Explicit Volatility Specification for the Linear Cheyette Model

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Abstract

In this paper we present an extension of the classical Hull-White framework for pricing single currency exotics, which allows for a more adequate fit to the swaption volatility smile. We first present a general framework based on the HJM model and then make a separability assumption on the instantaneous forward rate volatility, thus enabling a representation of the discount curve in a finite number of Markovian state variables.

We show a practical application of this family of models by analyzing calibration and pricing in the case where the forward rate volatility is a linear function of the short rate. By doing so, we provide a novel and parsimonious specification of the Cheyette model. Then for calibration purposes, we develop fast and accurate approximations for European swaptions, based on standard projection and averaging technics. We also improve the usual naïve mean state approximation by the use of Gaussian approximations. We give numerical examples.

Keywords: HJM, Cheyette, Separable volatilities, Skew, Averaging Technics

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1. Introduction

The Libor Market Model (LMM from now on) first introduced in Brace and Al (1997), either in its native lognormal specification or its extended stochastic volatility version as described in Piterbarg (2003) have now become standard tools in almost all reasonably advanced financial institutions. Whether used as risk-management models or solely for benchmarking purposes, they boast substantial advantages such as intuitive primary variables, i.e the forward rates, simplicity of calibration through analytical, respectively semi-analytical swaption pricing approximations and straighforward Monte-Carlo implementation. However they still exhibit shortcomings such as the absence of Markovian representation of the discount curve, a computational cost which is quadratic with the number of dates and the lack of information on how to interpolate discount factors for non primary dates. Although a lot of effort has been put into tactically addressing these issues, it seems that a more strategic solution has been provided with the emergence of a new family of models, first introduced by Cheyette (1992) and Ritchken and Sankarasubramanian (1995). These models dubbed Quasi-Gaussian or Cheyette model start from the HJM specification of instantaneous forward rates dynamics and assume separability of volatility. From now on we will favor the terminology Cheyette Models to describe such models.

Under the Cheyette assumptions, there exists a closed form representation of discount bond prices in a small number of Markovian state variables at any arbitrary time and the constraint on the instantaneous volatility is loose enough so that a wide variety of interest rate smiles can be reproduced. However the above works don't provide insight into how fast and accurate calibration can be obtained. This question has been partially solved in the work of Andreasen (2001), in the case of the single factor Cheyette Model and Andreasen (2005) for multifactor blended stochastic volatility Cheyette models. The latter borrows ideas already used in the calibration of Stochastic Volatility LMM as introduced in Piterbarg (2003), calculating volatility weights along the forward and optimally averaging parameters across time to derive efficient calibration routines. However in the LMM case, "along the forward" means using the initial value of forward rates whereas in the Cheyette model, the approximation requires computing the average of state variables under the annuity measure. In Andreasen (2005) this is done by naïvely neglecting the convexity of the drift in the state variable dynamics, which is hard to justify from a theoretical perspective. In this note we provide an improvement to this approximation whih relies in transfering to the Cheyette model, some exact statistical properties of the short rate in the Hull-White framework. Secondly Andreasen (2005) chooses to specify forward rate volatility state dependency in terms of arbitrarily chosen forward rates. We believe that this parameterization makes the calibration procedure convoluted. Instead we use a much more natural parameterization by choosing the short rate state variable as the key one to describe the volatility functional form. In this work we apply these two ideas to a one factor Cheyette model equipped with a linear volatility function. The generalization of our improvements to more factors or more sophisticated volatility specifications is direct. This work is organized as follows. In the next section we introduce our notation and briefly recall the HJM framework under a single factor

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setting. Then we introduced the specifics of the Cheyette model and derive a Markovian representation of the discount curve. In the third part we choose a linear parameterization for the forward rate volatility and derive efficient swaption pricing approximations. Then we give numerical examples of calibration accuracy. Last we investigate a new approach for improving the accuracy of the 'along the forward' approximation for the state mean.

2. Notations and a brief review of the HJM formalism

Here we recall the usual notations prevailing in Fixed Income modeling. We will denote the price at time t of a pure discount bond paying one unit of domestic currency at time T by $P(t,T)$. The instantaneous forward rate at time t maturing at time T will be denoted by $f(t,T)$. By construction, discount bond prices and instantaneous forward rates relate as follows:

$$P(t,T) = \exp\left(-\int_t^T f(t,s)\,ds\right) \Leftrightarrow f(t,T) = -\frac{\partial \ln\left(P(t,T)\right)}{\partial T}$$

The short rate prevailing at time t will be denoted by $r(t)$. By definition we have $r(t) = f(t,t)$.

Assuming the existence of a risk-neutral measure, we posit the dynamics of discount bond prices under this measure as:

$$\frac{dP(t,T)}{P(t,T)} = r(t)\,dt - \sigma(t,T)\,dW(t)$$

where:

- $\sigma(t,T)$ is the volatility of discount bonds on which we make no particular assumptions other than the usual regularity properties
- W is a standard one dimensional Brownian motion under the risk-neutral measure

Under these settings one can show easily that the dynamics of the instantaneous forward rates are completely determined and such that:

$$df(t,T) = \frac{\partial \sigma(t,T)}{\partial T}\sigma(t,T)\,dt + \frac{\partial \sigma(t,T)}{\partial T}\,dW(t)$$

We introduce the forward rate volatility $v(t,T) = \dfrac{\partial \sigma(t,T)}{\partial T}$ and rephrase the dynamics of forward rates as:

$$df(t,T) = v(t,T)\left(\int_t^T v(t,s)\,ds\right)dt + v(t,T)\,dW(t)$$

So far, we have summarized the HJM framework for instantaneous forward rates. In the next section we start to make additional assumptions on instantaneous volatilities to obtain the general one factor Cheyette model.

