out after the second barrier is touched or crossed. How would you replicate it statically using standard barrier and touch contracts?

Pay-Later with Premium in Foreign Currency The pay-later value in Equation (379) is measured in units of domestic currency. Does this change if the premium is specified to be paid in foreign currency? If no, argue why. If yes, specify how.

Pay-Later Digital Derive the pay-later value of a digital option.

Pay-Later Call Spread Derive the pay-later value of a call spread.

Pay-Later Up-and-Out Call How would you structure an up-and-out call whose premium is paid only if the spot is in-the-money at the expiration time and if it has not knocked out?

Chooser Option A *chooser option* lets the buyer decide at expiration time whether he wants to exercise a call with strike K or a put with strike K. Discuss how to find a market price and how to statically hedge it. (Hint: straddle.) Moreover, if the decision as to which of the options to take is made at time t strictly before the expiration time T, how would you price and hedge the chooser?

FIGURE 1.67 Nested double-no-touch ranges.

1.9 SECOND GENERATION EXOTICS (MULTIPLE CURRENCY PAIRS)

There are also a number of derivatives involving multiple currency pairs. They are referred to as *multi-currency* or *rainbow* derivatives. Some serve as FX hedging instruments for corporates, others occur as hedging instruments for institutions and hedge funds, and some in private banking.

1.9.1 Spread and Exchange Options

A spread option compensates a spread in exchange rates and pays off

$$\left[\phi\left(aS_T^{(1)} - bS_T^{(2)} - K\right)\right]^+. \tag{391}$$

This is a European spread put ($\phi = -1$) or call ($\phi = +1$) with strike $K > 0$ and the expiration time in years T. We assume without loss of generality that the weights a and b are positive. These weights are needed to make the two exchange rates comparable, as USD-CHF and USD-JPY differ by a factor of the size of 100. A standard for the weights are the reciprocals of the initial spot rates, i.e. $a = \frac{1}{S_0^{(1)}}$ and $b = \frac{1}{S_0^{(2)}}$.

Spread options are not traded very often in FX markets. If they are they are usually cash-settled. Exchange options come up more often as they entitle the owner to exchange one currency for another, which is very similar to a vanilla option, which is reflected in the valuation formula.

In the two-dimensional Black-Scholes model

$$dS_t^{(1)} = S_t^{(1)}\left[\mu_1 dt + \sigma_1 dW_t^{(1)}\right], \tag{392}$$

$$dS_t^{(2)} = S_t^{(2)}\left[\mu_2 dt + \sigma_2 dW_t^{(2)}\right], \tag{393}$$

$$\mathbf{Cov}\left[W_t^{(1)}, W_t^{(2)}\right] = \rho t, \tag{394}$$

with positive constants σ_i denoting the annual volatilities of the i-th foreign currency, ρ the instantaneous correlation of their log-returns, r the domestic risk-free rate, and risk-neutral drift terms

$$\mu_i = r - r_i, \tag{395}$$

where r_i denotes the risk-free rate of the i-th foreign currency, the value is given by (see [104])

$$\text{spread} = \int_{-\infty}^{+\infty} \text{vanilla}\left(S(x), K(x), \sigma_1\sqrt{1-\rho^2}, r, r_1, T, \phi\right) n(x)\, dx \tag{396}$$

$$S(x) \stackrel{\Delta}{=} aS_0^{(1)} e^{\rho\sigma_1\sqrt{T}x - \frac{1}{2}\sigma_1^2\rho^2 T} \tag{397}$$

$$K(x) \stackrel{\Delta}{=} bS_0^{(2)} e^{\sigma_2\sqrt{T}x + \mu_2 T - \frac{1}{2}\sigma_2^2 T} + K. \tag{398}$$

Notes

1. The integration can be done numerically, e.g. using the Gauß-Legendre algorithm with integration limits −5 and 5. The function vanilla (European put and call) can be found in Section 1.4.
2. The integration can be done analytically if $K = 0$. This is the case of *exchange options*, the right to exchange one currency for another.
3. To compute Greeks one may want to use homogeneity relations as discussed in [107].
4. In a foreign exchange setting, the correlation can be computed in terms of known volatilities. This can be found in Section 1.9.2.

Derivation of the Value Function We use Equation (7) for the value of vanilla options along with the abbreviations thereafter.

We rewrite the model in terms of independent new Brownian motions $W^{(1)}$ and $W^{(2)}$ and get

$$S_T^{(1)} = S_0^{(1)} \exp\left[(\mu_1 - \tfrac{1}{2}\sigma_1^2)T + \sigma_1\rho W_T^{(2)} + \sigma_1\sqrt{1-\rho^2}\,W_T^{(1)}\right], \tag{399}$$

$$S_T^{(2)} = S_0^{(2)} \exp\left[(\mu_2 - \tfrac{1}{2}\sigma_2^2)T + \sigma_2 W_T^{(2)}\right]. \tag{400}$$

This allows us to write $S_T^{(1)}$ in terms of $S_T^{(2)}$, i.e.

$$S_T^{(1)} = \exp\left[\hat{\mu}_1 + \frac{\sigma_1\rho}{\sigma_2}(\ln S_T^{(2)} - \hat{\mu}_2) + \sigma_1\sqrt{1-\rho^2}\,W_T^{(1)}\right], \tag{401}$$

$$\hat{\mu}_i \stackrel{\Delta}{=} \ln S_0^{(i)} + (\mu_i - \tfrac{1}{2}\sigma_i^2)T, \tag{402}$$

which shows that given $S_T^{(2)}$, $\ln S_T^{(1)}$ is normally distributed with mean and variance

$$\mu = \hat{\mu}_1 + \frac{\sigma_1\rho}{\sigma_2}(\ln S_T^{(2)} - \hat{\mu}_2), \tag{403}$$

$$\sigma^2 = \sigma_1^2(1-\rho^2)T. \tag{404}$$

We recall from the derivation of the Black-Scholes formula for vanilla options that (and in fact, for $\rho = 0$ this *is* the Black-Scholes formula)

$$\begin{aligned} &I\!E[(\phi(S_T^{(1)} - K))^+] \\ &= \phi\left[e^{\mu+\frac{\sigma^2}{2}}\mathcal{N}\left(\phi\frac{-\ln K + \mu + \sigma^2}{\sigma}\right) - K\mathcal{N}\left(\phi\frac{-\ln K + \mu + \sigma^2}{\sigma}\right)\right], \end{aligned} \tag{405}$$

which allows to compute the value of a spread option as

$$e^{-rT}\mathbb{E}[(\phi(aS_T^{(1)} - bS_T^{(2)} - K))^+] \tag{406}$$

$$= a\mathbb{E}\left[e^{-rT}\mathbb{E}\left[\left(\phi(S_T^{(1)} - (\frac{b}{a}S_T^{(2)} + \frac{K}{a}))\right)^+ \middle| S_T^{(2)}\right]\right] \tag{407}$$

$$= a \cdot \mathbb{E}\left[\text{vanilla}\left(S_0^{(1)} \exp\left\{\frac{\sigma_1\rho}{\sigma_2}(\ln S_T^{(2)} - \hat{\mu}_2) - \frac{1}{2}\sigma_1^2\rho^2 T\right\},\right.\right.$$

$$\left.\left.\frac{b}{a}S_T^{(2)} + \frac{K}{a}, \sigma_1\sqrt{1-\rho^2}, r, r_1, T, \phi\right)\right]$$

$$= \int_{\infty}^{\infty} \text{vanilla}\left(aS_0^{(1)} \exp\left\{\sigma_1\rho\sqrt{T}x - \frac{1}{2}\sigma_1^2\rho^2 T\right\},\right. \tag{408}$$

$$\left. b\exp\{\sigma_2\sqrt{T}x + \hat{\mu}_2\} + K, \sigma_1\sqrt{1-\rho^2}, r, r_1, T, \phi\right) n(x)\, dx$$

$$= \int_{-\infty}^{+\infty} \text{vanilla}\left(S(x), K(x), \sigma_1\sqrt{1-\rho^2}, r, r_1, T, \phi\right) n(x)\, dx. \tag{409}$$

Example We consider the example in Table 1.39. An investor or corporate believes that EUR/USD will outperform GBP/USD in 6 months. To make the exchange rates comparable we first normalize both exchange rates by dividing by their current spot and then want to reward the investor by one pip for each pip the normalized EUR/USD will be more than 20 pips higher than normalized GBP/USD.

1.9.2 Baskets

This section is joint work with Jürgen Hakala and appeared first in [67].

In many cases corporate and institutional currency managers are faced with an exposure in more than one currency. Generally these exposures would be hedged using individual strategies for each currency. These strategies are composed of spot transactions, forwards, and in many cases options on a single currency. Nevertheless, there are instruments that include several currencies, and these can be used to build a multi-currency strategy that is almost always more cost effective than the portfolio

TABLE 1.39 Example of a spread option.

	EUR	GBP	USD rate	3%
Spot in USD	1.2000	1.8000	Correlation	20%
Interest rates	2%	4%	Maturity	0.5 years
Volatility	10%	9%	Strike	0.0020
Weights	1/1.2000	1/1.8000	Value	0.0375 USD

of the individual strategies. As a prominent example we consider now basket options in detail.

Protection with Currency Baskets Basket options are derivatives based on a common base currency, say EUR, and several other risky currencies. The option is actually written on the basket of risky currencies. A basket option in Foreign Exchange markets is a European option granting its holder the right to exercise and upon exercise the holder pays/receives a portfolio of put currency amounts and receives/pays a call currency amount. In case of cash settlement this is economically equivalent to paying the difference between the basket cutoff value and the strike, if positive, for a basket call, or the difference between strike and basket value, if positive, for a basket put respectively at maturity. The risky currencies have different weights in the basket to reflect the details of the exposure.

For example, one can write down the payoff at maturity T of a basket call on two currencies USD and JPY as

$$\max\left(a_1\frac{S_1(T)}{S_1(0)} + a_2\frac{S_2(T)}{S_2(0)} - K, 0\right), \tag{410}$$

where $S_1(t)$ denotes the exchange rate of EUR-USD and $S_2(t)$ denotes the exchange rate of EUR-JPY at time t, a_i the corresponding weights, and K the basket strike. A basket option protects against a drop in both currencies at the same time. Individual options on each currency cover some cases that are not protected by a basket option, which occurs if one exchange rate falls more than the other exchange rate rises. In this case the basket would be out-of-the-money, but one of the individual options would be in-the-money. This is indicated by the shaded triangular areas in Figure 1.68, and that is why the portfolio of individual options would cost more than a basket. However, for the corporate the basket provides sufficient protection against the joint risk. If one exchange rate falls more than the other one rises, the corporate treasurer could buy the cheaper currency in the spot market and would make some extra profit, which may feel good, but this effect is just a potential bonus for which the treasurer would have paid when buying individual options rather than a basket.

Valuation of Basket Options in the Black-Scholes Model Basket options should be priced in a consistent way with plain vanilla options. In the Black-Scholes model we assume a log-normal process for the individual correlated basket constituents. A decomposition into uncorrelated constituents of the exchange rate processes

$$dS_i = \mu_i S_i\, dt + S_i \sum_{j=1}^{N} \Omega_{ij}\, dW_j \tag{411}$$

is the basis for pricing. Here μ_i denotes the difference between the foreign and the domestic interest rate of the i-th currency pair, dW_j the j-th component of independent Brownian increments. The covariance matrix is given by

$$C_{ij} = (\Omega\Omega^T)_{ij} = \rho_{ij}\sigma_i\sigma_j. \tag{412}$$

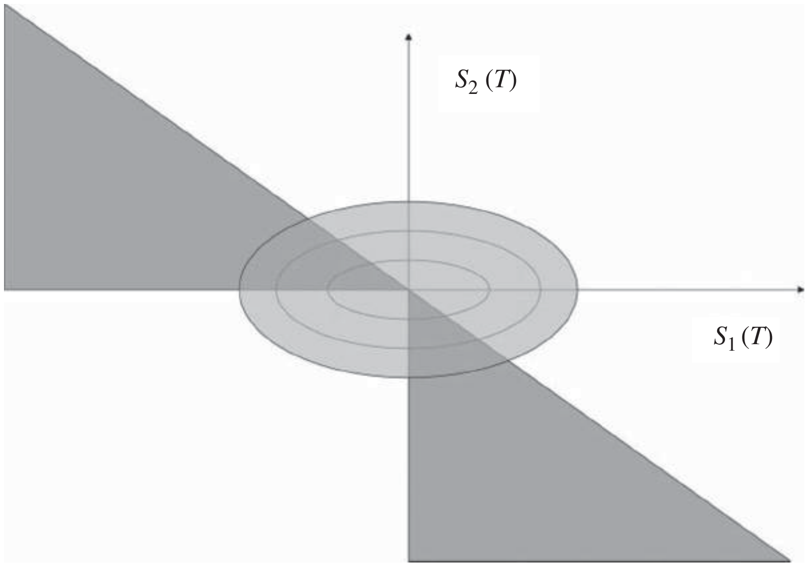


FIGURE 1.68 Protection with a basket option in two currencies. The ellipsoids connect the points that are reached with the same probability assuming that the forward prices are at the center.

Here σ_i denotes the volatility of the i-th currency pair and ρ_{ij} the correlation coefficients.

Exact Method. Starting with the uncorrelated components the pricing problem is reduced to the N-dimensional integration of the payoff. This method is accurate but rather slow for more than two or three basket components.

A Simple Approximation via Moment Matching assumes that the basket spot itself is a log-normal process with drift μ and volatility σ driven by a Wiener process $W(t)$,

$$dS(t) = S(t)[\mu\ dt + \sigma\ dW(t)] \tag{413}$$

with solution

$$S(T) = S(t)e^{\sigma W(T-t)+\left(\mu-\frac{1}{2}\sigma^2\right)(T-t)}, \tag{414}$$

given we know the spot $S(t)$ at time t. It is a fact that the sum of log-normal processes is not log-normal, but as a crude approximation it is certainly a quick method that is easy to implement. In order to price the basket call, the drift and the volatility of the basket spot need to be determined. This is done by matching the first and second moment of the basket spot with the first and second moment of the log-normal model for the basket spot. The moments of log-normal spot are

$$\mathbb{E}[S(T)] = S(t)e^{\mu(T-t)}, \tag{415}$$

$$\mathbb{E}[S(T)^2] = S(t)^2 e^{(2\mu+\sigma^2)(T-t)}. \tag{416}$$

We solve these equations for the drift and volatility,

$$\mu = \frac{1}{T-t} \ln \left(\frac{\mathbb{E}[S(T)]}{S(t)} \right), \tag{417}$$

$$\sigma = \sqrt{\frac{1}{T-t} \ln \left(\frac{\mathbb{E}[S(T)^2]}{S(t)^2} \right)}. \tag{418}$$

In these formulas we now use the moments for the basket spot,

$$\mathbb{E}[S(T)] = \sum_{j=1}^{N} \alpha_j S_j(t) e^{\mu_j(T-t)}, \tag{419}$$

$$\mathbb{E}[S(T)^2] = \sum_{i,j=1}^{N} \alpha_i \alpha_j S_i(t) S_j(t) e^{\left(\mu_i + \mu_j + \sum_{k=1}^{N} \Omega_{ki}\Omega_{jk}\right)(T-t)}. \tag{420}$$

The value is given by the well-known Black-Scholes-Merton formula for plain vanilla call options,

$$v(0) = e^{-r_d T} \left(f\mathcal{N}(d_+) - K\mathcal{N}(d_-) \right), \tag{421}$$

$$f = S(0)e^{\mu T}, \tag{422}$$

$$d_\pm = \frac{\ln \frac{f}{K} \pm \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \tag{423}$$

where $\mathcal{N}$ denotes the cumulative standard normal distribution function and r_d the domestic interest rate.

A More Accurate and Equally Fast Approximation. The previous approach can be taken one step further by introducing one more term in the Itô-Taylor expansion of the basket spot, which results in

$$v(0) = e^{-r_d T} \left(F\mathcal{N}(d_1) - K\mathcal{N}(d_2) \right), \tag{424}$$

$$F = \frac{S(0)}{\sqrt{1-\lambda T}} e^{\left(\mu - \frac{\lambda}{2} + \frac{\lambda\sigma^2}{2(1-\lambda T)}\right)T}, \tag{425}$$

$$d_2 = \frac{\sigma - \sqrt{\sigma^2 + \lambda \left(\left(1 + \frac{\lambda}{1-\lambda T} \right) \sigma^2 T - 2 \ln \frac{F\sqrt{1-\lambda T}}{K} \right)}}{\lambda\sqrt{T}}, \tag{426}$$

$$d_1 = \sqrt{1-\lambda T}\, d_2 + \frac{\sigma\sqrt{T}}{\sqrt{1-\lambda T}}. \tag{427}$$

The new parameter λ is determined by matching the third moment of the basket spot and the model spot. For details see [65]. Most remarkably, this major improvement in the accuracy requires only a marginal additional computation effort.

Correlation Risk Correlation coefficients between market instruments are usually not obtained easily. Either historical data analysis or implied calibrations need to be done. Implying the correlation from a traded instrument, however, will not produce the correlation, but the worst case assumption about correlation that somebody else has made, who does not know either what the correlation is. However, in the foreign exchange market the cross instrument is sometimes traded as well, for the example above the USD-JPY spot and options are traded, and the correlation in the Black-Scholes model can be determined from this contract. In fact, denoting the volatilities as in the tetrahedron in Figure 1.69, we obtain formulas for the correlation coefficients in terms of known market implied volatilities

$$\rho_{12} = \frac{\sigma_3^2 - \sigma_1^2 - \sigma_2^2}{2\sigma_1\sigma_2}, \tag{428}$$

$$\rho_{34} = \frac{\sigma_1^2 + \sigma_6^2 - \sigma_2^2 - \sigma_5^2}{2\sigma_3\sigma_4}. \tag{429}$$

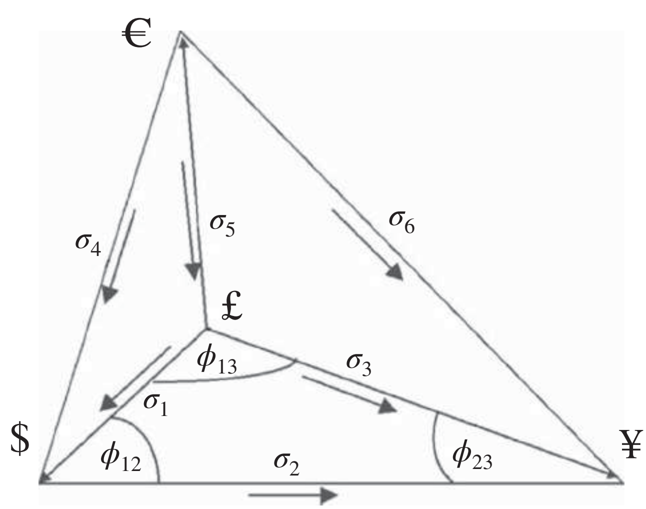


FIGURE 1.69 Relationship between volatilities σ (edges) and correlations ρ (cosines of angles) in a tetrahedron with four currencies and six currency pairs. The arrows mark the market standard quotation direction, i.e. in EUR-USD the base currency is USD and the arrow points to USD.

This method also allows hedging correlation risk by trading FX implied volatility. For details see [65]. While these relationships hold in a log-normal model for the at-the-money volatilities, De Col and Kuppinger [30] extend them to pricing multi-dimensional FX derivatives via stochastic local correlations. Zerolis explains how to picture volatility and correlation in [145].

Pricing Basket Options with Smile The previous calculations are all based on the Black-Scholes model with constant market parameters for rates and volatility. This can all be made time-dependent and can then include the term structure of volatility. If we wish to include the smile in the valuation, then we can either switch to a more appropriate model or perform a Monte Carlo simulation where the probabilities of the exchange rate paths are computed in such a way that the individual vanilla prices are correctly determined. This *weighted Monte Carlo approach* has been discussed by Avellaneda *et al*. in [5].

Practical Example We want to find out how much one can save using a basket option. We take EUR as a base currency and consider a basket of three currencies: USD, GBP, and JPY. We list the contract data and the amount of option premium one can save using a basket call rather than three individual call options in Table 1.40 and the market data in Table 1.41.

TABLE 1.40 Sample contact data of a EUR call basket put. The value of the basket is noticeably less than the value of three vanilla EUR calls.

Contract data	Strikes	Weights	Single option prices
EUR/USD	1.1390	33.33%	4.94%
EUR/GBP	0.7153	33.33%	2.50%
EUR/JPY	125.00	33.33%	3.87%
Sum		100%	3.77%
Basket price			2.90%

TABLE 1.41 Sample market data of 21 October 2003 of four currencies: EUR, GBP, USD, and JPY. The correlation coefficients are implied from the volatilities based on Equation (428) for the triangles and Equation (429) for the tetrahedra.

Vol	Spot	Correlation ccy pair	GBP/USD	USD/JPY	GBP/JPY	EUR/USD	EUR/GBP	EUR/JPY
8.80	1.6799	GBP/USD	1.00	−0.49	0.42	0.72	−0.15	0.29
9.90	109.64	USD/JPY	−0.49	1.00	0.59	−0.55	−0.21	0.41
9.50	184.17	GBP/JPY	0.42	0.59	1.00	0.09	−0.35	0.70
10.70	1.1675	EUR/USD	0.72	−0.55	0.09	1.00	0.58	0.54
7.50	0.6950	EUR/GBP	−0.15	−0.21	−0.35	0.58	1.00	0.42
9.80	128.00	EUR/JPY	0.29	0.41	0.70	0.54	0.42	1.00

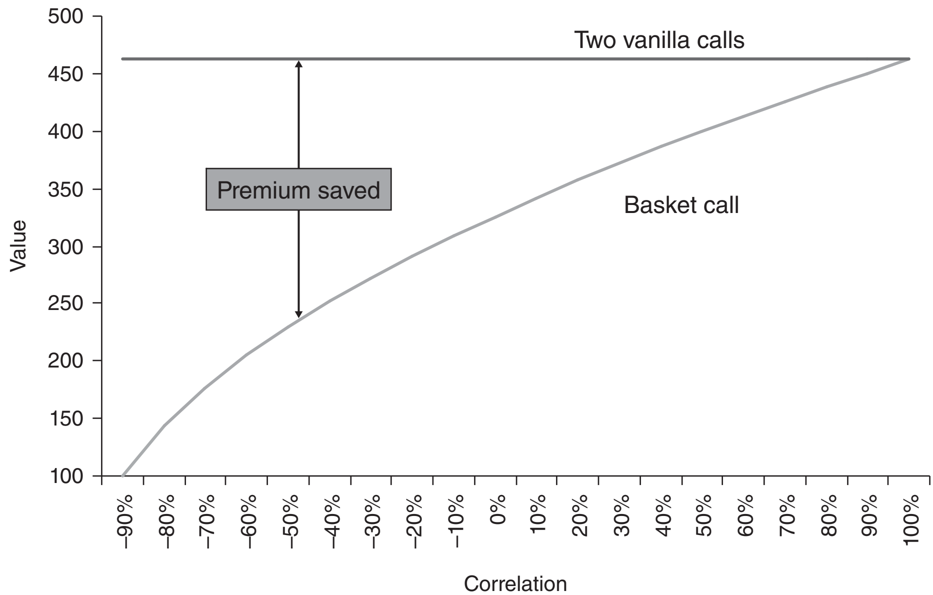


FIGURE 1.70 Amount of premium saved in a basket of two currencies compared with two single vanillas as a function of correlation: the smaller the correlation, the higher the premium savings effect.

The amount of premium saved essentially depends on the correlation of the currency pairs. In Figure 1.70 we take the parameters of the previous scenario, but restrict ourselves to the currencies USD and JPY. Note that graph is – while legally and mathematically correct – a sales slide. (Why?)

Conclusions Many corporate clients are exposed to multi-currency risk. One way to turn this fact into an advantage is to use multi-currency hedge instruments. We have shown that basket options are convenient instruments protecting against exchange rates of most of the basket components changing in the same direction. A rather unlikely market move of half of the currencies' exchange rates in opposite directions is not protected by basket options, but when taking this residual risk into account the hedging cost is reduced substantially. Note that for a treasurer with the underlying cash flow in the various currencies there is actually no risk. Risk comes in if a hedge fund sells the vanilla portfolio and buys the basket to generate net positive premium at inception and if then its correlation trade turns against the trader, and one of the vanilla options ends up deep in-the-money, whereas the basket is out-of-the-money. Another example of how to use currency basket options as part of a note is discussed in Section 2.6.2.

1.9.3 Outside Barrier Options

Outside barrier options are options in one currency pair with one or several barriers or window barriers in another currency pair. In general form the payoff can be written as

$$\left[\phi\left(S_T - K\right)\right]^+ I\!I_{\{\min_{0\le t\le T}(\eta R(t))>\eta B\}}. \tag{430}$$

This extends a European put or call with strike K by a knock-out barrier H in a second currency pair, called the *outer* currency pair. As usual, the binary variable ϕ takes the value +1 for a call and −1 for a put and the binary variable η takes the value +1 for a lower barrier and −1 for an upper barrier. I am still puzzled why outside barrier options trade, but this is definitely some funky correlation trade. On the other hand, why not?

Valuation We will now take a look at the valuation of the outside barrier option in the Black-Scholes model. The derivation is part of the exercises in integration, which you may or may not enjoy, but skipping it does not slow you down in reading the rest of the book. We let the positive constants σ_i denote the annual volatilities of the i-th asset or foreign currency, ρ the instantaneous correlation of their log-returns, r the domestic risk-free rate, and T the expiration time in years. In a risk-neutral setting the drift terms μ_i take the values

$$\mu_i = r - r_i \tag{431}$$

where r_i denotes the risk-free rate of the i-th foreign currency. Knock-in outside barrier options values can be obtained by the standard relationship *knock-in plus knock-out = vanilla*.

In the standard two-dimensional Black-Scholes model

$$dS_t = S_t\left[\mu_1 dt + \sigma_1 dW_t^{(1)}\right], \tag{432}$$

$$dR_t = R_t\left[\mu_2 dt + \sigma_2 dW_t^{(2)}\right], \tag{433}$$

$$\mathbf{Cov}\left[W_t^{(1)}, W_t^{(2)}\right] = \sigma_1\sigma_2\rho t, \tag{434}$$

Heynen and Kat derive the value in [74].

$$\begin{aligned} V_0 = {} & \phi S_0 e^{-r_1 T}\mathcal{N}_2(\phi d_1, -\eta e_1; \phi\eta\rho) \\ & - \phi S_0 e^{-r_1 T}\exp\left(\frac{2(\mu_2 + \rho\sigma_1\sigma_2)\ln(H/R_0)}{\sigma_2^2}\right)\mathcal{N}_2(\phi d_1', -\eta e_1'; \phi\eta\rho) \\ & - \phi K e^{-rT}\mathcal{N}_2(\phi d_2, -\eta e_2; \phi\eta\rho) \\ & + \phi K e^{-rT}\exp\left(\frac{2\mu_2\ln(H/R_0)}{\sigma_2^2}\right)\mathcal{N}_2(\phi d_2', -\eta e_2'; \phi\eta\rho), \end{aligned} \tag{435}$$

$$d_1 = \frac{\ln(S_0/K) + (\mu_1 + \sigma_1^2)T}{\sigma_1\sqrt{T}}, \tag{436}$$

$$d_2 = d_1 - \sigma_1\sqrt{T}, \tag{437}$$

$$d_1' = d_1 + \frac{2\rho\ln(H/R_0)}{\sigma_2\sqrt{T}}, \tag{438}$$

$$d_2' = d_2 + \frac{2\rho \ln(H/R_0)}{\sigma_2\sqrt{T}}, \tag{439}$$

$$e_1 = \frac{\ln(H/R_0) - (\mu_2 + \rho\sigma_1\sigma_2)T}{\sigma_2\sqrt{T}}, \tag{440}$$

$$e_2 = e_1 + \rho\sigma_1\sqrt{T}, \tag{441}$$

$$e_1' = e_1 - \frac{2\ln(H/R_0)}{\sigma_2\sqrt{T}}, \tag{442}$$

$$e_2' = e_2 - \frac{2\ln(H/R_0)}{\sigma_2\sqrt{T}}. \tag{443}$$

The bi-variate standard normal distribution $\mathcal{N}_2$ and density functions n_2 are defined by

$$n_2(x, y; \rho) \stackrel{\Delta}{=} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right), \tag{444}$$

$$\mathcal{N}_2(x, y; \rho) \stackrel{\Delta}{=} \int_{-\infty}^{x} \int_{-\infty}^{y} n_2(u, v; \rho)\, du\, dv. \tag{445}$$

Greeks For the Greeks, most of the calculations of partial derivatives can be simplified substantially by the homogeneity method described by Reiss and Wystup in [107], which states, for instance, that

$$V_0 = S_0 \frac{\partial V_0}{\partial S_0} + K \frac{\partial V_0}{\partial K}. \tag{446}$$

We list some of the sensitivities for reference.

delta (inner spot)

$$\begin{aligned}\frac{\partial V_0}{\partial S_0} &= \phi e^{-r_1 T} \mathcal{N}_2(\phi d_1, -\eta e_1; \phi\eta\rho) \\ &\quad - \phi e^{-r_1 T} \exp\left(\frac{2(\mu_2 + \rho\sigma_1\sigma_2)\ln(H/R_0)}{\sigma_2^2}\right) \mathcal{N}_2(\phi d_1', -\eta e_1'; \phi\eta\rho)\end{aligned} \tag{447}$$

dual delta (inner strike)

$$\begin{aligned}\frac{\partial V_0}{\partial K} &= -\phi e^{-rT} \mathcal{N}_2(\phi d_2, -\eta e_2; \phi\eta\rho) \\ &\quad + \phi e^{-rT} \exp\left(\frac{2\mu_2 \ln(H/R_0)}{\sigma_2^2}\right) \mathcal{N}_2(\phi d_2', -\eta e_2'; \phi\eta\rho)\end{aligned} \tag{448}$$

gamma (inner spot)

$$\frac{\partial^2 V_0}{\partial S_0^2} = \frac{e^{-r_1 T}}{S_0 \sigma_1 \sqrt{T}} \left[n(d_1) \mathcal{N}\left(\frac{-\phi \rho d_1 - \eta e_1}{\sqrt{1-\rho^2}} \right) \right.$$
$$\left. - \exp\left(\frac{2(\mu_2 + \rho\sigma_1\sigma_2)\ln(H/R_0)}{\sigma_2^2} \right) n(d_1') \mathcal{N}\left(\frac{-\phi\rho d_1' - \eta e_1'}{\sqrt{1-\rho^2}} \right) \right] \quad (449)$$

The standard normal density function n and its cumulative distribution function $\mathcal{N}$ are defined in (473) and (480). Furthermore, we use the relations

$$\frac{\partial}{\partial x} \mathcal{N}_2(x, y; \rho) = n(x) \mathcal{N}\left(\frac{y - \rho x}{\sqrt{1-\rho^2}} \right), \quad (450)$$

$$\frac{\partial}{\partial y} \mathcal{N}_2(x, y; \rho) = n(y) \mathcal{N}\left(\frac{x - \rho y}{\sqrt{1-\rho^2}} \right). \quad (451)$$

dual gamma (inner strike) Again, the homogeneity method described in [107] leads to the result

$$S^2 \frac{\partial^2 V_0}{\partial S_0^2} = K^2 \frac{\partial^2 V_0}{\partial K^2}. \quad (452)$$

1.9.4 Best-of and Worst-of Options

Options on the maximum or minimum of two or more exchange rates can be defined by their payoffs in their simple version as

$$\left[\phi \left(\eta \min(\eta S_T^{(1)}, \eta S_T^{(2)}) - K \right) \right]^+. \quad (453)$$

This payoff resembles a European put or call with expiration time T in years on the minimum ($\eta = +1$) or maximum ($\eta = -1$) of the two underlying exchange rates $S_T^{(1)}$ and $S_T^{(2)}$ with strike K. As usual, the binary variable ϕ takes the value $+1$ for a call and -1 for a put.

Valuation in the Black-Scholes Model In the two-dimensional Black-Scholes model

$$dS_t^{(1)} = S_t^{(1)} \left[\mu_1 dt + \sigma_1 dW_t^{(1)} \right], \quad (454)$$

$$dS_t^{(2)} = S_t^{(2)} \left[\mu_2 dt + \sigma_2 dW_t^{(2)} \right], \quad (455)$$

$$\mathbf{Cov}\left[W_t^{(1)}, W_t^{(2)} \right] = \sigma_1 \sigma_2 \rho t, \quad (456)$$

we let the positive constants σ_i denote the volatilities of the i-th foreign currency, ρ the instantaneous correlation of their log-returns, r the domestic risk-free rate. In a risk-neutral setting the drift terms μ_i take the values

$$\mu_i = r - r_i, \tag{457}$$

where r_i denotes the risk-free rate of the i-th foreign currency.

The value was published originally by Stulz in [122] and happens to be

$$\begin{aligned} & v(t, S_t^{(1)}, S_t^{(2)}, K, T, r_1, r_2, r, \sigma_1, \sigma_2, \rho, \phi, \eta) \\ & \quad = \phi \Bigg[S_t^{(1)} e^{-r_1\tau} \mathcal{N}_2(\phi d_1, \eta d_3; \phi\eta\rho_1) + S_t^{(2)} e^{-r_2\tau} \mathcal{N}_2(\phi d_2, \eta d_4; \phi\eta\rho_2) \\ & \qquad - K e^{-r\tau} \left(\frac{1-\phi\eta}{2} + \phi\eta\mathcal{N}_2(\eta(d_1 - \sigma_1\sqrt{\tau}), \eta(d_2 - \sigma_2\sqrt{\tau}); \rho) \right) \Bigg], \end{aligned} \tag{458}$$

$$\sigma^2 \triangleq \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2, \tag{459}$$

$$\rho_1 \triangleq \frac{\rho\sigma_2 - \sigma_1}{\sigma}, \tag{460}$$

$$\rho_2 \triangleq \frac{\rho\sigma_1 - \sigma_2}{\sigma}, \tag{461}$$

$$\tau \triangleq T - t, \tag{462}$$

$$d_1 \triangleq \frac{\ln(S_t^{(1)}/K) + (\mu_1 + \frac{1}{2}\sigma_1^2)\tau}{\sigma_1\sqrt{\tau}}, \tag{463}$$

$$d_2 \triangleq \frac{\ln(S_t^{(2)}/K) + (\mu_2 + \frac{1}{2}\sigma_2^2)\tau}{\sigma_2\sqrt{\tau}}, \tag{464}$$

$$d_3 \triangleq \frac{\ln(S_t^{(2)}/S_t^{(1)}) + (r_1 - r_2 - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \tag{465}$$

$$d_4 \triangleq \frac{\ln(S_t^{(1)}/S_t^{(2)}) + (r_2 - r_1 - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. \tag{466}$$

The bivariate standard normal distribution and density functions $\mathcal{N}_2$ and n_2 are defined in Equation (445) and Equation (444). I let you enjoy deriving the Greeks in the exercises.

Variations Options on the maximum and minimum can be generalized in various ways. For instance, they can be quantoed or have individual strikes for each currency pair. We consider some examples.

Multiple strike option. This variation of best-of/worst-of options deals with individual strikes, i.e. they pay off

$$\max_i \left[0; M_i(\phi(S_T^{(i)} - K_i))\right]. \tag{467}$$

Quanto best-of/worst-of options. These options come up naturally if an investor wants to participate in several exchange rate movements with a payoff in a currency other than the base currency.

Barrier best-of/worst-of options. One can also add knock-out and knock-in features to all the previous types discussed.

Application in Re-Insurance Suppose you want to protect yourself against a weak USD compared with several currencies for a period of one year. As USD seller and buyer of EUR, GBP, and JPY you need simultaneous protection against all three rising against the USD. Of course, you can buy three put options, but if you need only one of the three, then the premium can be considerably reduced, as shown in Table 1.42. We can imagine a situation like this if a re-insurance company insures ships in various oceans. If a ship sinks near the coast of Japan, the client will have to be paid an amount in JPY. The re-insurance company is long USD and assumes only one ship at most will sink in one year so is ready to take the residual risk of more than one sinking.

Since the accidents can occur any time, all options are of American style, i.e. they can be exercised any time. The holder of the option can choose the currency pair to exercise. Hence, he can decide for the one with the highest profit, even if the currency of accident is a different one. It would be difficult to incorporate and hedge this event insurance into the product, whence the protection needs to assume the worst case scenario that is still acceptable to the re-insurance company. For example, if the re-insurance company needs GBP and the spots at exercise time are at EUR/USD = 1.1200, USD/JPY = 134.00, and GBP/USD = 1.6400, you will find both the EUR and GBP constituents in-the-money. However, exercising in GBP would pay a net of 613,496.93 USD, exercising in EUR pays 6,666,666.67 USD. The client would then exercise in EUR, buy the desired GBP in the EUR/GBP spot market, and keep the rest of the EUR.

TABLE 1.42 Example of a triple strike best-of call (American style) with 100 M USD notional and one year maturity. Compared with buying vanilla options one saves 800,000 USD or 20%. All premiums are in USD.

Currency pair	Spot	Strikes	Vanilla premium	Best-of premium
EUR/USD	0.9750	1.0500	1.4 M	
USD/JPY	119.00	110.00	1.7 M	
GBP/USD	1.5250	1.6300	0.9 M	
		Total in USD	4.0 M	3.2 M

Application in Corporate and Private Banking Just like a *dual currency deposit* described in Section 2.4.1, one can use a worst-of put to structure a *multi-currency deposit* with a coupon even higher. We refer the reader to the exercises. This idea mostly comes up when volatilities are low and therefore selling options does not generate sufficient yield enhancement.

1.9.5 Other Multi-Currency Options

Generally, for multi-currency derivatives, there are no bounds to being creative. You get all colors and flavors. For this reason, multi-currency options are also sometimes referred to as *rainbow options*. Most of them have been more popular in equity markets.

Quanto Exotics In foreign exchange options markets cash-settle options can have payoffs in a currency different from the underlying currency pair. For instance, a USD/JPY call is designed to be paid in EUR, where the exchange rate for EUR/JPY is fixed upfront. Surely such quanto features can be applied to exotics as well. Same principles for quanto options apply as explained in Section 1.7.10.

Madonna, Pyramid, Montain Range, and Himalaya Options There are in fact derivatives where one might (rightfully) ask if this is meant to be serious or not. A few examples are contained in but are not limited to the following list.

Madonna option. This one pays the *Euclidian distance*,

$$\max\left[0; \sqrt{\sum_i (S_T^{(i)} - K_i)^2}\right]. \tag{468}$$

Pyramid option. This one pays the *maximum norm*,

$$\max\left[0; \sum_i |S_T^{(i)} - K_i| - K\right]. \tag{469}$$

Mountain range and Himalaya option. This type of option comes in various flavors and is rather popular in equity markets, so we will not discuss them here. A reference is the thesis by Mahomed [94].

In the next section we return to the real world of currency derivatives and deal with the correlation swap.

1.9.6 Correlation Swap

In a *correlation swap* two parties trade a fixed notional amount multiplied by the difference of a fixed correlation ρ_{fixed} and a prevailing historic correlation ρ_{historic} calculated by Equation (137). The payoff is simply

$$\text{correlation swap payoff} = \text{notional} \cdot (\rho_{\text{fixed}} - \rho_{\text{historic}}). \tag{470}$$

The fixed correlation that makes the transaction worth zero is called the *par correlation rate* or *fair correlation rate*.

Example of a Traded Correlation Swap On 25 June 2007 a correlation swap traded, terminating on 25 September 2007 on a notional of USD 19,563,090.00. The observation period was from and including the trade date to and including the termination date. The fixed rate was +82%, the floating rate based on prevailing historic correlation calculated by Equation (137). The first currency pair was GBP/NZD and the second one USD/NZD. As spot rate references the parties agreed on the source Reuters page **WMRSPOT11** using New York business days. Calculation date was 26 September 2007, and settlement date 27 September 2007. *Settlement differential* means the fixed rate minus the floating rate. The Settlement Differential is to be expressed as a percentage which may be a positive number or a negative number. Here is how the lawyers deal with minus signs: *Settlement Amount* means:

1. if the Settlement Differential is positive, then the Settlement Amount is an amount equal to the notional amount multiplied by the Settlement Differential and such amount shall be paid by the fixed rate payer to the floating rate payer on the settlement date;
2. if the Settlement Differential is negative, then the Settlement Amount is an amount equal to the notional amount multiplied by the Settlement Differential and such amount shall be paid by the floating rate payer to the fixed rate payer on the settlement date.

This traded as a zero-cost product. Obviously, the fixed rate payer has a small potential maximum gain of 18% of the USD notional and a much larger maximum potential loss of 182%. The fixed rate payer takes a view in USD and GBP both becoming stronger against NZD in the next three months. I am not sure on which analysis such a view is taken. Notice that the maximum potential loss exceeds the notional amount. So watch out: correlations can be negative!

Extensions One can put bounds on the best and/or worst case correlation. There are also correlation options instead of swaps.

1.9.7 Exercises

Exchange Option Compute the integral of the spread option for the special case of a zero strike in Equation (396) to get a closed form solution for the exchange option.

TABLE 1.43 Sample market ATM volatilities of four currencies: EUR, GBP, USD, and CHF.

Ccy pair	Volatility	Ccy pair	Volatility
GBP/USD	9.20%	EUR/USD	10.00%
USD/CHF	11.00%	EUR/GBP	7.80%
GBP/CHF	8.80%	EUR/CHF	5.25%

Implied Correlation Compute the correlation coefficients implied from the volatilities in Table 1.43. What are the upper and lower limits for the EUR/USD volatility to guarantee all correlation coefficients being contained in the interval $[-1, +1]$, assuming all the other volatilities are fixed?

Outside Barrier Option Value Formula Derive the value function of the outside barrier option in the Black-Scholes model, see Equation (435). We start with a triple integral. We treat the up-and-out call as an example. The value of an outside up-and-out call option is given in Section 24 in Shreve's lecture notes [119] by the integral

$$V_0 \stackrel{\Delta}{=} \frac{e^{-rT}}{\sqrt{T}} \int_{\hat{m}=0}^{\hat{m}=m} \int_{\hat{b}=-\infty}^{\hat{b}=\hat{m}} \int_{\tilde{b}=-\infty}^{\tilde{b}=\infty} F(\hat{b}, \tilde{b}) n\left(\frac{\tilde{b}}{\sqrt{T}}\right) f(\hat{m}, \hat{b})\, d\tilde{b}\, d\hat{b}\, d\hat{m}, \tag{471}$$

where the payoff function F, the normal density function n, the joint density function f, and the parameters m, b, $\hat{\theta}$, γ are defined by

$$F(\hat{b}, \tilde{b}) \stackrel{\Delta}{=} \left(S_0 e^{\gamma\sigma_2 T + \rho\sigma_2\hat{b} + \sqrt{1-\rho^2}\sigma_2\tilde{b}} - K\right)^+ \tag{472}$$

$$n(t) \stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}, \tag{473}$$

$$f(\hat{m}, \hat{b}) \stackrel{\Delta}{=} \frac{2(2\hat{m} - \hat{b})}{T\sqrt{2\pi T}} \exp\left\{-\frac{(2\hat{m} - \hat{b})^2}{2T} + \hat{\theta}\hat{b} - \tfrac{1}{2}\hat{\theta}^2 T\right\}, \tag{474}$$

$$m \stackrel{\Delta}{=} \frac{1}{\sigma_1} \ln \frac{L}{Y_0}, \tag{475}$$

$$b \stackrel{\Delta}{=} \frac{1}{\sigma_2} \ln \frac{L}{S_0}, \tag{476}$$

$$\hat{\theta} \stackrel{\Delta}{=} \frac{r}{\sigma_1} - \frac{\sigma_1}{2}, \tag{477}$$

$$\gamma \stackrel{\Delta}{=} \frac{r}{\sigma_2} - \frac{\sigma_2}{2} - \rho\hat{\theta}. \tag{478}$$

The goal is to write the above integral in terms of the bi-variate normal distribution function (445). For easier comparison we use the mapping of the notation in Table 1.44.

TABLE 1.44 Relating the notation of Heynen and Kat to the one by Shreve.

Heynen/Kat	Shreve	Heynen/Kat	Shreve
S_0	S_0	H	L
R_0	Y_0	K	K
σ_1	σ_2	μ_1	$r - \frac{\sigma_2^2}{2}$
σ_2	σ_1	μ_2	$r - \frac{\sigma_1^2}{2}$

The solution can be obtained by taking the following steps:

(a) Use a change of variables to prove the identity

$$\int_{-\infty}^{x} \mathcal{N}(az + B)n(z)\, dz = \mathcal{N}_2\left(x, \frac{B}{\sqrt{1+a^2}}; \frac{-a}{\sqrt{1+a^2}}\right), \tag{479}$$

where the cumulative normal distribution function $\mathcal{N}$ is defined by

$$\mathcal{N}(x) \stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\, dt. \tag{480}$$

A probabilistic proof is presented in [52].

(b) Extend the identity (479) to

$$\int_{-\infty}^{x} e^{Az}\mathcal{N}(az + B)n(z)\, dz = e^{\frac{A^2}{2}}\mathcal{N}_2\left(x - A, \frac{aA + B}{\sqrt{1+a^2}}; \frac{-a}{\sqrt{1+a^2}}\right). \tag{481}$$

(c) Change the order of integration in Equation (471) and integrate the $\hat{m}$ variable.

(d) Change the order of integration to make $\tilde{b}$ the inner variable and $\hat{b}$ the outer variable. Then use the condition $F(\hat{b}, \tilde{b}) \geq 0$ to find a lower limit for the range of $\tilde{b}$. This will enable you to skip the positive part in F and write Equation (471) as a sum of four integrals.

(e) Use (479) and (481) to write each of these four summands in terms of the bi-variate normal distribution function $\mathcal{N}_2$.

(f) Compare your result with the one provided by Heynen and Kat using Table 1.44.

Inside Barrier Option as Special Case of an Outside Barrier Option Derive the value function of an inside barrier option by viewing it as a special case of the outside barrier option in Equation (435).

TABLE 1.45 Sample short time series of two spots.

Spot 1	Spot 2
100	0.8
110	0.7
120	0.6
130	0.5

Greeks of Best-of/Worst-of Options Derive the Greeks of the best-of/worst-of value function in Equation (458). Hint: most of the calculations of partial derivatives can be simplified substantially by considering homogeneity properties described in [107].

Correlation Shocker Given the two time series of spot prices in Table 1.45, calculate the correlation (as it would be used in a correlation swap).

CHAPTER 2

Structured Products

2.1 FORWARD TRANSACTIONS

This section deals with various ways to enhance the final exchange rate a treasurer gets. We start by explaining the outright or vanilla forward. Enhancements can then be done in various ways and can be classified in forward products with guaranteed worst case and forward products without such a guaranteed worst case. While the former usually allow hedge accounting in the IAS 39 sense, the latter are subject to more uncertainties, but can produce more significant enhancements.

The forward transactions with worst case all work the same way. The corporate client engages in a forward contract with a final exchange rate that is worse than the market. Hence, he has some money left to invest in another product such as an option or a portfolio of derivatives. The portfolio is then chosen to match the client's view on the future development of the underlying exchange rate. There is unlimited freedom to choose such portfolios. We explain many popular structures.

Naturally, the forward transactions address primarily corporate treasurers, companies with international business and cash flows. However, the real target group are mid-size companies, because the large global players usually have enough knowledge, infrastructure, and staff to buy the components of the structures separately, whereas the mid-size corporate often prefers packaged solutions.

For the private or institutional investor, the forward products are of only limited use as they normally do not have the foreign exchange cash flow behind their investments. For this group similar strategies can be used by omitting the basic forward contracts with worst case and quanto the structures into the respective domestic currency with cash settlement.

There are also a number of contracts traded over the counter which I call exotic forward contracts. They normally trade at zero cost at inception of the trade and involve currencies to be physically exchanged at maturity and/or at points of time between inception and maturity. This is why they are generally considered to be forward-like contracts and are marketed and labeled under the term forward. However, amounts traded and levels at which currencies are converted depend on the spot price path, or on fixings in practice. We consider a few examples towards the end of this section.

FX Options and Structured Products, Uwe Wystup

2.1.1 Outright Forward

The outright forward is a zero-cost strategy to fix the exchange rate at a future date. It forms a risk-free basis of budget calculations. However, it does not leave any room for participation in any direction. The contract parameters of an outright forward are

1. Currency pair
2. Amount and currency to be sold or bought
3. Settlement date
4. Forward rate f

For example, a company agrees to sell 1 M EUR in six months and buys 1.18 M USD. In this case the currency pair is EUR/USD and the forward rate is 1.1800. Both parties have to stick to their agreement. However, one can of course unwind a forward contract at any time.

If S_t denotes the price of the underlying exchange rate, the payoff of the forward contract at time T is

$$S_T - f. \tag{1}$$

Hence the value at time zero is

$$e^{-r_f T} S_0 - e^{-r_d T} f. \tag{2}$$

The outright forward rate f is the rate that makes this zero cost, i.e.

$$f = f(T) = e^{(r_d - r_f)T} S_0, \tag{3}$$

which corresponds to the expected value of S_T under the risk-neutral measure.

Carry Trade The forward rate (3) is not a prediction of the spot rate at time T under the real-world measure. Figure 2.1 compares the mostly constant real-world expectation of the future spot with the market risk-neutral forward curve. Investing in this difference is called a *carry trade*. Mrs. Watanabe deposits her wealth in AUD because interest rates are higher in AUD than in JPY, and her accumulated wealth in AUD has AUD-JPY risk when measured in JPY. Mr. Penny takes a loan in CHF, invests in EUR, and faces EUR-CHF risk when paying back the loan. Such a foreign exchange loan is called a *floan*.

As time passes, the value of the forward contract is no longer necessarily zero. So one of the two counterparts is always in debt. This should be taken into consideration when computing the credit line for the counterpart.

Backwardation and Contango The function that maps the maturity T to the outright forward rate for this maturity is called the forward curve. A decreasing curve is called backwardation and an increasing forward curve is called a contango situation. There can be forward curves with humps, in which case neither of the two terms applies.

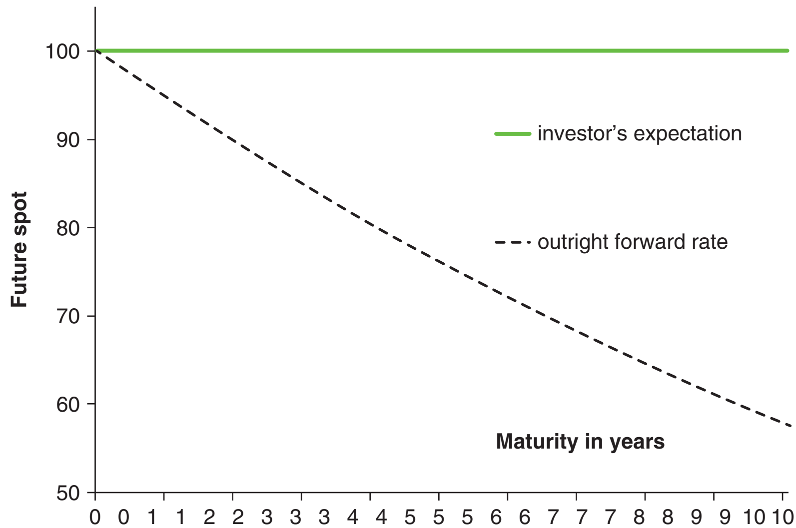


FIGURE 2.1 A carry trade is done if a market participant has a different expectation of the futures spot (shown by the constant straight line) from the risk-neutral outright forward rate (shown by the dotted curve following Equation (3), which could be EUR-CHF or AUD-JPY).

Put-Call Parity It is well known that a long forward can be replicated by a long call and a short put with the same strike and same notional. This replication is often called a *synthetic forward*. While an outright forward contract is usually zero cost for both parties, the synthetic forward can be used to generate a premium as the forward contract with a pre-agreed exchange rate below the outright forward rate is no longer a zero-cost contract. In practice, this premium is often replaced by giving the counterpart another portfolio of transactions (or simply a single option) to participate in his market view.

FX Swap An FX swap is a combination of a spot deal and a forward contract. A client agrees to buy 1 M EUR now at spot S_0 and sell it again on a pre-determined date at the outright forward rate f. Both trades are zero cost. The difference $f - S_0$ is called the FX swap rate. The first leg on an FX swap can also be a forward contract, with the second leg a forward contract settled on a later date.

The FX swap rate $f - S_0$ is usually multiplied by a factor of 10,000 to produce the so-called swap points, if the exchange rate is quoted to four decimal places. For example, assuming the spot rate for EUR/USD at 1.1800, the forward rate at 1.1750, then the swap points are −50. We are in a backwardation scenario. The negative sign is often dropped in market quotes, where a bid/offer quote of 51/49 indicates negative numbers, because the offer price must be higher than the bid price. If the exchange rate is quoted to two decimal places as in USD/JPY, then the FX swap rate is multiplied by a factor of 100 to get the swap points.

Non-Deliverable Forward The non-deliverable forward (NDF) is a forward with cash settlement, i.e. instead of exchanging two currencies at maturity, the net value $S_T - f$ (in domestic currency) or $(S_T - f)/S_T$ (in foreign currency) of this exchange is computed and the winner of the trade receives this net value as cash from the loser. It is possible in all traded underlying currency pairs, but most common in emerging markets and pegged currencies. Notice that the deliverable forward does not require a spot reference, but the NDF does: the counterparts need to agree on the source of the final spot rate, and the exact date and time and time zone they source it. The risk managing desk obviously has a settlement risk, because it is not guaranteed that the spot reference source is tradable. Therefore, NDFs usually have wider bid-offer spreads than regular delivered forward contracts. The less liquid the underlying, the wider the spread.

2.1.2 Participating Forward

The participating forward or *participator* is a very simple strategy consisting of two options with full protection of the upside (where spot goes against the treasurer's position) and partial participation on the downside (where spot goes in favor of the treasurer's position). A participating forward entitles the holder to buy an agreed amount (notional of the Call option) of a currency (say EUR) on a specified date (maturity) at a pre-determined rate if the exchange rate is above it at maturity. If the exchange rate is below the pre-determined rate at maturity, the holder must buy a second agreed amount (notional of the Put option) in EUR at the very same pre-determined rate. Therefore, entering into a participating forward provides full protection against rising EUR. The holder will typically exercise the option only if at maturity the spot is above the long strike at maturity.

Advantages

- Full protection against stronger EUR/weaker USD
- Zero-cost product

Disadvantages

- Participation in weaker EUR/stronger USD is limited

The participating forward can be composed in several ways. In the low-risk style, the notional of the sold put is smaller than the notional of the bought call, typically 50%. In this case, the outright forward rate is already in-the-money on the downside and partial participation has already started. In a high-risk style, the notional of the sold put is higher than the notional of the bought call, say by a factor of 2. In this case of leveraged notional amounts the risk on the downside is substantial and the strike of the call can be set much lower. Engaging in a product like this usually requires a strong bullish view of the counterpart. This high-risk style product is also sometimes called *leveraged forward*.

For example, a company wants to sell 1 M USD (notional of the Call option). At maturity:

1. If $S_T < K$, it is obliged to buy EUR for USD at rate K.
2. If $S_T > K$, it can buy EUR for USD at rate K.

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be fully protected against a stronger EUR. But it finds that the corresponding plain vanilla EUR call is too expensive and is prepared to take more risk in selling a put option.

In this case a possible form of protection that the company can use is to enter into a participating forward as exhibited in Table 2.1 and Figure 2.2.

If the company's market expectation is correct, it can buy EUR at maturity at a rate of 1.1700.

TABLE 2.1 Example of a participating forward.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put
Company sells	EUR put USD call
Maturity	1 year
Notional of the EUR Call option	USD 1,000,000
Notional of the EUR Put option	USD 500,000
Pre-agreed exchange rate	K = 1.1700 EUR-USD
Premium	0.00

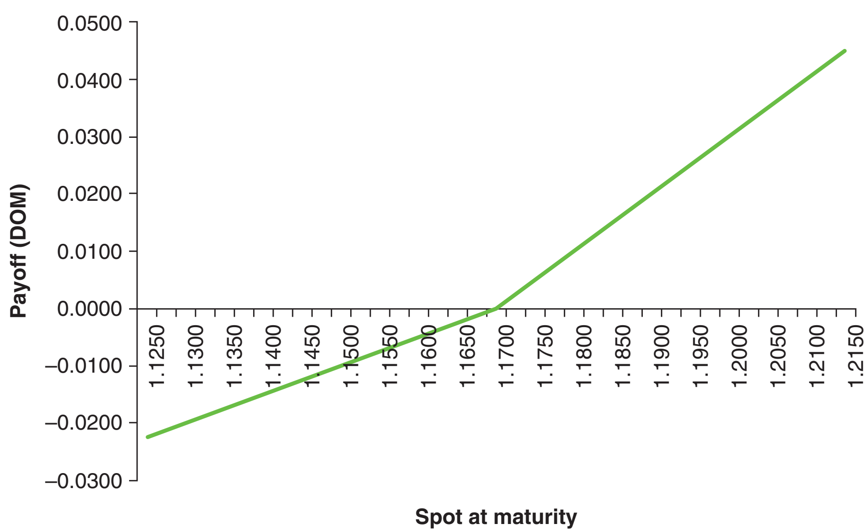


FIGURE 2.2 Payoff of a participating forward.

The drawback is that if the EUR-USD exchange rate will be below the rate of 1.1700 at maturity, it will be obliged to sell 500,000 USD at the rate of 1.1700. The company can sell the other 500,000 USD at the spot market and hence participate partially in a falling EUR (hence the name participating forward). For a treasurer with the underlying cash flow this situation is not risky; it just feels bad.

2.1.3 Participating Collar

The participating collar is like a participating forward except that the strikes are different. It mixes the idea of a risk reversal (collar) and a participating forward. A participating collar entitles the holder to buy an agreed amount (notional of the Call option) of a currency (say EUR) on a specified date (maturity) at a pre-determined rate (long strike) if the exchange rate is above it at maturity. If the exchange rate is below the strike of the short put at this time, the holder must buy a second agreed amount (notional of the Put option) in EUR at the short strike. Therefore, entering into a participating collar provides full protection against rising EUR. The holder will typically exercise the option only if at maturity the spot is above the long strike at maturity.

Advantages

- Full protection against stronger EUR/weaker USD
- Zero-cost product
- Participation in a weaker EUR/USD better than in a participating forward

Disadvantages

- Participation in weaker EUR/stronger USD is limited
- Protection rate closer than in a participating forward

The participating collar can be composed in several ways. In the low-risk style, the notional of the sold put is smaller than the notional of the bought call. In this case, the risk on the downside is comparatively small and hence the strike on the upside will normally not look like the greatest deal that comes down the pike. In a high-risk style, the notional of the sold put is higher than the notional of the bought call, say by a factor of 2. In this case of leveraged notional amounts the risk on the downside is substantial and the strike of the call can be set much lower. Engaging in a product like this usually requires a strong bullish view of the counterpart. This high-risk style product is also sometimes called a *leveraged collar*.

For example, a company wants to sell 1 M USD (notional of the Call option). At maturity:

1. If $S_T < K_1$, it is obliged to buy EUR for USD at rate K_1.
2. If $K_1 < S_T < K_2$, the company can trade at spot.
3. If $S_T > K_2$, it can buy the EUR for USD at rate K_2.

Example A company wants to hedge receivables from an export transaction in USD due in 12 months' time. It expects a stronger EUR/weaker USD. The company wishes to be fully protected against a stronger EUR. But it finds that the corresponding plain vanilla EUR call is too expensive and is prepared to take more risk in selling a put option. Furthermore the company is looking for a 50% participation in a cheaper EUR that is not already in-the-money at inception and does not want to enter into a participating forward. In this case a possible form of protection that the company can use is to enter into a participating collar, as exhibited in Table 2.2 and Figure 2.3.

If the company's market expectation is correct, it can buy EUR at maturity at the strike of 1.2000.

The drawback is that if the EUR-USD exchange rate will be below the strike of 1.1475 at maturity, it will be obliged to sell 500,000 USD at the strike of 1.1475. The company can sell the other 500,000 USD at the spot market and so participate partially in a falling EUR (hence the name participating collar).

2.1.4 Fade-In Forward

The fade-in forward is similar to a participating forward, whose participation percentage on the favorable side depends on the number of currency fixings traded above/below a pre-specified level or inside/outside a pre-defined range. We consider the example of a EUR seller/USD buyer selling EUR at rate K with a fade-in range $[R_L, R_H]$.

TABLE 2.2 Example of a participating collar with zero premium.

Spot reference	1.1500 EUR-USD	EUR Call notional	USD 1,000,000
Company buys	EUR call USD put	EUR Put notional	USD 500,000
Company sells	EUR put USD call	Call strike	K_2 = 1.2000 EUR-USD
Maturity	1 year	Put strike	K_1 = 1.1475 EUR-USD

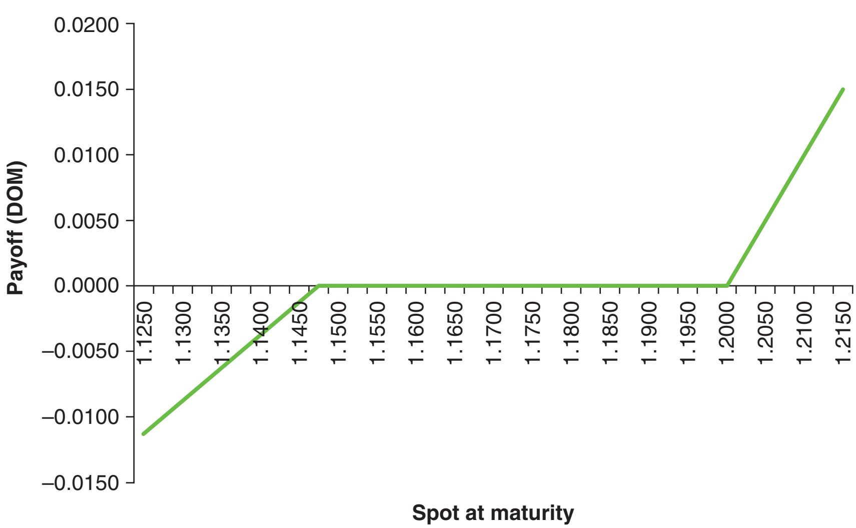


FIGURE 2.3 Payoff of a participating collar.

The participation percentage P is determined by the ratio of the number of fixings inside the range and the total number of fixings. If all fixings are inside the range, then $P = 100\%$. Conversely, if none of the fixings turn out to be in the fade-in range, then $P = 0\%$.

Advantages

- Protection against weaker EUR/stronger USD
- Participation in stronger EUR/weaker USD possible up to 100% (better than in the risk reversal or participating forward/collar)
- Zero-cost product

Disadvantages

- Participation in stronger EUR/weaker USD unknown at inception and at worst zero

Example For example, we consider a fade-in forward traded by a USD buying company on 27 May 2004, whose terms and conditions are listed in Table 2.3. The seller pays the notional amount to the company on the delivery date. The company pays the EUR amount to the seller on the delivery date. This EUR amount is determined as follows.

- If the spot at maturity is at or below the worst case, then the entire USD amount is converted into EUR using the worst case.
- If the spot at maturity is above the worst case, the company participates with $P\%$ in the favorable spot movement, i.e. $P\%$ of the USD amount is converted at the spot at maturity and $(1 - P\%)$ of the USD amount is converted at the worst case.

In the example considered the reference spot at maturity was the in-house fixing of the selling bank. For week days, where there is no EUR-USD fixing, the fixing of the previous weekday is used.

Composition The fade-in forward can be decomposed into a synthetic forward struck at the worst case and a long fade-in option struck at the worst case.

TABLE 2.3 Terms and conditions of a fade-in forward traded on 27 May 2004.

Company buys	USD
Company sells	EUR
Expiry date	1 July 2005
Delivery date	5 July 2005 (delivery settled)
Notional amount	USD 5,000,000
Corridor $R_L - R_H$	1.1200 – 1.2600 EUR-USD
Fixing calendar	all 286 week days starting from 28 May 2004 ending on 1 July 2005
Guaranteed worst case K	1.1800 EUR-USD
Participation level P	$\frac{\text{fixings inside range}}{\text{all fixings}} \cdot 100\%$
Premium	0.00

Alternatives/Extensions There are many structures ranging in the market under the label of a fade-in forward. The reason is that a fade-in forward can be generalized into fading versions of a participating forward or a participating collar or a risk reversal. Besides different exchange rates and leverage, one can further add knock-out levels $B_L \leq R_L$ and $B_H \geq R_H$ (see Section 1.8.3). Furthermore, one can assign different participation rules to the long leg and the short leg. Moreover, sometimes the term fade-*out* forward is used to describe the same story. Fading in with participation P is equivalent to fading out with participation $1 - P$. So watch out.

2.1.5 Knock-Out Forward

The knock-out forward is one of the many possibilities for improving the outright forward rate at zero cost by taking some risk. Instead of the company buying a long call and selling a short put with the same strike, notional, and maturity, we attach one or two knock-out barriers for both. For the EUR buyer a scenario is exhibited in Table 2.4 and Figure 2.4.

Advantages

- Treasurer buys EUR at 300 pips below the outright
- Zero-cost product

TABLE 2.4 Example of a knock-out forward with zero premium.

Spot reference	1.1500 EUR-USD	Maturity	1 year
Outright forward reference	1.1400 EUR-USD	Agreed rate	1.1000 EUR-USD
Notional	EUR 1,000,000	American KO	1.2900 EUR-USD

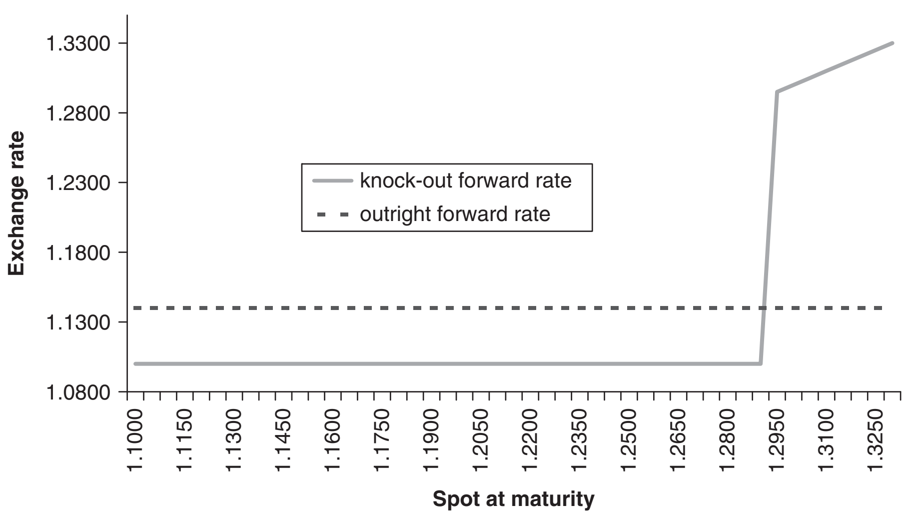


FIGURE 2.4 Final exchange rate of a knock-out forward for a EUR buyer.

Disadvantages

- Hedge is lost in case of knock-out
- No worst case scenario
- No participation in weaker EUR/stronger USD

The most noticeable risk factor here is the lack of a worst case. We only know a best case in advance. For example, EUR rising to 1.5000 USD would cause substantial trouble to the treasurer, because the knock-out event would occur and the treasurer would have to buy EUR at 1.5000, which is 40 big figures higher than his agreed forward rate of 1.1000 in case no knock-out event occurs. Trading this product requires a strong view of a rising EUR with an upper bound at the knock-out barrier. Nevertheless, this strategy is actively traded in the market. It can be viewed as a predecessor to the standard target forward with a fixing schedule consisting of only one fixing.

2.1.6 Shark Forward

The shark forward is also called *forward plus* or *forward extra* or *enhanced forward* or forward with profit potential. It is suitable for companies that want to fix a forward price while benefiting from a spot movement they believe in. It gives a certain range of profit potential, while *maintaining a worst case level* near the forward rate. It is composed of a synthetic forward contract with a forward price as the worst case and a reverse knock-out, whose payoff profile looks like a shark fin. The term "shark" is obviously avoided in sales efforts of all kinds. We distinguish two kinds of shark forward contracts:

(shark) forward plus: the buyer benefits from a favorable spot movement.

(shark) forward extra: the buyer benefits from a spot movement against his position.

Shark Forward Plus We consider the following example. Spot EUR/USD = 1.1200, three months, maturity, outright forward rate 1.1164, volatility 10.30%.

Exporter in the Euro zone: The seller of 1 M USD, who needs protection against a falling USD but expects the USD to rise, can get a worst case of 1.1200, equal to the current spot and only slightly worse than the outright forward rate, a participation level of 1.1200 and an American (European) style knock out trigger of 1.0772 (1.0951). The bid/ask prices of this strategy are −0.2075/0.0000% USD, i.e. the client buys the shark forward plus for zero cost and in case of unwind would have to pay 0.2075% of the USD notional. The exchange rate the client gets is min(worst case, spot at maturity), as we are looking at the following scenarios.

- If the spot at maturity is above 1.1200, then the client is entitled to sell 1 M USD at the worst case.
- If the spot at maturity is below 1.1200 and the trigger has not been touched, then the client can sell 1 M USD at the spot.
- If the trigger has been touched, then the client must sell 1 M USD at the worst case.

Importer in the Euro zone: The buyer of 1 M USD, who needs protection against a rising USD, but expects the USD to weaken, can get a worst case of 1.1100, one big figure below the current spot and only slightly below the outright forward rate, a participation level of 1.1100 and an American (European) style knock out trigger of 1.1712 (1.1495). The bid/ask prices of this strategy are −0.2425/0.0000% USD, i.e. the client buys the shark forward plus for zero cost and in case of unwind would have to pay 0.2425% of the USD notional. The exchange rate the client gets is max(worst case, spot at maturity), as we are looking at the following scenarios.

- If the spot at maturity is below 1.1100, then the client is entitled to buy 1 M USD at the worst case.
- If the spot at maturity is above 1.1100 and the trigger has not been touched, then the client can buy 1 M USD at the spot.
- If the trigger has been touched, then the client must buy 1 M USD at the worst case.

In both cases the client has a guaranteed worst case protection at zero cost and can participate in a favorable spot movement. The sales margin for this structure is 500.00 EUR. A comparison with an outright forward is illustrated in Figure 2.5.

Variations More generally, one can allow participation levels P different from the worst case W. This is sometimes called *forward plus plus* or *forward super plus* or *forward plus with extra strike*. The seller of 1 M USD, who needs protection against a falling USD but expects the USD to rise *significantly*, can get a worst case of $W = 1.1200$, equal to the current spot and only slightly worse than the outright forward rate, a participation level of $P = 1.1000$ and an American (European) style knock out trigger of 1.0582 (1.0744). The bid/ask prices of this strategy are −0.2475/0.0000% USD, i.e. the client buys the shark forward plus for zero cost and in case of unwind would have to pay 0.2475% of the USD notional. The exchange rate the client gets is

$$\min\left(W, \frac{1}{\frac{1}{S_T} + \frac{1}{W} - \frac{1}{P}}\right), \tag{4}$$

as we are looking at the following scenarios. Here S_T denotes the spot at maturity.

- If the spot at maturity is above the participation level 1.1000, then the client sells 1 M USD at the worst case.
- If the spot at maturity is below 1.1000 and the trigger has not been touched, then the client can sell 1 M USD at the rate

$$\frac{1}{\frac{1}{S_T} + \frac{1}{W} - \frac{1}{P}}. \tag{5}$$

- If the trigger has been touched, then the client must sell 1 M USD at the worst case.

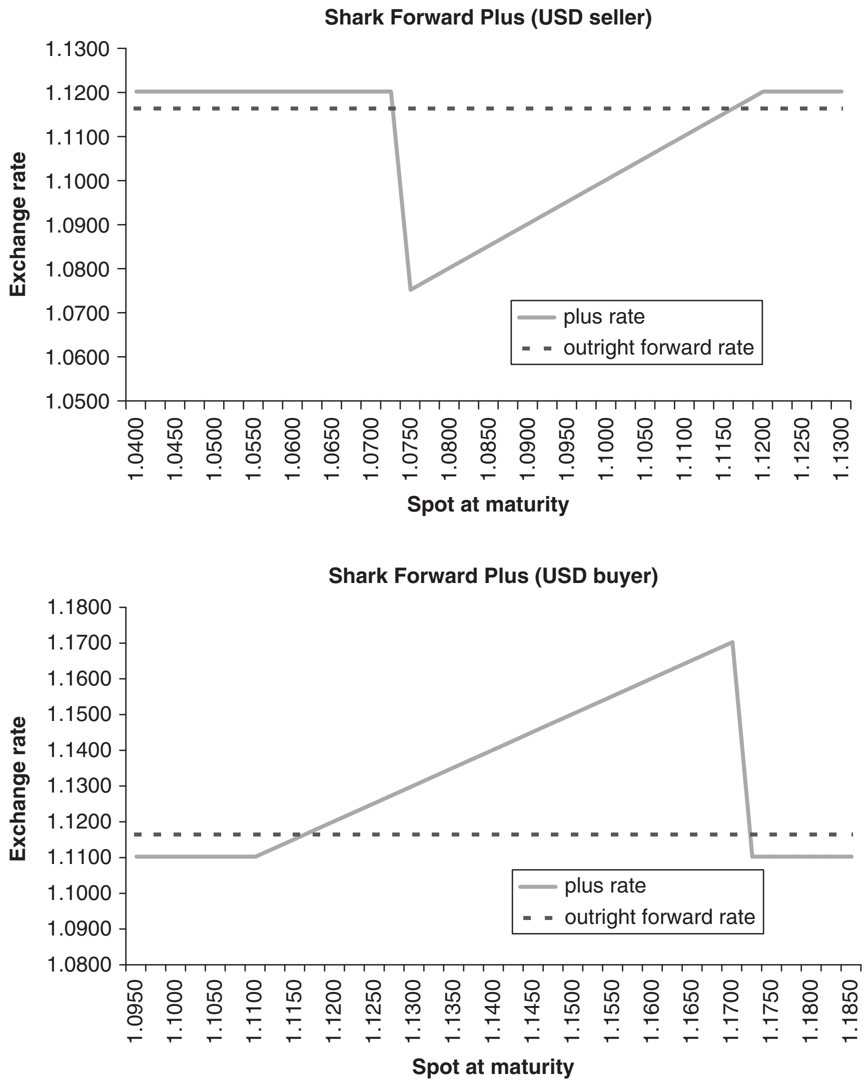


FIGURE 2.5 Comparison of final exchange rates for a shark forward plus: outright forward versus forward plus rate.

The buyer of 1 M USD, who needs protection against a rising USD but expects the USD to remain above the current spot, can get a worst case of 1.1100, one big figure below the current spot and only slightly below the outright forward rate, a participation level of 1.1200 and an American (European) style knock out trigger of 1.1830 (1.1632). The bid/ask prices of this strategy are −0.2900/0.0000% USD, i.e. the client buys the shark forward plus for zero cost and in case of unwind would have to pay 0.2900% of the USD notional. The exchange rate the client gets is

$$\max\left(W, \frac{1}{\frac{1}{S_T} + \frac{1}{W} - \frac{1}{P}}\right), \tag{6}$$

as we are looking at the following scenarios. As usual, S_T denotes the spot at maturity.

- If the spot at maturity is below 1.1200, then the client buys 1 M USD at the worst case.
- If the spot at maturity is above 1.1200 and the trigger has not been touched, then the client can buy 1 M USD at the rate

$$\frac{1}{\frac{1}{S_T} + \frac{1}{W} - \frac{1}{P}}. \tag{7}$$

- If the trigger has been touched, then the client must buy 1 M USD at the worst case.

In both cases the client has a guaranteed worst case protection at zero cost and can participate in a favorable spot movement. The margin for this structure is 500.00 EUR. The benefit of these participation levels is barriers that are further away from the current spot.

If the notional is specified in EUR rather than in USD, then the exchange rate

$$\frac{1}{\frac{1}{S_T} + \frac{1}{W} - \frac{1}{P}} \tag{8}$$

in Equations (4)–(7) must be replaced by

$$S_T + W - P. \tag{9}$$

For a USD buyer a comparison of a shark forward plus with extra strike, a standard shark forward and an outright forward is illustrated in Figure 2.6.

Concluding Comments A more aggressive version of the shark forward is discussed in Section 2.1.15 about the *double shark forward*. The shark forward version where the treasurer participates in a spot moving against his position is left to the exercises. A common argument brought forward against a shark forward is the sudden drop of the final exchange rate if the trigger is touched. The trade is then classified as risky, because when the trade is beginning to look interesting, it bounces back to the worst case. However, this criticism isn't fair, because it is done with hindsight. The decision as to which hedge

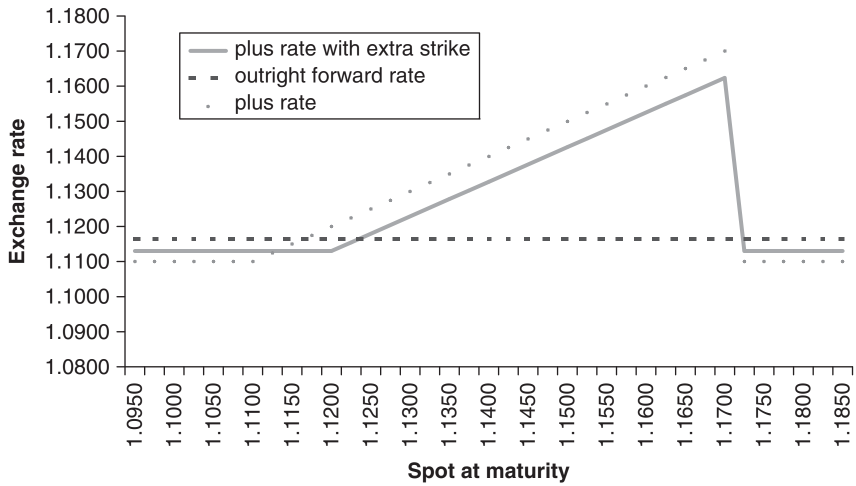


FIGURE 2.6 Comparison of final exchange rates for a shark forward plus: the forward plus worst case is the lowest. Using an extra strike or participation, one can increase either the knock-out barrier or – as shown here – move the worst case closer to the outright forward at the same price and allow less participation on the upside.

to employ must be taken at time zero. The shark forward is not an alternative to waiting. It has a really good and fully guaranteed worst case. The fair criticism should be comparing it to an outright forward contract.

2.1.7 Fader Shark Forward

A *fader forward plus* is a forward plus where the trigger/reset risk fades out linearly with the passage of time. The benefit over a standard forward plus is that the participation is proportionately retained on favorable spot moves up until the trigger event occurs. An example is provided in Table 2.5.

Analysis Compared with a forward plus, the fader forward plus seeks to mitigate the participation loss if the reset trigger trades. This is illustrated in the following example scenarios for a 12 months fader forward plus.

TABLE 2.5 Example of a fader forward plus.

Spot reference	1.2150 EUR-USD
Outright forward reference	1.2152 EUR-USD
Client sells notional	USD 1,000,000
Maturity	1 year
Worst case buy EUR (sell USD) on 100%	1.2250 EUR-USD
Best case buy EUR (sell USD) on 100%	1.1690 EUR-USD
Premium	0.00

Scenario 1: EUR/USD trades below 1.1690 in month 6 and fixes below 1.1690 on expiry. Result: 50% of the USD notional can be sold at 1.1690 and the remaining 50% at the worst case protection rate 1.2250.

Scenario 2: EUR/USD trades below 1.1690 in month 9 and fixes below 1.1690 on expiry. Result: 75% of the USD notional can be sold at 1.1690 and the remaining 25% at the worst case protection rate 1.2250.

Scenario 3: EUR/USD trades below 1.1690 in month 9 and fixes at 1.1800 on expiry. Result: 75% of the USD notional can be sold at 1.1800 and the remaining 25% at the worst case protection rate 1.2250.

Alternative An alternative could be a 50% fader forward plus, where the client participates on 50% of the notional.

- Improve the worst case on 100% to 1.2200
- Or improve the best case on 50% to 1.1390
- Maintain net zero cost

Composition The details of this structure are

- The client buys EUR (sells USD) forward at 1.2250
- The client buys a 1.2250–1.1690 fader EUR put spread (USD call spread) with a lower corridor and knock-out barrier at 1.1689

Advantages

- Absolute protection at 1.2250 on 100% of the notional
- The client can take advantage of favorable market movements down to a pre-determined level
- The participation will lock in with the passage of time a best case EUR buying (USD selling) rate of 1.1690
- Zero cost

Disadvantages

- If the pre-determined knock-out level trades relatively early in the tenor period with minimal accrual, the net achieved rate will be only slightly better than the worst case rate

Summary This hedge has been designed to provide an absolute level of protection with a potential to benefit from favorable moves in the spot market to a pre-determined level. This benefit accrues with time, such that even if the pre-determined level trades the net achieved exchange rate will be better than the outright forward.

The fader forward plus allows participation in favorable spot movements. Similarly, the *fader forward extra* allows participation if the spot moves against the client's position. In this case, the client trades a synthetic forward and buys a fader EUR call spread as for example indicated in Table 2.6. For information we list the details of the structure and the prices of the components in Table 2.7.

TABLE 2.6 Example of a fader forward extra.

Trade date	Oct 9 2004
Spot reference	1.2165 EUR-USD
Outright forward reference	1.2159 EUR-USD
Swap points reference	−6 EUR-USD
EUR interest rate	2.32%
USD interest rate	2.26%
ATM reference volatility	10.35%
Client sells notional	USD 4,800,000
Maturity/delivery date	Jul 27/29 2005
Fixing on each business day	from trade date to maturity date
Worst case buy EUR (sell USD) on 100%	1.2300 EUR-USD
Upper corridor and barrier	1.2801 EUR-USD
Fader style	keep 100% of the accrued amount
Client buys fader EUR call spread	1.2300–1.2800 EUR-USD
Premium	0.00
Sales margin	EUR 13,205
Delta hedge: the bank buys	EUR 3,768,784

TABLE 2.7 Pricing details of a fader forward extra.

Product	Strike	Barrier	Bank	Upper Corridor	Price in EUR
Forward	1.2300		sells		−43,187.23
Fader call	1.2300	1.2801	sells	1.2801	47,257.87
Fader call	1.2800	1.2801	buys	1.2801	−17,275.91

2.1.8 Butterfly Forward

A butterfly forward entitles the holder to buy a specified amount of a currency (say EUR) on a specified date (expiry) at a non-fixed preferential rate if the spot rate will be within a pre-defined range at any time during the entire period until expiry. A butterfly forward allows the holder to take advantage of both an appreciation and a depreciation of EUR-USD up to the pre-defined trigger levels. The butterfly forward is a zero-cost strategy like the outright forward. The worst case exchange rate is absolutely guaranteed but always slightly worse than the outright forward rate. It is suitable for the corporate treasurer who can't make up his mind whether to trade a shark forward plus or extra, i.e. his only view on the future development of the exchange rate is movement within the range.