3. The Cheyette model

So far we treated of fairly well known general results within the HJM framework. We now impose more specific conditions on the volatility structure of forward rates. These assumptions can be found in Cheyette (1992) and can be summarized as such: we assume that the instantaneous volatility function is the product of a function which solely depends on the maturity and a function of time and of a state vector θ which might be an arbitrary set of time t forward rates or derivables values. Therefore under these assumptions we can write without loss of generality that there exist two functions α, β such that:

$$v(t,T) = \alpha(T)\frac{\beta(t,\theta)}{\alpha(t)}$$

Based on that volatility specification we deduce the following expression for instantaneous forward rates:

$$f(t,T) = f(0,T) + \int_0^t \frac{\alpha(T)}{\alpha(s)}\beta(s,\theta)\left(\int_s^T \frac{\alpha(u)}{\alpha(s)}\beta(s,\theta)du\right)ds + \int_0^t \frac{\alpha(T)}{\alpha(s)}\beta(s,\theta)\,dW(s)$$

which we can rewrite as:

$$f(t,T) = f(0,T) + \int_0^t \frac{\alpha(T)}{\alpha(s)}\beta(s,\theta)\left(A(T)-A(s)\right)\frac{\beta(s,\theta)}{\alpha(s)}ds + \int_0^t \frac{\alpha(T)}{\alpha(s)}\beta(s,\theta)\,dW(s)$$

$$A(t) = \int_0^t \alpha(u)du$$

Forward rates can be more conveniently expressed as:

$$f(t,T) = f(0,T) + \frac{\alpha(T)}{\alpha(t)}\left(x(t) + \frac{A(T)-A(t)}{\alpha(t)}y(t)\right)$$
$$x(t) = \int_0^t \frac{\alpha(t)}{\alpha(s)}\beta(s,\theta)\left(A(t)-A(s)\right)\frac{\beta(s,\theta)}{\alpha(s)}ds + \int_0^t \frac{\alpha(t)}{\alpha(s)}\beta(s,\theta)dW(s) \quad (3.1)$$
$$y(t) = \int_0^t \left(\frac{\alpha(t)}{\alpha(s)}\right)^2 \beta^2(s,\theta)ds$$

In the special case of the short rate we get:

$$r(t) = f(t,t) = f(0,t) + x(t)$$

Therefore the variable $x(t)$ can be interpreted as a centered version of the short rate.

The dynamics of our state variables under the risk neutral measure are readily obtained:

$$\begin{aligned}
dx(t) &= \left(\frac{\alpha'(t)}{\alpha(t)}x(t) + y(t)\right)dt + \beta(t,\theta)dW(t) \\
dy(t) &= \left(\beta^2(t,\theta) + 2\frac{\alpha'(t)}{\alpha(t)}y(t)\right)dt \\
x(0) &= 0 \\
y(0) &= 0
\end{aligned} \quad (3.2)$$

Recalling the definition of the discount bond prices we also obtain the following reconstrution formula:

$$\begin{aligned}
P(t,T) &= \exp\left(-\int_t^T f(t,s)ds\right) = \exp\left(-\int_t^T\left(f(0,s) + \frac{\alpha(s)}{\alpha(t)}\left(x(t) + \frac{A(s)-A(t)}{\alpha(t)}y(t)\right)\right)ds\right) \\
&\Rightarrow \\
P(t,T) &= \frac{P(0,T)}{P(0,t)}\exp\left(-\frac{A(T)-A(t)}{\alpha(t)}x(t) - \frac{1}{2}\left(\frac{A(T)-A(t)}{\alpha(t)}\right)^2 y(t)\right)
\end{aligned} \quad (3.3)$$

Furthermore we add the assumption that the volatility dependence on maturity should actually appear through a dependence on time to maturity, which can be modeled by assuming:

$$\exists g \mid \forall (T,t): \qquad \frac{\alpha(T)}{\alpha(t)} = g(T-t)$$

Using simple ordinary differential equation knowledge we can prove easily that:

$$\exists! \kappa \in R \mid \frac{\alpha(T)}{\alpha(t)} = \exp\left(-\kappa(T-t)\right).$$

The forward rate volatility becomes:

$$v(t,T) = \exp\left(-\kappa(T-t)\right)\beta(t,\theta)$$

With these assumptions we have built a model which enables us to express the whole discount curve as a function of a parsimonious set of Markovian variables, thereby making the dimensionality of many pricing problems much easier to handle.

Indeed let us consider a European option which terminal payoff at time T can be expressed as a function of discount bond prices. We denote the payoff function as H and the price function at time t of this option as h. Then by non arbitrage argument it is well known that h is the solution to the following parabolic partial differential equation:

$$\begin{aligned} &\frac{\partial h}{\partial t} + \left(y(t) - \kappa x\right)\frac{\partial h}{\partial x} + \frac{1}{2}\beta(t,\theta)^2 \frac{\partial^2 h}{\partial x^2} + \left(\beta(t,\theta)^2 - 2\kappa y\right)\frac{\partial h}{\partial y} = \left(x + f(0,t)\right)h \\ &h(T,x,y) = H(x,y) \end{aligned} \tag{3.4}$$

This PDE can be efficiently solved in a two dimensional ADI scheme as described in Craig and Sneyd (1988). Extension of (3.4) to bermudan options through dynamics programing is straighforward.

We also note that the ubiquitous Hull-White model is just a particular case of the above framework with $\kappa > 0$ and $\beta(t,\theta) = \sigma(t)$ where σ is a deterministic function of time. Although the Hull White model proves to be attractive for its analytical tractability, it is unable to reproduce the swaption market implied smile and its usefulness tends to be limited to hybrid models. Therefore, from now on, we wish to consider Cheyette models with more flexible volatility structures. In the next section we suggest a possible solution to this problem.