Advantages

- Worst case W is guaranteed
- Zero-cost product

Disadvantages

- If the exchange rate will be out of the range $[L, H]$ at expiry, the final exchange rate will be worse than the outright forward rate

For example, the company wants to buy 10 M EUR. At maturity:

1. If $S_T > H$ or if $S_T < L$, the company would buy 10 M EUR at strike W (worst case).
2. If $H > S_T > L$, the company would buy 10 M EUR at strike S_T if $S_T \leq W$ or $2W - S_T$ if $S_T > W$.

Composition In the butterfly forward the corporate buys a synthetic forward (see Section 2.1.1) with strike at the worst case and buys a double-knock-out straddle with the same strike and barriers. Knock-out barriers can be either American or European style or Bermudan or windows.

Example A company wants to hedge receivables from an export transaction in USD due in three months' time and is considering hedging the currency risk. It expects EUR-USD to move either up or down but remain within a predefined range for the entire period.

In this case a possible form of protection that the company can use is to buy a butterfly forward. An example is given in Table 2.8 and Figure 2.7.

The final exchange rate is stated under the condition that the spot rate does not trade at or through either the 1.2000 EUR-USD or 1.1000 EUR-USD triggers at any time during the entire period until expiry.

The butterfly forward enables the company to buy EUR from its USD-receivables in three months' time at a rate of 1.1503 EUR-USD. This exchange rate is 35 points above the current forward rate of 1.1468 EUR-USD and represents its worst case scenario. This rate shall be its final exchange rate if the prevailing exchange rate at expiry is equal to the worst case of 1.1503 EUR-USD and/or if the spot rate reaches either trigger level of 1.2000 EUR-USD or 1.1000 EUR-USD at any time during the entire period until expiry.

TABLE 2.8 Example of a butterfly forward for a EUR buyer/USD seller.

Spot reference S_0	1.1500 EUR-USD
Outright forward reference	1.1468 EUR-USD
Notional	EUR 10,000,000
Maturity T	3 months
Worst case W	1.1503 EUR-USD
Upper trigger (American style) H	1.2000 EUR-USD
Lower trigger (American style) L	1.1000 EUR-USD
Final exchange rate	$W - \lvert W - S_T \rvert$ EUR-USD
Premium	0.00

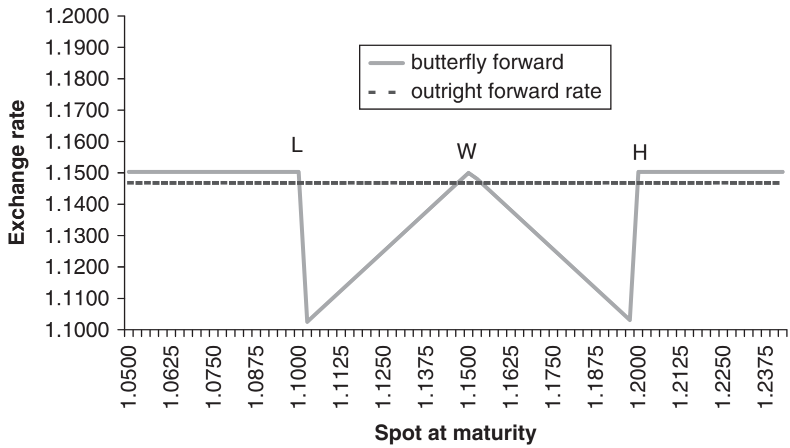


FIGURE 2.7 Final exchange rate of a butterfly forward for a EUR buyer.

If the prevailing exchange rate at expiry is above or below the worst case of 1.1503 EUR-USD and it never reaches the trigger levels during the entire period until expiry, then the company's final exchange rate will be calculated by subtracting the positive difference between the worst case and the spot reference at expiry from the worst case level. In this case the company can take advantage of both lower and higher exchange rates.

2.1.9 Range Forward

The range or bonus forward is another forward enhancement with guaranteed worst case. It entitles the holder to buy an agreed amount of a currency (say EUR) on a specified date (expiry) at a preferential rate (which will represent its best case scenario) if the spot rate is within a predefined range at expiry. Range forwards provide full protection for a foreign exchange transaction. The range forward is a zero-cost strategy like an outright forward. The holder will benefit from a best case scenario if the spot rate remains within a predefined range. However, if the spot rate is outside this predefined range, the holder will buy EUR at another rate, which represents the holder's worst case scenario. The worst case exchange rate is obviously slightly worse than the outright forward rate.

Advantages

- Worst case guaranteed
- Zero-cost product
- Worst case and best case known upfront

Disadvantages

- If the exchange rate will be out of the range at expiry, the final exchange rate will be worse than the outright forward rate

For example, the company wants to buy 10 M EUR. At maturity:

1. If $H < S_T$ or if $S_T < L$, the company would buy 10 M EUR at strike W (worst case).
2. If $H > S_T > L$, the company would buy 10 M EUR at strike B (best case).

Composition In the range forward the corporate treasurer trades a synthetic forward (see Section 2.1.1) with strike at the worst case and buys a double-no-touch.

Example An importer in the USA needs to sell USD/buy EUR in three months' time and is considering hedging the current risk. It expects that EUR-USD will end within a predefined range.

In this case a possible form of protection that the company can use is to trade a range forward as in Table 2.9 for example.

The range forward enables the holder to buy his EUR-liability in three months' time at a rate of 1.1600 EUR-USD. This exchange rate represents his worst case scenario. This rate will be his final exchange rate if the spot rate reaches or crosses trigger levels of either 1.1100 or 1.1700 at expiry. If the spot rate is within the predefined range of 1.1100 and 1.1700 at expiry then the final exchange rate will be the best case rate of 1.1303 EUR-USD at expiry. This is illustrated in Figure 2.8.

Alternatives The barriers can be American style barriers. Then the best case scenario happens if and only if the spot rate stays within the predefined range during the entire period until expiry. Usually this will lead to a wider range or a better best case. One can also think of one-sided touch transaction instead of a double-no-touch, or any touch transaction variations with window features, transatlantic double barriers, touch transactions with rebates. The principle is always the same. Flipping between a best case and a worst case is usually achieved with digital transactions, see also Section 2.5.5 on flip swaps, or Section 2.4.6 on tower deposits.

2.1.10 Range Accrual Forward

A range accrual or corridor or simply accrued forward is a forward contract in a currency pair, say EUR/USD, with a guaranteed worst case exchange rate. The worst case improves by a specified number of pips for each day between trade date and maturity

TABLE 2.9 Example of a range or bonus forward.

Spot reference	1.1500 EUR-USD
Outright forward reference	1.1471 EUR-USD
Notional	EUR 10,000,000
Maturity	3 months
Worst case	1.1600 EUR-USD
Best case	1.1303 EUR-USD
Range	1.1100–1.1700 EUR-USD
Premium	0.00

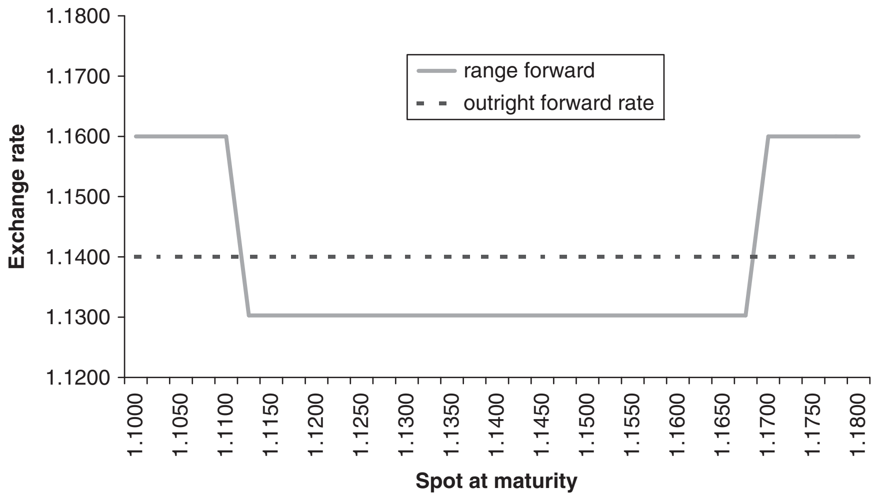


FIGURE 2.8 Final exchange rate of a range forward for a EUR buyer.

the EUR/USD fixing is inside a pre-determined range. In the standard resurrecting or European style range, the improvement does not happen on days when the fixing is outside the range, but the improvement can continue in the future. If all EUR/USD fixings are inside the range, then the exchange rate improves to the best case, which is above the outright forward rate. The standard range accrual forward is a zero-cost product and the worst case is worse than the outright forward rate. The product reflects a view of a quiet market phase ahead. If the view is correct then it can lead to a final effective exchange rate which is much better than the outright forward rate.

Advantages

- Worst case guaranteed
- Zero-cost product

Disadvantages

- No participation if the EUR/USD rate moves in the company's favor
- Many fixings outside the range may lead to an effective final exchange rate which is worse than the outright forward rate

Composition The range accrual forward works in a similar way to the range forward. The corporate treasurer buys a synthetic forward (see Section 2.1.1) with strike at the worst case, and instead of buying a double-no-touch, the client buys a corridor, see Section 1.8.2 and Figure 1.61.

Example A company wants to hedge receivables from an export transaction in USD due in three months' time. It expects the EUR/USD exchange rate to stay near the current spot over the next three months. The company wishes to use this view to enhance its

TABLE 2.10 Example of a range accrual forward.

Spot reference	1.1500 EUR-USD
Outright forward reference	1.1470 EUR-USD
Company	buys EUR sells USD
Notional	USD 1,000,000
Maturity	3 months
Worst case W	1.1550 EUR-USD
Best case	1.1265 EUR-USD
Resurrecting range	1.1400–1.1600 EUR-USD
USD pips per day (ppd)	3.49
Final exchange rate	$W/(1+ W\cdot$ days within the range $\cdot$ ppd/10,000)
Premium	0.00

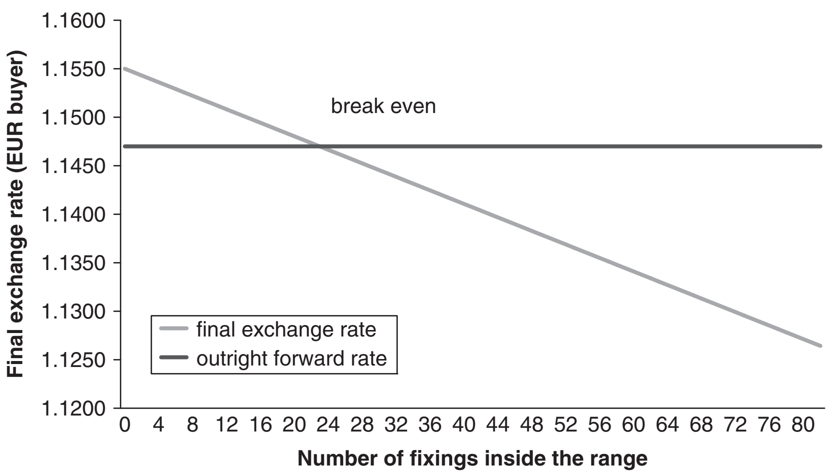


FIGURE 2.9 Final exchange rate of a range accrual forward for a EUR buyer/USD seller.

exchange rate, but would like to have a guaranteed worst case scenario. In this case a possible form of protection that the company can use is to enter into a range accrual forward as listed in Table 2.10 and illustrated in Figure 2.9 for example.

The range accrual forward enables the company to sell 1 M USD in three months' time at a rate of 1.1550 EUR-USD. This rate is 80 points above the current forward rate of 1.1470 EUR-USD and represents the worst case scenario. This rate is the final exchange rate if the spot is fixed outside the predefined range of 1.1400–1.1600 EUR-USD on every day until expiry. If the spot is fixed inside the range on some or all days until expiry, the worst case rate W will improve to the final exchange rate E, which is calculated by the following formula:

$$\frac{W}{1+\frac{WN_r p}{10{,}000}}, \tag{10}$$

where N_r denotes the number of days when the spot was fixed within the range and p the pips per day. For example, if the spot is fixed on 18 days within the predefined range until expiry, the final exchange rate will be

$$\frac{1.1550}{1 + 1.1550 \cdot 18 \cdot 3.49/10{,}000} = 1.1467 \text{ EUR-USD}.$$

This rate is equal to the current forward rate so that 18 days represents the break-even level. If, for example, spot is fixed on 50 days within the predefined range, the final exchange rate will be

$$\frac{1.1550}{1 + 1.1550 \cdot 50 \cdot 3.49/10{,}000} = 1.1322 \text{ EUR-USD}.$$

This is an improvement by 228 points compared with the worst case scenario and by 148 points compared with the current forward rate.

Alternatives Noticeably, the accrual range is very small, so this product appears difficult to sell from a psychological viewpoint. The reason is the resurrecting style range. If we change this to a non-resurrecting style range or American style, the improvement stops the first time the spot leaves the range. However, the company keeps the already accrued pips. This is more risky, but the range can be set wider.

Other alternatives include, of course, modifying the start and the end date of the range. For longer tenors it is advisable to use a series of forward start corridors, so the range will be reset around the future spot in, say, a quarterly schedule. This will give the client always fresh chances to accumulate forward enhancement pips even if the spot will move to a new level.

One important point to mention is to assure the schedule of fixings (daily, weekly, monthly, quarterly, ...) is fully agreed with the client. The safest way to go about this is to state the exact list of fixing days and agree before trading the contract. The other important issue is to clarify what happens if the fixing happens to be exactly on the lower or upper range. While a fair financial institution should treat such an instant always in favor of the client (tennis style), the front-office systems are sometimes programmed in a different way, namely counting fixings on the range as outside the range (golf style). For pricing it is of course irrelevant because in most continuous time models such an event has zero probability. However, a mismatch of agreement with the client and internal implementation can lead to losses in the options portfolio after the fixing is entered.

2.1.11 Accumulative Forward

An accumulative forward or *accumulator* is a (usually) zero-cost contract agreeing on a strip of future cash flows usually with some leverage and knock-out/knock-in conditions. The agreed exchange rate is better than the outright forward rate, but there is no guaranteed worst case. The accumulator has further uncertainty as the exact notional traded is typically not known at inception. It is popular in many countries in the world among many corporates. The trading parties agree on a payoff, say a plain forward, and specify ranges; with each fixing inside such a range a pre-determined notional amount of a specified currency for this payoff accumulates. There are many ways to build these contracts.

As an example we take an accumulative forward in EUR/USD, spot ref 1.1500. We let the expiry and delivery date be 12 months and take 250 ECB fixings. The client sells 200,000 USD at 1.1000 for each day the EUR/USD fixing is between 1.1000 and 1.2000. The client sells 400,000 USD at 1.1000 for each day the EUR/USD fixing is below 1.1000. In the extreme case of all fixing below 1.1000 the total amount accumulated would be 100 M USD. If the non-resurrecting knock-out level of 1.2000 is ever traded, then the accumulation stops but the client keeps 100% of the accumulated amount. This means that the amount accumulated becomes the notional of a forward contract with the pre-determined delivery date. It is up to the client to either keep or unwind the position and take the profit.

We notice here that there are many things to design.

Ranges. In principle arbitrarily many ranges can be set and for each range there is a notional amount that should be specified. They can even be overlapping. In our example we have three ranges, one from 0.0000 to 1.1000 with two times the notional, one from 1.1000 to 1.2000 with one times the notional, and one from 1.2000 to infinity with zero times the notional.

Knock-out Ranges. Knock-out ranges can be set anywhere and need not agree with the fade-in ranges.

Leverage. Usually the amount the client wishes to hedge is the basic notional. In our example it is 200,000 USD per business day. The client improves his exchange rate by taking risk if the spot moves against him, i.e. for the USD seller we consider here, a spot lower than 1.1000 would hurt. This loss is multiplied with a factor of 2 in our example. It is called the leverage of the accumulative forward. Any positive number is possible, common leverage factors are 1 and 2. One can be rather aggressive here to improve the desired low rate even more.

Resurrecting/non-resurrecting. Each range should be declared either resurrecting or non-resurrecting.

Amount kept in case of knock out. For resurrecting ranges, there is no knock-out. This is also commonly traded. However, the conditions are usually a bit worse than in the non-resurrecting case. Here the rate would be about 1.1250 instead of the 1.1000. We need to implement the two cases of *keep all* and *keep nothing* as they are different products. For general amounts to keep the *keep all* and *keep nothing* can be mixed by using notional amounts proportional to the mixing percentage.

Fixings. We need to specify the exact fixing schedule and the fixing source. We also need to clarify whether the daily amount on a day when the accumulative forward knocks out counts for the accumulated amount. The standard is that it does if the fixing is before the knock-out event on this day and it does not if the fixing is at or after the knock-out event.

Advantages

- Noticeable improvement of exchange rate
- Zero-cost product
- Allows numerous tailor-made features

Disadvantages

- No guaranteed worst case
- The amount traded is not fixed but depends of the spot movements, so it cannot be considered as a hedge and is unlikely to be eligible for hedge accounting in the IAS 39 sense.
- Generally risky product, entire hedge amount can be lost.

Composition The accumulative forward consists of fade-in calls and fade-in puts, possibly with extra knock-out ranges.

Comment Since this product does not have a worst case and can terminate early, it is usually just a part of a corporate hedging strategy along with outright forward or other enhanced forwards with guaranteed worst case. In case of a continuously rising exchange rate, the client keeps getting knocked out, may keep some of the accumulated amount as a forward contract, and can engage in a new accumulative forward with higher strike and ranges. This way the client always buys EUR at a rate less than the market at zero cost. So as an alternative to doing nothing the accumulative forward is worth considering. Compared with the outright forward, however, it can hurt the corporate budget a lot. So this may be a good add-on to more conservative hedging. Conversely, if the spot moves in the client's favor, the client is penalized, often in a multiple amount. Hence, it appears advisable to restructure the accumulative forward as soon as the spot moves below the strike. This means that this product needs a risk management policy and an attentive treasurer watching the market or a bank helping her to do this. Since the EUR seller sells EUR at a rate above the outright forward rate, the transaction is also called *sell-above-market (SAM)*. Similarly, since the EUR buyer buys EUR at a rate below the outright forward rate, the transaction is also called *buy-below-market (BBM)*. These terms are more frequently used in the equity derivatives context. The term *decumulator* also pops up once in a while referring to accumulator-like transactions, where at inception there is a full notional, which then decreases on the way by an amount per fixing day.

Extra Features

Rebates. As in the case of barrier options, one can pay a rebate amount in case of knock-out. Of course, this makes the product more expensive. Nevertheless, it makes it easier to sell a new product to the client who gets hurt by the knock-out event.

Fees. An accumulative forward need not necessarily trade at zero cost. A client can pay to improve the conditions or opt for a rather aggressive arrangement and even get a cash payment upfront.

Stripped Settlement. It is possible to settle the accumulated amounts at predetermined intermediate settlement dates. This basically means stripping the accumulative forward into a series.

Bounds on Amounts. There are ways to place upper bounds on the accumulated amount. In the example above, one could agree that the accumulation will stop when 75 M USD have accumulated. This is a *Parasian style* knock-out event. It can also be viewed as a *target feature*. We will revert to this idea in detail in Section 2.2 on target forwards.

Validity of Knock-out Barriers. Like in the shark forward, one can think of partial knock-out barriers.

Improved Rate on early Knock-out. It usually feels awkward to report a knock-out event to the client soon after inception of the trade. To avoid this one can agree on an exchange rate even below the improved one, say 1.0500 if the knock-out happens during the first month. Of course, this exchange rate is valid only for the amount accumulated up to the knock-out event. This can be included in the structure by the client buying a fade-in EUR call with *keep all* in case of knock-out and selling the same fade-in EUR call with *keep nothing*. Hence, if there is no knock-out, the two positions net each other, and conversely, if there is a knock-out, only the client receives a forward contract. The ranges should be chosen to match the basic version of the accumulative forward.

Digital Events. To improve the final exchange rate, one can include a digital event by agreeing to double the accumulated amount if the spot at expiration time is below the strike (and the accumulative forward has not knocked out until then). This can be achieved by the client selling another EUR put with the same specifications and ranges as the basic version and keeping nothing in case of knock-out. More generally, one can agree to an arbitrary multiple of the daily amount rather than doubling. And even more generally the digital strike may differ from the strike of the accumulative forward. For example, consider the following zero-cost structure for a EUR buyer/USD seller on spot reference of 1.2000 with a tenor of five months. The client buys one fold the EUR amount at 1.1700 between 1.1500 and 1.2500 and 2.3 fold below 1.1500, keeps 100% of the accumulated amount, and converts twice the accumulated amount if the spot is below 1.2000 at expiration time.

A Note on Pricing Fader contracts can be priced in closed form in the Black-Scholes model, see [65] for details. A tradable price can be obtained by an approximate hedge consisting of liquid first generation exotics, whose prices we know. The difference between the trading mid price and the TV (theoretical value in the Black-Scholes model using the at-the-money volatility) is called the *overhedge*. We then compose an approximate hedge, compute its overhedge, and take this overhedge as a proxy for the overhedge of the accumulator. This is very common practice among traders, particularly if they do not have access to an advanced model such as stochastic-local-volatility in their risk management system. In the simple case of a resurrecting fader, the contract merely consists of a sum of knock-out calls or puts, whose prices are known in the market. For the non-resurrecting accumulators, the approximation is slightly more complicated because the amount accumulated at the time of the knock-out event is not known in advance but has to be estimated.

Example We consider an accumulative forward in EUR/USD, at a spot reference of 0.9800 of September 24 2002. We let the tenor be 15 months. The client buys a total of 28 M EUR at an improved rate of 0.9150 with value date in 15 months. For each EUR/USD fixing between 0.9150 and 1.0500 the client accumulates 28 M EUR divided by the number of fixing days. For each EUR/USD fixing below 0.9150 the client accumulates twice this daily amount, such that in the extreme case of all fixing below 0.9150 the total amount accumulated would be 56 M EUR. If the non-resurrecting knock-out level of 1.0500 is ever traded, then the accumulation stops but the client keeps 100% of the accumulated amount.

The TV (theoretical value) of this structure can be calculated. Based on the volatility and interest rates term structure at that time it was about 400,000 EUR, which the client would have to receive. However, the hedge of the structure requires a massive overhedge, which can be computed as follows.

For the 0.9150 EUR calls reverse knock-out (RKO) at 1.0500 we determine the overhedge as the average of the maturities 6 to 15 months. We do the same for the 0.9150 EUR puts knock-out at 1.0500. The details are listed in Table 2.11.

On 28 M EUR 36 basis points (bps) are 101,000 EUR, which is the overhedge cost if we had to hedge the accumulator by buying the RKO calls. Similarly, on 56 M EUR 12 bps amount to 68,000 EUR, which is the overhedge cost for selling the KO puts. Both are priced at mid market, so fairly aggressive.

Next we need to worry about the cost of knock-out. If the knock-out level of 1.0500 is reached, the client has the right to buy the accumulated EUR amount at 0.9150 (as per the pre-determined value date), even though the spot is then at 1.0500. For the bank selling the accumulative forward this is substantial risk, which can be hedged by buying a 1.0500 one-touch with 15 months' maturity. The big unknown is the notional of this one-touch, which has to be approximated. First of all the amount at risk, called the *parity risk*, is the maximum intrinsic value

$$\begin{aligned} 1.0500 - 0.9150 &= 0.1350 \text{ USD per EUR} \\ &= 0.1286\% \text{ EUR}. \end{aligned} \tag{11}$$

TABLE 2.11 Overhedge of an accumulator using RKO EUR calls and KO EUR puts as an approximation.

	Basis points (in EUR)	
Tenor	**RKO calls**	**KO puts**
6 months	+25	−5
9 months	+40	−10
12 months	+40	−20
15 months	+40	−20
average	+36	−12

We approximate the time it takes to reach the parity level of 0.9150 by seven months. No math here, just guessing to have a number to work with. Next we need to estimate how much nominal will be accumulated within these seven months. Sometimes the spot fixing may be above and sometimes below 0.9150. We observe in the market that the price of a seven-month 0.9150 one-touch is 40%. We use this information to estimate that 40% of the fixings will be below 0.9150, accumulating 22.23 M EUR, and 60% of the fixings will be above 0.9150, accumulating 16.8 M EUR. The sum of these two amounts may be the total of 39.03 M EUR accumulated. The 15 months 1.0500 one-touch would cost 53.5%, whence the parity risk amount equals 39.03 M · 53.5% · 12.86% = 2.7 M EUR. The one-touch overhedge would be 2.7 M · 3% (mid market) = 81,000 EUR.

Finally, to hedge the vega of 206,000 EUR, we take the bid-offer spread of the price in volatilities and arrive at 206,000 · 0.15 (volatility bid-offer) = 31,000 EUR.

The total overhedge is now 101,000 + 68,000 + 81,000 + 31,000 = 281,000 EUR. This method is surely not fully exact but gives a very good approximating hedge and along its market price, which may very well compete with any more complicated model. The question as to whether to implement the hedge is a different matter. I consider the knock-out event and exploding Greeks the biggest source of risk. To compensate I would consider at least buying the 1.0500 one-touch, especially if the accumulator is in a separate book, i.e. separated from all other flow exotics.

The main risk in the accumulative forward to capture is the forward smile dynamics. A more scientific approach has been worked out by Baker *et al.* [7].

Example We consider an example of a GBP-EUR accumulative forward, whose terms and conditions are listed in Table 2.12. The ranges are illustrated in Figure 2.10.

Notional is computed by

$$\text{Notional} \cdot \left[\frac{\text{RangeDays}}{\text{Days}} + 2\frac{\text{TopDays}}{\text{Days}}\right], \tag{12}$$

TABLE 2.12 Terms and conditions of an accumulated forward in GBP-EUR.

Period	122 days
Start date	08-Apr-03
End date	24-Sep-03
Delivery date	26-Sep-03
Pre-agreed exchange rate	$K = 1.4200$
Knock-out barrier	1.5000
Client	sells EUR (buys GBP) at K
Notional	EUR 7,500,000.00
Notional per business day	EUR 61,475.41
Premium	Zero
EUR/GBP spot reference	1.4535 GBP-EUR
12mth outright forward	1.4463 (GBP 3.53%, EUR 2.45%)

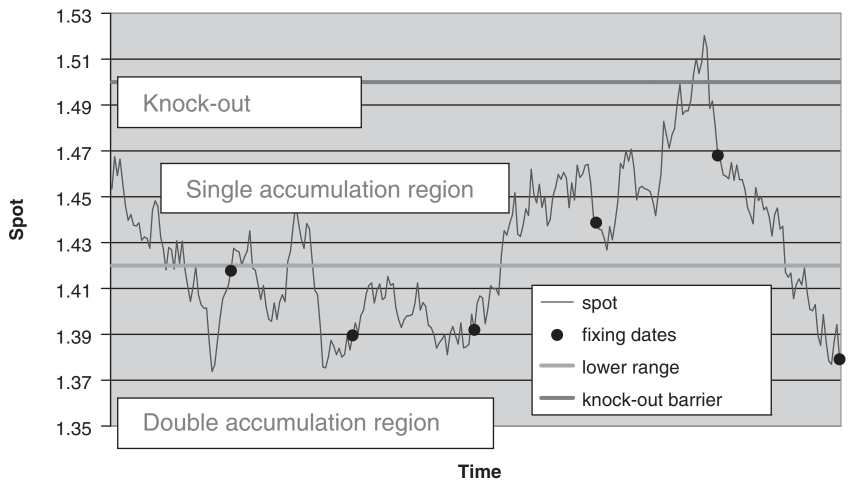


FIGURE 2.10 Ranges for an accumulative forward in EUR/GBP in GBP-EUR quotation.

Days is the number of all business days in the period.

RangeDays is the number of business days in the period that the reference rate fixes between the pre-agreed exchange rate and the knock-out barrier.

TopDays is the number of business days in the period that the reference rate fixes below the pre-agreed exchange rate.

Reference Rate is the EUR/GBP rate in GBP-EUR quotation by the European Central Bank published on Reuters page ECB37.

Settlement is physical on the delivery date.

Knock-Out Condition: If GBP-EUR trades at or above the knock-out barrier at any time, then the transaction terminates and the client sells 50% of the accumulated EUR amount up to the knock-out date if the knock-out occurs after 14h15 Frankfurt time. The GBP-EUR daily trading range will be determined by the bank and should a rate query arise, can be cross-referenced with three market making banks.

Example: If the GBP-EUR reference rate fixing remains in the range 1.4200 and 1.5000 for 110 days of the business days before trading at 1.5000, then the contract would terminate at that point with a notional of EUR 3,381,147.55, which the client sells at 1.4200 for settlement on the delivery date (26-Sep-03).

2.1.12 Boomerang Forward

The following zero-cost transaction could serve as an alternative for corporates, particularly those that trade accumulators. As an example we consider a treasurer buying EUR 10 M (selling USD) at a spot reference of 1.2000 and one-year outright forward reference of 1.2000.

- If EUR/USD remains above $B = 1.1349$ for the $T = 1$ year period, the bank will pay $(S(T) - 1.1790) \cdot 10/S(T)$ M EUR to the treasurer (cash settlement). Or in the case of delivery settlement, the treasurer can buy EUR at 1.1790, which is much better than the initial spot and the initial outright forward rate.
- If EUR/USD trades below B at any time during that year period, the treasurer pays $(1.2190 - S(T)) \cdot 10$ M USD to the bank, unless spot is above 1.2190 at maturity, in which case the bank pays $(S(T) - 1.2190) \cdot 10/S(T)$ M EUR to the treasurer. In other words, in case of knock-in at B the treasurer is locked into a forward contract with strike 1.2190. This is also his guaranteed worst case.

The boomerang forward can be structured as follows. The treasurer

1. buys 1Y 1.1790 EUR call USD put KO at 1.1349,
2. sells 1Y 1.2190 EUR put USD call RKI at 1.1349,
3. buys 1Y 1.2190 EUR call USD put KI at 1.1349,

all with a notional of EUR 10 M. The good feature here as compared with accumulators is the existence of a guaranteed worst case. The zero-cost strategy hurts the client only if the spot falls significantly, so the barrier is hit and the client is faced with a much higher exchange rate than the prevailing spot. However, to be fair, we must compare the final exchange rate to the outright forward rate at inception, and their difference is less than 2 big figures.

2.1.13 Amortizing Forward

A treasurer can enhance a company's effective foreign exchange rate at zero cost by taking risk but still participate in the exchange rate moving in its favor. One of the possible solutions can be an amortizing forward, which provides the treasurer with a worst case exchange rate, but if the spot rate moves against him the notional of the forward contract decreases following a pre-specified amortization schedule, shown in Figure 2.11.

For contract parameters maturity in years T, strike K, and knock-out barrier B, notional of the underlying N, payment schedule $0 = t_0 < t_1 < t_2, \dots, t_n = T$, the payoff is

$$F(S, K, B, t_i, N) \stackrel{\Delta}{=} N \sum_{m=1}^{n} (S_{t_m} - K) \prod_{i=1}^{m-1} \left[\min \left[1, \left(\frac{S_{t_i} - B}{K - B} \right)^{+} \right] \right]. \tag{13}$$

We denote by $v(t, x)$ the value of the amortizing forward at time t if the spot S_t takes the value x. The value at time zero is given by

$$v(0, S_0) = \mathbb{E}[e^{-r_d \tau} F(S, K, B, t_i, N)]. \tag{14}$$

A closed form solution or numerical integration based valuation are both possible in the Black-Scholes model. A straightforward exercise to determine a benchmark for the value would be a Monte Carlo simulation. The standard trading convention is to determine B for a desired worst case K in such a way that the fair value is slightly negative, so the transaction can be traded at zero cost for the treasurer.

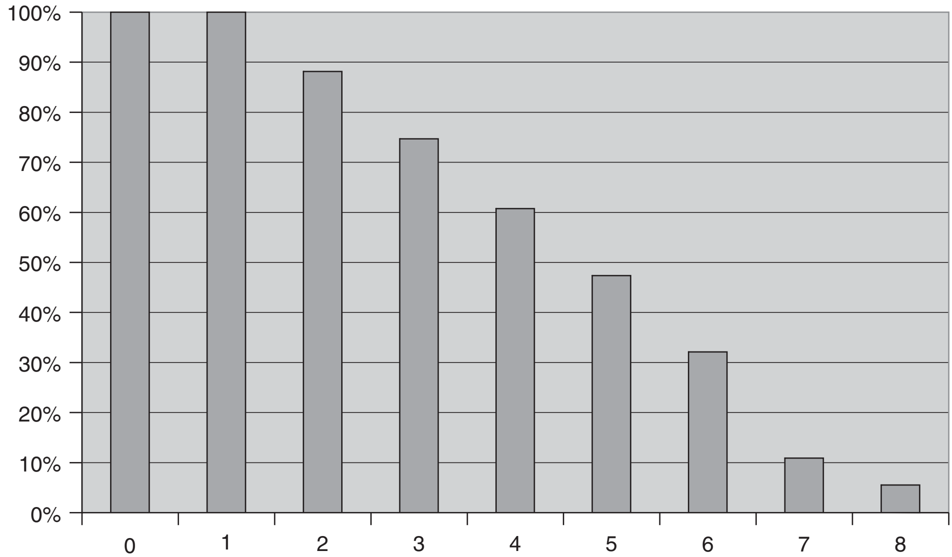


FIGURE 2.11 Amortization schedule of an amortizing forward contract for a EUR buyer/USD seller.

Example We consider a EUR buyer/USD seller for a total initial notional of $N = 10$ M USD for a period of two years with quarterly settlements. This means that at the end of each following eight quarters the corporate treasurer sells USD. A worst case $K = 1.1650$ is guaranteed and is taken slightly worse than the two-year outright forward rate. If the spot moves up over the next two years, then the treasurer can benefit from the guaranteed worst case. The worst case exchange rate is always going to be the rate at which the notional is traded. The only changing parameter is the notional. If the spot goes up, the full notional is traded. If the spot moves down, then the notional decreases proportional to the distance of the spot from the worst case, i.e. by the factor

$$\left(\frac{S_{t_i} - B}{K - B}\right), \tag{15}$$

but only if the spot at time t_i is below the worst case K and above the barrier B. If any of the S_{t_i} is below B then all the future notionals are zero, thus the contract terminates early. Consequently, when the exchange rate depreciates, the worst case is binding, but only for a smaller notional, and the more the spot movement hurts the treasurer, the less notional is required to trade. An example is provided in Table 2.13.

We illustrate a possible scenario in Table 2.14 and Figure 2.11. At the last quarter, for example, the treasurer is obliged to sell 600,000 USD at 1.1650 even though the market spot is at 1.0200. However, the treasurer can sell the remaining 9.4 M USD at spot.

TABLE 2.13 Example of an amortizing forward.

Spot reference	1.1500 EUR-USD
Outright forward reference	1.1566 EUR-USD
Notional	USD 10,000,000
Maturity	2 years
Worst case	1.1650 EUR-USD
Barrier	0.8700 EUR-USD
Schedule	Quarterly
Premium	0.00

TABLE 2.14 Possible amortization schedule.

Quarter	1	2	3	4	5	6	7	8
Spot reference	1.18	1.13	1.12	1.11	1.10	1.07	0.97	1.02
Notional traded in %	100	88	75	61	47	32	11	6

2.1.14 Auto-Renewal Forward

Once more, we consider a client who wants to enhance his forward exchange rate at zero cost by taking up a portion of controlled risk. The auto-renewal forward is a strategy consisting of a series of forward contracts which is automatically renewed. For contract parameters maturity in years T, levels K_1 and K_2, renewal barrier B, payment schedule $0 = t_0 < t_1 < t_2, \dots, t_n = T$, the payoff is

$$F(S, K_1, K_2, B, \{t_i\}, N) \triangleq \sum_{N=1}^{J} \prod_{n=0}^{N-1} I\!I_{\{S_{t_{ln}} > B\}} H\left(K_1, K_2, \{S\}_{t_{ln+1}, \dots t_{ln+l}}\right),$$

$$H\left(K_1, K_2, \{S\}_{t_j, \dots t_k}\right) \triangleq \sum_{i=j}^{k} \Big[(K_1 - S_{t_i})^+ + (K_2 - S_{t_i})^+ I\!I_{\{S_{t_j} > K_1\}} \\ -2(S_{t_i} - K_2)^+\Big]. \tag{16}$$

We denote by $v(t, x)$ the value of the auto-renewal forward at time t if the spot S_t takes the value x. Its risk-neutral time zero value is given by

$$v(0, S_0) = I\!E[e^{-r_d T} F(S, K_1, K_2, B, t_i, N)]. \tag{17}$$

The contract parameters K_1, K_2, and B are usually determined in such a way that the market value at inception of the contract is slightly negative, so it can be traded at zero cost with the client.

Example Taking a USD/JPY spot ref 120.00, we consider a company receiving 1 M USD per quarter for the next three years. The company wishes to exchange the USD into JPY at an enhanced rate, i.e. the higher the better. However, a worst case exchange rate for the USD seller should be guaranteed. In this case the conditions of the auto-renewal forward could be set as follows.

- The company sells 1 M USD at the end of each quarter of the first year.
- If the exchange rate at the end of the quarter is below 110.00, then the company sells at 110.00.
- If the exchange rate at the end of the quarter is between 110.00 and 125.00, then the company sells at 125.00.
- If the exchange rate at the end of the quarter is above 125.00, then the company sells twice the amount at 125.00.
- If at the end of the first year, the exchange rate is above 120.00, then the strategy is automatically renewed for another year.
- The maximum number of years will be three.

In this example, the contract parameters are $K_1 = 110.00$, $K_2 = 125.00$, $B = 120.00$, $J = 3$, $l = 4$. Usually, J denotes the number of years and l the number of delivery dates per year. So if $l = 4$, then the company has agreed on quarterly delivery.

2.1.15 Double Shark Forward

The double shark forward is structured just like a shark forward (see Section 2.1.6). In the case of a USD buyer, the treasurer enters into a transaction that is equivalent to buying a EUR put with strike equal to the worst case and selling a reverse knock-in EUR call. It happens frequently that the barrier of the reverse knock-in is too close to the spot and therefore likely to be hit. In the case of such a barrier event the treasurer would be left with the worst case. If we want to move the knock-in barrier further away and still have a zero-cost product, we can increase the notional of the reverse knock-in call, commonly by a factor of 2

Example We consider a Euro zone company that needs to buy 10 M USD in eight months due to an import transaction. A double shark forward (plus) example is presented in Table 2.15.

The terms and conditions mean that the treasurer is protected against falling EUR with a worst case of 1.1250 at no cost. At the same time, the treasurer can participate in a rising EUR all the way up to 1.3080. The only crucial risk is that the amount

TABLE 2.15 Example of a double shark forward (plus).

Spot reference	1.1500 EUR-USD
Outright forward reference	1.1460 EUR-USD
Notional	USD 10,000,000
Maturity	8 months
Worst case	1.1250 EUR-USD
Knock-in barrier (European style)	1.3080 EUR-USD
Premium	0.00

will be doubled if EUR/USD trades above 1.3080 on the expiration date. This event is considered extremely unlikely, but if it happens, then the treasurer has to buy twice the USD notional. For example, if the spot ends at 1.3250, the corporate is obliged to buy 20 USD at 1.1250 even though the market is at 1.3250. This may cause a loss if the treasurer does not have the USD 20 M but needs to buy the other half in the market. It would mean a loss of 20 big figures on a notional of 10/1.1250 M EUR, which is approximately EUR 1.34 M. Thus, if the unlikely knock-in event happens, the damage is disastrous if the treasurer does not have the cash he is required to trade. The safer method would be to use the transaction's double amount as the underlying USD amount to buy. This implies that the treasurer has an insurance for USD 10 M USD and the other half is unhedged. Still a lot better than having no hedge at all.

2.1.16 Forward Start Chooser Forward

Another possible forward enhancement with guaranteed worst case would be to trade a synthetic forward with worst case as strike, pretty much like most of the forward catalog products. Here the client buys a forward start chooser option, i.e. the strike will be set in the future on the forward start date and the client can decide to take either a call or a put. One can additionally allow forward starting barriers. This product is useful if the corporate treasurer will have a market view only at a later stage, namely at the forward start date.

2.1.17 Free Style Forward

If the corporate treasurer has no view on the exchange rate at all and is willing to leave the profit maximization to the bank, then the treasurer can trade a worst case forward contract with the bank. In turn, the bank is taking proprietary trading positions on behalf of the corporate client and lets him participate in any gains created from there. This requires a substantial trust factor, as the client has no way to double-check what the bank does. The bank would in turn try to maximize the effective final exchange rate for the client to keep him happy and ready for the next trade. This approach is a mix of FX risk management with a worst case exchange rate and an asset management approach.

2.1.18 Boosted Spot/Forward

The boosted spot/forward is a zero-cost spot/forward deal with a rate better than the market, which is financed by selling an option, usually an out-of-the-money vanilla. The improved spot/forward rate is often chosen to match the strike of the sold vanilla option.

Advantages

- Improved exchange rate for spot/forward deal
- Zero-cost product

Disadvantages

- Client needs a credit line to go short the option
- Unlimited risk in the short option position

This is a product that usually looks good in the beginning and after that it becomes a continuous worry about the option staying out-of-the-money It is not suitable for hedge-accounting but a rather speculative strategy.

Example We consider a EUR buyer, USD seller, who gains three big figures in his spot deal by selling a 12 month EUR put, see Table 2.16.

Clearly the cost of the spot deal is

$$1{,}000{,}000 \left(\frac{1}{1.1200} - \frac{1}{1.1500} \right) \text{EUR} = 23{,}292 \text{ EUR}.$$

On a 10% volatility, 1% USD interest rate, and 2% EUR interest rate, the EUR put pays 23,433 EUR, so the sales margin would be 141 EUR.

The philosophy behind this structure is that the client wants to buy EUR, so as a sales representative one might argue as follows. If the spot moves up or stays above 1.1200, then the client should be happy having bought the EUR for less than the market at no cost. If the spot moves down below 1.1200, then the client just gets a few more EUR at the improved rate. The improved rate must be compared to the spot at inception, not to the prevailing spot rate. For large downward movements, however, this may cause a loss if the client type is an investor without the underlying cash flow. In this case one could limit the risk on the downside by introducing a floor or a barrier. However, we would then have to increase the notional or the strike. For the corporate treasurer with the underlying cash flow, having to buy EUR, this could be a good deal to buy EUR below market at inception and possibly buy some more EUR later at a rate that is still better than the outright forward observed at inception. However, in practice, we see this type of structure traded in the inter bank market rather than as a corporate hedging strategy.

TABLE 2.16 Example of a boosted spot. And yes: the client sells both USD and a USD call.

Spot reference	1.1500 EUR-USD
Improved spot rate	1.1200 EUR-USD
Client sells	USD 1,000,000
Client sells	EUR put USD call
Notional	750,000 EUR
Maturity	12 months
Strike	1.1200 EUR-USD
Premium	0.00

2.1.19 Flexi Forward/Time Option

A flexi forward or time option or American style forward contract entitles *and* requires the holder to buy/sell a pre-determined amount of a currency at a pre-determined strike K within a given time period $[T_1, T_2]$ in the future. The holder can choose times in this period to exercise parts of the total amount, where he shouts, which is why this type of contract is also called a *shout forward*. The right to choose the time of the foreign exchange transaction supports the term "time option," although the contract is not considered a currency option. The fact that the holder has to trade makes it a forward contract with a flexible transaction time, which is why the contract is also called "flexi forward." One may think of it as an American style forward contract. Obviously, the optimal transaction time would apply to the entire amount.

Valuation Mathematically, the value at time zero is given by

$$v(0) = \sup_{\tau \in [T_1, T_2]} I\!E \left[e^{-r_d \tau} \phi (S_\tau - K) \right], \tag{18}$$

where τ is a stopping time. As in the case of the outright forward, the strike K^* is usually determined by requiring $v(0) = 0$. Now, based on a simple arbitrage argument, taking EUR/USD as a default exchange rate, we observe

$$K^* \geq \max_{t \in [T_1, T_2]} f(t) \text{ for the EUR buyer} \tag{19}$$

$$K^* \leq \min_{t \in [T_1, T_2]} f(t) \text{ for the EUR seller,} \tag{20}$$

where $f(t)$ denotes the outright forward rate for value date t. As a matter of fact, the inequalities can be strict. However, it is advisable to verify that the front-office system contains a correct implementation of this product. The optimal time to transact is mathematically difficult to determine; however, the buy-side strategy is usually not optimal but need-based. Only the sell-side has to consider the worst case from her point of view. Flexi forward trades have very little FX vega risk. In the absence of stochastic interest rates, the client ends up dealing at the worst rate between T_1 and T_2.

Advantages

- Complete flexibility in the transaction schedule
- Zero-cost product
- Guaranteed worst case

Disadvantages

- Worst case is usually not better than any outright forward rate over the time horizon
- No participation in the spot moving in the client's favor
- Cumbersome for the client to determine the optimal exercise region

2.1.20 Strike Leverage Forward

In June 2007 we are looking at a GBP-USD (cable) spot reference of $S_0 = 2.0000$. Herbert, our sample client buying a *strike leverage forward*, is a GBP seller and USD buyer, so would benefit from GBP-USD going up. The following view is reflected in the strategy: Herbert wants to lock in high levels of the spot and take advantage of the spot fixing near the highs. He expects GBP-USD to be fixing above 2.0500 very few times or never over the next two years. We illustrate a sample transaction in Table 2.17.

2.1.21 Escalator Ratio Forward

We consider the situation of a treasurer buying EUR selling GBP over the coming 12 months. As an exporter in the Euro zone she would like to reduce her FX risk of rising EUR/GBP rates as much as possible. She considers the risk of a moderately stronger EUR as higher than the risk of weaker EUR. Generally, she is interested in achieving substantially lower EUR buying rates than the ones that are currently on a historic high. Under these assumptions she would like to realize an attractive EUR buying rate for nominal as large as possible. As a solution she could use an *escalator ratio forward*: she reduces her FX risk of rising EUR/GBP substantially, however, the strategy does not guarantee to cover the entire exposure over the entire 12 months. The main

TABLE 2.17 Example transaction of a strike leverage forward traded in June 2007.

Spot/average forward reference	2.0000 / 1.9550 GBP-USD
Best/worst case	2.0000 / 1.9500 GBP-USD
Expiry dates	Mid month from May 2009 to Dec 2009 with the possibility to extend to Jan 2010 to Aug 2010 (8 dates with the possibility of an additional 8)
	At each expiry from mid-month May 2009 to Dec 2009
Client buys	GBP put USD call in USD 10,000,000
Client sells	GBP call USD put in USD 12,500,000
The strike K	starts from 2.0000 and decreases by 0.0025 for every weekly fixing above 2.0500, down to a worst case of 1.9500
The strike K	$\max(1.9500, 2.0000 - 0.0025 \times n)$
n	number of fixings above 2.0500
Total number of fixings	100 weekly **1FED** fixings
First fixing	in week 1, then weekly
Last fixing	beginning of May 2009, so before the first expiry the strike will be fixed for all expiry dates
Extension condition	If the **1FED** fixing on 15 May 2009 is above 2.0500, then additionally leveraged forwards trade
Client buys	GBP put USD call in USD 10,000,000
Client sells	GBP call USD put in USD 12,500,000
Expiry dates	mid Jan 2010 until mid Aug 2010 (8 expiry dates)
Strike K	same as for the previous 8 expiry dates
Premium	0.00

purpose is to supplement an existing hedging strategy of the FX exposure to improve the exchange rate. The escalator ratio forward comprises monthly purchases of EUR at a worst case rate of 0.8750 via a strip of leveraged (ratio 1:2) forward contracts. This worst case can gradually improve (i.e. be lowered) on each fixing date provided the EUR/GBP fixing is above 0.8250. This escalator-like mechanism reduces the EUR buying rate. Once the reduction reaches a rate of 0.6500, the current and all further transactions are canceled. Details are given in Table 2.18. We illustrate a scenario analysis in Table 2.19.

TABLE 2.18 Indicative terms and conditions of an escalator ratio forward.

Maturity	12 months maximum
Monthly nominal	EUR 500,000 / EUR 1,000,000
Premium	zero
EUR/GBP spot reference	0.8750
EUR/GBP forward reference	0.8720 (average over next 12 months)
Initial exchange rate K_0	0.8750 (30 GBP pips worse than forward ref)
Target exchange rate	0.6500 (2,250 pips better than the forward ref)
Escalator barrier B	0.8250
Fixing F_i	monthly ECB Fixing in each month i, $i = 1, \ldots, 12$
Effective exchange rates K_i	reset monthly based on previous month effective strike via $K_i = K_{i-1} - \max[0, F_i - B]$ to be applied in month i
Early termination condition	As soon as the effective exchange rate K_i reaches or exceeds the target exchange rate, no settlement takes place at that fixing date and on all following fixing dates
Ratio (leverage)	if the structure has not terminated: if $F_i \geq K_i$, you buy EUR 500,000 at K_i on the fixing date if $F_i < K_i$, you buy EUR 1,000,000 at K_i on the fixing date

TABLE 2.19 Sample scenario of an escalator ratio forward, designed for an exporter in the Euro zone in December 2008.

Month	Fixing	Exchange rate calculation	K_i	Transaction
1	0.8700	0.8750 – (0.8700 – 0.8250)	0.8300	Buy EUR 0.5 M at 0.8300
2	0.8000	no change	0.8300	Buy EUR 1 M at 0.8300
3	0.8300	0.8300 – (0.8300 – 0.8250)	0.8250	Buy EUR 0.5 M at 0.8250
4	0.8700	0.8250 – (0.8700 – 0.8250)	0.7800	Buy EUR 0.5 M at 0.7800
5	0.9100	0.7800 – (0.9100 – 0.8250)	0.6950	Buy EUR 0.5 M at 0.6950
6	0.9200	0.6950 – (0.9200 – 0.8250)	0.6000	Early termination
7	0.9300			No further transactions
8	0.8600			
9	0.7700			
10	0.8900			
11	0.9400			
12	0.8600			

Advantages

- Noticeable reduction of the FX exposure in EUR/GBP to guaranteed rates between 0.6500 and 0.8750
- Designed as an add-on to an existing hedging strategy at no further cost
- If EUR/GBP trades in a range over the next couple of months, the structure terminates and a similar one can be traded at similar conditions

Disadvantages

- Not suitable as 100% hedge of cash flows against a persistent and long-term rise of EUR/GBP
- Not suitable as a stand-alone hedge
- Notional to be traded unknown upfront

Variations and Assessment Modifications are possible, for example, one can change the ratio factor (leverage factor); in fact, one could have a term structure of the ratio factor, i.e. an individual ratio factor for each maturity. The structure combines the idea of a target feature explained in Section 2.2 with a step technique. The structure is not statically decomposable into simple building blocks. It is a typical structure used in an environment where the structuring desk uses a payoff language.

2.1.22 Intrinsic Value Ratio Knock-Out Forward

Here is a sample structured forward transaction frequently offered in 2008. It combines the idea of a leveraged accumulative forward with the target feature in Section 2.2.

We consider the basic situation of an exporter in the EUR zone. The treasurer needs to sell GBP and buy EUR monthly over the next 12 months and is looking for a hedge against a rising EUR/GBP exchange rate. In the mid term the treasurer believes it is more likely that EUR will go weaker moderately than stronger against sterling. However, the short-term view is that EUR/GBP will trade in a range above 0.7725. Sounds like a typical situation for the "do nothing – wait for a better spot" strategy. The treasurer would rather wait a bit to gain from a stronger pound than lock in an unfavorable forward series at current spot level of 0.8050. Instead of doing nothing the treasurer might consider the intrinsic value ratio knock-out forward. It is a strategy that reduces the FX risk of rising EUR/GBP without guaranteeing all the notionals over the complete time horizon. In return, the treasurer can already lock in a lower EUR/GBP rate. The deal is to be viewed as an alternative to waiting rather than playing it safe. The structure comprises monthly purchases of EUR at an attractive guaranteed rate of 0.7725 with 1:2 ratio. Gains caused by favorable currency transactions will be accounted for and accumulated, whereas losses are ignored in the calculation of the accumulative gain. As soon as the accumulative gain reaches a pre-defined target, the current and all remaining transactions are terminated. Details are given in Table 2.20.

We illustrate a sample scenario in Table 2.21.

TABLE 2.20 Indicative terms and conditions of an intrinsic value ratio knock-out forward.

Maturity	12 months maximum
Monthly nominal	EUR 500,000 / EUR 1,000,000
Premium	zero
EUR/GBP spot reference	0.8050
EUR/GBP forward reference	0.8075 (average over next 12 months)
Exchange rate	$K = 0.7725$ (350 GBP pips better than forward ref)
Monthly gain	$\mathrm{MG}_i = \max[0, F_i - K]$
Fixing F_i	monthly ECB fixing in each month i, $i = 1, \ldots, 12$
Cumulative gain after X months	$\mathrm{CG} = \sum_{i=1}^{X} \mathrm{MG}_i$
Target	0.1200 GBP per EUR
Early termination condition	As soon as CG reaches or exceeds the target If the target is reached on a fixing date, no settlement takes place at that fixing date and on all following fixing dates
Ratio (leverage)	if the structure has not terminated: if EUR/GBP fixes at or above 0.7725, you buy EUR 500,000 for GBP at 0.7725 on the fixing date if EUR/GBP fixes below 0.7725, you buy EUR 1,000,000 for GBP at 0.7725 on the fixing date

TABLE 2.21 Sample scenario of an intrinsic value ratio knock-out forward, designed for corporate treasury hedging in September 2008.

Month	Exchange rate	Fixing	MG	CG	Transaction (on the fixing date)
1	0.7725	0.8000	0.0275	0.0275	Buy 0.5 M EUR at 0.7725
2	0.7725	0.7925	0.0200	0.0475	Buy 0.5 M EUR at 0.7725
3	0.7725	0.7975	0.0250	0.0725	Buy 0.5 M EUR at 0.7725
4	0.7725	0.8075	0.0350	0.1075	Buy 0.5 M EUR at 0.7725
5	0.7725	0.7900	0.0175	0.1250	Early termination
6	0.7725	0.7800			No more transactions
7	0.7725	0.7750			
8	0.7725	0.7825			
9	0.7725	0.7650			
10	0.7725	0.7600			
11	0.7725	0.7525			
12	0.7725	0.7500			

Advantages

- Reduction of the FX risk EUR/GBP at a guaranteed rate of 0.7725
- Hedge for a transition phase and add-on to an existing hedge portfolio to maximize the final EUR amount
- If EUR/GBP stays in the range in the next couple of months, the gain can be realized and a similar transaction can be traded at the same level as a subsequent hedge

Disadvantages

- No 100% hedge against long-term high EUR/GBP rates
- Not suitable as stand-alone hedge

Variations With the intrinsic value ratio knock-out forward one could also offer an even better strike for the earlier maturities and accept worse strikes for the later maturities (in case there is no knock-out). This product is indeed primarily meant to replace a pure waiting strategy for a lower EUR-GBP spot. Further variations can be added as in the case of accumulators, including knock-out levels. This is quite obviously a target forward running under a different name. Hence, all variations presented in Section 2.2 apply.

2.1.23 Tender Linked Forward

The *tender linked forward* is a structure that turns into a forward contract if the client wins a tender. If the client does not win the tender then no settlements take place. What happens is linked to an event independent of the FX spot that happens with an unknown probability. For example, consider a client that is a EUR based company and tenders large projects in USD. To hedge this, the company uses an insurance-like transaction that guarantees it a specified forward rate in n months. If the company does not win the tender it has to pay only 5% of the premium; however, if the company wins the tender the full premium must be paid. In general the premium is equal to an n month vanilla option. We consider an example in Table 2.22.

To clarify, the contractual forward rate is identical to the outright (market) forward rate. The deal is that the client pays 105,000 EUR at inception and in three months pays nothing else and ends up with no rights or obligations if the tender is lost, but pays a total of 2.1 M EUR (i.e. another EUR 1,995,000) if the tender is won and then will sell USD 100 M at 1.3883 no matter where the spot is, i.e. enters a forward contract, not a USD put option. The probability of winning the tender is not known. I leave it to you to figure out how to hedge this. Enjoy!

TABLE 2.22 Example of a tender-linked forward contract.

Company sells	USD 100 M
Maturity	3 months
Spot reference	1.3900 EUR-USD
Forward rate	1.3883 EUR-USD
Vanilla premium	EUR 2.1 M (USD put EUR call) to be paid if tender is won
Premium	EUR 105,000 to be paid if tender is lost
ATM volatility	13.95%
RR	1.114%
BF	0.383%

TABLE 2.23 Terms and conditions of a *contingent rebate* structure.

Currency pair	EUR-USD
Nominal	USD 1,000,000.00
Maturity	4 months from now
Strike	1.2000
Range	1.1500–1.2500
Type of barrier	American
Premium	EUR 28,000 to be paid after two working days

2.1.24 Exercises

Contingent Rebate Structure Given a market situation of EUR-USD spot 1.2000 and four-months forward of 1.1960 and terms listed in Table 2.23.

For your EUR buy/USD sale in four months you have guaranteed a worst case scenario of 1.2000 EUR-USD. This rate is 0.40 USD cents above the current outright forward of 1.1960 EUR-USD.

- In case the EUR-USD exchange rate at maturity is above 1.2000, you exercise your right to sell your USD at 1.2000.
- You may participate arbitrarily, if the EUR-USD exchange rate decreases.
- If the EUR-USD exchange rate stays within the specified range for the entire lifetime of the structure, your premium will be returned to you.

How would you structure this product? And how much sales margin would it generate for a volatility of 10%, EUR interest rate of 3% and USD interest rates such that the forward comes out correctly, if it is priced in the Black-Scholes model?

Structured Forward with Improved Exchange Rate Given a market situation of EUR-USD spot 1.2000 and four-months forward of 1.9960 and terms listed in Table 2.24.

You have ensured a worst case of 1.2060 EUR-USD for your EUR purchase/USD sale in four months. This exchange rate is 1.00 USD cents above the current outright forward of 1.1960 EUR-USD. The worst case will be used in one of the following cases.

- If at maturity the EUR-USD exchange rate is below 1.2060 EUR-USD, then the USD notional will be exchanged into EUR with the worst case rate of 1.2060 EUR-USD.
- If the EUR-USD exchange rate ever trades once at or above the trigger of 1.2300 EUR-USD, then the strategy changes into the obligation to buy EUR and sell USD at a rate of 1.2060 EUR-USD.
- If until maturity the exchange rate EUR-USD never touches or crosses 1.2300 EUR-USD, and if at maturity the spot reference is above 1.2060, then the final exchange rate is computed using the above formula.

TABLE 2.24 Terms and conditions of a structured forward. The final amount of EUR applies only in the case that the reference spot at maturity is above 1.2060 EUR-USD and the trigger 1.2300 has not been touched or crossed during the lifetime of the structure.

Currency pair	EUR-USD
Nominal	N = USD 1,000,000.00
Maturity	T = 4 months from now
Worst case exchange rate	$W = 1.2060$
Trigger	$B = 1.2300$
Trigger type	American
Final realized exchange rate	Nominal / final amount of EUR
Final amount of EUR	$N\left[\frac{1}{W}+\left(\frac{1}{W}-\frac{1}{S_T}\right)\right]$

How would you structure this product? And how much sales margin would it generate for a volatility of 10%, EUR interest rate of 3%, and USD interest rates such that the forward comes out correctly, if it is priced in the Black-Scholes model?

Flip Forward Given a market situation of EUR-USD spot 0.9000 and four-months forward of 0.8960 and terms listed in Table 2.25.

- For your EUR-purchase/USD-sale in four months you have a preliminary exchange rate of 0.8850.
- This rate is 1.10 USD cents better than the outright forward 0.8960 EUR-USD and will be used if the trigger is not touched or crossed at maturity and spot is above the preliminary exchange rate at maturity.
- If the EUR-USD exchange rate touches or crosses the trigger on the maturity date, then you must sell your USD at a rate of 0.9050.

How would you structure this product? And how much sales margin would it generate for a volatility of 10%, EUR interest rate of 3%, and USD interest rates such that the forward comes out correctly, if it is priced in the Black-Scholes model?

Structured Forward with Doubling Option Given a market situation of EUR-USD spot 0.8900 and six-months forward of 0.8850 and terms listed in Table 2.26.

TABLE 2.25 Terms and conditions of a flip forward.

Currency pair	EUR-USD
Nominal	USD 1,000,000.00
Maturity	4 months from now
Preliminary exchange rate	0.8850
Worst case	0.9050
Trigger	0.9200
Trigger type	European – valid at maturity only

TABLE 2.26 Terms and conditions of a structured forward with *doubling option*: the bank has the right to double the notional at maturity.

Nominal	USD 1,000,000.00
Maturity	6 months from now
Worst case	0.8850 EUR-USD
American style knock-in trigger K	0.9850 EUR-USD

You have locked your USD purchase in six months at a rate of 0.8850 EUR-USD. This is your worst case scenario. This rate is also identical with today's outright forward.

The potential forward rate of 0.8850 EUR-USD will be used if one of the following cases will occur.

- The EUR-USD exchange rate at maturity is below 0.8850 EUR-USD. In this case you buy USD 1,000,000 at a rate of 0.8850 EUR-USD.
- The EUR-USD exchange rate touches or crosses the trigger 0.9850 EUR-USD at some time before maturity. In this case you are bound to a forward contract with the worst case rate of 0.8850 EUR-USD.

If the EUR-USD exchange rate never touches or crosses the trigger 0.9850 EUR-USD until maturity, and if at maturity the exchange rate is above 0.8850, you do not use the forward contract but buy the USD at the exchange rate spot at maturity. This way you can participate in a rising exchange rate up to a maximal level of 0.9849 EUR-USD.

At maturity the bank has the right to double the nominal amount, if EUR-USD has ever traded above the trigger 0.9850 EUR-USD before maturity *and* EUR-USD trades above 0.8850 EUR-USD at maturity. In this case you would buy USD 2,000,000 at the rate of 0.8850 EUR-USD.

How would you structure this product? And how much sales margin would it generate for a volatility of 10%, EUR interest rate of 3%, and USD interest rates such that the forward comes out correctly, if it is priced in the Black-Scholes model?

Forward with Knock-Out Chance Consider the following terms and conditions of a structured forward contract with a knock-out chance exhibited in Table 2.27. It is meant for an importer in the Euro zone selling EUR and buying USD. The treasurer wants a structure with a worst case. Decompose the structure into standard building blocks of vanilla and barrier options, specifying all parameters such as strikes, barriers, nominals, nominal currencies, maturities. Plot a graph illustrating the effective final exchange rate compared with the outright forward rate as a function of the final spot price. Discuss the pros and cons.

The treasurer assumes that EUR will more likely go weaker than stronger, but would like to participate in a weaker EUR when buying the USD. In this strategy the treasurer starts with a worst case exchange rate of 1.2700. If the EUR/USD spot never trades at or below the lower barrier then the treasurer's effective final exchange rate improves proportionally to the EUR/USD drop, which means it will be higher than the worst case and the treasurer will receive more USD for his EUR nominal.

TABLE 2.27 Indicative terms and conditions of a forward with knock-out chance.

Maturity	12 months
Nominal	EUR 5,000,000
Premium	zero
EUR/USD spot reference	1.2950
EUR/USD forward reference	1.2890 (12 months outright forward)
Initial exchange rate K_0	1.2700 (190 USD pips worse than forward ref)
Knock-out barrier B	1.0000 continuously observed
Strategy	at maturity you sell EUR 5,000,000 at K_0 or better
Knock-out chance	if EUR/USD trades above B all the time and spot final S_T is at K_0 of below, your final exchange rate improves by 50% of your opportunity cost
Effective final exchange rate	$K_0 + 50\% \times (K_0 - S_T)$

Power Reset Forward Consider the following terms and conditions of a *power reset forward* contract exhibited in Table 2.28. It is meant for an exporter in the Euro zone buying EUR and selling USD. The treasurer wants a structure with a worst case. Decompose the structure into standard building blocks of vanilla and barrier options, specifying all parameters such as strikes, barriers, nominals, nominal currencies, maturities. Generate a table listing the effective final exchange rate for a few scenarios. Discuss the pros and cons.

The treasurer expects EUR to become significantly weaker during the next nine months and then stabilize. She wants to participate in a weaker EUR observed in nine months. In this strategy the treasurer starts with a worst case exchange rate of 1.3250. If the EUR/USD spot will be below initial spot then the treasurer's effective final exchange rate improves proportionally to the EUR/USD drop, which means it will be lower than the worst case and the treasurer will need less USD for her EUR nominal.

TABLE 2.28 Indicative terms and conditions of a power reset forward.

Maturity	18 months
Monthly nominal	EUR 5,000,000 / EUR 10,000,000
Premium	zero
EUR/USD spot reference S_0	1.2950
EUR/USD forward ref	1.2860 (18 months outright forward)
Initial exchange rate K_0	1.3250 (390 USD pips worse than forward ref)
Reset strike R	1.2950 equal to current spot
Strike reset date t_R	9 months
EUR/USD fixing S_{t_R}	observed on Reuters ECB37 on t_R
EUR/USD spot final $_T$	WMR
Final exchange rate K_f	if $S_{t_R} \geq R$ then $K_f = K_0$ if $S_{t_R} < R$ then $K_f = K_0 - (R - S_{t_R})$
Leverage	if $S_T \geq K_f$ at maturity, you buy EUR 5 M if $S_T < K_f$ at maturity, you buy EUR 10 M

Time Option Replication with American Options You might be tempted to replicate a long time option as a long American style call and a short European or American style put. Convince yourself that this is wrong by stating an example where the replication and the time option differ.

Fade-In Forward What are the building blocks needed to structure the fade-in forward listed in Table 2.3? Try to find the relevant market data for this day and re-evaluate this deal to find out how much margin the bank made.

Kiwi Forward On a kiwi spot ref of 0.7500 a treasurer buys NZD 6 M at 0.7000 in a (zero-cost) boosted 1-year forward transaction, and accepts that in one year she may have to buy the same amount at 0.7000 even if spot is then lower. The outright forward rate for a NZD buyer is 0.7200. Assume a flat volatility of 10%, a NZD deposit rate of 4.155%, a USD deposit rate of 0.000%. Which option component is used to replicate this scenario if the bank's sales margin is USD 10,000?

1. Treasurer sells a one-year NZD put/USD call on NZD 6 M with strike 0.7000 at a premium (deferred by one year) of at least 2.8571% USD.
2. Treasurer sells a one-year NZD put/USD call on NZD 6 M with strike 0.7000 at a premium (deferred by one year) of at least 3.0952% USD.
3. Treasurer sells a one-year NZD put/USD call on NZD 6 M with strike 0.7200 at a premium (deferred by one year) of at least 2.5641% USD.
4. Treasurer sells a one-year NZD call/USD put on NZD 6 M with strike 0.7000 at a premium of at least 2.8571% USD.

2.2 TARGET FORWARDS

Now let us look at the class of *target redemption products*, whose notional amount increases until a certain gain is reached. A common example is a *target redemption forward (TRF, TARF, tarf)*. Unlike a structured forward with worst case, which can often be decomposed into building blocks such as a synthetic forward with the exchange rate being the worst case, and a participation building block, the class of target forwards usually offer the client an exchange rate that is better than the current spot or outright forward rate, but does not guarantee a worst case. Target forwards terminate early once a pre-specified target is reached, and therefore the nominal amount to be exchanged is typically not known at inception. When target products are traded as a note, they are often referred to as *target redemption note (TARN, tarn)*.

Tarns started trading from around 2004 in FX markets. The idea has been taken from the structured interest rate derivatives market. To get started, we provide a description and an example.

2.2.1 Plain Target Forward

We consider a TRF in which a client sells EUR and buys USD at a much higher rate than current spot or forward rates. The key feature in this product is that the client has a total target profit that, once hit, knocks out all future settlements (in the example below, all weekly settlements), locking the gains registered until then. The idea is to place the strike over 5.50 big figures above spot to allow the client to quickly accumulate profits and have the trade knocked out after five or six weeks. The client will start losing money if EUR-USD starts fixing above the pre-agreed exchange rate. On a spot reference of 1.4760 EUR-USD consider a one-year TRF in which the client sells EUR 1 M per week at 1.5335, subject to a KO condition: if the sum of the client profits reaches the target, all future settlements are canceled. We let the target be 0.30 (i.e. 30 big figures), measured weekly as

$$\text{Profit} = \max(0, 1.5335 - \text{EUR-USD-Spot-Fixing}). \tag{21}$$

As usual this type of target forward is also traded at zero cost. We illustrate a profit scenario in Table 2.29. Since the TRF is similar to an accumulator, the name *target accumulator* is also used.

Critical Judgment A target forward like this seems to be too good to be true. The client gets a zero-cost contract, and beats the outright forward every week. And if EUR-USD stays where it is, and the target profit is reached, so the transaction terminates, the client can do a new TRF with similar conditions. So where is the problem? We will learn more about it by differentiating clients: investors/speculators vs. treasurers.

Client Type Investor: For this client type there is no underlying cash flow to hedge. In this case it may be in fact more suitable to agree on cash settlement of the profit at maturity or at the early termination time, whichever comes first. In any case, the investor's profit is limited by the target, whereas the potential loss is unlimited. A simple

TABLE 2.29 Scenario illustration of a target redemption forward. In the non-capped type the last week's profit will be 4.85 big figures. In the capped type the accumulated profit is capped at 0.30, so the client accumulates only the last 3.75 big figures and the trade terminates. In traders' jargon it "knocks out," although there is no knock-out barrier or barrier event. Each forward will be settled physically every week until trade terminates (if target is reached). Note that a loss in one of the weeks does *not* lead to a reduction of the accumulated profit.

Week	Fixing	Profit	Accumulated profit
1st	1.4800	+0.0535	0.0535
2nd	1.4750	+0.0585	0.1120
3rd	1.4825	+0.0510	0.1630
4th	1.4900	+0.0435	0.2065
5th	1.4775	+0.0560	0.2625
6th	1.5400	−0.0065	0.2625
7th	1.4850	+0.0485	0.3110

simulation of the valuation of the TRF shows that under standard initial market assumptions the profit of 30 big figures is reached with a high probability, whereas large losses are reached with a small probability. This allows the product to have zero cost at inception. By the way, the losses do not reduce the accumulated profit. The accumulated profit grows on a fixing when there is a profit, but stays at the same level if there is a loss. An investor can easily incur a loss of 500 big figures (or more) if the target is not reached and EUR-USD spot fixes above the pre-agreed exchange rate of 1.5335 many weeks in a row. For example, if the spot fixes at 1.6335, then the client encounters a loss of 10 big figures. If this happens 50 times (i.e. for 50 weeks after a large initial up move), then the loss will sum up to 500 big figures. And it is clear now that the loss in unlimited because the spot can fix arbitrarily high.

Client Type Treasurer: For this client type we assume there is an underlying cash flow to hedge, i.e. the treasurer receives EUR 1 M every week from his underlying business and would then exchange these Euros at the pre-agreed exchange rate every week. Now, if the spot moves up but stays below 1.5335, and terminates the TRF, then the treasurer will have traded at spot levels that are much better for her than using the outright forward or the spot, and all of that at zero cost. The hedge (viewing the TRF as a hedge here) will be lost, but at the time of early termination the spot is either high, which puts the EUR-selling treasurer in a comfortable position, or the EUR-USD spot is low, and then she can proceed with a new TRF or any other hedge idea. If she keeps doing that for the whole year, she will typically be better off than doing nothing and always trading at spot. Therefore, the sell-side likes to recommend the TRF as a zero-cost alternative to not hedging at all. And obviously, the sell-side makes more profit on a TRF than on a super-transparent outright forward.

Risk Assessment: We have seen that the TRF can be very risky for a client type investor. It acts like selling a lottery ticket. However, for a client type treasurer it can be viewed as a reasonable hedging instrument. It is not fair to say that all target forwards are too risky and therefore should not be traded. The risk for the treasurer is that it requires attention, in the sense that the hedges may need to be re-established on the way. A strip of outright forwards would also serve as a hedging instrument for the incoming weekly cash flow. It would require no attention, no justification, provide perfect hedge accounting, but at the end of the day does not beat the spot. For an investor a TRF is about as risky as an outright forward with the total amount, i.e. sum of all the weekly settlement amounts. Even in an outright forward, there is unlimited risk if spot moves against the investor.

Product Booking/Classification Since a tarf is an agreement on a series of future cash flows in two currencies, it is possible to think of a tarf as a variant of a cross currency swap. Unlike in a standard crossie, there is no exchange of notionals in the beginning and the end; furthermore, the interest rates are fixed rather than fixing derived. Frequencies are usually significantly shorter in a tarf. And there are overall conditions on the target profit and features like puttability, which refers to the right to close out the tarf at pre-specified conditions. Therefore, one way to seek product approval internally in a bank is to view the tarf as a crossie with special features, or exotic crossie. This view may also enable the market participants to book the tarf in an existing system. Special features may not be reflected properly, leave alone valuation and risk figures. But never mind. Gotta get the deal done and worry about the nitty-gritty details later – probably not good advice in this new regulatory regime.

2.2.2 Leveraged Target Forward

As *beating the forward* is the key philosophy of the target forward product class, a common way to make the agreed exchange rate appear even more attractive is to introduce a leverage factor, similar to the way accumulators are structured: double (200%) the amount if the spot fixing is on the wrong side of the agreed exchange rate (strike). The factor 2 or 200% is the leverage factor and is a contractual parameter, so 200% is just an example. A leveraged target forward may also be called a target profit forward, as in the previous section. Let us consider a sample term sheet:

Currency and Notional: EUR 1,000,000

Trade Date: 16 Oct 2012

Last Delivery Date: 18 Oct 2013 (subject to Target Event)

Strike: 1.3500 USD per EUR

Target: 0.3000

Leverage: 200.00%

Positive Intrinsic Value: max(0, Strike − Fixing)

Target Event: A Target Event occurs if the sum of all the positive intrinsic values (meaning only fixings at or below the relevant Strike are taken into account) for all previous forward transactions is equal or larger than the Target. As soon as the Target Event occurs, the transaction that caused the Target Event and all the following ones are knocked out.

Analysis: The client enters into a strip of forward transactions to sell the relevant EUR Notional against USD at the relevant Strike price subject to the Target Event. The Leverage indicates that the relevant Notional of the transaction is multiplied by the relevant Leverage if the spot fixed strictly above the relevant Strike for this Maturity.

EUR/USD: Expressed in USD per 1 EUR as will be determined at any time by the Calculation agent and this determination will be final except in case of manifest error.

Fixing: Expressed in USD per EUR as published on Reuters ECB37 (or any successor thereto). The fixing dates are listed in Table 2.30.

Upfront Premium: Zero-Cost Strategy

Calculation Agent: The bank

Business Days: Target, London, New York

Date Format: All Dates are expressed in the form dd/mm/yyyy (day/month/year).

TABLE 2.30 Fixing table of a monthly target redemption forward, M denoting the month.

M	Fixing dates	Delivery dates	M	Fixing dates	Delivery dates
1	Fri 16-Nov-2012	Tue 20-Nov-2012	7	Thu 16-May-2013	Mon 20-May-2013
2	Mon 17-Dec-2012	Wed 19-Dec-2012	8	Mon 17-Jun-2013	Wed 19-Jun-2013
3	Wed 16-Jan-2013	Fri 18-Jan-2013	9	Tue 16-Jul-2013	Thu 18-Jul-2013
4	Tue 19-Feb-2013	Thu 21-Feb-2013	10	Fri 16-Aug-2013	Tue 20-Aug-2013
5	Mon 18-Mar-2013	Wed 20-Mar-2013	11	Mon 16-Sep-2013	Wed 18-Sep-2013
6	Tue 16-Apr-2013	Thu 18-Apr-2013	12	Wed 16-Oct-2013	Fri 18-Oct-2013

Note that in this version of the tarf, the transaction at the time of the fixing that triggers the termination is not carried out, so the target is not filled up. In fact, there are typically and generally three ways to handle the settlement at the time of the fixing that triggers the termination:

full pay: the full transaction is settled, even if the profit exceeds the target; however, no further settlements will take place;

capped: the last settlement will be capped to the target and no further settlements will take place;

no pay: if the target is reached, then the settlement on the day when the fixing triggers the termination does not take place, and no further settlements will take place.

We show the pricing results in Table 2.31, calculated using the market data listed in Table 2.32 for interest rates and swap point, and Table 2.33 for the volatility smile. The latter also contains bucketed sensitivities of ATM volatility, risk reversals, and butterflies. Bucketed interest rate sensitivities are shown in Table 2.34.

TABLE 2.31 Pricing results of a monthly target redemption forward. For the digital risk we assume a digital risk range of 0.025. For the pin risk we assume a pin risk range of 0.005. Spot reference is EUR/USD 1.2964.

Risk	Value	Coefficient	Total (EUR)
PV	−80,491.92	1	−80,491.92
Aega	−53,826.30	0.23	12,380.05
Rega	−16,728.67	0.2	3,345.73
Sega	−30,706.41	0.2	6,141.28
Digital risk	6,617.84	1	6,617.84
Delta	−7,944,278.72	0.000255	1,562.63
Hedging cost			30,047.53
Final price			−50,444.39

TABLE 2.32 Market data used for pricing a monthly target redemption forward: discount factors (DF), deposit rates, and swap points for EUR/USD. Spot reference is EUR/USD 1.2964.

Date	USD DF	Depo	EUR DF	Depo	Swap
Thu 18-Oct-2012	0.999993	0.12%	1.000008	−0.15%	0.19
Fri 19-Oct-2012	0.99999	0.12%	1.000012	−0.15%	0.285
Thu 25-Oct-2012	0.999956	0.18%	1.000027	−0.11%	0.91
Thu 01-Nov-2012	0.999908	0.21%	1.00004	−0.09%	1.702
Thu 08-Nov-2012	0.99986	0.22%	1.000053	−0.08%	2.495
Mon 19-Nov-2012	0.999784	0.23%	1.000073	−0.08%	3.74
Tue 18-Dec-2012	0.999477	0.30%	1.000035	−0.02%	7.24
Fri 18-Jan-2013	0.999227	0.30%	1.000186	−0.07%	12.44
Tue 19-Feb-2013	0.998463	0.45%	0.999706	0.09%	16.14
Mon 18-Mar-2013	0.997963	0.49%	0.999467	0.13%	19.54
Thu 18-Apr-2013	0.99849	0.30%	1.000314	−0.06%	23.689
Mon 20-May-2013	0.998224	0.30%	1.000341	−0.06%	27.489
Tue 18-Jun-2013	0.997984	0.30%	1.000389	−0.06%	31.239
Thu 18-Jul-2013	0.997707	0.31%	1.000411	−0.05%	35.139
Fri 18-Oct-2013	0.996862	0.31%	1.000421	−0.04%	46.289
Tue 21-Jan-2014	0.995915	0.32%	1.000439	−0.03%	58.889
Tue 22-Apr-2014	0.994924	0.34%	1.00023	−0.02%	69.138
Mon 20-Oct-2014	0.992683	0.37%	0.999876	0.01%	93.938
Mon 19-Oct-2015	0.986434	0.46%	0.99634	0.12%	130.187
Tue 18-Oct-2016	0.976009	0.61%	0.98837	0.29%	164.186
Wed 18-Oct-2017	0.95995	0.82%	0.975181	0.50%	205.685
Thu 18-Oct-2018	0.939077	1.05%	0.955251	0.76%	223.285
Fri 18-Oct-2019	0.91634	1.25%	0.93512	0.96%	265.684
Mon 19-Oct-2020	0.890609	1.46%	0.912495	1.15%	318.582
Mon 18-Oct-2021	0.865367	1.62%	0.890578	1.29%	377.681
Tue 18-Oct-2022	0.837988	1.78%	0.866409	1.44%	439.679

Further Variations of a Target Forward Most trading and risk management systems use a payoff language, mostly combined with a Monte Carlo based pricing procedure. Therefore, it is easy to implement a term structure of all contractual parameters, which includes notional, the leverage or notional factor, strike, or even additional barriers.

2.2.3 Target Profit Forward

In the early days of trading tarfs we realized very quickly the problem of an infinite potential loss for the client type investor. The industry reacted to it by flooring the potential loss, even in so-called vanilla tarfs. Let us consider this product in detail. The following tarf has a leverage factor, which some market participants may consider a "vanilla" feature. One can obviously argue about that. Much of this section is based on a presentation by Deutsche Bank from 2008 [8].

TABLE 2.33 Volatility matrix and bucketed risk of a monthly target redemption forward: Aega, Rega, and Sega are in USD. Spot reference is EUR/USD 1.2964.

Date	ATM	10RR	25RR	10BF	25BF	Aega	Rega	Sega
Wed 17-Oct-2012	11.75%	−0.08%	−0.07%	0.52%	0.15%	1.30	−208.26	779.12
Tue 23-Oct-2012	8.30%	−0.07%	−0.04%	0.57%	0.17%	307.45	213.26	34.56
Tue 30-Oct-2012	8.20%	−0.36%	−0.23%	0.53%	0.16%	−386.75	36.25	−1,607.43
Tue 06-Nov-2012	8.25%	−0.45%	−0.29%	0.52%	0.16%	166.60	130.57	610.09
Thu 15-Nov-2012	8.41%	−0.63%	−0.37%	0.51%	0.15%	−922.12	130.45	−4,698.29
Fri 14-Dec-2012	8.52%	−1.17%	−0.67%	0.64%	0.21%	−1,104.67	−647.57	−5,577.72
Wed 16-Jan-2013	8.46%	−1.49%	−0.83%	0.73%	0.24%	−14,221.89	−2,979.54	−8,457.66
Tue 16-Apr-2013	9.02%	−2.26%	−1.24%	0.89%	0.28%	−29,703.21	−9,577.90	−11,971.58
Tue 16-Jul-2013	9.41%	−2.47%	−1.34%	0.98%	0.32%	−17,277.95	−6,735.02	−8,489.71
Wed 16-Oct-2013	9.72%	−2.82%	−1.52%	1.23%	0.39%	−6,421.36	−2,052.75	−2,603.20
Thu 16-Oct-2014	10.17%	−3.04%	−1.65%	1.15%	0.35%	111.82	0.00	0.00
Thu 15-Oct-2015	10.40%	−3.20%	−1.75%	1.08%	0.32%			
Fri 14-Oct-2016	10.70%	−3.37%	−1.85%	1.03%	0.32%			
Mon 16-Oct-2017	10.88%	−3.46%	−1.90%	1.01%	0.33%			
Wed 16-Oct-2019	10.93%	−3.55%	−1.95%	0.94%	0.32%			
Fri 14-Oct-2022	11.13%	−3.84%	−2.10%	0.82%	0.30%			
Thu 14-Oct-2027	12.33%	−4.30%	−2.35%	0.40%	0.26%			

TABLE 2.34 Bucketed interest rate risk of a monthly target redemption forward: both rhos are in USD. Spot reference is EUR/USD 1.2964.

Date	USD rho	EUR rho	Date	USD rho	EUR rho
2012-10-18	410.45	−410.20	2013-01-18	−1,809.70	1,780.82
2012-10-19	−0.19	0.22	2013-02-19	−2,870.97	2,844.46
2012-10-25	1.90	−1.89	2013-03-18	−3,680.70	3,703.07
2012-11-01	61.02	−60.97	2013-04-18	−4,674.95	4,837.44
2012-11-08	−60.43	60.51	2013-05-20	−5,490.57	5,629.10
2012-11-19	−649.18	601.62	2013-06-18	−4,935.05	5,123.97
2012-12-18	−1,178.80	1,127.36	2013-07-18	−11,093.84	11,709.29
			2013-10-18	−14,008.58	14,861.36

Client Background

- Client has recurring EUR/USD exposure where they need to sell USD/buy EUR on some periodic basis for at least the next year and is looking to manage this short EUR/USD exposure.
- Client is looking to outperform current EUR/USD buying levels, i.e. the client has a target rate well below current par forwards.
- Client is willing to monetize certain indifferences to achieve a better EUR purchasing rate, e.g.:
  - she is flexible over how many USD cash flows will be converted;
  - she is flexible over the hedge ratio;
  - she is willing to cap her total gain from the transaction.

- Some key suitability issues:
  - trade maturity should not exceed maximum tenor for USD exposure (one year in our example, depending on how recurrent the FX exposure is);
  - the maximum converted amount should not exceed actual EUR/USD exposure (hedge ratio < 100%).
- To avoid exposing the client to unlimited potential losses on their EUR/USD purchase strategy, the strategy has an implicit MTM cap at 7.50% of maximum cumulative EUR notional. Throughout this example the maximum cumulative EUR notional is 12 times the EUR notional. This means that the target forward is puttable: at all stages the client has the right to walk away from the trade and cancel all remaining settlements at the cost of 7.50% of EUR maximum cumulative notional.

Indicative Terms We assume a EUR/USD spot reference of 1.4600 and a one-year par or outright forward reference of 1.4500.

Description and Analysis

- Description: every month, provided the target profit condition has not been met:
  1. If EUR/USD > strike, the client buys EUR, sells USD at strike on 50% of the EUR notional;
  2. If EUR/USD ≤ strike, the client buys EUR, sells USD at strike on 100% of the EUR notional.
- The target profit condition works as follows: The cumulative profit is observed monthly and if it exceeds the target profit, all subsequent cash flows, including the breaching cash flows, are terminated.
- If the client's view is realized and EUR/USD trades in a range around current spot (i.e. above strike) she will initially buy EUR/sell USD at a rate six big figures better than the current 12-month par forward.
- Client flexibilities are incorporated in the following way:

  Tenor: If the target profit condition is met, the client converts USD on only some of the 12 cash flows;

  Hedge Ratio: If EUR/USD falls below strike, the client is converting USD into EUR on 100% of EUR notional;

  Capped Profit: Client's profit is capped at 0.3000 USD per EUR (EUR/USD 30 big figures).

Indicative Terms

Terms: At each expiry, provided the target profit condition has not been met:

- If EUR/USD > strike, client buys EUR, sells USD at strike on 50% of EUR notional;
- If EUR/USD < strike, client buys EUR, sells USD at strike on 100% of EUR notional.

Expiry Dates: Monthly, for up to 12 months.

Strike: 1.3900.

Cumulative Profit: Initially zero; at each expiry, provided the target profit condition has not been met:

- If EUR/USD > strike the cumulative profit increases by [EUR/USD – strike];
- If EUR/USD ≤ strike the cumulative profit is unchanged.

Target Profit Condition: If the cumulative profit exceeds the target profit, all subsequent cash flows (including the breaching cash flow) are knocked out.

Target Profit: 0.30 USD per EUR.

Sensitivity Analysis Let us consider the key Greeks:

Delta:

- Initial delta of the transaction is 30% of maximum cumulative EUR notional;
- In other words, initial MTM sensitivity to EUR/USD moves will be equivalent to that of a par forward on 30% of the maximum cumulative EUR notional.

Vega:

- Client is initially short volatility on the transaction since she is betting on the exchange rate remaining stable;
- Every 1% increase in volatility implies a loss on the TPF of 0.20% of the maximum cumulative EUR notional;
- In other words, for each 1% increase in implied volatility strike discount vs current spot increases by 90 pips (0.0090 USD per EUR);
- As a result trade parameters may improve when EUR/USD implied volatility rises.

Simulated Trade Maturity:

- Average simulated trade maturity is seven months;
- Depending on the model one may compute the probabilities of the trade terminating at each month (and not before) as well as the probability of the tarf not terminating at all;
- For example, there is about a 20% probability the trade will terminate three months after trade date.

Booking Considerations and Dynamic Replication Clients can usually request timely valuations and sensitivity analysis from the bank on all traded tarfs to monitor risk and MTM. Ideally, the bank operations will provide trade valuations priced separately from their sales and trading desk. However, some clients may also need to have an internal booking of the structure but are currently only able to book vanilla and barrier options. As a quick fix, target profit forwards can be booked or replicated "dynamically" as a combination of short vanilla option positions and long European reverse-knockout option positions. Let's consider an example. On the trade date, the client would book:

1. Short put positions for 12 expiry dates on 100% of notional;
2. Long a European reverse knock-out call option for the first expiry only, strike 1.3900, KO Level 1.6900 (strike + remaining profit) on 50% of notional.

At the first expiry, suppose EUR/USD fixes at 1.4522. This earns the client 6.22 big figures profit; the remaining profit is reduced to 23.78 big figures. Now, the client amends the booking to:

1. Short put positions for the remaining 11 expiry dates on 100% of the EUR notional.
2. Long a European reverse knock-out call option for the next expiry only, strike 1.3900, KO level 1.6278 (strike + remaining profit) on 50% of the EUR notional.

The client continues to amend the booking like this at each expiry. This method of booking is a simple but conservative approximation, assuming after six expiry dates the trade terminates, but the client has booked short put positions without knock-out features. A more accurate method would be to book an out-of-the-money discrete knock-out feature on the puts that correspond to the barrier on the respective European reverse knock-out calls.

Variants that Address Major Concerns The features of this TPF may give rise to some concern. In the following we outline some of the concerns and also provide examples of how the financial industry modifies the TPFs accordingly. The credit concern is the most prominent. Therefore, we follow the typical approach taken in the financial industry and assume that all trades below include a puttable feature at 7.50% of maximum cumulative EUR notional.

Credit Lines/Unlimited Loss This is an issue for so-called *credit vegetarians*. Trading a TPF without the underlying cash flow, means that the losses are potentially unlimited. To circumvent this one may make the TPF *puttable*: the client has the right to walk away at all stages for a fixed price. This obviously requires a corresponding implementation on the quant side and may not be available at all banks. The puttability feature in this TPF means that at any stage during the trade, the client has the right (but not the obligation) to unwind all remaining cash flows and pay a predetermined amount of 54% of EUR (monthly) notional (i.e. 4.50% of EUR maximum cumulative notional). This feature not only allows the client to cap the mark-to-market losses they may have, it also limits the bank's potential credit exposure to the client. The cost of a puttable feature at a *puttable strike* of 4.50% of the maximum cumulative EUR notional would be about one big figure, i.e. a strike of 1.4000 instead of 1.3900.

Accounting To ensure a simple situation for accounting purposes one may use a *window TPF*: The rationale is that the client is worried about the possible accounting treatment of a vanilla TPF. We assume that the client reports on a semi-annual basis and enters the trade at the start of their reporting period. For the first six expiry dates, as per a regular TPF and provided the target profit condition has not been met:

- If EUR/USD > Strike, client buys EUR, sells USD at strike on 50% of the EUR notional.
- If EUR/USD ≤ Strike, client buys EUR, sells USD at strike on 100% of the EUR notional.

Window Feature – in six months, if the target profit has not been reached, the remaining cash flows are replaced by a USD annuity equal to then 6m outright forward – strike on 100% of the EUR notional. This means a constant USD annuity; alternatively one could contractually agree on cash flows of type 1m outright forward – strike, 2m outright forward – strike, … , 6m outright forward – strike. Economically, if the trade has not knocked out in 6m, the client will have no remaining FX exposure apart from the dollar annuity. The client could then enter into a *reset trade* consisting of a strip of outright forwards for which they would seek hedge accounting. The combination of the annuity and these outright forwards would be economically equivalent to a strip of forwards struck at strike. For instance, if the target profit is not reached in the first six months, from then on the client is synthetically long a strip of vanilla forwards. This reset or window feature may raise the initial strike to about 1.4050.

Timing Sometimes timing the trade time is tricky. If the spot moves against the client's favor right after trade time, then the client obviously would have liked to get a better strike. In other words, the client is concerned about *pulling the trigger* on a TPF when EUR/USD could retrace to much lower levels in the initial part of the trade, which is referred to as a timing concern given current volatile markets. One can capture this feature by signing up for a *second chance TPF* or a *shout TPF*: in a second chance TPF, the client can benefit from improvement in strike of 1.3400 if the market moves away in the short run. If at the first expiry EUR/USD has fallen to 1.4000 or below, the client benefits from a six big figure improvement in strike. In the shout TPF, the client has the right to "shout" until the first expiry. If the shout feature is exercised, the strike is reset to the EUR/USD spot prevailing at that time. If the shout feature is not exercised, the initial strike will remain. Both these features allow the client to participate should EUR/USD substantially depreciate shortly after the trade date. Both features require the initial strike to be put higher to about 1.4000 to ensure the tarf is still a zero-cost structure.

Actual Tenor Suppose a client wishes to prevent a very early termination of the TPF, she should opt for a *digital TPF*, which means that a given number of conversion dates are guaranteed; the trade terminates based on the number of profit occurrences. This is also referred to as a *counter tarf* or a *discrete target accumulator*. This may raise the initial strike to about 1.4100. It is a bit higher, because in a counter the potential profit for the client is not limited. If the client still wants to accumulate profits after a first target is reached, a solution could be a *resetting strike TPF*: if the target profit condition is met, the strike and accumulated profit are reset for subsequent flows and the trade does not terminate. If you want to go completely bananas you may do this multiple times. For instance, if the target profit is reached, for subsequent expiry dates the strike is reset to the EUR/USD prevailing spot minus 4.5 big figures, capped at 1.6000, and cumulative profit is reset to zero. The target profit condition can take place on several occasions over the life of the trade. As a result, independently of the number of strike resets, the client is guaranteed to have 12 cash flows. This may raise the initial strike to about 1.4000.

Participation Another example for the greedy or the payoff-language freak is a *market following TPF*: here the client is concerned about the lack of participation in any favorable market development, i.e. if EUR/USD depreciates creating losses on the TPF. In a market following TPF, each time the market fixes out-of-the-money the client's strike improves by a pre-determined offset amount. For instance, if at any expiry EUR/USD spot is below the strike, the strike is reduced by 0.5 big figures. This may raise the initial strike to about 1.4100.

Disclaimer Who reads disclaimers? Or instruction manuals? Well, actually, it is quite entertaining. Look at this extract taken from a bank:

> The transaction(s) or products(s) mentioned herein may not be appropriate for all investors and before entering into any transaction you should take steps to ensure that you fully understand the transaction and have made an independent assessment of the appropriateness of the transaction in the light of your own objectives and circumstances, including the possible risks and benefits of entering into such transaction. You should also consider seeking advice from your own advisers in making this assessment. If you decide to enter into a transaction, you do so in reliance on your own judgment. The information contained in this document is based on material we believe to be reliable; however, we do not represent that it is accurate, current, complete, or error free. Assumptions, estimates and opinions contained in this document constitute our judgment as of the date of the document and are subject to change without notice. Any projections are based on a number of assumptions as to market conditions and there can be no guarantee that any projected results will be achieved.

One can't read this often enough. In my opinion it is a good exercise to calculate the worst case scenario in each product and define an exit strategy at the beginning of the transaction and strictly follow it, especially if the tarf is traded as an investment product on a margined account.

2.2.4 Pivot Target Forward (PTF)

A pivot target forward belongs to the class of target forwards. Again we set up a sequence of forward FX transactions based on a fixing schedule, count the profits, and have the trade stop once the profit reaches a pre-specified target. Losses do not offset the profits in the accumulated profit counting scheme. This product is a bet on the spot staying in a range near initial spot. If the spot leaves this range, and does not come back, then potential losses are unlimited, whereas potential profits are always limited to the target.

The idea is to generate a zero-cost product by paying a small profit with a high probability and causing a catastrophic loss with a small probability. On average it is worth zero, or more precisely, worth a negative amount to the client that corresponds to the sales margin for the bank and cost of the trade.

TABLE 2.35 Term sheet of a pivot target forward in USD-CAD.

Notional Amount	USD 1,000,000
High Strike	1.0545
Pivot	1.0000
Low Strike	0.9455
Observation Dates	As defined in Schedule
Settlement Dates	23 July, 2009
CAD Settlement Amount	If FX Rate ≤ Pivot: USD Notional Amount · (FX Rate – Low Strike); if FX Rate > Pivot: USD Notional Amount · (High Strike – FX Rate); if Settlement Amount is positive, Party B will deliver the CAD Settlement Amount to Party A; if the Settlement Amount is negative, Party A will deliver the CAD Settlement Amount to party B.
Positive Intrinsic Value	With respect to each Observation Date, Day 1: max(0, CAD Settlement Amount); Day 2 to expiry: Positive Intrinsic Value of previous day + max(0, CAD Settlement Amount)
Target Cap Level	CAD 1,500,000
Target Knock Out Event	If at an Observation Date (the Knock out observation date), the positive intrinsic value at the Observation Date is equal to or exceeds the Target Cap Level, this transaction shall be automatically terminated and thereafter no future CAD Settlement Amounts shall be calculated. The settlement arising at the Knock Out Observation Date will be adjusted as follows:
Adjusted Settlement Amount	Target Cap Level-Positive Intrinsic Value of Previous Day Party B will pay Party 1 Adjusted Settlement Amount
FX Rate	The Mid WMR USD/CAD spot exchange on each applicable observation date evidenced at 4:00 p.m. London Time on Reuters Page **WMRSpot35** expressed as the number of units of CAD per unit of USD as determined by the calculation agent at its sole discretion.
Day Adjustment	Modified Following
Calculation Agent	Party B

Terms of a Traded Pivot Target Forward in USD-CAD Let us look at an example of a transaction where a bank traded a pivot target forward with a HNWI on 22 July 2008. The term sheet is displayed in Table 2.35.

Individual Terms: Each of the FX transactions to which this confirmation relates has the following individual terms:

In the confirmation there is normally a list of each fixing day. This will be too boring to fill this book. Basically, it starts with the first fixing day of 23 July 2008 and ends with the 22 July 2009, with daily fixings except weekends.

Representations: Each party represents to the other party as of the date that it enters into this Transaction that (absent a written agreement between the parties that expressly imposes affirmative obligations to the contrary for this Transaction):

(i) Non-Reliance. It is acting for its own account, and it has made its own independent decisions to enter into this Transaction and as to whether the Transaction is appropriate or proper for it based upon its own judgment and upon advice from such advisers as it has deemed necessary. It is not relying on any communication (written or oral) of the other party as investment advice or as a recommendation to enter into this Transaction, it being understood that information and explanations related to the terms and conditions of this Transaction shall not be considered to be investment advice or a recommendation to enter into the Transaction. No communication (written or oral) received from the other party shall be deemed to be an assurance or guarantee as to the expected results of this Transaction.

(ii) Assessment and Understanding. It is capable of assessing the merits of and understanding (on its own behalf or through independent professional advice), and understands and accepts the terms and conditions and risks of this Transaction. It is also capable of assuming, and assumes, the risks of the Transaction.

(iii) Status of Parties. The other party is not acting as a fiduciary for or adviser to it in respect of this Transaction.

(iv) Early Termination Provisions: this Transaction shall early terminate and cancel in whole on such Fixing Date (an "Early Termination Event"). Following such Early Termination Event, the parties shall be relieved of all further payment obligations under the Transaction described herein, except for

1. the obligation of the bank to pay Counterparty on the Coupon Payment Settlement Date (a) the positive Coupon Payment Amounts as determined on the Fixing Dates preceding the Early Termination Event and (b) the Adjusted Coupon Payment Amount. (For the avoidance of doubt, this net payment shall be a payment of CAD 1,500,000) and
2. the obligation of Counterparty to pay the bank on the Coupon Payment Settlement Date the absolute amount of the negative Coupon Payment Amounts as determined on the Fixing Dates preceding the Early Termination Event.

(v) Definitions. "Fixing Exchange Rate" shall mean the mid CAD/USD[1] rate, expressed as the amount of Canadian Dollar per one United States Dollar for settlement in one Business Day, as displayed at approximately 4:00 p.m. London Time for that Fixing Date by WM Company on the applicable Reuters page, or such other symbol or page that may replace such symbol or page for the purpose of displaying such exchange rate; provided, however, that if such pages are no longer published or is not published as of the designated time and date, and no replacement symbol or page is designated, the Calculation Agent shall determine such affected Fixing Exchange Rate in good faith and in a commercially reasonable manner.

(vi) Business Days. Business Days applicable to each applicable Fixing Date: shall mean all days on which The WM Company, through it's currency market data

[1]If you wonder why the bank did not write USD/CAD then, so do I.

services, publishes spot rates for the relevant currency pair (the dates on which such services will not be provided may be found on its internet website page, http://www.wmcompany.com/). Business Days applicable to Coupon Payment Settlement Date: Toronto

Critical Judgment This pivot target forward traded at zero cost and subsequently caused a substantial loss during the financial crisis in October 2008. The client type was an investor (without the underlying cash flow). The way a transaction like this sells is based on human psychology: there is a small gain with a high probability and a disastrous loss with a low probability. The buyer hopes that the small probability is zero. The transaction becomes like selling a lottery ticket. Many humans have problems with interpreting probabilities. Suppose you board a plane and you are told that you will get 50% off the price of your ticket if you fly in the plane which has been classified as crashing with 0.1% probability. Would you fly? What about if you get 90% discount if the plane crashes with 0.01% probability? In reality what one should consider is not only the probability of the disaster but also whether the disaster is an acceptable worst case. We list sample scenarios of a pivot target forward in Figure 2.12. Human psychology furthermore does not easily capture the change of the probability distribution over time. In Figure 2.13 we illustrate the density of the short term, which indicates that a profit occurs almost surely, and compare it with the long term, which shows that as time passes the loss scenarios will become more and more likely.

2.2.5 KIKO Tarn

Generally, there is no end to the variants of TARNS. One can easily introduce all kind of barriers: European and American barriers, knock-in and knock-out barriers. In this section we consider a case study of an actual trade of an AUD-JPY knock-in-knock-out (kiko) TARN between an American investment bank and an Indonesian HNWI on

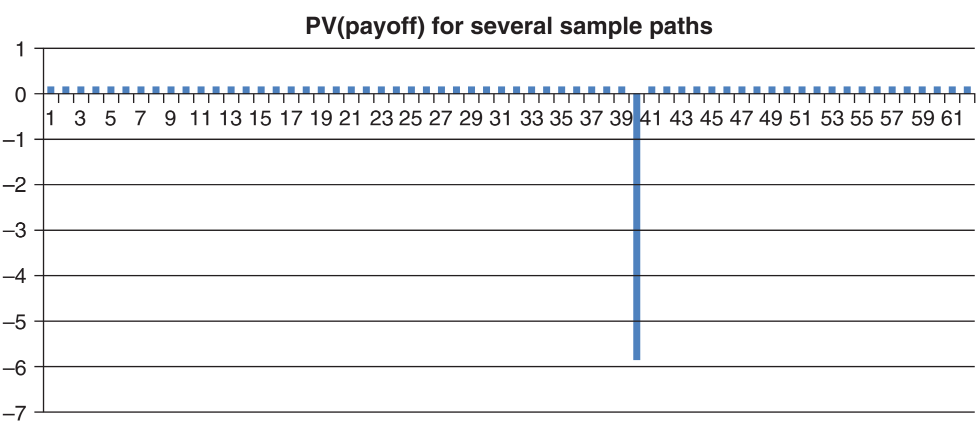


FIGURE 2.12 P&L scenarios pivot target forward.

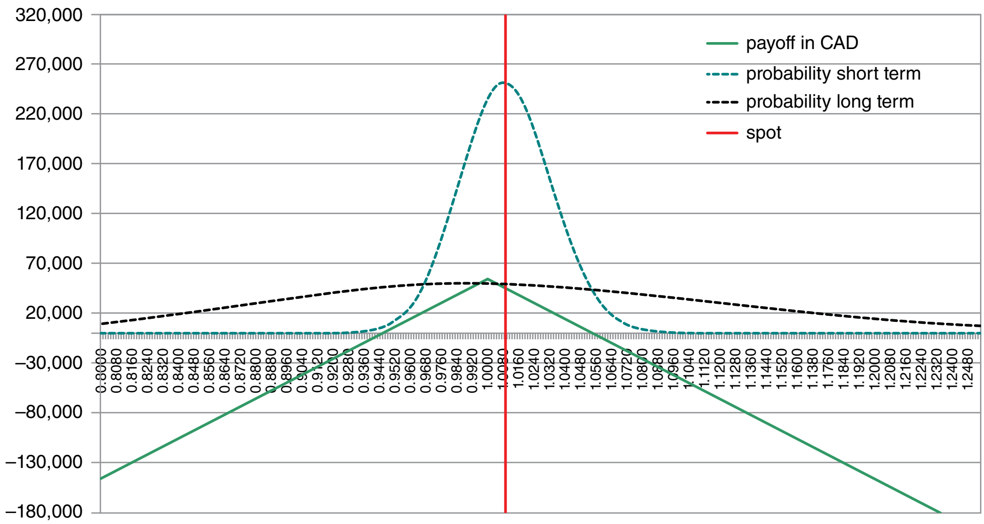


FIGURE 2.13 Pivot target forward payoff and psychology.

27 June 2008, the summer before the Lehman bankruptcy. Normally, TARNS are zero-premium strategies. Unusually, in this case, the investor receives an upfront premium of AUD 400,000.

KIKO Tarn Product Description This FX TARN obliges the investor to purchase some amount of AUD and pay in JPY at the fixed rate of exchange of 97.50 JPY per AUD (the "strike price"), once a fortnight for 52 weeks. The strike price is set below the market price prevailing at the trade date which was 101.99. A "knock-in level" is additionally defined, fixed at 89.50. The amount of AUD that the investor is obliged to purchase each week is either 2,000,000 if the exchange rate is above the strike, or 4,000,000 if the exchange rate is below the knock-in. However, if the exchange rate is above 103.40 (the "knock-out level"), then the strategy is terminated and no further purchases take place. The knock-out condition applies to all 52 weeks, and does not affect past deliveries.

An additional feature in the TARN is a cap on gains. For each purchase of AUD 2,000,000 (when the exchange rate is above the strike), the gain is realized immediately and paid in JPY. Should these gains exceed JPY 68,000,000 in aggregate, then the strategy is terminated and no further purchases take place. We outline the key features as follows.

Objective: To profit from a stable AUD/JPY exchange rate.

View: Range-bound (market stays between strike and knockout until maturity).

Risk: Exposed to higher volatility. If the exchange rate rises above the knock-out, no further purchases happen and the profit to date is locked in, thereby limiting the profit potential. If the exchange rate falls below the knock-in, the size of weekly purchases doubles and at a rate that is then far above the market, i.e. underwater. Either of these conditions is a negative for the investor.

Potential Gain: Maximum gain is JPY 68,000,000 or approximately AUD 657,640.

Potential Loss: Maximum loss is theoretically AUD 104,000,000 in the event that the exchange rate falls to zero without ever rising above the knockout or triggering the cap on gains. Unlike equity, for a major currency pair this is not a feasible scenario. The all-time low of 55.00 would represent a loss of AUD 80,363,636. This is not far from the low of 57.12 for 2007–2008.

Leveraged: Yes: ratio of options sold to options bought is 2×.

Margined: Yes: amplifies gains and losses, and reduces the ability of the investor to hold to maturity.

Generally a kiko TARN is a derivative investment on one underlying similar to an accumulator. The essential difference is that the profits are capped by a maximum amount called the *target*. The investor receives profits if the spot goes up, and makes a loss if the spot goes down. Cash flows are specified and occur usually on a sequence of currency fixing dates. Typically, fixings are chosen daily, weekly, fortnightly, or monthly. The product terminates either if a knock-out barrier is reached or if the client's accumulated profit reaches a pre-specified *target*. There are many different variations traded in the market.

I would like to continue with our example of such an investment. The product starts on June 27 2008 and ends on July 6 2009. Let $K = 97.50$ denote the strike, $B = 103.40$ the knock-out barrier, and $L = 89.50$ the additional lower knock-in barrier. There are 26 fixing dates during this one-year contract in a fortnightly sequence, starting from July 11 2008. Let S_i denote the fixing of the AUD-JPY exchange rate on date i. The client receives the amount

$$\text{REC} = \begin{cases} 2 \times (S_i - K) & \text{if } S_i \geq K \\ 4 \times S_i & \text{if } S_i \leq L \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

and pays the amount

$$\text{PAY} = \begin{cases} 4 \times K & \text{if } S_i \leq L \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

in million of JPY. This is illustrated in Figure 2.14.

The fortnightly payments terminate automatically,

1. if a currency fixing is at or above B,
2. or if the accumulated profit reaches the target of 68 M JPY.

The accumulated profit is the sum of the fortnightly profits

$$\text{PROFIT} = 2 \times \max(S_i - K, 0) \tag{24}$$

in million of JPY. This means twice the difference of the fixing S_i and the strike K, provided that this difference is positive.

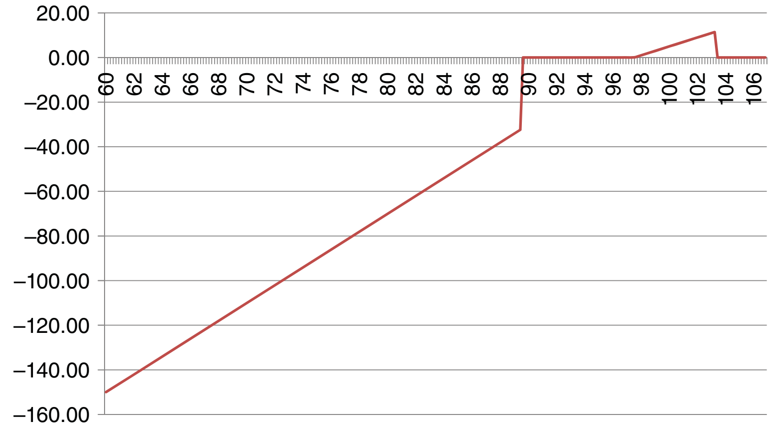


FIGURE 2.14 Illustration of profits and losses in a long kiko TARN (target redemption range accrual note). A range of possible AUD-JPY fixings is plotted on the x-axis. The y-axis denotes the profit and loss per fixing in million of JPY.

Scenario Analysis The best case each fortnight happens when the spot is just before the barrier B, which would be

$$\begin{aligned}\text{MAXPROFIT} &= 2 \times \max(B - K, 0)\\ &= 2 \times \max(103.40 - 97.50, 0)\\ &= 2 \times \max(5.90, 0)\\ &= 2 \times 5.90\\ &= 11.8 \text{ M JPY}.\end{aligned}$$

If there was no target, then the maximum profit for all the 26 fortnights would be $11.8 \times 26 = 306.8$ M JPY. However, because of the target the total profit is limited to 68 M JPY. The figures shows clearly the *limited* upside potential and the *unlimited* downside. For example, should the AUD-JPY spot go down to 80.00, then the investor would make a loss of 70 M JPY or equivalently 875,000 AUD each fortnight. In this case the total loss would be AUD 875,000 × 26 = AUD 22,750,000.

The worst case happens on the downside, where the investor is getting long 26 × AUD 4,000,000 vs JPY at a rate of 97.50, which will happen if all 26 fixings occur below 89.50. Although the nominal "notional" is AUD 2,000,000, this TARN can potentially create an FX position of AUD 104,000,000 at 97.50, and thus its delta, or exposure, would be of the same order of magnitude if the spot rate falls early on towards 89.50. Note that the lowest exchange rate ever observed was 55; the lowest of the move in 2008 was 57. Compared with Figure 2.14, Figure 2.15 illustrates how the gains and losses accumulate through time taking all 26 fixings into account and assuming that the TARN does not terminate early. The *limited gains* as opposed to the *unlimited losses* are clearly observable.

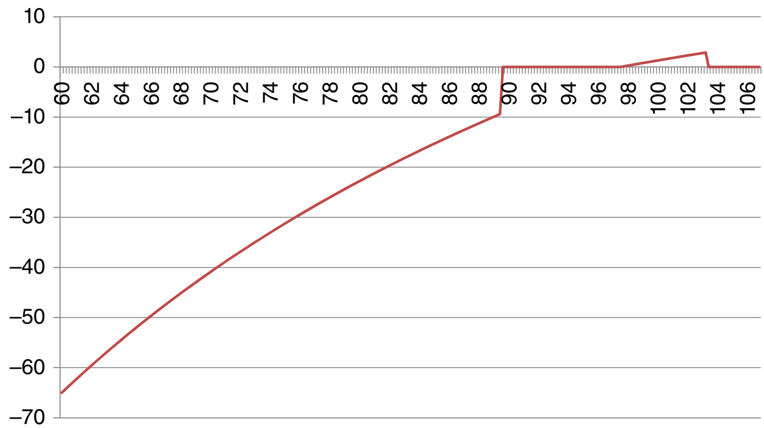


FIGURE 2.15 Illustration of profits and losses in a long kiko TARN (target redemption range accrual note). A range of possible AUD-JPY fixings is plotted on the x-axis. The y-axis denotes the profit and loss in AUD summed over all 26 fixings.

The nature of the trade is the hope that the spot rises and leads to an early knock-out either by reaching the target profit or by hitting the barrier B.

Finally, I would like to point out that although usually TARNS trade at zero cost at inception (i.e. neither the buyer nor the seller pays anything), the one considered here contains an upfront payment of 400,000 AUD from the bank to the client. Typically an upfront premium of a normally zero-cost structure is used in the market to cover losses encountered by other investments.

2.2.6 Target Forwards in the Media

There are numerous reports in the media on target forwards. The sell-side media is usually pro-active and tends to report very positively about the various innovations in tarfs in new underlying currency pairs and new add-on features. The buy-side media likes to report when something goes wrong, i.e. when some investor has made a huge loss on trading target forwards. It is to be expected in any carry trade that small profits occur for a while, and when market regimes shift and many want to leave the carry trade at the same time, huge losses can arise.

In the November 2007 issue of *Structured Products* [114] we find that the range of complexity in target forwards has already become very high and that trading volume of target forwards has indeed grown substantially:

> In addition to emerging markets currencies, HSBC has also completed more than USD 1 billion of transactions with G10 currencies of varied types of target redemption forwards contracts for the institutional market. Investors can profit from currency movements by buying or selling a currency at a rate better than the forward rate, and the trade will be redeemed when the target level of profit has been accrued. HSBC has structured best-of, worst-of features, American knockouts and increased the leverage by using strikes based on previous movements in the spot rate – a snowball structure.

Randall Dodd reports in his article *Playing with Fire – Firms Across the Spectrum of Emerging Markets Entered into Exotic Derivative Contracts that Caused Massive Losses* [39] how many corporates and HNWIs have lost a fortune when trading kikos or tarfs on a margined account.

Nunn and Sinha report in a statement sponsored by BNP Paribas in Risk-Magazine [99] how target forwards work and how they can be used by liability managers and investors. This sell-side article is very well worth reading.

It was reported in *Derivatives Week* in March 2010 that USD/CNY target redemption forwards find favor [2]. Apparently pegged currency pairs do not stop these structured forwards from trading. Target forwards have generally been very popular in Asia, especially in Taiwan.

It was reported in *Derivatives Week* in January 2011 that Standard Chartered Bank had begun selling USD/CNH target forwards deliverable in CHY to corporates looking to cover either currency depending on their exposures [3]. This structure was a new one for this cross as the OTC derivatives market for CNH, which is CHY deliverable to a bank in Hong Kong, had just begun to develop.

2.2.7 Valuation and Hedging of Target Forwards

Generally, the market risk of target forwards will be hedged using the usual delta, vega, gamma, and rho hedges one would also apply to hedging first generation exotic options. Since there is a fixing schedule, the handling and post-trade valuations are considerably more complex than those for first generation exotic options. Some target forwards have digital features in the sense that their value jumps if the spot reaches a pre-specified level or a level that causes an early termination. In the FX TRF market valuation consensus has drifted eventually towards including the cost of digital risk in TARNS, although how different banks quantify said risk still seems to vary.

Tarfs are most commonly priced with Monte Carlo simulation methods these days. It is not surprising as the Monte Carlo approach offers the biggest flexibility to modifying payoffs and also to change the underlying models, such as going from stochastic-local volatility (SLV) to stochastic-local volatility with jumps (JSLV) or one with stochastic rates. Monte Carlo also extends to two-factor models in a straightforward way. There is a market consensus that local volatility (LV) is good enough for TARNs in general, and therefore a LV-based PDE approach with an extra layer counting the accumulated profit might be a better choice, especially when you need to show smooth Greeks. Volmaster uses purely Monte Carlo + GPUs for the purpose of flexibility. Actually I expect any pure pricing platform (Volmaster, Pricing Partners, Numerix, Fincad, …) to use Monte Carlo almost whatever the model and the payoff, simply because it is much easier to code as a generic engine. However, for bank position keeping systems with exotics in volumes it might be another story, particularly if smooth Greeks are required.

Semi-Static Replication of Target Forwards with Vanilla Options and Exotics There is a way to replicate target forwards semi-statically with a portfolio of vanilla options and first generation exotics. I call it semi-static because the portfolio needs to be changed at each fixing date. Since there may be many fixing dates, this does not appear to be practically feasible. However, there is something to learn from the replication, so let's take a look.

Building Blocks. Target forwards have an accruing *intrinsic value counter*

$$\text{IVC}(t_i) = \begin{cases} \sum_{j=1}^{i} \max(S(t_i) - K_i, 0) \text{ for call payoffs} \\ \sum_{j=1}^{i} \max(K_i - S(t_i), 0) \text{ for put payoffs} \end{cases} \tag{25}$$

As soon as $\text{IVC}(t_i) > L$ for a given target level L, the structure terminates. Hence, through the IVC the payoff at time t_i for FX spot $S(t_i)$ becomes a function of the FX spot $S(t_{i-1})$ prevailing at time t_{i-1}; the whole structure is path-dependent and for exact pricing and risk management an appropriate exotics model is required. The good news is: the nested vanilla-style payoffs can be approximately replicated by making assumptions about the future FX spot. The working assumption is to replace the unknown future spot price by the known outright forward rate.

The Expected Hedge Exit Time. Target forwards are usually in-the-money for a client at trade inception. A negative theoretical value is due to the fact that according to the FX forward curve most fixings of the client's long leg are expected to be out-of-the-money and/or the short leg from a client's perspective is leveraged by a factor λ. Nevertheless, the expected fixing date at which the IVC is reached can be before the date at which the forward crosses the pre-agreed exchange rate of the client's long leg. We focus on replicating the structure up to the date at which the target level is expected to be reached, or the forward $F(t_i)$ is beyond the pre-agreed exchange rate of the client's long leg. We define the expected hedge exit time as

$$t_E = \begin{cases} \min\{t_i | \ I\hat{V}C(t_i) \geq L \vee F(t_i) < K_i^{long}\} \text{ for a buyer TRF} \\ \min\{t_i | \ I\hat{V}C(t_i) \geq L \vee F(t_i) > K_i^{long}\} \text{ for a seller TRF} \end{cases},$$

where K_i^{long} denotes the pre-agreed exchange rate of the long leg from a client perspective and $I\hat{V}C(t_i) = \sum_{j=1}^{i} \max(F(t_i) - K_i, 0)$. The idea of the replication is to cover the most likely outcome of the trade life-cycle from a risk-neutral perspective. If the risk manager has a different view on the future FX spot, t_E can be modified accordingly. In the example of a plain target forward in Section 2.2.1, we assume the interest rates for both USD and EUR to be flat 4%, so all forward rates are equal to the initial spot, i.e. $F(t_i) = 1.4760$. This generates a profit of 575 pips each week. It will take six weeks for the intrinsic value counter to exceed the target, i.e. $t_E = 6/52$.

Final Payout Versions Each payoff can be the final payoff depending on the level of the FX spot (for most target forwards). We define the distance D_i to target level as

$$D_i = L - IVC(t_{i-1}). \tag{26}$$

Market participants typically use three different final payoff styles:

Full final payout if $\mathrm{IVC}(t_i) > L$. This can be replicated by a plain vanilla call or put with strike K_i^{long}.

Capped final payout $\mathrm{IVC}(t_i) > L$. This can be replicated by a call spread or put spread with strikes K_i^{long} and $K_i^{long} \pm D_i$.

No final payout if $\mathrm{IVC}(t_i) > L$. This implies digital risk and can be replicated by a vanilla call or put spread with strike K_i^{long} and a European knock-out with level $K_i^{long} \pm D_i$.

Building Blocks for Hedging The standard versions of tarfs can be replicated with ratio forward contracts or ratio collars (version 5 in Table 2.36) with leverage λ. The risk manager has the option to additionally approximate the target feature by implementing a call spread for a buyer tarf and a put spread for a seller tarf. In a version where the seller agrees on a fixed number of payments with capped intrinsic value G up to fixing date t_G, provided the structure is in-the-money for the client, the risk manager can set the limiting strike for the vanilla spread exactly to $K_i \pm G$, see version 6 in Table 2.36. Depending on the risk manager's appetite for vanilla risk one could omit either the vanilla replication of the short leg and/or the limiting vanilla trades on the long leg. This might also be recommendable as the limiting strike of the vanilla spread would need to be adjusted following the reduction of the distance to the target level D_i, and consequently crossing bid-ask and causing transaction costs.

Overview of Different Popular Styles Table 2.36 refers to a seller tarf and provides examples of semi-static replication portfolios of common versions of tarfs.

Following the example of a plain target forward in Section 2.2.1 with $\lambda = 1$, the replicating portfolio would consist of a long EUR put with strike 1.5335, a short EUR call with strike 1.5335, and a short EUR put with strike 1.2335 (= 1.5335 − 0.3000), all with 1 M EUR notional and maturity of one week. For the first week, this is a perfect

TABLE 2.36 Semi-static replication portfolio of common versions (V) of seller tarfs. Version 2 generalizes the standard version 1 to a term-structure of strikes; version 4 uses a knock-in ratio forward.

V	Long	Limit(Long)(optional)	Short (optional)
1	$\sum_{t_i=t_1}^{t_E} Put(K, t_i)$	$-\sum_{t_i=t_1}^{t_E} Put(K - D_1, t_i)$	$-\lambda \sum_{t_i=t_1}^{t_E} Call(K, t_i)$
2	$\sum_{t_i=t_1}^{t_E} Put(K_i, t_i)$	$-\sum_{t_i=t_1}^{t_E} Put(K_i - D_1, t_i)$	$-\lambda \sum_{t_i=t_1}^{t_E} Call(K_i, t_i)$
3	$\sum_{t_i=t_1}^{t_E} Put(K_i, t_i)$	none	$-\lambda \sum_{t_i=t_1}^{t_E} Call(K_i, t_i)$
4	$\sum_{t_i=t_1}^{t_E} Put(K_i, t_i)$	$-\sum_{t_i=t_1}^{t_E} Put(K_i - D_1, t_i)$	$-\lambda \sum_{t_i=t_1}^{t_E} KICall(K_i, B_{up}, t_i)$
5	$\sum_{t_i=t_1}^{t_E} Put(K_i^{long}, t_i)$	$-\sum_{t_i=t_1}^{t_E} Put(K_i^{long} - D_1, t_i)$	$-\lambda \sum_{t_i=t_1}^{t_E} Call(K_i^{short}, t_i)$
6	$\sum_{t_i=t_1}^{t_G} Put(K_i^{long}, t_i)$	$-\sum_{t_i=t_1}^{t_G} Put(K_i^{long} - G, t_i)$	$-\lambda \sum_{t_i=t_1}^{t_G} Call(K_i^{short}, t_i)$

replication. The same replication of a EUR put spread and a EUR call will be set up for the subsequent six weeks, using lower strikes 1.2910, 1.3485, 1.4060, 1.4635, and 1.5210. These strikes are obtained by adding 0.0575, which is the weekly additional profit assuming all future fixings are equal to the current spot. We continue increasing the strikes as long as they stay below the pre-agreed exchange rate of 1.5335. Since the short EUR put with the low strike 1.2335 is very far out-of-the-money it really appears reasonable to just omit it from the replication. After the first week is over, the fixing of the first week will be known, and possibly the strikes of the short EUR puts will have to be replaced to keep the replication exact. In practice, one may keep the existing replication as an approximation, provided the spot does not fix too far away from the forward rates determined at inception. From this semi-static replication we deduce that

- the risk management of a plain target forward is not more complicated than risk managing a vanilla portfolio, even if the leverage factor is different from one;
- in the unleveraged case, the vega of the tarf is initially small but is concentrated around the strikes of the short EUR put; since the selling bank is long this EUR put, the bank is typically vega long, and vega approaches the spot like an avalanche as time passes; this provides an intuitive explanation as to why vega is initially small and becomes larger close to the expected hedge exit time, see the bucketed aega, rega, and sega in Table 2.33;
- in the leveraged case, the selling bank is also vega long at the pre-agreed exchange rate, which is created by the long EUR call; every week, one of the options in replication expires, so total vega decreases, an effect sometimes referred to as *vega bleed*;
- the replication would require quite a number of replacements to keep it exact, which is consequently normally not done in practice.

Scenarios The best outcome of the replicating strategy is obtained if the target level is reached quickly at a time $t_i < T_E$:

- The client achieves maximum profit from the structure.
- The remaining hedges can be canceled out.

The worst outcome would be obtained if the FX spot remains slightly in-the-money, but the target level L is never obtained.

- The initial approximate hedge is not sufficient and additional hedges might be needed, leading to transaction costs.
- The risk manager might prefer to resort to dynamic hedging in this case after t_E has been exceeded.

Valuation of Target Forwards on Pricing Platforms Many platforms offer the valuation of target forwards. For example, Bloomberg's OVML. Figure 2.16 shows the explanation of a tarf in Bloomberg. Figure 2.17 shows a sample valuation of a pivot target forward in Bloomberg.

FIGURE 2.16 Bloomberg screen shot tarf explanation.

FIGURE 2.17 Bloomberg screen shot: OVML pivot target forward.

2.2.8 Exercises

KIKO TARN Referring to the following transaction, what would be the impact on the pre-agreed exchange rate of trading this for zero premium? That is, value the below, and then solve for the pre-agreed exchange rate to make it zero premium. Underlying: GBP/JPY, spot ref 205.50, client buys GBP 1,000,000 (cash settled) at the pre-agreed exchange rate 198.50 if fixing is below, buys GBP 2,000,000 (deliverable) if fixing is above 198.50, knock-in 190.50 (European, observed on each fixing date), knock-out 206.50 (continuously observed on whole structure), tenor 1 year, 26 fixings every 2 weeks, target on profit JPY 34,000,000. This final profit settlement is reduced to bring the total accumulated profit back to the target amount.

This *kiko TARN* terminates either if the accumulated profit exceeds the target, or if the spot fixing is at or above the knock-out barrier. Accumulated profit is the running total of the cash settled JPY amount. You may use a constant parameter Black-Scholes model for simplicity. Assume the same spot reference, and scenario 1 market data of volatility 11.7%, rate JPY 0.37%, rate GBP 0.96%; rerun your calculation for scenario 2 market data of volatility 11.7%, rate JPY 1.00%, rate GBP 6.00% reflecting the rates in 2008, which is when this kiko TARN traded.

As an alternative consider the following variant, as it would be a less risky version: the lower barrier is a knock-out rather than a knock-in.

Leveraged EUR/USD Target Forward Consider the terms and conditions of the target forward in Figure 2.18.

1. What is the maximum potential profit for the client?
2. Assuming at trade time the EUR/USD swap points were 46, which of the following is probably the initial EUR/USD spot reference at trade time: 1.3750, 1.2964, 1.3500, or 1.2045?
3. After selling the target forward, the trader on the FX derivatives desk
   a) buys about 12 M EUR to delta hedge and is vega short.
   b) sells about 8 M USD to delta hedge and is vega short.
   c) has a large long vega position but no significant delta.
   d) buys about 8 M EUR to delta hedge and is vega long.
   Which one is correct?
4. What is the maximum potential loss for the buyer? Assume the buyer is an investor with the underlying cash flow.
5. Suppose the client is not happy with the 200% leverage and you change the leverage to 150%. If you still want to offer the target forward as a zero-cost strategy and keep the same sales margin, which contractual parameters can you change and in which direction?
6. If EUR/USD moves to 1.2900 and then stays there, when does the target forward terminate and what is the profit for the buyer?
7. Suppose the buyer will accumulate a profit of USD 260.000 and EUR/USD moves to 1.3500, and the investor wants to unwind the target forward. What will happen?

Target Forward

Client:	
Trade Id:	
Currency and Notional:	EUR 1,000,000
Trade Date:	16/10/2012
Last Delivery Date:	18/10/2013 (subject to Target Event)
Strike:	1.3500 USD per EUR
Target:	0.3000
Leverage:	200.00%
Positive Intrinsic Value:	Max(0, Strike - Fixing)
Target Event:	A Target Event occurs if the sum of all the positive intrinsic values (meaning only fixings **at or below** the relevant **Strike** are taken into account) for all previous forward transactions is **equal or larger** than the Target. As soon as the Target Event occurs, the transaction that caused the Target Event and all the following ones are knocked out.
Analysis:	The client enters into a strip of forward transactions to **sell** the relevant **EUR Notional** against USD at the relevant **Strike** price subject to the Target Event. The Leverage indicates that the relevant Notional of the transaction is multiplied by the relevant Leverage if the spot fixed **strictly above** the relevant **Strike** for this Maturity.
EUR/USD:	Expressed in USD per 1 EUR as will be determined at any time by the Calculation agent and this determination will be final except in case of manifest error.
Fixing:	Expressed in USD per EUR as published on Reuters ECB37 (or any successor thereto).
Upfront Premium:	Zero Cost Strategy
Calculation Agent:	
Business Days:	Target, London, New York
Date Format:	All Dates are expressed in the form dd/mm/yyyy (day/month/year).

Fixing Table:

	Fixing Dates	**Delivery Dates**
1	16/11/2012	20/11/2012
2	17/12/2012	19/12/2012
3	16/01/2013	18/01/2013
4	19/02/2013	21/02/2013
5	18/03/2013	20/03/2013
6	16/04/2013	18/04/2013
7	16/05/2013	20/05/2013
8	17/06/2013	19/06/2013
9	16/07/2013	18/07/2013
10	16/08/2013	20/08/2013
11	16/09/2013	18/09/2013
12	16/10/2013	18/10/2013

FIGURE 2.18 EUR/USD target forward sample term sheet.

2.3 SERIES OF STRATEGIES

Series of strategies are often applied when the corporate treasurer expects regular cash flows over a period of time. Essentially all (structured) products can be decomposed into a series or a *strip*. The process of creating a series from a strategy with a given maturity into one with many maturities is also called *stripping*. Everything is possible, so that I can't really present a complete list of series of structures. It starts with a simple series of forward contracts, continued with series of vanillas, spreads, risk reversals, straddles, strangles, seagulls, etc. For the forward series in turn, one can just package a series of, say, 12 outright forward contracts for one year, with monthly settlement dates. The individual legs would all be zero cost, hence the entire series would also be zero cost. The alternative is to find the average exchange rate that works for all 12 legs and still produces a zero-cost structure known as a *par forward*. In the case of the forward contracts, the average is just the average of the outright forward rates.

Generally, any accumulative forward, target forward, or cross currency swap can also be considered a series of strategies.

For a zero-cost series of risk reversals for example, where we want to use only one and the same strike for all the calls and one strike for all the puts, we need to find these strikes by iteration. In the following we would like to give a few examples of how these series can be worked out and what the alternatives are.

2.3.1 Shark Forward Series

For the shark forward series many different variations are possible.

Identical Shark Forwards Here we do a shark forward series, whose parameters for each tenor are the same except the maturity date. This is sometimes called the *Milano strategy*. For example, we take 12 shark forwards, one maturing and settling in each month. The major disadvantage of this is that the barrier for the shorter maturities is comparatively far away from the spot, so the components with short maturity in the structure become unreasonably expensive. There are ways to circumvent this weakness, as we show now.

Identical Shark Forwards with Time Dependent Barriers Here we can simply design a step function for the knock-out barrier, starting with a barrier closer to the spot for the shortest leg and then move the barrier further away from the spot with longer maturities. To keep it simple for the client, the steps of the barrier moves are usually kept constant.

Window Shark Forwards Another common way to structure series of shark forwards is to use window barriers for the knock-out barrier components. This means that the barrier is valid only for the time between the last maturity and the actual maturity of the leg. Of course, window barriers and stepping barriers can be combined.

Forwards with Single Compensation Payment This is more than just stripping existing structures with varying maturities. Here we take a series of synthetic forward contracts, usually with the same fixed exchange rate and notional amounts. The fixed exchange rate represents the guaranteed worst case scenario for the treasurer. The value of the entire series leaves money to invest into an option representing the treasurer's market view. In the case of a shark forward the structure contains a reverse knock-out. The entire structure is usually zero cost.

TABLE 2.37 EUR/USD outright forward rates as of May 3 2004 for spot reference 1.1900.

Value date	Outright forward rate	Value date	Outright forward rate
05/31/04	1.1892	03/31/05	1.1844
06/30/04	1.1882	04/29/05	1.1844
07/30/04	1.1873	05/31/05	1.1844
08/31/04	1.1866	06/30/05	1.1845
09/30/04	1.1860	07/29/05	1.1847
10/29/04	1.1856	08/31/05	1.1850
11/30/04	1.1852	09/30/05	1.1854
12/31/04	1.1849	10/31/05	1.1858
01/31/05	1.1847	11/30/05	1.1864
02/28/05	1.1845	12/30/05	1.1870

Example We consider an exporter in the EUR zone expecting incoming cash flow of 1 M USD at the end of each month for the following 20 months, representing a total notional amount of 20 M USD. The treasurer seeks a low exchange rate to buy EUR. In this concrete case the outright forward rates for EUR/USD as of May 3 2004 are listed in Table 2.37. We consider the following alternatives.

Wait for better times. This means that the treasurer does not enter into any hedge but waits for the spot to come down. This strategy is free but carries enormous risk. Suppose the exchange rate during the following 20 months is at 1.2500 on average, then the corporate treasurer encounters a total loss of 867,660 EUR. On the other hand, the corporate can fully participate in a falling EUR.

20 outright forward contracts. This is the zero-cost fully safe hedging strategy. However, the corporate treasurer cannot benefit from a falling EUR.

20 EUR call options. This is the full coverage solution with full participation in a falling EUR. However, the drawback is, of course, that it is expensive. For strike 1.2000, for instance, this strip would cost 550,000 EUR.

20 risk reversals. This covers the corporate treasurer's risk of a rising EUR and allows some decent participation in a falling EUR at zero cost. For instance, one could set the strike of the EUR calls at 1.2500 and the strike of the EUR puts at 1.1300.

Series of forwards with a single compensation payment. Here we could generate a worst case of 1.2000, which is only 143 pips worse than the average outright forward rate. By giving up these 143 pips the corporate treasurer would buy a EUR put with strike 1.2000 and American style barrier 1.0500. We let the notional of this reverse knock-out put coincide with the total notional amount of 20 M USD, although this is in no regard binding. Even the strike could be lowered to allow for a lower knock-out trigger if needed. Now, if the trigger is ever touched, the EUR RKO put is knocked out, and the treasurer is left with the guaranteed worst case. If the trigger is not touched, then the treasurer participates in a cheaper EUR and receives a compensation payment. As a result his overall exchange rate is approximately equal to the EUR/USD spot at maturity. This strategy is free, a well-balanced mixture of safety and participation opportunities, allows for hedge accounting, can be confirmed as a forward contract, and can be unwound any time at market conditions.

2.3.2 Collar Extra Series

Let us assume a corporate treasurer who needs to buy EUR in three months and in six months from an income in JPY. He would like to hedge against rising EUR/JPY and have a worst case of 130.00 EUR-JPY guaranteed. However, if EUR-JPY trades at least once below 128.00 in each respective time period, he is willing to buy EUR at a rate below the worst case but near the current outright forward rate. He finds buying two vanilla calls too expensive and is looking for a zero-cost strategy. We list the indication term sheet in Table 2.38.

We note that this is a series of two collar extra transactions. The knock-in barrier is valid for each tenor individually. The knock-in event happens if at least once between trade time and expiry EUR/JPY trades at or below 128.00. If the knock-in event occurs, the treasurer has locked in a forward with the improved case. It is called improvement because it is better than the guaranteed worst case. However, it is worse than trading a new outright forward at the time of knock-in. This feature makes the structure worth zero cost. If the knock-in event does not occur, then the treasurer participates in a falling EUR/JPY rate until 128.01.

Composition For each tenor, the treasurer enters a strategy that is equivalent to

1. buying a EUR call with strike at the worst case, down-and-out at 128.00,
2. selling a EUR put with strike at the improved case, down-and-in at 128.00,
3. buying a EUR call with strike at the improved case, down-and-in at 128.00.

TABLE 2.38 Example term sheet of a collar extra series.

Spot reference	133.00 EUR-JPY
3-month outright forward	132.30 EUR-JPY
6-month outright forward	131.60 EUR-JPY
Treasurer sells	JPY
Treasurer buys	10,000,000.00 EUR per maturity
Guaranteed worst case	133.00 EUR-JPY
3-month improved case	132.30 EUR-JPY
6-month improved case	131.60 EUR-JPY
Knock-in condition for improved case	128.00 EUR-JPY (American style)
Premium	0.00

2.3.3 Exercises

USD Call Strip Compose a strip of EUR put USD call vanillas for one year, one for each month with the same strike K. Assume a flat interest rate of 3% for USD and 2.5% for EUR without bid-offer spread, an offer volatility of the trading desk of 10%, and a current spot of 1.2000. Let the monthly notional be 200,000.00 USD. The treasurer is willing to pay 30,000 EUR premium for the strip. Where should the strike be for a sales margin of 2,000 USD? What is the delta hedge you as a seller have to do? Where should the spot go if sales is more greedy and wants a sales margin of 3,000 USD? For the sake of simplicity you may assume times to maturity as multiples of 1/12.

2.4 DEPOSITS, LOANS, BONDS, AND CERTIFICATES

There is a huge market for deposits and loans for investors, corporate treasury as well as retail. With permanently low interest rates for the major currencies many investors seek ways to enhance coupons, which is usually achieved by taking a speculative short position in the financial markets. Deposits can be linked to many underlyings. Convertible and reverse convertible bonds are popular structures linking deposits to equity markets. There have been offers for savings accounts where the interest rate is linked to the result of a lottery. Hypovereinsbank has come up with a coupon linked to the result of soccer matches. We also find inflation or weather-linked notes. In this book we present structured deposits and loans linked to the foreign exchange markets. All of them trade and more are expected to come as the hunger for yield enhancements is never ending. Yield can be enhanced by selling options (short vega) or even selling correlation risk.

Deposits can be generally categorized into *capital guaranteed* and *non-capital-guaranteed* products. In the latter the investor waives part of the coupon the money market would pay and takes a position that is equivalent to buying a derivative instrument matching her market view. In the former, the investor usually is in a role equivalent to selling an option to increase the coupon and then hopes that the holder of the option will not exercise the option.

Legal formats of deposits are typically OTC deposits, certificates, notes, or bonds.

2.4.1 Dual Currency Deposit/Loan

A dual currency deposit (DCD) is a very popular and liquidly traded standard FX-linked investment that works like a *reverse convertible bond* in the equity markets. Investors searching for higher yields trade it and take a position which is equivalent to selling a vanilla call on the deposit currency, which is also a put on another currency, to receive a higher coupon and then hope that the option they sold will be out-of-the-money at maturity. Some banks also name it *enhanced deposit* or *dual currency investment (DCI)*.

The product works like this. An investor deposits an amount in, say, EUR for a fixed time horizon, usually from one day up to one year, and instead of the market interest rate she receives a higher coupon. For example, she invests 5 M EUR for one month (we take 30/360 years here), where the market rate is 3.00 %, which would return 12,500 EUR interest payment. The higher and fully guaranteed coupon is then, for example, 5.00% = 20,833 EUR. As this cannot be a free lunch, the investor is taking the following risk. At a EUR-CHF spot reference of 1.4662 she chooses a pre-agreed conversion rate of 1.4700. If at maturity the spot stays below 1.4700 (strong CHF), then the investor is paid back the full notional in EUR. Conversely, if the spot at maturity is above 1.4700 (weak CHF), then the notional is converted into CHF 7.35 M. So the investor is always paid back the weaker currency. The interest rate with the enhanced coupon of 5.00% is paid in either case.

Advantages

- Guaranteed higher coupon than market
- Liquid product, thus sales margins are comparatively small
- An exporting company in the country of the other currency needs to buy the other currency anyway and can use the pre-agreed conversion rate, which is used as the strike of the short option, as a budget rate
- Investor can take a short option position

Disadvantages

- No capital guarantee: if the other currency becomes worthless, then the investor can lose the entire capital
- The investor receives a higher coupon at the cost of taking *unlimited* exchange rate risk, see Figure 2.19
- Investor has to pay tax on the enhanced coupon, but cannot tax-deduct the risk on the upside

Composition

- The investor deposits 5 M EUR for 1 month
- The investor sells a EUR call CHF put with strike at the pre-agreed conversion rate 1.4700

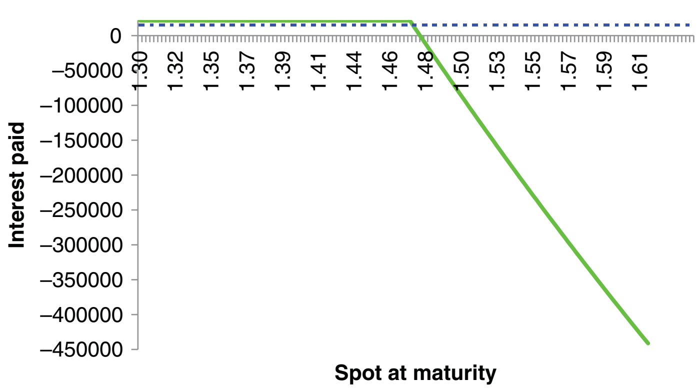


FIGURE 2.19 Comparing the market interest rate (dotted line) with the enhanced interest rate of a dual currency deposit.

- Spot reference 1.4662
- Premium: 11,631 EUR (deferred to end of the deposit)
- This yields 12,500 EUR + 11,631 EUR = 24,131 EUR
- The bank pays only 20, 833 EUR
- → Margin: 3,298 EUR

In general, the formula relating the notional amount N, the bid deferred premium of the vanilla in units of the underlying P, the sales margin M, the market interest rate r_M (bid side), and the enhanced interest rate r_E, the deposit horizon of d days is given by

$$N \cdot r_E \cdot \frac{d}{360} = N \cdot r_M \cdot \frac{d}{360} + P - M, \tag{27}$$

which can be solved easily for any quantity desired. Note that for GBP the day-count convention for deposits is act/365 days rather than act/360.

For valuation, booking, and accounting purposes, note that the premium of the short option must be paid at delivery of the deposit. The DCD is an ideal way for a bank to buy options from its clients without taking any credit risk. Since the bank has the deposit amount, the DCD does not lead the bank into a situation of suffering from a defaulting client (but probably a complaining client). Conversely, the investor is taking the credit default risk of the issuer as in any other deposit. Surely, the better the rating of the issuer, the lower the coupon. This extends to the DCD. The exchange rate risk can be reduced by taking a strike sufficiently far away from the spot. However, selling such a far out-of-the-money option will not enhance the coupon significantly. The unlimited risk on the upside can be limited by selling an up-and-out call (see Section 1.7.3) or a call spread (see Section 1.6.1) rather than a vanilla call. But even that would give only a small coupon enhancement. Yet another alternative would be to take a smaller notional for the option, so the conversion risk would apply to only, say, 50% of the invested capital. Again, lowering risk would imply a smaller coupon.

The investor is obviously taking risk on the upside. What if the investor believes in an upward trend and would rather take risk on the downside? One can then take the other currency. But if she wants to invest in the same currency, a similar structure as in the dual currency deposit does not work in a straightforward way. Of course, the investor can always sell a put on the deposit currency. However, at maturity, the payoff of the put would have to be cash-settled in the deposit currency and this payoff then subtracted from the deposit amount. In fact, the payoff of the put may exceed the notional of the deposit. We will revisit this challenge in the case studies in Section 2.8.4 on the inverse DCD. An alternative might be the *turbo deposit* as explained in Section 2.4.5. The case of unwinding such a dual currency deposit is treated in the exercises.

Dual Currency Loan One can turn this deposit into a loan, where the client borrows EUR at a subsidized interest rate but has to pay back the other currency if the EUR decreases below the pre-agreed conversion rate. So in the case of a dual currency loan, the client would take a view on a strengthening loan currency. Her position would be equivalent to selling a EUR put. This product is by far less popular as the issuers are taking the credit default risk of both the loan and of the long option position. An example can be worked out in the exercises.

2.4.2 Performance-Linked Deposits

A performance-linked deposit is a deposit with a participation in an underlying market. The standard is that a GBP investor waives her coupon that the money market would pay and instead takes a position that is equivalent to buying a EUR-GBP call with the same maturity date as the coupon, strike K, and notional N in EUR. These parameters have to be chosen in such a way that the offer price of the EUR call equals the money market interest rate plus the sales margin. The strike is often chosen to be the current spot. The notional is often a percentage p of the deposit amount A, such as 50% or 25%. The coupon paid to the investor is then a pre-defined minimum coupon plus the participation

$$p \cdot \frac{\max[S_T - S_0, 0]}{S_0}, \tag{28}$$

which is the return of the exchange rate viewed as an underlying asset, where the investor is protected against negative returns. So, obviously, the seller's hedge is selling a EUR call GBP put with strike $K = S_0$ and notional $N = pA$ GBP or $N = pA/S_0$ EUR. So if the EUR goes up by 10% against the GBP, the investor gets a coupon of 10p% in addition to the minimum coupon. The structure does not work if money market rates are insignificantly less higher or even lower than the guaranteed interest rate. In particular, in an environment of negative interest rates, a capital guarantee structure with a participation does not work. In such a case one would have to consider lowering the capital guarantee to less than 100%.

Advantages

- Possible higher yield than market
- Liquid product, thus sales margins are comparatively small
- Capital is guaranteed, possibly partially guaranteed
- Worst case coupon and return are known upfront

Disadvantages

- Possible lower coupon than market
- Investor has to pay tax on the best case coupon, but cannot tax-deduct the risk on receiving the worst case coupon
- Participation formulas can often appear misleading to investors
- Construction does not work if the money market interest rate of the investment currency is already low

Example We consider the example shown in Table 2.39. This means, for example, that if the EUR-GBP spot fixing is 0.7200, the additional coupon would be 0.8571%. The break-even point is at EUR-GBP 0.7467, so this product is suitable for a very strong EUR bullish view. For a weakly bullish view an alternative would be to buy an up-and-out call with barrier at 0.7400 and 75% participation, where we would find the best case to be EUR-GBP 0.7399 with an additional coupon of 4.275%, which would lead to a total coupon of 6.275%.

TABLE 2.39 Example of a performance-linked deposit, where the investor is paid a minimum coupon below market plus 30% of the EUR-GBP return.

Notional	5,000,000 GBP
Horizon/spot date	1 June 2005 / 3 June 2005
Maturity	2 Sept 2005 (91 days)
Number of days	91
Money market reference rate	4.00% p.a. act/365
EUR-GBP spot reference	0.7000
Minimum rate	2.00% p.a. act/365
Additional coupon	$30\% \cdot \frac{100 \max[S_T - 0.7000, 0]}{0.7000}$ p.a. act/365
S_T	EUR-GBP fixing on August 31 2005
Fixing source	ECB

Composition

- From the money market we get 49,863.01 GBP at the maturity date.
- The investor takes a position that is equivalent to buying a EUR call GBP put with strike 0.7000 and with notional 1.5 M GBP or EUR 2,142,857.
- The offer price of the call is 20,609.70 GBP, assuming an offer volatility of 6.0% and a EUR money market rate of 2.50%.
- The deferred premium is 20,815.24 GBP.
- The investor receives a minimum payment of 24,931.51 GBP.
- Subtracting the deferred premium and the minimum payment from the money market leaves a sales margin of 4,116.27 GBP (as of the delivery date).
- Note that the option used in the composition must be cash-settled.

Variations There are many variations of the performance-linked notes. Of course, one can think of European style knock-out calls, or window-barrier calls. For a participation in a downward trend, the investor can take a position that is equivalent to buying puts. One of the frequent issues in Foreign Exchange, however, is the deposit currency being different from the domestic currency of the exchange rate, which is quoted in FOR-DOM (foreign-domestic), meaning how many units of domestic currency are required to buy one unit of foreign currency. So if we have a EUR investor who wishes to participate in a EUR-USD movement, we have a problem, the usual *quanto confusion* that can drive anybody up the wall in FX on various occasions. What is the problem? The payoff of the EUR call USD put

$$[S_T - K]^+ \tag{29}$$

is in domestic currency (USD). Of course, this payoff can be converted into the foreign currency (EUR) at maturity, but at what rate? If we convert at rate S_T, which is what we could do in the spot market, then the investor buys a vanilla EUR call. But here, the investor receives a coupon given by

$$p \cdot \frac{\max[S_T - S_0, 0]}{S_T}. \tag{30}$$

If the investor wishes to have performance of Equation (28) rather than Equation (30), then the payoff at maturity is converted at a rate of 1.0000 into EUR, and this rate is set at the beginning of the trade. This is the *quanto factor*, and the vanilla is actually a *self-quanto* vanilla, i.e. a EUR call USD put, cash-settled in EUR, where the payoff in USD is converted into EUR at a rate of 1.0000. This self-quanto vanilla can be valued with smile by calculating the implied time-T density of the spot from the vanilla volatility surface and integrating the payoff against this density. A similar problem comes up if the investor wishes to use Equation (30) for a USD coupon. Such a construction happened, for instance, in *swap 4175*, see Section 2.5.7.

There need to be similar considerations if the currency pair to participate in does not contain the deposit currency at all. A typical situation is a non-USD investor, who wishes to participate in the gold price, which is measured in USD, so the investor needs to buy a XAU call USD put quantoed into his investment currency, say EUR. So the investor is promised a coupon as in Equation (28) for a XAU-USD underlying, where the coupon is paid in EUR; this implicitly means that we must use a quanto plain vanilla with a quanto factor of 1.0000. For the valuation of quanto options see Hakala and Wystup [65] or Section 1.7.10.

2.4.3 Tunnel Deposit/Loan

Unlike the dual currency deposit, the *tunnel deposit* or *double-no-touch linked deposit* or *range deposit* is fully capital guaranteed. The maturities vary from one month up to one year. The investor receives a minimum coupon below the market, which is often taken to be 0.00%, if the underlying exchange rate leaves a pre-defined range at least once between trade time and expiration time. Conversely, if the exchange rate stays inside the pre-defined range at all times, then the investor receives a coupon above the market rate. Hence, the tunnel deposit reflects a view on a sidewards movement of the underlying exchange rate. The investor waives parts of her market coupon and takes a position that is equivalent to going long a double-no-touch, whose notional is in the deposit currency, and the amount is the difference between the best case and the worst case interest rate payments. Usually, the exchange rate contains the currency of the deposit and another currency. If the deposit currency is not in the exchange rate, then the payoff has to be quantoed into the deposit currency, which is possible but not very common.

Advantages

- Possible higher coupon than market
- Liquid product, thus sales margins are comparatively small
- Capital is guaranteed
- Worst case coupon is known upfront

Disadvantages

- Possible lower coupon than market
- Consequently, if the money market interest rate is low or negative, the construction does not work or appears unattractive
- Investor has to pay tax on the best case coupon, but can not tax-deduct the risk on receiving the worst case coupon

TABLE 2.40 Example of a tunnel deposit. The minimum rate is paid if EUR-USD leaves the range, the maximum rate is paid if EUR-USD stays inside the range.

Notional	2,000,000 EUR
Start date	June 3 2005
Maturity	Sept 5 2005
Number of days	94
Money market reference rate	2.50% act/360
Minimum rate	1.00% act/360
Maximum rate	4.20% act/360
Range	1.1750–1.3000 EUR-USD
Range valid up to	Sept 1 2005 10:00 a.m. N.Y.

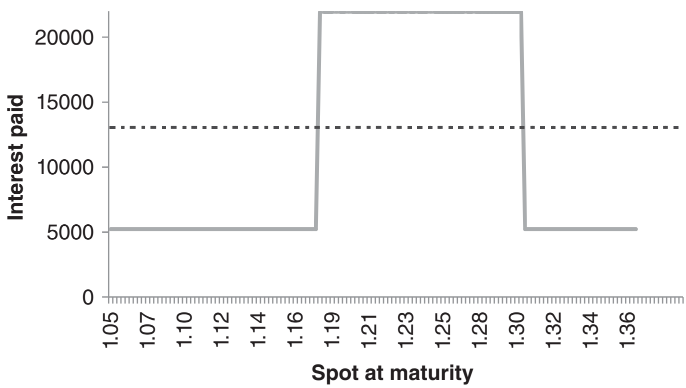


FIGURE 2.20 Best case and worst case interest rate of a tunnel deposit compared with the market interest rate (dotted line).

Example We consider the example shown in Table 2.40 and illustrate the interest rate payments in Figure 2.20.

Composition

- We obtain 13,056.67 EUR from the money market.
- The investor has a position that is equivalent to buying a EUR-USD double-no-touch with range 1.1750–1.3000 and with payout 16,711.11 EUR. This notional must match the interest rate difference of the maximum and the minimum rate, which is

$$2{,}000{,}000.00 \cdot (4.20\% - 1.00\%) \cdot \frac{94}{360} = 16{,}711.11.$$

- The offer price of the double-no-touch is 37.00%, assuming a volatility of 10.0% and a USD rate of 2.70%.
- To defer the premium to the maturity date we use the rate 2.65%.

- The deferred premium becomes 37.2506%, which is 6,225.89 EUR.
- The investor receives a minimum payment of 5,222.22 EUR.
- Subtracting the deferred premium and the minimum payment from the money market leaves a sales margin of 1,608.56 EUR.
- The investor receives a maximum payment of 21,933.33 EUR.

In general, the formula relating the notional deposit amount N, the offer deferred premium of the double-no-touch in units of the underlying P, the sales margin M, the market interest rate r_M (bid side), the minimum interest rate r_0, and the deposit horizon of d days is given by

$$N \cdot (r_M - r_0) \cdot \frac{d}{360} = M + P, \tag{31}$$

which can be solved easily for any quantity desired. Note that for GBP the day-count convention for deposits is act/365 days rather than act/360.

Variations and Remarks This tunnel deposit can be altered in various ways. Instead of a double-no-touch the investor could take different views and implement these with another transaction such as a single-no-touch, a single-one-touch, a window double-one-touch, a digital call or put, or European style double-no-touch.

The capital guarantee in an environment of low or negative rates cannot be structured in a tunnel deposit. A possible way forward is to reduce the capital guarantee from 100% to a lower percentage.

One can also design this as a loan, where the client is charged a best case interest rate below market if the underlying exchange rate remains inside a range, but a worst case interest rate above market otherwise. We refer the reader to the exercises.

2.4.4 Corridor Deposit/Loan

The corridor deposit works in a very similar way to a tunnel deposit. The capital is guaranteed, the investor is also guaranteed a worst case coupon, which is often taken to be 0%. The investor then takes the view that a currency pair is trending sideways, i.e. she chooses a currency pair out of which one is the deposit currency. Then the investor and the issuer agree on a fixing schedule, usually consisting of N business days, and a fixing source such as ECB or FED. At maturity, the number of fixings inside a pre-defined range k will be counted. The coupon paid to the investor will then be $\frac{k}{N}$ times the maximum coupon C. So the worst case is $k = 0$, where the investor is left with the minimum coupon. The best case is $k = N$, where the investor gets the full coupon C.

Advantages

- Possible higher coupon than market
- Capital is guaranteed
- Worst case coupon is known upfront
- Unlike the tunnel deposit, price spikes beyond the range do not result in the worst case coupon, but will lead only to losing $\frac{1}{N}$ of the coupon

Disadvantages

- Possible lower coupon than market
- Investor has to pay tax on the best case coupon, but cannot tax-deduct the risk on receiving a coupon below market
- Since resurrecting style corridors are expensive, the range or the maximum coupon often do not appear to be overwhelmingly attractive to the investor

Example We consider the example shown in Table 2.41 and illustrate the interest rate payments in Figure 1.61.

Composition

- Money from the money market is 49,863.01 GBP.
- The investor's position is equivalent to buying a GBP-USD resurrecting corridor with range 1.7750–1.8250 and with notional 49,863.01 GBP. This notional must match the interest rate difference of the maximum and the minimum rate, which is

$$5{,}000{,}000.00 \cdot (6.00\% - 2.00\%) \cdot \frac{91}{365} = 49{,}863.01.$$

- The offer price of the corridor is 40.00%, assuming a volatility of 9.0% and a USD rate of 2.50%.
- To defer the premium to the maturity date we use the rate 4.00%.
- The deferred premium becomes 40.3989%, which is 20,144.11 GBP.
- The investor receives a minimum payment of 24,931.51 GBP.
- Subtracting the deferred premium and the minimum payment from the money from the money market leaves a sales margin of 4,787.40 GBP.

TABLE 2.41 Example of a corridor deposit. The minimum rate is paid in any case. The coupon paid at maturity in GBP is $(6\% - 2\%)\frac{k}{N}$, where k is the number of fixings inside the range.

Notional	5,000,000 GBP
Start date	June 3 2005
Maturity	Sept 2 2005 (91 days)
Number of days	91
Money market reference rate	4.00% act/365
Minimum rate	2.00% act/365
Maximum rate	6.00% act/360
Range	1.7750–1.8250 GBP-USD
Range style	resurrecting
First fixing	June 6 2005
Last fixing	August 31 2005
Total number of fixings N	61
Fixing source	ECB using the ratio of EUR-USD and EUR-GBP

- The investor receives a maximum payment of 74,794.52 EUR.
- The amount of interest paid per fixing inside the range is 817.43 GBP.

In general, the formula relating the notional deposit amount N, the offer deferred premium of the corridor in units of the underlying P, the sales margin M, the market interest rate r_M (bid side), the minimum interest rate r_0, and the deposit horizon of d days is given by

$$N \cdot (r_M - r_0) \cdot \frac{d}{360} = M + P, \tag{32}$$

which can be solved easily for any quantity desired. Note that for GBP the day-count convention for deposits is act/365 days rather than act/360.

Variations and Remarks This corridor deposit can be altered in various ways. Instead of a two-sided range the investor could go for any type of range bet, such as a one-sided corridor or a non-resurrecting style corridor, where the number of fixings inside the range will stop increasing if the continuously observed exchange rate or the first fixing is at or outside the range. The ranges can also be set afresh at fixed times in the future depending on the spot in the future, so the investor's position would be equivalent to going long a series of forward start corridors. Generally, bands need not be constant, but can be functions of time, mostly linear or exponential in practice.

The capital guarantee in an environment of low or negative rates cannot be structured in a corridor deposit. A possible way forward is to reduce the capital guarantee from 100% to a lower percentage.

One can also design this as a loan, where the client is charged a best case interest rate below market if all the fixings of the underlying exchange rate remain inside a range, but a worse case interest rate above market otherwise, where the interest rate to be paid increases linearly with the number of fixings outside the pre-defined range. We refer the reader to the exercises.

2.4.5 Turbo Deposit/Loan

The trend is your friend. We are looking at an (alleged) up-trend market such as EUR-CHF displayed in Figure 2.21 and would like to invest EUR in a strategy which could generate an interest rate higher than the market if the trend persists. This can be done with the turbo deposit.

At maturity, the investor then receives his capital plus

- a minimum rate (usually 0%) if the EUR-CHF spot at cutoff time is below a level K_1,
- the EUR market rate if the EUR-CHF spot at cutoff time is between K_1 and K_2,
- twice the EUR market rate if the EUR-CHF spot at cutoff time is above K_2.

Example We display an example term sheet of a turbo deposit in Table 2.42.

The way this works is that the market interest rate payment for the EUR deposit is not paid to the investor but is split up in a sales margin and the premium for two digital

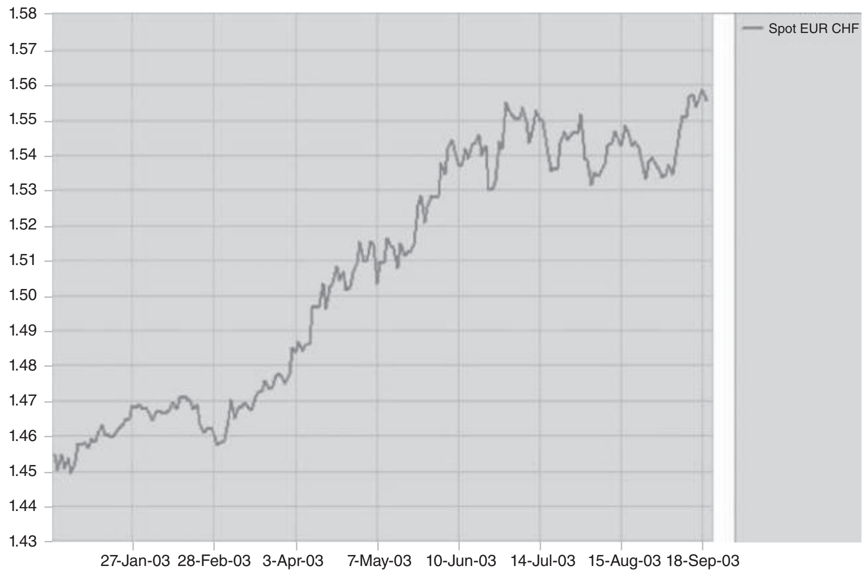


FIGURE 2.21 EUR-CHF spot during the first three quarters of 2003. Source: SuperDerivatives

TABLE 2.42 Example of a turbo deposit.

Spot reference	1.5550 EUR-CHF
6-months reference deposit rate	2.11% p.a. act/360
Notional	EUR 5,000,000
Start value date	24-Sep-2003
Expiration date	22-Mar-2004, 10:00 a.m. New York time
Maturity date	24-Mar-2004
Lower level K_1	1.5100 EUR-CHF
Upper level K_2	1.5800 EUR-CHF
Double interest rate	4.22% p.a. act/360
Single interest rate	2.11% p.a. act/360
Minimum interest rate	0.00% p.a. act/360
Interest rate payment currency	EUR

EUR-CHF call options, one with strike K_1 and one with strike K_2. The investor takes a position that is equivalent to going long these two digital calls. The notional currency of the digital calls is in the deposit currency. The notional amount is the interest payment in EUR, which is here equal to

$$5{,}000{,}000.00 \cdot 0.0211 \cdot \frac{180}{360} = 52{,}750.00.$$

The premium for the digital calls is paid at the maturity date of the deposit, since we do not have any cash flow other than the notional of the deposit at the beginning. This product will usually look attractive if the forward curve is decreasing, because then the two digital calls are comparatively cheap. Similar *turbo*-type structures are popular in conjunction with interest rate swaps, see for example Section 2.5.3. Similarly, to implement a view on a downward trend, one can flip the structure, so the investor would take a position that is equivalent to buying two digital puts.

2.4.6 Tower Deposit/Loan

The tower deposit takes the idea of the tunnel deposit and develops it further for the investor with a higher risk appetite. The investor receives a guaranteed capital and a worst case coupon below market, which is often taken to be 0% (or a negative coupon if market is already near or below 0%), and then builds up higher coupons like in a tower depending on which range the underlying exchange rate remains. Hence, the tunnel deposit reflects a view on a sidewards movement of the underlying exchange rate. The investor waives parts of her market coupon and takes a position that is equivalent to going long a portfolio of nested double-no-touch contracts, whose notional is in the deposit currency, and whose amounts are based on the differences between the interest rate payments of two successive coupons. Usually, the exchange rate contains the currency of the deposit and another currency. If the deposit currency is not in the exchange rate, then the payoff has to be quantoed into the deposit currency, which is possible but not very common. Figure 2.22 shows an example of three ranges in a tower shape. Since the ranges are nested, there are other names for this structured deposit, such as *multiple range deposit* or *onion deposit* or *tetris bond* or *wedding cake*. A quantitative analysis on onion deposits can be found in Ebenfeld *et al.* [41].

Advantages

- Possible much higher coupon than market
- Capital is guaranteed
- Worst case coupon is known upfront

Disadvantages

- Possible lower coupon than market
- Investor has to pay tax on the best case coupon, but cannot tax-deduct the risk on receiving the worst case coupon

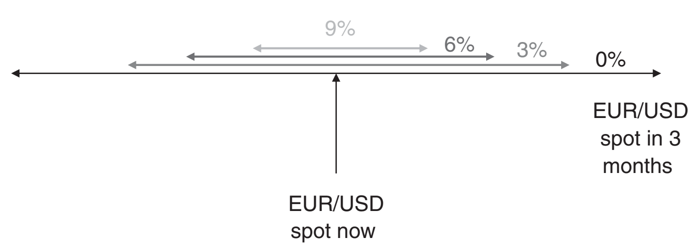


FIGURE 2.22 Design of a tower deposit with three ranges.

TABLE 2.43 Example of tower deposit. The minimum rate is paid if EUR-USD leaves the widest range, the rates for the ranges are paid if EUR-USD stays inside the respective range.

Notional	3,000,000 EUR
Tenor	3 months (91 days)
EUR-USD spot reference	0.8800
Market reference rate	3.34% act/360
Minimum rate	0.00% act/360
Rate for range 0.8500–0.9150	3.00% act/360
Rate for range 0.8600–0.9050	6.00% act/360
Rate for range 0.8650–0.9000	9.00% act/360

- Since the investor's position is equivalent to a portfolio of long first generation exotics, the sales margins tend to be higher as for a dual currency deposit or a tunnel deposit
- Looks attractive only if the time to maturity is at least three months and the interest rate of the deposit currency is at least 3.0% higher than the guarantee

Example We consider the example shown in Table 2.43.