4. Linear Cheyette model

In this parameterization we postulate a linear volatility for the state variable so that its dynamics can be written as follows:

$$
\begin{aligned}
&dx(t) = \left(-\kappa x(t) + y(t)\right)dt + \left(a(t)x(t) + b(t)\right)dW(t) \\
&dy(t) = \left(\left(a(t)x(t) + b(t)\right)^2 - 2\kappa y(t)\right)dt \\
&x(0) = 0 \\
&y(0) = 0
\end{aligned}
\qquad (4.1)
$$

It is intuitively clear that the slope parameter a will control the volatility skew whereas volatility parameter b will control the ATM volatlity level. We now need to come up with a fast and smooth calibration routines. For this purpose we first establish some approximate dynamics for the swap rate.
Let us first consider a swap agreement fixing at time T_0 and paying at time $(T_i)_{1\leq i\leq N}$.
Using Ito's lemma, it is clear that the exact dynamics of the underlying forward swap rate S under the annuity measure are:

$$
\begin{aligned}
&dS(t) = \eta(t, x(t))\, dW^A(t) \\
&\eta(t, x(t)) = \frac{\partial g}{\partial x}(t, x(t), y(t)) \times \left(a(t)x(t) + b(t)\right) \\
&S(t) = g(t, x(t), y(t)) = \frac{P(t, T_0) - P(t, T_N)}{A(t)} \\
&A(t) = \sum_{i=1}^{N} P(t, T_i)\,\delta_i
\end{aligned}
$$

where:

- A is the annuity process
- W^A is a standard Brownian motion under the annuity measure
- The annuity measure is the measure where the numeraire is the annuity process

We wish to price swaptions efficiently. To this end we suggest approximating the forward swap rate dynamics by a displaced lognormal diffusion expressed in the following format:

$$
dS(t) \simeq \Sigma(t)\left(\beta(t)S(t) + \left(1 - \beta(t)\right)S(0)\right)dW^A(t) \quad (4.2)
$$

By matching the first and second moment of $dS(t)$ around the expectations of x and y we can get exact expressions for proxy spot volatility Σ and blending β by solving analytically system (4.3):

$$\begin{aligned}
&\Sigma(t)S(0)=\eta(t,\bar{x}(t))=\frac{\partial g}{\partial x}(t,\bar{x}(t),\bar{y}(t))(a(t)\bar{x}(t)+b(t))\\
&\Sigma(t)\beta(t)\frac{\partial g}{\partial x}(t,\bar{x}(t),\bar{y}(t))=\frac{\partial \eta}{\partial x}(t,\bar{x}(t))=\frac{\partial^2 g}{\partial x^2}(t,\bar{x}(t),\bar{y}(t))(a(t)\bar{x}(t)+b(t))+a(t)\frac{\partial g}{\partial x}(t,\bar{x}(t),\bar{y}(t))\\
&\bar{x}(t)=E^A[x(t)]\\
&\bar{y}(t)=E^A[y(t)]
\end{aligned} \tag{4.3}$$

which gives:

$$\begin{aligned}
&\Sigma(t)=\frac{\frac{\partial g}{\partial x}(t,\bar{x}(t),\bar{y}(t))(a(t)\bar{x}(t)+b(t))}{S(0)}\\
&\beta(t)=S(0)\frac{\frac{\partial^2 g}{\partial x^2}(t,\bar{x}(t),\bar{y}(t))(a(t)\bar{x}(t)+b(t))+a(t)\frac{\partial g}{\partial x}(t,\bar{x}(t),\bar{y}(t))}{(a(t)\bar{x}(t)+b(t))\left(\frac{\partial g}{\partial x}(t,\bar{x}(t),\bar{y}(t))\right)^2}
\end{aligned} \tag{4.4}$$

where $E^A[\ ]$ denotes the expectation operator under the annuity measure.

Using the Girsanov theorem, one can show that the dynamics of state variables under the annuity measure are given by:

$$\begin{aligned}
&dx(t)=\left(-\kappa x(t)+y(t)+\frac{\partial \ln A}{\partial x}(t)(a(t)x(t)+b(t))^2\right)dt+(a(t)x(t)+b(t))dW^A(t)\\
&dy(t)=\left((a(t)x(t)+b(t))^2-2\kappa y(t)\right)dt\\
&x(0)=0\\
&y(0)=0
\end{aligned}$$

We deduce that the expected state variable solves the following ODE:

$$\begin{aligned}
&d\bar{x}(t)=\left(-\kappa\bar{x}(t)+\bar{y}(t)+E^A\left[\frac{\partial \ln A}{\partial x}(t)(a(t)x(t)+b(t))^2\right]\right)dt\\
&d\bar{y}(t)=\left(E^A\left[(a(t)x(t)+b(t))^2\right]-2\kappa\bar{y}(t)\right)dt\\
&\bar{x}(0)=0\\
&\bar{y}(0)=0
\end{aligned} \tag{4.5}$$

However, because of the expectation terms in (4.5) the above ODE system is not yet an ODE system explicit in $(\bar{x}(t),\bar{y}(t))$. To simplify our problem we make the following approximation in line with

Andreasen (2005):

$$E^A\left[\frac{\partial \ln A}{\partial x}(t)\left(a(t)x(t)+b(t)\right)^2\right] \simeq \frac{\partial \ln A}{\partial x}(t)\bigg|_{x=\bar{x}(t),\, y=\bar{y}(t)} E^A\left[\left(a(t)x(t)+b(t)\right)^2\right] \quad (4.6)$$

$$E^A\left[\left(a(t)x(t)+b(t)\right)^2\right] \simeq \left(a(t)\bar{x}(t)+b(t)\right)^2 \quad (4.7)$$