Composition

- Money from the money market is 25,328 EUR.
- The investor takes a position that is equivalent to buying three EUR-USD double-no-touch contracts with payoff 22,750 EUR. This notional matches the interest rate differences of 3.00%, which is for example

$$3{,}000{,}000.00 \cdot (9.00\% - 6.00\%) \cdot \frac{91}{360} = 22{,}750. \tag{33}$$

- The deferred offer prices of the double-no-touch contracts are
  1. Range 0.8650–0.9000: 1,149 EUR
  2. Range 0.8600–0.9050: 2,948 EUR
  3. Range 0.8500–0.9150: 7,873 EUR
  assuming a volatility of 10.0% and a USD rate of 2.70%.
- The total price to pay for the portfolio of the three double-no-touch contracts is hence 11,970 EUR.
- Subtracting this price from the money market leaves a sales margin of 13,358 EUR.

Modifications There are many ways to modify the tower deposit.

1. The portfolio can be composed of an arbitrary number of double-no-touch contracts, but also single-touch or touch contracts with partial barriers are possible.

2. One can think of a partial (90%) capital guarantee, which would yield higher coupons.
3. One can use forward start barriers or reset barriers.
4. The same structure works for a loan.
5. The tower deposit usually does not work well for low-yield currencies – unless the capital guarantee is reduced. If the investor is willing to take risk on the capital, then one can combine the dual currency deposit with the tower deposit

Tower Deposit as a Note As an example of a yield enhancement we consider a one-year tower note in EUR, which is just a listed version of the tower deposit. It is designed for investors who believe EUR/USD will trade in a range during the next year.

The investor prefers to have a note issued by an AA bank under the *European Medium Term Note (EMTN)* program. The indicative terms are listed in Table 2.44.

2.4.7 FX-linked Bonds

The idea of a DCD can be extended to a longer-term bond, where a coupon above market is paid, and the notional is possibly converted to another currency at maturity. The notional conversion can further depend on a knock-in event.

As an example we consider a 5Y bond. Given a 5Y funding spread of an issuing bank of 3M-EURIBOR plus 52 basis points and a 5Y swap rate of −0.2650%, we consider a bond with EUR 10 M notional and 5Y maturity. The bond pays an annual coupon (act/act unadjusted following). The notional at maturity is either 100% of the initial EUR notional if the condition is satisfied, or a USD amount converted with the pre-specified EUR-USD conversion rate.

The following variants are common.

Vanilla: The annual coupon is 1.52% and the condition is that the EUR-USD spot at maturity is lower than the conversion rate 1.3000. This is structured by the investor selling a vanilla EUR call/USD put with strike 1.3000 for a price of 7.32% EUR.

TABLE 2.44 Example of a tower note. Ranges are American style.

Spot reference	1.2350 EUR-USD
1 year EUR rate reference	2.20%
Payment date	1 month after the trade date
Maturity date	1 year after the payment date
Principal amount	EUR 3,000,000
Issue price	99.00%
Redemption price	100%
Coupon 4.00% p.a.	if EUR-USD stays inside the range 1.1500–1.3200
Coupon 2.28% p.a.	if EUR-USD stays inside the range 1.1300–1.3500
Coupon 1.50% p.a.	if EUR-USD stays inside the range 1.1100–1.4000
Coupon 0.00% p.a.	if EUR-USD trades outside the range 1.1100–1.4000
Basis	act/360

European Knock-in: The annual coupon is 1.97% and the condition is that EUR-USD does not trade at or above the upper knock-in barrier of 1.5000 at maturity, otherwise the notional is converted at a rate of 1.0000 at maturity. This is structured by the investor selling a EUR call/USD put with strike 1.0000 and European up-and-in barrier at 1.5000 for a price of 9.60% EUR.

American Knock-in: The annual coupon is 2.65% and the condition is that EUR-USD does not trade at or above the upper knock-in barrier of 1.5000 during the entire lifetime, otherwise the notional is converted at a rate of 1.0000 at maturity. This is structured by the investor selling a EUR call/USD put with strike 1.0000 and American up-and-in barrier at 1.5000 for a price of 13.05% EUR

Note that the coupons are typically about 1/5 of the option price because the 5Y interest rate and term structure of interest rates is assumed close to zero in this example. The structures are meant for retail investors and include about 1% sales margin for the bank, plus possibly a bid-offer spread for the options desk. Prices are calculated on a horizon date Sept 2016, spot 9 Sept 2016, EUR-USD spot ref 1.1247, 5Y volatilities between 10% and 11.5%.

2.4.8 FX-Express Certificate

The certificates industry has also discovered FX as an asset class. Basically, the popular underlying single stocks and stock indices are replaced by a currency pair. As an example we consider a retail *two-way express certificate*, which RBS had issued in 2007 with ISIN DE000RBS0412. The terms and conditions are listed in Table 2.45, and sample spot scenarios are exhibited in Figure 2.23.

The two-way express certificate consists of a bond and nested payoff structure. It is not possible to decompose it into simple first generation exotics. Therefore, pricing and risk management are usually done via Monte Carlo simulation or PDEs.

TABLE 2.45 Two-way express certificate: terms and conditions. Early termination occurs if the fixing is outside the barriers, otherwise extended to next fixing date; coupons in case of early termination are 8.75 EUR, 12.25 EUR, 15.75 EUR, 19.25 EUR, 22.75 EUR; at maturity: if all fixings are between the barriers including the last one, then 90 EUR are returned.

Currency pair	EUR/USD
Initial price	100 EUR
Initial spot date	9 Nov 2007
Maturity	16 Feb 2011
Barriers	92% and 108% of initial spot
Fixing source	RBSFIX01 3pm London time
Fixing dates	9 Feb 2009, 17 Aug 2009, 16 Feb 2010, 16 Aug 2010, 16 Feb 2011

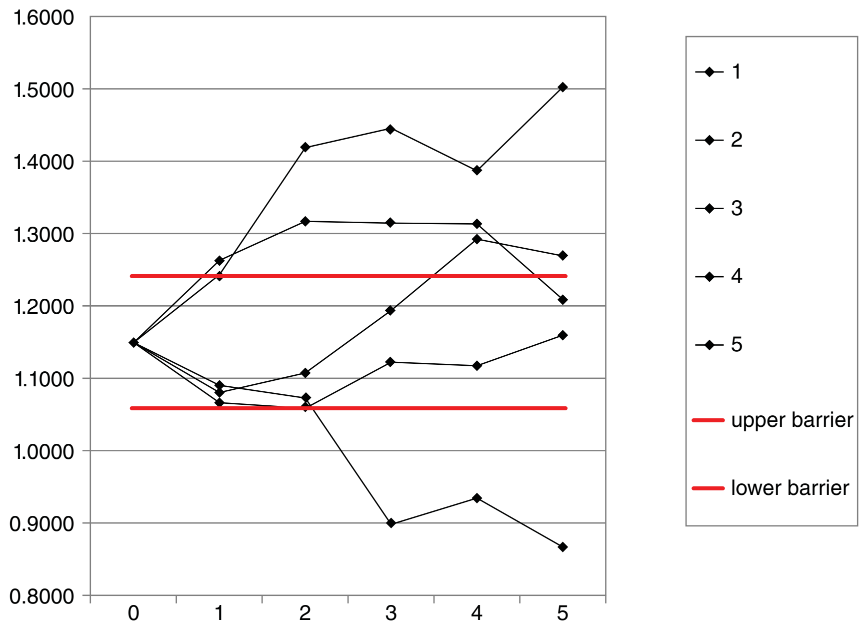


FIGURE 2.23 Two-way express certificate: five possible scenarios.

2.4.9 Exercises

Dual Currency Deposit Consider a simplified market in EUR-USD with spot 1.0500, EUR continuous interest rate of −1%, USD continuous interest rate of 2%, flat volatility of 10%. An investor wants to deposit EUR 1 M for six months at a coupon of 5% p.a. Assuming the bank wants to generate EUR 2,000 sales margin, where should the conversion rate be? You may assume $T = 0.5$ years and omit bid-offer spreads.

Unwinding a Dual Currency Deposit Consider a client who has traded a dual currency deposit, notional in USD 1 M, against CHF, with a pre-agreed conversion rate of 1.2000 USD-CHF. He has agreed on a coupon of 6.00%. Now there are 60 days left to go until maturity, the spot has gone up to 1.2200, volatility to 9.50%–9.70%, 60 days USD interest rate is 3.00%–3.10%, 60 days CHF interest rate is 1.20%–1.30%. The investor is afraid of the USD further rising and wants to unwind his DCD. Compute how much the bank can pay the client if the bank wants to charge a sales margin of 1,000.00 USD, carefully distinguishing which bid and ask rate to use. You may assume all interest rates are continuous, time to maturity as 2/12.

Performance-Linked Deposit Consider the performance-linked deposit in Section 2.4.2. Assuming a minimum interest rate of 0% and a desired sales margin of 3,000.00 GBP, what would be the percentage of the participation p? If instead of the call the client would take a position that is equivalent to buying a call spread with upper strike 0.7500 to cap the participation, what would be the participation p? What would you suggest to an investor who insists on 100% participation ($p = 100\%$)? How about a *ratio call spread*, where the notional of the sold EUR call is twice the notional of the bought EUR call, which means that the participation actually goes down after the upper strike and becomes 0% for a strike of 0.8000? Is this still capital guaranteed?

2.5 INTEREST RATE AND CROSS CURRENCY SWAPS

An interest rate swap is a very common bank product. Two counterparties agree on a currency and a notional amount in this currency and on a schedule of payment dates. On these payment dates there will be cash flows between the counterparties. The standard is that one pays a fixed coupon and the other pays a floating coupon based on the LIBOR (London Interbank Offer Rate). This standard interest rate swap is a tool to hedge uncertainties of future interest rates. The fixed rate is normally chosen in such a way that the total value of the swap is zero (or leaves some margin for the bank), and is then called the *par swap rate*. There can also be floating payments for both the counterparties, if for example A pays to B quarterly and B to A annually. Relevant for each swap contract is the exact specification of the payment schedules (interest rate fixing dates and value dates), the interest rate and day-count convention, the source of the LIBOR rate, possible fees, exchange of notional amounts. However, this is not a book about interest rate products or the valuation or modeling issues of the very same. The contribution of this book is about how to link such swaps to foreign exchange products. Such a combination becomes then an FX-linked swap if the FX part of the combination is just an add-on or a true hybrid FX-IR product if the FX and IR (interest rate) part cannot be separated. We start with an introduction to cross currency swaps. In the following we concentrate on some popular FX-linked swaps.

2.5.1 Cross Currency Swap

In a cross currency swap the two cash flow schedules and notional amounts are in different currencies. Other than that, a cross currency swap is just like an interest rate swap. It can also be viewed as a series of FX forward contracts. The principle of cash flows is illustrated in Figure 2.24.

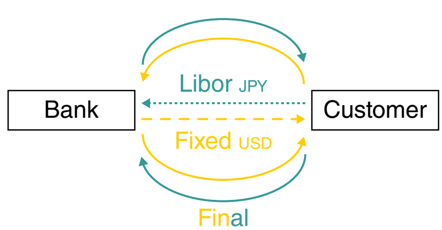


FIGURE 2.24 A cross currency swap as a dual currency trade, e.g. USD and JPY. Two parties exchange cash flows in different currencies.

An initial reference for the valuation of cross currency swaps is [17]. The default motivation to enter into a cross currency swap is the desire to pay interest in a currency with lower rates such as JPY or CHF and receive interest in a currency with higher rates. This is free! However, it works only if the notional amounts are agreed to be exchanged at the maturity date of the swap, so both counterparties have an open foreign exchange position, namely a long-term FX forward. Low interest rates are hence bought by taking this risk. This is a carry trade. Since the risk for both counterparties is high, market has moved to use *resettable* cross currency swaps, where the P&L is settled on the pay dates on the way and new FX rates are set.

Advantages

- Beneficial interest rate payments, i.e. receiving higher coupons or paying lower coupons than market
- Liquid and transparent product, thus sales margins are comparatively small
- Zero-cost product
- (Partial) protection of the notional possible

Disadvantages

- Long-term FX risk equivalent to FX forwards
- Long-term view on the FX rate impossible

Example We consider a situation where

- EUR interest rates are higher than JPY interest rates,
- a treasurer expects the EUR-JPY exchange rate to be near the current spot or even higher in five years,
- the treasurer is looking for an opportunity to receive EUR payments at zero cost.

The terms of a cross currency swap to reflect this are listed in Table 2.46.

If the treasurer's market expectation is correct, then she receives 1.90% of the EUR notional every year for free. In case of a stronger EUR in five years there are chances of additional profit, if EUR-JPY will be above 135.50.

TABLE 2.46 Example of a cross currency swap in EUR-JPY.

Currency pair	EUR-JPY
Current spot	135.50 EUR-JPY
Notional	EUR 10,000,000 and JPY 1,355,000,000
Exchange of notionals	Only at maturity
Maturity	5 years from now
You receive EUR interest rates	1.90% annually
You pay JPY interest rates	0.00% annually
Notional for protection	JPY 1,355,000,000
Strike for protection	100.00
You receive for protection	EUR put JPY call expiring in 5 years

Her risk is that if the EUR-JPY exchange rate will be below 135.50 at expiry and she is bound to buy 1,355 M JPY at the EUR-JPY spot. However, the bank guarantees a worst case exchange rate of 100.00.

If she is willing to give up this worst case protection, the bank will pay annual interest rates of 2.50% in EUR instead of 1.90%.

We will now discuss some more details about crossies and then present a set of examples of how swaps (and cross currency swaps) can be combined with FX transactions.

Basis Spreads Basis spread (margin) M is also called *financial bias*. It is an adjustment applied to interest rates to make a *basis swap* worth zero. A basis swap is a cross currency swap with both legs paying LIBOR in the respective currency. The basis spread M is usually applied to all currencies on the non-USD-leg. Figure 2.25 shows the concept of the cash flows.

Figure 2.26 illustrates how these spreads are quoted in the market.

For example, in a USD-JPY cross currency swap, the USD investor pays USD and receives JPY + M, where M is chosen so the swap trades at zero cost. This has an interesting consequence. We need to work with two different discount factor curves:

With basis spread margin: we use the curve with M to bootstrap the discount factors used for option pricing. We need to do this because a cross currency swap is basically a series of FX forward contracts. These forward contracts are the ones trading in the market. In order to avoid arbitrage, the put-call parity must hold. This can be guaranteed only if the forward contracts are priced correctly.

Without basis spread margin: we use the curve without M for pricing interest rate contracts within the one currency.

For this reason, when building a discount factor curve for EUR and USD, one usually takes the EUR interest rate instruments and builds the EUR curve and then uses the forward points from the liquid forward market in EUR-USD to construct an