In which case the ODE system is approximated by:

$$\begin{aligned}
d\bar{x}(t) &\simeq \left(-\kappa\bar{x}(t)+\bar{y}(t)+\frac{\partial \ln A}{\partial x}(t)\bigg|_{x=\bar{x}(t),\, y=\bar{y}(t)}\left(a(t)\bar{x}(t)+b(t)\right)^2\right)dt \\
d\bar{y}(t) &\simeq \left(\left(a(t)\bar{x}(t)+b(t)\right)^2-2\kappa\bar{y}(t)\right)dt \\
\bar{x}(0) &= 0 \\
\bar{y}(0) &= 0
\end{aligned} \quad (4.8)$$

ODE System (4.8) can be solved efficiently using a Runge-Kutta scheme. Once we know the approximate blending and volatility functions β, Σ we don't yet have analytical treatment for swaption pricing. The first step to this effect is to add a new approximation which consists in replacing the time dependent blending displaced diffusion with a constant blending displaced diffusion (keeping volatility time dependent). Therefore we want to build a model of the form:

$$dS(t) \simeq \Sigma(t)\left(\bar{\beta}S(t)+\left(1-\bar{\beta}\right)S(0)\right)dW^A(t) \quad (4.9)$$

which prices options expiring at time T_0 as close as possible to the proxy model (4.2). As demonstrated in Piterbarg (2005), a suitable candidate is such that:

$$\bar{\beta} = \frac{\int_0^{T_0}\beta(t)\Sigma^2(t)\left(\int_0^t \Sigma^2(s)ds\right)dt}{\int_0^{T_0}\Sigma^2(t)\left(\int_0^t \Sigma^2(s)ds\right)dt}$$

Under the last approximation, pricing a swaption boils down to using a shifted version of the Black formula. Indeed one can prove easily that the price of a payer swaption with strike K and expiry T_0 under model dynamics (4.9) is:

$$
\begin{aligned}
&C\left(T_0,K\right)=\frac{A(0)}{\bar{\beta}}C^{BS}\left(T_0,S\left(0\right),\bar{\beta}K+\left(1-\bar{\beta}\right)S\left(0\right),\bar{\beta}\Sigma^{BS}\right)\\
&\Sigma^{BS}=\sqrt{\frac{\int_0^{T_0}\Sigma^2\left(t\right)dt}{T_0}} \qquad (4.10)\\
&\bar{\beta}>0
\end{aligned}
$$

where $C^{BS}\left(T,F,K,\sigma\right)$ is the undiscounted Black-Scholes call option formula for expiry *T*, forward *F*, strike *K* and volatility σ defined by:

$$
\begin{aligned}
&C^{BS}\left(T,F,K,\sigma\right)=FN\left(d_1\right)-KN\left(d_2\right)\\
&d_1=\frac{\ln\left(\frac{F}{K}\right)}{\sigma\sqrt{T}}+\frac{1}{2}\sigma\sqrt{T}\\
&d_2=d_1-\sigma\sqrt{T}
\end{aligned}
$$

Of course formula (4.10) should be adapted in the case $\bar{\beta}\leq 0$ but this is left as an exercise for the reader. Our approximation's computation time is instantaneous and can be used to obtain fast calibration of the linear Cheyette model to European swaption volatility. In the next section we give numerical examples of this approximation.

5. Numerical examples

In graph 5.1 we compare the swaption price approximation presented in section 4. to a Craig-Sneyd finite differences pricing scheme (400 states x 40 time steps) based on PDE (3.4) after model calibration to Japanese Yen ATM semianual coterminal swaptions covering a 15 year period. In these numerical experiments, for the sake of simplicity, we imposed a constant slope coefficient.