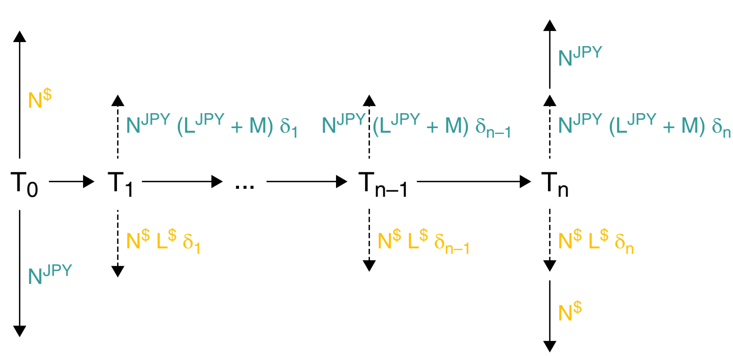


FIGURE 2.25 Basis swaps and basis spread margin concept. The margin M is constant over all interest rate pay dates. L refers to LIBOR.

```
11:29 09NOV09    ICAP                              UK69580          ICAB1
Basis Swaps - All currencies vs. 3m USD LIBOR - Also see <ICAB2>
        REC/PAY          REC/PAY         REC/PAY           REC/PAY         REC/PAY
          EUR              GBP             JPY               CHF             SEK
1 Yr -19.25/-29.25  -08.00/-18.00  -21.50/-31.50     -16.75/-26.75 -20.50/-30.50
2 Yr -18.00/-23.00  -12.00/-17.00  -24.00/-34.00     -18.75/-24.75 -14.00/-20.00
3 Yr -16.00/-21.00  -13.75/-18.75  -26.00/-36.00     -19.25/-25.25 -06.00/-12.00
4 Yr -14.00/-19.00  -14.75/-19.75  -27.25/-37.25     -20.75/-26.75  00.00/-06.00
5 Yr -12.75/-17.75  -15.75/-20.75  -28.00/-38.00     -21.25/-27.25 +03.50/-02.50
7 Yr -11.00/-16.00  -17.75/-22.75  -27.75/-37.75     -20.50/-26.50 +07.00/+01.00
10Yr -9.000/-14.00  -19.75/-24.75  -27.25/-37.25     -19.00/-25.00 +09.00/+03.00
15Yr -0.750/-5.750  -19.25/-24.25  -25.50/-35.50     -08.50/-14.50 +18.50/+12.50
20Yr +4.750/-0.250  -14.50/-19.50  -23.75/-33.75     -03.00/-09.00 +21.50/+15.50
30Yr +8.750/+3.750  -08.00/-13.00  -19.50/-29.50     +01.00/-05.00 +23.50/+17.50
40Yr +9.250/+4.250  -06.50/-11.50                         DKK            NOK
50Yr +9.500/+4.500  -06.25/-11.25               1Yr -48.50/-58.50 -18.00/-28.00
                                                2Yr -44.50/-54.50 -14.00/-24.00
**FOR OIS BASIS <ICAB5>**                       3Yr -39.50/-49.50 -10.00/-20.00
                                                4Yr -35.50/-45.50 -08.00/-18.00
                                                5Yr -31.50/-41.50 -07.00/-17.00
** Call Brendan McVeigh,                        7Yr -26.50/-36.50 -07.00/-17.00
Marcus Kemp or Simon Payne                     10Yr -22.50/-32.50 -07.00/-17.00
on +44(0)207 532 3660 **                       15Yr -13.50/-23.50 -06.50/-16.50
                                               20Yr -07.00/-17.00 -05.00/-15.00
                                               30Yr -03.00/-13.00 -03.00/-13.00
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FIGURE 2.26 Basis swaps and spreads of November 9, 2009. Source: Reuters page ICAB1

artificial USD discount factor curve and uses these curves for option pricing. This way the basis spread is already built in the curves. Building individual curves based on rates products would generate a non-tradable forward curve and hence violate the put-call parity.

Reasons for the existence of M could be liquidity constraints, investors' preferences in certain currencies, probability of the country's currency collapsing.

Hedging the basis spread is difficult as market participants tend to have all the same position.

The History of Basis Swaps This section is based on the results of the thesis by Claudia Zunft [147]. While basis spreads have been around for a long time, it took the 2008 financial crisis to bring them to everybody's attention. The spreads nearly exploded, as suddenly everybody thought it would be better in the long run to have USD. This is illustrated in Figure 2.27 and Figure 2.28.

Classic Interest Rate Parity Interest rate parity is the result of an arbitrage argument used to derive forward foreign exchange rates. Define

S_0: spot exchange rate of dollars per unit of foreign currency

r_d: T-year default-free dollar spot (i.e. zero-coupon) rate of interest

r_f: T-year default-free foreign spot (i.e. zero-coupon) rate of interest

f: forward exchange rate of dollars per unit foreign currency for delivery in T years

Now consider Table 2.47 of today transactions. It is assumed throughout that there is no counterparty default risk in forward or swap contracts.

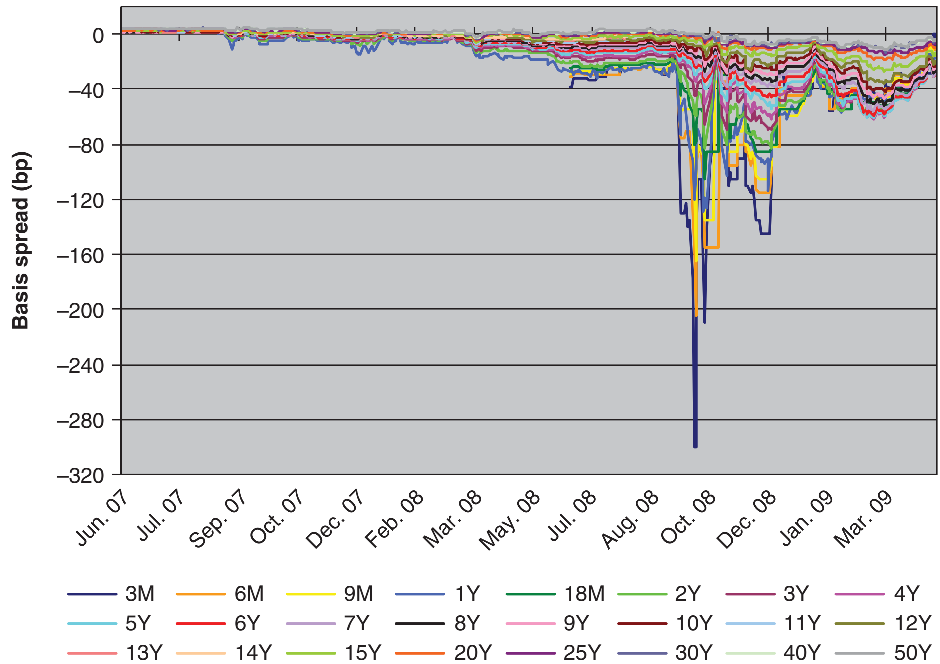

FIGURE 2.27 Basis spread history in EUR-USD. Source: Bloomberg

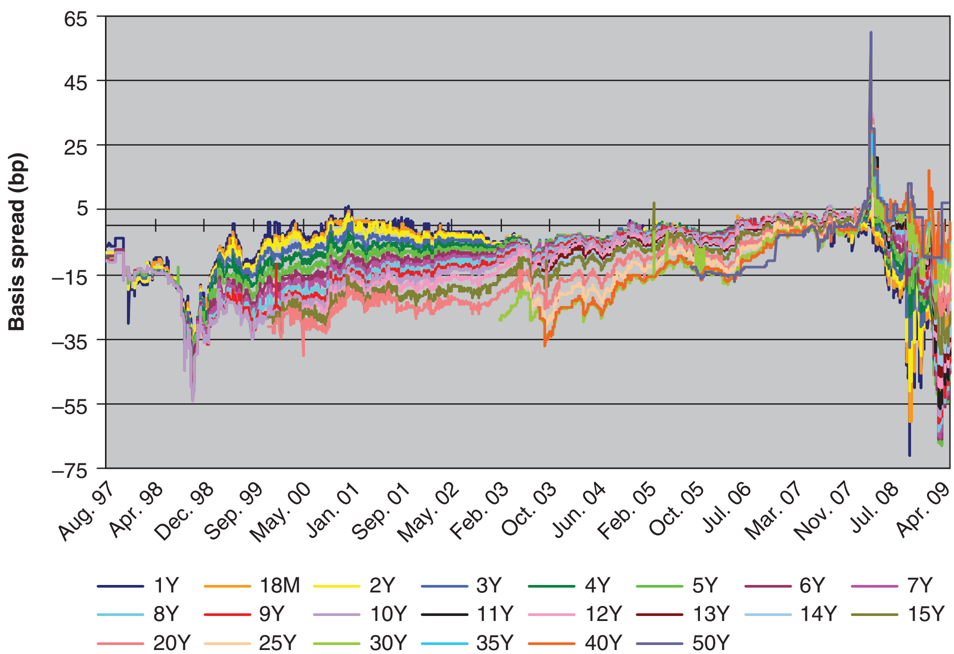

FIGURE 2.28 Basis spread history in USD-JPY. Source: Bloomberg

TABLE 2.47 The arbitrage argument for classic interest rate parity.

Transaction	Today DOM	Time T DOM	Today FOR	Time T FOR
Borrow S_0 DOM	$+S_0$	$-S_0(1+r_d)^T$		
Sell S_0 DOM, buy FOR	$-S_0$		$+1$	
Invest FOR			-1	$(1+r_f)^T$
Sell FOR forward		$+f(1+r_f)^T$		$-(1+r_f)^T$
Total	0		0	0

Because these transactions neither generate nor require cash today, ruling out arbitrage requires that they neither generate nor require cash at time T either. Mathematically,

$$-S_0(1+r_d)^T + f(1+r_f)^T = 0. \tag{34}$$

Solving for the forward price, we obtain

$$f = S_0 \frac{(1+r_d)^T}{(1+r_f)^T}. \tag{35}$$

In practice: market participants observe spot rates implied from swap rates rather than from default-free spot rates. Traders quote basis swaps to trade the difference.

Interest Rate Parity with Basis Spread Margin Included Following Tuckman and Porfirio [130], a cross-currency basis swap is essentially an exchange of a floating rate note in one currency for a floating rate note in another currency.

For example, on March 4 2003 CAD traded for 0.677 USD and a trader might have agreed to the following cross-currency basis swap:

- Pay 1 CAD and receive 0.677 USD at initiation
- Receive three-month CDOR3 plus 10 basis points on 1 CAD and pay three-month USD LIBOR quarterly on 0.677 USD for three years
- Receive 1 CAD and pay 0.677 USD at expiration

We now derive the interest rate parity with basis spread margin included, using the following notation.

R_d: T-year dollar par swap rate with fixed flows paid quarterly

R_f: T-year foreign par swap rate with fixed flows paid quarterly

Y_d: T-year dollar zero-coupon swap rate

Y_f: T-year foreign zero-coupon swap rate

L_t^d: three-month dollar LIBOR at time t

TABLE 2.48 The arbitrage argument for interest rate parity with cross currency basis swap.

Transaction	Today DOM	Interim DOM	Time T DOM	Today FOR	Int. FOR	Time T FOR
Spot FX	$-S_0$			$+1$		
Ccy swap	$+S_0$	$-S_0(L_t^d + X)$	$-S_0$	-1	L_t^f	$+1$
Par swap		S_0X				
Rec fix		$-[S_0X/R_d]L_t^d$				
Pay fl on $[S_0X/R_d]$						
Zero coupon swap		$S_0\left(1+\frac{X}{R_d}\right)L_t^d$	$-S_0\left(1+\frac{X}{R_d}\right)$			
Rec fl on $S_0\left(1+\frac{X}{R_d}\right)$			$\times[(1+Y_d)^T-1]$			
Zero coupon swap					$-L_t^f$	$(1+Y_f)^T-1$
Pay floating						
Sell FOR forward			$+f(1+Y_f)^T$			$-(1+Y_f)^T$
Total	0	0		0	0	0

L_t^f: three-month foreign LIBOR at time t

X: a swap of three-month dollar LIBOR plus X is fair against three-month foreign LIBOR

The derivation of the first parity relationship, using par swaps, zero-coupon swaps, and a three-month cross-currency basis swap, is explained in Table 2.48. Interim cash flows are annualized.

Ruling out arbitrage opportunities implies that the time-T dollar payoff is zero, i.e.

$$-S_0\left[(1+X/R_d)(1+Y_d)^T - X/R_d\right] + f(1+Y_f)^T = 0. \tag{36}$$

Solving for the forward rate,

$$f = S_0\frac{(1+Y_d)^T}{(1+Y_f)^T}\left[1 + X\frac{(1+Y_d)^T-1}{R_d(1+Y_d)^T}\right]. \tag{37}$$

The obvious advantage of Equation (37) over the classic parity condition (35) is that (35) depends on swap rates and on the cross-currency market basis swap spread. By contrast, the classic condition depends on the generally unobservable default-free rates of interest.

Vanilla Options with Basis Spreads Using continuous rates, we can generalize the values of vanilla options to

$$\text{vanilla}(t, S_0, \sigma, r_d, r_f, X; K, T, \phi) = \phi e^{-r_d\tau}\left[f\mathcal{N}(\phi d_+) - K\mathcal{N}(\phi d_-)\right] \tag{38}$$

$$\tau \stackrel{\Delta}{=} T - t \tag{39}$$

$$f = S_0 e^{(r_d - r_f)\tau}\left[1 + X\frac{e^{r_d\tau} - 1}{R_d e^{r_d\tau}}\right] \tag{40}$$

$$d_\pm \triangleq \frac{\ln\frac{f}{K} \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}. \tag{41}$$

The relationship between the zero coupon swap rates Y and r is as usual given by

$$(1 + Y)^T = e^{rT}. \tag{42}$$

The par swap rate R_d can be backed out of

$$1 = \sum_{i=1}^{n} R_d \mathrm{DF}(t_i)\Delta_i + \mathrm{DF}(T) \tag{43}$$

$$\Longleftrightarrow R_d = \frac{1 - \mathrm{DF}(T)}{\sum_{i=1}^{n} \mathrm{DF}(t_i)\Delta_i} \tag{44}$$

$$\Longleftrightarrow R_d = \frac{1 - e^{-r_d(T)T}}{\sum_{i=1}^{n} e^{-r_d(t_i)t_i} 0.25} \tag{45}$$

Outlook: Cross Currency Swap with Bermudan Cancellation Right Crossies can have more general features. For example, we consider the terms and conditions of a 30-year cross currency swap with Bermudan cancellation right. Notionals are JPY 17,500,000,000 and EUR 127,000,000, start date March 1 2006. Initially counterparty A pays JPY notional; counterparty A receives EUR notional. Counterparty A pays six-month EURIBOR + 35 bps act/360 on the EUR notional. Counterparty A receives: 2.76% on JPY notional payment dates, which are semi-annually on March 1 and September 1 starting on September 1 2006. For the final exchange counterparty A receives JPY notional; counterparty A pays EUR notional. The end date is March 1 2036. Callable Bermudan option: counterparty B has the right to call the swap in whole on March 1 2018. When the swap is called the amounts that are exchanged are counterparty A receives JPY notional; counterparty A pays EUR notional, i.e. it closes at zero premium.

The callability is meant to allow counterparty B to back out of the deal if EUR interest rates are lower and expected to be low from 2018. However, termination of the crossie may also imply counterparty B is to receive EUR worth less than the JPY amount at inception. The right to cancel depends on the view of the dependence of EUR interest rates and the EUR-JPY exchange rate.

2.5.2 Hanseatic Swap

We will deal now with a more realistic example about how to protect the final exchange rate risk. The *Hanseatic swap* is a cross currency swap with partial protection of the final exchange notional, a trading strategy to lower interest rate payments on the one hand and to protect the FX risk arising from the exchange of notionals at maturity on the other hand. As we have seen above, the standard cross currency swap carries a lot of exchange rate risk at the end. This product helps to reduce this risk at zero cost and still allows the lowering of interest rate payments. The extreme upside participation is given up and used to finance a partial downside protection.

Advantages

- Beneficial interest rate payments, i.e. receiving higher coupons or paying lower coupons than market
- Zero-cost product
- (Partial) protection of the notional included

Disadvantages

- Final exchange rate risk not *fully* covered
- Long-term view on the FX rate impossible, i.e. suitable for rather short-term contracts

Example We take EUR-CHF and show in Figure 2.29 and Table 2.49 how a corporate client can lower his interest rate payments in EUR obligations by 1.05% for six months, as long as EUR-CHF stays in a range around the current spot.

- If the exchange rate EUR-CHF rises but stays below the upper knock-in level of 1.6050 during the next six months, the client can participate in the exchange rate and obtain an interest rate advantage of 1.05% p.a.
- If the exchange rate EUR-CHF ever touches or crosses 1.6050 during the next six months, the client is obliged to buy CHF at 1.5670 at maturity and can no longer participate in a stronger EUR.
- If the exchange rate EUR-CHF remains above the lower knock-out level of 1.5100 during the next six months, the client is entitled to buy CHF at 1.5670. This is his protection against currency losses.
- If the exchange rate EUR-CHF ever trades at or below the lower knock-out level of 1.5100 during the next six months, the client's protection is lost and he needs to buy CHF at the spot price at maturity.

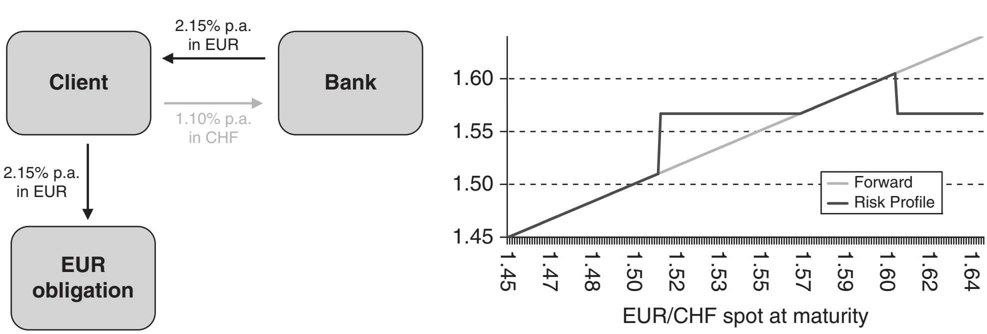

FIGURE 2.29 Cash flows and currency risk protection for a Hanseatic cross currency swap.

TABLE 2.49 Example of a Hanseatic cross currency swap in EUR-CHF. Both levels are of American style, i.e. observed continuously over time.

Period	6 months
Notional	EUR 5,000,000 CHF 7,835,000
The client receives	2.15% p.a. in EUR 30/360
Fixing and payment	quarterly
The client pays	1.10% p.a. in CHF 30/360
Fixing and payment	quarterly
Spot reference	1.5624 EUR-CHF
Currency protection	
Strike	1.5670 EUR-CHF
Upper knock-in-level	1.6050 EUR-CHF
Lower knock-out-level	1.5100 EUR-CHF
Premium	0.00 EUR

- In summary, if the exchange rate EUR-CHF stays inside the range 1.5100–1.6050, the client's interest rate is reduced by 1.05% for six months. Additionally his notional exchange is protected and he may even participate in a rising EUR.

Composition To generate a zero-cost overlay on top of an existing cross currency swap, the client sells participation and buys protection. In detail:

1. The basis for this structure is a six-months cross currency swap in EUR-CHF with strike 1.5670 worth 32,000 EUR.
2. The client sells a six-months EUR call CHF put with strike 1.5670, up-and-in at 1.6050 and receives a premium of 26,375 EUR (=0.5275% EUR).
3. The client buys a six-months EUR put CHF call with strike 1.5670, down-and-out at 1.5100 and pays a premium of 30,875 EUR (=0.6175% EUR).
4. Overall the structure is offered at zero cost to the client, thus the sales margin is 27,500 EUR.

Critical Judgment This is obviously a yield-enhancer for the attentive user and meant to be a short-term transaction of up to one year – because of the continuously observed barrier options. Spot hitting the lower barrier is the biggest risk for the client. So if the EUR-CHF spot moves down substantially, one may think of an early close-out – materializing the loss – or buying the protective EUR put CHF call on the way. However, I do want to point out that this protection overlay with the two barrier options should not be viewed as the key risk driver. The key risk is the cross currency swap in the first place. A trade one should not do if one is not convinced about the exchange rate staying where it is. If you start with a carry trade, you cannot blame the barrier options if it blows up. The barrier options merely provide a free add-on to protect the small risk (of EUR-CHF declining), but the big risk (of EUR-CHF declining substantially) remains. Both counterparts must make sure they can live with all market scenarios and have a risk management policy in place for early termination based on their risk profile.

2.5.3 Turbo Cross Currency Swap

The turbo cross currency swap is a cross currency swap *without* any protection of the final exchange notional. It doubles the idea of participation on the upside as well as the risk on the downside. Hence it is a hedging instrument for those who take a serious view on a stable or rising exchange rate. So as in the *turbo deposit* described in Section 2.4.5, the client chooses three ranges, a lower range with a worst case interest rate payment, a middle range – usually around the current spot – with an interest rate payment attractive enough to enter into the swap, and an upper range with a very low, ideally zero, interest rate. These ranges are valid at each interest cash flow date. This product usually works well in a forward backwardation market such as EUR-CHF or EUR-JPY and can be marketed easily in situations where exchange rates are near their historic low, because then it appears likely to turn into a rising exchange rate in the long run. That is if one believes in mean reversion of exchange rates. The product generates attractive coupons for the client and worthwhile margins for the bank for long tenors.

Advantages

- Beneficial interest rate payments, i.e. receiving higher coupons or paying lower coupons than market
- Zero-cost product
- Turbo effect can bring the coupon down to zero or close to zero

Disadvantages

- Very risky as final exchange rate risk not covered
- Long-term view on the FX rate impossible, i.e. once the exchange rate moves far into the worst case range, it is likely to stay there

Example We take EUR-CHF and show in Figure 2.30 and Table 2.50 how a corporate client can lower his interest rate payments in EUR obligations to 0.5% for four years, as long as EUR-CHF moves and stays in a range above the current spot.

Composition

1. The basic product is a 4-year EUR-CHF cross currency swap with final exchange of the notionals. This is at par with the CHF semi-annual rate at 1.72%. To structure this deal, we start with a swap where the client pays the worst case 4.00% in CHF to the bank, thus the swap should be worth more than zero. In fact, let's assume the fair market value of the swap is 240,000 EUR. From this amount the bank provides the turbo participation and takes a sales margin, so the turbo cross currency swap can be offered at zero cost to the client.
2. The client goes long a strip of eight European digital EUR-CHF calls with strike 1.4300, two for each of the four years. The notional of the digital calls must be in CHF and the amount must be the cash flow to get from the worst case 4.00% down to the middle case 2.00%, so it must be 3,670,000 CHF · (4.00–2.00)%·180/360 = 36,700 CHF.

TABLE 2.50 Example of a turbo cross currency swap in EUR-CHF.

Maturity	4 years
Notional	2,500,000 EUR / 3,670,000 CHF
Fixed exchange rate	1.4680
Spot reference	1.4677
4-year par interest rate	1.72% in CHF
Amortization	none
The bank pays in EUR	fix 3.20%
The bank receives in CHF	fix 4.00% if EUR-CHF < 1.4300 fix 2.00% if EUR-CHF is between 1.4300 and 1.5000 fix 0.50% if EUR-CHF ≥ 1.5000
Cutoff of the FX options	10:00 a.m. N.Y. time two business days before the end of each period
Coupon payments	semi-annually
Basis	30/360, modified following, unadjusted
Premium	none

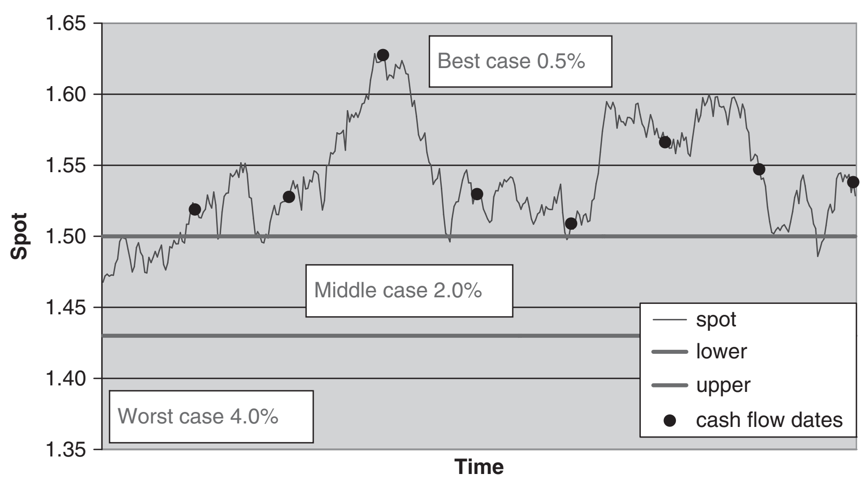

FIGURE 2.30 Three ranges for a turbo cross currency swap in EUR-CHF. In this sample path the structure works out well for the client as all the future spot fixings are in the upper range, thus he would pay the best case interest rate of only 0.50% in all of the eight periods.

3. Furthermore, the client goes long a strip of another eight European digital EUR-CHF calls with strike 1.5000. Their notional is computed similarly by 3,670,000 CHF · (2.00–0.50)%·180/360 = 27,525 CHF.
4. The total offer price for these strips is 170,000 EUR, thus there are 70,000 EUR left for the bank as a sales margin.
5. For the hedge, the trader needs to buy 1,800,000 EUR from the spot desk, and trade the cross currency swap with worst case with the swap desk.

Variations Obviously, there can be many ways to vary this type of a swap, for example:

- The swap itself can have floating rates on the payer as well as on the receiver side, and the worst and best cases can be built as absolute differences from these floating rates.
- The swap itself can be amortizing. Consequently the notionals of the digital EUR-CHF calls are no more constant but rather decreasing with maturity. Very small notional amounts of each ticket can make the overall structure unattractive, as the fixed costs for each ticket lead to a high offer price of the strip.
- There is no restriction to have three ranges: there can be two, in which case we are dealing with a *flip swap* as in Section 2.5.5, or more than three. The only issue is that if we increase the number of ranges, then the notional amounts for the digital calls become very small and the costs for each ticket can lead to the strips being very expensive.
- Instead of simple digital options, one can use more exotic options, such as digital options with additional (window) knock-out barriers, which would make the ranges or the interest rates even more attractive. One can also imagine forward start digital options, where ranges are set afresh at each cash flow date. Of course, the strikes need not be constant over time. If the client wishes to incorporate a (linear) trend, this can easily be done. However, the structure with *constant* strikes usually works out attractive *because* the forward curve tends downwards, so placing the ranges along the forward rate will make the structure unattractive, as the digital options would then become more expensive.

Critical Judgment As the name indicates, the turbo crossie amplifies the carry trade idea in the crossie, so it is meant for strong carry trade believers. As with any product with unlimited risk, the parties involved need to have a risk management policy in place, in particular the corporate client should not sign up for this trade as a buy-and-hold strategy. The conditions of the trade tend to appear attractive as the digital call strips are a bet against the forward, which is why they are comparatively cheap. The way to sell it is to interpret the worst case as the interest rate the client would pay in his accounting currency, the middle case as the crossie par rate, both of which don't really hurt, and view the best case as zero or close to that and what the client actually wants (or can afford) – and present the sales slide from Figure 2.30.

2.5.4 Buffered Cross Currency Swap

While the turbo cross currency swap doubles the benefit from a rising exchange rate, the buffered cross currency swap buffers the effect of a falling exchange rate. If the exchange rate depreciates, then the client gets caught at the end. If this happens, he wants to pay less interest – at least. The overall reason for a client to enter a cross currency swap is saving on the interest payments. This is achieved by taking a final exchange rate risk. Thus, the client takes a view against the forward. If this view turns out to be correct, then the turbo cross currency swap adds a higher leverage. If the view turns out to be incorrect, then the buffered cross currency swap compensates for the pain of a loss. A carry trade for wimps, one might say.

This effect is achieved by the client buying two strips of digital puts instead of digital calls. The rest of the structure is identical to the turbo cross currency swap.

Of course, for a downward-sloping forward curve, structuring an attractive turbo is much easier than structuring a buffered cross currency swap, because the swap with the worst case costs the same in both types, whereas the digital put strips are significantly more expensive than the digital call strips. For this reason they are not traded very often. You either want a carry trade or you don't. An upward-sloping forward curve would make it unreasonable to enter into a cross currency swap in the first place because the interest rates would then be higher and the client hence not motivated.

2.5.5 Flip Swap

A flip swap can be a single currency interest rate swap or a cross currency swap where the client receives a fixed or floating interest rate and pays either a best case below par or a worst case above par. The worst and best case depend on a foreign exchange rate to trade below or above a pre-specified strike and will be set at the end of each period. As in the turbo or buffered cross currency swap the structure works best if the client takes a view against the outright forward curve, i.e. in case of forward backwardation, he would pay the best case if the exchange rate is *above* the strike and worst case otherwise.

Advantages

- Beneficial interest rate payments, i.e. receiving higher coupons or paying lower coupons than market
- Zero-cost product
- Worst case known in advance

Disadvantages

- Drastic interest payment improvement difficult if the client wants to have a strike such that at inception of the trade she is deep in the best case (a turbo crossie could solve this drawback)
- Long-term view on the FX rate impossible, i.e. once the exchange rate moves far into the worst case range, it is likely to stay there

Example We take EUR-CHF and show in Figure 2.31 and Table 2.51 how a corporate client can lower his interest rate payments in CHF obligations by 0.5% for nine years, as long as EUR-CHR stays above a strike significantly below the current spot. This flip swap is usually added to an existing interest rate payment plan, that can hereby be lowered if the client is willing to take a certain long-term FX view. It is attractive as we start in a best case region, the worst case region appears far away (in our example near

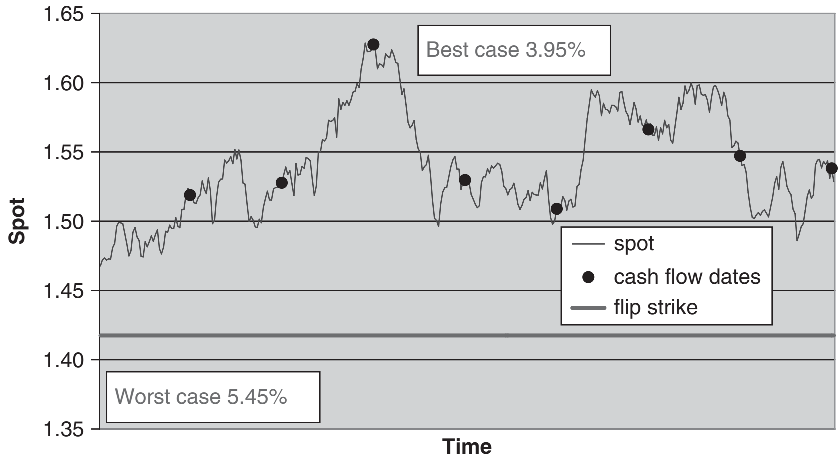

FIGURE 2.31 Two ranges for a flip swap in EUR-CHF. In this sample path the structure works out well for the client as all the future spot fixings are in the upper range, so he would pay the best case interest rate of only 3.95% in all periods.

TABLE 2.51 Example of a flip swap in EUR-CHF.

Maturity	9 years
Notional	4,420,200.00 CHF
Strike	1.4175
Spot reference	1.4671
Amortization	none
The bank pays in CHF	fix 4.45%
The bank receives in CHF	fix 5.45% if EUR-CHF < 1.4175
	fix 3.95% if EUR-CHF ≥ 1.4175
Fixing of the FX options	10:00 a.m. N.Y. time
	two business days before the end of each period
Coupon payments	annually
Basis	30/360, modified following, unadjusted
Premium	none

the historic low of EUR-CHF), and even if the exchange rate falls below the strike, the interest rate burden is just 1.0% higher than what the client receives.

Composition

1. The basic product is a nine-year CHF swap with two fixed legs, where – assuming a worst case – the bank receives more than it pays, so the swap should be worth more than zero. In fact, the swap is worth, say, 335,000 CHF, which is about 228,000 EUR. From this amount the bank provides the possibility to reduce the worst case

to the best case (and takes a sales margin), so the flip swap can be offered at zero cost to the client.

2. The client goes long a strip of nine European digital EUR-CHF calls with strike 1.4175, one for each of the nine years. The notional of the digital calls must be in CHF and the amount must be the cash flow to get from the worst case 5.45% down to the best case 3.95%, so it must be 4,420,200 CHF · (5.45–3.95)% · 360/360 = 66,303 CHF.
3. The total offer price for this strip is 196,000 EUR, thus there are 32,000 EUR left for the bank as a sales margin.
4. For the hedge the trader needs to buy 1,400,000 EUR from the spot desk, and trade the CHF swap with worst case with the swap desk.

Variations Obviously, there can be many ways to vary this type of a swap, for example:

- The swap itself can have floating rates on the payer as well as on the receiver side, and the worst and best cases can be built as absolute differences from these floating rates.
- The swap itself can be amortizing. Consequently the notionals of the digital EUR-CHF calls are no more constant but rather decreasing with maturity. Very small notional amounts of each ticket can make the overall structure unattractive, as the fixed costs for each ticket lead to a high offer price of the strip.
- There is no restriction to have two cases: there can be three, in which case we are dealing with a *turbo* or a *buffered* (cross currency) swap as in Section 2.5.3 and Section 2.5.4, or more than three. The only issue is that if we increase the number of cases, then the notional amounts for the digital calls become very small and the costs for each ticket can lead to the strips being very expensive.
- Instead of simple digital options, one can use more exotic options, such as digital options with additional (window) knock-out barriers, which would make the strike or the interest rates even more attractive. One can also imagine forward start digital options, where the strike is set afresh at each cash flow date. Of course, the strike need not be constant over time. If the client wishes to incorporate a (linear) trend, this can easily be done.

Critical Judgment The flip swap is a soft carry trade, and therefore not as risky. A view against the forward is taken, but the client can plan with the worst case and be happy if she gets the best case instead. A worst case interest rate at one time has no effect on subsequent times. The risk policy in place must ensure that the client can survive the worst case at all payment dates.

2.5.6 Corridor Swap

The corridor swap or *bonus* swap (compare with the *bonus forward* in Section 2.1.9) is very similar to the turbo or buffered (cross currency) swap. In all three products, there are three ranges. In the corridor swap the client pays a best case interest rate below par if an exchange rate is inside a pre-specified corridor and a worst case above par otherwise.

Advantages

- Beneficial interest rate payments, i.e. receiving higher coupons or paying lower coupons than market
- Zero-cost product
- Start in the best case

Disadvantages

- Very risky as final exchange rate risk not covered
- Long-term view on the FX rate impossible, i.e. once the exchange rate moves far up or down, then it is likely to stay there and worst case applies
- Ranges are often unattractive as position is taken only halfway against the forward curve

Example We take EUR-CHF and show in Figure 2.32 and Table 2.52 how a corporate client can lower his interest rate payments in EUR obligations to 0.8% for three years, as long as EUR-CHR stays inside a range around the current spot. The attractive part of this deal is that the worst case is not as far away from the par rate as the best case.

Composition

1. The basic product is a three-year EUR-CHF cross currency swap with final exchange of the notionals. This is at par with the CHF semi-annual rate at 1.72%. To structure this deal, we start with a swap that pays the worst case 2.40% in CHF to the bank, so the swap should be worth more than zero. In fact, the swap is worth, say, 72,000 EUR. From this amount the bank provides the corridor participation and

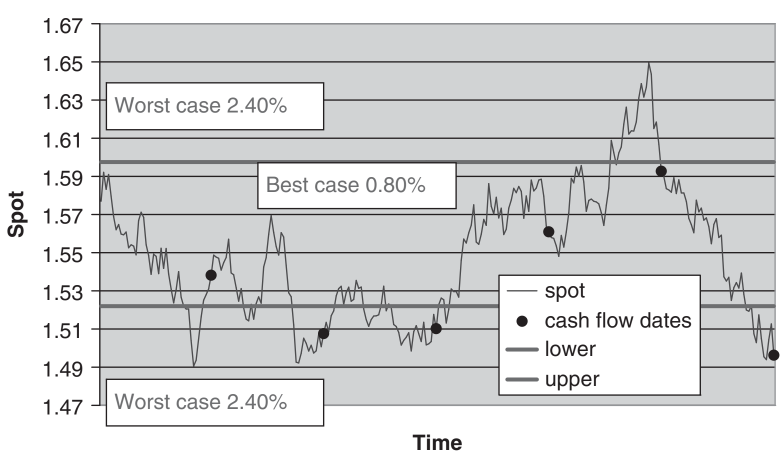

FIGURE 2.32 Corridor ranges for a corridor cross currency swap in EUR-CHF. In this sample path the structure works out well for the client for three future spot fixings where she would pay the best case and not so well for the other three spot fixings where she would pay the worst case.

TABLE 2.52 Example of a corridor cross currency swap in EUR-CHF.

Maturity	3 years
Notional	3,942,500 CHF / 2,500,000 EUR
Strike	1.5770
Spot reference	1.5770
3 year par interest rate	1.72% in CHF
Amortization	none
The bank pays in EUR	fix 2.80%
The bank receives in CHF	fix 2.40% if EUR-CHF $\notin$ [1.5220, 1.5975]
	fix 0.80% if EUR-CHF $\in$ [1.5220, 1.5975]
Fixing of the FX options	10:00 a.m. N.Y. time
	two business days before the end of each period
Coupon payments	semi-annually
Basis	30/360, modified following, unadjusted
Premium	none

takes a sales margin, so the corridor currency swap can be offered at zero cost to the client.

2. The client goes long a strip of six European digital EUR-CHF calls with strike 1.5220, two for each of the three years. The notional of the digital calls must be in CHF and the amount must be the cash flow to get from the worst case 2.40% down to the best case 0.80%, so it must be 3,942,500 CHF · (2.40–0.80)% · 180/360 = 31,5400 CHF.
3. Furthermore, the client goes short a strip of six European digital EUR-CHF calls with strike 1.5975, two for each of the three years. Their notional is 31,5400 CHF as for the long digital calls.
4. The total offer price for this strip of digital calls is 52,000 EUR, so there are 30,000 EUR left for the bank as a sales margin.
5. In this case there is only a small or no delta hedge for the FX options required as the long and the short positions cancel the necessary spot hedges.

Variations Obviously, there can be many ways to vary this type of a swap, see for example the variations of the *turbo cross currency swap* of Section 2.5.3. One variation is to reset the ranges depending on future spot levels. This overcomes the difficulty of stating any reasonable long-term view on FX levels. We discuss an example in Section 2.5.9.

Critical Judgment The cross currency swap is a carry trade, and the ranges have no impact on the final exchange of nominals. As usual any product with unlimited potential loss requires a risk policy including an exit strategy in place before we transact it.

2.5.7 Currency Related Swap (CRS)

The currency related swap or *CRS* (compare with the *performance-linked deposits* in Section 2.4.2) is generally an interest rate (cross currency) swap, where the floating coupons are linked to the performance of a currency exchange rate. The term does not specifically refer to one payoff-structure, but is rather used as a generic term for swaps with FX-related payoffs, much like the PRDCs. We consider a very common example of transactions in the first decade, where the investor lowers his interest rate obligations in EUR as long as the EUR-CHF fixings observed on the payment days stay above a certain pre-defined threshold K. If EUR-CHF fixes below K, then the interest rate the investor needs to pay is a monotone function of the difference of the spot fixing and K. This is obviously a carry trade where the investor assumes a scenario where EUR-CHF does not decrease over the lifetime of the swap, obviously reflecting a view against the EUR-CHF forward (assuming CHF interest rates being lower than EUR interest rates).

Advantages

- Beneficial interest rate payments, i.e. coupons are lower than market
- Zero-cost product

Disadvantages

- Very risky as coupon size is not limited
- Long-term view on the FX rate impossible, i.e. once the exchange rate moves far below threshold, then it is likely to stay there

Example We take EUR-CHF and show in Figure 2.33 and Table 2.53 how a treasurer can lower his interest rate payments in EUR obligations by 2.09% for 10 years, as long as

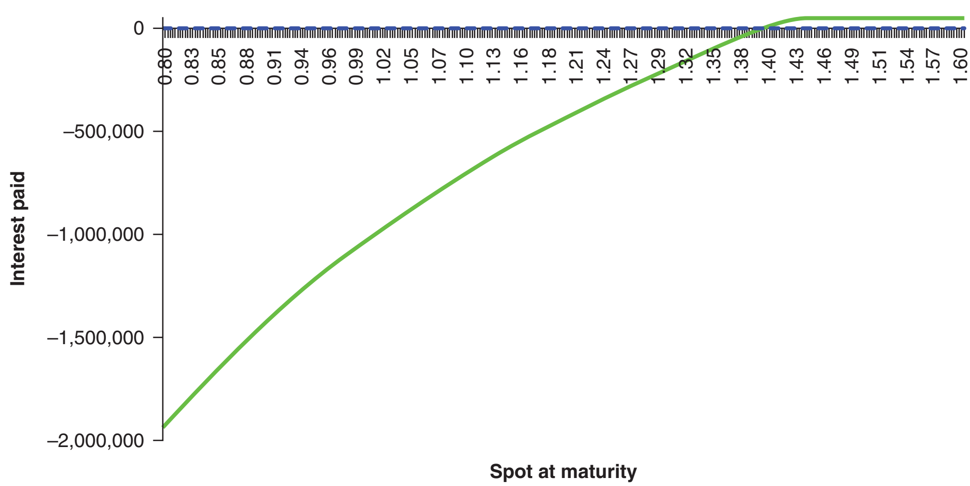

FIGURE 2.33 Currency related swap in EUR-CHF. We show the payoff (in EUR) of each of the 21 embedded EUR put CHF call options. Note that the benefit of saving EUR 25,250 if spot is above the strike is achieved by taking an unlimited risk if spot goes lower. Measured in EUR, the loss is a non-linear function of spot and the maximum potential loss is unbounded. Even for a spot of 1.1000 the interest rate amount the investor needs to pay to the bank is EUR 709,114, which is almost 30 times higher than the maximum possible profit.

TABLE 2.53 Example of a currency related swap in EUR-CHF. This example traded between a family office and a leisure-oriented bank on 5 Sept 2008, starting on 9 Sept 2008, maturing on 31 Dec 2018. Maturity days were 31 Dec and 30 June, starting on 31 Dec 2008. Spot reference is unknown, Bloomberg shows 1.5960 for end of day; 10Y-EUR-CHF forward is about 18 big figures down; EUR-CHF ATM volatilities between 5% and 7%.

Maturity	10 years
Notional	5,000,000 EUR
Strike K	1.4350
Spread s	2.09%
Amortization	none
The bank pays in EUR	6-month EURIBOR $+s$
The bank receives in EUR	6-month EURIBOR $+\left(\frac{K-F}{F}\right)^+$
Fixing of the FX options	F: ECB fixing on Reuters page ECB37 two business days before the end of each period
Calendar	TARGET
Coupon payments	semi-annually
Basis	365/360, modified following, unadjusted
Premium	none

EUR-CHF stays above a pre-specified level called the strike. Many treasurers and family offices have been attracted by this type of swap because of the high spread (2.09% in this example) and the fact that the deal is a zero-cost transaction (at inception). If the investor furthermore believes that EUR-CHF will not fall substantially or is talked into believing this, then it feels like a free lunch.

Market value at inception was around EUR 274,000 assuming a spot reference of EUR-CHF 1.5960. However, the exact spot reference at trading time is unknown. This lack of information causes a large uncertainty about the market value, because the structure has about EUR 14 M delta in EUR/CHF. If we assume a volatility in EUR-CHF of 5.777% in a Black-Scholes model, we would obtain a 95% confidence interval of spots with one day time to go of about 1.5866 – 1.6055, so basically 200 pips uncertainty within a trading day. The exact range of spots on the trading day can be inferred from a tick data analysis. This range can be smaller or bigger than the confidence interval. With a 14 M delta it implies that value of the embedded options changes by EUR 140.000 for a 100 pip change in spot, or EUR 280.000 for a 200 pip change in spot. Therefore, without knowing the traded spot reference we need to assume an uncertainty of $\pm$ EUR 280,000. Obviously there are many other factors contributing to the market value, but delta is certainly the biggest.

Composition

1. Swap with semi-annual fixed payments in the size of the spread. Obviously the floating interest rate that is part of the terms is completely irrelevant. One can put it in to make the investor feel as though they are trading a familiar product.
2. Twenty-one vanilla EUR put CHF call options with strike K and maturities matching the cash flow dates in the swaps, and expiry dates two business days before.

The notionals of these FX options are approximately EUR 250,000, exact amount depends on the respective day count fractions for the half years. The payoff formula of a put option in the terms is literally visible, except that one may wonder why the payoff is divided by the spot reference at expiry. Now remember, *FX is all about currencies*: applying the payoff of strike minus spot to a EUR notional will generate an amount in CHF. But the interest rate is supposed to be paid in EUR. Therefore, the CHF payoff is divided by the prevailing EUR-CHF spot reference to convert it into a EUR payoff. Quite natural. No quanto, nothing complicated.

The composition is comparable to a dual currency deposit, where the investor "enhances" his yield by selling a vanilla option, see Section 2.4.1. This idea of "yield enhancement" or "optimization" is now repeated 21 times. However, note that DCDs are mostly short dated, so the FX risk is comparatively small, whereas CRSs are mostly long dated, and regime shifts of FX rates can cause a disaster.

Variations Obviously, there can be many ways to vary this type of a swap, see for example the variations of the *turbo cross currency swap* in Section 2.5.3. A reasonable variation may be using a put spread rather than a single put to place a bound on the maximum potential loss. However, in that case the overall conditions will appear less appetizing. Another possible variation will automatically be introduced if the same coupon formula is applied to a swap with a notional in domestic currency: In our example, if the notional of the swap is in CHF, and the coupons are in CHF, then the payoff is quantoed into CHF. An example is swap 4175 traded between the city of Linz in Austria with BAWAG P.S.K. on February 12 2007, whose terms are shown in Table 2.54.

TABLE 2.54 Terms of the quanto currency related swap 4175 in EUR-CHF; traded 12 February 2007, starting on 14 February 2007, maturing on 15 April 2017. Maturity days were 15 April and 15 October, starting on 15 April 2007. Spot reference is unknown, probably near 1.6238; 10Y-EUR-CHF forward 1.3991; EUR-CHF ATM volatilities average around 3%.

Maturity	10 years
Notional	195,000,000 CHF
Strike K	1.5400
Fix s	0.065%
Amortization	none
The bank pays in CHF	6-month CHF LIBOR, 2.18% for first period
The bank receives in CHF	$s + \left(\frac{K-F}{F}\right)^+$
Fixing of the FX options	F: ECB fixing on Reuters page ECB37 two business days before the end of each period
Calendar	TARGET
Coupon payments	semi-annually
Basis	30/360, modified following, unadjusted
Premium	none

This swap is composed of a fixed-floating interest rate swap and 21 EUR put CHF call quanto plain vanilla options.

Legal Consequences Swap 4175 and many other currency related swaps that traded between banks and family offices and city/community treasurers ended up in court. The lawyers argue about the embedded risk and which client type is entitled to take it and which level of risk advisory banks should have provided. The legal framework for these transactions has been amended to protect treasurers from so-called speculation. One must note that any financial decision is some form of speculation. Even a decision not to hedge can be viewed as speculation. Any type of speculation has become illegal in many jurisdictions. This provides enormous room for interpretation and ammunition for more legal disputes.

Real Quanto CRS Currency related swaps whose coupons depend in a way on a formula that looks like an option payoff are obviously not restricted to vanilla and self-quanto vanilla. As a common example of a structured transaction we consider a 10-year transaction in CNY with quarterly coupons depending on USD-JPY spot S_T.

Every quarter the investor receives N in CNY and pays in CNY

$$N \cdot \begin{cases} \max\left[0, \frac{64-S_T}{60}\right] & \text{if } S_T < 60 \\ 0 & \text{if } S_T \geq 60 \end{cases}, \tag{46}$$

as long as the USD-JPY spot S stays below 120. This is a structure containing quanto barrier options in USD-JPY quantoed into CNY.

2.5.8 Double-No-Touch Linked Swap

The double-no-touch linked swap picks up the idea of a *tunnel deposit* of Section 2.4.3. A swap or cross currency swap is set up as a basic product and the interest rate payments for the client are a worst case above par if a currency pair ever hits or leaves a pre-specified range between inception of the trade and maturity time and a best case below par otherwise.

Advantages

- Beneficial interest rate payments, i.e. receiving higher coupons or paying lower coupons than market
- Zero-cost product
- Start in the best case
- Guaranteed worst case known in advance

Disadvantages

- Long-term view on the FX rate impossible, i.e. suitable only for maturities up to one year
- Tunnel sometimes unattractive as position is taken only halfway against the forward curve

TABLE 2.55 Example of a EUR-CHF 9-month double-no-touch linked 2-year swap in EUR.

Maturity	2 years
Notional	5,000,000 EUR
Spot reference	1.5388
Amortization	none
The bank pays in EUR	fix 2.60% p.a.
The bank receives in EUR	fix 3.70% p.a. if EUR-CHF $\notin [1.4990, 1.5810]$
	fix 1.00% p.a. if EUR-CHF $\in [1.4990, 1.5810]$
Range valid up to	10:00 a.m. N.Y. time in 9 months
Coupon payments	annually
Basis	act/360, modified following, adjusted
Premium	none

Example We take EUR-CHF and show in Table 2.55 how a corporate client can lower his interest rate payments in EUR obligations from 2.60% to 1.00% for one year if EUR-CHF stays inside a range around the current spot for the next nine months. The attractive part of this deal is that the worst case is not as far away from the par rate as the best case.

Composition

1. The basic product is a two-year EUR swap, where the bank pays a fixed rate of 2.60% and receives a worst case fixed rate of 3.70%, which is worth, say, 107,000 EUR. From this amount the client goes shopping and buys the tunnel participation and leaves a sales margin with the bank, so the swap can be offered at zero cost.
2. The client goes long two nine-month EUR-CHF double-no-touch with range 1.4990–1.5810, with *deferred* EUR cash settlement, one in one year and the other one in two years. The notional of each double-no-touch must be in EUR and the amount must be the cash flow to get from the worst case 3.70% down to the best case 1.00%, so it must be 5,000,000 EUR $\cdot$ (3.70 − 1.00)% $\cdot$ 365/360 = 136,875 EUR.
3. The total offer price of the two double-no-touch options is 71,000 EUR, so there are 36,000 EUR left for the bank as a sales margin.
4. Pricing the double-no-touch options with deferred delivery may not be available in some front-office applications. However, one can price the double-no-touch with standard non-deferred delivery and take this as a super-replication because the options desk can keep the potential payoff for free between expiration and delivery and would thus make some extra profit (assuming non-negative interest rates). However, the booking of the trades should have the correct delivery dates of the double-no-touch options.

5. In this case the FX options desk buys 400,000 EUR from the spot desk to delta hedge the double-no-touch transactions. This EUR swap is fixed-fixed with practically no risk, whence the valuation is easy and the hedge is not urgent.

Variations With this basic idea laid out one can think of a whole bunch of variations. We can take other touch transactions instead of a double-no-touch. We can apply the idea to swaps with a floating leg, to cross currency swaps, swaps with amortization. The only constraint is that the first interest rate payment must be after the expiration of the double-no-touch. Using a deferred delivery allows a combination of rather long-term products such as swaps with rather short-term products such as double-no-touch. One can also think of nested ranges with corresponding coupons just like the tower deposits in Section 2.4.6.

2.5.9 Range Reset Swap

The range reset swap is based on the idea of a *corridor swap* as explained in Section 2.5.6. Investors taking a position in FX rates remaining inside a pre-defined range are often hit by unexpected market moves, specially if the time intervals are long. For this reason range trades are often more attractive if the range is reset around future spot levels in the future. This is implemented in the range reset swap, which is an interest rate swap where the investor pays LIBOR or a fixed rate in one currency and receives a best case or a worst case interest payment in another (or the same) currency depending on whether the FX spot remains inside a range for the time interval under consideration. Resetting the ranges is usually only marginally more expensive than fixing the range for all times, particularly if the forwards are not too far away from the current spot, but gives a lot more security to the investor. We consider an example listed in Table 2.56.

The way this works is that we price a swap floating leg against a zero fixed leg to get the amount of EUR we can spend. From this amount the bank subtracts the sales margin and buys six EUR-USD forward start corridors.

2.5.10 Exercises

Basis Swap Quotes Consider the ICAB1 quotes of basis Swaps of 9 November 2009 in Figure 2.26. An investor entering a 15Y EUR/USD cross currency swap, where he pays the EUR nominal at inception and pays the USD Nominal at maturity

1. receives USD LIBOR +5.75% and pays EUR LIBOR;
2. pays USD LIBOR and receives EUR LIBOR −0.0075%;
3. pays USD LIBOR −0.75 basis points and receives EUR LIBOR −5.75 basis points;
4. pays USD LIBOR and receives EUR LIBOR −0.75%.

Which one is correct?

TABLE 2.56 Indicative terms and conditions as of 11 August 2004 or a EUR-USD range reset swap. Ranges are American style. The best case is paid for each period in which EUR/USD stays inside the pre-defined range between the fixing date and the expiry date.

Spot reference	1.2700 EUR-USD
1 year EUR rate reference	2.20%
Trade date	11 August 2004
Start date	13 August 2004
Maturity date	13 August 2007
Principal amount	USD 10,000,000
Up-front fee	1.85% of the Notional
Redemption price	100%
The bank receives	6 Month LIBOR – 2 bps, act/360
The bank pays best case	5.00% p.a. semi-annually, 30/360
The bank pays worst case	0.00%
Fixing of the barriers (pre-defined range)	At fixing date
Upper barrier	108% of the fixing spot
Lower barrier	92% of the fixing spot
	Fixing date and expiry date
Period 1	11 Aug 2004–10 Feb 2005
Period 2	10 Feb 2005–11 Aug 2005
Period 3	11 Aug 2005–9 Feb 2006
Period 4	9 Feb 2006–10 Aug 2006
Period 5	10 Aug 2006–9 Feb 2007
Period 6	9 Feb 2007–9 Aug 2007
Fixing spot	Reuters ECB37
Fixing and payment business days	London & New York

2.6 PARTICIPATION NOTES

Participation or *performance* notes are usually listed and capital guaranteed products, where the investor invests a certain amount of a currency, waives parts of the market interest, and participates in a certain market sector. These types of notes are generally more popular in equity and index markets, where even retail investors have a view. Foreign Exchange is only recently being discovered as an *asset class*. Only very commonly known exchange rates like EUR/USD or the gold price XAU/USD are considered suitable from a marketing point of view. We discuss a few examples.

2.6.1 Gold Participation Note

We consider a capital guaranteed five-year note in EUR with a coupon based on the gold price (XAU/USD). Let us assume an investor believes in a rising gold price and wishes to invest an amount N in EUR. At maturity the investor receives

- 100% of the invested capital,
- +95% participation via Min(gold price return, 60%) in USD.

This type of participation specification usually causes a lot of confusion or discussion since in FX it is never so clear which currency we are talking about. Generally, only a formula helps, which makes these notes a bit difficult to retail clients. In our case the payoff formula is

$$N \cdot \left[100\% + 95\% \frac{\max(0, \min(\text{XAU-USD}(T) - \text{XAU-USD}(0), 60))\%}{\text{EUR-USD}(T)}\right]. \tag{47}$$

This means, for instance, that if we start with a gold price of 400 USD per ounce and end with 450 USD per ounce, and the EUR/USD exchange rate at maturity T is 1.0000, then the investor receives the invested capital and 95% of 50%, i.e. 47.5% one-time coupon, which is quite a good deal. The maximum loss will be a 0% return, a coupon below the market interest rate, which happens if the gold price does not rise. The maximum gain in this case is 95% of 60%, which is 57%. An additional confusion is caused by the conversion into EUR. In this case, the conversion is taken at the maturity date T, which means that the investor is exposed to the risks of EUR/USD spot movements. Other offers could be a EUR/USD exchange rate to be fixed in advance, which would make the structure a *quanto* capped call performance note, and the EUR/USD exchange rate risk rests with the issuer. The terms and conditions are summarized in the term sheet in Table 2.57. Overall, as you can see this is not so easy to explain to a retail client. To support it, one can use historic time series as in Figure 2.34.

Here are some arguments for a possible rising gold price I presented in the first edition of this book.

1. The price of gold has moved above a *25-year low* and is now expected to break out to the upside.

TABLE 2.57 Sample indicative terms and conditions of a gold performance note as of 21 February 2003. Note that OPTREF no longer exists; one would have to take a different fixing source now.

Investment type	Bond
Issuer	to be specified
First trading day	to be set
Fixing of the initial spot reference	on the first trading day
Maturity	5 years after the first trading day
Notional N	EUR 10,000,000.00
Initial price	100%
Capital guarantee	100%
Underlying	Gold price XAU-USD
Gold price source	Reuters Instrument Code (RIC) XAU=
EUR-USD fixing source	Reuters page OPTREF
Payoff at maturity	Formula (47)
XAU-USD(0)	Gold price on the first trading day
XAU-USD(T)	Gold price one week before maturity
EUR-USD(T)	EUR-USD spot reference one week before maturity

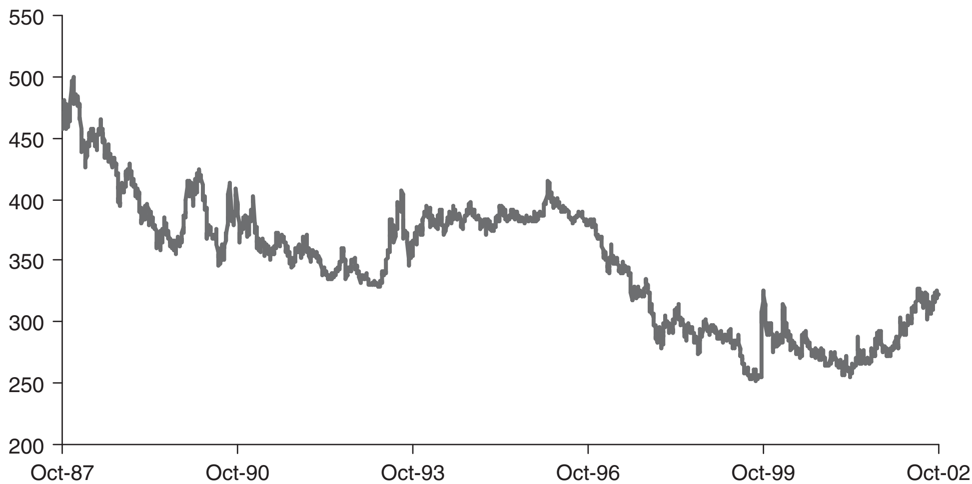

FIGURE 2.34 History of the ask price of gold in USD from 1987 to 2002. Source: Bloomberg

2. Consumption of gold has exceeded production for the past ten years, and when the low price of gold fell below production cost, many *gold mines reduced production*.
3. Gold price is highly *de-correlated from the equity and bond markets*: the correlation between gold price and the DJI index is equal to −0.30 over the last four years.
4. In such a climate of market uncertainty, gold constitutes a good way to *diversify your investments*.

Of course, it is a matter of the quality of the market research groups to support such arguments, and at the very end, it is a matter of belief for the investor. Now that I am working on the second edition of this book, I can see that the gold performance note wasn't such a bad idea. How markets can change!

2.6.2 Basket-Linked Note

Now we analyze how to structure a USD-denominated note, where the investor participates in the performance of a currency basket of the four currencies EUR, AUD, GBP, and CHF against USD. Following the basket payoff in Equation (410) we use the weights and normalizers in Table 2.58.

This way we form an index $I(t)$, and the note could look like this. The issuer pays

$$\min[1.18; \max[0; I(T) - 1.06]] \tag{48}$$

times the notional, which is essentially a capped basket call. Taking a maturity of one year and a notional of 10,000,000.00 USD, we obtain 220,000.00 USD from the money market, assuming a USD rate of 2.20%. The basket with strike 1.06 would cost 1.4% USD, the one with strike 1.18 would generate 0.1% USD, which leaves us with a total cost of 1.3% USD. Since the USD interest is not paid to the investor, the issuer makes 0.9% USD sales margin. Alternatively, he could guarantee a minimum interest rate of 0.5% and keep only 0.4% of sales margin.

TABLE 2.58 Currency pairs, normalizers, weights, sample spots, and values of a basket of four used to structure a basket-linked performance note. The summands are computed as $a_j \frac{S_j(T)}{S_j(0)}$, the index value $I(T)$ being the sum of these four summands, the basket payoff following Equation (410).

Currency pairs	Sample spots $S_j(T)$	Weights a_j	Normalizers $S_j(0)$	Summands
EUR/USD	0.9800	25%	1.0400	0.265306122
AUD/USD	0.5610	25%	0.6500	0.289661319
GBP/USD	1.5300	25%	1.6000	0.261437908
CHF/USD	0.6725	25%	0.7000	0.260223048
			Index	107.66% USD

In the sample spot scenario of Table 2.58, the client would be paid an interest of 1.66%, which is below par money market and hence an effective loss. If the index shoots up to 118% or more, the investor is paid an interest of 12%, which is way above par money market.

2.6.3 Issuer Swap

An issuer swap is not a structured product. The issuer of a bond or a note or any listed derivative is usually a bank or corporate. Along with the issuer goes its credit default risk. A low-rated bank or a small cap will tend to have a higher default risk than a central bank. Therefore, coupons of low-rated issuers will be higher than coupons of high-rated issuers. It can happen that a bank wants to issue a bond or a note but cannot do this because either its credit rating is too low or it has legal or marketing constraints that do not allow it to issue a certain note. In this case, however, the product can still be sold under the name of another issuer. In this case the issuers have to be swapped and the *credit spread* has to be paid from the lower-rated to the higher-rated issuer. In such a case the entire product of a bank is sold to the issuer, and the bank then buys it back for distribution. This swap of issuers is called an *issuer swap*.

2.6.4 Moving Strike Turbo Spot Unlimited

The moving strike turbo spot is a product which is often issued as a certificate or listed note. It is automatically renewed at pre-specified times, say monthly, unless it is canceled. It can be canceled by the holder or the issuer at these renewal dates. If the contract is canceled, it pays to the holder

$$[\phi(S - K_t)]^+, \tag{49}$$

where K_t denotes a moving strike and as usual $\phi = +1$ in the case of a call and $\phi = -1$ in the case of a put. The strike is updated following the rule

$$K_{t_{i+1}} = K_{t_i} e^{(r_d + \phi r - r_f)(t_{i+1} - t_i)}, \tag{50}$$

where r_d, r_f are the domestic and foreign continuously compounded rates respectively and r is some positive rate. As soon as the strike K_t is touched, the product is exercised with value 0.

The idea behind this product is to enable an investor to invest into the spot of an underlying in a note format. The investor participates in a linear way in rising and falling spot rates. The alternative would be to buy the underlying with a stop-loss strategy if the spot moves down to K_t. However, this would require a lot of cash and short selling the underlying in case of a desired participation in a downward movement. In the certificate format only a quantity of the size of the payoff needs to be invested. Besides that the default risk is also minimized for both parties. These features combined with the simple payoff structure have made this product extremely popular, not only in foreign exchange markets.

Pricing of a Moving Strike Turbo Spot Unlimited Given that in t_1 the contract can be canceled, the price in t_1 must be

$$[\phi(S_{t_1} - K_{t_1})]^+ \tag{51}$$

unless K_t has been touched. As an example we consider the call version. In $t_0 < t_1$ the value of a down-and-out call with a barrier equal to the strike can be determined in the case of zero drift $r_d - r_f$, because a forward contract with strike K_{t_0} would be a perfect static hedge. For this reason the adjustment rate r is introduced to generate a zero drift. The value of the forward contract is

$$v_{t_0} = e^{-r_d(t_1-t_0)} I\!E\left(S_{t_1} - K_{t_1}\right) = S_{t_0} e^{-r_f(t_1-t_0)} - K_{t_0} e^{(r-r_f)(t_1-t_0)}. \tag{52}$$

Provided $r_f \geq 0$ and r is chosen greater than 0 we have

$$v_{t_0} < (S_{t_0} - K_{t_0}). \tag{53}$$

The right side of this inequality is the quoted offer price.

Hedging the Moving Strike Turbo Spot Unlimited In a backwardation scenario of the forward curve, the issuer does not need to worry, as earning sales margin comes for free because the hedge of the down-and-out call using the forward is cheaper than the down-and-out call even in the case of $r = 0$. The issuer just pockets the cost of carry. However, in a contango situation the use of r becomes crucial as the issuer would otherwise pay the cost of carry. It has been interesting to observe how much time it took for various banks in 2003/2004, when the EUR-USD forward curve switched from backwardation to contango, to discover the reasons for their losses in dealing with the turbo notes.

2.7 HYBRID FX PRODUCTS

A real hybrid FX product is a transaction whose terms and conditions depend on more than one asset class where the components of the asset classes cannot be separated. We have seen that forwards, deposits, and swaps can be enhanced by starting with a worst case and then buying FX options or series of these to participate in certain FX market movements. These structures have an FX component that is separable from the basic product. This is like Lego. You take the building blocks, possibly from different asset classes, and build a product of these Lego blocks. For hybrids it is different. Examples for real hybrid products where FX is one of the asset classes include but are not limited to the following.

Long-term FX options. The interest rate risk of long-term FX options is so prominent that we can no longer work with the Black-Scholes or any one factor model assuming deterministic interest rates. We need to rather model the future interest rates in the two currencies as a stochastic process. Modeling both rates along with the exchange rates requires at least a three-factor model.

Options with deferred delivery. Usually the delivery date of options is two business days after the maturity date. However, it can happen that we need to settle the cash flows of an exercised option at a much later date. For example, consider a client buying a six-month double-no-touch, whose payoff is supposed to enhance the interest rates of a five-year swap with semi-annual cash flows. In such a case the delivery date may be four and a half years after the maturity of the double-no-touch. This cash on hold is subject to interest rates in the future, such that modeling the value of such a deferred delivery double-no-touch no longer just depends on the exchange rate. Here we cannot separate the interest rate risk of the FX-derived payoff because we do not know at inception whether there is going to be any cash flow. The asset classes IR (interest rates) and FX (foreign exchange) are not separable.

Interest rate products with a knock-in or knock-out barrier in FX.

Equity products with a knock-in or knock-out barrier in FX.

Credit products with a knock-in or knock-out barrier in FX.

Derivatives in a non-FX market quantoed into another currency.

The most prominent types of FX hybrids are FX vanilla options with a long tenor and so-called *power reverse dual currency bonds (PRDC)*. There are also a number of quanto products and other hybrids. We will cover these in the next sections in some detail.

2.7.1 Long-Term FX Options

Long-term options are hidden hybrids. In fact, the way their prices are quoted in the market is just like short-dated ones: implied volatility assuming a Black-Scholes model. This insinuates that FX spot volatility is the driving source of risk. Well guess what: it isn't.

2.7.2 Power Reverse Dual Currency Bonds

A PRDC is a generic name for an entire class of transactions (bonds and swaps) whose coupons depend on an exchange rate or are quantoed into other currencies, and/or whose notionals may be converted into other currencies. A detailed overview can be found for example in Baum [11]. The reason PRDCs came up was the low-interest environment in Japan along with the pension funds in Japan that had promised high yields to their investors. Now how to boost the coupons? We have learned that generating returns is normally done via carry trades or selling options or selling option-like features (callability). Essentially, all of these approaches tend to increase the risk of the investor. The risk has been amplified further, because practically all pension funds ended up having the same positions.

Trade Features We start with a very common example of a long dated (typically 20Y to 30Y) dual currency trade in USD and JPY, in the format of a swap:

1. Principal exchange of USD 1 M against JPY 115 M at inception.
2. Financing (funding) coupons depending on LIBOR, e.g. 6M JPY-LIBOR.
3. Power (structured) coupon depending on FX rate, e.g. USD/JPY.
4. Reverse principal exchange of JPY 115 M against USD 1M at maturity.

The *power coupon* depends on the FX rate. As a simple example we consider a call spread

$$\text{power coupon} = N \cdot \min\left(\max\left(F\frac{S_T}{S_0} + M, \text{Floor}\right), \text{Cap}\right)\tau, \tag{54}$$

where N denotes the notional, F the fixed rate, M the margin, T the coupon fixing date, S_0 the initial spot rate, S_T the spot rate at the coupon fixing date, and $\tau = T - t$ the tenor. For example, with an initial spot of $S_0 = 120$ in USD-JPY, the payoff

$$\text{power coupon} = \min\left(\max\left(24\%\frac{S_T}{120.00} - 18\%, 0\%\right), 6\%\right) \tag{55}$$

represents a call spread with coupon of 0% for USD-JPY spot below 90.00, 6% for USD-JPY spot above 120.00, and linear interpolation in between, see Figure 2.35.

The main reasons why a PRDC with a power coupon is considered attractive by the buy-side are that the first coupon(s) is (are) typically guaranteed, and that the buy-side receives high coupons if USD-JPY does not decrease, i.e. if JPY does not strengthen. In the first decade of 2000 USD interest rates were higher than JPY interest rates, so the USD-JPY forward curve is in backwardation. The buy-side taking a view in USD-JPY *not* going backwards is essentially a carry trade, 90% of all trade ideas. Furthermore, the buy-side investor receives USD coupons, for which she would receive more JPY as the forward curve indicates. The principle of the cash flows of a power reverse dual are illustrated in Figure 2.36.

Soon after the principal idea of a power coupon had spread, the greed-driven path to even higher coupons took its natural course. One way is to make the power reverse dual callable by the issuing bank, typically Bermudan style with a right to call on the coupon dates. However, since it is very difficult to predict for the buy-side when the bank is likely to call the bond, a more transparent early termination based on an FX spot hitting a pre-specified barrier is introduced. This feature is referred to as *auto-callable*. Needless

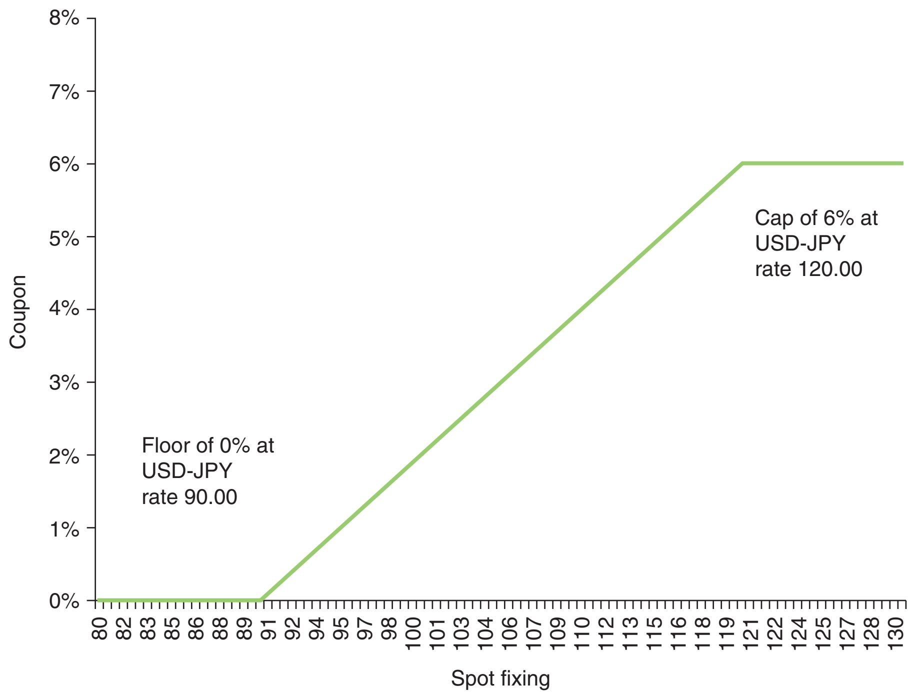

FIGURE 2.35 PRDC power coupon via USD-JPY call spread.

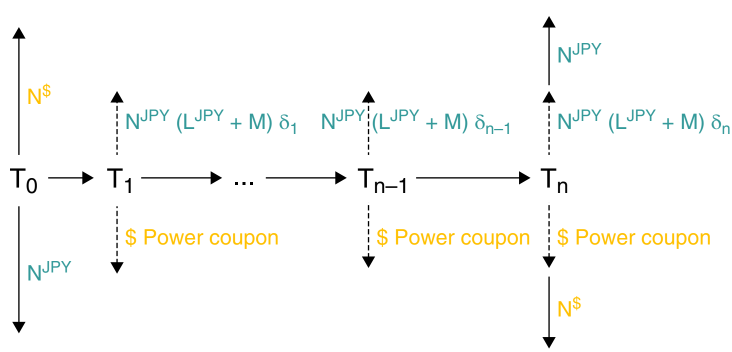

FIGURE 2.36 PRDC power coupon via USD-JPY call spread.

to mention that multiple coupons, auto-callability, and Bermudan callability can and have been combined. To pick up an even higher carry effect, one can also structure a PRDC with USD funding and power coupons depending on AUD/JPY. AUD/JPY is known to be the carry trade currency Japanese housewives have applied, commonly and generally referred to as Mrs. Watanabe.

Complex PRDCs The interest in higher coupons does not end with features such as callability. One can add other flavors, increase complexity, and enhance coupons ever further. For example, a common yield enhancement approach is based on target features, similar to those in Section 2.2.

Power Reverse Dual Target Redemption Note (PRD TARN) This structure terminates early as soon as a pre-specified *target coupon* is reached. For instance, revisiting the power coupon in Equation 55, assume the first coupon is 4% and the target coupon is 10%, then a spot $S_T = 120$ would imply a coupon of 6%, so the total accrued coupon is 4% + 6% = 10%, which causes the structure to terminate early after two coupon payments. There are the obvious variants as to how to handle overshoots, which we do not need to get into here. The risk for the investor is that she may have to reinvest the notional at an even worse market interest rate in JPY (re-investment risk). The seller – hopefully – has no particular risk as he has hedged his USD-JPY exposure, and will then be happy to suggest a new structure. If the second coupon is 5% (for spot 115), then the total accrued coupon is 9%, and the structure continues. If the USD-JPY spot drops to 92 and stays there or below, then all future coupons will be zero, the total coupon does not accrue, and the target will never be reached. This is in fact the higher risk for the investor, as she might be stuck with a 30-year contract and zero coupons for the last 28 years, and possibly receiving USD in 30 years that are worth substantially less in JPY than at inception (nominal FX risk). However, it is this additional risk for the investor that allows more attractive power coupons altogether.

Power Reverse Dual Double TARN This structure generalized the idea of a power coupon in combination with a target feature. We now have several coupons

$$\text{power coupon} = \begin{cases} C_1 & \text{if } S_T < L \\ C_2 & \text{if } S_T \geq L \end{cases}, \tag{56}$$

$$C_i = \min\left(\max\left(F_i \frac{S_T}{S_0} + M_i, \text{Floor}_i\right), \text{Cap}_i\right), i = 1, 2. \tag{57}$$

Power Ball In this structure the coupons follow a snowball system and are nested like

$$C_T = \min\left(\max\left((C_{T-1} + F)\frac{S_T}{S_0} + M, \text{Floor}\right), \text{Cap}\right). \tag{58}$$

FX TARN In this structure we use a multiple coupon as in Equation (56) along with a coupon accrual counter

$$C_T^{\text{accr}} = C_{T-1}^{\text{accr}} + \sum_{i=1}^{2} \alpha_i(S_T)C_i, \tag{59}$$

where α_i are pre-specified weighting functions of the spot. The structure terminates once the accrued coupon C_T^{accr} reaches a pre-specified target.

Chooser TARN In this structure the coupons follow the PRD pattern, for example

$$
\begin{aligned}
C_1 &= 8.0\%, \\
C_T &= 1.0\% \cdot \min\left(\max\left(F \cdot W_T + M, \text{Floor}\right), \text{Cap}\right), && (60) \\
W_T &= \min\left[\frac{S_T^{\text{AUD-JPY}}}{S_0^{\text{AUD-JPY}}}, \frac{S_T^{\text{USD-JPY}}}{S_0^{\text{USD-JPY}}}\right]. && (61)
\end{aligned}
$$

Each coupon depends on (at least) two different currency pairs, usually taken as the *worst of* the two. As usual, the coupons in a TARN accrue up to pre-specified target, for instance 25%.

Hedging of PRDCs Let me make a few comments on the hedging of PRDCs. We consider the simple variant with the power coupon following a call spread in USD-JPY as in Equation (55). The sell-side would be mainly concerned with the FX options embedded in the power coupon, i.e. hedging the short vega of the lower strike option and the long vega of the higher strike option. The vega position of the short call spread (sell-side view) is illustrated in Figure 2.37.

In Japan, many of these types of PRDCs traded in the 1990s and the first decade, and obviously the banks were only sellers and pension funds the buyers. Consequently, practically all market participants on the sell-side had similar positions and since the sell-side typically hedges the market risk of the derivatives portfolios, they would all be short FX vega for lower strikes and long maturities and needed to compensate this by

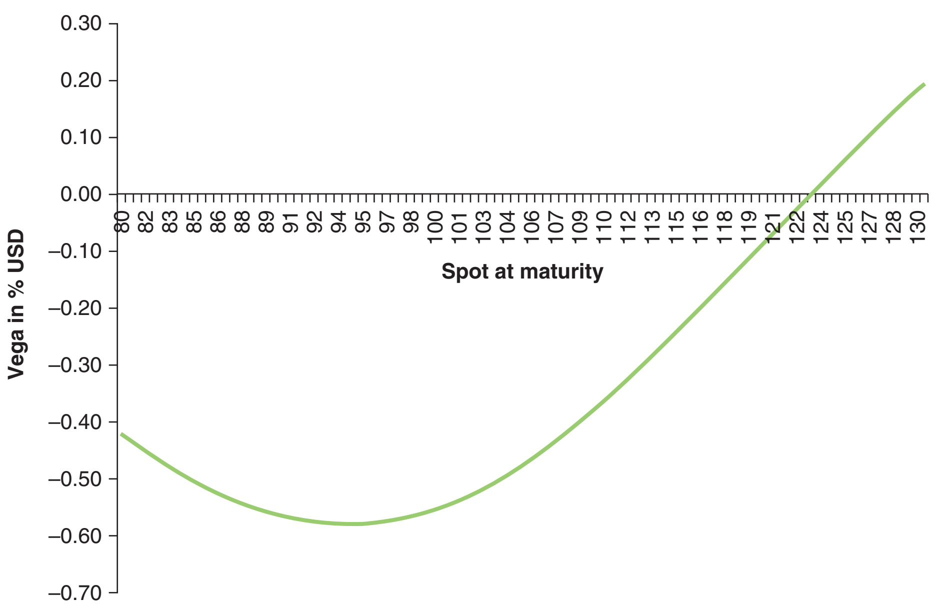

FIGURE 2.37 Sell-side vega of a PRDC power coupon via a 10-year USD-JPY call spread.

going long vega for lower strikes in USD-JPY. This implied a heavy skew in USD-JPY for many years, merely driven by supply and demand, especially because long-term FX vega is illiquid and therefore bid-offer spreads wide. Risk reversals were strongly negative, a market phenomenon that did not indicate that spot would go down. However, when JPY strengthened and USD weakened in the second decade, and many of the PRDCs were called or auto-called, the vega positions were unwound and the USD-JPY smile changed its pattern to a risk reversal closer to zero. Moreover, the hedging situation also suffered from significant cross effects, because FX vega moves as the FX forward moves and the FX forward moves are driven by FX spot, interest rates, and their correlation.

Based on the expected cost of the hedge, there are a number of factors determining the price of PRDCs, including

- FX smile and skew;
- correlation between FX rates and interest rates;
- interest rate swaption skew;
- the number of factors for interest rate dynamics.

A pricing model – and its implied hedging parameters – would typically be a three-factor model. For more details I refer to [141].

2.7.3 Hybrid Forward Contracts

Hybrid FX forward contracts are forward contracts with a non-separable link to another asset class, usually interest rates. We consider two examples.

CMS Spread-Linked FX Forward In this hybrid forward contract the counterparts trade a forward contract with a *hybrid strike*

$$K = M - (2S_T + 5 \cdot \text{spread}) \tag{62}$$

$$\text{spread} = \text{JPYCMS20Y} - \text{JPYCMS2Y}, \tag{63}$$

where S_T is the USD-JPY spot at maturity, M a pre-specified "margin," and the spread depends on the difference of the 20-year and the 2-year constant maturity swap (CMS) in JPY. Pricing and risk management of a contract like this obviously require a multi-factor model for interest rates and the FX spot, with the respective dependence (correlation if you want).

Long-Term Knock-Out Forward Series A knock-out forward (or a series thereof) was introduced in Section 2.1.5. For a short-term product used in treasury, this can be decomposed into vanilla and barrier options in one FX spot as an underlying. Nothing hybrid. However, the moment FX derivatives are long-term contracts, they do require a multi-factor hybrid model, because the effects of interest rates and FX rates are no longer separable. As an example we consider a series of knock-out forwards over a time span of 15 years, where every month the counterparties agree to exchange currencies as follows. The investor receives N in USD and pays in JPY

$$N \cdot \begin{cases} 5 & \text{if } 105.25 \leq S_T \\ S_T & \text{if } 77.50 \leq S_T < 105.25 \\ 84.85 \cdot \frac{84.85}{S_T} & \text{if } 77.50 < S_T \end{cases}, \tag{64}$$

as long as the USD-JPY spot S_t stays below 116.50 at all times.

2.7.4 Dual Asset Range Accrual Note

Notes and certificates are generally intended for private banking or retail clients. The incentive is to pick up an extra yield if you like the trade idea. Since any payoff can be valued with a Monte Carlo approach, there is basically no limit to creating notes. Serious questions like what are suitable models and how to get any grip on the correlations are often conveniently ignored.

As an example you might get inspired by a 13-month dual asset range accrual note whose terms are summarized in Table 2.59. Not a hedging instrument for a treasurer, but never mind.

A few funny names have come up with such notes. This one would be called a "DARAN." Range accrual notes are generally referred to as "RANs." If the payoff depends on the worst of several underlyings staying in a range it might be referred to as a "WORAN." You continue!

TABLE 2.59 Example of a dual asset range accrual note. The number of fixing days, where both EUR-CHF and 12-month EURIBOR are fixed inside the corridor, is denoted by n.

Spot reference	1.5300 EUR-CHF
EUR rate reference	2.20%
Issue price	100%
Capital guarantee	100%
Notional	10,000,000 EUR
Maturity	13 months
Coupon	$3.75\% \cdot \frac{n}{d}$ p.a., paid once at maturity
Total number of fixing days	d
Conventions	30/360, modified following, adjusted
12 month EURIBOR corridor	2.25–3.25%
EUR-CHF corridor	1.5100–1.5700
EUR-CHF fixing source	Reuters page ECB37
EURIBOR fixing source	Reuters page Euribor01

2.8 TREASURY CASE STUDIES

2.8.1 FX Protection for EM Currencies with High Swap Points

Here is a typical problem in a market with a strong forward, as you typically observe in emerging markets (EM). A treasurer is looking to protect an investment in Brazil worth about USD 40 M. Obviously protecting USD/BRL with a synthetic forward with the exchange rate at current spot or a USD call BRL put is extremely expensive. The question is which alternative ideas might reduce the cost of protection.

Assume the treasurer buys 40 USD sells BRL in 12 months in a market scenario exhibited in Table 2.60.

Suppose the treasurer's goal was to buy USD at a spot as low as possible with a zero-cost strategy. Nice goal, ain't it, but this is extremely difficult as the forward works against the treasurer's goal, so it is clear upfront that we cannot expect wonders. Looking at the high RR indicates that we can bring down the cost by selling OTM calls.

Seagull As a first start we can consider a seagull, see Section 1.6.7. Why so? Here we go: since buying a USD call is the primary item on the shopping list, and selling an OTM call could lower the cost, we could try a call spread. However, a call spread is not free of costs. We can make it zero cost by selling a USD put.

- Treasurer sells a USD put 2.2600 (current spot, assuming this rate is good enough for the treasurer as it has just fallen by seven big figures).
- Treasurer buys USD call 2.4200 (692 pips better than outright forward par).
- Treasurer sells USD call 2.5000 (high strike is more than five big figures above the one-year historic high).

The bid/offer from the trading desk using a default volatility bid/offer spread of 0.25% for ATM may quote −88,894/−9,294 USD. Sales can then trade this at zero cost with 9,294 USD sales margin (assuming no other credit charges apply). The treasurer is protected at the level 2.4200 and if spot goes even higher then he can always buy USD eight big figures below spot. This trade definitely beats the outright forward while preserving zero cost. What do you think?

Worst Case Structures None of the forward structures with worst case (see Section 2.1.6 and the following sections) would look attractive because the worst case would have to be even higher than the outright forward, i.e. around 2.5000, which feels miles away

TABLE 2.60 USD-BRL market on 28 March 2014. Source: SuperDerivatives.

Spot ref	2.2600	ATM	13.625%
Swap points	2292	RR	4.55%
Forward	2.4892	BF	0.83%
USD rates	0.275%		
BRL rates	10.298%	Spot 1 Y high	2.4550

from current spot. The treasurer would prefer an improved forward rate, which is lower than the outright forward rate in this case of a USD buyer. If that was to be guaranteed, it would not come for free.

Ratio Call Spread One would have to check whether the treasurer insists on a zero-cost strategy or whether he may be willing to pay 2% for the protection. If zero-cost is a must, we can work with a leverage approach. Looking at the RR that massively favors the upside, we can consider a ratio call spread (see Section 1.6.1), which allows the level of 2.4200 to be lowered but introduces more risk if spot goes really high. However, I would carefully consider whether it is necessary to lower the protection level for greater risk.

Target Forward How about a target forward (see Section 2.2)? If the treasurer has the underlying cash (BRL) to sell, then it is not that risky. On current spot ref 2.2600 he would buy every month USD 3,333,333 at 2.3600 for a total of 12 monthly fixings. Target could be a generous 8 M BRL. He could hence massively beat the 1Y forward at zero cost. If it terminates early, then spot is low and he can then do another one of these target forwards or simply an outright as a follow-up hedge. This is priced in SuperDerivatives at bid −181,809 USD and offer −86,640 USD. Therefore, sales credit could be around 86k USD if traded at zero cost. Why does this work so well without using any leverage? Most likely, there will be only a few settlements in the beginning when the outright forward rate is still worse (i.e. lower) than 2.3600. Not satisfied? I leave it to you to make this more juicy by adding leverage and other flip-flops from the trick box.

2.8.2 Exit Strategies for a Sick Floan

The idea of a carry trade will catch anyone down the pike at some point in life. Besides Mrs. Watanabe, there are numerous *carriators*[2] ranging from corporate treasurers to hedge funds, but also municipality treasury departments. Anybody would like to take a loan in a currency with low interest rates. Revisit Figure 2.1 for an illustration. The following example (as reported in [142]) is about a EUR based county, in fact it could have been the county I live in. But the problem is a global one, and EUR is not really the worst of the currencies. Taking a loan in a foreign currency – I call this a *floan* – improves interest amounts in the short term at the cost of a – huge – FX risk at maturity. The hot news is that maturity is often past the next election period or after retirement of the person in charge. The pressure to act and engage in low interest rate payments is omnipresent because the public sector is mostly already bankrupt at inception. Floans are considered short-term liquidity bridges and never make it to the balance sheet (or there is no balance sheet). Other sick floans are expected to arise in many of the EM currencies including INR, CNH, BRL, ZAR. The concept of a floan is illustrated in Figure 2.38.

How does one exit a CHF loan that turned bad? After the SNB removed the guarantee level of EUR-CHF 1.2000 on 15 January 2015, many floans in CHF that had already gone bad in the last few years caused another substantial loss for municipalities

[2]Somebody entering into a carry trade

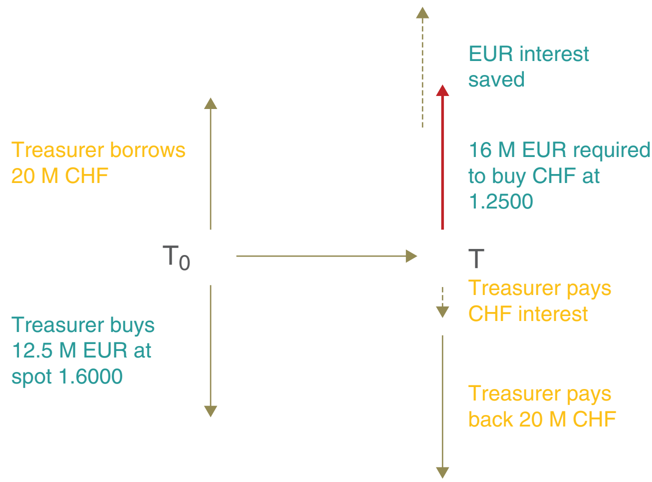

FIGURE 2.38 Concept of a floan: ignoring interest payments on the way, the treasurer borrows CHF at time T_0, converts into EUR at EUR-CHF spot S_0. At maturity T he needs to buy back CHF at the prevailing spot S_T. The interest rate he saves is in theory identical to the higher EUR cash required if the prevailing spot is equal to the forward $f(T)$. The treasurer enters the floan because he expects $S_T \geq f(T)$ when he takes the decision at T_0. His risk is that $S_T < f(T)$, and the losses arising from this risk are potentially unlimited.

and corporates in many countries in Europe. Spot dropped below 1.0000 for a short time and then gradually recovered, as illustrated in Figure 2.39. The question is, what can be done? There are several ways to handle the situation.

1. Strategy 0: do nothing – wait for better times
2. Strategy 1: outright forward
3. Strategy 2: call on the foreign currency
4. Strategy 3: vanilla zero-cost structure
5. Strategy 4: exotics zero-cost structure
6. Strategy 5: sue the bank
7. Strategy 6: reduce coupon further by trading a currency related swap and then sue the bank

Let us consider an example of a floan of 20 M CHF expiring in six months, on 27 November 2015, and consider a working spot reference EUR-CHF of 1.0400 on 23 May 2015. Other market data used: swap points −0.00510, forward 1.0349, EUR deposit 0.055%, CHF deposit −0.906%, ATM 8.950%, 25-delta risk reversal −2.80% favoring EUR puts, butterfly 0.45%. Contract data: spot date 27 May 2015, expiry 25 November 2015, delivery 27 November 2015. We compare the following strategies and illustrate them in Figure 2.40.

Strategy 1 – Outright Forward The treasurer trades an outright forward contract at zero cost at a rate of 1.0340, which means the treasurer pays nothing now and buys back 20 M CHF in six months for EUR 19,342,360. This strategy freezes the future exchange

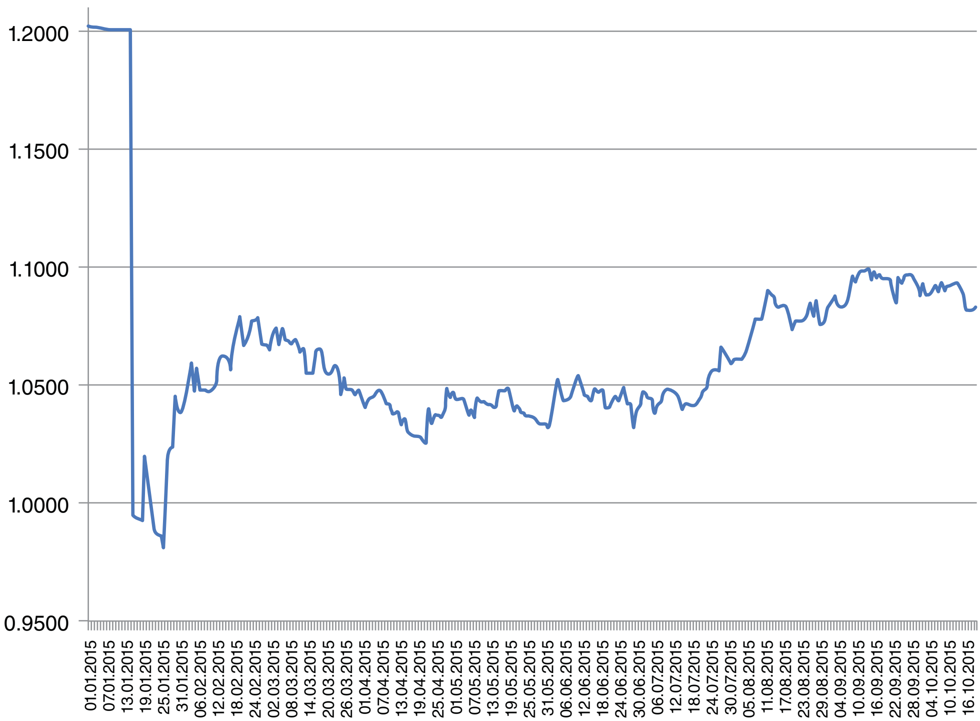

FIGURE 2.39 EUR-CHF drop and recovery in 2015.

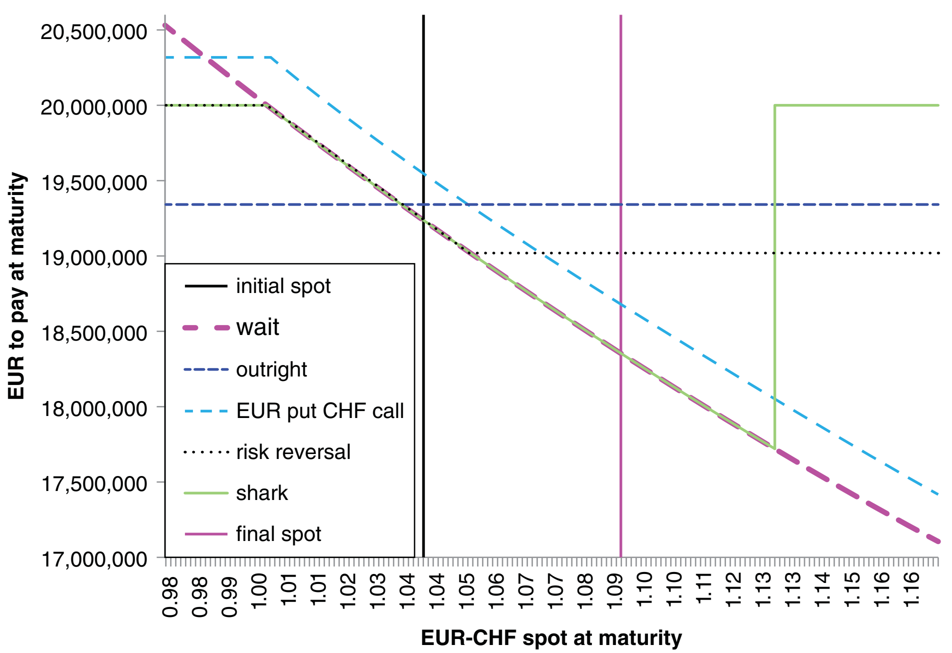

FIGURE 2.40 Comparison of exit strategies of a sick floan in EUR-CHF. The treasurer needs to buy CHF 20 M for EUR in six months to pay back his floan. His goal is to minimize the EUR amount required to buy CHF. Initial spot 1.0400 and terminal spot 1.0900 shown in vertical lines.

rate, so that on the one hand the treasurer will not lose any more EUR on his CHF floan, but on the other hand will not be able to participate in EUR going stronger in six months.

Strategy 2 – EUR Put The treasurer buys a EUR put CHF call strike 1.0000 at a current offer price of 1.597% EUR, so EUR 319,400 now and in six months has the right to buy back 20 M CHF for 20 M EUR, but could also buy back the 20 M CHF for less EUR should EUR-CHF rise. For example, if EUR-CHF trades at 1.0800 in six months, then the treasurer needs only 18.5 M EUR to buy back the 20 M CHF, so she would save a noticeable amount of EUR 842,360 compared with the outright in strategy 1. Subtracting the initial cost of the option the treasurer would still be able to save more than EUR 500,000, compared with strategy 1. However, the initial cost of the strategy puts many treasurers off, or the funds for the premium are just not available, or it is impossible to get a sign-up majority in the responsible committees. From a markets' point of view, we notice that the risk reversal in EUR-CHF is at −2.80% favoring the CHF calls, which is extremely high and makes all CHF calls much more expensive. This is because the majority of the market participants are afraid of a further drop of EUR-CHF in the next six months. Therefore, we consider two more strategies.

Strategy 3 – Risk Reversal The treasurer trades a zero-cost cylinder/collar/risk reversal, which means she buys the CHF call EUR put at strike 1.0000 and sells the EUR call CHF put strike 1.0510. The effect is that now she does not need to pay any premium, is protected at a level of 1.0000, and can participate in EUR-CHF rising to 1.0510, but no further. If spot in six months trades between 1.0000 and 1.0510, she would buy 20 M CHF at the prevailing spot rate. If the spot is lower than 1.0000, she would buy the 20 M CHF at a rate of 1.0000, so pay a total of EUR 20 M. If the spot is higher than 1.0510, she would buy the 20 M CHF at a rate of 1.0510, so for EUR 19,029,495. While this is a zero-cost strategy with a guaranteed worst case, it still does not appear to be a good deal because the participation in the upside is rather limited to 1.1 big figures. Therefore, we consider an alternative using a barrier option.

Strategy 4 – Shark Forward The treasurer trades a zero-cost forward extra (try to avoid the term shark when dealing with county treasurers), which means that she is guaranteed a worst case exchange rate of 1.0000 in EUR-CHF in six months. If EUR-CHF trades between 1.0000 and 1.1280 during the next six months, then the treasurer can trade at the prevailing spot rate in six months, so the higher the better. If EUR-CHF trades at or above 1.1280 at least once during the next 6 months, then the treasurer will buy back the 20 M CHF at rate 1.0000. The best case is a spot of 1.1279, in which case she needs to spend only EUR 17,732,068, which would leave her with a profit of EUR 2,267,932 compared with the worst case. This is a classic flow product for mid-caps. In a current market scenario with high EUR-CHF volatilities, it would make sense to consider a barrier option as part of the strategy because the knock-out feature will make a barrier option rather inexpensive. This strategy is often criticized for forcing the exchange rate back to the worst case when it has risen too much in the treasurer's favor. It is inappropriately labeled as speculation. The actual speculation is the floan without a risk policy. This comparison with a prevailing high spot rate is not fair because the strategy has to be chosen at time zero, not at maturity. And at time zero, the rate to compare is the outright forward rate.

Legal Limitations There are many more strategies. Which one to take does not depend only on market considerations but also on legal limitations for the acting treasurer. After the EUR-CHF drop politicians have lined up to brag with a program to prohibit derivatives, especially if the derivatives are used for speculation. The problem here is where to draw the line. A forward extra using a barrier option as a building block may be considered speculation but in my opinion is actually the best deal of the four strategies above. Prohibiting derivatives appears to be like a rule to generally forbid cars for everybody just because there are some irresponsible untrained individuals who cannot drive. The only solution to the problem in a society with developed financial markets is

Education Education Education

Recap of the Outcome The press reports that county treasurer opted for strategy 1.[3]

In the last quarter of 2015 the EUR-CHF drop had calmed down a bit, and it's time to turn back and check the performance of the various strategies. As a recap of the above strategies, let us apply a spot reference of 1.0900 in November 2015.

Strategy 1: Treasurer buys back 20 M CHF for EUR 19,342,360, independent of the prevailing spot.

Strategy 2: Applying the prevailing EUR-CHF spot rate of 1.0900, the treasurer needs only 18.3 M EUR to buy back the 20 M CHF, so she would save a noticeable amount of EUR 993,736 compared to the outright in strategy 1. Subtracting the initial cost of the option the treasurer would still be able to save more than EUR 674,336, compared with strategy 1.

Strategy 3: Now that the spot has moved up, the treasurer would buy 20 M CHF at the upper collar rate of 1.0510 and needs to spend EUR 19,029,495. With this zero cost strategy with a guaranteed worst case she would have saved EUR 312,864.

Strategy 4: EUR-CHF has not traded at or above 1.1280 since inceptions, however, has risen in favor of the treasurer, she can by CHF 20 M at 1.0900, for which she needs to spend EUR 18,348,624, which saves her EUR 993,734 compared with the outright forward, the strategy 1, as it is also a zero-cost strategy.

Let's recap: at the beginning of the year, when we were faced with an exaggeration in the EUR-CHF exchange rate, we wanted protection on the downside, but also to

[3] http://www.wiesbadener-tagblatt.de/lokales/rheingau/landkreis/rheingau-taunus-kreis-steigt-aus-krediten-in-schweizer-franken-aus_15330402.htm

keep the option open to participate in a recovering EUR-CHF. All the strategies above did serve this purpose except strategy 1, the outright forward. Politicians and media had pushed affected floan takers into outright forward contracts, which may create a show-effect for politics but turns out to be the strategy with zero savings, while all the others – involving derivatives – would have done a much better job.

New laws about county finances are on their way where derivatives are to be banned as "speculative instruments." Politicians are scared to use derivatives as media have created a sentiment that derivative instruments are speculative and labeled as devilatives. However, with a simple forward extra, the county could have saved about EUR 1 M per CHF 20 M if they had had the patience to listen and the courage to apply modern risk management instruments and a political environment that would provide the scope to act wisely. The missing million to pay for child care and old age homes has to now be sourced again from the tax payer and the county head had the nerve to run for re-election.

Interestingly, the press argues that the county should have gone for

Strategy 5: Wait for better times, i.e. do nothing and just wait for EUR-CHF to rise.

This is obviously a suggestion with hindsight. Strategy 5 does not satisfy the protection against a further EUR-CHF drop. So even doing nothing in this case is in my opinion speculation and hence – under new laws – illegal.

So here I am back to my ceterum censeo: the only solution to the problem in a society with developed financial markets is

Education Education Education

2.8.3 Trade Ideas for FX Risk Management in View of Brexit

February 2016 [140]. In these days where we all mainly regulate, validate, justify, and document, what is left to do for the actual business? Derivatives are labeled as devilatives; however, in view of a possible Brexit, let me revisit some of the current hedging strategies. There is a market for structured forwards, and the rising volatilities are one motivation to go back and prepare some hedges.

The Market EUR-GBP is rising, back to 0.7800 and vols are up to 11% 1Y ATM, from about 6% in 2014. An exporter in the Euro zone receiving sterling would be naturally concerned about a weaker British pound. Huge losses can occur if FX risk is not hedged. Let's assume a spot reference of 0.7800 EUR-GBP, as observed on 22 February 2016. Riskies are up from −1% three months ago to +2%, so market sentiment is a substantial fear of EUR-GBP rising further. Time to act, I would say.

GBP selling / EUR buying Treasurer Let's assume a treasurer selling GBP 10 M every month for the next 12 months, buying EUR. What can be done?

Outright Forward: the treasurer sells GBP 120 M in one year at the forward rate 0.7900 and would then receive EUR 152 M. This hedge is the safe game; however, it does not allow the treasurer to participate in a stronger GBP should Britain not exit the EUR, which is one of the realistic scenarios (Brinexit).

Vanilla Option: the treasurer buys a EUR call GBP put option struck at 0.8000. This means he has the right to sell his GBP 120 M in one year and receive EUR 150 M upon exercise. The initial premium is about EUR 6 M. Worst case occurs for all spots at or above 0.8000, where the treasurer receives EUR 2 M less than in the outright forward, less the initial premium in fact EUR 8 M less. However, the advantage of the option is that the treasurer can participate in a stronger GBP. Should EUR-GBP drop to 0.7000 the treasurer would receive EUR 171 M, which is EUR 13 M more than with the outright forward, even after deducting the option premium.

Target Forward: the treasurer sells GBP 10 M every month at 0.7600 and receives EUR 13.17 M. This is the non-leveraged zero-cost version. However, the total profit is limited to 20 big figures. This means: should the spot reference after the first month be 0.7800, the treasurer would make a profit of 0.7800-0.7600 = 0.02, i.e. two big figures. To continue, if spot rises to 0.7900 at the end of the second month, the treasurer would be three big figures better than the market, and would lock in an accumulated profit of two plus three = five big figures. The contract terminates as soon as the accumulated profit reaches 20 big figures. This can happen, for example, if the spot refs in the following five months are all 0.7900. In this case the target forward would terminate early after seven cash flow dates. The treasurer would then have sold a total of GBP 70 M and received EUR 92 M, and could then trade another zero-cost target forward for the remaining five months where he would again beat the forward by two big figures and would generate another EUR 65 M, a total of EUR 157 M. That is EUR 5 M more than in the outright forward, the zero-cost alternative. A good deal for the treasurer, one might say. However, one must appreciate that target forwards are not a buy-and-hold strategy. Sounds too good to be true? Where is the problem? Good news first: should EUR-GBP drop further, then the treasurer would be much better off with the target forward, because it would not terminate early and the treasurer could sell all his sterling at 0.7600 and receive EUR 158 M in total, 6 M more than with the outright forward. However, this profit is limited. With the option the treasurer would be able to have unlimited profit potential as a result of EUR-GBP dropping. However, one might reasonably doubt that the drop will be so massive. The bad news: if GBP goes very weak very quickly, i.e. EUR-GBP rises to levels of 0.8500 the target forward would perform worse than the outright forward and worse than the option. In any case the sell-side has a huge interest in trading target forwards as they generate a bit more margin, are not really capital intensive and can be renewed as soon as they terminate.

Forward Extra: the treasurer locks in a worst case of 0.8000, which is only one big figure worse than the outright forward. He can participate in a favorable lower spot all the way down to 0.7150 and would receive EUR 168 M at best, EUR 150 M at worst as in the option based hedge. All other scenarios are in between, provided EUR-GBP doesn't trade below 0.7150 in the next year. The worst case of EUR 150 M is guaranteed in either case. If the treasurer expects a stronger pound, but still wants to have a guaranteed worst case in a zero-cost strategy, the forward extra could be an alternative worth considering.

Summary All hedges work and have their pros and cons; all of them would pass the IFRS 9 requirements and have been around now for many years.

For the risk-warehousing the sell-side view, the key to success with the FX product range is a reliable technology with state-of-the-art models and implementation of pricing and risk management figures. A necessary condition is a proper FX volatility surface, which we will revert to in [141]. For the buy-side, it is important to compare the various approaches, appreciate the need for a risk policy and its implementation.

Finally, note that the problem is not Brexit-specific. There are repeatedly situations when a company exports into a country whose currency is about to devalue. The above listed hedging approaches can then be adapted accordingly.

2.8.4 Inverse DCD

One of the questions I am asked a lot is how to structure an inverse DCD. Picking up on the idea in Section 2.4.1, a DCD or DCI is a dual currency deposit or dual currency investment. The key idea is easy: an investor deposits 10 M USD for six months and receives an enhanced coupon above market, e.g. 3.5% p.a. in USD, but accepts that his notional may be converted to 10 M EUR. The issuer would then always return the currency that is worth less, i.e. 10 M USD if the EUR-USD exchange rate in six months is above 1.0000 or 10 M EUR if the EUR-USD exchange rate in six months is below 1.0000. Effectively, the investor sells a USD call EUR put to the issuer, and the premium for this option is used to pay an additional interest amount above market (and a sales margin, of course).[4]

Comparison to other Asset Classes DCDs have been popular for many decades. In other asset classes, say in equity/bond markets, the product is referred to as a *reverse convertible bond*. The same principle applies: the investor sells a USD call STOCK put to the issuer with the effect to receive a coupon above market and taking the risk of his cash investment to be returned as shares of stock if the stock price falls. In currency markets, EUR takes the role of the stock.

Reasons for Trading DCDs The buy-side is looking for a higher coupon, so is essentially greed-driven, while accepting the risk of losing the entire notional in case of the hypothetical case that the EUR is not worth anything when measured in USD. Institutional investors sometimes have to invest in higher-yield products because they have themselves promised their investors higher yields (pension funds). The sell-side is very happy they can buy options from clients without credit risk. DCDs have become in fact a flow product.

[4]Market data used on 29 Nov 2016: EUR-USD spot 1.0500, 6M USD MM 0.992%p.a., 6M EUR MM −0.831%p.a., ATM 10.472%, 25RR 2.617% in favor of EUR puts, 25BF 0.355%, volatility for strike 1.0000 12.09% offer, sales margin of 10 bps.

Technical Requirements for Trading DCDs as Flow Products What is needed to offer this product to a larger client base? A trading desk that can do deposits and vanilla FX options, a system that can price, risk manage and administer the transactions. On the quantitative side, we need a really solid FX volatility surface, in particular, since DCDs tend to be short dated products, a careful management of trade events, varying geographic trade activity, handling of different cuts. Essentially, a professional issuer requires a technology of interpolation and extrapolation on the FX volatility surface with all bells and whistles, ideally with a proper financial market data platform.

How to Invert the DCD Trade Idea What can an investor do who wants to participate in EUR-USD staying below a pre-specified level K instead of participating in EUR-USD staying above a pre-specified level K? Well, she could simply invest in 10 M EUR instead of $N = 10$ M USD. But this is a non-satisfactory answer for an investor who wants to still invest in USD. Essentially she needs an inverse DCD. How to make it work? If we let S_T denote the final spot price in USD-EUR, then for the standard DCD, we could say that the investor receives N if $S_T < K$, and NK/S_T USD otherwise. In the case of an inverse DCD, we could then say analogously that the investor receives N if $S_T > K$, and NS_T/K USD otherwise.

Why can't we structure the inverse DCD by the bank buying a vanilla USD put, if the regular DCD can be structured by the bank buying the vanilla USD call? It is a common sense thing: the USD call is worth at most the USD notional, which is the same notional as the deposit notional N; consequently, the final amount to return to the investor can't go negative. However, the value and payoff of a USD put is unbounded (when measured in USD); consequently the inverse DCD structured with a vanilla USD put would potentially have an overall negative value. This is indicated by the dotted gray line in Figure 2.41.

To make the inverse DCD work, the bank would have to buy N/K *self-quanto* USD put EUR call options, indicated by the dashed black notional in Figure 2.41. I am sure you won't need more than one night in the pub to check this out. However, the self-quanto needs more work on the quant side and more flexibility of the system. In fact, it needs a fool-proof extrapolation of the volatility surface on the low deltas. The self-quanto effect comes in because we pretend the investor deposits EUR in a standard DCD, in which she would be short a vanilla EUR call USD put; but in fact, she deposits USD in an inverse DCD, in which she would then have to be short a self-quanto EUR call USD put.

Summary The inverse DCD requires a non-vanilla self-quanto option to structure it in a way that is analogous to the standard DCD. This is not a liquid flow product, and this is why many banks do not offer the inverse DCD. An alternative way to structure the inverse DCD using only vanilla options can be done as an exercise.

2.8.5 Exercises

Inverse DCD Structure the inverse DCD for a EUR investor using only vanilla options, i.e. assume the investor deposits N EUR, receives a coupon above market, and participates in a stable or rising EUR-USD exchange rate and loses parts of his EUR notional if

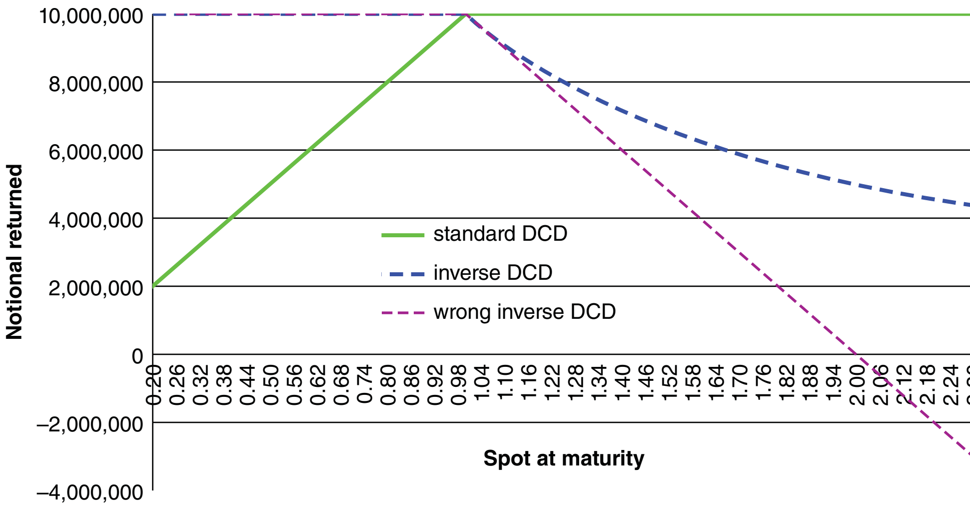

FIGURE 2.41 USD notional returned for a DCD. In the standard case the investor sells a USD call EUR put. In the inverse case selling USD put EUR call would potentially lead to a negative value. One way to correct this would be the self-quanto USD put EUR call.

EUR-USD ends up below a level K at maturity. Ensure that the investment has a capital guarantee of 0%, i.e. the notional to be returned does not go negative.

Risk Reversal Case Study in EUR-USD Consider a treasurer who needs to buy 10 M USD in six months. The underlying market is EUR-USD on Jan 12 2017: Spot ref 1.0500, USD money market rate 1.112%, EUR money market rate −0.732%, ATM volatility 10.156%, 25-delta risk reversal −1.769%, 25-delta butterfly 0.313%. All of these refer to a six-months maturity (182 days). We assume all market input without term structure and have no bid-ask spreads. Structure a Risk Reversal: treasurer buys a USD call EUR put with strike $K_p = 0.9875$, treasurer sells a USD put EUR call with strike K_c.

1. Determine the strike K_c in EUR-USD that would generate a value of zero of the Risk Reversal. Prepare the version with the sales margin both in TV (theoretical value), where both the options are valued with the ATM-volatility, and a second version where the values reflect the smile in the EUR-USD market. You may use parabolic interpolation (Malz [95]) for simplicity.
2. Determine a Risk Reversal that trades at zero cost but leaves the selling bank with a sales margin of EUR 3,000. Option values should reflect the smile in the EUR-USD market. You may use parabolic interpolation (Malz) for simplicity.
3. Discuss the impact of the smile on the overall conditions of the Risk Reversal. In particular, discuss which market situations typically lead to attractive conditions of a Risk Reversal from a treasurer's point of view.

4. Explain how a spot delta (premium-unadjusted) would be calculated in the model with smile, and particularly in the parabolic smile, and compare it to the TV delta (calculated with the ATM-volatility). Which delta hedge does the selling bank need to do at inception?
5. What would be the bid price of the Risk Reversal (the offer price is EUR −3,000 by assumption) if we assume a bid-offer spread of 0.20% in ATM-volatility on the trading desk? Discuss both versions, one where the spread applies to both legs of the Risk Reversal, and the other more aggressive version where the spread is applied to only one of the two legs and the other leg is quoted at choice (no spread). Note that the ATM-volatility spread translates to a spread in EUR. We assume for simplicity that the same spread in EUR is then applied to all other vanilla options, independent of their moneyness.

CHAPTER 3

Hedge Accounting

3.1 HEDGE ACCOUNTING UNDER IAS 39

In this section we will provide an overview of Hedge Accounting under IAS 39 and then test the effectiveness of a Forward Plus in a case study. This is joint work with Sebastian Krug based on [87]. A list of frequently used abbreviations can be found in Table 3.1.

3.1.1 Introduction

Globalization in business is progressing. Not only do companies deliver their products or services to many other parts of the world but investors also act on a global basis. For investors, it is absolutely necessary to obtain information about the companies in which they wish to invest. The financial reporting of the target firms helps investors reach an investment decision. But national rules for the preparation of financial statements differ and investors would need to gain knowledge about the accounting guidelines of all countries they want to invest in. Obviously this is not possible. Accounting rules that are applicable for all companies, no matter what their home country is, would simplify the whole issue. International Accounting Standards aim to give exactly this sort of financial transparency to the users of the financial reporting.

IAS 39 provides accounting rules for financial instruments. This standard has to be applied for all companies reporting under IAS from 1 January 2005 onwards.

One topic of IAS 39 deals with derivatives. Derivatives are mainly used for two purposes: speculation and hedging. In some national accounting rules, derivatives are not included in the balance sheet but only in the footnotes. This issue is critical because derivatives may influence the income of the company considerably. The most famous example of this was the collapse of Barings Bank. Nick Leeson speculated illegally with derivatives. He caused a loss of 619 Mio GBP which led to Barings' bankruptcy in February 1995. IAS 39 provides rules on how to include derivatives in companies' balance sheet and profit & loss.

> "The objective of this standard is to establish principles for recognizing and measuring financial assets, financial liabilities and some contracts to buy or sell non-financial items. Requirements for presenting and disclosing information about financial instruments are set out in IAS 32 Financial Instruments: Disclosure and Presentation." (IAS 39.1)

FX Options and Structured Products, Uwe Wystup

TABLE 3.1 List of abbreviations for hedge accounting relevant material.

AG	Application guidance
BA	Basis adjustment
BC	Basis for conclusions
Δ	delta
ΔBC	change in fair value of the basis contract
ΔHI	change in fair value of the hedging instrument
DO	Dissenting opinions
Ed.	Editor
HFV	Hedge fair value
HAC	Hedge amortized costs
IAS	International accounting standard
IE	Illustrative example
IFRS	International financial accounting standard
IG	Guidance on implementing
IN	Introduction
OTC	Over the counter
p.a.	per year
R^2	coefficient of determination
(S)FAS	(statements of) Financial accounting standards
US-GAAP	United States generally accepted accounting principles
VRM	Variance reduction measure

Hedging risks is without any doubt economically meaningful. For accounting purposes, the treatment of a hedge relationship is not easy. IAS 39 provides rules for the treatment of hedge relationships. This is the most discussed accounting standard ever. The tight prerequisites that have to be fulfilled cumulatively in particular often make companies face serious accounting problems.

Therefore, the topic of this section is to explain the most critical point in the context of hedge accounting under IAS 39 – the test for effectiveness.

Section 3.1.2 and Section 3.1.3 provide relevant basic information about financial instruments, which assets and liabilities belong to this category, under which conditions they are recognized in the balance sheet, and how they are initially and subsequently measured.

In Section 3.1.4 general topics of hedge accounting are discussed. Section 3.1.5 deals with possible methods for testing hedge effectiveness.

A case study for effectiveness using a forecast transaction, which is hedged with a shark forward, is performed in Section 3.1.6. This is a very common exemplary situation because most companies do business abroad and face foreign exchange risks. Within this framework, the question as to whether a structured foreign exchange derivative might satisfy the strict criteria of hedge accounting under IAS 39 will be answered.

3.1.2 Financial Instruments

Overview Hedge Accounting can be applied only for financial assets and financial liabilities. For this reason the following section gives the basic facts about financial instruments. In Section 3.1.3 the accounting rules, recognition, and measurement of financial instruments will be explained. The appropriate standards which deal with these topics are IAS 32 and IAS 39.

IAS 32 deals with the presentation and disclosure of financial instruments. It does not give information about the recognition or measurement of financial instruments. IAS 39 gives the recognition and measurement rules for most of the financial instruments. Exceptions that are not covered by IAS 39 are named in IAS 39.2–7.

The aim of IAS 32 is to state the significance of financial instruments to the entity's financial position and financial performance to the users of the financial reporting, see IAS 32.1. Principally the application of IAS 39 is not limited to certain types of entities, see [50], p. 226. All entities no matter what size, industry or company's legal structure are within the scope of this standard. There are exceptions for the usage of IAS 39 to some financial items. These exceptions are listed in IAS 39.2 and IAS 32.4:

- Interests in subsidiaries, associates and joint ventures that are accounted for under IAS 27, IAS 28 and IAS 31; "Investments in subsidiaries, associates and joint ventures, that are consolidated, equity accounted or proportionately consolidated under IAS 27, IAS 28 and IAS 31 respectively [...] are excluded from the scope of IAS 32 and IAS 39." Nevertheless IAS 39 applies to derivatives on an interest in a subsidiary, associate or joint venture if the derivative does not meet the definition of an entity's equity instrument. IAS 39 also applies to derivatives that are held by the reporting entity on interests in subsidiaries, associates and joint ventures that are not owned by the reporting entity but by another party, see [46], p. 204.
- Rights and obligation under leases that are recognized and measured under IAS 17 are not regulated by IAS 39.

  However, lease receivables recognized by the lessor as well as finance lease payables recognized by the lessee are subject to the de-recognition under IAS 39. For the lease receivables the impairment under IAS 39 also has to be applied, see [46], p. 203. Derivatives that are embedded in leases are also subject to an application of IAS 39 (see IAS 39.2(b)(iii)). Financial leases are defined as financial instruments and fall therewith into the scope of IAS 32 (IAS 32.AG 9).
- Employers' rights and obligations under employee benefit plans that apply to IAS 19.
- Rights and obligations arising under insurance contracts.

  However, insurance contracts do not meet the regulation of IAS 32 and IAS 39, it does not mean that insurance companies do not have to apply those standards. These companies have to apply the standards to all their financial instruments that do not meet the definition of an insurance contract as it is described in IFRS 4. Derivatives that are embedded into an insurance contract that is not an insurance contract itself fall again into the scope of IAS 39 (IAS 32.4 (d)).
- Financial instruments issued by the entity including options and warrants that meet the definition of an equity instrument in IAS 32.

  However, the holder of these financial instruments has to apply IAS 39, if the instruments do not fulfill the classification as interest in a subsidiary, associate or joint venture.
- Financial guarantees including letters of credit and other credit default contracts that provide insurance against the default of a specified debitor.

However, financial guarantees that compensate in reaction to changes in a specified interest rate, commodity price, credit rating, foreign exchange rate or other underlying variables fall into the scope of IAS 39 because they have the characteristics of a derivative, see IAS 39.3 or [103], p. 164.

- Acquirers' contracts for contingent consideration in a business combination.
- Contracts that require a payment based on climatic, geological or other physical variables (IAS 39.AG 1).

  As those contracts have the characteristics of an insurance contract where the transfer of the financial risk is not automatically included, they are not in the scope of IAS 39.
- Loan commitments that cannot be settled net in cash or another financial instrument.

  These loan commitments are not within the scope of IAS 39 as a special rule to IAS 39.4.

In IAS 39.5 and IAS 32.8 it is mentioned that contracts to buy or sell a non-financial item that can be settled net in cash or another financial instrument, or by exchanging financial instruments, shall be treated under the mentioned standards. But this is the case only if the contract does not have the purpose of covering the receipt or delivery of a non-financial item with the entity's expected purchase, sale, or usage requirements, so-called *regular way contracts*. Further details and requirements on this issue are mentioned in IAS 32.9 and IAS 39.6 as well as in IAS 39, BC 24.

General Definition A financial instrument is according to IAS 32.11 "[...] any contract that gives rise to a financial asset of one entity and a financial liability or equity instrument of another entity" (IAS 32.11).

Within this context an entity can be an individual, a partnership as well as an incorporated body, a trust or a government agency (IAS 32.14).

Such a contract is an agreement between two or more parties that has clear economic consequences which can usually be enforceable by law (IAS 32.13).

The term financial instrument means primary instruments and derivative instruments. A financial instrument has to contain a contractual obligation. Other obligations that are not due to a contract can therefore not be a financial instrument, e.g. a tax asset or tax liability.

Physical assets, leased assets and intangible assets are not financial assets, because the control over one of these assets creates the possibility to generate cash out of this asset, but it does not give the enforceable right to receive a benefit in terms of cash or another financial asset.

Prepaid expenses are also not financial assets (IAS 32.AG 10, IAS 32.AG 11 and IAS 32.AG 12). The Guidance on Implementing IAS 39 also clarifies that physical precious metals do not belong to the financial instruments, because there is no enforceable right to receive cash or another financial asset in exchange of that metal (IAS 39.IG B1).

Financial Assets A financial asset is defined as any asset that is

- cash;
- an equity instrument of another entity;

- a contractual right such as
  1. to receive cash or another financial asset from another entity; or
  2. to exchange financial assets or financial liabilities with another entity under conditions that are potentially favorable to the entity; or
- a contract that will or may be settled in the entity's own equity instruments and is
  1. a non-derivative for which the entity is or may be obliged to receive a variable number of the entity's own equity instruments; or
  2. a derivative that will or may be settled other than by the exchange of a fixed number of the entity's own equity instruments (IAS 32.11).

Common examples are trade accounts receivable, notes receivable, loans receivable, and bonds receivable. These examples are mentioned in the *Application Guidance* of IAS 32 (see IAS 32.AG 4).

Financial Liabilities A financial liability is any liability that is

- a contractual obligation:
  1. to deliver cash or another financial asset to another entity; or
  2. to exchange financial assets of financial liabilities with another entity under conditions that are potentially unfavorable to the entity; or
- a contract that will or may be settled in the entity's own equity instrument and is:
  1. a non-derivative for which the entity is or may be obliged to deliver a variable number of the entity's own equity instruments; or
  2. a derivative that will or may be settled other than by exchange of a fixed amount of cash or another financial asset for a fixed number of the entity's own equity instruments (IAS 32.11).

Common examples are trade accounts payable, notes payable, loans payable, and bonds payable. These examples are mentioned in the Application Guidance of IAS 32 (IAS 32.AG 4).

Liabilities like deferred revenue in advance, prepaid expenses, and most warranty obligations are not financial liabilities (see [46], p. 205). This is due to the fact that the outflow of economic benefits associated with these items is more the delivery of goods and services rather than a contractual obligation to pay cash or another financial asset (IAS 32.AG 11).

Offsetting of Financial Assets and Financial Liabilities As stated in IAS 1.33, it is generally not allowed to set off assets and liabilities. Assets and liabilities should be presented separately from each other, unless another International Accounting Standard requires an offsetting (IAS 1.33). This requirement is given in IAS 32:

> "A financial asset and a financial liability shall be offset and the net amount presented in the balance sheet when, and only when, an entity:
>
> **(a)** currently has a legally enforceable right to set off the recognized amounts;
> **(b)** intends either to settle on a net basis, or to realize the asset and settle the liability simultaneously" (IAS 32.42).

This regulation does not restrict the number of financial instruments to offset. It speaks of *two or more separate financial instruments* (IAS 32.43). Further, the right to offset has to be a legally enforceable right. The pure intention of one or both parties is not sufficient for offsetting. In the other case of a legally enforceable right to settle on a net basis but without the intention of doing so, the entity has to present the instruments separately and has to disclose the effect on the entity's credit risk exposure in its notes (IAS 32.46 and IAS 32.47).

IAS 32.49 gives conditions when offsetting is inappropriate and therefore forbidden:

- several different financial instruments are used to imitate the features of a single financial instrument (synthetic instrument);
- financial assets and financial liabilities have the same primary risk exposure, but involve different counterparties;
- (financial) assets are pledged as collateral;
- financial assets are set aside for the purpose of discharging an obligation (for example a sinking fund arrangement);
- obligations are expected to be recovered from a third party under an insurance contract (IAS 32.49).

If settlement is done simultaneously by a clearing house, the cash flows may be seen as a single net amount and are therefore allowed to be set off (IAS 32.48). A master netting agreement does not provide the right to offset unless the general criteria mentioned under IAS 32.42 are met (IAS 32.50).

Equity Instruments

> *"An equity instrument is any contract that evidences a residual interest in the assets of an entity after deducting all of its liabilities." (IAS 32.11)*

Equity instruments include shares, warrants and other items that do not bear a contractual obligation for the issuing entity to deliver cash or another financial asset or to exchange financial assets under potentially unfavorable conditions. These items are excluded from the scope of IAS 39 for the issuing entity. This is not the case for the holder of such items, and IAS 39 applies (see IAS 39.2 (e)).

A financial instrument that was issued by an entity is only allowed to classify this instrument as an equity instrument rather than a financial liability if, and only if, the following two conditions are simultaneously met.

(a) "The instrument includes no contractual obligation:
   a. to deliver cash or another financial asset to another entity; or
   b. to exchange financial assets or financial liabilities with another entity under conditions that are potentially unfavorable to the issuer.
(b) If the instrument will or may be settled in the issuer's own equity instruments, it is:
   a. a non-derivative that includes no contractual obligation for the issuer to deliver a variable number of its own equity instruments; or

b. a derivative that will be settled only by the issuer exchanging a fixed amount of cash or another financial asset for a fixed number of its own equity instruments" (IAS 32.16).

The issuer of a financial instrument has to take care to classify the instrument in accordance with the substance of the contractual arrangement and the appropriate definitions when distinguishing between financial asset, financial liability or equity instrument for first time recognition ([103], p. 170 and IAS 32.15). It is important to note that the substance of the financial instrument rather than its legal form governs its classification on the entity's balance sheet. Substance and legal form are not always consistent (IAS 32.18). Sometimes a financial instrument has the legal character of an equity instrument and the economic character of a financial liability or vice versa. An example for this inconsistency of legal form and substance is a preferred share, see IAS 32.18 (a) for further explanation.

In the case that an entity is able to settle its contractual obligation by delivering its own equity shares, it depends on the arrangement of the delivery of the shares whether this contract has to be recognized as equity or liability. If the payment is a fixed amount of equity shares the contractual obligation is treated as equity. In the case that the amount of equity shares that are required to fulfill the obligation vary with the changes in the fair value of the shares, the contractual obligation is treated as a liability (IAS 32.21).

Compound Financial Instruments The issuer of a non-derivative financial instrument has to evaluate whether it contains an equity instrument component and a financial liability component. If this is the case, the components shall be classified separately as financial liability, financial asset or equity instrument in accordance with IAS 32.15 (see IAS 323.28).

An example for such an instrument is a convertible bond, that entitles the holder to receive a fixed amount of equity instruments of the issuer in exchange for the bond. In this case the first component is a financial liability to pay cash or other financial assets to the holder of the instrument. The second component is a call option on equity shares of the issuing entity (see IAS 32.29). The classification into the two components also has to be maintained in the case that the probability of an exercise of this call option has changed. The contractual obligation to fulfill the payments remains until it is sunk through conversion, maturity, or some other transaction (IAS 32.30). In order to allocate the initial carrying amount of the instrument to the components, the issuer has to determine first the value of the liability and then in a second step subtract this value from the overall value of the combined instrument. The residual is the initial carrying amount of the equity instrument component (IAS 32.31). No gain or loss should arise from the process of splitting the initial value of the combined instrument into its components.

Derivatives In general a derivative is an instrument whose value is determined by changes of the underlying asset or variable. Derivatives are mainly used to give protection against changes in commodity prices, interest rates, or exchange rates. They are of great importance to modern risk management. Derivatives can also be used for speculation purposes. The key characteristic of a derivative is the leverage effect it can offer which can result in extraordinary gain or losses combined with the characteristic of only a small initial investment.

The recognition of derivatives on the balance sheet has been a major issue for the standard setters. Specifically they should be displayed in a way that gives a fair view of the economic situation. Historically, derivatives are often treated as off-balance-sheet transactions in many national accounting standards. The fact that derivatives often have an initial value of near to zero as well as the fact that many accounting policies display unrealized losses but not unrealized gains does not simplify the situation.

Derivative instruments can occur as conditional or absolute derivatives. Futures, forwards, and swaps are examples for absolute derivative instruments, because the obligations out of the derivative have to be fulfilled by both parties at the negotiated conditions. Options are conditional derivative instruments. The option holder has the right but not the obligation to demand for fulfillment of the contract (see [44], p. 42).

Definition 3.1.1 *In IAS 39.9 a derivative is defined as following:*
A derivative is a financial instrument or other contract within the scope of this standard [. . .] with all three of the following characteristics:

(a) Its value changes in response to the change in a specified interest rate, financial instrument price, commodity price, foreign exchange rate, index of prices or rates, credit rating or credit index, or other variable, provided in the case of a non-financial variable that the variable is not specific to a party to the contract (sometimes called the underlying*).*
(b) It requires no initial net investment or an initial net investment that is smaller than would be required for other types of contracts that would be expected to have a similar response to changes in market factors.
(c) It is settled at a future date.

Typical examples of derivative contracts are presented in Figure 3.1.

For the definition of a derivative there is no difference whether the contract is settled on a gross basis or on a net basis. For the example of an interest rate swap this means, that there is no difference between whether the parties pay the interest payment to each other or if settlement takes place on the net basis (IAS 39.IG B3).

For clarification it is important to mention that the term *underlying* as mentioned in Definition 3.1.1 of a derivative in the respective standard does not refer to an asset or liability in the balance sheet. It is a variable that creates changes in the value of a contract (see [46], p. 205).

Another aspect that is mentioned in IAS 39.9 is the “smaller” initial net investment. There is no further quantification of the term “smaller.” It can be interpreted as an amount relative to the investment that would be required to do a direct investment in a primary instrument that has the same or similar characteristics as the derivative (see [46], p. 205). The margin requirements that have to be met for derivatives like futures do not count to the net initial investment amount as they have the characteristic of a collateral (IAS 39.IG B10). If the net initial investment for a derivative is almost equal to the direct investment, it could be problematic to meet the requirements

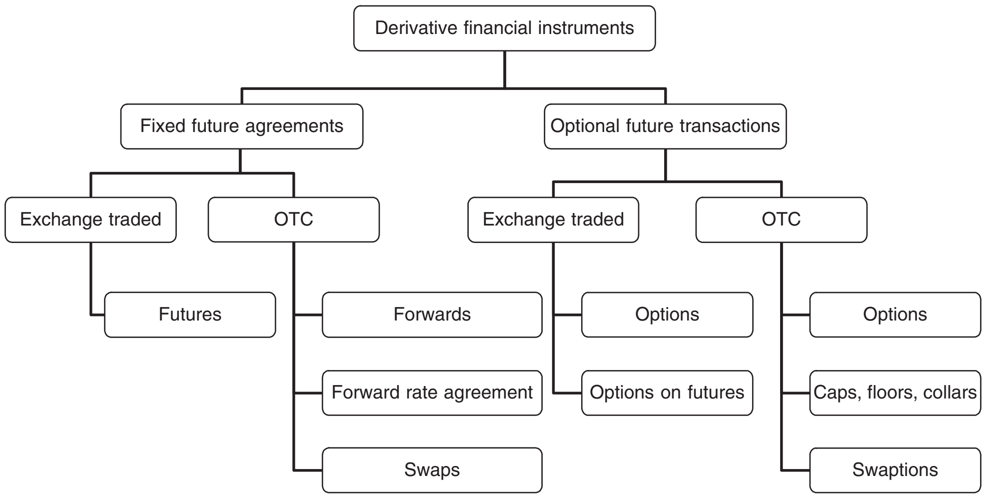

FIGURE 3.1 Typical derivative contracts, see e.g. [4], p. 137.

of a derivative according to IAS 39. IAS 39.IG B9 states an example for not meeting the requirement. KPMG also states their interpretation of this specific topic in [46], p. 208:

> In the case that a call option may have a very low exercise price so that the option premium paid is nearly equivalent to the amount that would be paid to acquire the underlying asset outright instead of the option, the derivative does not meet the requirements according to IAS 39 to have a small net initial investment. Such options should be treated as a purchase of the underlying asset and not as derivative.

Another example for the possible failure of the "smaller net investment test" is stated in the *Guidance on Implementing of IAS 39*. This example refers to a partial prepayment within a swap contract (see IAS 39.IG B4 and B5).

Concerning the third requirement of IAS 39.9 "settled at a future date" the settlement can take different forms. Even if an option is likely not to be exercised, for example because it is deep out-of-the-money it meets the requirement of future settlement. Expiration at maturity is also a form of settlement.

An exemption of the derivative treatment can occur if there is a commitment to buy or sell non-financial items. Usually these contracts are treated as derivatives unless the contract was entered into for the entity's purchase, sale, or usage requirements. This was discussed earlier in Section 3.1.2 referring to IAS 39.5.

Regular way contracts are contracts that will be settled within a time frame that is regulated or established by market conventions. These contracts are not treated as derivatives in the time between trade day and settlement date. In the event that there is a delay in the settlement procedure, IAS/IFRS do not give any guidance as to whether to treat the contract as derivative or not.

KPMG states in [46], p. 207 that

> In our view, a delay would not preclude the use of the regular way exemption if the contract requires delivery within the time frame established by the convention in the market, and the delay is caused by a factor that is outside the control of the entity.

Following this argument, the treatment as derivative has to be applied between trade day and settlement date if the delay is caused by the entity or if the time period between trade and settlement negotiated between the entities deviates from the normal settlement period.

Embedded Derivatives An embedded derivative is a part of a hybrid instrument which also contains a non-derivative host contract. Typical for these hybrid instruments is the fact that a part of the cash flows of the combined instrument varies in a way that is similar to a stand-alone derivative (IAS 39.10). A derivative that is attached to another financial instrument and that is independent on a contractual basis or has another counterpart is not an embedded derivative but an independent derivative, see [103], p. 177. In order to ensure that the principle of measuring derivatives at fair value is not avoided, the embedded derivative has to be detached from the host contract and has to be treated on a stand-alone basis if certain conditions are fulfilled.

An embedded derivative shall be separated from the host contract and accounted for as a derivative under this standard if and only if

(a) the economic characteristic and risks of the embedded derivative are not closely related to the host contract;
(b) a separate instrument with the same terms as the embedded derivative would meet the definition of a derivative; and
(c) the hybrid instrument is not measured at fair value with changes in fair value reported in income (IAS 39.11).

If an entity is required to separate an embedded derivative from its host contract but is not able to measure the fair value of the derivative separately at acquisition or at subsequent reporting dates, it shall treat the whole hybrid instrument as a financial asset or financial liability that is held for trading, see IAS 39.12. The classification of financial instruments will follow in Section 3.1.2.

In the case that the entity is unable to determine reliably the carrying amount of the embedded derivative on the basis of its term of conditions, then the carrying amount of the embedded derivative is the difference between the carrying amounts of the combined instrument and the host contract (IAS 39.13).

An embedded derivative that does not have the character of an option and that has to be detached from the host contract is to detach in a way that the fair value of the derivative is zero at first recognition. If the embedded derivative has ceteris paribus the character of an option, the fair value of the host instrument is the residual amount after separating the embedded derivative (IAS 39.AG 28). In the case that a hybrid instrument contains more than one embedded derivative they shall be treated like a single compound derivative. This is not the case if the derivatives are independent, refer to different risk exposure, and can be separated from each other. In that case the derivatives shall be recognized not only separated from the host instrument but also separated from each other (IAS 39.AG 29). In the *Application Guidance of IAS 39*, several examples are given for not closely related economic characteristics and risks and closely related respectively.

The not closely related are:

- an investment in a note or bond that is convertible into shares of the issuer, or another entity;
- an option to extend the remaining term of a debt instrument at an interest rate that is unequal to the market rate at the time of extension;
- a call, put, or prepayment option in a debt instrument that is exercisable at an amount other than the amortized cost of the instrument;
- equity- or commodity-indexed principal or interest payments;
- an instrument that the holder has an option to put back to the issuer for an amount based on an equity or commodity price or index;
- an equity instrument that the issuer has an option to call;
- an embedded credit derivative that allows the holder to transfer the credit risk of an asset to another party, see [46], p. 233 and IAS 39.AG 30;
- an embedded foreign currency derivative is not closely related if:
  1. the currency is the functional currency of one of the parties to the contract;
  2. the currency is routinely used in international commerce for that good or service; or
  3. the currency is commonly used in business transactions in the economic environment in which the transaction takes place (IAS 39.AG 33(d)).

The closely related are:

- an interest rate derivative that changes the interest payable on a debt instrument, if it could not increase the holder's initial return by more than twice what it would have been without the derivative and does not result in a rate of return that is double or more the market return for an instrument with the same terms as the host contract;
- a fixed rate note with an embedded fixed or floating swap;
- an option to extend the maturity of debt at market rates at the time of extension;
- a call, put or prepayment option at amortized cost in a debt instrument;
- an embedded cap (floor) on an interest rate or the purchase price of an asset, provided that the cap (floor) is out-of-the-money when it is issued and not leveraged;
- a prepayment option in an interest-only or principal-only strip, as long as the original financial instrument did not contain any embedded derivative and the strip does not contain any terms not originally present in the host contract;
- certain inflation-linked lease payments;
- a foreign currency derivative that provides interest or principal payments denominated in a foreign currency;
- a foreign currency derivative that provides finance lease payments in a foreign currency provided that the embedded foreign currency derivative is not leveraged and does not contain an option feature;
- a natural gas supply contract that is indexed to another energy source, if there is no spot price for natural gas in the environment in which the entity operates, see [46], p. 233 and IAS 39.AG 33.

Entities may have a problem in identifying embedded derivatives, especially those that are not included in financial instruments. The requirements related to the recognition of embedded derivatives will cause problems for entities adopting IFRS for the first time. Often the entity will not be aware of the identification of embedded derivatives and will not have the knowledge of complex hybrid instruments (see [40]).

Classification of Financial Instruments All financial instruments are classified into one of the following categories on initial recognition. IAS 39 distinguishes between four categories of financial assets and two categories for financial liabilities.

Financial assets:

- Financial assets or financial liabilities at fair value through profit and loss.
- Held-to-maturity investments.
- Loans and receivables.
- Available-for-sale assets.

Financial liabilities:

- Financial assets or financial liabilities at fair value through profit and loss.
- Other liabilities.

Financial Assets or Liabilities at Fair Value through Profit and Loss The category "financial asset or financial liability at fair value through profit and loss" contains two sub-categories. The first one is any financial asset or financial liability that was designated to this category when it was initially recognized. The second sub-category includes all financial instruments that are held for trading purposes.

A financial asset or financial liability is classified as held-for-trading if it is:

1. "acquired or incurred principally for the purpose of selling or repurchasing it in near term;
2. part of a portfolio of identified financial instruments that are managed together and for which there is evidence of a recent actual pattern of short-term profit-taking; or
3. a derivative (except for a derivative that is designated and effective hedging instrument.)" (IAS 39.9)

The term trading is explained in the *Application Guidance of IAS 39*:

> It is generally reflected by active and frequent buying and selling under the objective to generate profits from short-term price fluctuations or the dealer's margin (see IAS 39.AG 14). In other words, the turnover and the average holding period of financial assets in a portfolio indicates the trading intention (see [46], p. 209).

According to KPMG, an investment in an actively managed fund that is managed by an independent third party, would not fall into the classification of held-for trading automatically, although the fund is traded actively (see [46], p. 209).

Financial instruments that are held for a longer time period but that are part of a portfolio, which has the intention to realize short-term profits, are still held-for-trading instruments (see IAS 39.IG B11).

IAS 39.AG 15 specifies further financial liabilities that have to be classified as held-for-trading:

- "derivative liabilities that are not accounted for as hedging instruments;
- obligations to deliver financial assets borrowed by a short seller [...];
- financial liabilities that are incurred with the intention to repurchase them in the near term [...]; and
- financial liabilities that are part of a portfolio of identified financial instruments that are managed together and for which there is evidence of a recent pattern of short-term profit-taking."

A liability should not be considered as held-for-trading due to the fact that it funds trading activities.

For the sub-category of financial assets or financial liabilities that were designated to the category "financial asset or financial liability at fair value through profit and loss," there are almost no restrictions concerning the designation of different financial assets or financial liabilities to that category. One constraint is that this designation is solely possible at first recognition of an asset or liability. The designation is irrevocable afterwards, i.e. the asset or liability cannot be re-designated to another category during its life. Another limitation regarding the assets and liabilities that are able to be designated to the mentioned category exists for equity instruments that do not have a quoted market price in an active market, and whose fair value cannot be measured reliably (see IAS 39.9, IAS 39.46(c), IAS 39.AG 80 and IAS 39.AG 81). This regulation gives the companies the opportunity to measure most financial assets and financial liabilities at its fair value and to recognize profits and losses directly.

Held-to-Maturity Investments that qualify for the category "held-to-maturity" have to be non-derivative financial assets with fixed or determinable payments and fixed maturity. Further the entity must have the positive intention and ability to hold the assets until maturity (see IAS 39.9).

As demanded by IAS 39.AG 25 the entity has to assess its intention and ability not only at first recognition but at each balance sheet date (see IAS 39.AG 25). An asset with variable interest payments like a floating rate note may also qualify for this classification (IAS 39.AG 17). Consistent with the criteria mentioned above, the following instruments cannot be qualified as held-to-maturity:

- Equity securities.
- An investment that the investor intends to hold for an undefined period or that does not have fixed or determinable payments.
- An investment that the investor stands ready to sell in response to changes in market conditions.
- A perpetual debt instrument that will pay interest in perpetuity.
- An instrument that is redeemable at the option of the issuer at an amount significantly below amortized cost.
- An instrument that is putable by the holder, because the put feature is inconsistent with the intention to hold the investment to maturity.

- An asset that an entity does not have adequate resources to hold to maturity.
- An asset that is subject to legal constraints that unable the entity to hold the asset to maturity ([46], p. 210 based on IAS 39.9 and IAS 39.AG 17–25).

Further assets that are allocated to any other classification cannot be classified as "held-to-maturity" at the same time (IAS 39.9).

Consequently the following instruments may meet the definition:

- A fixed maturity debt security that bears interest at a fixed or variable rate.
- A fixed maturity debt security even if there is a high risk of non-payment, provided that the security's contractual payments are fixed or determinable and the other criteria for classification are met.
- A perpetual debt instrument that will pay interest for a specified period only;
- A debt instrument that is callable by the issuer, as long as substantially all of the carrying amount would be recovered if the call were exercised.
- Shares with a fixed maturity (or callable by the issuer) that are classified as liabilities by the issuer ([46], p. 210).

The entity is not allowed to classify any financial asset to the "held-to-maturity" group if it has reclassified or sold more than an insignificant amount of "held-to-maturity" investments before maturity. This restriction applies for the current financial year as well as the two preceding years. This means if the entity has reclassified more than an insignificant amount in the year zero, it is allowed to classify assets again in the year three to the "held-to-maturity" group. But there are exceptions that do not fall into this "tainting rule," as follows:

- When the asset was sufficiently close to maturity or the asset's call date that changes in market interest rates no longer had a significant effect on the asset's value.
- Sales after collecting substantially all of the principal.
- Sales due to an isolated non-recurring event that is beyond the entity's control and which it could not reasonably have anticipated ([46], p. 211).

If a sale or reclassification results in tainting, all assets that are currently in the "held-to-maturity" group must be reclassified as available for sale until the classification is possible again within the third year after tainting (see IAS 39.52).

IAS 39 does not give further definition of what is "more than insignificant". An appropriate range could be 10–15% of all "held-to-maturity" assets, see [6], p. 411.

Loans and Receivables *Loans and receivables* are non-derivative financial assets with fixed or determinable payments that are not quoted in an active market (IAS 39.9). Excluded from this category are:

- "Loans and receivables" that are quoted in an active market.
- "Loans and receivables" that are actively and frequently purchased or originated and sold with the intention of generating profit from short-term fluctuation in price or dealer's margin.

- "Loans and receivables" for which the entity may not recover substantially all its initial investment for reasons other than credit deterioration.
- "Loans and receivables" that are designated as "fair value through profit and loss" or "available-for-sale" ([46], p. 212).

Common examples for "loans and receivables" are trade receivables, loan assets, deposits held in banks and non-listed debt instruments (IAS 39.AG 26).

Available-for-Sale This category includes all financial assets that are not assigned to any of the previous mentioned categories. Financial assets that the entity intends to hold to maturity or a loan or a receivable are also allowed to be classified in this category on initial recognition ([46], p. 212).

Other Liabilities All liabilities that are not "held-for-trading" nor categorized "at fair value through profit and loss" fall into the category "other liabilities or non-trading liabilities."

3.1.3 Evaluation of Financial Instruments

In this section we deal with the question how financial assets and financial liabilities shall be recognized. Further, if recognition has taken place, we will illustrate which value has to be applied on the balance sheet at initial recognition and for the subsequent balance sheet dates.

Initial Recognition

> *"An entity shall recognize a financial asset or financial liability on its balance sheet when, and only when the entity becomes a party to the contractual provisions of the instrument." (IAS 39.14)*

The connection between recognition and the contractual rights or obligations has the effect that all rights or obligations arising from derivatives have to be recognized on the balance sheet. An exception from this rule are derivatives that prevent a transfer of financial assets from being accounted for as a sale (IAS 39.AG 34).

As the transfer does not qualify for de-recognition for the transferring party, the other party is not allowed to recognize the financial item (IAS 39.AG 50).

The transfer of cash from one party to another party as collateral for another transaction between these parties leads to a de-recognition of the transferring party and a recognition of the receiving party of the collateral as asset (IAS 39.IG D1).

A non-derivative financial instrument that meets the definition of a "regular way purchase or sale" shall be recognized and de-recognized either on the trade day or on the settlement date (see IAS 39.38). Trade day specifies the day on which an entity commits itself to buy or sell an asset. Settlement day is the date the asset is delivered to or by an entity (see IAS 39.AG 53, IAS 39.AG 55–56).

The chosen method has to be applied consistently to all assets and liabilities that belong to this category. It is important to note that for this purpose the sub-categories "held-for-trading" and assets or liabilities designated "at fair value through profit and loss" form a separate category (IAS 39.AG 53).

A contractual right or obligation that permits net settlement of a change in the value of the contract does not belong to the "regular way contracts". Such contracts are accounted for as a derivative in the time period between trade day and settlement day (IAS 39.AG 54). IAS 39 distinguishes further between unconditional rights and obligations and firm commitments that are conditional on other obligations. Unconditional receivables and payables are recognized as an asset or a liability directly at the time an entity becomes party to a contract. In the case of a firm commitment to purchase or sell goods for instance, these goods are not recognized until at least one of the parties has performed its part of the agreement, i.e. one party has paid for the goods or the other has shipped or delivered (IAS 39.AG 35 (a) and (b)).

As mentioned in the beginning of this section, derivatives are recognized at the moment an entity becomes a party to a contract. This means for derivatives with option character that they are recognized with their value. However, forward contracts are often negotiated in a way that the net fair value of the rights and obligations of this contract is zero. If the net fair value is unequal to zero, the forward contract is recognized as asset or liability. Consequently, as the value of the underlying may change during the time of the forward contract, the contract may be recognized as an asset at some point in its life and as a liability at another point in its life, see IAS 39.AG 35 (c) and (d) and IAS 39.AG 66.

Planned future transactions, independent of the likelihood to take place are not recognized as an asset or a liability since there is no actual right or obligation (IAS 39.AG 35 (e)).

Initial Measurement At initial recognition of a financial asset or financial liability an entity shall measure it at fair value. Additionally, if the financial asset or liability is not at fair value through profit or loss, transaction costs that are directly attributable to the transaction shall be added to the fair value for financial assets or subtracted from the fair value for financial liabilities (IAS 39.43). Transaction costs within the framework of IAS 39 include incremental costs directly attributable to acquiring or issuing a financial instrument such as fees and commissions paid to agents, advisers, brokers or dealers, levies by regulatory agencies and securities exchanges as well as transfer taxes and duties. Not included in the transaction costs are debt premiums or discounts, financing costs or internal administrative or holding costs (IAS 39.AG 13).

For the financial instruments that are measured at fair value through profit and loss and that do not belong to the category of "held-for-trading," transaction costs are shown directly in that reporting period (IAS 39.IG E1.1).

Transaction costs that might arise at disposal of financial assets or repayment of financial liabilities are subject for consideration neither at initial measurement nor at subsequent measurement (IAS 39.48).

The fair value is generally the transaction price that corresponds to the given or received item. For a financial asset or financial liability this is mostly a quoted price in an active market. If part of the consideration given or received is for something other than the financial instrument, the fair value of the financial instrument has to be estimated using valuation techniques (see IAS 39.AG 64). If the entity has immediate access to different markets it has to invoke the most advantageous quoted price (IAS 39.AG 71). IAS 39.AG 72 extends the specification of appropriate prices to bid prices for long positions and to ask prices for short positions. If the market is not active for the entity, it has to use a valuation technique to determine the fair value. This could include recent arm's length transactions as well as discounted cash flow analysis or option pricing models. The valuation technique has to incorporate all factors market participants would consider for pricing and should be consistent with general accepted pricing methodologies for pricing financial instruments (IAS 39.AG 74–75). Generally, it is not appropriate to recognize any gain or loss on the initial recognition of a financial instrument since the best evidence of the fair value is presumed to be the transaction price.

For equity instruments that do not have a quoted market price and for which other reasonable estimations for the fair value cannot be applied to get a reasonable fair value, the equity instrument is measured at cost less impairment. Similar procedures are used for the measurement of derivative financial liabilities that can only be settled by physical delivery of such unquoted equity instruments (see IAS 39.46, IAS 39.AG 80).

Subsequent Measurement For subsequent measurement IAS 39 proposes a so called "mixed model" approach, where depending on the classification into a category the method of subsequent measurement varies. For hedged items there are special regulations which will be examined later. As financial assets and financial liabilities are treated differently under IAS 39, their subsequent measurements will be discussed separately in the following.

Subsequent Measurement of Financial Assets For subsequent measurement of financial assets, these items are classified according to the categories defined in IAS 39.9. Particularly, these are

- financial assets at fair value through profit and loss;
- held-to-maturity investments;
- loans and receivables; and
- available-for-sale financial assets (IAS 39.45).

Financial assets that are measured at fair value through profit and loss are measured subsequently at fair value (IAS 39.46). As the name of the category leads to suppose, all changes in the fair value, i.e. the realized as well as the unrealized, are recognized in the income statement at once (IAS 39.55 (a)). This category is composed of assets "held-for trading," derivative assets and those assets that were designated to this category at initial recognition.

Subsequent to initial measurement "held-to-maturity" investments are measured at amortized cost using the effective interest method. The carrying amount reported as amortized cost is the initially measured amount at initial recognition minus principal repayments, cumulative amortization and any reduction for impairment. The effective interest rate is the rate that allocates the interest income over the relevant period so that it exactly discounts the estimated future cash payments through the expected life of the financial instrument (IAS 39.9).

Gains and losses of "held-to maturity" investments are recognized in profit or loss when the item is de-recognized or impaired and through the amortization process.

"Loans and receivables" are treated the same way as the "held-to-maturity" investments at amortized cost using the effective interest method (IAS 39.46).

"Available-for-sale" assets are measured at fair value. Differently to the "at fair value through profit or loss" category, gains and losses arising as a result of re-measurements are shown directly in equity. This is not the case for impairment losses and foreign exchange gains or losses. At de-recognition the cumulative gain or loss that was previously recognized in equity shall then be recognized in profit or loss (IAS 39.55 (b)).

Investments in equity instruments that do not have a quoted price in an active market and whose value cannot be measured reliably shall be measured at cost. The same measurement rule applies for derivatives that are linked to the mentioned equity instruments and must be settled by delivery of the same (IAS 39.46).

For further clarification the subsequent measurement of financial assets is summarized in Table 3.2.

Another important aspect is the treatment of value changes resulting from reclassification of financial assets.

> "Transfers to or from the fair value through profit or loss category from or to any other category are prohibited" (see [46], p. 216).

In the case of a tainting of the "held-to-maturity" portfolio, all items of this class have to be reclassified to "available-to-sale". Adjustments arising from re-measurement from amortized cost to fair value have to be recognized in equity at the date of transfer. The category of equity is the same as that used for revaluations of "available-for-sale" items (see IAS 39.51 and IAS 39.55(b)).

A transfer from the category "available-for-sale" to "held-to-maturity" is possible in the case that a previously tainting period is over or if there is a change in the intention or ability to hold the financial asset to maturity. A reclassification of measurement at fair value to cost is permitted only in the case that the fair value can no longer be determined reliably. The fair value that was measured directly prior to the transfer becomes the new "cost" in the new category. The difference between the new "cost" and the maturity amount is amortized for instruments that are carried at amortized cost. Therefore, the effective interest method has to be applied. Cumulative gains or

TABLE 3.2 Subsequent measurement of financial assets.

Financial assets	Measurement	Change in carrying amount	Impairment test
At fair value through profit & loss	Fair value	Income statement	No
Loans & receivables	Amortized costs	Income statement	Yes
Held-to-maturity investments	Amortized costs	Income statement	Yes
Available-for-sale assets	Fair value	Equity	Yes

losses that were recognized previously in equity are transferred to income divided over the period to maturity of the asset (see [46], p. 216).

Impairment of Financial Assets If there is an objective that the carrying amount of a financial asset exceeds its recoverable amount, the asset is impaired and an impairment loss has to be recognized. The objective evidence of an impairment has to be the result of an event that occurred after the asset's initial recognition and has to have an impact on the asset's estimated future cash flows. Losses that are expected as a result of future events are not recognized. At each balance sheet date it is necessary to assess whether there is objective evidence that any financial asset not measured at fair value through profit or loss is impaired or noncollectable (see IAS 39.58 and IAS 39.59).

> "Evidence that a financial asset may be impaired include
>
> - Significant financial difficulty of the issuer;
> - Payment defaults;
> - Renegotiation of the terms of an asset due to financial difficulty of the borrower;
> - Significant restructuring due to financial difficulty or expected bankruptcy;
> - Disappearance of an active market for an asset due to financial difficulties; or
> - Observable data indicating that there is a measurable decrease in the estimated future cash flows from a group of financial assets since their initial recognition, although the decrease cannot yet be identified with the individual asset in the group" ([46], p. 222).

It is important to note that recognition of an impairment loss is not restricted to situations that are considered to be permanent.

For a debt security, impairment takes place if there is an indication that the originally anticipated cash flows from the instrument are not recoverable. Therefore a change in the market interest rate that would result in a change of the fair value of the instrument is not an indication for impairment as long as the cash flows of the instrument are recoverable ([46], p. 222). For investments in equity instruments a significant or prolonged decline in the fair value below its cost is objective evidence of impairment (IAS 39.61). As IAS 39 does not give detailed guidance of the terms "significant" and "prolonged," one can believe that a decline in excess of 20 percent below cost should be regarded as significant and a time period of three quarters of a year should be regarded as prolonged ([46], p. 223).

If there is not sufficient data available in order to measure the appropriate impairment loss reliably, the entity should use its experience to estimate the amount (IAS 39.62).

For financial assets carried at amortized cost, that are "loans and receivables" and "held-to-maturity" investment, the impairment loss is recognized in the income statement. The impairment loss is the difference between the carrying amount and the recoverable amount. The recoverable amount is calculated by discounting the estimated future cash flows at the original effective interest rate. The carrying amount can be reduced either directly or through an allowance account (IAS 39.63). A reversal of an impairment loss for assets that are carried at amortized cost is recognized in the income statement with a corresponding increase in the carrying amount either directly or by the allowance account. The reversal is limited to the amount that does not state more than what the assets amortized cost would have been in the absence of an impairment (IAS 39.65).

For financial assets that are measured at cost the impairment loss is calculated as the difference between the carrying amount and the present value of estimated future cash flows discounted at the current market rate for similar financial assets. A reversal of impairment is not allowed (IAS 39.66).

For "available-for-sale" investments, changes in fair value are recognized directly in equity. If there is objective evidence for an impairment, the cumulative loss that had been recognized directly in equity shall be removed from equity and recognized in profit or loss. The impairment amount is the difference between acquisition cost and the current fair value less any impairment loss that was recognized in profit or loss previously. Impairment losses of investments in equity instruments shall not be reversed through profit or loss but are a revaluation and recognized again directly in equity ([46], p. 225).

For investments in debt instruments, the reversal of impairment is recognized in profit or loss if there is evidence that the increase is attributable to an event that occurred after the impairment loss was recognized (IAS 39.67–70).

Subsequent Measurement of Financial Liabilities All financial liabilities shall be measured subsequently at amortized cost using the efficient interest method. Gains and losses are recognized in profit or loss via the amortization process. Excluded from the amortized cost measurement are those financial liabilities that are measured at fair value through profit and loss. These items, including derivatives that are liabilities, are measured at fair value. Realized and unrealized gains and losses are recognized in income in the period in which they take place. Another exception are financial liabilities that arise when a transfer of a financial asset does not qualify for de-recognition and that are accounted for using the continuing involvement approach (see IAS 39.47).

De-recognition To achieve de-recognition of a financial asset either the contractual rights to the cash flows from the financial asset expire or the entity transfers the financial asset and certain criteria must be met with respect to the transfer (IAS 39.17). In that context, a transfer is characterized as the transfer of the contractual rights to receive the cash flows out of the financial asset or retaining the contractual rights to receive the cash flows but assumes a contractual obligation to pay to other recipients (IAS 39.18). The latter named sort of transfer is also called a *pass-through arrangement*. It has to fulfill certain criteria in order to meet the character of a transfer in the sense of IAS 39.17, namely:

- there is no obligation to pay amounts to the transferee unless the entity collects equivalent amounts from the original asset;
- the entity is prohibited from selling or pledging the original asset under the term of the pass-through arrangement; and
- the entity is obliged to remit all cash flows it collects without material delay, see [46], p. 227 based on IAS 39.19.

The next appropriate step after ensuring that a transfer has taken place is to examine whether the transfer qualifies to meet the criteria for de-recognition:

- If an entity transfers substantially all risks and rewards, the financial asset is de-recognized. If substantially all of the asset's risks and rewards are retained, the asset is not de-recognized.
- If some but not substantially all of the asset's risks and rewards are transferred and the control of the asset is transferred as well, the asset is de-recognized.
- If some but not substantially all of the asset's risks and rewards are transferred and the control of the asset is not transferred, the asset is not de-recognized. The entity continues to recognize the transferred asset to the extent of its continuing involvement in the asset (IAS 39.20).

The term "substantially" is not explained in a quantitative way in IAS 39. Another question to be answered is whether the entity has control over the asset. Having control is described as the transferee having the practical ability to sell the asset unilaterally without the need to impose additional restrictions on the transfer (IAS 39.23). That means that if the transferee (that is the party that has received the asset by transfer) has the ability to sell the asset it has the control and, therewith, the control is not retained by the transferring entity. As a consequence, the transferring entity has to de-recognize the asset if substantially all the risks and rewards of the asset were transferred and the entity does not have the control any more. If an entity transfers an asset entirely in a way that qualifies for de-recognition and retains the right to service the financial asset for a fee, it shall recognize the servicing contract either as servicing asset for the case that the received fee is expected to compensate for the servicing or as servicing liability in the case the fee is not expected to compensate for the servicing (IAS 39.24).

On de-recognition of a financial asset the difference of the carrying amount and the sum of the consideration received and any cumulative gains or losses that had been recognized directly in equity shall be recognized in profit or loss (IAS 39.26).

If only parts of a larger asset are transferred, the de-recognition rules according to IAS 39.27 have to be applied. In this case the carrying amount of the entire asset before the transfer is allocated between the sold and retained portions based on their relative fair values on the date of the transfer ([46], p. 229).

If a transfer does not result in de-recognition because the entity has retained substantially all the risks and rewards of ownership of the transferred asset, the entity shall continue to recognize the entire asset and shall additionally recognize a financial liability for the consideration received (IAS 39.29). The treatment of value changes in the financial liability should be consistent with those of the asset it refers to, i.e. the gains and losses are taken both into income or equity.

The extent of a continuing involvement and therewith the new carrying amount of a transferred asset that did not qualify for de-recognition depends on the extent the entity is exposed to risk, reward, and control the transferring entity has retained in the asset on the one hand and on the way the asset was measured previous to the transfer. IAS 39 provides a wide range of guidelines depending on different situations that are out of the scope of this book, see IAS 39.30–37 for further explanation.

An entity shall de-recognize a financial liability when it is extinguished, i.e. the obligation is discharged, canceled or expired (IAS 39.39). The difference between the carrying amount of the financial liability and the total consideration received shall be recorded in income (IAS 39.41). If only parts of the financial liability are sold, the entity has to allocate the carrying amount prior to the transfer on the basis of relative fair values like the method explained earlier for financial assets (IAS 39.42).

When a liability is restructured or refinanced by substantial modification of the terms, the transaction is accounted as extinguishment of the existing debt (with gain or loss) and the recognition of a new financial liability (new debt) at fair value. Terms are substantially different if the discounted present value of the cash flows under the new terms using the effective interest rate of the original instrument and including all fees differs at least 10% from the old instrument (see IAS 39.AG 62 and IAS 39.40).

3.1.4 Hedge Accounting

Overview In various business areas, e.g. financial institutions, industrial businesses, or service providers, it is normal business practice to enter into credit transactions with fixed or variable interest rates. Sometimes these or other transactions are also denominated in foreign currency. As a result the companies are exposed to various market risks. In order to steer these market risks, companies enter into derivative contracts. The process that compensates for the risks a company is involved in due to a contract, an obligation, or by even not yet accounted future transactions by entering into an opposite behaving transaction is called hedging, see [115], p. 3. Hedging can therefore be seen as a risk management strategy against certain market risks. With hedging, companies try to compensate market value movements or changes in cash flows of the hedged item by taking a position in a hedging instrument whose value or change in cash flows changes as far as possible in the opposite direction so that an offsetting of gains and losses is reached. Therewith hedging is a structure to eliminate risk but also limits the chance to participate in favorable value or cash flow movements of the hedged item, see [102], p. 4.

While hedging describes a risk management strategy, hedge accounting describes a method of accounting. It is a method that matches the applicable accounting methods of both, the hedged item and the hedging instrument that would normally be unequal, see [45], p. 14.

Hedge accounting recognizes the offsetting effects on profit or loss of changes in the fair values of the hedging instrument and the hedged item (IAS 39.85). Entities are free to use hedge accounting, but it is permitted only when strict documentation and effectiveness testing requirements are met ([46], p. 235).

The regulations concerning hedge accounting are the most discussed regulations of IAS 39. The first time adoption of this standard does not only require an examination of all existing hedging relations and their accounting treatment, but also has consequences for risk management, see [44], p. 96.

The necessity of the regulations for hedge accounting results in the dissimilar treatment of the different categories of financial instruments within IAS 39. Hedging instruments always belong to the category "trading," which is a sub-category of "financial instruments at fair value through profit and loss." Therefore these financial instruments are always measured at fair value; changes in the fair value are shown in income. The categories "held-to-maturity" and "loans and receivables" are measured at amortized cost, the category "available-for-sale" is measured at fair value but without showing the changes in fair value in income. Regulations to hedge accounting make an exception to usual accounting treatment for financial instruments. The special regulations for hedge accounting are needed to match the timing of offsetting gains and losses of the hedged item and the hedging instrument. It is important to note that within hedge accounting following IAS 39 the accounting method of the hedged item follows the accounting method of the hedging instrument and not vice versa.

The following example will clarify the purpose of hedge accounting. Suppose that an entity has a loan and a swap that hedges the risk exposure of that loan perfectly. If hedge accounting is not applied, the measurement for the loan would be at amortized cost but the measurement of the swap would be at fair value through profit and loss. In the income statement a gain in fair value of the loan would not appear, but the corresponding loss of the swap would be measured in income. Applying hedge accounting, both items would be measured at fair value through profit and loss, so that the gain resulting from the loan as well as the loss resulting from the swap would be displayed in income and offset, see [102], p. 5.

Types of Hedges IAS 39 distinguishes between three types of (micro-) hedge relationships. These are the *fair value hedge*, the *cash flow hedge*, and the *hedge of a net investment* in a foreign operation. The regulations for hedging of a portfolio of financial assets or financial liabilities (macro-hedge) will not be discussed in this book.

Fair Value Hedge A fair value hedge is a hedge of changes in the fair value of a recognized asset or liability, an unrecognized firm commitment, or an identified portion of such an asset, liability, or firm commitment that is attributable to a particular risk (IAS 39.86 (a)). IAS 39 gives some examples for fair value hedges:

- A hedge of interest rate risk associated with a fixed rate interest bearing asset or liability.
- A hedge of a firm commitment to purchase an asset or incur a liability (see IAS 39.AG 102).
- A forward currency contract hedging a foreign currency receivable or payable, including debt.

A hedge of a foreign currency risk on a firm commitment may be accounted for as fair value hedge or as cash flow hedge (IAS 39.87).

In the case of a fair value hedge, hedge accounting accelerates income recognition of value changes of the hedged item in order to match the gain or loss with the income of the hedging instrument. The hedging instrument is measured at fair value whereas the changes in fair value are recognized in income. The hedged item is also measured at fair value with respect to the hedged risk. This is also true if the hedged item is usually measured in another way. Adjustments to the carrying amount of the hedged item related to the hedged risk are recognized in income although the usual treatment would recognize the change directly in equity (IAS 39.89). Summing up the regular valuation treatment of the hedged item is overruled and adjusted so that it is in accordance with the measurement rules of the hedging instrument. It is important to note that only the changes in fair value attributable to the hedged risk are recognized in income.

For a fair value hedge of a firm commitment, with hedge accounting, a change in fair value of the firm commitment results in an asset or a liability during the time period of the hedge relationship. When the hedged transaction is finally recognized, the previous recognized amount with respect to the fair value of the commitment is transferred to adjust the initial measurement of the underlying transaction (see IAS 39.93 and IAS 39.94).

The adjustment to the carrying amount of the hedged item within a fair value hedge often results in a measurement that is neither at cost nor at fair value. It is more a mixture of both approaches. This phenomenon occurs due to the fact that the adjustment is made only for changes that are attributable to the hedged risk and not all changes in value. It occurs only during the time the item is hedged and is also limited to the extent that the item is hedged (IAS 39.90). This type of adjustment makes the identification and separate measurement of all risks factors that may influence the value of the hedge item necessary.

Within fair value hedges, any ineffectiveness is reported automatically in income, as the gain or loss of the hedged item, as well as the corresponding gain or loss of the hedging instrument, are recognized immediately in income. An ineffectiveness of the hedge results in a net position that is unequal to zero.

Cash Flow Hedge A cash flow hedge is a hedge of the exposure to variability in cash flows that is attributable to a particular risk associated with recognized asset or liability or a highly probable forecast transaction and that could affect profit or loss, see [46], p. 246. Examples of common cash flow hedges are given in the following list:

- Hedges of floating rate interest-bearing instruments.
- Hedges of currency exposure on foreign-currency denominated future operating lease payments.
- Hedges in highly probable forecast transactions.

The probability of forecast transactions is not defined in IAS 39; however, a likelihood of 80 to 90 percent gives a common interpretation.

Abstractly forecast transactions fit better to the fair value hedges. However, they are treated as cash flow hedges since the treatment as fair value hedge would lead to the recognition of an asset or liability before the entity becomes party of a contract, which would actually not meet the definition of the IFRS framework.

Within a cash flow hedge, the hedging instrument is measured at fair value where the effective portion of changes in its fair value is recognized directly in a separate component of equity. This is called the hedging reserve. Any ineffectiveness of the hedging instrument is recognized in profit or loss (IAS 39.95). In the case that the hedged risk is a foreign currency risk and the hedging instrument is a non-derivative, the gains or losses of the foreign currency are recognized directly in equity.

The amount that is recorded in equity has to be adjusted to the lesser of absolute amounts of either

- the cumulative gain or loss on the hedging instrument from inception of the hedge; or
- the cumulative gain or loss in fair value of the expected future cash flows on the hedged item from inception of the hedge (IAS 39.96 (a)).

In order to reduce complexity for accounting of forecast transactions, IAS 39 allows basis-adjustments. The carrying amount of a non-financial asset or liability can be adjusted by the prior accumulated amount in equity when the forecast transaction is realized. Therefore, the associated gains or losses from the hedging relationship are removed from equity and included in the initial recognition of the asset or liability, resulting out of the forecast transaction. The same rule is applicable when forecast transactions become firm commitments for which fair value hedge accounting is the appropriate hedge accounting method (IAS 39.98). The other choice is to leave the accumulated gains or losses in equity and transfer it to the income statement at the time the asset or liability affects income for example through sale or depreciation. The chosen policy must be applied consistently to all cash flow hedges (IAS 39.99). This choice is relevant only to transactions involving non-financial items. For financial assets or financial liabilities the amount deferred in equity remains there and is recognized in the period when the financial asset or financial liability affects profit or loss (IAS 39.97).

Hedges of a Net Investment A hedge of a net investment is a hedge of the currency exposure on a net investment in a foreign operation using a derivative or a monetary item, see [46], p. 247. All criteria concerning hedge accounting stated in IAS 39.88 apply equally for this kind of hedge. The hedged risk is the foreign currency exposure on the carrying amount of the net investment.

For accounting, the hedge of a net investment shall be treated similarly to the cash flow hedge. This means in detail that the hedging instrument is measured at fair value. The effective portion of the gains or losses on the hedging instrument is recognized in equity. Here it is called a currency translation reserve. Ineffectiveness is recognized instantly in income. When the net investment is sold, the cumulative amount recognized in the currency translation reserve is transferred to income and adjusts the result of disposal (IAS 39.102).

Basic Requirements Hedge accounting is tightly linked to a couple of requirements that all have to be fulfilled. These requirements are:

- There is a written documentation at the inception of the hedge that identifies the hedging instrument, the hedged item and the risk being hedged; the risk management objective and strategy for undertaking the hedge; and how effectiveness will be measured.
- The effectiveness of the hedge can be measured reliably.
- The hedge is expected to be highly effective.

- The hedge is assessed and determined to be highly effective on an ongoing basis throughout the hedge relationship.
- For a hedge of a forecast transaction, the transaction is highly probable and creates an exposure that ultimately could affect profit or loss, see [46], p. 236.

In the following subsections the most crucial facts will be discussed in more detail.

Hedging Instruments Generally only derivatives qualify as hedging instruments. But there are limitations and exceptions to consider. An exception is the hedging of foreign currency risks. Here also non-derivative financial assets or financial liabilities may qualify (see IAS 39.72). Limitations exist with regard to written options. Written options incur a potential loss that is by far greater than the potential gain in value of the hedged item. Therefore, a written option is not limiting the profit or loss potential of the hedged item. Written options can qualify as hedging instruments only in the case that they are designated to offset a purchased option. This purchased option may be part of a structured product (IAS 39.AG 94). A derivative does not have to be designated as hedging instrument at the time of first recognition. A designation might occur at a later point in the derivative's life (IAS 39.IG F 3.9). But a designation can occur only on a prospective basis; a retrospective designation is not possible. Once designated as hedging instrument to a certain hedged item the hedging instrument remains in this relationship as long as the derivative is outstanding. In other words, once designated as hedging instrument a derivative has to stay designated for the remaining time to maturity (IAS 39.75). On the other hand it is possible to hedge only a portion of the hedged item's life (IAS 39.IG F 2.17).

It is possible to designate only a portion of a derivative as a hedging instrument or the other way around it is also possible to designate more than one derivative (or even parts of them) to hedge one item. This is also true if some of the derivatives' effects offset as long as none of them is a written option (see IAS 39.75 and IAS 39.77).

A designation as hedging instrument has the prerequisite that the fair value of the derivative is measurable reliably.

One hedging instrument may be designated to hedge against more than one different risk under the presumption that the risks can be identified clearly, the effectiveness of the hedge can be demonstrated, and it must be possible to designate the derivative to the different risk positions (IAS 39.76).

Generally, a hedging instrument can only be measured at fair value as a whole. Therefore only entire derivatives or percentage portions can be designated as hedging instruments. It is assumed that the factors that cause changes in the fair value of the derivative are interdependent. Exceptions exist for options and forwards. For options it is possible to separate the intrinsic value from the time value. It is possible to exclude the time value from the hedging relationship and to consider only the intrinsic value.

Dynamic hedging strategies may qualify for hedge accounting as well, incorporating the intrinsic value and the time value. As a result of this, a delta-neutral hedging strategy may qualify for hedge accounting under the assumption that all other requirements especially the documentation of the strategy are fulfilled (IAS 39.IG F 1.9).

For forwards it is possible to separate the fair value in the interest element and the spot price. It is possible to designate only the interest element of a forward as hedging instrument (IAS 39.74).

If some terms of the hedging instrument and the hedged item differ from each other, a hedge relationship still may qualify for hedge accounting. The entity has to demonstrate that this hedge relationship is highly effective in terms of there being a strong correlation between the hedged item and the hedging instrument. Additionally, all other requirements stated in IAS 39.88 have to be fulfilled (IAS 39.AG 100).

Intra-company derivative transactions can be part of a hedge accounting relationship only if the risk associated was transferred one-to-one to an external party, so that the external transaction can be designated as hedge transaction. The intra-company transaction has the purpose of clarifying the relationship between the hedged item and the external hedging transaction. This issue is critical for banks that mainly use a centralized treasury department to allocate the risk exposure, see [42], p. 419.

Hedged Item A hedged item creates a risk exposure that will affect the income of the entity. A hedged item can be a single recognized financial asset or financial liability, an unrecognized firm commitment, a highly probable forecast transaction, or a net investment in a foreign operation. Additionally, a hedged item can be a group of above mentioned possible items with the same risk characteristics or in a portfolio hedge of interest rate risk only, a portion of the portfolio of financial assets or financial liabilities that share the risk being hedged (IAS 39.78). It is not possible to designate a derivative as hedged item. "Held-to-maturity" investments may never be a hedged item in a hedge relationship against interest rate risk or prepayment risk. This is because the entity has the intention to hold this item until maturity and changes in the fair value arising from those risks do not have to bother the entity. But "held-to-maturity" investments may be designated as hedged items for foreign exchange risk or credit rating risk (IAS 39.AG 95).

It is not possible to hedge against general business risk (IAS 39.AG 110). Further, the entity's own equity instruments, associated entities and subsidiaries cannot be the hedged item (see IAS 39.AG 99 and IAS 39.IG F 2.7).

For intra-company transactions the same rules apply as for the hedging instruments. An external hedged item that corresponds with the internal item must exist (IAS 39.80). Generally, the hedged item's fair value or change in cash flow has to be measurable reliably with respect to the hedged risk.