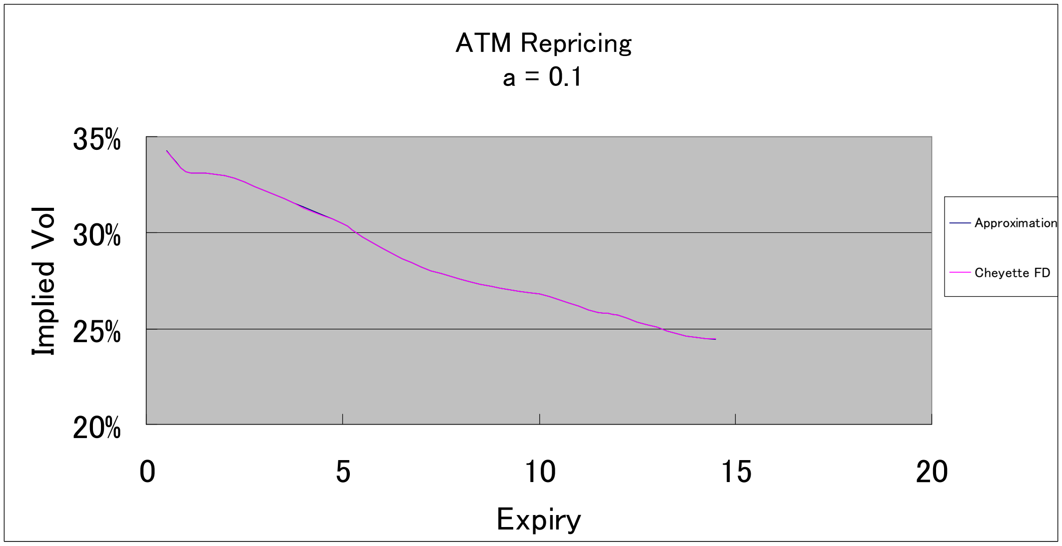

Figure 5.1.a

Repricing of coterminal ATM JPY swaptions

a = 0.1

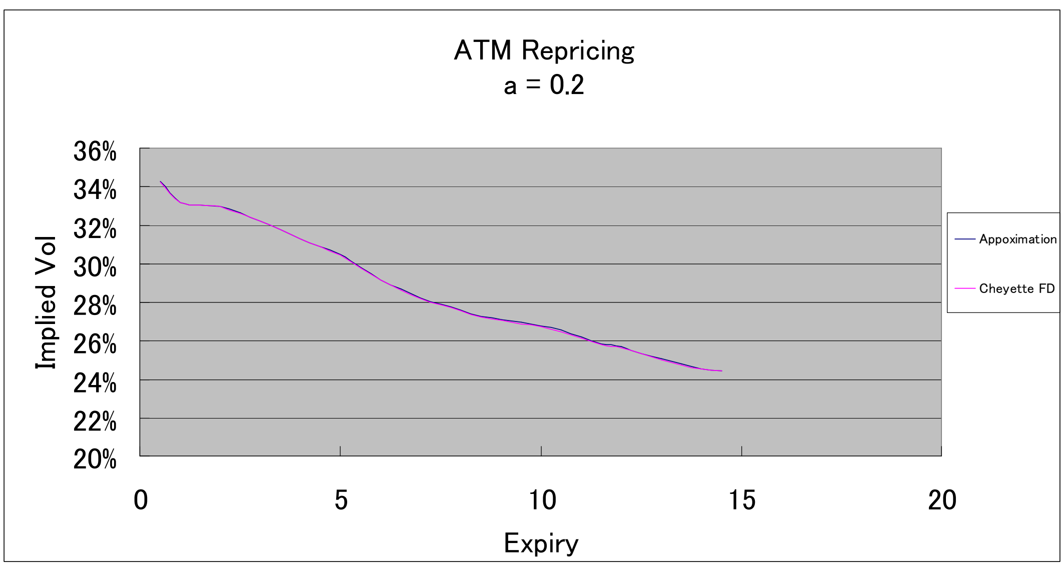

Figure 5.2.b

Repricing of coterminal ATM JPY swaptions

a = 0.2

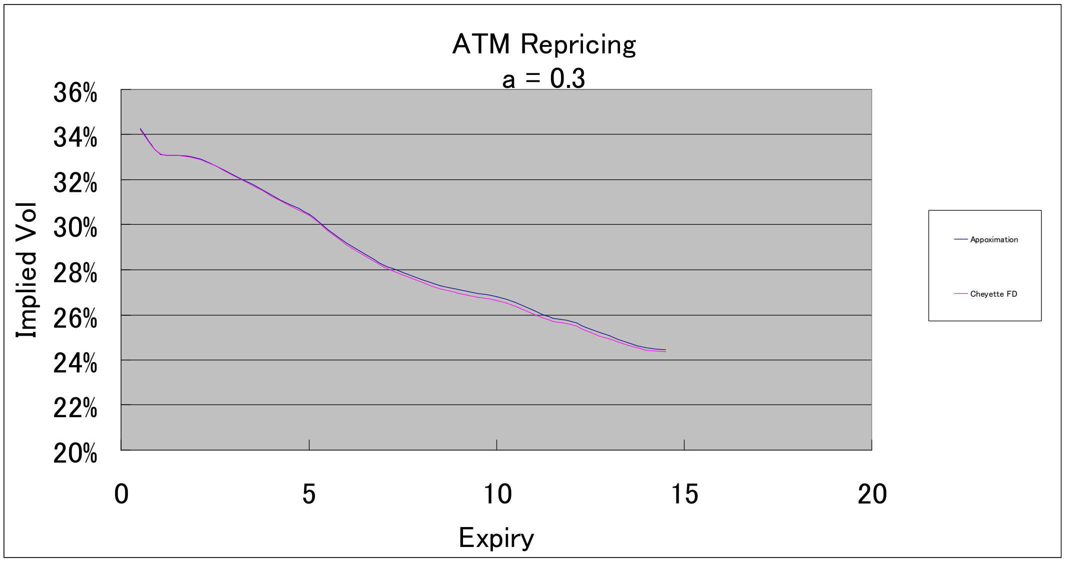

Figure 5.1.c

Repricing of coterminal ATM JPY swaptions (15 year period)

a = 0.3

It is clear that for various levels of the slope coefficient the swaption pricing approximation presented in section 4 does a great job at reproducing the corresponding PDE pricing since the error in all three cases is on average a few basis points of Black volatility, with a maximum of 17 basis points for a slope coefficient a = 0.3 and the 10Y x 5Y swaption.

Although the analytical approximation for swaption pricing is quite satisfactory, we are not completely happy with the crude one we used to estimate the state mean (x, y). In the next section we describe an improvement to the ODE (4.8).

6. State mean accuracy improvement

The fact that we neglect the convexity adjustement in the ODE system (4.8) seems a little gross. To improve this approximation one might first notice that for the Hull and White model, i.e. $a \equiv 0$, the following idendity is exact under the risk-neutral measure:

$$\begin{aligned} &\operatorname{var}\left(x(t)\right) = y(t) = \overline{y}(t) \\ &\Leftrightarrow \overline{x}_2(t) = E\left[x^2(t)\right] = y(t) + \overline{x}^2(t) \end{aligned} \qquad (4.11)$$

By assuming that indentity (4.11) holds true under the annuity measure and combining it with ODE (4.5) and approximation (4.6), the new approximate ODE for the mean state becomes:

$$
\begin{aligned}
d\overline{x}(t) &\simeq \left(-\kappa\overline{x}(t) + \overline{y}(t) + \frac{\partial \ln A}{\partial x}(t)\Big|_{x=\overline{x}(t),\, y=\overline{y}(t)} \left(\left(a(t)\overline{x}(t) + b(t) \right)^2 + a^2(t)\overline{y}(t) \right) \right) dt \\
d\overline{y}(t) &\simeq \left(\left(a(t)\overline{x}(t) + b(t) \right)^2 - \left(2\kappa - a^2(t) \right)\overline{y}(t) \right) dt \qquad (4.12) \\
\overline{x}(0) &= 0 \\
\overline{y}(0) &= 0
\end{aligned}
$$

In the following graphs we compare the accuracy of the numerical mean of state variables obtained by Monte-Carlo simulation under the annuity measure associated with swap 1Yx14Y using JPY market data as of 23 August 2011. and their ODE approximations (4.8) and (4.12). It is clear that the Hull-White approximation (4.12) outperforms the naïve ODE (4.8),