Macro-hedges that hedge an overall net position instead of a specified hedged item do not qualify for hedge accounting under IAS 39 (see IAS 39.AG 101). If the portfolio to be hedged involves exclusively interest rate risk and is not a net position of assets and liabilities, the portion hedge may be designated in terms of an amount of a currency instead of the individual items (IAS 39.81A).

> "It is possible to designate only a portion of the cash flows or fair value as a hedged item [. . .]. [. . .] once the partial designation is made, hedge effectiveness is measured on the basis of the hedged exposure." (See [46], p. 236 for further explanation and IAS 39.AG 99A and B)

There are no restrictions on the timing of designation or de-designation of a hedged item. Consequently it is possible to hedge an item after initial recognition and also merely for a portion of its period to maturity (IAS 39.IG F 2.17).

Coming back to portfolios as hedged items, this form of hedging relationship is solely possible if all items in the portfolio vary proportionally in response to changes in the hedged risk (IAS 39.78). Therefore it is important to take care of the items grouped in a portfolio. For loan portfolios, for example, it is necessary to group items in terms of time to maturity because long-term instruments are more volatile to changes in the yield curve than short-term instruments due to a longer duration, see [97], p. 84. Equity portfolios cannot qualify for portfolio hedging as the different equity shares do not react in the same way to the portfolio value movements (IAS 39.IG F 2.20).

Formal Designation and Documentation In order to qualify for hedge accounting, each hedging relationship consisting of hedged item and hedging instrument has to be formally designated and documented at inception of the hedge. Further, the nature of the hedged risk, the entity's risk management objective and strategy, as well as the assessment method used to prove hedge effectiveness have to be documented (IAS 39.88(a)). The documentation cannot be applied retrospectively. As a result, hedging relationships already existing that are not documented in the demanded extent do not qualify for hedge accounting.

It is important to clearly define interest rate exposure for the hedge item and the hedging instrument since changes in interest rates may result in a series of reactions, see [124], p. 271.

Hedge Effectiveness In a first step the entity has to decide against which risk it is willing to hedge the hedged item. The hedged risk must be able to affect the income statement (IAS 39.AG 110). Financial items can be hedged against exposure to a single or a combination of the single risks that are measurable. These risks include market prices, interest rates or a component of interest rates, foreign currency rates or credit rates. A non-financial item can only be hedged against either all of its risks or currency risk only (see IAS 39.82). As stated above, the hedged risk must be specific and measurable. However, hedging against general business risk does not qualify for hedge accounting (IAS 39.AG 110).

If a hedge is not perfect, the gain or loss on the hedging instrument will differ from the gain or loss on the hedged item. This difference is called *hedge ineffectiveness* ([46], p. 241). In order to qualify for hedge accounting, a hedge must be highly effective. The entity has to prove that this high effectiveness is expected to be met (prospectively) and that it is actually given (retrospectively). Only in the case when both, the **prospective** and the **retrospective** high effectiveness are given, the hedge can qualify for hedge accounting. The hedge has to be effective at inception and throughout the life of the hedge. The offsetting has to be in a range of 80–125% for the retrospective effectiveness. For prospective effectiveness there is no exact range required, but the adoption of the same range is in line with the audit companies. For the hedged item only changes in fair value or cash flows that are attributable to the hedged risk are considered for the assessment of the hedge effectiveness. This is important as the change in the full fair value may incorporate changes attributable to other than the hedged risk. Therefore the determination of the so called hedge fair value, that is the change in fair value only due to the hedged risk has to be performed in order to get the appropriate basis for assessment. IAS does not give further guidance for the determination of the hedge fair value. For the hedging instrument the full fair value has to be adopted. A separation of parts of the instrument is only possible for options and forwards as mentioned in IAS 39.74 and discussed earlier in Section 3.1.4.

In IAS 39.88 (c) the standard demands that the effectiveness of the hedge must be measurable reliably. Further, the effectiveness has to be proven on an ongoing basis. The minimum frequency to test for effectiveness is at each balance sheet date including interim reports (IAS 39.AG 106). However, it is advisable that the test is performed more often so that ineffectiveness is identified early enough and the entity can adjust or rebalance the hedge to minimize the impact of the ineffectiveness. Generally, if an entity does not meet the criteria concerning effectiveness any more, hedge accounting is discontinued from the last date on which the hedge still met the criteria of being effective (IAS 39.AG 113). This is a clear incentive to perform the test for effectiveness more often than minimally required by the standard for hedges where passing the test is assumed to be critical. Most of the hedges that fulfill the requirement regarding the effectiveness to be in the demanded range will not offset perfectly. The actual ineffectiveness must be recognized in income in the period of time in which it occurs although it is in the range of 80–125%, see IAS 39.95 (b) and IAS 39.102 (b).

Testing for hedge effectiveness can be performed on a period-by-period or a cumulative basis (IAS 39.107, IAS 39.IG F 4.2 and IAS 39.IG F 4.4). Further the entity can choose whether to assess the effectiveness on a pre-tax or after-tax basis. Testing on a cumulative basis may have the advantage that hedge accounting may be continued even if one period does not fulfill the requirement as long as previous periods compensate this ineffectiveness and the effectiveness is expected to remain over the life of the hedge relationship.

IAS 39 does not specify methods to use for measuring hedge effectiveness. The methods should be in line with the risk management strategy of the entity. The method that is applied has to be specified in the hedge documentation.

An entity may use different methods for different types of hedges.

> "Different approaches may be used to measure prospective and retrospective effectiveness for a single hedge relationship." ([46], p. 242)

If an entity hedges less than 100% of the exposure of an item, it must designate this percentage amount as hedged item and measure ineffectiveness based on the change in the designated exposure only (IAS 39.AG 107A).

Whichever method is used to determine hedge effectiveness, it should be consistent for similar transactions and over time.

IAS 39 does not specify methods to use for assessing hedge effectiveness. However, there are some methods that are commonly used. For the prospective effectiveness test these are the comparisons on a historical basis or the calculation based on sensitivities. For the retrospective effectiveness test the *dollar-offset method*, the *variance reduction measure* or the *regression analysis* are commonly used methods.

In practice, the strict limits of effectiveness will limit the number of hedges that can qualify for hedge accounting significantly since most of the hedges will not be set up as perfect hedges. Even perfect hedges in the economic view will not always meet the definition of a highly effective hedge.

Stopping Hedge Accounting Hedge accounting must be stopped prospectively if the hedged transaction is no longer highly probable, the hedging instrument expires, is sold, terminated, or exercised. Further, if the hedged item is sold, settled, or disposed in another way, or the hedge is no longer highly effective, hedge accounting must be stopped as well, see [46], p. 246. At the date the hedge accounting is stopped, the entity has to assess hedge effectiveness and report ineffectiveness in the income statement.

The hedging instrument and the hedged item are subsequently accounted for according to their usual treatment in IFRS. In the case that the hedging relationship is terminated due to ineffectiveness, hedge accounting will be stopped at the date at which effectiveness was proven the last time (IAS 39.AG 113). Therefore it could make sense to test for effectiveness more often than at each balance sheet date. If an entity identifies the event or change in circumstances that caused insufficient effectiveness for the hedging relationship and if the entity could demonstrate that the hedging relationship was effective prior to this event, the entity discontinues hedge accounting from the date of the event on (IAS 39.AG 113).

IAS 39 does not give guidance on the question whether hedge accounting is discontinued only for ending of the retrospective effectiveness or for ending of the prospective effectiveness as well.

In practice, the circumstance of failing the effectiveness test only slightly may not automatically lead to discontinuation of hedge accounting. If the ineffectiveness is merely temporary and prospective effectiveness is proven the overall effectiveness can be assumed. In this case the ineffectiveness is ignored due to insignificance, see [115], p. 11.

At the end of a fair value hedge, any adjustments to the carrying amount of the hedged item will be reversed at the time the item is sold or depreciated. If the item is normally measured at amortized cost, the adjustment is amortized to profit or loss by adjusting the hedged item's effective interest rate when amortization begins ([46], p. 248). Cumulative gain or loss recognized in equity arising from a cash flow hedge remain in equity until the transaction occurs, if the transaction is still expected to take place. If the transaction is not expected to occur any more, the cumulative gain or loss is transferred to equity at once (IAS 39.101). For cumulative gain or loss previously recognized in equity of a hedge of a net investment in a foreign operation the amount is kept in equity until the investment is sold (IAS 39.102).

3.1.5 Methods for Testing Hedge Effectiveness

IAS 39 does not give guidance on specific methods to apply for testing hedge effectiveness. It states only that the method should be in line with the risk management strategy. This may have the consequence that the method applied should be in line with the risk management strategy for a specific transaction. However, the chosen method should be applied consistently for similar transactions. The choice of the method is of great importance because the implementation may influence the result as to whether a hedging relationship is effective or not.

Fair Value Hedge According to IAS 39.AG 105 the entity has to perform a prospective test for effectiveness. Here the entity has to prove that the changes in fair value or cash flows of the hedged item and the hedging instrument will compensate in future. In this section, we discuss three possible methods to prove the prospective effectiveness.

The first method is the historical review. Within this method the fair value of the hedging instrument and the hedge fair value of the hedged item are determined based on historical data sets. For the hedged item, the hedge fair value is determined. This means that only the changes in fair value due to the hedged risk are incorporated into the item's value. If the changes in (hedge) fair value of the hedging instrument and the hedged item have compensated each other in the past, the prospective effectiveness is considered guaranteed. The availability of the required market data is critical for this method.

Another method is the calculation of sensitivities. The hedge fair value of the hedged item and the fair value of the hedging instrument are examined for the case that the hedged risk factor changes by a predefined amount. A typical example is the calculation of the basis point value for interest rate related items. If the changes in value offset to a great extent, the hedge relationship is considered to be prospectively effective, see [102], p. 13.

A third method is the *critical term match* (IAS 39.AG 108). This is not equal to the *short-cut method* that is allowed under the regulation of US-GAAP. Following US-GAAP in a hedge relationship where the term, the notional amount, the dates of interest payment etc. match perfectly, a 100% effectiveness is assumed. This is not allowed under IAS. However, for the test of prospective effectiveness, a similar alleviation is allowed in IAS. If the critical terms of the hedged item and the hedging instrument are the same, it is likely that the changes in fair value or cash flows offset entirely. This is valid for the evaluation of the prospective effectiveness at the beginning of the hedging relationship as well as during the hedging relationship. The critical term match is not permitted for the retrospective testing of effectiveness, see [115], p. 8.

For the retrospective testing of effectiveness, we now explain the *dollar-offset method* and common statistical methods.

The *dollar-offset method* sets the change in hedge fair value of the hedged item in relation to the change in fair value of the hedging instrument. The *dollar-offset method* can be applied either on a period-by-period basis or on a cumulative basis, see [49], pp. 43–44. The cumulative method may have the advantage of compensating ineffectiveness of single periods. The continuity in application of a method is of relevance.

The relevant hedge effectiveness is given if the relation

$$\text{effectiveness} = \frac{\text{change in hedge fair value of hedged item}}{\text{change in fair value of hedging instrument}} \tag{1}$$

is between -1.25 and -0.8. The result must be negative because the changes shall offset each other. The advantage of this method is that it is easy and comprehensive. This is of special importance as the entities have to prove effectiveness on their own. The auditor is not allowed to perform the effectiveness test for the entity. A disadvantage of the *dollar-offset method* is the fact that even for economically effective hedging relationships, the effectiveness requirements are not always met due to problems with small numbers. The problem occurs, if the change in value is small. Hailer and Rump discuss possible solutions for that problem in [63], pp. 599–603. Figure 3.2 illustrates the problem. We consider a system of coordinates with the change in hedge fair value of the hedged item (ΔBC) on the x-axis and the change in fair value of the hedging instrument (ΔHI) on the y-axis. For a perfect offsetting the hedging relationship is a straight line with a slope of -1 ($\Delta BC = -\Delta HI$, Figure 3.2a).

However, to be effective according to IAS 39 it is not necessary to offset 100%. The tolerance level of the effectiveness is between 80% and 125%. This means that the coordinates must be within the cones between the lines $\Delta BC = -\frac{4}{5}\Delta HI$ and $\Delta BC = -\frac{5}{4}\Delta HI$ (Figure 3.2b).

The tolerance level is very small near the intersection of these lines. One method presented by Hailer and Rump to solve this problem is an approximation of the boundary function so that the interval is unchanged for large numbers and so that the cones overlap for small numbers. This is done with the formula

$$\Delta HI = \pm 0.225 \cdot \sqrt{(\Delta BC)^2 + c} - 1.025 \cdot \Delta BC. \tag{2}$$

For the diagram in Figure 3.2c, the value of c is set equal to 10. The resulting diagram shows the positive effect of the approximation and gives a possible solution to the problem of small numbers. Figure 3.2d combines Figure 3.2b and Figure 3.2c to show the overlapping for small numbers where the area further apart from the origin stays almost unchanged.

A second common method to measure hedge effectiveness is the *variability reduction method*. The scope of this method is to examine whether a reduction in variability of the fair value is obtained through the hedging relation. For this purpose, the variability of the hedged item's fair value on a sole basis is compared with the variability of the fair value of the combined hedge position (consisting of the hedge fair value of the hedged item and the fair value of the hedging instrument), see [48], p. 103. The measure used for this comparison is the variance. The smaller the variance of the hedging relationship relative to the unhedged item, the more effective the hedge. The formula for the variance reduction is given by

$$\text{effectiveness} = 1 - \frac{\mathbf{var}(\text{hedging relationship})}{\mathbf{var}(\text{hedge fair value hedged item})}. \tag{3}$$

Effectiveness according to IAS 39 holds if the result of the *variance reduction measure* is above 0.96, see [83], p. 96.

The question whether to prefer changes solely of the period since the last measurement or the total change on a cumulative basis does not arise, since the method does not use changes but the fair value itself.

In practice, this method is not used very often, because it is more complicated and less illustrative than the *dollar-offset method*, see [43], pp. 54–55.

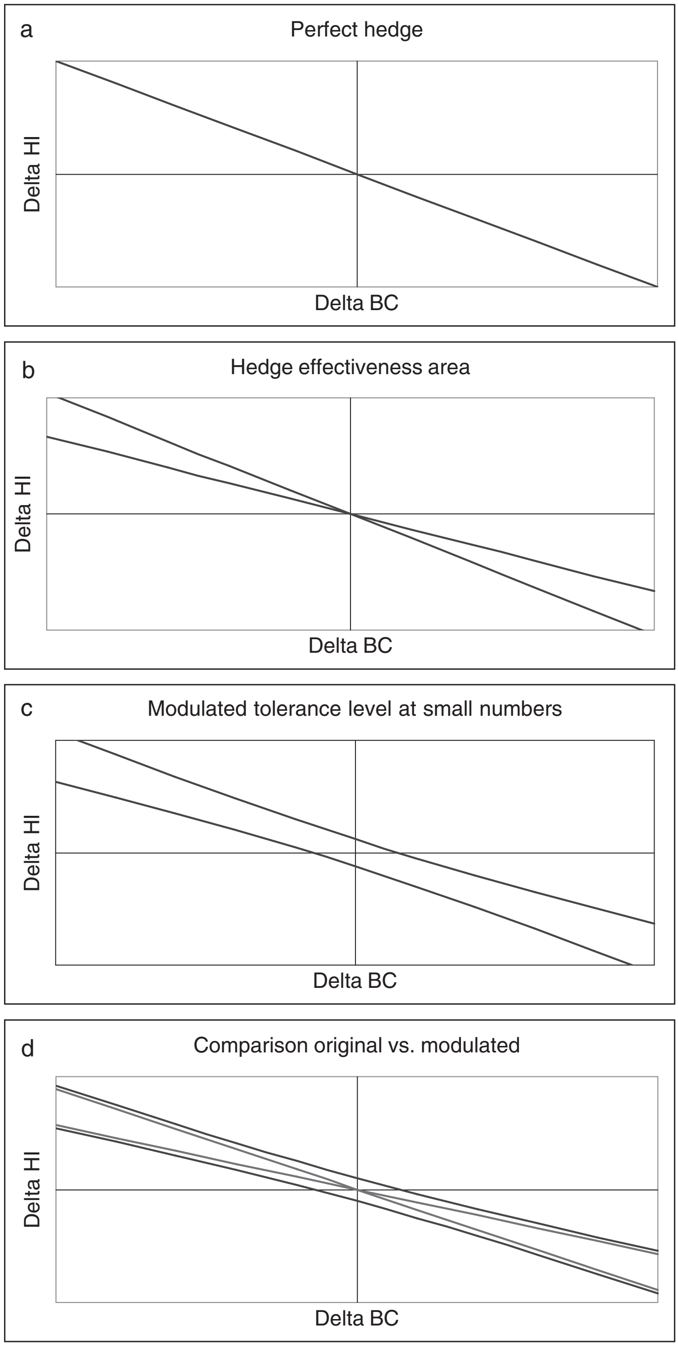

FIGURE 3.2 Dollar-offset and solution for small numbers. From top to bottom: (a) perfect hedge, (b) hedge effectiveness area, (c) Hailer/Rump tolerance bounds, (d) combination of (b) and (c).

A variation of this method uses the variability of the changes ([48], p. 103) or the standard deviation ([83], pp. 93–99) instead of the variance of the fair values.

A third very common method to determine hedge effectiveness is the *regression analysis*. It is a statistical method to test for hedge effectiveness. Within the context of a *regression analysis* the relation between a depended variable and one or more independent variables is examined. The basis for this effectiveness test is the change in value of the hedged item and the hedging instrument in the past. For testing hedge effectiveness, a linear regression is sufficient in most cases. Therefore the regression model

$$y = \alpha + \beta x \tag{4}$$

is used, where y denotes the change in fair value of the hedging instrument and x denotes the change in hedge fair value of the hedged item. The interception of the regression line with the y-axis is α. The coefficient β is the slope of the regression line. It is calculated using the correlation between y and x and the standard deviations of these two variables,

$$\beta = \mathbf{corr}(y, x)\frac{\mathbf{stdev}(y)}{\mathbf{stdev}(x)}. \tag{5}$$

In this context the interception term α is the average amount per period, by which the change in value differs between the hedged item and the hedging instrument.

The slope of the regression line should be -1 for a perfect hedge relation. As IAS does not ask for perfect offsetting, the slope may be in the interval $[-1.25; -0.8]$. Fulfilling the slope argument is not enough to prove effectiveness. Even if the scattering of values is very wide, the slope can be within the desired interval. Therefore the coefficient of determination, R^2, also plays a role in determination whether the hedge is effective. In this simple linear regression model, R^2 is the squared correlation of the underlying variables and gives an indication about the quality of how the dots fit the regression line. Generally, a coefficient of determination of at least 0.8 is considered to interpret a hedge as highly effective, see [84], p. 63.[1]

In order to achieve reliable results it is necessary to have a sufficient number of observations available as historic data. Therefore, a decision regarding the frequency of observations has to be done, i.e. daily, weekly or monthly. For statistically reliable results, a minimum of 25–30 observations should be considered ([84], p. 63). If this is not the case, for example at the beginning of a hedging relationship, the entity has to search for alternative methods. One alternative might be to generate synthetic observations of a point in time prior to hedge designation. Another alternative is the use of the *dollar-offset method* until the minimum required data set is available. For the later alternative this change in testing has to be documented prior to application to avoid failure of hedge effectiveness due to a forbidden change in methods, see [102], p. 16.

An advantage of the *regression analysis* is the better results compared with the *dollar-offset method*. Disadvantages arise through complexity and the treatment of outliers.

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[1] Note that R^2 takes values in the interval $[0, 1]$.

Balance Sheet Treatment of the Fair Value Hedge As far as the requirements for hedge accounting are fulfilled, the changes in fair value of the hedging instruments as well as the changes in fair value attributable to the hedged risk have to be reported in income in the period they occur. Changes in fair value of the hedged item lead to an adjustment in the carrying amount.

Changes in fair value of the hedged item and the changes in fair value of the hedging instrument are reported directly in income. Preferably, the entity has a line called hedging result where the result of the hedge is reported. Ineffectiveness is shown automatically as a net position as the changes in fair value of the hedge relationship are both shown in income. The portion that is not compensated, remains as net position or ineffectiveness respectively.

The *hedge fair value (HFV)* is the fair value of the hedged item that is determined on the basis of the hedged risk. Other risk factors that are not hedged do not contribute to determining the HFV. IAS 39 does not give guidance for the calculation of the HFV. The adjustment of the hedged item's carrying amount to the HFV has to be amortized recognizing the amortization in income. The amortization may begin at the earliest at designation of the hedge but not later than at the end of the hedge (see IAS 39.92).

For interest-bearing instruments one common possibility is to keep the actual credit spread at designation of the hedge constant. Therewith, the HFV at designation is the full fair value or cost. The HFV has to be determined for all hedges that have an interest rate risk as defined hedge risk. The HFV should be a clean price. Therefore it has to be adjusted eventually for accrued interest payments. For subsequent calculation of the HFV the current interest term structure is applied adjusted by the credit spread which is kept constant from designation of the hedge.

For the hedging instrument the HFV has to be calculated likewise. Applying IAS 39.AG 107, it is possible to assign only the intrinsic value of an option or the spot price component of a forward contract as HFV ([115], pp. 13–14).

In the case that equity prices or foreign exchange risks are being hedged with options or forward contracts using the possibility of omitting the time value according to IAS 39.AG 107, a distinctive feature has to be borne in mind. If the option is out-of-the-money and the time component is not included in the hedge, the option does not have a HFV and can therefore not have an offsetting effect. If the option is in-the-money, it is possible to determine the HFV which will offset fluctuations in the stock price.

For financial assets that are measured at amortized cost, the *hedge amortized costs (HAC)* have to be calculated. Therefore it is important to distinguish whether the hedge relationship was designated at first recognition of the hedged item or if the designation took place at a later point in time. For the first case, the HAC are equal to the amortized costs. For the other case, the HAC are calculated as the difference between the full fair value at designation and the repayment amount, distributed to the remaining time to maturity using the effective interest rate method. The amortization of this difference amount is not shown in income ([115], p. 15).

The *basis adjustment (BA)* is the amount by which the carrying amount of the hedged item has to be adjusted in order to reflect the gain or loss recognized in income that incurred due to the hedged risk in the framework of a fair value hedge.

The BA for the hedging of interest rate risk is calculated as the difference between the HFV and the HAC at a specific point in time.

For loans and receivables, as well as for all other liabilities that are measured at amortized cost, the carrying amount is calculated by adding the BA to the amortized cost. For securities of the category "available for sale," the carrying amount is the full fair value. The BA is added or subtracted from the amortized cost and recognized in income. For the hedging of stock prices and foreign exchange risks the calculation is much simpler. The BA is the difference between the current HFV and the HFV at the last date of evaluation.

Cash Flow Hedge As stated above, IAS 39 does not give guidance on the methods to use for testing for effectiveness. Although the *cash flow hedge* transforms variable cash flows into fixed cash flows, the way to measure effectiveness uses the fair value as in the fair value hedge. Therefore the changes in fair value of the hedging instrument and the hedged item will be examined. The methods to do this are the same as for the *fair value hedge*, namely the *dollar-offset method* or statistical methods such as the *variability reduction method* and *regression analysis*.

The American counterpart to IAS 39, FAS 133 allows a short-cut method similar to the critical term match for testing effectiveness. This method is not allowed under IAS. However, this short-cut method is not always allowed under US-GAAP either, but gives information about alternative methods to use. These methods belong to the family of *dollar-offset methods* and might be allowed under IAS for cash flow hedges as well ([115], p. 17).

The first method is the *change in fair value method*. Here the cumulative absolute change in fair value of the future cash flows of the hedged asset or liability is compared with the cumulative absolute change in full fair value of the hedging instrument. For testing effectiveness, the values are set in relation to each other. If the result is within the interval [0.8; 1.25], a retrospective effectiveness holds. An inefficient amount arises if the cumulative absolute change in full fair value of the hedging instrument is greater than the cumulative absolute change in fair value of the future cash flows. This difference amount has to be recognized in income. This difference amount may have a significant size, although the hedge relationship is perfect from an economic perspective ([115], p. 20).

Example Assume an entity has a variable interest rate liability with a notional amount of 1,000,000 (3M EURIBOR). This liability is hedged with an interest rate swap with the same notional amount so that the entity receives 3M EURIBOR and pays 5% fixed interest rate. From an economic point of view, this hedge is perfect. However, there may arise an inefficiency from an accounting perspective. The fair value of the variable interest rate liability is calculated via the actual fixed EURIBOR and the forward rates. These cash flows are discounted with the appropriate zero rates. The value of the swap is calculated as the net position of the present value of the variable leg and the fixed leg. The variable leg uses the same calculation as for the variable interest rate liability. The fixed leg present value is the fixed rate multiplied with the notional amount and discounted with the appropriate zero rates.

The change in full fair value of the swap can be greater than the change in fair value of the cash flows. This would result in an ineffectiveness according to IAS 39 and would be recognized in income.

The second method is the *change in variable cash flow method*. This method is also described in the context of a variable interest rate bearing instrument and an interest rate swap. As the method's name already hints, only the variable side is of importance. For effectiveness testing, the method assumes that only the variable leg of the swap substantiates the cash flow hedge. Therefore, changes in fair value of the swap that are attributable to the fixed leg do not play a role for the hedge of the variable interest rate cash flows arising from the hedged item. In other words, changes in the swap's fair value attributable to the fixed leg are not included in testing effectiveness of the cash flow hedge. This is in line with the definition in IAS 39, to measure effectiveness of a cash flow hedge. A limitation for the *change in variable cash flow method* is the prerequisite that the fair value of the hedging instrument must be zero or at least almost zero at designation to the hedge.

Effectiveness is tested in the way that the accumulated changes in fair value of the variable leg of the hedging instrument (swap) is compared to the change in fair value of the expected cash flows of the hedged financial instrument in future. Ineffectiveness has to be recognized in income for the case that the change in fair value of the hedging instrument relating to the variable leg is greater than the change in fair value of the future expected cash flows. If the change in fair value of the hedging instrument is smaller than the corresponding change of the cash flows, ineffectiveness is not recognized in income.

Independent from the measurement of the effectiveness, the hedging instrument is recognized with full fair value in the balance sheet. The financial instrument that generates the expected cash flows in future is accounted for following the appropriate rules normally applying to this sort of instrument, see [115], pp. 20–21.

A third method is called the *hypothetical derivative method*. Within this method the effectiveness is measured by comparing the change in fair value of a fictitious perfect hedging instrument with the change in fair value of the real hedging instrument. This fictitious hedging instrument replicates the relevant terms of the hedged cash flows with its variable leg, see [102], p. 38. Therefore the change in fair value of this instrument is the "deputy" for the cumulative changes in fair value of the expected future cash flows of the hedged transaction. The measurement of the hypothetical derivative is based on the appropriate market conditions. At inception, the fair value of the hypothetical derivative must be zero. Further, the hypothetical derivative should compensate completely the hedged cash flows.

The fair value of the real hedging instrument does not necessarily have to be zero at inception to the hedge.

Ineffectiveness has to be recognized in income in the case that the cumulative absolute change in fair value of the real hedging derivative is greater than the cumulative absolute change in fair value of the hypothetical hedging instrument.

In order to fulfill effectiveness retrospectively the relation of the cumulative change in fair value of the two hedging derivatives must be in the range between 0.8 and 1.25, see [115], p. 22.

Balance Sheet Treatment of the Cash Flow Hedge If a cash flow hedge fulfills the general requirements according to IAS 39.88, the recognition takes place as described below. The transaction on which the variable cash flows are based is recognized following the general rules. There is no difference in accounting for that instrument whether it is part of a hedging relationship or not.

The hedging instrument is measured at full fair value where the change in fair value is separated into an effective and an ineffective portion. The portion of the gain or loss on the hedging instrument that is determined to be effective is recognized directly in equity. The ineffective portion is recognized in income (IAS 39.95).

The portion of the gain or loss that is recognized in equity has to be adjusted at each balance sheet date to the smaller of the following two amounts (measured in absolute terms):

- "The cumulative gain or loss on the hedging instrument from inception of the hedge; or
- The cumulative change in fair value (present value) of the expected future cash flows on the hedged item from inception of the hedge" (IAS 39.96 (a)).

As long as the hedge is effective in terms of having an effectiveness of 80% to 125%, the ineffectiveness is treated as follows. If the cumulative gain or loss of the hedging instrument is smaller than the cumulative change in fair value of the expected future cash flows, the difference amount is recognized in equity. If the cumulative gain or loss of the hedging instrument is greater, the difference amount is recognized in income, see [102], p. 38.

The amounts that were recognized previously in equity as ineffectiveness, are de-recognized and shown in income at the time the hedged transaction is recognized on the balance sheet or in income.

3.1.6 Testing for Effectiveness – A Case Study of the Forward Plus

Having illustrated the theoretical concept of hedge accounting under IAS 39, including its requirements and possible methods, the following case study will demonstrate the practical implementation of testing for effectiveness for a *shark forward plus* introduced in Section 2.1.6. It is assumed that a EUR-zone based exporter (= USD seller) will have a forecast transaction in USD that has a worth of EUR 100,000,000. He will receive the USD amount in six months. Knowing about the variability in exchange rates, the exporter is willing to fix the exchange rate now. He is aware that he needs protection against a rising EUR-USD exchange rate. At the same time he knows that a falling exchange rate improves his position. The exporter believes that the exchange rate is going to decline moderately, i.e. move in a favorable direction. However, he is not willing to spend money for a possible better position. Therefore he enters into a forward plus contract. The included forward is not fair, i.e. the exchange rate fixed representing his *worst case*, is worse than the one he would receive by entering into an outright forward contract. On the other hand, he has the chance to participate in moderate favorable exchange rate movements. If the exchange rate drops sharply and hits the knock-out barrier, the exporter will again receive the *worst case* rate specified in the unfair forward contract.

TABLE 3.3 Specifications for the case study of the forward plus.

Spot exchange rate in time t_0	1.3000 EUR/USD
USD interest rate r_d	2.00% p.a.
EUR interest rate r_f	2.20% p.a.
Volatility σ	10.00% p.a.
Time to maturity	180 days
Day count	365 days
Notional amount	EUR 100,000,000.00

The key data used for the case study are shown in Table 3.3.

All calculations, on which the data shown here are based upon, are performed in the spread sheet named hedgeaccounting.xls, which can be found on the book web page.

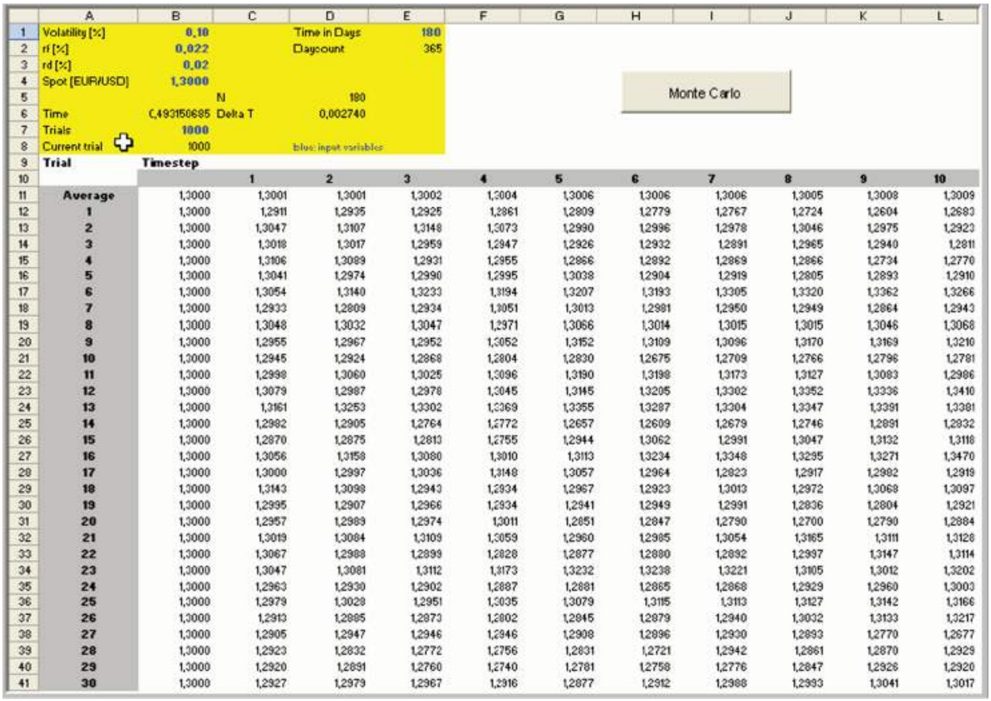
Simulation of Exchange Rates At inception of a hedge relationship, the entity has to prove that the hedge is going to be effective in the future. For a structured product like the *American style Forward Plus*, not only the exchange rate at the end of the hedge decides whether the hedge relationship is effective or not. Instead, the path itself is crucial, due to the barrier option included in the product. For this purpose a Monte Carlo simulation is performed to show possible exchange rate paths.

Therefore, the current spot exchange rate is the starting point of the simulation. The spot rate of the next observation point, that is one day later, is modeled by a geometric Brownian motion,

$$S_{t_{i+1}} = S_{t_i} \cdot \exp\left[\sigma\sqrt{\Delta t}Z + \left(r_d - r_f - \frac{1}{2}\sigma^2\right)\Delta t\right], \tag{6}$$

where $S_{t_{i+1}}$ represents the next simulated spot exchange rate, S_{t_i} the current spot rate, r_d and r_f denote the two local interest rates for the USD and the EUR respectively as specified in Table 3.3, and σ denotes the volatility of the exchange rate. The time interval is denoted by $\Delta t = t_{i+1} - t_i$ and Z is a standard-normal random variable (see [65], p. 3). Following this model the exchange rate is simulated on a daily basis in which the next exchange rate depends only on the preceding exchange rate, see [12], p. 48. Therefore, Δt is one day. The model assumes that exchange prices are determined every day and every day of the year is a business day. Figure 3.3 shows a part of the Monte Carlo simulation as it is used to test for effectiveness in the example. The simulation has the dimension of 180 time steps as the hedge relationship is determined for 6 months and 1000 simulated paths. All calculations are performed for every single path. At the end, the average over the final calculation value is determined in order to obtain a representative result.

In Figure 3.4, 150 of the 1000 simulated exchange rate paths are displayed. As is clearly visible, there is a certain concentration in the range of 1.2000 to 1.4000. However, outliers are also included in this selection of simulated paths.



	A	B	C	D	E	F	G	H	I	J	K	L
1	Volatility [%]	0,10		Time in Days	180							
2	rf [%]	0,022		Daycount	365							
3	rd [%]	0,02										
4	Spot [EUR/USD]	1,3000										
5			N	180								
6	Time	C,493150685	Delta T	0,002740								
7	Trials	1000										
8	Current trial	1000		blue: input variables								
9	Trial	Timestep										
10			1	2	3	4	5	6	7	8	9	10
11	Average	1,3000	1,3001	1,3001	1,3002	1,3004	1,3006	1,3006	1,3006	1,3005	1,3008	1,3009
12	1	1,3000	1,2911	1,2935	1,2925	1,2861	1,2809	1,2779	1,2767	1,2724	1,2604	1,2683
13	2	1,3000	1,3047	1,3107	1,3148	1,3073	1,2990	1,2996	1,2978	1,3046	1,2975	1,2923
14	3	1,3000	1,3018	1,3017	1,2959	1,2947	1,2926	1,2932	1,2891	1,2965	1,2940	1,2811
15	4	1,3000	1,3106	1,3089	1,2931	1,2955	1,2866	1,2892	1,2869	1,2866	1,2734	1,2770
16	5	1,3000	1,3041	1,2974	1,2990	1,2995	1,3038	1,2904	1,2919	1,2805	1,2893	1,2910
17	6	1,3000	1,3054	1,3140	1,3233	1,3194	1,3207	1,3193	1,3305	1,3320	1,3362	1,3266
18	7	1,3000	1,2933	1,2809	1,2934	1,3051	1,3013	1,2981	1,2950	1,2949	1,2864	1,2943
19	8	1,3000	1,3048	1,3032	1,3047	1,2971	1,3066	1,3014	1,3015	1,3015	1,3046	1,3068
20	9	1,3000	1,2955	1,2967	1,2952	1,3052	1,3152	1,3109	1,3096	1,3170	1,3169	1,3210
21	10	1,3000	1,2945	1,2924	1,2868	1,2804	1,2830	1,2675	1,2709	1,2766	1,2796	1,2781
22	11	1,3000	1,2998	1,3060	1,3025	1,3096	1,3190	1,3198	1,3173	1,3127	1,3083	1,2986
23	12	1,3000	1,3079	1,2987	1,2978	1,3045	1,3145	1,3205	1,3302	1,3352	1,3336	1,3410
24	13	1,3000	1,3161	1,3253	1,3302	1,3369	1,3355	1,3287	1,3304	1,3347	1,3391	1,3381
25	14	1,3000	1,2982	1,2905	1,2764	1,2772	1,2657	1,2609	1,2679	1,2746	1,2891	1,2832
26	15	1,3000	1,2870	1,2875	1,2813	1,2755	1,2944	1,3062	1,2991	1,3047	1,3132	1,3118
27	16	1,3000	1,3056	1,3158	1,3080	1,3010	1,3113	1,3234	1,3348	1,3295	1,3271	1,3470
28	17	1,3000	1,3000	1,2997	1,3036	1,3148	1,3057	1,2964	1,2823	1,2917	1,2982	1,2919
29	18	1,3000	1,3143	1,3098	1,2943	1,2934	1,2967	1,2923	1,3013	1,2972	1,3068	1,3097
30	19	1,3000	1,2995	1,2907	1,2966	1,2934	1,2941	1,2949	1,2991	1,2836	1,2804	1,2921
31	20	1,3000	1,2957	1,2989	1,2974	1,3011	1,2851	1,2847	1,2790	1,2700	1,2790	1,2884
32	21	1,3000	1,3019	1,3084	1,3109	1,3059	1,2960	1,2985	1,3054	1,3165	1,3111	1,3128
33	22	1,3000	1,3067	1,2988	1,2899	1,2828	1,2877	1,2880	1,2892	1,2997	1,3147	1,3114
34	23	1,3000	1,3047	1,3081	1,3112	1,3173	1,3232	1,3238	1,3221	1,3105	1,3012	1,3202
35	24	1,3000	1,2963	1,2930	1,2902	1,2887	1,2881	1,2865	1,2868	1,2929	1,2960	1,3003
36	25	1,3000	1,2979	1,3028	1,2951	1,3035	1,3079	1,3115	1,3113	1,3127	1,3142	1,3166
37	26	1,3000	1,2913	1,2885	1,2873	1,2802	1,2845	1,2879	1,2940	1,3032	1,3133	1,3217
38	27	1,3000	1,2905	1,2947	1,2946	1,2946	1,2908	1,2896	1,2930	1,2893	1,2770	1,2677
39	28	1,3000	1,2923	1,2832	1,2772	1,2756	1,2831	1,2721	1,2942	1,2861	1,2870	1,2929
40	29	1,3000	1,2920	1,2891	1,2760	1,2740	1,2781	1,2758	1,2776	1,2847	1,2926	1,2920
41	30	1,3000	1,2927	1,2979	1,2967	1,2916	1,2877	1,2912	1,2988	1,2993	1,3041	1,3017

FIGURE 3.3 Screenshot of a Monte Carlo simulation. The variables on which the simulation is based are shown in the shaded area in the upper left corner. The bold numbers can be varied. Generally, for a prospective test for effectiveness, all simulated exchange rate paths are used, except the path with the title "Average."

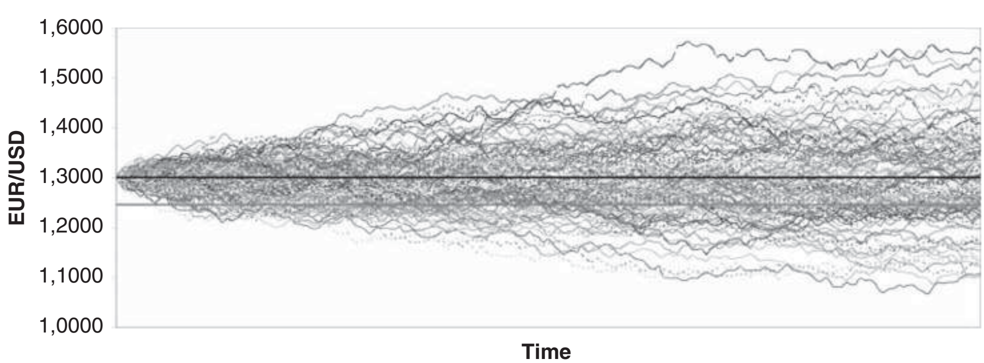


FIGURE 3.4 150 simulated paths of the exchange rate.

Calculation of the Shark Forward Plus Value The data gathered from the Monte Carlo simulation is used to calculate the value of the forward plus. In this case the (bank's) client is an exporter and the participation takes place in the case of a falling EUR-USD rate. The client's components of the shark forward plus are a long EUR forward position (long EUR call and short EUR put) and a long EUR put with a reverse knock-out barrier (American style). The valuation of these options is performed according to the formulas for the theoretical value given in [65].

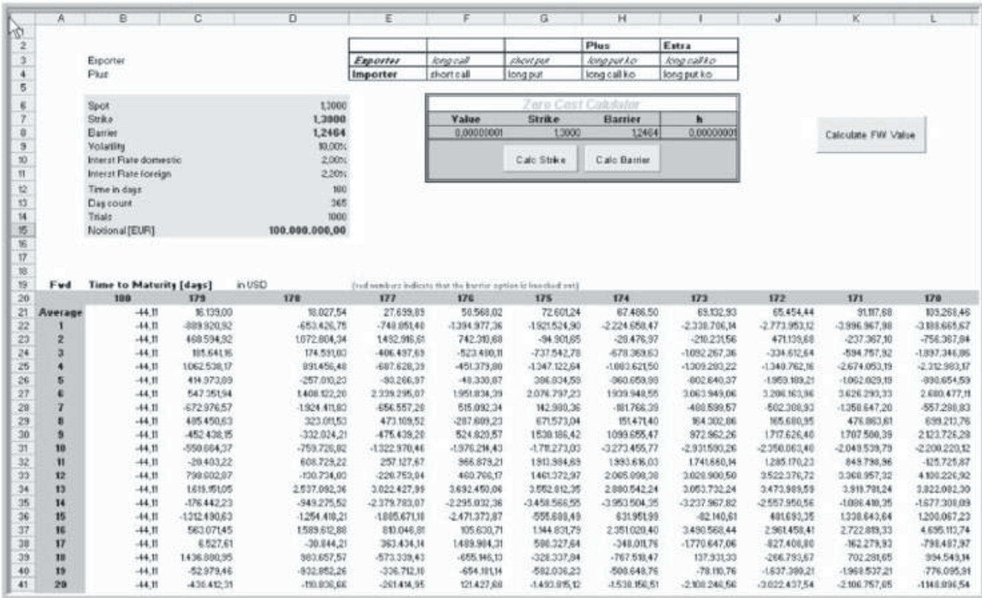
In the second step, the specifications of the barrier option and the notional amount of the hedge transaction have to be determined. For the hedge transaction, it is important that the initial expenditure for the shark forward plus is at zero cost, see Section 2.1.6. In the example, it is assumed that the strike of all options is equal to the spot price at initiation of the hedge relationship S_0. Therefore, the adjustable variable to obtain zero cost is the knock-out barrier. In order to find out the zero-cost generating barrier without calculating all prices, a *zero cost calculator* is included in the worksheet. It is important to note that the *Solver* in Microsoft Excel 2003 does not deliver reasonable results if the starting value for the zero-search is out of an acceptable range.[2] Generally, it is virtually impossible to determine the barrier in a way that the costs for the shark forward plus are exactly zero. The fact that an exchange rate is quoted as a four-digit number further limits the chance to obtain a zero cost. Therefore, the definition of a small range of acceptable costs seems to be sensible. In fact, costs of USD 50 for hedging a transaction worth EUR 100,000,000 is virtually zero in this context. In practice, the value is actually not zero, but the sales margin of the bank. This is set as a target quantity of say 2,000 EUR, but due to rounding it will end up being 1,973.90 EUR. The appropriate knock-out barrier B can be entered into the cell for the calculation of payoffs based on all exchange rate paths. If the barrier is hit or passed, the barrier option becomes worthless from this point onwards for the remaining time to maturity of the Forward Plus. If this occurs, the value of the Forward Plus is shown in light gray in black-white mode. Moreover, the option pricing formulas do not work at maturity. The options have their intrinsic value as payoff. For the Forward Plus the payoff is given by

$$(S_T - K) + (K - S_T)^+ \mathbb{I}_{\{S(u) > B, 0 \leq u \leq T\}}, \tag{7}$$

where K denotes the strike and worst case. Within this payoff formula, it is apparent that a problem might occur at a later point of the test for effectiveness: in the case that the barrier option is not knocked out and the spot exchange rate is within the participation interval at maturity, the payoff of the Forward Plus is zero. Figure 3.5 shows the calculation sheet for the Forward Plus values.

Recalling Figure 3.4, which shows a selection of simulated exchange rate paths, Figure 3.6 now includes the specifications of the Shark Forward Plus contract. The bold black line indicates the strike exchange rate representing also the client's worst case and the bold gray line shows the knock-out barrier of the long reverse knock-out put option. As clearly visible, several of the simulated paths hit or cross the barrier so that the Shark

[2]Putting the initial barrier value 0.0200–0.0400 EUR/USD below the strike might lead to a reasonable result.



Exporter		
Plus		
Spot		1,3000
Strike		1,3000
Barrier		1,2464
Volatility		10,00%
Interst Rate domestic		2,00%
Interst Rate foreign		2,20%
Time in days		180
Day count		365
Trials		1000
Notional [EUR]		100.000.000,00

			Plus	Extra
Exporter	long call	short put	long put ko	long call ko
Importer	short call	long put	long call ko	long put ko

Zero Cost Calculator

Value	Strike	Barrier	h
0,00000001	1,3000	1,2464	0,00000001

Calc Strike Calc Barrier

Calculate FW Value

Fwd Time to Maturity [days] in USD (red numbers indicats that the barrier option is knocked out)

	180	179	178	177	176	175	174	173	172	171	170
Average	-44,11	16.139,00	18.027,54	27.699,85	50.568,02	72.601,24	67.486,50	69.132,93	65.454,44	91.117,68	105.268,46
1	-44,11	-889.920,92	-653.426,75	-748.851,40	-1.394.977,36	-1.921.524,90	-2.224.658,47	-2.338.706,14	-2.773.953,12	-3.996.967,98	-3.188.665,67
2	-44,11	468.594,92	1.072.804,34	1.492.916,61	742.310,68	-94.901,65	-28.476,97	-210.231,56	471.139,68	-237.367,10	-756.367,84
3	-44,11	185.641,16	174.591,00	-406.497,69	-523.480,11	-737.542,78	-678.369,63	-1.092.267,36	-334.612,64	-594.757,92	-1.897.346,86
4	-44,11	1.062.538,17	891.456,48	-687.628,39	-451.379,80	-1.347.122,64	-1.083.621,50	-1.309.280,22	-1.340.762,16	-2.674.053,19	-2.312.983,17
5	-44,11	414.973,09	-257.810,23	-80.266,97	-48.330,87	396.834,59	-960.659,98	-802.640,37	-1.959.189,21	-1.062.029,19	-890.654,59
6	-44,11	547.351,94	1.408.122,20	2.339.295,07	1.951.834,39	2.076.797,23	1.939.948,55	3.063.949,06	3.286.163,96	3.626.290,33	2.680.477,11
7	-44,11	-672.976,57	-1.924.411,80	-656.557,20	515.092,34	142.980,36	-181.766,39	-480.599,57	-502.308,93	-1.350.647,20	-557.290,83
8	-44,11	485.450,63	323.011,53	473.109,52	-287.609,23	671.573,04	151.471,40	164.302,86	165.680,95	476.863,61	699.213,76
9	-44,11	-452.438,15	-332.024,21	-475.439,20	524.820,57	1.538.186,42	1.099.655,47	972.962,26	1.717.626,40	1.707.580,39	2.123.726,28
10	-44,11	-550.664,37	-759.726,82	-1.322.970,46	-1.976.214,43	-1.718.273,03	-3.273.455,77	-2.931.590,26	-2.350.063,40	-2.049.539,79	-2.200.220,12
11	-44,11	-20.403,22	608.729,22	257.127,67	966.879,21	1.913.984,89	1.993.616,03	1.741.660,14	1.285.170,23	849.798,96	-125.725,87
12	-44,11	799.002,07	-100.734,00	-220.753,04	460.796,17	1.461.372,97	2.065.098,30	3.020.900,50	3.522.376,72	3.360.957,32	4.100.226,92
13	-44,11	1.619.151,05	2.537.092,36	3.022.427,99	3.692.450,06	3.552.812,35	2.880.542,24	3.063.732,24	3.473.989,59	3.918.791,24	3.822.082,30
14	-44,11	-176.442,23	-949.275,52	-2.379.783,07	-2.295.032,36	-3.458.566,55	-3.953.504,35	-3.237.967,82	-2.557.950,56	-1.086.410,35	-1.677.308,09
15	-44,11	-1.312.490,63	-1.254.410,21	-1.885.671,18	-2.471.373,87	-555.680,49	631.951,99	-82.140,61	481.693,35	1.338.643,64	1.200.067,23
16	-44,11	563.071,45	1.589.612,88	810.046,81	105.630,71	1.944.831,79	2.351.020,40	3.490.568,44	2.961.458,41	2.722.889,33	4.695.113,74
17	-44,11	6.527,61	-30.844,21	363.434,14	1.489.984,31	586.327,64	-348.011,76	-1.770.647,06	-827.408,80	-162.279,93	-798.487,97
18	-44,11	1.436.880,95	983.657,57	-573.339,43	-655.146,13	-328.337,84	-767.518,47	137.931,33	-266.793,67	702.281,65	994.549,14
19	-44,11	-52.979,46	-932.852,26	-336.712,10	-654.181,14	-582.036,23	-508.648,76	-78.110,76	-1.637.380,21	-1.968.537,21	-776.095,91
20	-44,11	-436.412,31	-110.836,66	-261.414,95	121.427,68	-1.490.815,12	-1.538.156,51	-2.108.246,56	-3.022.437,54	-2.106.757,05	-1.146.896,54

FIGURE 3.5 Screenshot: calculation of Shark Forward Plus values. Inside the box, the knock-out barrier for the given specification can be calculated so that the initial value is near zero without calculating all paths and time steps.

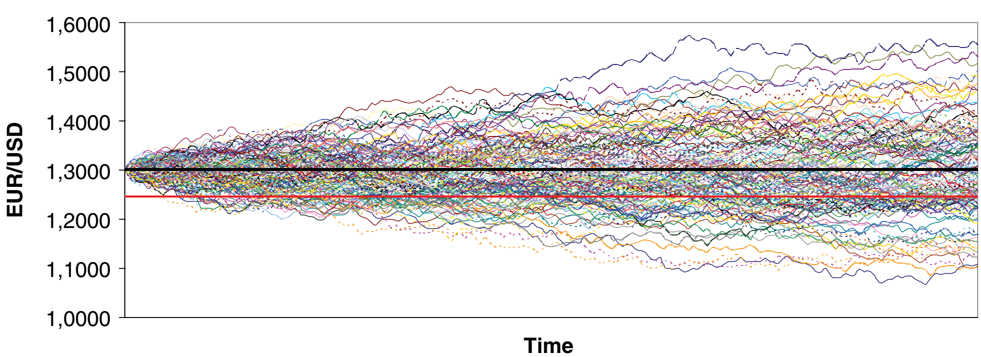


FIGURE 3.6 150 simulated paths of the exchange rate including strike and barrier of Forward Plus.

Forward Plus loses its possibility to participate in favorable exchange rate movements. This issue is shown in Figure 3.7. It shows the first simulated exchange rate paths as shown in Figure 3.5 for the last ten days of the simulation period.

Calculation of the Forward Rates The next worksheet displayed in Figure 3.8 calculates the forward exchange rates for the current point in time until the specified time to maturity.

10	9	8	7	6	5	4	3	2	1	0
1.083.403,44	1.049.149,36	1.008.918,89	967.617,64	903.250,19	829.222,84	772.873,41	709.125,27	605.789,46	506.095,72	342.325,20
-22.063.101,26	-21.037.117,62	-21.159.889,34	-21.061.976,75	-21.030.837,33	-20.877.558,46	-19.884.229,07	-19.434.251,71	-19.270.097,83	-19.420.210,86	-19.197.930,17
4.164.223,05	3.696.962,33	4.182.929,14	3.499.992,77	2.538.497,38	1.996.025,77	2.189.685,16	2.316.187,20	3.532.911,82	3.236.600,29	2.498.527,35
8.488.751,96	7.299.892,36	7.430.228,33	7.188.700,61	7.851.699,91	7.022.217,89	7.335.672,47	7.084.997,65	6.555.659,23	7.886.329,90	7.235.559,88
6.287.453,46	5.727.053,03	5.448.896,69	5.395.457,55	5.356.070,37	5.380.466,19	5.686.621,20	4.834.655,72	6.574.674,73	7.450.114,08	6.237.746,78
3.607.195,33	4.490.047,63	4.877.493,17	4.751.701,16	5.173.654,40	5.454.262,81	6.211.639,77	7.536.993,66	7.389.971,82	8.308.048,62	8.116.329,29
17.197.886,31	16.800.797,35	15.738.399,63	16.323.548,96	16.372.970,55	17.767.232,90	16.419.103,24	16.078.658,67	16.903.460,54	16.420.622,72	16.405.438,34
-11.104.431,11	-10.729.097,00	-10.478.278,71	-10.035.501,72	-9.691.000,26	-10.073.386,04	-9.552.069,39	-9.945.731,47	-9.681.299,92	-10.122.543,05	-9.907.524,27
-4.042.897,62	-3.475.812,77	-3.329.979,39	-2.934.322,75	-2.385.768,76	-2.685.628,61	-1.638.821,47	-2.312.223,47	-2.722.459,08	-2.072.254,52	-1.312.095,02
9.275.421,87	7.938.985,46	7.846.306,86	8.358.530,93	7.326.121,83	7.448.417,12	6.890.882,95	7.023.208,15	7.065.862,66	7.711.965,44	8.900.743,27
-6.026.463,28	-5.976.941,64	-4.978.311,36	-4.825.427,87	-3.795.125,72	-4.515.198,00	-4.928.310,13	-5.056.363,62	-5.471.735,05	-3.998.161,90	-3.965.340,27
-3.016.905,78	-4.016.111,28	-3.404.073,35	-3.018.832,19	-3.090.382,13	-3.784.278,54	-3.217.343,73	-3.487.237,37	-4.202.076,15	-3.853.954,88	-3.701.054,40
1.485.332,16	1.212.453,54	759.826,60	1.065.102,20	676.793,91	364.583,45	93.017,75	33.430,03	37.402,03	14.934,93	0,00
7.204.565,07	6.920.923,75	7.335.179,31	6.993.803,21	6.645.898,68	5.852.203,44	6.202.797,93	6.511.181,57	6.944.262,25	7.425.064,21	7.457.894,45
-16.277.642,86	-16.324.785,05	-15.970.957,08	-16.088.148,35	-15.405.550,23	-16.279.979,34	-16.650.703,67	-14.699.713,91	-14.221.332,85	-13.965.835,67	-13.559.021,29
1.235.634,94	790.686,79	1.066.652,78	587.137,57	491.501,26	570.060,54	1.292.501,21	1.717.931,35	853.547,68	154.631,92	547.813,62
9.812.644,85	9.585.280,54	8.532.459,65	8.815.187,49	9.602.373,25	9.778.222,05	10.266.561,29	11.228.488,65	11.288.216,19	11.364.401,17	10.912.213,10
-7.503.383,21	-8.488.842,84	-7.832.011,27	-8.638.208,81	-8.756.555,51	-8.944.463,70	-9.568.037,33	-9.465.214,31	-10.344.005,35	-10.274.634,12	-10.181.250,99
-6.735.719,71	-5.895.062,11	-6.766.557,59	-7.042.251,00	-7.784.938,77	-8.005.812,75	-8.429.288,74	-8.535.130,17	-8.883.252,64	-8.541.988,71	-7.510.526,59
-5.466.264,17	-5.695.715,40	-5.730.441,72	-6.545.483,50	-6.994.604,20	-5.757.325,07	-5.240.235,74	-6.455.525,08	-7.067.812,62	-7.371.477,20	-7.206.597,99
-14.736.538,56	-13.835.073,98	-13.668.523,47	-13.735.128,99	-14.226.367,68	-14.643.049,54	-14.337.157,31	-14.547.467,60	-14.966.614,77	-15.668.434,30	-15.849.291,30
14.755.646,70	15.045.483,41	14.896.904,80	14.648.255,70	14.947.333,58	15.154.166,40	15.748.587,88	16.492.234,02	17.415.952,06	17.017.103,49	17.615.796,42
-1.763.705,13	-1.015.586,91	-40.532,40	211.239,53	654.046,52	1.414.996,65	1.739.166,71	1.490.239,86	2.354.075,75	2.869.274,91	2.099.263,99
3.293.640,09	2.878.449,55	2.173.912,62	1.191.043,41	1.496.251,70	1.441.162,52	619.300,88	874.167,69	590.002,25	721.309,40	0,00
-13.916.238,20	-14.607.298,44	-14.957.293,12	-14.705.369,34	-13.845.825,71	-14.553.754,60	-15.683.253,37	-15.725.489,06	-15.610.104,25	-16.230.117,82	-16.707.342,83
7.308.389,51	8.002.690,67	7.563.267,32	7.795.782,74	7.762.899,99	7.460.678,54	8.614.924,25	8.214.447,87	8.573.271,82	8.329.177,34	7.545.403,18
-5.778.803,78	-5.535.645,51	-5.741.101,86	-4.828.658,86	-6.052.981,57	-5.764.545,71	-6.187.915,41	-5.936.216,60	-4.822.295,16	-4.335.845,58	-4.205.252,91
-5.295.154,67	-4.909.252,46	-5.465.422,72	-5.378.818,56	-6.269.299,24	-5.511.993,77	-5.026.006,63	-3.882.842,72	-3.474.599,58	-3.307.148,34	-2.561.402,66
4.572.804,83	4.345.827,42	5.100.443,51	6.343.058,21	7.364.681,83	7.263.706,08	6.286.195,67	5.764.292,20	5.800.138,06	6.096.872,37	6.377.301,65
-17.432.822,33	-17.308.859,79	-17.344.198,32	-17.161.040,60	-16.933.665,42	-17.147.564,76	-17.470.039,09	-19.504.906,19	-19.814.041,90	-20.052.362,93	-19.480.097,81
4.853.438,93	5.383.175,11	6.084.323,21	7.478.662,72	7.546.060,40	6.738.911,82	7.851.122,80	7.742.909,53	6.780.365,97	7.576.165,90	8.191.246,14
-484.755,60	155.341,96	-55.963,97	-4.050,96	152.552,70	-1.001.523,42	-1.519.817,86	-1.016.160,78	-432.270,64	210.853,81	876.746,78
3.884.997,23	3.585.723,65	3.348.196,69	2.967.370,82	2.764.593,01	3.079.324,52	2.975.359,86	3.846.000,72	3.501.051,52	4.608.722,70	5.345.341,68
701.814,73	282.905,05	183.671,87	94.422,39	-156.969,35	18.501,63	103.488,22	28.086,28	42.339,42	54,86	0,00
-2.335.816,34	-3.481.107,87	-2.339.566,16	-1.565.082,50	-2.571.412,83	-1.911.329,73	-904.284,73	-1.747.654,81	-2.414.340,37	-2.472.229,78	-2.347.966,98
-6.720.836,94	-5.777.680,40	-5.556.617,89	-6.392.949,29	-5.600.556,47	-6.278.415,64	-5.378.090,07	-5.111.761,86	-5.600.443,31	-5.892.359,70	-6.959.309,37
1.551.432,19	2.498.860,21	1.480.750,12	2.176.556,38	2.627.792,79	2.754.226,69	3.824.992,73	5.133.765,18	5.023.737,39	5.070.817,77	4.478.996,61

FIGURE 3.7 Screenshot: calculation of Shark Forward Plus values at maturity. Red (light gray in black-white mode) numbers indicate that the barrier was hit along the path.

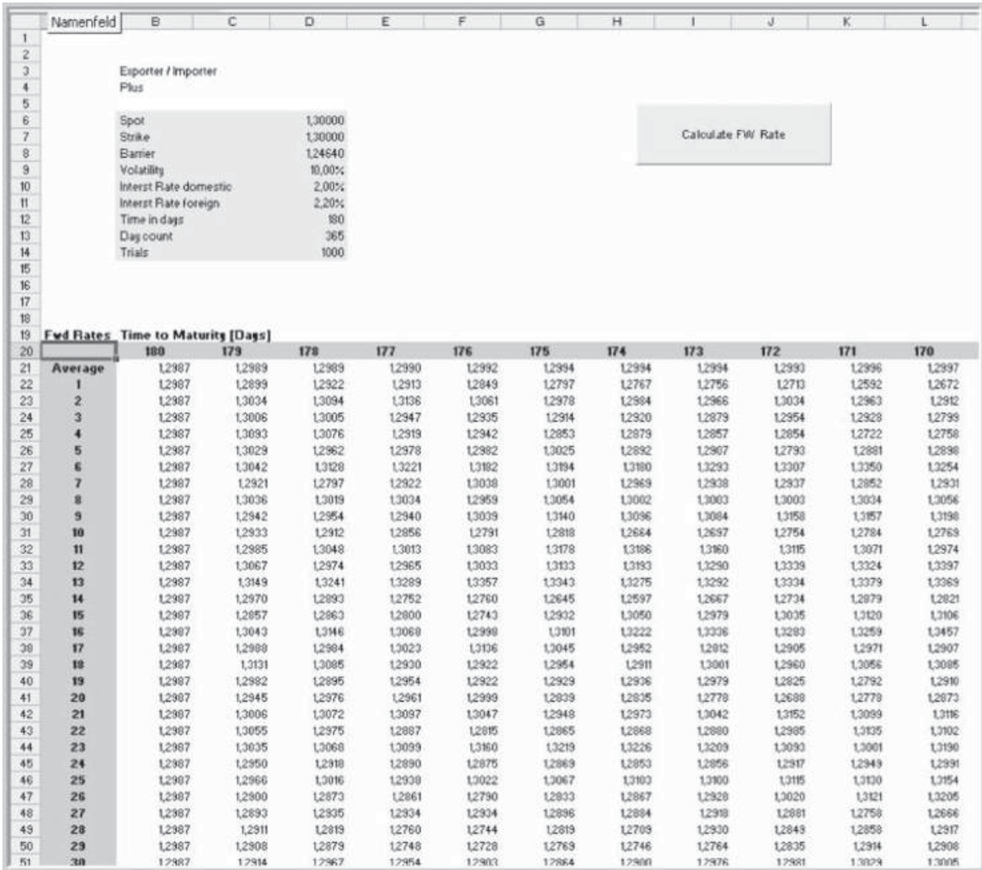
The time period of the forward rates is reduced from column to column by one day. The forward rate is not required for the calculation of the Shark Forward Plus. But it is necessary to evaluate the forecast transaction the Shark Forward Plus will hedge. The calculation is performed according to the usual formula

$$f = S_0 e^{(r_d - r_f)T}, \tag{8}$$

where f and S_0 denote the current forward rate and the current spot exchange rate respectively. The remaining time period until the forecast transaction takes place is denoted by T.

As the time to maturity shortens, the forward rate converges towards the current spot rate. As visible in Figure 3.8, there are no more specifications needed for the calculations in this worksheet.

Calculation of the Forecast Transaction's Value The calculation of the forecast transaction's value uses the information given in the preceding calculations. At initiation of the hedge relationship, the value is set to zero as the standard value. For the remaining points in time, the value is calculated as the difference between the strike of the Forward Plus and the current forward exchange rate. This difference multiplied with the notional amount represents the time value of the forecast transaction during the hedge relationship,



Exporter / Importer
Plus

Spot	1,30000
Strike	1,30000
Barrier	1,24640
Volatility	10,00%
Interst Rate domestic	2,00%
Interst Rate foreign	2,20%
Time in days	180
Day count	365
Trials	1000

Calculate FW Rate

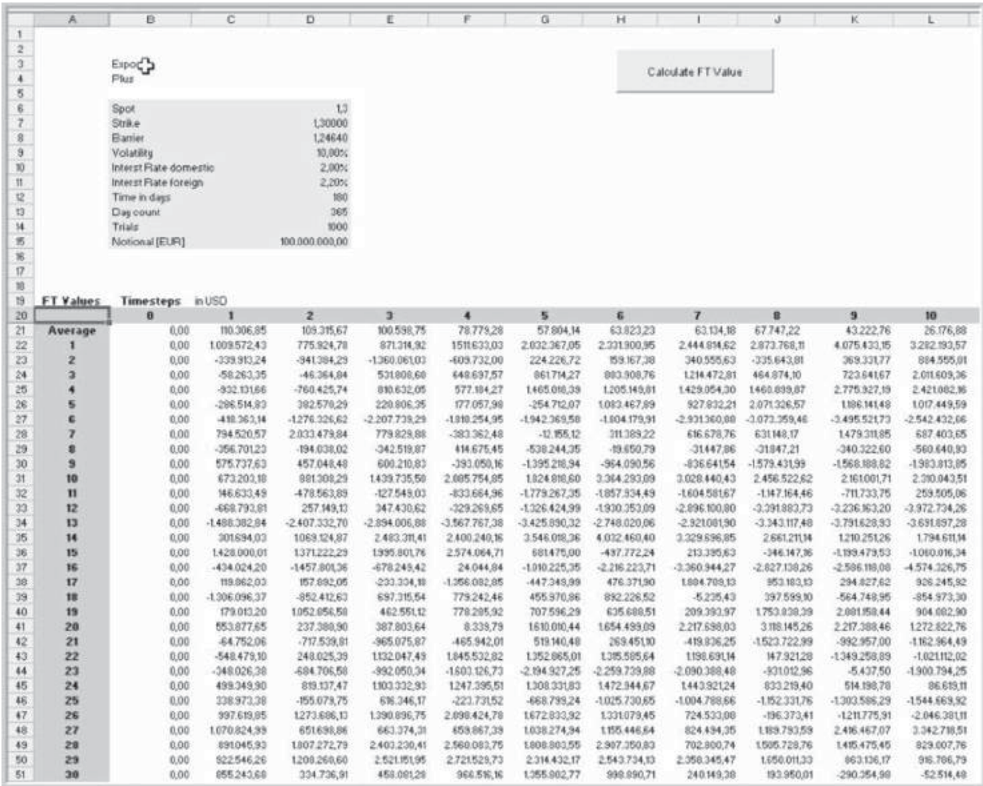
Fwd Rates Time to Maturity [Days]

	180	179	178	177	176	175	174	173	172	171	170
Average	1,2987	1,2989	1,2989	1,2990	1,2992	1,2994	1,2994	1,2994	1,2993	1,2996	1,2997
1	1,2987	1,2899	1,2922	1,2913	1,2849	1,2797	1,2767	1,2756	1,2713	1,2592	1,2672
2	1,2987	1,3034	1,3094	1,3136	1,3061	1,2978	1,2984	1,2966	1,3034	1,2963	1,2912
3	1,2987	1,3006	1,3005	1,2947	1,2935	1,2914	1,2920	1,2879	1,2954	1,2928	1,2799
4	1,2987	1,3093	1,3076	1,2919	1,2942	1,2853	1,2879	1,2857	1,2854	1,2722	1,2758
5	1,2987	1,3029	1,2962	1,2978	1,2982	1,3025	1,2892	1,2907	1,2793	1,2881	1,2898
6	1,2987	1,3042	1,3128	1,3221	1,3182	1,3194	1,3180	1,3293	1,3307	1,3350	1,3254
7	1,2987	1,2921	1,2797	1,2922	1,3038	1,3001	1,2969	1,2938	1,2937	1,2852	1,2931
8	1,2987	1,3036	1,3019	1,3034	1,2959	1,3054	1,3002	1,3003	1,3003	1,3034	1,3056
9	1,2987	1,2942	1,2954	1,2940	1,3039	1,3140	1,3096	1,3084	1,3158	1,3157	1,3198
10	1,2987	1,2933	1,2912	1,2856	1,2791	1,2818	1,2664	1,2697	1,2754	1,2784	1,2769
11	1,2987	1,2985	1,3048	1,3013	1,3083	1,3178	1,3186	1,3160	1,3115	1,3071	1,2974
12	1,2987	1,3067	1,2974	1,2965	1,3033	1,3133	1,3193	1,3290	1,3339	1,3324	1,3397
13	1,2987	1,3149	1,3241	1,3289	1,3357	1,3343	1,3275	1,3292	1,3334	1,3379	1,3369
14	1,2987	1,2970	1,2893	1,2752	1,2760	1,2645	1,2597	1,2667	1,2734	1,2879	1,2821
15	1,2987	1,2857	1,2863	1,2800	1,2743	1,2932	1,3050	1,2979	1,3035	1,3120	1,3106
16	1,2987	1,3043	1,3146	1,3068	1,2998	1,3101	1,3222	1,3336	1,3283	1,3259	1,3457
17	1,2987	1,2988	1,2984	1,3023	1,3136	1,3045	1,2952	1,2812	1,2905	1,2971	1,2907
18	1,2987	1,3131	1,3085	1,2930	1,2922	1,2954	1,2911	1,3001	1,2960	1,3056	1,3085
19	1,2987	1,2982	1,2895	1,2954	1,2922	1,2929	1,2936	1,2979	1,2825	1,2792	1,2910
20	1,2987	1,2945	1,2976	1,2961	1,2999	1,2839	1,2835	1,2778	1,2688	1,2778	1,2873
21	1,2987	1,3006	1,3072	1,3097	1,3047	1,2948	1,2973	1,3042	1,3152	1,3099	1,3116
22	1,2987	1,3055	1,2975	1,2887	1,2815	1,2865	1,2868	1,2880	1,2985	1,3135	1,3102
23	1,2987	1,3035	1,3068	1,3099	1,3160	1,3219	1,3226	1,3209	1,3093	1,3001	1,3190
24	1,2987	1,2950	1,2918	1,2890	1,2875	1,2869	1,2853	1,2856	1,2917	1,2949	1,2991
25	1,2987	1,2966	1,3016	1,2938	1,3022	1,3067	1,3103	1,3100	1,3115	1,3130	1,3154
26	1,2987	1,2900	1,2873	1,2861	1,2790	1,2833	1,2867	1,2928	1,3020	1,3121	1,3205
27	1,2987	1,2893	1,2935	1,2934	1,2934	1,2896	1,2884	1,2918	1,2881	1,2758	1,2666
28	1,2987	1,2911	1,2819	1,2760	1,2744	1,2819	1,2709	1,2930	1,2849	1,2858	1,2917
29	1,2987	1,2908	1,2879	1,2748	1,2728	1,2769	1,2746	1,2764	1,2835	1,2914	1,2908
30	1,2987	1,2914	1,2967	1,2954	1,2903	1,2864	1,2900	1,2976	1,2981	1,3029	1,3005

FIGURE 3.8 Screenshot: calculation of forward rates.

see [100], p. 34. This calculation makes sense considering the following explanations. The transaction is hedged at initiation of the hedge relationship. The *worst case* exchange rate the entity obtains, is the strike of the options the Forward Plus consists of. If the entity hedges the forecast transaction at a later point in time with an outright forward, it would of course fix the exchange rate at the current forward rate. So the difference between the two rates multiplied with the notional amount represents the value gained or lost through not hedging at a later point in time. The resulting figure displays the value of the forecast transaction. Figure 3.9 shows an excerpt of the exemplary calculation results.

Dollar-Offset Ratio – Prospective Test for Effectiveness As described in Section 3.1.5, the *Dollar-Offset Ratio* is an easy way to measure hedge effectiveness. On the other hand this method also involves weaknesses. One weakness is that for small changes in value, a resulting ratio that is out of the range accepted as effective might occur. In this case, the ineffectiveness is high relative to the change in value from an accounting



Expo
Plus

Calculate FT Value

Spot	1,3
Strike	1,30000
Barrier	1,24640
Volatility	10,00%
Interst Rate domestic	2,00%
Interst Rate foreign	2,20%
Time in days	180
Day count	365
Trials	1000
Notional [EUR]	100.000.000,00

FT Values **Timesteps** in USD

	0	1	2	3	4	5	6	7	8	9	10
Average	0,00	110.306,85	109.315,67	100.598,75	78.779,28	57.804,14	63.823,23	63.134,18	67.747,22	43.222,76	26.176,88
1	0,00	1.009.572,43	775.924,78	871.314,92	1.511.633,03	2.032.367,05	2.331.900,95	2.444.814,62	2.873.768,11	4.075.433,15	3.282.193,57
2	0,00	-339.913,24	-941.384,29	-1.360.061,03	-609.732,00	224.226,72	159.167,38	340.555,63	-335.643,81	369.331,77	884.555,01
3	0,00	-58.263,35	-46.364,84	531.808,60	648.697,57	861.714,27	883.908,76	1.214.472,81	464.874,10	723.641,67	2.011.609,36
4	0,00	-932.131,66	-760.425,74	810.632,05	577.184,27	1.465.018,39	1.205.149,81	1.429.054,30	1.460.899,87	2.775.927,19	2.421.082,16
5	0,00	-286.514,83	382.579,29	220.806,35	177.057,98	-254.712,07	1.083.467,89	927.832,21	2.071.326,57	1.186.141,48	1.017.449,59
6	0,00	-418.363,14	-1.276.326,62	-2.207.739,29	-1.818.254,95	-1.942.369,58	-1.804.179,91	-2.931.360,88	-3.073.359,46	-3.495.521,73	-2.542.432,06
7	0,00	794.520,57	2.033.479,84	779.829,88	-383.362,48	-12.155,12	311.389,22	616.678,76	631.148,17	1.479.311,85	687.403,65
8	0,00	-356.701,23	-194.038,02	-342.519,87	414.675,45	-538.244,35	-19.650,79	-31.447,86	-31.847,21	-340.322,60	-560.640,93
9	0,00	575.737,63	457.048,48	600.210,83	-393.050,16	-1.395.218,94	-964.090,56	-836.641,54	-1.579.431,99	-1.568.188,82	-1.983.813,85
10	0,00	673.203,18	881.308,29	1.439.735,58	2.085.754,85	1.824.818,60	3.364.293,09	3.028.440,43	2.456.522,62	2.161.001,71	2.310.043,51
11	0,00	146.633,49	-478.563,89	-127.549,03	-833.664,96	-1.779.267,35	-1.857.934,49	-1.604.581,67	-1.147.164,46	-711.733,75	259.505,06
12	0,00	-668.793,81	257.149,13	347.430,62	-329.269,65	-1.326.424,99	-1.930.353,09	-2.896.100,80	-3.391.883,73	-3.236.163,20	-3.972.734,26
13	0,00	-1.488.382,84	-2.407.332,70	-2.894.006,88	-3.567.767,38	-3.425.890,32	-2.748.020,06	-2.921.081,90	-3.343.117,48	-3.791.628,93	-3.691.897,28
14	0,00	301.694,03	1.069.124,87	2.483.311,41	2.400.240,16	3.546.018,36	4.032.460,40	3.329.696,85	2.661.211,14	1.210.251,26	1.794.611,14
15	0,00	1.428.000,01	1.371.222,29	1.995.801,76	2.574.064,71	681.475,00	-497.772,24	213.395,63	-346.147,36	-1.199.479,53	-1.060.016,34
16	0,00	-434.024,20	-1.457.801,36	-678.249,42	24.044,84	-1.010.225,35	-2.216.223,71	-3.360.944,27	-2.827.138,26	-2.586.118,08	-4.574.326,75
17	0,00	119.862,03	157.892,05	-233.334,18	-1.356.082,85	-447.349,99	476.371,90	1.804.709,13	953.183,13	294.827,62	926.245,92
18	0,00	-1.306.096,37	-852.412,63	697.315,54	779.242,46	455.970,86	892.226,52	-5.235,43	397.599,10	-564.748,95	-854.973,30
19	0,00	179.013,20	1.052.856,58	462.551,12	778.295,92	707.596,29	635.688,51	209.393,97	1.753.838,39	2.081.158,44	904.082,90
20	0,00	553.877,65	237.380,90	387.803,64	8.338,79	1.610.010,44	1.654.499,09	2.217.698,03	3.118.145,26	2.217.388,46	1.272.822,76
21	0,00	-64.752,06	-717.539,81	-965.075,87	-465.942,01	519.140,48	269.451,10	-419.836,25	-1.523.722,99	-992.957,00	-1.162.964,49
22	0,00	-548.479,10	248.025,39	1.132.047,49	1.845.532,82	1.352.865,01	1.315.585,64	1.198.691,14	147.921,28	-1.349.258,89	-1.021.112,02
23	0,00	-348.026,38	-684.706,58	-992.050,34	-1.603.126,73	-2.194.927,25	-2.259.739,88	-2.090.388,48	-931.012,96	-5.437,50	-1.900.794,25
24	0,00	499.349,90	819.137,47	1.103.332,93	1.247.395,51	1.308.331,83	1.472.944,67	1.443.921,24	833.219,40	514.198,78	86.619,11
25	0,00	338.973,38	-155.079,75	616.346,17	-223.731,52	-668.799,24	-1.025.730,65	-1.004.788,66	-1.152.331,76	-1.303.586,29	-1.544.669,92
26	0,00	997.619,85	1.273.686,13	1.390.896,75	2.898.424,78	1.672.833,92	1.331.079,45	724.533,08	-196.373,41	-1.211.775,91	-2.046.381,11
27	0,00	1.070.824,99	651.698,86	663.374,31	659.867,39	1.038.274,94	1.155.446,64	824.494,35	1.189.793,59	2.416.467,07	3.342.718,51
28	0,00	891.045,93	1.807.272,79	2.403.230,41	2.560.083,75	1.808.803,55	2.907.350,83	702.800,74	1.505.728,76	1.415.475,45	829.007,76
29	0,00	922.546,26	1.208.260,60	2.521.151,95	2.721.529,73	2.314.432,17	2.543.734,13	2.358.345,47	1.650.011,33	863.136,17	916.786,79
30	0,00	855.243,68	334.736,91	458.081,29	964.516,16	1.355.902,77	998.890,71	240.149,38	193.950,01	-290.354,98	-52.514,48

FIGURE 3.9 Screenshot: calculation of the forecast transaction's value.

perspective, although the hedge is economically highly effective. Recall that the *Dollar-Offset Ratio* is calculated using Equation (1). Therefore, in a first step the changes in values for both the Shark Forward Plus and the forecast transaction have to be calculated. The change in value is determined by subtracting the previous value from the current value. This is done for every time step and all simulated exchange rate paths. In the second step, the appropriate changes of the specified point in time are set in a ratio according to Equation (1). The result of this represents the one-day period *Dollar-Offset Ratio*.

An exception from this method is the value at initiation of the hedge. At the first date of observation, a previous value does not exist. Therefore, the ratio is set to zero. For the second point in time the ratio should be very near to minus one. This is due to the fact that the hedged item has a value of zero at the previous point in time and the value of the Forward Plus should also be near zero in the previous point in time as it has zero cost. As one can see in Figure 3.10, the second column is headed by "Cumulative." In this column the cumulative *Dollar-Offset Ratio* is shown. It is calculated using the ratio of the final value of the forecast transaction and the final payoff of the Forward Plus.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2													
3								Calculate					
4	trials	1000											
5	Timesteps	180											
6													
7	Path	Cumulative	180	179	178	177	176	175	174	173	172	171	170
8	1	-1,00	0,00	-0,99	-1,00	-0,99	-0,99	-0,99	-0,99	-0,99	-0,98	-0,98	-0,98
9	2	-1,00	0,00	-1,00	-1,00	-1,00	-1,00	-0,98	-1,00	-0,99	-1,00	-0,99	-0,99
10	3	-1,00	0,00	-1,08	-0,99	-1,00	-1,00	-0,98	-0,99	-0,99	-0,99	-0,99	-0,99
11	4	-1,00	0,00	-1,00	-0,99	-0,99	-0,99	-0,99	-0,99	-1,01	-0,99	-0,98	-0,99
12	5	-1,00	0,00	-1,00	-0,99	-0,97	-0,99	-0,99	-0,98	-0,99	-0,99	-0,98	-0,99
13	6	-1,00	0,00	-1,00	-1,00	-1,01	-0,99	-1,01	-1,00	-1,00	-1,00	-1,01	-1,00
14	7	-1,00	0,00	-0,99	-0,99	-0,99	-1,00	-1,00	-0,99	-1,06	-0,99	-0,99	-0,99
15	8	-1,00	0,00	-1,00	-0,99	-1,00	-0,99	-1,00	-0,92	-0,29	-0,99	-0,99	-0,98
16	9	-1,00	0,00	-0,99	-1,00	-0,99	-1,00	-1,00	-1,01	-1,00	-1,11	-1,00	-1,00
17	10	-1,00	0,00	-1,00	-0,99	-0,99	-0,98	-0,99	-0,98	-0,98	-0,98	-0,99	-0,99
18	11	-1,00	0,00	-0,99	-1,00	-0,99	-1,00	-0,99	-1,01	-1,00	-1,00	-1,00	-0,99
19	12	0,00	0,00	-1,00	-1,00	-0,99	-1,00	-1,00	-1,00	-1,00	-1,01	-1,01	-0,99
20	13	-1,00	0,00	-1,00	-1,00	-1,01	-1,02	-1,01	-1,00	-1,00	-1,01	-1,02	-1,01
21	14	-1,00	0,00	-0,99	-0,99	-0,98	-0,98	-0,98	-0,98	-0,98	-0,99	-0,99	-0,99
22	15	-1,00	0,00	-0,98	-0,99	-0,99	-0,99	-0,99	-1,00	-0,99	-1,00	-1,01	-0,99
23	16	-1,00	0,00	-1,00	-1,00	-1,00	-1,00	-1,00	-1,00	-1,01	-1,01	-1,01	-1,01
24	17	-1,00	0,00	-1,02	-0,99	-1,00	-1,00	-1,00	-0,99	-0,99	-0,99	-0,99	-0,99
25	18	-1,00	0,00	-1,00	-1,00	-1,00	-0,99	-0,99	-0,99	-1,00	-0,99	-0,99	-0,99
26	19	-1,00	0,00	-0,99	-0,99	-0,99	-0,98	-0,98	-0,99	-0,99	-0,99	-0,99	-0,98
27	20	-1,00	0,00	-0,99	-1,00	-0,99	-0,99	-1,00	-0,99	-0,98	-0,98	-0,99	-0,99
997	990	0,00	0,00	-1,00	-1,00	-1,00	-1,01	-1,01	-1,01	-1,01	-1,01	-1,01	-1,02
998	991	-1,00	0,00	-1,00	-0,99	-1,00	-1,00	-0,99	-0,99	-1,00	-1,00	-1,00	-1,00
999	992	-1,00	0,00	-1,00	-1,00	-1,00	-1,00	-0,99	-1,00	-1,00	-1,00	-1,00	-1,00
1000	993	-1,00	0,00	-1,00	-0,98	-1,00	-1,00	-1,00	-0,99	-1,00	-1,00	-1,00	-1,00
1001	994	-1,00	0,00	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-0,98	-0,99	-0,99
1002	995	0,00	0,00	-1,00	-1,00	-0,98	-1,00	-1,00	-1,01	-1,00	-1,00	-1,00	-0,99
1003	996	-1,00	0,00	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-1,00	-0,99	-0,99
1004	997	-1,00	0,00	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-1,00
1005	998	-1,00	0,00	-1,00	-0,99	-1,00	-1,00	-0,99	-0,99	-0,99	-0,98	-0,98	-0,99
1006	999	0,00	0,00	-1,00	-1,00	-0,99	-0,99	-0,99	-0,98	-0,98	-0,98	-0,98	-0,98
1007	1000	-1,00	0,00	-1,00	-0,99	-1,01	-1,00	-1,03	-0,99	-1,00	-0,99	-1,00	-1,00
1008													
1009	Average	-0,92	0,00	-0,99	-0,99	-0,99	-0,99	-0,99	-0,99	-1,00	-0,99	-0,98	-1,00

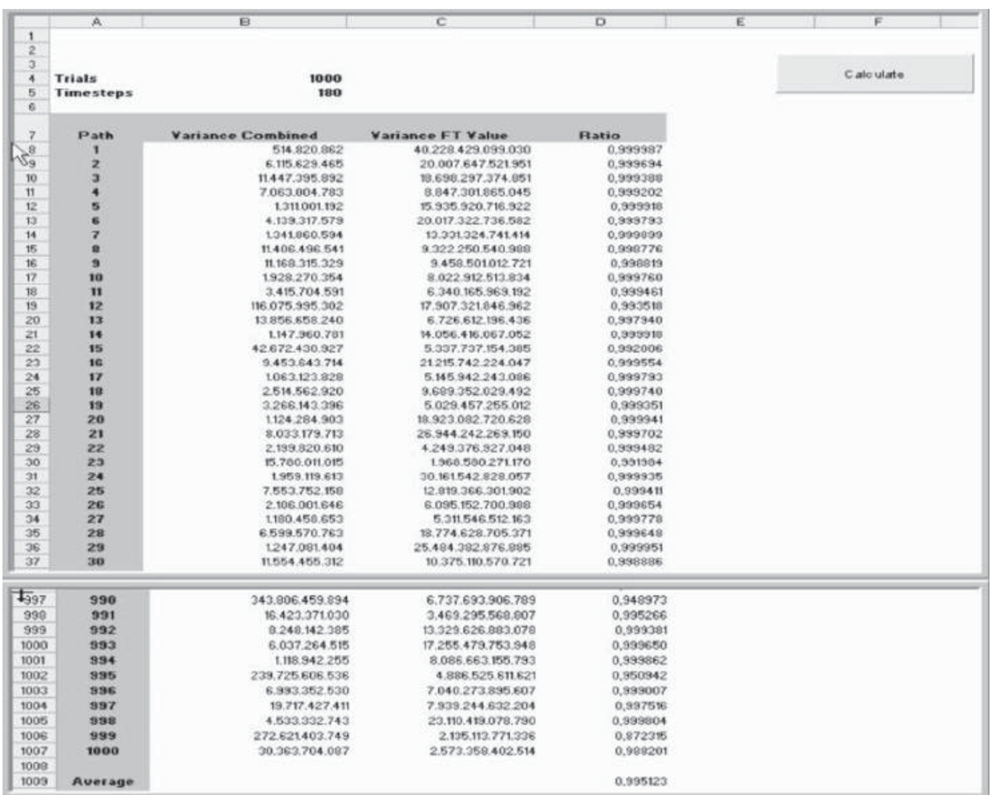
FIGURE 3.10 Screenshot: prospective Dollar-Offset Ratio.

As described above, a problem may arise at this stage if the barrier option was not hit and the spot exchange rate is within the participation range. In this case the final payoff would be zero and the cumulative *Dollar-Offset Ratio* is not defined due to a division by zero. In this case, the value of the *Dollar-Offset Ratio* is set to zero in the spread sheet. This problem only arises for the Shark Forward Plus, but not for a Shark Forward Extra.This is because of the offsetting option position within the participation range. For the Shark Forward Extra, the barrier option enforces the option positions that replicate the forward contract, see Section 2.1.6.

For the calculation of the prospective hedge effectiveness per time period, the *Dollar-Offset Ratios* of the single paths are averaged for every point in time. For the expected cumulative *Dollar-Offset Ratio* the cumulative *Dollar-Offset Ratios* of each path are averaged. As one can see the expected cumulative *Dollar-Offset Ratio* is -0.92. This is within the acceptable range of $[-1.25; -0.8]$ and would prove the prospective hedge effectiveness for the given example according to this method.

Averaging over the average *Dollar-Offset Ratios* per time step would come to another result. The ratio is -1.50, and the hedge would fail according to this criterion. So the cumulative ratio leads to a better result in terms of effectiveness due to an offsetting effect of outliers from the accepted range. The day-by-day average *Dollar-Offset Ratio* is not calculated automatically in the spreadsheet.

Variance Reduction Measure – Prospective Test for Effectiveness The calculation according to the *variance reduction measure* (VRM) involves multiple steps. In the first step, the changes in value of the hedged item, that is the forecast transaction, are observed. Now the variance of the change in value is calculated per exchange rate path over the observed time period. In the next step, the changes in value of the combined hedge relationship, i.e. the changes in value of the hedging instrument and the changes in value of the forecast transaction are added up for each day. The variance is calculated also from the joint change in value (hedging relationship). This variance should be much smaller because the aim of the hedge is that the changes in value are offset for the most part. The VRM is calculated by Equation (3). This procedure is repeated for every exchange rate path from the Monte Carlo simulation. Finally, the average of the calculated VRM ratios gives an indication of the prospective hedge effectiveness. This is shown in the last line of Figure 3.11. We notice that the ratio is always near 1, indicating that the variance is reduced to a large extent. The literature does not give any information as to which threshold value should be reached. A value of 0.96 would be equal to an offset of 80% of the changes in value, see [83], p. 96. Other authors merely mention that the reduction must be specified in order to be useful, see [48], p. 44 and [48], p. 103.



	A	B	C	D
1				
2				
3				
4	**Trials**	**1000**		
5	**Timesteps**	**180**		
6				
7	**Path**	**Variance Combined**	**Variance FT Value**	**Ratio**
8	**1**	514.820.862	40.228.429.099.030	0,999987
9	**2**	6.115.629.465	20.007.647.521.951	0,999694
10	**3**	11.447.395.892	18.698.297.374.851	0,999388
11	**4**	7.063.004.783	8.847.301.865.045	0,999202
12	**5**	1.311.001.192	15.935.920.716.922	0,999918
13	**6**	4.139.317.579	20.017.322.736.582	0,999793
14	**7**	1.341.860.594	13.331.324.741.414	0,999899
15	**8**	11.406.496.541	9.322.250.540.988	0,998776
16	**9**	11.168.315.329	9.458.501.012.721	0,998819
17	**10**	1.928.270.354	8.022.912.513.834	0,999760
18	**11**	3.415.704.591	6.340.165.969.192	0,999461
19	**12**	116.075.995.302	17.907.321.846.962	0,993518
20	**13**	13.856.658.240	6.726.612.196.436	0,997940
21	**14**	1.147.960.781	14.056.416.067.052	0,999918
22	**15**	42.672.430.927	5.337.737.154.385	0,992006
23	**16**	9.453.643.714	21.215.742.224.047	0,999554
24	**17**	1.063.123.828	5.145.942.243.086	0,999793
25	**18**	2.514.562.920	9.689.352.029.492	0,999740
26	**19**	3.266.143.396	5.029.457.255.012	0,999351
27	**20**	1.124.284.903	18.923.082.720.628	0,999941
28	**21**	8.033.179.713	26.944.242.269.150	0,999702
29	**22**	2.199.820.610	4.249.376.927.048	0,999482
30	**23**	15.760.011.015	1.968.580.271.170	0,991984
31	**24**	1.959.119.613	30.161.542.828.057	0,999935
32	**25**	7.553.752.158	12.819.366.301.902	0,999411
33	**26**	2.106.001.646	6.095.152.700.988	0,999654
34	**27**	1.180.458.653	5.311.546.512.163	0,999778
35	**28**	6.599.570.763	18.774.628.705.371	0,999648
36	**29**	1.247.081.404	25.484.382.876.885	0,999951
37	**30**	11.554.455.312	10.375.110.570.721	0,998886
997	**990**	343.806.459.894	6.737.693.906.789	0,948973
998	**991**	16.423.371.030	3.469.295.568.807	0,995266
999	**992**	8.248.142.385	13.329.626.883.078	0,999381
1000	**993**	6.037.264.515	17.255.479.753.948	0,999650
1001	**994**	1.118.942.255	8.086.663.155.793	0,999862
1002	**995**	239.725.606.536	4.886.525.611.621	0,950942
1003	**996**	6.993.352.530	7.040.273.895.607	0,999007
1004	**997**	19.717.427.411	7.939.244.632.204	0,997516
1005	**998**	4.533.332.743	23.110.419.078.790	0,999804
1006	**999**	272.621.403.749	2.135.113.771.336	0,872315
1007	**1000**	30.363.704.087	2.573.358.402.514	0,988201
1008				
1009	**Average**			0,995123

FIGURE 3.11 Screenshot: prospective variance reduction measure.

The average reduction in variance amounts to 99.51% in this example. This means that the hedge fulfills the prospective test for effectiveness.

Regression Analysis – Prospective Test for Effectiveness The third method that can be used to test the hedge relationship for effectiveness is the *Regression Analysis*. For this method, the changes in value of the forecast transaction (hedged item) and the Forward Plus (hedging instrument) are employed. For the linear regression model we use

$$y = \alpha x + \beta + \epsilon, \tag{9}$$

where y denotes the change in fair value of the hedging instrument and x denotes the change in hedge fair value of the hedged item. The interception of the regression line with the $y-$axis is α. The term β is the slope of the regression line and ϵ represents the error term. With the regression the change in value of the hedging instrument is explained by the change in value of the forecast transaction.

The slope of the regression line should be close to minus one. In order to deem the hedge as effective in accordance with IAS 39, R^2, which is a measure of how good the regression line explains the data points, should be at least 80%.

For the prospective test for hedge effectiveness, the regression parameters α, β, and R^2 are calculated for each exchange rate path. The next appropriate step is to average the regression parameters over the different paths. The averages are the expected values for the parameters and give an indication for the anticipated hedge effectiveness. As mentioned in Section 3.1.5, it is important to have a sufficient number of data points per path in order to obtain reasonable results. Figure 3.12 shows a selection of results for the *Regression Analysis* from the given example.

The Average line shows the expected values. The prospective hedge effectiveness holds according to the *Regression Analysis*. The slope of the regression line is −0.9911 which is sufficiently close to minus one. The measure of determination is $R^2 = 99.55\%$ and thereby clearly above the necessary threshold to qualify as highly effective hedge in combination with the slope coefficient.

Result According to the *Dollar-Offset Method*, which seems to be the method most likely to fail, we have calculated a *cumulative Dollar-Offset ratio* of −0.92. This is clearly in the range of [−1.25; −0.8] which is categorized as highly effective. Taking a closer look at the calculated results, one can see that the ratios nearly always have values of −1.00. In some cases the values of the cumulative ratios are zero. This is the case if the barrier of the Forward Plus has not been reached and the spot exchange rate is within the participation range at maturity, i.e. the barrier option has an intrinsic value at maturity. In this case the ratio cannot be calculated, because this would be a division by zero. As such the ratio is defined as zero. In practice, this means that the exporter receives a gain from the forecast transaction, but on the other side does not suffer a loss from the Forward Plus. For the exporter, this is an ideal scenario, but from an accounting perspective the hedge is at risk of overcompensation and being ineffective.

Looking at the *day-by-day Dollar-Offset ratio* the hedge fails the test for effectiveness. The average over the paths and the time result in a Dollar-Offset ratio of −1.50.

	A	B	C	D	E
1					
2					
3					
4	**Trials**	**1000**			Calculate
5	**Timesteps**	**180**			
6					
7		**Slope**	**Intercept**	**R-Squared**	
8	**1**	-0,9962	-406,38	0,9997	
9	**2**	-0,9899	139,50	0,9994	
10	**3**	-1,0147	-587,90	0,9988	
11	**4**	-0,9826	598,37	0,9994	
12	**5**	-0,9992	37,04	0,9998	
13	**6**	-0,9902	886,12	0,9997	
14	**7**	-0,9980	-107,76	0,9997	
15	**8**	-1,0127	92,44	0,9992	
16	**9**	-0,9758	1.191,97	0,9992	
17	**10**	-1,0018	40,13	0,9997	
18	**11**	-1,0002	4,43	0,9997	
19	**12**	-0,9527	8.390,75	0,9827	
20	**13**	-0,9948	214,80	0,9984	
21	**14**	-1,0032	239,87	0,9997	
22	**15**	-0,9364	192,79	0,9877	
23	**16**	-0,9913	526,18	0,9997	
24	**17**	-0,9976	-136,08	0,9998	
25	**18**	-1,0061	251,55	0,9997	
26	**19**	-1,0043	169,46	0,9996	
27	**20**	-0,9981	-168,80	0,9997	
997	**990**	-0,9765	20.749,31	0,9817	
998	**991**	-0,9791	335,58	0,9971	
999	**992**	-1,0045	-123,62	0,9996	
1000	**993**	-0,9887	784,03	0,9996	
1001	**994**	-0,9994	-26,48	0,9997	
1002	**995**	-0,9680	11.446,98	0,9666	
1003	**996**	-0,9695	878,76	0,9984	
1004	**997**	-0,9584	458,57	0,9984	
1005	**998**	-0,9876	944,24	0,9995	
1006	**999**	-0,9896	17.460,68	0,9611	
1007	**1000**	-0,9624	397,44	0,9881	
1008					
1009	**Average**	-0,9911	1.251,28	0,995501	
1010					

FIGURE 3.12 Screenshot: prospective Regression Analysis.

This is clearly out of the range regarded as being highly effective. So the *Dollar-Offset Ratio* can testify the effectiveness requirements only on a cumulative basis.

Using the VRM, high effectiveness is achieved. The hedge reduces the variance for the combined position compared with the unhedged position by more than 99.50% on average. The worst result of variance reduction is 76.31%. This is below the threshold of 96%, but there are only 36 paths out of 1000 that fail to reduce the variance by at least 96%. For the third method, the *Regression Analysis*, the crucial figures are R^2 and the slope. For R^2, the regression results lie between 85.51% and 99.99%. For the slope of the regression line the results are in a range between -1.1022 and -0.8918. This is slightly more than 10% from the perfect value, but is overall a satisfactory result. The average values are -0.9911 for the slope coefficient and 0.995501 for R^2. This average result gives a clear indication that the hedge is highly effective on a prospective basis.

Summing up, three of the observed four methods result in the hedge fulfilling the test for high prospective effectiveness. Only the *Dollar-Offset Method* on a *day-by-day basis* fails to meet the required effectiveness. Considering this circumstance, it is questionable whether a *Dollar-Offset ratio on a day-by-day basis* makes sense at all. But this result is not surprising. As mentioned in Section 3.1.5 and in the literature, the *Dollar-Offset Method* in general and especially in the period-by-period Dollar-Offset Method is likely to fail.

As entities have to choose one method for testing for effectiveness, it is advisable to choose either the VRM or the *regression analysis*, as these methods appear to give more reasonable and stable results.

Retrospective Test for Effectiveness For the retrospective test for hedge effectiveness, the methods can be applied in the same manner. The only difference is that the exchange rate path does not have to be simulated but is given. Therefore, there is only one path which has to be examined. The results obtained cannot be averaged. For this reason the effectiveness results may be different. As the real exchange rate path is not known in this example, one of the simulated paths is selected randomly as the path that really happened during the time period of the hedge. In the following, four paths are examined, see Figure 3.13. They represent four basic scenarios which might happen at maturity. The first scenario is that the barrier option is knocked out during the hedge and the end value of the Shark Forward Plus is negative. The second scenario is that the barrier is knocked out, as in the first scenario, but the Shark Forward Plus has a positive value at maturity. The third scenario is that the barrier is not knocked out and the Shark Forward Plus has a value of zero at maturity. The last scenario is that the barrier was not knocked out and the Shark Forward Plus value is positive at the end of the hedge.

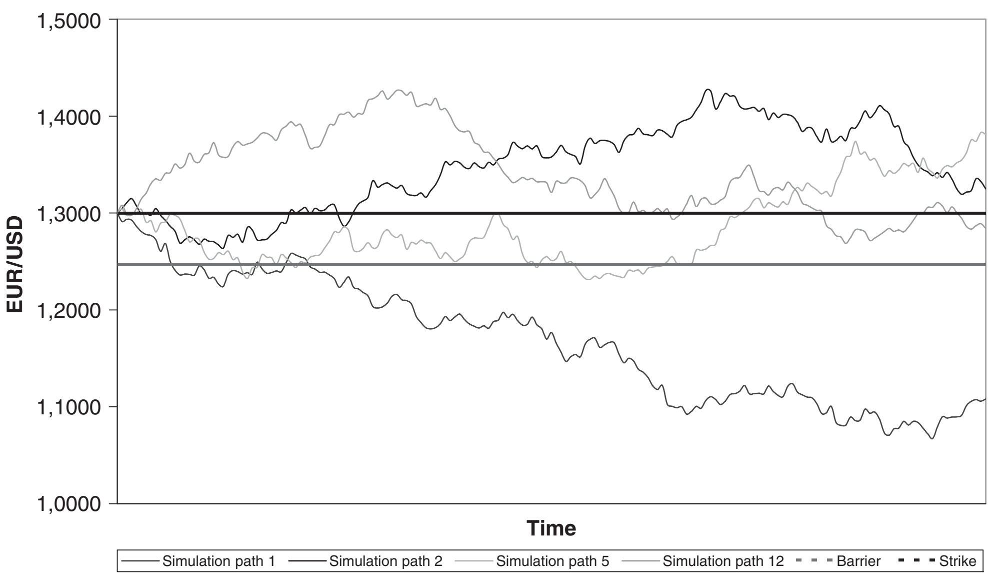


FIGURE 3.13 Selected paths for the retrospective test for effectiveness.

This selection is just an example of possible scenarios. However, the effectiveness is dependent on the exchange rate path, not only on the value of the derivative at maturity. The criteria whether the barrier option is knocked out or not and the value at maturity are possibilities for categorization.

Scenario 1, Simulation Path 1 In the first scenario the barrier option is knocked out. The value of the Shark Forward Plus is negative at maturity of the derivative. Testing the effectiveness using the *Dollar-Offset Method*, the hedge can be classified as highly effective. As visible in Figure 3.14, the cumulative Dollar-Offset ratio is −1.00.

Even the *day-by-day Dollar-Offset ratio* would qualify for a highly effective hedge relationship as all values are within the range [−1.25; −0.8], except the ratio for the first day. There, the ratio is set to zero as the value of the forecast transaction is zero per definition.

For the VRM, the result definitely proves high effectiveness. The hedge reduced the variance nearly 100%. This is demonstrated in Figure 3.15.

5	Timesteps	180					
6	Selected Path:	1					
7	Nr.	FV Value	Delta FV Value	FT Value	Delta FT Value	Ratio	
178	170	-22.063.101,26	1.208.090,59	22.075.074,57	-1.210.009,47	-1,00	
179	171	-21.037.117,62	1.025.983,64	21.047.392,22	-1.027.682,35	-1,00	
180	172	-21.159.889,34	-122.771,71	21.169.075,36	121.683,14	-0,99	
181	173	-21.061.976,75	97.912,58	21.069.977,11	-99.098,25	-1,01	
182	174	-21.030.837,33	31.139,42	21.037.684,46	-32.292,65	-1,04	
183	175	-20.877.558,46	153.278,88	20.883.222,66	-154.461,80	-1,01	
184	176	-19.884.229,07	993.329,39	19.888.544,71	-994.677,94	-1,00	
185	177	-19.434.251,71	449.977,36	19.437.415,11	-451.129,60	-1,00	
186	178	-19.270.097,83	164.153,88	19.272.188,90	-165.226,22	-1,01	
187	179	-19.420.210,86	-150.113,02	19.421.264,50	149.075,61	-0,99	
188	180	-19.197.930,17	222.280,69	19.197.930,17	-223.334,34	-1,00	**-1,00**

FIGURE 3.14 Screenshot: cumulative Dollar-Offset Ratio path 1.

	A	B	C	D	E	F	G	H
1								
2								
3							Calculate	
4								
5	Timesteps	180						
6	Selected Path:	1						
7	Time	Combined	Variance	FT Value	Variance	Ratio		
8	0	-44,11	514.820.862	0,00	40.228.429.099.030	**0,99998720**		
9	1	119.651,52		1.009.572,43				
10	2	122.498,03		775.924,78				
11	3	122.463,52		871.314,92				
12	4	116.655,67		1.511.633,03				
13	5	110.842,15		2.032.367,05				
14	6	107.242,48		2.331.900,95				
15	7	106.108,48		2.444.814,62				
16	8	99.814,99		2.873.768,11				
17	9	78.465,17		4.075.433,15				
18	10	93.527,90		3.282.193,57				
19	11	57.296,22		5.120.815,52				
20	12	54.655,69		6.023.846,49				
21	13	58.623,25		6.499.641,46				
22	14	57.542,84		6.418.114,27				
23	15	57.445,24		6.445.885,00				

FIGURE 3.15 Screenshot: variance reduction measure path 1.

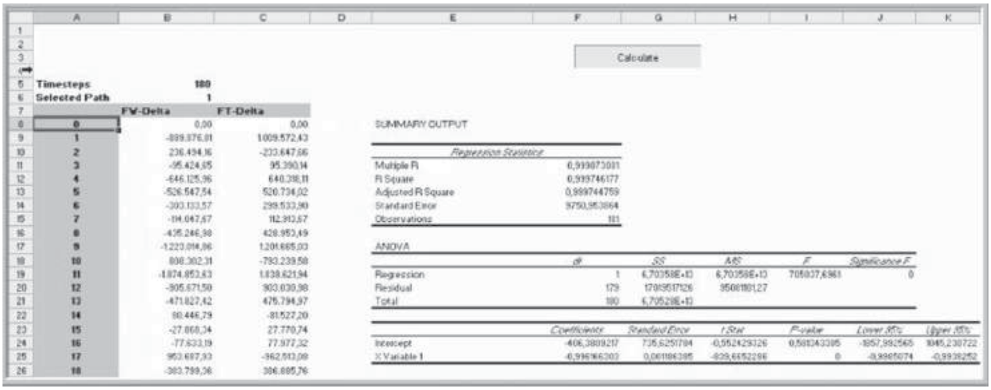


FIGURE 3.16 Screenshot: regression analysis path 1.

The third method introduced for testing the hedge effectiveness, the *Regression Analysis*, leads to the same result. For the first path, the slope is −0.9962 and thereby close to minus one. The crucial measure R^2 explains 99.97% of the data points. The full *Regression Analysis* result can be seen in Figure 3.16.

Summing up, all methods to test for effectiveness come up with a positive assessment result for simulation path one.

Scenario 2, Simulation Path 5 In the second scenario, path five is selected. Within this path the barrier option is knocked out during the hedge. The value of the derivative is positive at the end of the derivative's life.

Following the *period-by-period Dollar-Offset Method*, the hedge would not qualify for hedge accounting. Not only the first ratio, which is zero for the above mentioned reason, but also the second ratio is out of the accepted range. All other ratios would fulfill the demands. The crucial value that would have terminated the hedge accounting treatment in the very beginning of the hedge relationship is shown in Figure 3.17.

Applying the *cumulative Dollar-Offset Method*, the hedge is highly effective. The cumulative Dollar-Offset ratio is −1.00. This is shown in Figure 3.18.

Using the VRM for testing the hedge effectiveness, the calculated ratio of 0.9999 is a clear sign towards high effectiveness. This is illustrated in Figure 3.19.

The *Regression Analysis* also confirms the high effectiveness that is necessary to qualify for hedge accounting. The slope is calculated with a value of −0.9991 and $R^2 =$ 99.98%. The full regression is displayed as a screen shot in Figure 3.20.

Scenario 3, Simulation Path 12 In scenario three, the barrier option is not knocked out. At the end of the hedge relationship the spot exchange rate is below the strike, so that the value of the Forward Plus is zero.

Testing the effectiveness with the *day-by-day Dollar-Offset ratio*, hedge accounting treatment has to be denied. There are multiple cases in which the ratio is not within the

	A	B	C	D	E	F	G	H	I	J
1										
2										
3									Calculate	
4										
5	**Timesteps**	**180**								
6	**Selected Path:**	**5**								
7	**Nr.**	**FV Value**	**Delta FV Value**	**FT Value**	**Delta FT Value**		**Ratio**			
8	**0**	-44,11	0,00	0,00	0,00		0,00			
9	**1**	414.973,89	415.018,00	-286.514,83	-286.514,83		-0,69			
10	**2**	-257.010,23	-671.984,12	382.570,29	669.085,11		-1,00			
11	**3**	-93.266,97	163.743,26	220.806,35	-161.763,94		-0,99			
12	**4**	-48.330,87	44.936,10	177.057,98	-43.748,37		-0,97			
13	**5**	386.834,59	435.165,46	-254.712,07	-431.770,04		-0,99			
14	**6**	-960.659,99	-1.347.494,58	1.083.467,89	1.338.179,95		-0,99			
15	**7**	-802.640,37	158.019,63	927.832,21	-155.635,68		-0,98			
16	**8**	-1.959.189,21	-1.156.548,84	2.071.326,57	1.143.494,36		-0,99			
17	**9**	-1.062.029,19	897.160,01	1.186.141,48	-885.185,09		-0,99			
18	**10**	-890.654,59	171.374,61	1.017.449,59	-168.691,89		-0,98			
19	**11**	-114.356,34	776.298,25	249.191,19	-768.258,40		-0,99			
20	**12**	-225.098,47	-110.742,14	360.022,51	110.831,32		-1,00			
21	**13**	-609.684,90	-384.586,43	742.036,05	382.013,54		-0,99			
22	**14**	-1.783.966,06	-1.174.281,16	1.902.572,45	1.160.536,40		-0,99			
23	**15**	-2.990.775,91	-1.206.809,84	3.089.769,98	1.187.197,52		-0,98			

FIGURE 3.17 Screenshot: dollar-offset ratio path 5.

5	**Timesteps**	**[illegible]0**					
6	**Selected Path:**	**5**					
7	**Nr.**	**FV Value**	**Delta FV Value**	**FT Value**	**Delta FT Value**	**Ratio**	
178	**170**	3.607.195,33	-542.556,41	-3.609.152,90	543.076,11	-1,00	
179	**171**	4.490.047,63	882.852,31	-4.492.240,59	-883.087,69	-1,00	
180	**172**	4.877.493,17	387.445,54	-4.879.610,61	-387.370,03	-1,00	
181	**173**	4.751.701,16	-125.792,01	-4.753.506,09	126.104,52	-1,00	
182	**174**	5.173.654,40	421.953,24	-5.175.338,81	-421.832,73	-1,00	
183	**175**	5.454.262,81	280.608,41	-5.455.742,58	-280.403,77	-1,00	
184	**176**	6.211.639,77	757.376,96	-6.212.987,94	-757.245,36	-1,00	
185	**177**	7.536.993,66	1.325.353,89	-7.538.220,49	-1.325.232,55	-1,00	
186	**178**	7.389.971,82	-147.021,85	-7.390.773,73	147.446,76	-1,00	
187	**179**	8.308.048,62	918.076,81	-8.308.499,38	-917.725,65	-1,00	
188	**180**	8.116.329,29	-191.719,33	-8.116.329,29	192.170,08	-1,00	**-1,00**

FIGURE 3.18 Screenshot: cumulative dollar-offset ratio path 5.

demanded range. Using the cumulative ratio as the method to test for hedge effectiveness, the accepted range is missed as well. This is shown in Figure 3.21.

If the VRM is used to prove hedge effectiveness, the result is different. Following this method, the reduction of the variance is 99.35%. This is sufficiently high to qualify for hedge effectiveness and very close to the previously examined paths which have proven effectiveness.

The *Regression Analysis* also comes to the result that hedge effectiveness holds. The slope of the regression line is −0.9527 and R^2 is 98.27%. Compared with the previously examined paths and the one in the last scenario, these are the worst results for the regression parameters as well as for the Variance Reduction ratio. However, they are still good enough to qualify for hedge accounting treatment of the examined path.

Calculate

Timesteps	180				
Selected Path:	5				
Time	**Combined**	**Variance**	**FT Value**	**Variance**	**Ratio**
0	-44,11	1.311.001.192	0,00	15.935.920.716.922	**0,99991773**
1	128.459,07		-286.514,83		
2	125.560,05		382.570,29		
3	127.539,38		220.806,35		
4	128.727,11		177.057,98		
5	132.122,52		-254.712,07		
6	122.807,89		1.083.467,89		
7	125.191,84		927.832,21		
8	112.137,36		2.071.326,57		
9	124.112,29		1.186.141,48		
10	126.795,01		1.017.449,59		
11	134.834,86		249.191,19		
12	134.924,04		360.022,51		
13	132.351,15		742.036,05		
14	118.606,39		1.902.572,45		
15	98.994,07		3.089.769,98		

FIGURE 3.19 Screenshot: Variance reduction measure path 5.

Calculate

Timesteps	180	
Selected Path	5	
	FV-Delta	**FT-Delta**
0	0,00	0,00
1	415.010,00	-286.514,83
2	-671.984,12	669.085,11
3	163.743,26	-161.763,04
4	44.936,10	-43.748,37
5	435.165,46	-431.770,04
6	-1.347.494,58	1.338.179,95
7	150.019,63	-155.635,69
8	-1.156.548,84	1.143.494,36
9	897.160,01	-885.185,09
10	171.374,61	-168.691,89
11	776.290,25	-768.258,40
12	-110.742,14	110.831,32
13	-394.586,43	392.013,54
14	-1.174.281,16	1.160.536,40
15	-1.296.009,84	1.187.197,52
16	366.408,30	-359.253,91
17	-127.069,20	125.294,37
18	-1.302.476,56	1.276.481,56

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0,999876086
R Square	0,999752047
Adjusted R Square	0,999750661
Standard Error	10867,81735
Observations	181

ANOVA

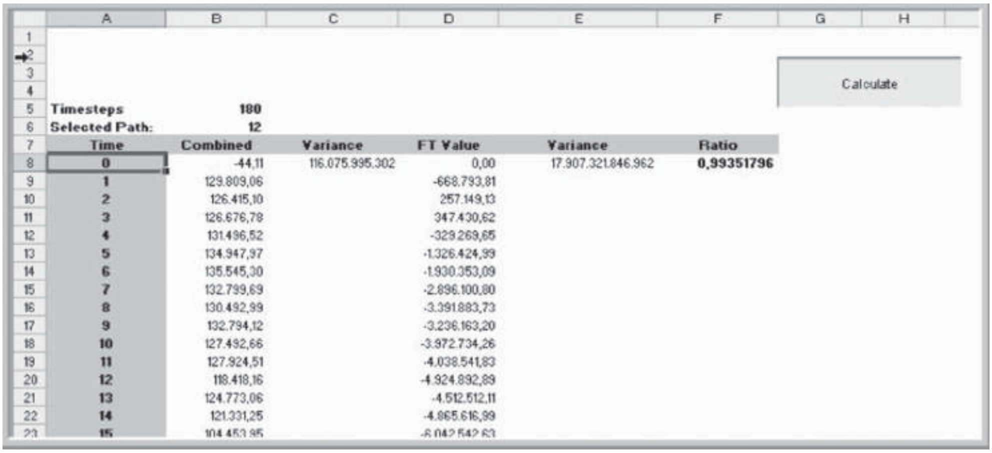
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	8,52433E+13	8,52433E+13	721731,0208	0
Residual	179	21141592241	118109453,3		
Total	180	8,52644E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	37,03650045	809,5183046	0,045751292	0,963559482	-1560,38863	1634,461707
X Variable 1	-0,999179434	0,001176131	-849,5475389	0	-1,001500358	-0,99685863

FIGURE 3.20 Screenshot: regression analysis path 5.

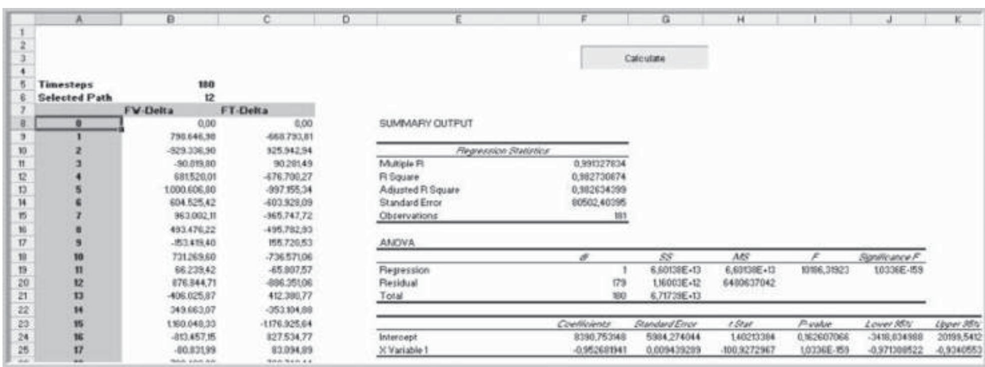
Timesteps	180					
Selected Path:	12					
Nr.	**FV Value**	**Delta FV Value**	**FT Value**	**Delta FT Value**	**Ratio**	
170	1.485.332,16	297.377,73	-1.068.078,51	-477.486,64	-1,61	
171	1.212.453,54	-272.878,62	-719.537,99	348.540,52	-1,28	
172	759.826,60	-452.626,94	-30.689,05	688.848,94	-1,52	
173	1.065.102,20	305.275,60	-616.004,76	-585.315,71	-1,92	
174	676.793,91	-388.308,29	-34.113,78	581.890,97	-1,50	
175	364.583,45	-312.210,46	546.572,73	580.686,52	-1,86	
176	93.017,75	-271.565,70	1.385.549,55	838.976,82	-3,09	