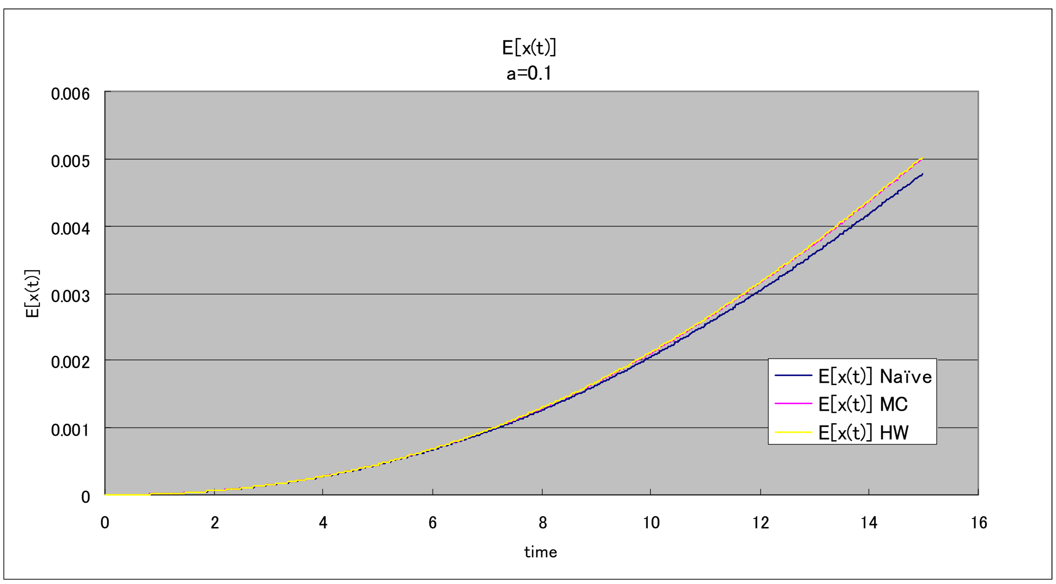

Figure 6.1.a

Accuracy Analysis of mean state *x*

a=0.1

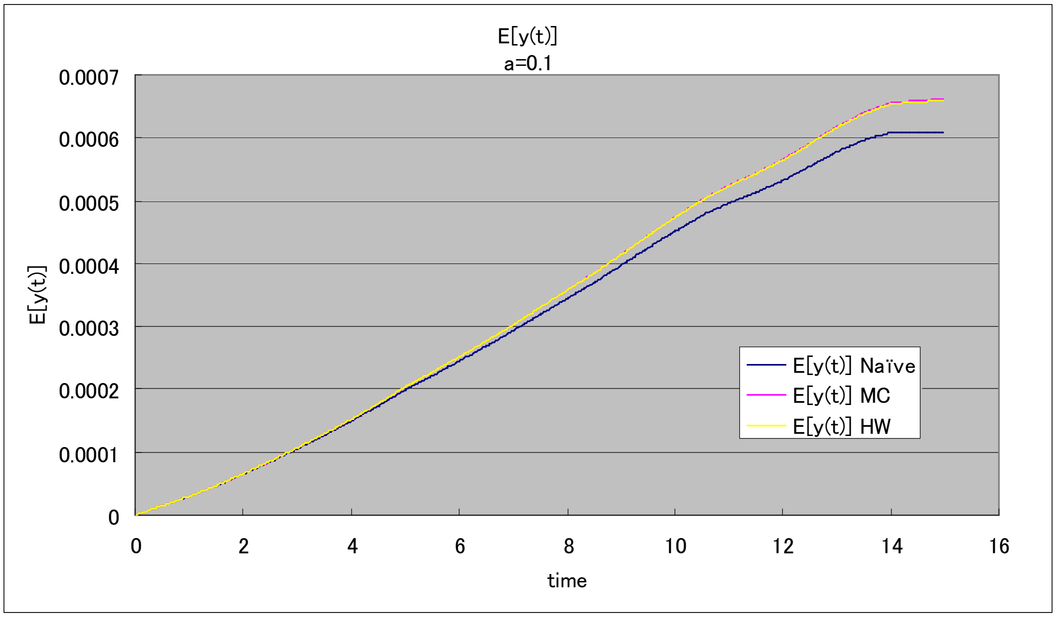

Figure 6.1.b

Accuracy Analysis of mean state *y*

a=0.1

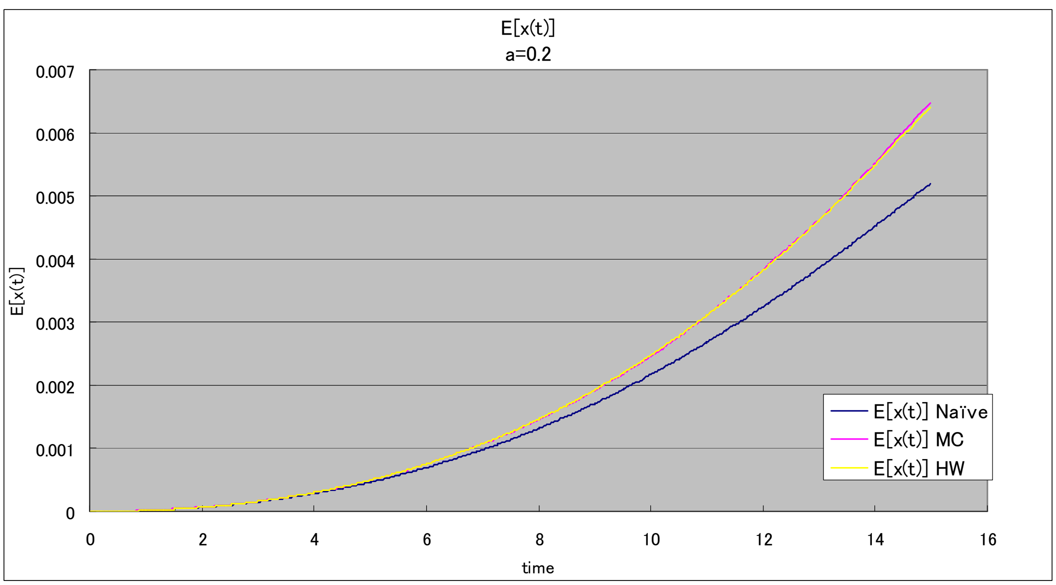

Figure 6.1.c Accuracy Analysis of mean state *x*

a=0.2

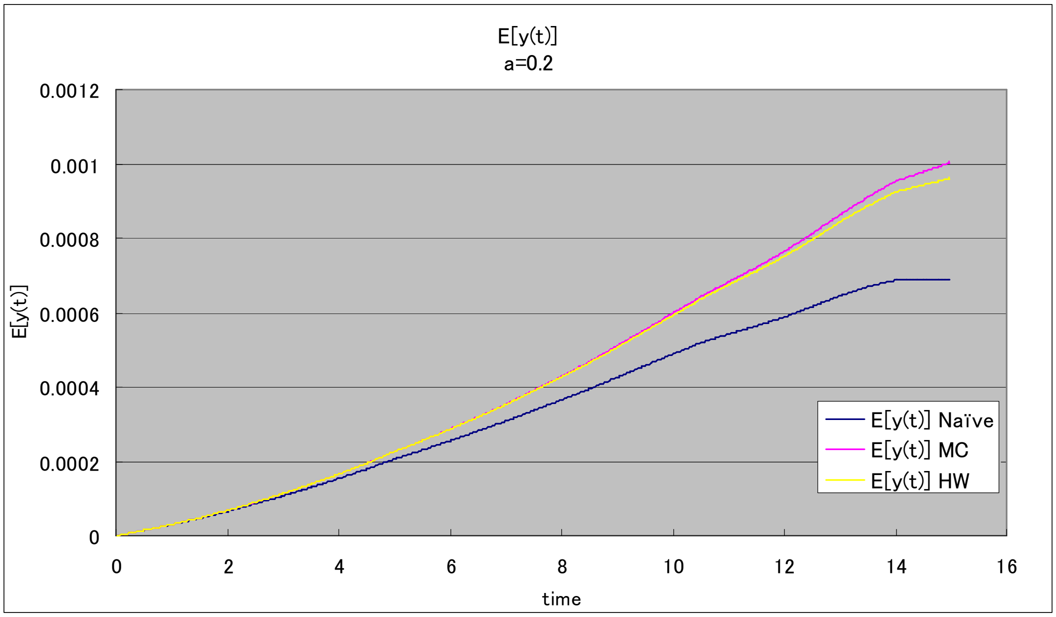

Figure 6.1.d

Accuracy Analysis of mean state *y*

a=0.2

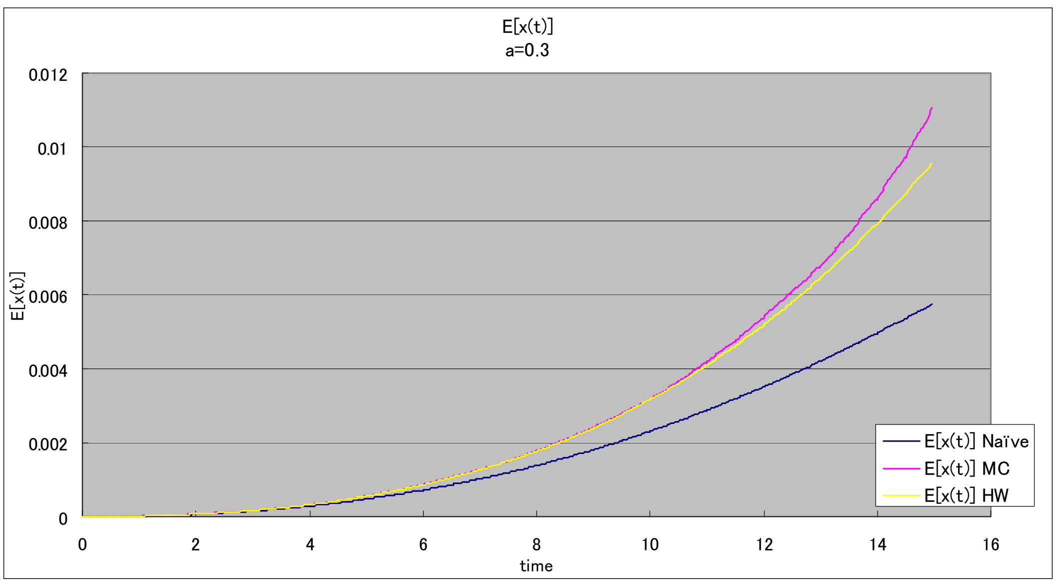

Figure 6.1.e

Accuracy Analysis of mean state *x*

a=0.3

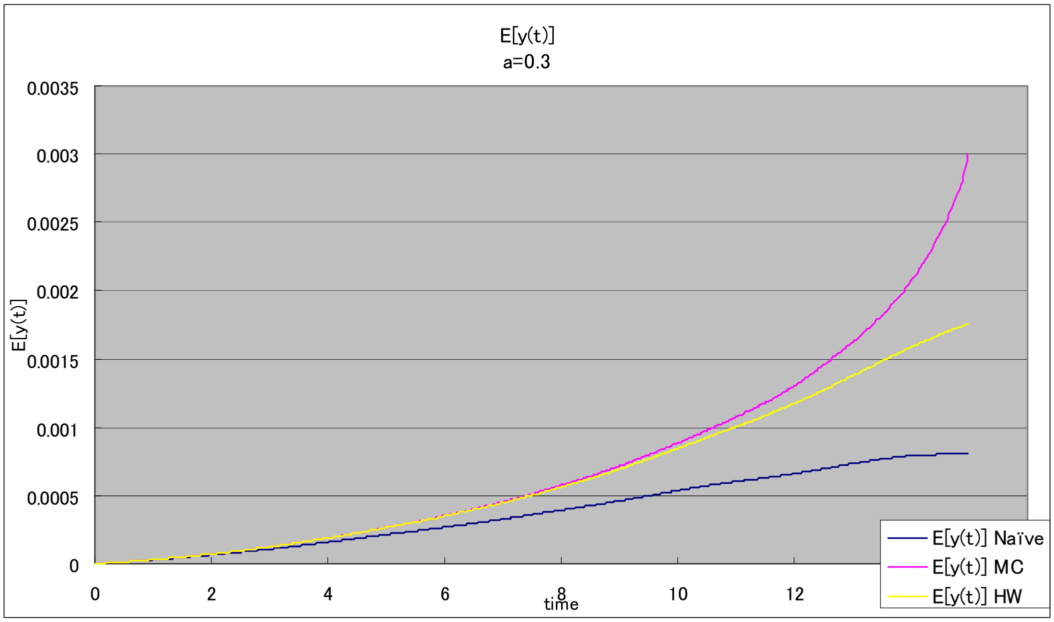

Figure 6.1.f

Accuracy Analysis of mean state *y*

a=0.3

So the Hull-White approximation has obviously drastically improved the accuracy of the 'along the forward' approximation in the Cheyette model. The next logical step would be to use the new state means to peform the projection (4.3). We tried this approach and it turns out that in practice using the naïve mean produces a more accurate swaption pricing than the Hull and White approximation. Graph 6.2 shows a numerical example based on the JPY market as of 23 August 2011 for a 6Mx15Y seminanual coterminal calibration of how both swaption pricing approximations with different state means compare to finite difference pricing. It seems the Hull-White mean approximation is able to perform better at the short end but produces larger swaption repricing at the long end. This behaviour seems to be persistent across a wide range of parameters. Therefore at this stage, since calibration is a craft more than a science, based on empirical results, one would be more tempted to use the naïve mean rather than the improved one. However the improved mean approximation may prove useful in other contexts.

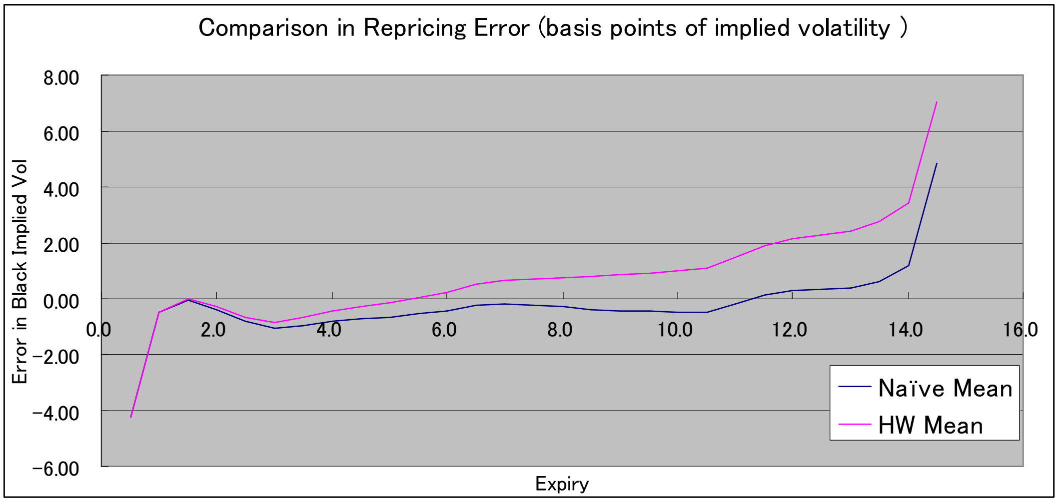

Figure 6.2

Accuracy Comparison between naïve and Hull-White approximation

7. Conclusion

In this work, we have adapted work started in Andreasen (2001) and Andreasen (2005) by making the Cheyette volatility parameterization function more parimonious and by improving the accuracy of the so called "along the forward" concept. These two improvements can easily be extended to stochastic volatility or multifactor Cheyette model. This is left as future research.

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