177	33.430,03	-59.587,72	1.670.749,07	285.199,52	-4,79	
178	37.402,03	3.972,00	1.311.698,07	-359.051,00	-90,40	
179	14.934,93	-22.467,10	1.099.277,47	-212.420,60	9,45	
180	0,00	-14.934,93	1.594.112,52	494.835,05	-33,13	**0,00**

FIGURE 3.21 Screenshot: cumulative dollar-offset ratio path 12.



Timesteps	180				
Selected Path:	12				
Time	Combined	Variance	FT Value	Variance	Ratio
0	-44,11	116.075.995.302	0,00	17.907.321.846.962	**0,99351796**
1	129.809,06		-668.793,81		
2	126.415,10		257.149,13		
3	126.676,78		347.430,62		
4	131.496,52		-329.269,65		
5	134.947,97		-1.326.424,99		
6	135.545,30		-1.930.353,09		
7	132.799,69		-2.896.100,80		
8	130.492,99		-3.391.883,73		
9	132.794,12		-3.236.163,20		
10	127.492,66		-3.972.734,26		
11	127.924,51		-4.038.541,83		
12	118.418,16		-4.924.892,89		
13	124.773,06		-4.512.512,11		
14	121.331,25		-4.865.616,99		

FIGURE 3.22 Screenshot: variance reduction measure path 12.



Timesteps	180	
Selected Path	12	
	FV-Delta	FT-Delta
0	0,00	0,00
1	798.646,38	-668.793,81
2	-929.336,90	925.942,94
3	-90.019,80	90.281,49
4	681.520,01	-676.700,27
5	1.000.606,80	-997.155,34
6	604.525,42	-603.929,09
7	963.002,11	-965.747,72
8	493.476,22	-495.782,93
9	-153.419,40	155.720,53
10	731.269,60	-736.571,06
11	66.239,42	-65.807,57
12	876.844,71	-886.351,06
13	-406.025,87	412.380,77
14	349.663,07	-353.104,88
15	1.160.048,33	-1.176.925,64
16	-813.457,15	827.534,77
17	-80.831,99	83.094,89

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0,991327834
R Square	0,982730674
Adjusted R Square	0,982634399
Standard Error	80502,40395
Observations	181

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	6,60138E+13	6,60138E+13	10186,31923	1,0336E-159
Residual	179	1,16003E+12	6480637042		
Total	180	6,71739E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	8390,753148	5984,274044	1,40213384	0,162607066	-3418,034988	20199,5412
X Variable 1	-0,952681941	0,009439289	-100,9272967	1,0336E-159	-0,971308522	-0,9340553

FIGURE 3.23 Screenshot: regression analysis path 12.

The results of the VRM and the *Regression Analysis* are shown in Figure 3.22 and Figure 3.23.

Scenario 4, Simulation Path 2 The last scenario examines simulation path two. Within this path, the barrier option is not knocked out. The value of the Shark Forward Plus is positive at maturity.

Assessing the hedge effectiveness with the *day-by-day Dollar-Offset Method*, the preferred accounting treatment cannot be employed. There are several ratios that are out of the accepted range. It is important to note that these ratios depend on the simulation path and are not necessarily characteristic for this scenario. The *cumulative Dollar-Offset Method* results in a ratio of −1.00, which would qualify for hedge accounting in contrast to the previously mentioned method. The result is displayed in Figure 3.24.

5	Timesteps	180					
6	Selected Path:	2					
7	Nr.	FV Value	Delta FV Value	FT Value	Delta FT Value	Ratio	
178	170	4.164.223,05	291.568,41	-4.142.049,98	-311.586,71	-1,07	
179	171	3.696.962,33	-467.260,72	-3.667.080,39	474.969,58	-1,02	
180	172	4.182.929,14	485.966,81	-4.173.250,02	-506.169,63	-1,04	
181	173	3.499.992,77	-682.936,38	-3.481.598,94	691.651,08	-1,01	
182	174	2.538.497,38	-961.495,38	-2.487.535,92	994.063,02	-1,03	
183	175	1.996.025,77	-542.471,61	-1.919.406,48	568.129,43	-1,05	
184	176	2.189.685,16	193.659,38	-2.156.046,18	-236.639,70	-1,22	
185	177	2.316.187,20	126.502,04	-2.304.673,89	-148.627,71	-1,17	
186	178	3.532.911,82	1.216.724,62	-3.533.260,47	-1.228.586,58	-1,01	
187	179	3.236.600,29	-296.311,52	-3.236.775,72	296.484,74	-1,00	
188	180	2.498.527,35	-738.072,95	-2.498.527,35	738.248,37	-1,00	**-1,00**

FIGURE 3.24 Screenshot: cumulative dollar-offset ratio path 2.

	A	B	C	D	E	F	G	H
1								
2								
3							Calculate	
4								
5	Timesteps	180						
6	Selected Path:	2						
7	Time	Combined	Variance	FT Value	Variance	Ratio		
8	0	-44,11	6.115.629.465	0,00	20.007.647.521.951	**0,99969434**		
9	1	128.681,68		-339.913,24				
10	2	131.420,05		-941.384,29				
11	3	132.855,58		-1.360.061,03				
12	4	132.578,69		-609.732,00				
13	5	129.325,08		224.226,72				
14	6	130.690,41		159.167,38				
15	7	130.324,07		340.555,63				
16	8	135.495,87		-335.643,81				
17	9	131.964,67		369.331,77				
18	10	128.187,18		884.555,01				
19	11	121.853,24		1.506.210,21				
20	12	115.586,36		2.011.868,66				
21	13	95.871,68		3.212.865,04				
22	14	102.277,67		2.888.104,82				
23	15	108.148,63		2.587.463,57				

FIGURE 3.25 Screenshot: variance reduction measure path 2.

Using the VRM to test for hedge effectiveness, the result is, as in all previously examined simulation paths clearly above the threshold to testify high effectiveness. The value of the calculated ratio is 0.9997. This is displayed in Figure 3.25.

Following the *Regression Analysis*, the necessary hedge effectiveness to qualify for hedge accounting holds as well.

The regression parameters slope and R^2 take the values −0.9899 and 99.94%, respectively. The complete *Regression Analysis* result is displayed in Figure 3.26.

3.1.7 Conclusion

IAS 39 formulates broad regulations for the treatment of financial instruments. These include the recognition as well as the initial and subsequent measurement of financial

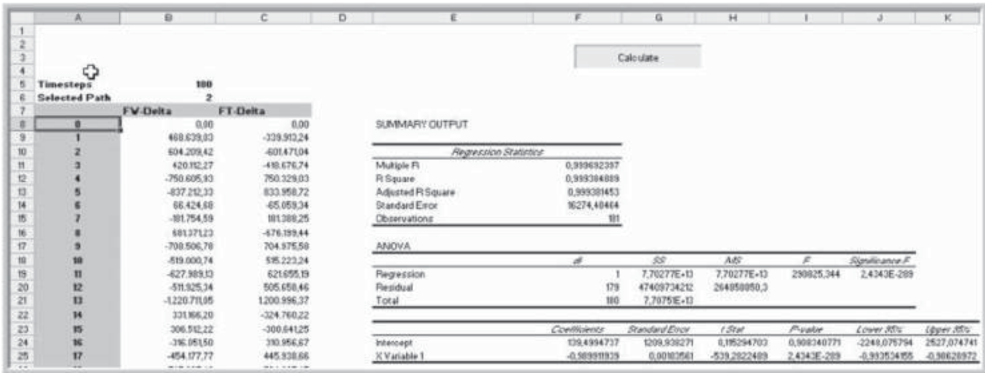


FIGURE 3.26 Screenshot: regression analysis path 2.

assets and financial liabilities. In Section 3.1.2 and Section 3.1.3 the most important regulations concerning financial instruments under IAS were introduced and made comprehensible. These regulations form the basis of the main topic of this section: hedge accounting. This is an important topic for most companies. Hedge accounting means that the hedged item, which is a financial asset or a financial liability, is treated in another way under certain circumstances. The relation of the hedged item and the hedging instrument is in the scope. The regulations concerning hedge accounting partly overrule the general regulations for financial instruments accounting and are again extensive. In Section 3.1.4 and Section 3.1.5, these regulations were reviewed in more depth. The crucial part of the substantial prerequisites for hedge accounting treatment is the high effectiveness, which has to be proven to exist on a prospective as well as on a retrospective basis.

This issue and possible methods for measurement of the effectiveness were discussed in theory and in examples. In the case study, the question as to whether the prospective and retrospective hedge effectiveness exists is tested in the framework of a cash flow hedge for a foreign exchange forecast transaction that is hedged with a Shark Forward Plus.

As an overall result, the hedge effectiveness for the example could be confirmed for both the prospective and the retrospective hedge effectiveness. One critical issue is the choice of an appropriate method to test for the effectiveness. The VRM as well as the *Regression Analysis* deliver stable results for both tests. The *Dollar-Offset Method* is likely to fail the test, especially when using the period-by-period method instead of the cumulative method.

Summing up, hedge accounting is possible for the Shark Forward Plus. However, the choice of the method to test for effectiveness is crucial for the result. For the entity it is also a question of the effort that is required to set up the more complicated methods like the VRM or *Regression Analysis*, compared with the simpler *Dollar-Offset Method*. Within this context the effort is often rewarded with a better result for the desired hedge effectiveness.

3.1.8 Relevant Original Sources for Accounting Standards

1. International Accounting Standards Board,
International Accounting Standard 1, Financial Instruments:
Disclosure and Presentation, (1997), as at July 1997
2. International Accounting Standards Board,
International Accounting Standard 32, Financial Instruments:
Disclosure and Presentation, (2004), as at 31 March 2004
3. International Accounting Standards Board,
International Accounting Standard 39, Financial Instruments:
Recognition and Measurement, (2004), as at 31 March 2004
4. International Accounting Standards Board,
Guidance of Implementing International Accounting Standard 39,
Financial Instruments: Recognition and Measurement, (2004), as of 31 March 2004

3.2 HEDGE ACCOUNTING UNDER IFRS 9

In this section we will provide an overview about hedge accounting under IFRS 9 and then test the effectiveness of a Forward Plus in a case study. This is based on Kazmaier [85]. New hedge accounting principles have been published by *International Financial Reporting Standards* in IFRS 9 on *Financial Instruments* [1]. The *International Accounting Standards Board (IASB)* has determined that IFRS 9 "shall be applied by all entities to all types of financial instruments."

3.2.1 Hedge Effectiveness

IFRS 9 defines hedge (in)effectiveness as follows:

> Hedge effectiveness is the extent to which changes in the fair value or the cash flows of the hedging instrument offset changes in the fair value or the cash flows of the hedged item (for example, when the hedged item is a risk component, the relevant change in fair value or cash flows of an item is the one that is attributable to the hedged risk). Hedge ineffectiveness is the extent to which the changes in the fair value or the cash flows of the hedging instrument are greater or less than those on the hedged item.[3]

The major difference and advantage of IFRS 9 in comparison with IAS 39 is that the effective component can be accounted for in OCI, whereas the ineffective part is booked directly into P&L. IAS 39 did not allow a split into OCI and P&L in the case of ineffectiveness. In addition, as a result of the regression analysis, the 80–125% range, was not supposed to be exceeded. In cases where values were below 80% or above 125%, the complete amount would be qualified as ineffective. The next section explores hedge effectiveness in more detail.

[3]cf. IASB, 2015, IFRS 9.B6.4.1

3.2.2 Documentation and Qualifying Criteria

The new hedge accounting requirements, including the hedge effectiveness assessment, must be documented. A summary of all the documentation and qualification requirements for IFRS 9 for documentation is outlined below in direct quotations as this is the central requirement of the standard. As auditors have the last word on the hedge effectiveness assessment, this shall help identify the minimum analysis requirements for hedge effectiveness assessment and which requirements need to be verified by the auditor. A hedging relationship can qualify for hedge accounting only if all of the following criteria of IFRS 9.6.4.1 are met:[4]

(a) the hedging relationship consists only of eligible hedging instruments and eligible hedged items.
(b) at the inception of the hedging relationship there is formal designation and documentation of the hedging relationship and the entity's risk management objective and strategy for undertaking the hedge. That documentation shall include identification of the hedging instrument, the hedged item, the nature of the risk being hedged and how the entity will assess whether the hedging relationship meets the hedge effectiveness requirements (including its analysis of the sources of hedge ineffectiveness and how it determines the hedge ratio).
(c) the hedging relationship meets all of the following hedge effectiveness requirements:
  (i) there is an economic relationship between the hedged item and the hedging instrument (see paragraphs B6.4.4–B6.4.6);
  (ii) the effect of credit risk does not dominate the value changes that result from that economic relationship (see paragraphs B6.4.7–B6.4.8); and
  (iii) the hedge ratio of the hedging relationship is the same as that resulting from the quantity of the hedged item that the entity actually hedges and the quantity of the hedging instrument that the entity actually uses to hedge that quantity of hedged item. However, that designation shall not reflect an imbalance between the weightings of the hedged item and the hedging instrument that would create hedge ineffectiveness (irrespective of whether recognised or not) that could result in an accounting outcome that would be inconsistent with the purpose of hedge accounting (see paragraphs B6.4.9–B6.4.11).[5]

Furthermore, IFRS 9.B6.5.21 states: "When re-balancing a hedging relationship, an entity shall update its analysis of the sources of hedge ineffectiveness that are expected to affect the hedging relationship during its (remaining) term (see paragraph B6.4.2). The documentation of the hedging relationship shall be updated accordingly."

3.2.3 Case Study: Shark Forward

We consider an example where party A is exposed to an appreciating USD relative to EUR. Although the entity wants to hedge against an increasing EUR/USD rate, it wishes

[4]cf. IASB, 2015, IFRS 9.6.4.1
[5]cf. IASB, 2015, IFRS 9.6.4.1

TABLE 3.4 Example of shark forward plus as a basis for IFRS 9 hedge accounting.

Spot reference	1.0800 EUR-USD
Outright forward reference	1.0700 EUR-USD
Party A sells, bank B buys	USD 100,000,000
Settlement	Delivery
Maturity	6 months
Worst case	1.0800 EUR-USD
American style trigger	1.0500 EUR-USD
Premium	0.00

to be flexible enough to participate in favorable exchange rates. The entity is willing to accept an exchange rate below the outright forward rate as the worst case scenario. To hedge this FX exposure, A may enter into a shark forward plus (see Section 2.1.6) with the terms shown in Table 3.4.

Generally, the shark forward plus (also called forward plus, forward extra, enhanced forward, or forward with profit potential) is suitable for entities that want to fix a forward price while they can still benefit from a spot movement in which they take a view. This type of instrument provides some potential for limiting possible losses with the level near the forward rate. On the maturity day, party A sells USD 100 M to bank B. The exchange rate applied depends on the path of the spot during the time till maturity. The following scenarios are possible at maturity:

1. If the spot at maturity is above 1.0800 EUR-USD or if the trigger 1.0500 EUR-USD has been breached during the lifetime of the contract, party A sells the notional at the worst case of 1.0800 EUR-USD.
2. If the spot at maturity is between the trigger rate of 1.0500 EUR-USD and 1.0800 EUR-USD, and the trigger of 1.0500 EUR-USD has not been breached during the lifetime of the contract, the spot rate at maturity applies.
3. If the spot at maturity is equal to or smaller than the trigger of 1.0500 EUR-USD, the worst case of 1.0800 EUR-USD applies.

The shark forward plus enables a participation down to the trigger level and guarantees a worst case rate which is slightly higher than the market outright forward rate. If the spot rate at maturity is lower than the trigger level, the entity does not participate in a favorable spot. For a better understanding of this hedging relationship, we summarize some relevant points before we present the minimum documentation requirements.

Hedge Effectiveness

Economic Relationship The critical terms match. In comparison with a plain vanilla forward, the shark forward plus allows the hedging entity to participate in a favorable spot movement until the trigger is touched. However, the hedging instrument and the hedged item do move in opposite directions.

Sources of Ineffectiveness If the exchange rate for the evaluation of the shark forward plus changes, the hedging instrument and the hedged item will not move with the same value in opposite directions.

Decomposition This is about designating the hedging instrument in its entirety or considering the embedded synthetic forward contract separately. The shark forward plus can be treated in exactly the same way as the plain vanilla forward. To date, it is not clear from the standard or the commentaries how exactly the embedded synthetic forward contract could be separated in practice. Therefore, party A chooses to consider the shark forward plus in its entirety.

Qualitative Hedge Effectiveness Assessment The critical terms match, as in the case of a vanilla option, until the barrier is breached, and as in the case of an outright forward contract after the barrier is breached. The probability of breaching depends on the model applied and the probability measure in use. In the worst case, if the spot is lower than the trigger rate, the critical terms are still closely aligned.

Quantitative Hedge Effectiveness Assessment The instrument's value is path-dependent. The economic relationship exists (qualitative proof) and at hedge initiation the shark forward plus is fully effective and can be compared with a plain vanilla call instrument at that point. Assuming the spot moves below the trigger rate and causes the instrument to lock in the worst case, the effect for hedge accounting is no different than with an outright forward contract. Using a Monte Carlo simulation for conducting a regression analysis could provide details about the prospective hedge effectiveness as we have seen in Section 3.1. As a deviation of 1% could make the hedge ineffective, a Monte Carlo simulation could offer more exact values. However, this calculation would not necessarily give any added value in proving hedge effectiveness for this product under IFRS 9. The hedge ineffective part of the relationship between the hedged item and the hedging instrument can be calculated via the dollar-offset method in exactly the same way as with a plain vanilla call option. If the auditor is not clear about the worst case effect and the impact on P&L, a scenario analysis could help a better understanding of the best and worst case valuation to determine how much the movements could impact P&L. If the auditor sets a bright line for risk management, assuming 65–140% (which is 80–125% under IAS 39), no Monte Carlo simulation is necessary in the first place. The dollar-offset method in combination with the worst case scenario could give a result. If the dollar-offset method were to result in 60%, party A could first think about setting a trigger at a more favorable level or could continue quantitative testing with IAS 39 using a Monte Carlo simulation. In my opinion, the minimum requirement does not include a quantitative hedge effectiveness assessment. Using the dollar-offset method to calculate the ineffectiveness satisfies the minimum requirement and could be complemented by a scenario analysis for the worst case.

Minimum Documentation Requirements According to IFRS 9 We now present the minimum documentation requirements for the shark forward plus. The same would apply to a vanilla USD put option, except for the passages highlighted in bold.

Risk Management Strategy and Objective for Undertaking the Hedge The FX risk management strategy is to decrease P&L volatility. The hedging instruments to be used are options and forwards (**with and without participation**). The risk management objective is to protect against an increase in EUR-USD of the highly likely sale of a machine XYZ for USD 100 M. The risk management objective is in line with the risk management strategy.

Type of Hedge Cash flow hedge

Nature of Risk being Hedged FX exposure

Identification of the Hedged Item USD 100 M sale of machine XYZ expected to be delivered on 30 June and to be paid for on 30 June. The forecast sale being highly probable, the contract is signed on 1 January and the hedge is started on 1 January. The liquidity of the entity is guaranteed to finalize the machine on time and the customer has a consistent previous history of paying for similar-sized machines. The entity is able to produce the machine by its expected delivery date. For the avoidance of doubt, the ensuing receivables will not be part of the hedging relationship. The hedged item is eligible for hedge accounting.

Identification of the Hedging Instrument The FX shark forward plus contract is a derivatives instrument. The main terms of this contract are a notional of USD 100 M, **a 1.0800 EUR-USD worst case, a 1.0500 EUR-USD trigger** and a six-month maturity. The term sheet and/or deal confirmation should be enclosed to simplify the documentation. The counterparty to the shark forward plus is bank B. The credit risk associated with this counterparty is considered to be very low. The hedging instrument is eligible for hedge accounting.

Hedge Effectiveness Assessment Party A performs the hedge effectiveness assessment at hedge inception, at each reporting date and when the circumstances of the hedging relationship change significantly. To assess whether there is an economic relationship between the hedged item and the hedging instrument, a qualitative assessment is always performed supported by a scenario analysis. The critical terms method is applied. The critical terms of the hedged item and the hedging instrument match. **In the worst case scenario of the hedging instrument, the critical terms are still closely aligned.** The credit risk of the counterparty of the hedging instrument will be continuously monitored. The hedge's effective and ineffective parts will be determined by comparing changes, from the start of the hedging relationship, in the fair value of the hedging instrument to changes in the fair value of a hypothetical derivative. The terms of the hypothetical derivative will be equal to those of the forecast cash flow. The effective part of the hedge will be booked into *other comprehensive income (OCI)* and reclassified to P&L when the cash inflow of the hedged item takes place which is accounted for in P&L. The ineffective part of the hedge will be recognized in P&L. The forward points of both the hedging instrument and the expected cash flow are included in the assessment.

Overall Assessment The hedging relationship was considered effective as all the following requirements were met:

1. There is an economic relationship between the hedged item and the hedging instrument as the critical terms (nominal amount, maturity, and the underlying) match. Based on the qualitative assessment, supported by a scenario analysis, party A concludes that the change in fair value of the hedged item is expected to be substantially offset by the change in fair value of the hedging instrument. This result affirms that the hedged item and the hedging instrument generally move in opposite directions.
2. The effect of credit risk did not dominate the value changes resulting from the economic relationship as the credit ratings of both party A and bank B were considered sufficiently strong.
3. The 1:1 hedge ratio of the hedging relationship is the same as that resulting from the quantity of the hedged item that party A actually hedges and the quantity of the hedging instrument that party A actually uses to hedge that quantity. The hedge ratio is not intentionally weighted to create imbalances and hence ineffectiveness.

Advantages

1. Zero-cost structure
2. Guaranteed worst case
3. Participation in favorable spot movement
4. Easy handling for hedge accounting
5. High offset possible

Disadvantages

1. Limited participation in favorable spot movement
2. Worst case less favorable than the market outright forward rate

3.2.4 Conclusion and Outlook

E&Y states in [143], p. 3, that under the new standard, hedge effectiveness testing will be simpler as it will only be required on a prospective basis. In comparison with IAS 39, IFRS 9 does not require retrospective testing and in general no quantitative hedge effectiveness testing; there is no challenge to demonstrate effectiveness within a defined quantitative band. In particular the IAS 39 tolerance band of 80–125% no longer applies. The requirement is to demonstrate an economic relationship between the hedged item and the hedging instrument with consideration of credit risk and that the designated hedge ratio is appropriate. The cases made clear that more qualitative documentation is required. There exist exceptions for more complex hedging instruments which can cause unlimited losses, where quantitative methods are useful. There have been some suggestions that IFRS 9 only requires qualitative testing, but this cannot be confirmed as the entity always needs to use the dollar-offset method to calculate the ineffective part. In particular, the fact that the bright line and obligatory quantitative testing have been, more or less, abandoned is a great incentive to get more instruments qualified under hedge accounting. As seen in the example of the shark forward plus, it would be easier for even a barrier product or a structured and exotic product in general to achieve hedge effectiveness under IFRS 9 than was the case with IAS 39. With IFRS 9, only the fair value changes and the ineffective part need to be calculated.

CHAPTER 4

Foreign Exchange Markets

In this chapter we cover several topics relevant for the FX derivatives market, including the traders' rule of thumb to price exotics with vanna and volga, issues on systems, bid-ask spreads, and a glossary and some hopefully useful summary tables.

4.1 VANNA-VOLGA PRICING

Vanna-volga pricing is a traders' rule of thumb to determine the cost of risk managing the volatility risk of exotic options with vanilla options. This cost is then added to the theoretical value in the Black-Scholes model and is called the *overhedge*. The method has been described in [93] and [136]. We explain the rule and then consider an example of a one-touch introduced in Section 1.7.4.

Delta and vega are the most relevant sensitivity parameters for foreign exchange options maturing within one year. A delta-neutral position can be achieved by trading in the spot. Changes in the spot are explicitly modeled in the Black-Scholes model. Therefore, model and practical trading have very good control over spot change risk. The more sensitive part is the vega position. A change in vega is not part of the Black-Scholes model. Market participants need to trade other options to obtain a vega-neutral position. However, even a vega-neutral position is subject to changes of spot and volatility. For this reason, the sensitivity parameters *vanna* (change of vega due to change of spot, *d-vega-d-spot*) and *volga* (change of vega due to change of volatility, *d-vega-d-vol*) are of special interest. The plots for vanna and volga for a vanilla option are displayed in Figure 4.1 and figure 4.2 respectively. The risk of changing interest rates is typically ignored, because the rule is used for short-term exotics where volatility risk dominates over the interest rate risk. In this section we outline how the cost of such a vanna and volga exposure can be used to obtain indication for the price and an exotic derivative that is closer to the market than their *TV*, the theoretical value calculated in the Black-Scholes model using the ATM volatility.

4.1.1 Cost of Vanna and Volga

We fix the rates r_d and r_f, the time to maturity T and the spot x and define

$$\text{cost of vanna} \triangleq \text{Vanna ratio} \times \text{overhedge of RR}, \tag{1}$$

$$\text{cost of volga} \triangleq \text{Volga ratio} \times \text{overhedge of BF}, \tag{2}$$

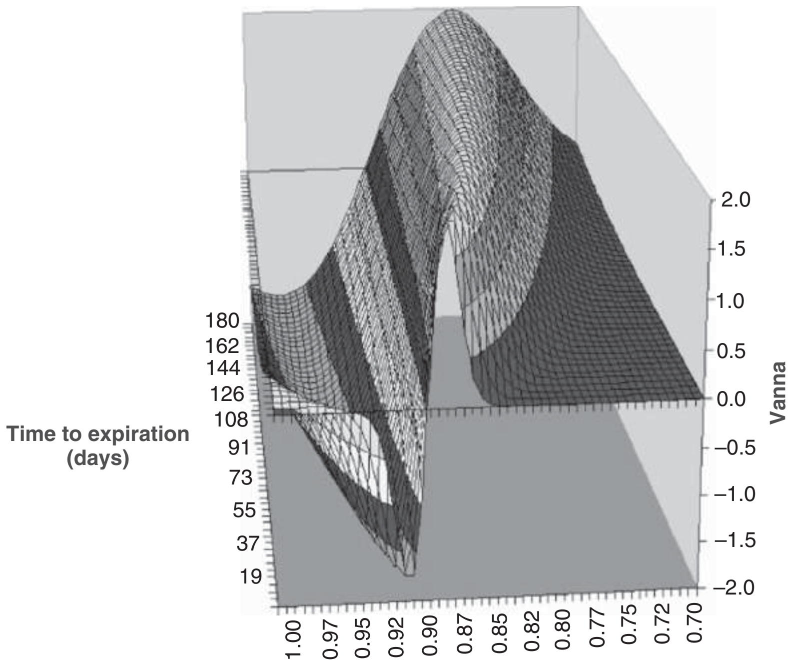


FIGURE 4.1 Vanna of a vanilla option as a function of spot and time to expiration, showing the skew symmetry about the at-the-money line.

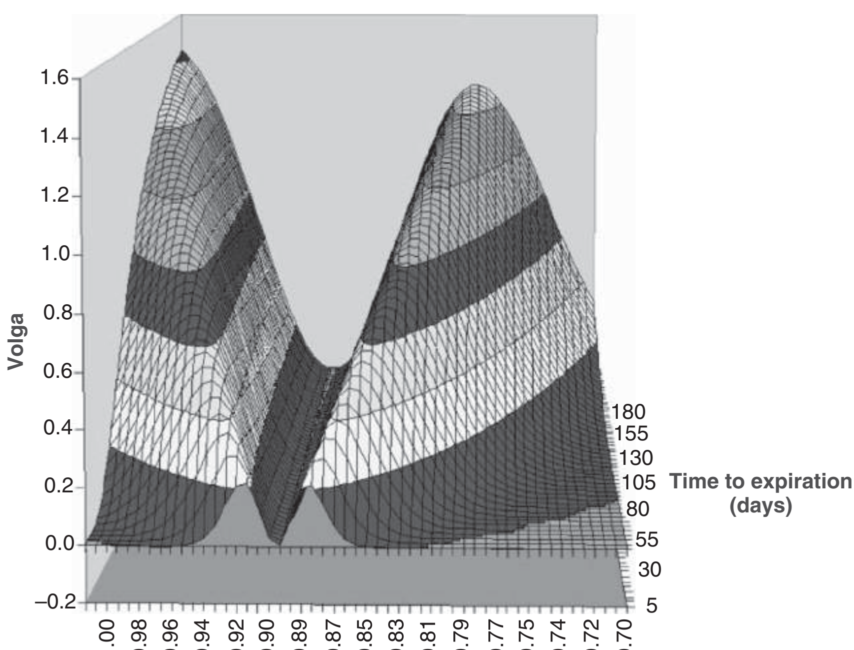


FIGURE 4.2 Volga of a vanilla option as a function of spot and time to expiration, showing the symmetry about the at-the-money line.

$$\text{Vanna Ratio} \stackrel{\Delta}{=} B_{\sigma x}/\text{RR}_{\sigma x}, \tag{3}$$

$$\text{Volga Ratio} \stackrel{\Delta}{=} B_{\sigma\sigma}/\text{BF}_{\sigma\sigma}, \tag{4}$$

$$\text{overhedge of RR} \stackrel{\Delta}{=} [\text{RR}(\sigma_\Delta) - \text{RR}(\sigma_0)], \tag{5}$$

$$\text{overhedge of BF} \stackrel{\Delta}{=} [\text{BF}(\sigma_\Delta) - \text{BF}(\sigma_0)], \tag{6}$$

where σ_0 denotes the at-the-money volatility and σ_Δ denotes the wing volatility at the delta pillar Δ, B denotes the value function of a given exotic contract. The values of risk reversals and butterflies are defined by

$$\text{RR}(\sigma) \stackrel{\Delta}{=} \text{call}(x, \Delta, \sigma, r_d, r_f, T) - \text{put}(x, \Delta, \sigma, r_d, r_f, T), \tag{7}$$

$$\text{BF}(\sigma) \stackrel{\Delta}{=} \frac{\text{call}(x, \Delta, \sigma, r_d, r_f, T) + \text{put}(x, \Delta, \sigma, r_d, r_f, T)}{2} - \frac{\text{call}(x, \Delta_0, \sigma_0, r_d, r_f, T) + \text{put}(x, \Delta_0, \sigma_0, r_d, r_f, T)}{2}, \tag{8}$$

where vanilla$(x, \Delta, \sigma, r_d, r_f, T)$ means vanilla$(x, K, \sigma, r_d, r_f, T)$ for a strike K chosen to imply $|\text{vanilla}_x(x, K, \sigma, r_d, r_f, T)| = \Delta$ and Δ_0 is the delta that produces the at-the-money strike. The volatility σ_Δ used in the butterfly refers to the market strangle volatility σ_{Str} explained in Equation (150). To summarize we abbreviate

$$\text{c}(\sigma_\Delta^+) \stackrel{\Delta}{=} \text{call}(x, \Delta^+, \sigma_\Delta^+, r_d, r_f, T), \tag{9}$$

$$\text{p}(\sigma_\Delta^-) \stackrel{\Delta}{=} \text{put}(x, \Delta^-, \sigma_\Delta^-, r_d, r_f, T), \tag{10}$$

and obtain

$$\text{cost of vanna} = \frac{B_{\sigma x}}{\text{c}_{\sigma x}(\sigma_\Delta^+) - \text{p}_{\sigma x}(\sigma_\Delta^-)} \left[\text{c}(\sigma_\Delta^+) - \text{c}(\sigma_0) - \text{p}(\sigma_\Delta^-) + \text{p}(\sigma_0)\right], \tag{11}$$

$$\text{cost of volga} = \frac{2B_{\sigma\sigma}}{\text{c}_{\sigma\sigma}(\sigma_\Delta^+) + \text{p}_{\sigma\sigma}(\sigma_\Delta^-)} \left[\frac{\text{c}(\sigma_\Delta^+) - \text{c}(\sigma_0) + \text{p}(\sigma_\Delta^-) - \text{p}(\sigma_0)}{2}\right], \tag{12}$$

where we note that volga of the butterfly should actually be

$$\frac{1}{2}\left[\text{c}_{\sigma\sigma}(\sigma_\Delta^+) + \text{p}_{\sigma\sigma}(\sigma_\Delta^-) - \text{c}_{\sigma\sigma}(\sigma_0) - \text{p}_{\sigma\sigma}(\sigma_0)\right], \tag{13}$$

but the last two summands are close to zero and are hence conveniently ignored. The *vanna-volga adjusted value* of the exotic is then

$$B(\sigma_0) + p \times [\text{cost of vanna} + \text{cost of volga}]. \tag{14}$$

A division by the spot x converts everything into the usual quotation of the price in % of the underlying currency. The cost of vanna and volga is often adjusted by a number

$p \in [0, 1]$, which is often taken to be the risk-neutral no-touch probability. The reason is that in the case of contracts that can knock out, the hedge is not needed any more once the contract has knocked out. The exact choice of p depends on the product to be priced, and would be equal to one if the product has a path-independent payoff. Taking $p = 1$ be default would lead to overestimated overhedges for double-no-touch options as pointed out in [93]. In fact, it is possible to use different p's for vanna and volga.

4.1.2 Observations

1. The overhedges are linear in butterflies and risk reversals. In particular, there is no cost of vanna if the risk reversal is zero and no cost of volga if the butterfly is zero.
2. The overhedges are linear in vanna and volga of the given exotic contract.
3. It is not clear up front which target delta to use for the butterflies and risk reversals. We take a delta of 25% merely on the basis of its liquidity.
4. The prices for vanilla options are consistent with the input volatilities as shown in Figure 4.3, Figure 4.4, and Figure 4.5.
5. The method assumes a zero volga of risk reversals and a zero vanna of butterflies. This way the two sources of risk can be decomposed and hedged with risk reversals and butterflies. However, the assumption is actually not exact. For this reason, the method should be used with a lot of care and causes traders and financial engineers to keep adding exceptions to the standard method. The consistency checks do in fact show that the dots are in some cases slightly off the curve, which can always be corrected by making the dots big enough.

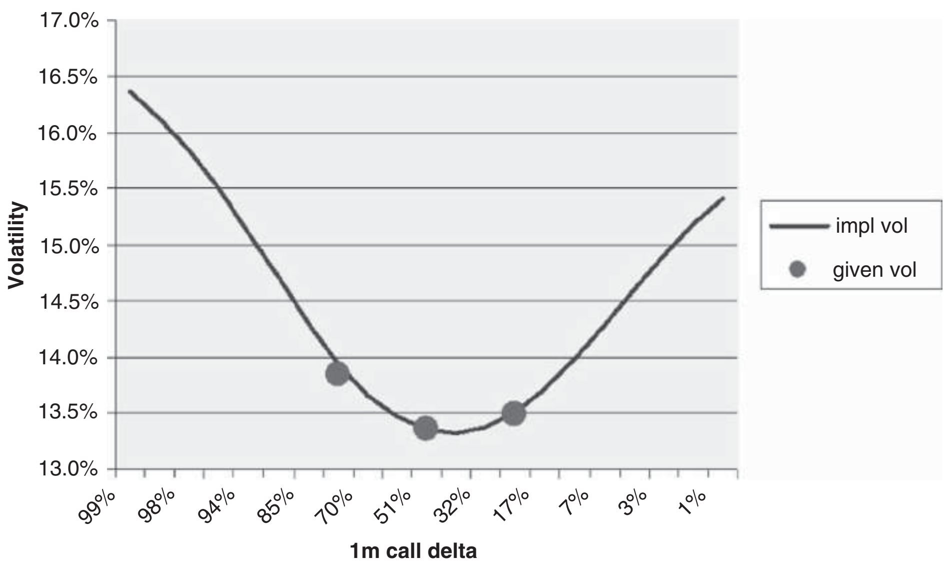


FIGURE 4.3 Consistency check of vanna-volga-pricing. Vanilla option smile for a one month maturity EUR/USD call, spot = 0.9060, $r_d = 5.07\%$, $r_f = 4.70\%$, $\sigma_0 = 13.35\%$, $\sigma_\Delta^+ = 13.475\%$, $\sigma_\Delta^- = 13.825\%$.

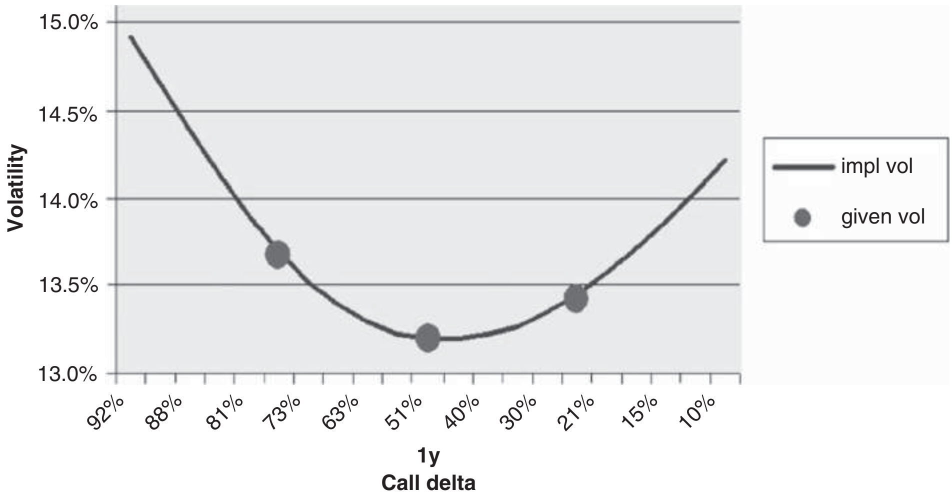


FIGURE 4.4 Consistency check of vanna-volga-pricing. Vanilla option smile for a one-year maturity EUR/USD call, spot = 0.9060, $r_d = 5.07\%$, $r_f = 4.70\%$, $\sigma_0 = 13.20\%$, $\sigma_\Delta^+ = 13.425\%$, $\sigma_\Delta^- = 13.575\%$.

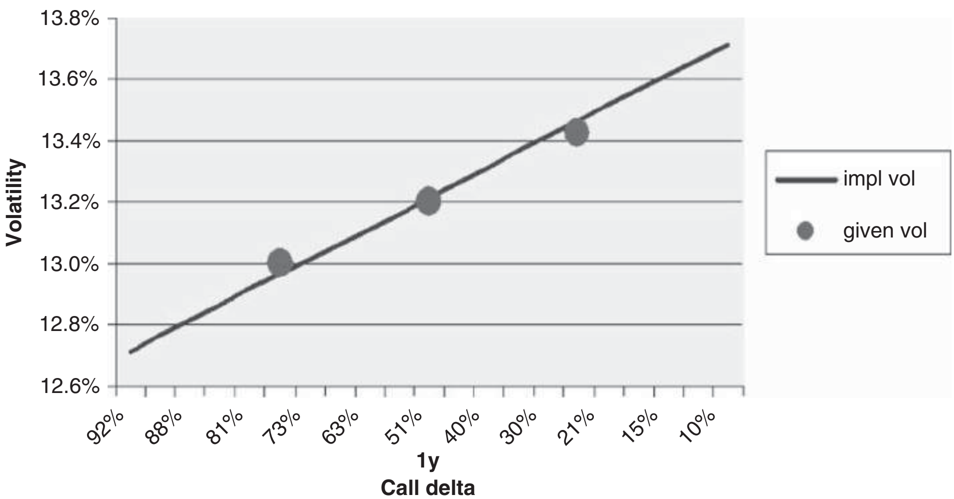


FIGURE 4.5 Consistency check of vanna-volga-pricing. Vanilla option smile for a one-year maturity EUR/USD call, spot = 0.9060, $r_d = 5.07\%$, $r_f = 4.70\%$, $\sigma_0 = 13.20\%$, $\sigma_\Delta^+ = 13.425\%$, $\sigma_\Delta^- = 13.00\%$.

4.1.3 Consistency Check

A minimum requirement for the vanna-volga pricing to be correct is the consistency of the method with vanilla options. We show in Figure 4.3, Figure 4.4, and Figure 4.5 that the method does in fact yield a typical foreign exchange smile shape and produces the correct input volatilities at-the-money and at the delta pillars. In fact, one can use the vanna-volga-pricing approach as an interpolation and extrapolation method to construct a smile curve, as suggested by Castagna and Mercurio in [26]. We will now consider the consistency in the following way. Since the input consists only of three volatilities (at-the-money and two delta pillars), it would be too much to expect that the method produces correct representation of the entire volatility smile curve. We can only check if the values for at-the-money and target-Δ puts and calls are reproduced correctly. In order to verify this, we check if the values for an at-the-money call, a risk reversal, and a butterfly are priced correctly. Surely we only expect approximately correct results. Note that the number p is taken to be 1, which agrees with the risk-neutral no-touch probability for vanilla options.

For an at-the-money call vanna and volga are approximately zero, so there are no supplements due to vanna cost or volga cost.

For a target-Δ risk reversal

$$\mathrm{c}(\sigma_\Delta^+) - \mathrm{p}(\sigma_\Delta^-) \tag{15}$$

we obtain

$$\begin{aligned}
\text{cost of vanna} &= \frac{\mathrm{c}_{\sigma x}(\sigma_\Delta^+) - \mathrm{p}_{\sigma x}(\sigma_\Delta^-)}{\mathrm{c}_{\sigma x}(\sigma_\Delta^+) - \mathrm{p}_{\sigma x}(\sigma_\Delta^-)} \left[\mathrm{c}(\sigma_\Delta^+) - \mathrm{c}(\sigma_0) - \mathrm{p}(\sigma_\Delta^-) + \mathrm{p}(\sigma_0)\right], \\
&= \mathrm{c}(\sigma_\Delta^+) - \mathrm{c}(\sigma_0) - \mathrm{p}(\sigma_\Delta^-) + \mathrm{p}(\sigma_0) \qquad (16) \\
\text{cost of volga} &= \frac{2[\mathrm{c}_{\sigma\sigma}(\sigma_\Delta^+) - \mathrm{p}_{\sigma\sigma}(\sigma_\Delta^-)]}{\mathrm{c}_{\sigma\sigma}(\sigma_\Delta^+) + \mathrm{p}_{\sigma\sigma}(\sigma_\Delta^-)} \\
&\quad \left[\frac{\mathrm{c}(\sigma_\Delta^+) - \mathrm{c}(\sigma_0) + \mathrm{p}(\sigma_\Delta^-) - \mathrm{p}(\sigma_0)}{2}\right], \qquad (17)
\end{aligned}$$

and observe that the cost of vanna yields a perfect fit and the cost of volga is small, because in the first fraction we divide the difference of two quantities by the sum of the quantities, which are all of the same order.

For a target-Δ butterfly

$$\frac{\mathrm{c}(\sigma_\Delta^+) + \mathrm{p}(\sigma_\Delta^-)}{2} - \frac{\mathrm{c}(\sigma_0) + \mathrm{p}(\sigma_0)}{2} \tag{18}$$

we obtain analogously a perfect fit for the cost of volga and

$$\begin{aligned}
\text{cost of vanna} &= \frac{\mathrm{c}_{\sigma x}(\sigma_\Delta^+) - \mathrm{p}_{\sigma x}(\sigma_0) - [\mathrm{c}_{\sigma x}(\sigma_0) - \mathrm{p}_{\sigma x}(\sigma_\Delta^-)]}{\mathrm{c}_{\sigma x}(\sigma_\Delta^+) - \mathrm{p}_{\sigma x}(\sigma_0) + [\mathrm{c}_{\sigma x}(\sigma_0) - \mathrm{p}_{\sigma x}(\sigma_\Delta^-)]} \\
&\quad \left[\mathrm{c}(\sigma_\Delta^+) - \mathrm{c}(\sigma_0) - \mathrm{p}(\sigma_\Delta^-) + \mathrm{p}(\sigma_0)\right], \qquad (19)
\end{aligned}$$

which is again small.

TABLE 4.1 Common abbreviations for FX derivatives and structured products.

VAN	vanilla	DOT	double one-touch
FWD	forward contract	DNT	double no-touch
DIG	European style digital	IOT	instant one-touch
KO	(regular) knock-out	TT	two-touch
KI	(regular) knock-in	TARN	target redemption note
RKO	reverse knock-out	TARF	target forward
RKI	reverse knock-in	TPF	target profit forward
DKO	double knock-out	PTF	pivot target forward
DKI	double knock-in	DCD	dual currency deposit
KIKO	knock-in-knock-out	FVA	forward volatility agreement
OT	one-touch	RAC	range accrual
NT	no-touch		

You may convince yourself as an exercise that the consistency can actually fail for certain parameter scenarios. This is one of the reasons, why the vanna-volga-pricing method has been criticized repeatedly by a number of traders and quants. We summarize the abbreviations for first generation exotics and structured products in Table 4.1.

4.1.4 Adjustment Factor

The factor p has to be chosen in a suitable fashion. Since there is no mathematical justification or indication, there is a lot of dispute in the market about this choice. Moreover, the choices also may vary over time. Most importantly, one must ensure consistency of the method amongst the product range. All the relationships among exotics shown in Figure 4.10 and Table 4.7, which can be found at the very end of the book, must be respected to avoid inconsistencies and arbitrage.

4.1.5 Volatility for Risk Reversals, Butterflies, and Theoretical Value

To determine the volatility, vanna, and volga for the risk reversal and butterfly, the convention is the same as for the building of the smile curve, hence the 25% delta risk reversal retrieves the strike for 25% delta call and put using the spot delta of the relevant delta type (excluded or included depending on the currency pair) and calculates vanna and volga of these options using the corresponding volatilities from the smile.

The theoretical value of the exotics is calculated using the ATM–volatility retrieving it with the same convention that was used to build the smile.

4.1.6 Pricing Barrier Options

For regular knock-out options one can refine the method to incorporate more information about the global shape of the vega surface through time.

We chose M future points in time as $0 < a_1\% < a_2\% < \cdots < a_M\%$ of the time to expiration. Using the same cost of vanna and volga we calculate the overhedge for the regular knock-out with a reduced time to expiration. The factor for the cost is the no-touch probability within the remaining times to expiration $1 > 1 - a_1\% > 1 - a_2\% > \cdots > 1 - a_M\%$ of the total time to expiration. Some desks believe that for at-the-money strikes the long time should be weighted higher and for low delta strikes the short time to maturity should be weighted higher. The weighting can be chosen (rather arbitrarily) as

$$w = \tanh[\gamma(|\delta - 50\%| - 25\%)] \tag{20}$$

with a suitable positive γ. For $M = 3$ the total overhedge is given by

$$\mathrm{OH} = \frac{\mathrm{OH}(1 - a_1\%) \cdot (1 + w) + \mathrm{OH}(1 - a_2\%) + \mathrm{OH}(1 - a_3\%) \cdot (1 - w)}{3}. \tag{21}$$

Which values to use for M, γ, and the a_i, whether to apply a weighting, and what kind varies for different trading desks.

An additional term can be used for single barrier options to account for slippage in the stop–loss of the barrier. The theoretical value of the regular barrier option is determined with a barrier that is moved by four basis points and, 50% of that adjustment is added to the price if it is positive. If it is negative it is omitted altogether. For reverse knock-out options, moving the barrier and barrier bending is a common approach. The theoretical foundation for such a method is explained in [117]. Reverse knock-out options can also be priced using the static replication listed in Table 4.7, which can be found at the very end of the book.

4.1.7 Pricing Double Barrier Options

Double barrier options behave in a similar way to regular knock-out options for a spot near the out-of-the-money barrier and more like reverse knock-out options for a spot close to the in-the-money barrier. Therefore, it appears reasonable to use vanna-volga pricing for the corresponding regular knock-out to determine the overhedge for a spot closer to the out-of-the-money barrier and for the corresponding reverse knock-out for a spot closer to the in-the-money barrier. As a border one may use the mean between strike and the in-the-money barrier.

4.1.8 Pricing Double-No-Touch Contracts

For double-no-touch contracts with lower barrier L and higher barrier H at spot S one can use the overhedge

$$\mathrm{OH} = \max\{\text{Vanna-Volga-OH};\ \delta(S - L) - \mathrm{TV} - 0.5\%;\ \delta(H - S) - \mathrm{TV} - 0.5\%\}, \tag{22}$$

where δ denotes the delta of the double-no-touch (DNT). The DNT can also be statically replicated by double-knock-out options (DKOs), see Table 4.7 at the very end of the book, and hence the consistent vanna-volga price of a DNT can be inferred from the corresponding DKOs.

4.1.9 Pricing Path-Independent Contracts

Contracts with path-independent payoffs can always be approximated or even perfectly replicated by a portfolio of vanilla options. Consequently, their value can be calculated using the MTM of the approximating or replicating portfolio. We consider two examples.

European Digital Options Digital options are priced using the overhedge of the call or put spread with the corresponding volatilities, see Section 1.7.2. The vanna-volga approach can be used, but is not necessary. What really matters is the choice of the interpolation/extrapolation method when constructing the vanilla smile. Vanna-volga is only one of the many such methods.

European Barrier Options European barrier options (EKO) are valued using the MTMs of vanilla and European digital options and the relationship

$$\text{EKO}(\phi, K, B) = \text{vanilla}(\phi, K) - \text{vanilla}(\phi, B) - \text{digital}(B)\phi(B - K). \tag{23}$$

Therefore, as for European digitals, the same principle applies: it all depends on the vanilla smile construction and the interpolation/extrapolation method in place.

4.1.10 No-Touch Probability

The no-touch probability is linked to the value of the no-touch, see Equation (199), and similarly for no-touch probabilities in the case of two barriers. Note that the price of the no-touch can be either calculated using the TV or using an iteration for the no-touch probability. This means that the value of the no-touch used to compute the no-touch probability is itself based on vanna-volga-pricing. Which one is better is hard to say. The latter is the one that appears more consistent and is analytically tractable. One can further challenge the choice of the payoff currency of a no-touch. The probability we infer from the no-touch price depends on this choice, and there is no clear answer which one is preferred. As a compromise one might average the two resulting probabilities.

4.1.11 The Cost of Trading and its Implication on the One-Touch MTM

Now let us take a look at an example of vanna-volga pricing in its simple version. We consider one-touch contracts, which hardly ever trade at TV. The MTM is the sum of the TV and the overhedge. Typical examples are shown in Figure 4.6, one for an upper touch level in EUR-USD, one for a lower touch level.

Clearly there is no overhedge for one-touch contracts with a TV of 0% or 100%, but it is worth noting that a low-TV one-touch can be twice as expensive as their TV, sometimes even more. In between, note that the overhedge can be and usually is positive or negative. Turning the top graph in Figure 4.6 around and letting the TV on the x-axis increase from right to left, and then pasting both the LHS and RHS graph together would typically look like a mustache. A *mustache graph* is a common method

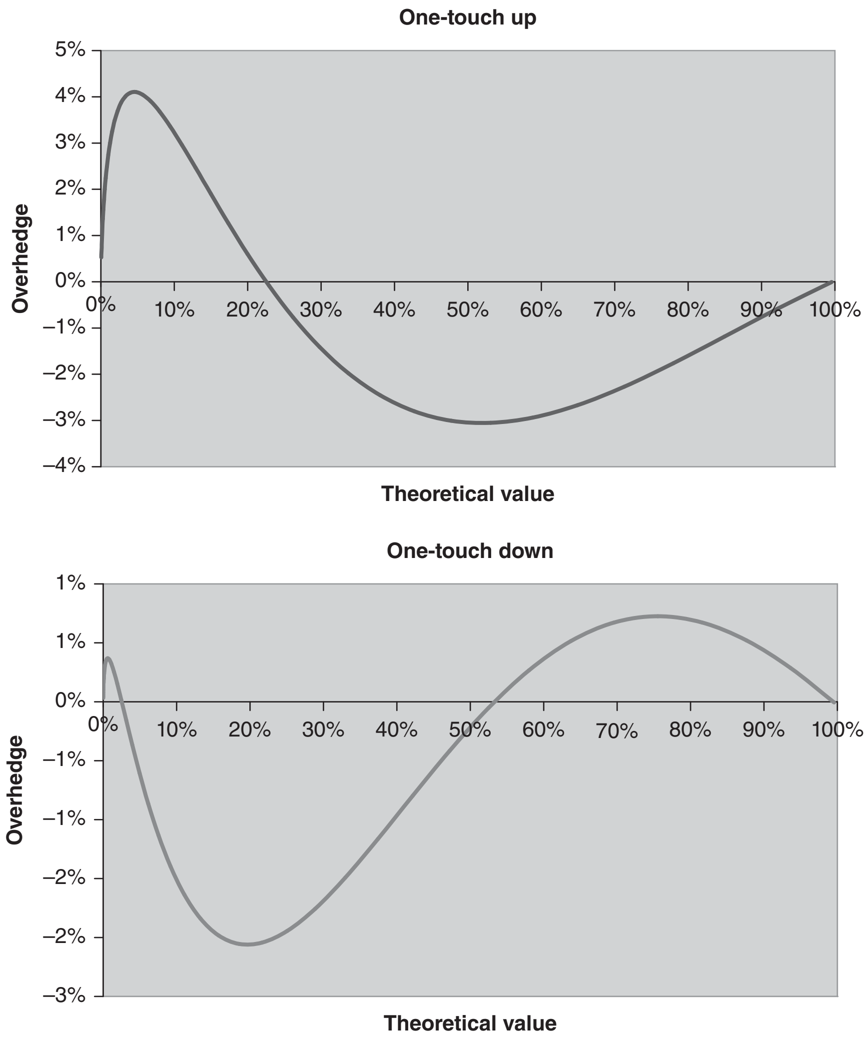


FIGURE 4.6 Overhedge of a one-touch in EUR-USD for both an upper touch level (graph above) and a lower touch level (graph below), based on vanna-volga pricing.

to visualize the deviation of a model value or a market price from the TV. It shows the smile effect. Unlike the smile effect for vanilla options, which is mostly visualized in terms of the implied volatility, the smile effect for exotics is commonly visualized on the value space, because there is no guarantee for a one-to-one correspondence of volatility and value of a contract. Such a correspondence is only guaranteed for values of contracts with convex path-independent payoffs.

The overhedge arises from the cost of risk managing the one-touch. In the Black-Scholes model, the only source of risk is the underlying exchange rate, whereas the volatility and interest rates are assumed constant. However, volatility and rates are themselves changing, so the trader of options is exposed to unstable vega and rho

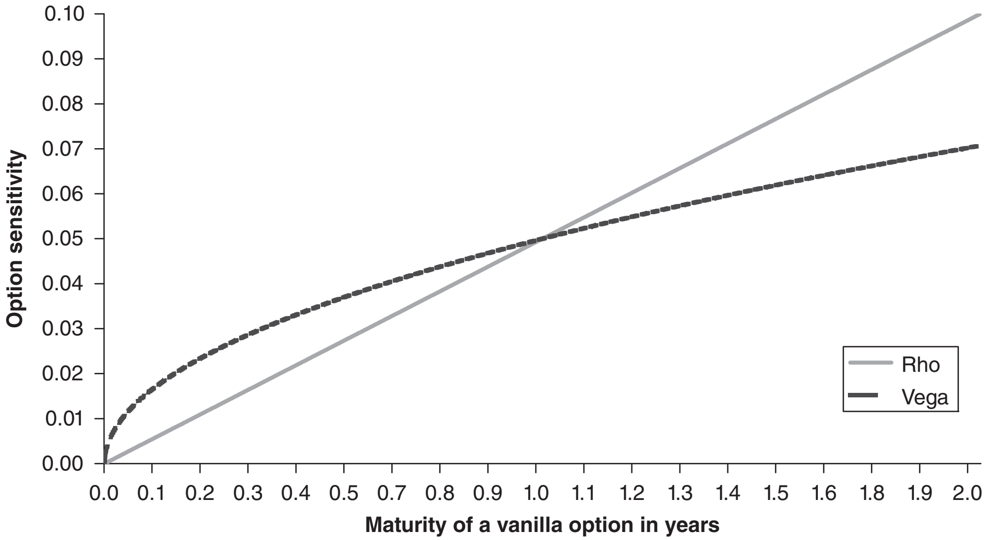


FIGURE 4.7 Comparison of interest rate and volatility risk for a vanilla option. The volatility risk behaves like a square root function (see Equation (24)), whereas the interest rate risk is close to linear (see Equation (29)). Therefore, short-dated FX options have higher volatility risk than interest rate risk.

(change of the value with respect to volatility and rates). For short dated options, the interest rate risk is negligible compared to the volatility risk as shown in Figure 4.7. Hence the overhedge of a one-touch is a reflection of a trader's cost arising from the risk management of his vega exposure.

4.1.12 Example

We consider a one-year one-touch in USD/JPY with payoff currency USD. As market parameters we assume a spot of 117.00 JPY per USD, JPY interest rate 0.10%, USD interest rate 2.10%, volatility 8.80%, 25-delta risk reversal −0.45%,[1] 25-delta butterfly 0.37%.[2] The notion of risk reversals and butterflies is explained in Section 1.5.3.

The touch level is 127.00, and the TV is at 28.8%. If we now hedge only the vega exposure, then we need to consider two main risk factors.

1. The change of vega as the spot changes, often called vanna
2. The change of vega as the volatility changes, often called volga or volgamma or vomma

[1]This means that a 25-delta USD call is 0.45% cheaper than a 25-delta USD put in terms of implied volatility.
[2]This means that a 25-delta USD call and 25-delta USD put is on average 0.37% more expensive than an at-the-money option in terms of volatility.

To hedge this exposure we treat the two effects separately. The vanna of the one-touch is 0.16%, the vanna of the risk reversal is 0.04%. So we need to buy 4 (=0.16/0.04) risk reversals, and for each of them we need to pay 0.14% of the USD amount, which causes an overhedge of −0.6%. The volga of the one-touch is −0.54%, the volga of the butterfly is 0.03%. So we need to sell 18 (=−0.54/0.03) butterflies, each of which pays us 0.23% of the USD amount, which causes an overhedge of −4.1%. Therefore, the overhedge is −4.7%. However, we will get to the touch level with a risk-neutral probability of 28.8%, in which case we would have to pay to unwind the hedge. Therefore the total overhedge is −71.2%*4.7% = −3.4%. This leads to a mid market price of 25.4%. Bid and offer could be 24.25%–26.75%. There are different beliefs among market participants about the unwinding cost. Other observed prices for one-touch contracts can be due to different existing vega profiles of the trader's portfolio, the risk appetite, a hidden additional sales margin, or even the overall condition of the trader in charge.

4.1.13 Further Applications

The method illustrated above shows how important the current smile of the vanilla options market is for the pricing of simple exotics. Similar types of approaches are commonly used to price other exotics. For long-dated contracts the interest rate risk will take over the lead in comparison to short-dated options where the volatility risk is dominant.

4.1.14 Critical Assessment

While the vanna-volga-pricing approach is very intuitive and goes back to determining the cost of hedging for the trading desk, it is prone to generate inconsistencies, as many decisions have to be taken how to use it. It was very popular in the days of pricing platforms coming up at the beginning of the first decade. The calculation time required for values, bid-offer spreads and Greeks is low, which added to its popularity. Traders would still consider a vanna-volga price as a guideline for several first generation exotics. However, the market consensus in the second decade has converged to using the stochastic-local volatility model class to consistently run a valuation in both front-office and all back-office and reporting units of a bank, or are trying to achieve this goal.

4.2 BID-ASK SPREADS

Bid-ask or bid-offer spreads are the price quotes for sellers and buyers of financial assets and derivatives respectively. The spread indicates the sales margin a trading desk earns: the wider the spread, the higher the risk, and/or margin of the product. Wide spreads can also indicate a lack of liquidity or increased risk. Different markets have different spreads. The inter bank market has the tightest spreads, because the banking community normally knows very well how much financial products should cost. Spreads turn to be slightly wider for corporate and institutional clients and very wide for retail clients. There is no fixed rule on how bid-ask spreads should be set up.

In an e-commerce FX options trading environment it is important to set some rules how to compute spreads automatically. One starts with simple and liquid products like the vanilla and one-touch contracts and sets up some rules to derive spreads for other exotics from these basic spreads. For example, it can be done as follows.

4.2.1 Vanilla Spreads

The spreads for vanilla options are usually specified as an ATM *volatility spreads* (spread in terms of implied volatility); they vary over tenor and currency pair. The vanilla *price spreads* are calculated for the maturity pillars {1w, 2w, 1m, 2m, 3m, 6m, 9m, 1y, 18m, 2y} using the ATM volatility and ATM spread, the spot and the respective rates. A corresponding spread matrix expresses half the vanilla spread in terms of volatility for the maturity pillar. In order to calculate the vanilla ATM spread for a maturity between maturity pillars, one can interpolate linearly between the values of vanilla spread. For a given maturity pillar, the spreads for other levels of moneyness are systematically calculated. Keeping the spread constant in terms of volatility will make the price spread smaller on the wings. Remember that vega attains its maximum at-the-money. This is not intended. A trading desk will rather want to keep the price spread similar across all levels of moneyness. On the one hand, risk is reduced for low vega options, so reducing the price spread a bit as a monotone function of vega has become the market standard. The corresponding spreads in terms of volatility will then naturally widen on the wings. However, if we let them widen too much, the volatility may touch or cross the zero line. Therefore, spreads in terms of volatility are kept constant in the far wings, say from the 5-delta region; and spreads are not calculated automatically any more outside the 2-delta region. The resulting spread logic is displayed in Figure 4.8.

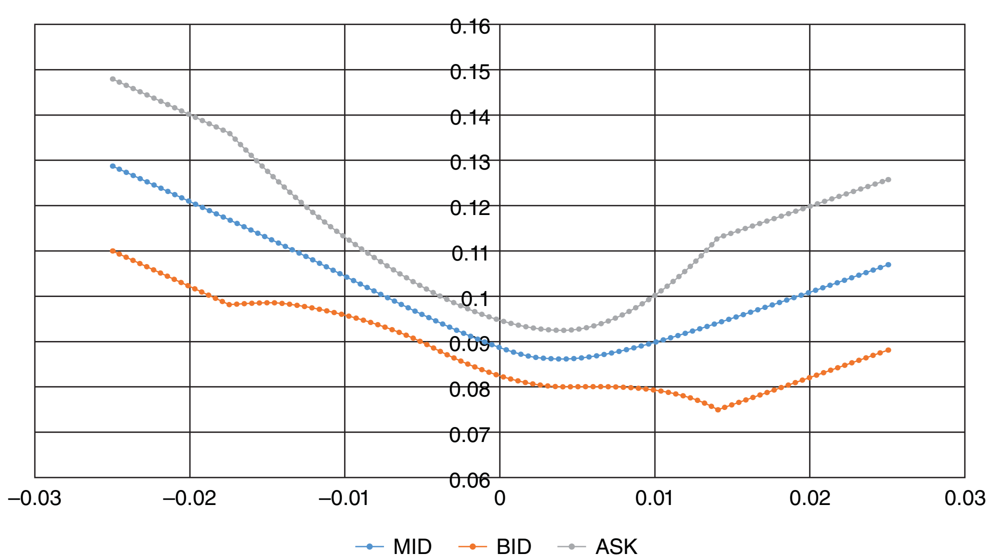


FIGURE 4.8 Vanilla bid-ask spreads on log-moneyness space in implied volatilities.

4.2.2 Spreading Vanilla Structures

The spreading rule for a single vanilla option does not necessarily translate to a portfolio of vanilla options. For example, in a straddle, the vega exposure of the two vanillas contained in it add up, and therefore, the spread of the straddle is expected to be between the single vanilla spread and twice that spread, and in fact, should be closer to twice. In a risk reversal, one option is long and the other one short. Therefore, the short option's vega reduces the long option's vega. In this case the price spread of the structure is often the same as the price spread of the vanilla. One option is quoted in bid-ask, the other is quote at *choice*. Spreading has become an art, as desks need to cover the operational cost – and ideally actually make money – and simultaneously quote aggressive bid-ask prices to increase the likelihood of getting the trade.

4.2.3 One-Touch Spreads

The spreads for one-touch contracts are often constant across tenors and touch levels. Examples of one-touch spreads in basis points (bps) are exhibited in Table 4.2. For one-touch spreads 1 bp is 0.01% of the foreign currency *or* of the domestic currency, based on the notional currency specified.

These spreads are subject to changes and have to be maintained by traders according to market conditions and situation of the existing book of options. The one-touch spreads should be entered manually in the system configuration with the possibility to change them. However, there may be phases lasting for many months of trading where these spreads do not have to be changed. One can also specify no-touch spreads. One must also keep in mind that the spreads can be set differently depending on the customer group.

4.2.4 Spreads for First Generation Exotics

A very simple – rather educational – example of spreads for first generation exotics is exhibited in Table 4.3. Vanilla spread means the equivalent vanilla spread for the same time to maturity, while the maximum intrinsic value (IV) at the barrier for a call option is $\mathrm{IV} = \max\left[\frac{B-K}{B}, 0\right]$ and for a put option is $\mathrm{IV} = \max\left[\frac{K-B}{B}, 0\right]$. For a double barrier call we use the IV for the upper barrier, while for a double barrier put the IV for the lower barrier. We divide by the barrier B to express the maximum intrinsic value in foreign currency and hence put it on the same scale with basis points.

Regular barrier options are out-of-the-money at hitting time, whereas reverse barrier options are in-the-money at that time. Therefore, a barrier call is reverse when the

TABLE 4.2 Spreads for one-touch contracs.

EUR/USD	200	EUR/JPY	350
EUR/CHF	300	USD/JPY	250
EUR/GBP	250	GBP/USD	200

TABLE 4.3 Spreads for first generation exotics.

OT/NT	OT spread
DOT/DNT	$1.5 \cdot$ OT spread
Regular barrier	$1.5 \cdot$ Vanilla spread and 7 bps $\leq$ spread $\leq$ 11 bps
Reverse barrier	Max{$1.5 \cdot$ vanilla spread, OT spread $\cdot$ IV barrier}
Double barrier	Max{$1.5 \cdot$ vanilla spread, OT spread $\cdot$ IV barrier}

barrier is larger than the strike, while a barrier put is reverse when the barrier is lower than the strike. The regular barrier is spread wider than the vanilla, although it can be semi-statically replicated with a risk reversal. However, this replication still leaves the trader with future skew risk which is accounted for by a higher spread.

4.2.5 Minimal Bid-Ask Spread

A minimum spread should be chosen to cover the cost of the ticket. We may assume this to be 400 EUR for vanilla options and 1200 EUR for exotic options. If the bid-ask spread is less than the minimal spread, then it is widened to ensure the minimum. If the bid price is less than zero, then both bid price is floored at zero and the ask price should converge to the ticket cost.

4.2.6 Bid-Ask Prices

For vanilla options the bid and ask prices are calculated as a symmetric interval around the mid market price. For exotics this principle no longer applies. For example, selling a reverse-knock-out EUR put USD call requires the trading desk to be massively EUR long right above the barrier, and in case of knock-out unwind the delta hedge, sell the EUR at a lower spot, possibly with slippage effects. This will cause a loss and hence the expected cost must be charged by increasing the offer price of the RKO put. Delta hedging the same but long RKO put does not cause the same loss, in fact, it might generate a profit, because the trader sells EUR high and buys back low when delta hedging and EUR-USD goes down. One can build an entire theory on spreading vanillas, exotics, and structured products. Several software companies hold patents or would like to hold patents on their spreading logic.

4.3 SYSTEMS AND SOFTWARE

There are many pricing and risk management systems and tools. However, there are not many vendor systems with a dedicated focus on FX. A risk management system with a dedicated FX focus is FENICS. For pricing across a wide range of exotics, SuperDerivatives is widely spread among banks as well as the buy-side. Both use proprietary models for their vanilla volatility surface construction and exotics pricing. It is browser based. A modern pricing system is Volmaster, a software as a service (SAAS) with an SLV model also showing the respective model Greeks. Bloomberg offers volatility surfaces as part of their market data supply and a pricing tool (OVML) on the Bloomberg platform. The user can choose between a variety of models. Most of the top tier banks have developed their in-house platforms and are making a front-end available to their clients. One of the first was the UBS trader, now gradually being replaced by the Neo platform. Other such sell-side platforms include Merlin (CS), Barx (Barclays), Kristall (Commerzbank), Autobahn (Deutsche Bank), Citi, JP Morgen, BNP Paribas, etc. Rolling out these platforms to the sales teams and the buy-side clients allows market takers to trade vanillas and exotics electronically. Consequently, vendor platforms are becoming less relevant. The bigger vendor risk management systems banks use for FX derivatives include Murex, Kondor+, and Calypso. Murex has traditionally been strong in FX and has even developed its own SLV model which they call the *Tremor*. Pricing libraries are also available from MathFinance, Numerix, and Fincad.

4.3.1 Position Keeping

For running an FX Options book, many banks use their in-house systems. Particularly, larger banks tend to build front-office systems themselves. The software company Murex offers a fully working front-office application including pricing, structuring, portfolio analysis, back-office functionalities. The big plus point is really the back-office part, where all deals can be watched, exercised, early terminated, expired, confirmed, etc. One of the problems is of course the pricing of contracts, particularly of exotics. As this is mostly proprietary bank information, vendor systems do not even attempt to cover exotic options in a full range but rather offer interfaces for users to connect their own pricing libraries, e.g. via *flexible deals*, which is a hull of a derivative contract, whose details and interfaces have to be programmed by experts.

4.3.2 Reference Prices and Volatilities

Since FX derivatives are mostly OTC contracts their prices are not directly observable. Buy-side institutionals see the most, as they can sign up for the sell-side platforms. On the sell-side the top market makers deliver their prices and volatilities daily to Markit. Markit's *Totem* service collects these feeds, cleans them and returns an average to their members. If you are a member of the market maker's club, you can get the best source of volatilities as per end of day. Public systems often provide outdated and/or faulty data.

4.3.3 Straight Through Processing

Straight through processing (STP) means that when a trade is done, the entire *booking* and back-office machinery is fully automated. After trading in the *front office* a trade has to be booked, i.e. stored in the database of all live trades, the details have to be checked by the *middle office*, the trade confirmation has to be written up, signed and sent to the counterpart by the *back office*. A complete treatment of FX back-office procedures can be found in the thesis by Hervas-Zurita in [73]. The issues concerning counterparty credit and limits have to be added to the respective counterpart, so it is clear how much of the credit limit is still available for a potential future trade. *Risk controlling* has to be informed about the market risk and credit risk of the trade, double check it and determine if extra cash needs to be set aside. Legal issues have to be checked, like compliance and counterparty origin. Furthermore, the trade has to appear correctly in the balance sheets of the bank. The sales margins of respective sales people, structurers, traders, and branches have to be booked in the respective profit centers to keep track of the success of staff and departments. Necessary hedges of the deals should be automatically executed and also booked and confirmed. Only if all these things happen automatically rather than manually can a bank make a product into a flow business, lower the spreads or prices, and attract new deals. Any kind of manual interference with deals management slows down the process, increases labor costs. It has become a job called *derivatives trading process engineering* to ensure all these processes run smoothly and clarify with all the parties involved if a new product can find its way through all the applications and necessities running in a bank. Overall, the need for STP shows how dominantly important a working system infrastructure is for a bank. The key requirement for e-commerce and a successful FX business is a fully state-of-the-art volatility surface.

4.3.4 Disclaimers

Usually issuers and banks insert a disclaimer at the bottom of their term sheets to avoid legal problems if an investor buys a structure and gets hit by the market. It could look like this.

There are significant risks associated with the product described above including, but not limited to, interest rate risk, price risk, liquidity risk, redemption risk, and credit risk. Investors should consult their own financial, legal, accounting, and tax advisers about the risk associated with an investment in these products, the appropriate tools to analyze that investment, and the suitability of that investment in each investor's particular circumstances. No investor should purchase the product described above unless that investor understands and has sufficient financial resources to bear the price, market, liquidity, structure, redemption, and other risks associated with an investment in these products.

The market value can be expected to fluctuate significantly and investors should be prepared to assume the market risks associated with the product under consideration.

4.4 TRADING AND SALES

Option traders trade options and are supposed to risk manage their option portfolios and generate profit for their desk. Sales sell options (among other things) and are supposed to generate sales margin for their desk. If these desks are separate profit centers, then fights are immanent in the system. When a contract is traded, there is always a discussion about how much of the mark up is the sales margin, and how much is the trading margin. It is a job for the management of the bank to set up clear rules on how these profits should be split among desks. For structures there are often structurers in the middle who also claim their share, which makes it even more challenging.

In the following we briefly describe the different areas of trading and sales.

4.4.1 Proprietary Trading

Proprietary trading primarily means actively taking risky positions, i.e. designing a portfolio of financial products aiming for prosperous growth. The management sets the degree of risk appetite. Since always one counterpart is long and another one short, this will tend to work well on average in 50% of all cases. Successful desks will continue, loss-generating desks will disappear, the causers often being promoted to higher positions. After the financial crisis of 2008 prop trading has essentially moved to hedge funds, as banks are required to take as little risk as possible or equivalently are required to keep capital reserves for risky trades. In this sense, regulators are actually hindering banks from being profitable. Furthermore, liquidity dries up so market bubbles and anomalies are more likely to occur.

4.4.2 Sales-Driven Trading

Another approach of trading is taking in the positions originating from sales activities and risk managing these. This approach is often preferred by smaller enterprises or by very risk averse institutions. Of course, one can mix sales-driven and proprietary trading.

4.4.3 Inter Bank Sales

As the name tells, an inter bank sales desk trades with other banks. As one can imagine, the counterparts being experts in financial products will quote each other *very tight* bid-offer spreads. This market is very fast and profit is generated by the sheer number of traded products.

4.4.4 Branch Sales

Banks entertaining a branch network usually have sales desks responsible for the branch clients. The products are often standardized and profits are generated by the turnover on the one hand and the sales skills of the branch employees on the other. As branch clients are not always able to verify market prices of structured products, a noticeable sales margin can often be hidden. A common strategy is to sell zero-cost structures with attractive conditions, with obviously a negative market value at inception, and potential risk for the buy-side. Structured forwards with a known worst case are particularly popular.

4.4.5 Institutional Sales

Institutional sales means selling FX hedging or investment products to big institutions such as funds, pension funds, hedge funds, insurance companies, municipalities, or government sectors. It lies in the nature of these clients that they incline to trade only with very highly rated banks. For the government and insurance sector, structured products are difficult to sell as there are often regulatory restrictions. Hedge funds on the other hand are keen on trading complex structures and volatility products.

4.4.6 Corporate Sales

Multi-national companies (MNCs) are often served from a bank's head office rather than a branch. They often need foreign exchange rate protection, whence they prefer trading spot, forward, and vanilla options. Structured products trade sometimes depending on many factors, such as country, risk appetite, knowledge of the corporate treasurer, credit lines. Several corporates know the FX market very well and are able to buy the components of structured products and structure the desired position themselves. That way they can save on the sales margin for the bank, as they buy only liquid ingredients that are quoted with tight bid-offer spreads. For mid-caps accumulators and target forwards are particularly popular.

4.4.7 Private Banking

The private banking sector uses the full blast of structured products for the purposes of yield enhancement. A standard investment is the dual currency deposit (DCD) introduced in Section 2.4.1. It has become a flow product. Some of the more aggressive banks have also flooded innocent investors with portfolios of accumulators and KIKO-tarns and more. Obviously, private banking clients trading zero-cost structures have to maintain a margin account. During the financial crisis many positions went under water very quickly and accounts had to be force-closed, with the obvious result of endless litigation. It appears that the global market for structured FX products is growing still.

4.4.8 Retail FX Derivatives

The private investor who wants to participate in FX markets and bet on certain events certainly has numerous opportunities to buy listed FX derivatives offered by many banks and online platforms. Besides vanilla options, we find many exotic derivative securities, mostly barrier options, touch options, range notes, participation notes, and power options. Since the investor pays and receives domestic currency, listed FX options are often quantoed.

4.4.9 Exchange Traded FX Derivatives

Traditionally, there have not been many transactions of FX derivatives on exchanges. In light of the counterparty credit risk and collateralization requirements, this picture is slowly changing. Several exchanges have started offering FX vanilla options.

4.4.10 Casino FX Products

Trading FX digitals or touch contracts on online platforms has become very popular. This is a market of its own kind because maturities are often very short, in fact, can be just a few minutes. You can squeeze a life time of betting into just one week. In fact, even a weekend, as there are platforms that offer trading on random indices, generated for example by a discrete geometric Brownian motion with zero drift and a constant volatility. This market is open on the weekends.

4.4.11 Treasury

As an incentive for treasury managers to consider hedging FX exposure at all, one can consider an example of 100,000 EUR at risk. Suppose the EBIT (annual earnings before interest and tax) is 5%, then the ratio of 100,000 EUR and the EBIT of 5% would be 2,000,000 EUR, which is the extra amount of profit the company needs to raise if the FX exposure goes bad. Since, in addition, banks might reject an extra credit of 2 Mio, it becomes clear that immediate action needs to be taken.

4.4.12 Fixings and Cutoffs

Currency fixings are official reference exchange rates published by allegedly credible institutions or organizations. They are usually published for each business day. Conversely, a cutoff marks the end of a trading day and the beginning of a new trading day. Most common examples are the 3 p.m. Tokyo cut and the 10 a.m. New York cut. Traditionally, for currency options, the cutoff time is the time on a trading day by which the holder of an option can exercise the option. Exercising it triggers a cash flow, i.e. the holder receives the call currency amount and pays the put currency amount. For this cash flow, a reference spot S_T is not needed. A reference spot is only required if an option is contractually cash-settled or to calculate bespoke payoff-based derivatives values and settlement amounts. For such contracts, the reference is usually a currency fixing. Market participants using currency fixings include but are not limited to index providers, asset managers, non-financial corporates, and dealers.

Fixing Sources Fixings can be sourced from various providers. They differ in the way they are calculated, by the sources the provider uses to calculate a fixing, and by how transparent the provider is about the fixing calculation.

ECB is a daily fixing published by the European Central Bank (ECB) on the Reuters page ECB37, with delay, and only against EUR. Complaints from the trading community about the delay in publication animated the ECB to increase the delay even further. In fact, institutions that publish fixings as a reference for the valuation of financial contracts need to comply with a number of rules, which the ECB has deliberately decided not to fulfill, therefore purposely discouraging the use of their fixings. No comments.

WMRSPOT refers to the daily fixing by World Market, previously managed by the State Street Company, acquired by Thomson Reuters on 1st April 2016. This is known as the 4pm London fixing.

FEDFX refers to the noon FX fixing of the Federal Reserve Bank. Note is it not FEDEX.

BFIX refers to the Bloomberg fixing, which is also published on http://www.bloomberg.com/markets/currencies/fx-fixings.

OPTREF used to take the average of the four largest German banks: Deutsche, Dresdner, Commerzbank, and Hypovereinsbank; later it was based on Commerzbank and Deutsche. Hypovereinsbank left on 16 Oct 2013. OPTREF ceased to exist per 31 Dec 2013.

Moreover, many central banks in many countries of the world publish their own currency fixings. Fixings are usually published with a delay to prevent manipulation of the fixing rate.

Independent of the official fixings, each bank uses an in-house fixing and prefers to use this as a reference for their client transactions. For the bank, this is better because the fixing can be controlled and is close to tradable. For the client this can be problematic as it may not be transparent.

Fixing Calculations Since 2013, many improvements have been made. The International Organization of Securities Commissions (IOSCO) published a guideline for financial benchmarks and conducted periodical reviews on the key benchmark rates used in the industry. This framework pushed the reference rate providers to publish their methodology, be more transparent and keep a stronger connection with the investment world to make their products more relevant.

A detailed description can be found in Correia [36].

Reuters Fixing Input rates are selected from different platforms accordingly to the tradability of the currency. The three sources are *Thomson Reuters Matching*, *Electronic Broking Service (EBS)*, and *Currenex*.

As an example for the spot rate we consider the Trade Currencies methodology: The process of producing the WM/Reuters benchmark rates can be decomposed into the following steps (in the scenario where enough valid trades are in the pool):

1. The fix period is set at five minutes. Snapshots of trade executed and bid and offer order rates are taken every second, starting 2 minutes 30 seconds before and ending 2 minutes and 30 seconds after the fixing time.
2. At every snapshot, a single traded rate will be captured for each source. The trade will be a bid or offer depending on if it is a buy or sell trade. For the same time period, a bid and offer order will be extracted for each source.
3. A spread will be computed from the order rates of this source and applied to its trade order to get the corresponding bid or offer trade rate.
4. The trades will be subject to specific validations. Two reasons for excluding the data point could be the absence of a new trade since last snapshot or that the trade falls outside the best bid or best offer captured. All valid trades across the different sources will be pooled together.
5. A median trade bid and a median trade offer are calculated separately. From these results a mid-rate is defined.
6. A standard spread will be applied to this mid-rate to compute a new bid and a new offer rate. Those rates will be published as the benchmark rate if they fulfill a specific "tolerance check threshold" that might trigger a request for review by a staff member.

The description is outlined in Reuters' FX benchmark manual [112], and on it's website financial.thomsonreuters.com/benchmarks.

Bloomberg Fixing Cleaning and selecting inputs using the BGN methodology is the first step of the process. The main product used in the industry is the spot reference rate. BFIX takes a snapshot of those BGN ticks with an interval of 30 minutes. This procedure is repeated the whole day and the rates are published 15 seconds after the fixing time.

The methodology used to compute the BFIX is a Time-Weighted Average Price (TWAP) of the BGN mid rates. This average is taking into account data *before and after* the fixing time. Every BFIX rate is divided in multiple slices of one second. The inputs for every slice are the geometric mid rates of the BGN bid and ask ticks. The TWAP is then computed based on those data points, using a triangular formula, as exhibited in Figure 4.9. The peak of the triangle is the fixing time and carries a weight of 10%. Around this peak, weight of every slice is decreasing linearly. As a result, the pre-fix time weight approximately 88.52% and the post-fix time 1.48%.

This common rule varies according to the time window assigned to the TWAP. This time frame is determined according to the currency. BFIX rates for currencies for the G10 countries (AUD, CAD, CHF, EUR, GBP, JPY, NOK, NZD, SEK, and USD) use a TWAP that starts 300 seconds before the fix and ends six seconds after the fix time. All other currencies have a window starting 900 seconds before and ending six seconds after. The description is outlined in Bloomberg's BFIX manual [16].

Overall, it is important to note the difference between the BFIX rate and the WM/Reuters reference rate for this same time. This implies not only that the methodology differs but also that the selection of the sources are different from one platform to the other. This can have a relatively important impact on reference rates for currency pairs that do not have a very liquid market.

ECB Fixing The ECB fixing does not meet the IOSCO guidelines of transparency.

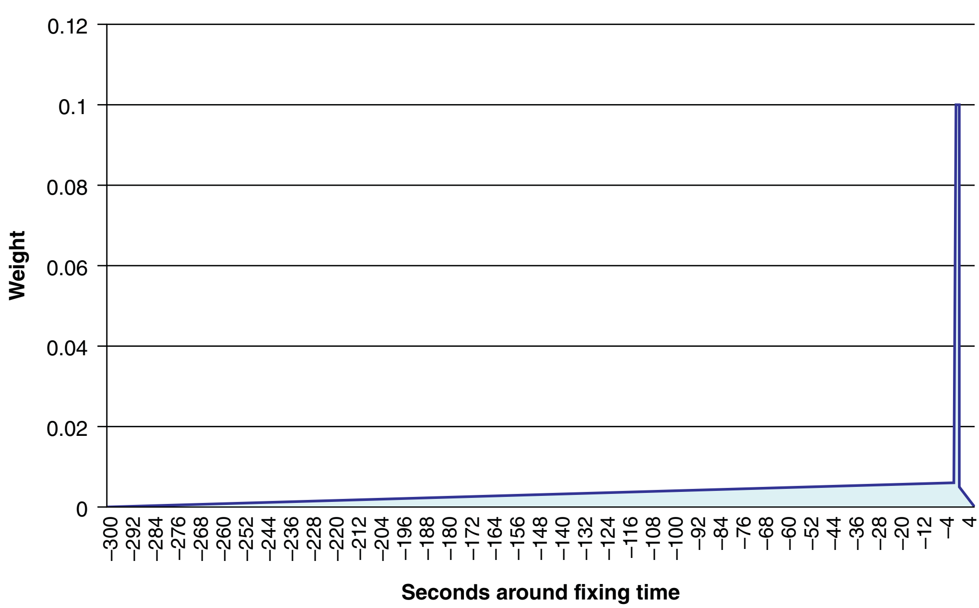


FIGURE 4.9 BFIX TWAP weights assigned to the 306 snapshots.

4.4.13 Trading Floor Joke

We conclude this section with a story describing very accurately the difference between a trader and a sales representative.

A guy in a balloon had fallen asleep and when he woke up wanted to find out where he was. So he approached the earth and asked the nearest person on the ground he found, "Excuse me, sir, do you know where I am?" The man on the ground replied, "Yes, your are 45° and 17′ north, 14° and 03′ west and 2.55 meters above ground zero." The man in the balloon was stunned and answered, "You must be an FX trader." – "Indeed, yes I am, but how do you know?" – "You appear very busy, you speak loud and fast, you tell me a lot of detailed but useless information, and I still don't know where I am." – "And you must be in FX sales, right?" – "I am, but now how did you know?" – "Very obvious: you fall asleep in the middle of the day, you have risen to your current position due to a lot of hot air, you don't know where you are, you don't understand what I am telling you, have no clue what's going, and as a cherry on the cake, you make me feel like it's my fault."

4.5 CURRENCY PAIRS

A currency pair is – obviously – a pair of two currencies. A list of currencies and their ISO codes is provided in the next section. Generally one can take any two different currencies to generate a currency pair. However, mostly a pair is usually considered when we trade one for the other, i.e. when a foreign exchange rate exists. Most currencies trade only against USD or against G10, which is why the number of relevant pairs is not as big as it could be in theory.

4.5.1 ISO 4217 Currency Code List

We follow the exposition of http://www.xe.com/iso4217.php.

Table 4.4 and Table 4.5 provide a list of global currencies and the three-character currency codes that are generally used to represent them. Often, but not always, this code is the same as the *ISO 4217 standard*. The ISO – or International Organization for Standardization – is a worldwide federation of national standards bodies, see http://www.iso.org. In most cases, the currency code is composed of the country's two-character internet country code plus an extra character to denote the currency unit. For example, the code for Canadian dollars is simply Canada's two-character internet code (CA) plus a one-character currency designator (D).

We have endeavored to list the codes that, in our experience, are actually in general industry use to represent the currencies. This list no longer contains obsolete Euro zone currencies.

The standard quotation for most currencies against the USD is foreign currency for USD and domestic for the other one. Exceptions to this rule are indicated by the currency code in gray. This standard is implemented in data providers like Reuters or Bloomberg. The only difference in the two occurs for SBD, which is the domestic currency in Reuters but foreign currency in Bloomberg.

TABLE 4.4 Currency codes, part one, sorted by the three-letter code.

AED	United Arab Emirates, Dirhams	*EUR*	Euro Member Countries, Euro
AFN	Afghanistan, Afghanis	*FJD*	Fiji, Dollars
ALL	Albania, Leke	*FKP*	Falkland Islands (Malvinas), Pounds
AMD	Armenia, Drams	*GBP*	United Kingdom, Pounds
ANG	Netherlands Antilles, Guilders (also called Florins)	GEL	Georgia, Lari
AOA	Angola, Kwanza	GGP	Guernsey, Pounds
ARS	Argentina, Pesos	GHS	Ghana, Cedis
AUD	Australia, Dollars	*GIP*	Gibraltar, Pounds
AWG	Aruba, Guilders (also called Florins)	GMD	Gambia, Dalasi
AZN	Azerbaijan, New Manats	GNF	Guinea, Francs
BAM	Bosnia and Herzegovina, Convertible Marka	GTQ	Guatemala, Quetzales
BBD	Barbados, Dollars	GYD	Guyana, Dollars
BDT	Bangladesh, Taka	HKD	Hong Kong, Dollars
BGN	Bulgaria, Leva	HNL	Honduras, Lempiras
BHD	Bahrain, Dinars	HRK	Croatia, Kuna
BIF	Burundi, Francs	HTG	Haiti, Gourdes
BMD	Bermuda, Dollars	HUF	Hungary, Forint
BND	Brunei Darussalam, Dollars	IDR	Indonesia, Rupiahs
BOB	Bolivia, Bolivianos	ILS	Israel, New Shekels
BRL	Brazil, Brazil Real	IMP	Isle of Man, Pounds
BSD	Bahamas, Dollars	INR	India, Rupees
BTN	Bhutan, Ngultrum	IQD	Iraq, Dinars
BWP	Botswana, Pulas	IRR	Iran, Rials
BYR	Belarus, Rubles	ISK	Iceland, Kronur
BZD	Belize, Dollars	JEP	Jersey, Pounds
CAD	Canada, Dollars	JMD	Jamaica, Dollars
CDF	Congo/Kinshasa, Congolese Francs	JOD	Jordan, Dinars
CHF	Switzerland, Francs	JPY	Japan, Yen
CLP	Chile, Pesos	KES	Kenya, Shillings
CNY	China, Yuan Renminbi	KGS	Kyrgyzstan, Soms
COP	Colombia, Pesos	KHR	Cambodia, Riels
CRC	Costa Rica, Colones	KMF	Comoros, Francs
CUP	Cuba, Pesos	KPW	Korea (North), Won
CVE	Cape Verde, Escudos	KRW	Korea (South), Won
CZK	Czech Republic, Koruny	KWD	Kuwait, Dinars
DJF	Djibouti, Francs	*KYD*	Cayman Islands, Dollars
DKK	Denmark, Kroner	KZT	Kazakhstan, Tenge
DOP	Dominican Republic, Pesos	LAK	Laos, Kips
DZD	Algeria, Algeria Dinars	LBP	Lebanon, Pounds
EEK	Estonia, Krooni	LKR	Sri Lanka, Rupees
EGP	Egypt, Pounds	LRD	Liberia, Dollars
ERN	Eritrea, Nakfa	LSL	Lesotho, Maloti
ETB	Ethiopia, Birr		

TABLE 4.5 Currency codes, part two, sorted by the three-letter code.

LTL	Lithuania, Litai	*SHP*	Saint Helena, Pounds
LVL	Latvia, Lati	SKK	Slovakia, Koruny
LYD	Libya, Dinars	SLL	Sierra Leone, Leones
MAD	Morocco, Dirhams	SOS	Somalia, Shillings
MDL	Moldova, Lei	SPL	Seborga, Luigini
MGA	Madagascar, Ariary	SRD	Suriname, Dollars
MKD	Macedonia, Denars	STD	São Tome and Principe, Dobras
MMK	Myanmar (Burma), Kyats	SVC	El Salvador, Colones
MNT	Mongolia, Tugriks	SYP	Syria, Pounds
MOP	Macau, Patacas	SZL	Swaziland, Emalangeni
MRO	Mauritania, Ouguiyas	THB	Thailand, Baht
MUR	Mauritius, Rupees	TJS	Tajikistan, Somoni
MVR	Maldives, Rufiyaa	TMM	Turkmenistan, Manats
MWK	Malawi, Kwachas	TND	Tunisia, Dinars
MXN	Mexico, Pesos	*TOP*	Tonga, Pa'anga
MYR	Malaysia, Ringgits	TRY	Turkey, New Lira
MZN	Mozambique, Meticais	TTD	Trinidad and Tobago, Dollars
NAD	Namibia, Dollars	TVD	Tuvalu, Tuvalu Dollars
NGN	Nigeria, Nairas	TWD	Taiwan, New Dollars
NIO	Nicaragua, Cordobas	TZS	Tanzania, Shillings
NOK	Norway, Krone	UAH	Ukraine, Hryvnia
NPR	Nepal, Nepal Rupees	UGX	Uganda, Shillings
NZD	New Zealand, Dollars	USD	United States of America, Dollars
OMR	Oman, Rials	UYU	Uruguay, Pesos
PAB	Panama, Balboa	UZS	Uzbekistan, Sums
PEN	Peru, Nuevos Soles	VEF	Venezuela, Bolivares Fuertes
PGK	Papua New Guinea, Kina	VND	Viet Nam, Dong
PHP	Philippines, Pesos	VUV	Vanuatu, Vatu
PKR	Pakistan, Rupees	*WST*	Samoa, Tala
PLN	Poland, Zlotych	XAF	Communauté Financière Africaine BEAC, Francs
PYG	Paraguay, Guarani	XAG	Silver, Ounces
QAR	Qatar, Rials	XAU	Gold, Ounces
RON	Romania, New Lei	XCD	East Caribbean Dollars
RSD	Serbia, Dinars	XDR	International Monetary Fund (IMF) Special Drawing Rights
RUB	Russia, Rubles	XOF	Communauté Financière Africaine BCEAO, Francs
RWF	Rwanda, Rwanda Francs	XPD	Palladium Ounces
SAR	Saudi Arabia, Riyals	XPF	Comptoirs Français du Pacifique Francs
SBD	Solomon Islands, Dollars	XPT	Platinum, Ounces
SCR	Seychelles, Rupees	YER	Yemen, Rials
SDG	Sudan, Pounds	ZAR	South Africa, Rand
SEK	Sweden, Kronor	ZMK	Zambia, Kwacha
SGD	Singapore, Dollars	ZWD	Zimbabwe, Zimbabwe Dollars

TABLE 4.6 Chinese yuan currency symbols, (*) only by legal entities resident in China.

CNX	Off-shore Chinese yuan	NDF-based
CNH	Chinese Yuan in HK	Deliverable in Hong Kong
CNY	On-shore Chinese yuan	Deliverable domestically (*)

Chinese Yuan For Chinese yuan, it is important to distinguish the currency symbols, so this is a recap of typical conventions in Table 4.6.

4.6 THINGS TO REMEMBER

As a general guide I have placed a list of common replication strategies in Table 4.7, a list of common rules of thumb in Table 4.8, and the pedigree of vanilla and exotic contracts in Figure 4.10 at the end of the book for easy reference. Besides that, this is what I keep preaching:

1. FX is all about currencies.
2. Clarify what ATM means.
3. Use d_+ and d_-, otherwise chances are you will miss out all the symmetries.
4. Volga of a long vanilla option can be negative.
5. Theta of a long vanilla option can be negative.
6. The terms option and derivative are not synonyms.
7. Correlation is typically unknown and widely misunderstood.
8. All fixings are manipulated.
9. Zero-cost products have a negative market value.
10. The risk in a derivative depends heavily on the client type: treasurer vs. investor/speculator.
11. Regulation should be regulated.
12. Simuliert – nix kapiert.
13. TV does not stand for television in the context of FX options.
14. Nokkies are typically harder than Gnocchi.

4.7 GLOSSARY

delta-gap is the difference of the delta position on a barrier before the barrier event and after the barrier event.

EM refers to emerging markets, typically with a high contango when used as domestic currency against USD or EUR.

Eventualtermingeschäft (ETG) is a common German term for a structured forward or forward contract with conditional payoff.

Floan is a foreign exchange loan, i.e. a loan in a foreign currency.

G10 Currencies USD, EUR, JPY, GBP, CHF, AUD, NZD, CAD, SEK, NOK. *G11 currencies* is a superset of the G10 currencies plus Danish krone (DKK).

HNWI is an abbreviation for a *high net-worth individual*, a client type in private banking.

maturity/tenor refers to the lifetime of a financial security. There are constant maturity financial products like bonds, futures, where the maturity date is fixed and does not change with time. Conversely, there are constant tenor financial products like FRAs, swaps, options, where the maturity date moves along with time in a pre-trade quotation. For example, a 6M option is a constant tenor product quoted by traders before it trades. Once it trades, it obviously also becomes an option with a constant maturity.

MTM stands for *marked to market* and refers to the mid-market value (without bid-offer-spread) of a derivative contract using an industry-probed or market consensus model, a current synchronous set of input market data and a robust calibration of the model to the market data. Consequently, the value depends on the input market data, the model and the calibration. However, the choice of these cannot be completely arbitrary. It must follow common industry practice, as the idea is to determine a realistic market price.

on-the-run/off-the-run refers to the state of a bond. A newly issued bond is called on the run until the next bond is issued and is hence called off the run.

pin risk refers to the cost occurring for a trader when delta-hedging a short gamma portfolio or product. Gamma is used as a measure of transaction costs when dynamically hedging a derivative payoff by trading in the underlying spot market. High gamma occurs especially for payoffs with kinks or jumps and short time to maturity. An exploding gamma looks like a pin and causes high costs. Pin risk is the risk that the spot price moves towards the high-gamma zone during the life time of a contract.

puttability is the right to sell (back) a financial contract at pre-specified conditions. In turn, *callability* refers to the right to buy (back) a financial contract at pre-specified conditions.

RIC is a Reuters Instrument Code, e.g. EUR3M= for the EUR-USD three-months outright forward rate. It usually provides live market quotes bid, offer, mid as well as other relevant information. RICs are single value results and can be integrated into applications and cells in a spread sheet.

slippage describes the effect that a trader who needs to buy or sell a currency at a specific spot level may not be able to do this, because the spot moves faster through this spot level than she can trade. A typical example is when she needs to unwind a delta-hedge position in a foreign currency the first time the spot trades at a barrier. In this situation selling/buying the FX position may not be possible at exactly the spot level equal to the barrier. When the spot slips though the barrier very quickly, the spot may be faster than a trader can react. Even in case of a spot order, it is possible that the order can't be executed at the barrier level due to lack of liquidity. Slippage is particularly important if the FX position to trade is large, i.e. at a delta gap when hedging a barrier or touch contract. The problem of slippage increases as the underlying instrument is less liquid.

SLV is an abbreviation for stochastic-local volatility model, a mix of a local and a stochastic volatility model. Sometimes the order is LSV, representing local-stochastic volatility model, which refers to the same model class.

strike refers to the ratio of the call currency amount and the put currency amount in an option. It is the level at which the option holder can trade if he decides to exercise.

TARGET is an abbreviation for *Trans-European Automated Real-time Gross Settlement Express Transfer*. This system is open on European settlement days. Business days are often counted following the number of open days in the TARGET system.

Target is a feature of target forwards referring to the maximum profit that when reached lets the target forward terminate early. Not to be confused with *TARGET* system used for settlement days.

TV refers to the theoretical value of a contract calculated in the Black-Scholes model with constant market variables and the ATM volatility.

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TABLE 4.7 Common replication strategies and structures.

digital(ϕ, K)	$\lim_{n\to\infty} n[\text{vanilla}(K) - \text{vanilla}(K+\phi/n)]$
knock-in	vanilla − knock-out
EKO(ϕ, K, B)	vanilla(ϕ, K) − vanilla(ϕ, B) − digital$(\phi, B)\phi(B-K)$
EDKOCall(K, L, H) $(K < L < H)$	call(L) − call(H) + $(L-K)$digital(L) − $(H-K)$digital(H)
vanilla(K)	digital$_{for}$ − $K \cdot$ digital$_{dom}$
RKO(ϕ, K, B)	KO$(-\phi, K, B)$ − KO$(-\phi, B, B)$ + $\phi(B-K)$NT(B)
(D)OT	e^{-rT} − (D)NT
barrier$(S, r_d, r_f, \sigma, K, B, T, t, \phi, \eta)$	$\text{barrier}\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{K}, \frac{1}{B}, T, t, -\phi, -\eta\right) \cdot S \cdot K$
DOT$_{for}(L, H, S_0, r_d, r_f, \sigma)$	$S_0\text{DOT}_{dom}\left(\frac{1}{H}, \frac{1}{L}, \frac{1}{S_0}, r_f, r_d, \sigma\right)$
DNT$_{dom}(L, H)$	[DKOCall$(K = L, L, H)$ + DKOPut$(K = H, L, H)$]/$[H - L]$
DNT$_{for}(L, H)$	$[H \cdot$ DKOCall$(K = L, L, H)$ + $L \cdot$ DKOPut$(K = H, L, H)]$/ $[(H-L)S_0]$
DNT$_{for}(L, H)$	DKOCall$(K = 0, L, H)$
EDNT(L, H)	digital(L) − digital(H)
two-touch(L, H)	OT(L) + OT(H) − DOT(L, H)
second DNT$(A < B < C < D)$	DNT(A, C) + DNT(B, D) − DNT(B, C)
KIKO(ko, ki)	KO(ko) − DKO(ko, ki)
forward(K)	call(K) − put(K)
paylater premium	vanilla/digital
(ratio) spread(ϕ)	vanilla(K, ϕ) − ratio $\cdot$ vanilla$(K + \phi \cdot$ spread, $\phi)$
risk reversal	call(K_+) − put(K_-)
straddle(K)	call(K) + put(K)
strangle	call(K_+) + put(K_-)
butterfly	call(K_+) + put(K_-) − call$_{ATM}$ − put$_{ATM}$
shark forward	forward + RKO
bonus forward	forward + DNT
butterfly forward	forward + DKO straddle
accrued forward	forward + corridor
participating forward	call − P%put
fade-in forward	forward + fade-in vanilla
dcd$(r >$ market$)$	deposit$(r =$ market$)$ − vanilla
range deposit$(r >$ market$)$	deposit$(r <$ market$)$ + DNT
performance note$(r_{max} >$ market$)$	deposit$(r <$ market$)$ + call

TABLE 4.8 Common approximating rules of thumb.

KO(ϕ, K, B)	vanilla(ϕ, K) − vanilla$(-\phi, K')$(= 0 for $S_0 = B$)
OT	$2 \cdot$ digital
Asiangeo(S_0, K, σ)	$\sqrt{\frac{S_0}{K_+}}\text{vanilla}\left(S_0, K\sqrt{\frac{K_+}{S_0}}, \frac{\sigma}{\sqrt{3}}\right) \approx \text{vanilla}\left(S_0, K, \frac{\sigma}{\sqrt{3}}\right)$

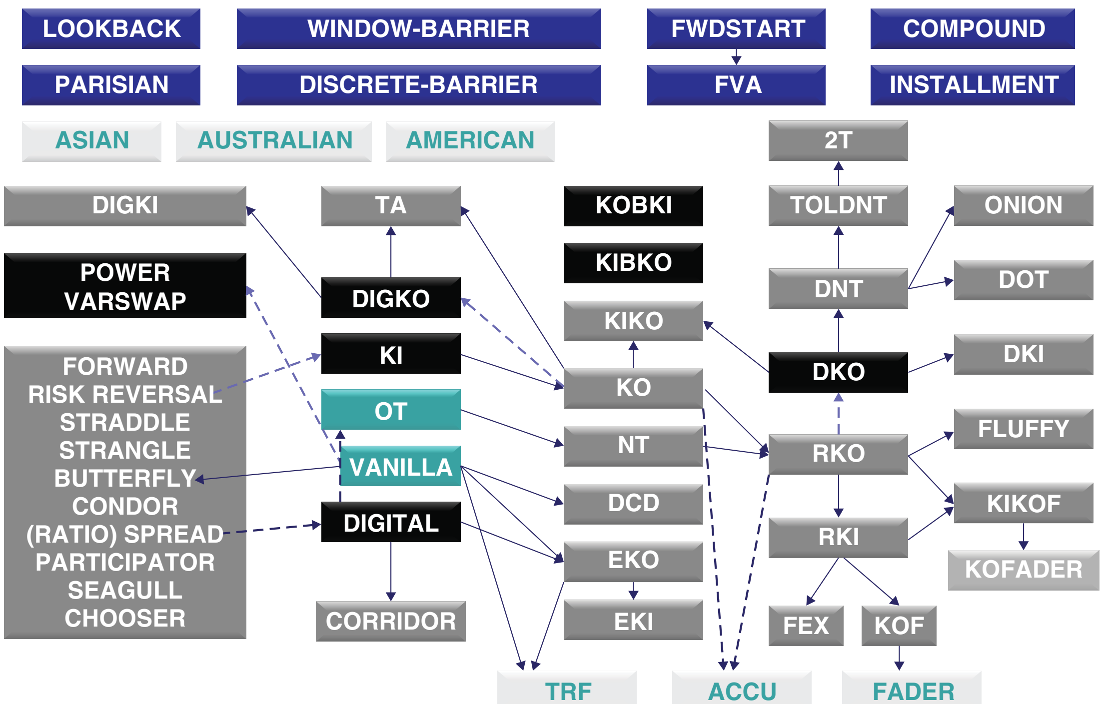

FIGURE 4.10 Pedigree of FX options, exotics and structured products. Dotted lines resemble approximate replications, full lines resemble static replication. Key building blocks are vanillas and the one-touch.