Draft

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1 Introduction

$$\frac{dS_t^-}{S_t^-} = (r_t - q_t - b_t - \lambda_t \kappa)dt + \sigma(t, S_t^-)dW_t + (Y(t) - 1)dN_t$$

where S_t^- is the pre-default stock value, $\kappa = E(Y(t)-1)$ is the expected relative jump size, q_t is the continuous dividend rate, while b_t is the stock borrowing rate and $N(t)$ is a simple Poisson process with parameter λ_t

if $Y(t) = 0$ for all t, it is a jump-to-ruin model:

$$\frac{dS_t^-}{S_t^-} = (r_t - q_t - b_t + \lambda_t)dt + \sigma(t, S_t^-)dW_t - dN_t$$

$$\begin{aligned} P(\text{S defaults in interval}\,[t, t+dt] &= 1 - \exp\left(-\lambda_t \cdot dt\right)) \\ &\approx \lambda_t \cdot dt \qquad\qquad \text{since } dt \ll 1 \end{aligned}$$

$$\{\text{S defaults in interval}\,[t, t+dt]\} \;\text{ if } U \leq \lambda_t \cdot dt$$

$$\lambda_t \cdot (1-R) = \text{CDS}\,(t)$$

The payoff can be summarized in the following equation:

$$\text{Payoff} = max\left[\alpha_r(t)S_t + Acc, K_{put}, min\left(\bar{C}_t, K_{call} + Acc\right)\right]$$

Acc is the accrued interest up to time t at the current accrual period. K_{put} and K_{call} are the strike prices accordingly. And $\alpha_r(t)$ is the prevailing conversion ratio at time t (different from initial conversion ratio due to things such as dividend protection and accretion).

Most put clause doesn't give the bond holder the accrued interest usually, thus the lack of Acc in the put term (this is of course, subject to the term sheet)

if we take away the accrual interest for simplicity, the equation can be re-written as:

$$\text{Payoff} = min\left[max(\bar{C}_t, K_{put}, \alpha_r(t)S_t), max\left(K_{call}, K_{put}, \alpha_r(t)S_t\right)\right]$$

For path i, at time T_{k-1}, the CB has value of $V\left(i, T_{k-1}\right)$,
with the following auxiliary values:

$$\begin{aligned}\text{Pathwise Continuation Value} : C\left(i, T_{k-1}\right) \\ &= \text{Sum of PV at } T_{k-1} \text{ for coupons paid between } [T_{k-1}, T_k] \\ &+ DF(T_{k-1}, T_k) \cdot (\mathrm{V}\left(i, T_k\right))\end{aligned}$$

Estimated Continuation Value: $\hat{C}\left(i, T_{k-1}\right) = \overrightarrow{\beta}_{k-1}^{T} \psi\left(S_{i,k-1}\right)$
Recovery Value: $V_{recovery}\left(i, T_{k-1}\right) = \text{Recovery Ratio} \cdot (\text{Pre-defaulted Value or Par})$
$\overrightarrow{\beta_k}$

not used below

$$\begin{aligned}\text{Pathwise Continuation Value} : C\left(i, T_{k-1}\right) \\ &= \text{Sum of PV at } T_{k-1} \text{ for coupons paid between } [T_{k-1}, T_k] \\ &+ DF(T_{k-1}, T_k) \cdot (\mathrm{V}\left(i, T_k\right))\end{aligned}$$

Payoff Value: $h\left(i, T_{k-1}\right) = max\left[\alpha_r(t)S_t + Acc, K_{put}, min\left(\bar{C}_t, K_{call} + Acc\right)\right]$
Recovery Value: $V_{recovery}\left(i, T_{k-1}\right) = \text{Recovery Ratio} \cdot \text{Par}$
$\overrightarrow{\beta_k}$
Estimated Continuation Value: $\hat{C}\left(i, T_{k-1}\right) = \overrightarrow{\beta}_{k-1}^{T} \psi\left(S_{i,k-1}\right)$

1.0.1 Exercise Region

$\alpha_r(t) = \alpha_r(t, \Omega(t, S_t)) = 0$
if not in the conversion region (CoCo)/Conversion period

$K_{put} = K_{put}(t) = 0$
if not in the put period

$K_{call} = K_{call}(t, \Omega(t, S_t)) = +\infty$
if not in the call period/soft-call provision is not satisfied

$$V\left(i, T_{k-1}\right) = \begin{cases} h(i, T_{k-1}) & \text{if not default, } h(i, T_{k-1}) \geq \hat{C}\left(i, T_{k-1}\right) \\ C\left(i, T_{k-1}\right) & \text{if not default, } h(i, T_{k-1}) < \hat{C}\left(i, T_{k-1}\right) \\ V_{recovery}\left(i, T_{k-1}\right) & \text{if default} \end{cases}$$

$$V\left(i, T_{k-1}\right) = \begin{cases} V_{nondefault}\left(i, T_{k-1}\right) & \text{if not default within } dt \\ V_{recovery}\left(i, T_{k-1}\right) & \text{if default within } dt \end{cases}$$

$$V_{nondefault}\left(i, T_{k-1}\right) = \begin{cases} \bar{V}_{call,cont}\left(i, T_{k-1}\right) & \text{if } \hat{V}_{call,cont} = M \\ V_{put}\left(i, T_{k-1}\right) & \text{if } V_{put} = M \\ V_{conv}\left(i, T_{k-1}\right) & \text{if } V_{conv} = M \end{cases}$$

Where

$$\begin{aligned} M &= max(V_{conv}, V_{put}, \hat{V}_{call,cont}) \\ \hat{V}_{call,cont} &= min(V_{call}, \hat{C}(i, T_{k-1})) \\ \bar{V}_{call,cont} &= min(V_{call}, C(i, T_{k-1})) \end{aligned}$$

As stated previously, the estimated continuation value $\hat{C}$ given by state-variables polynomial is used only to decide the exercise boundary, where the pathwise continuation value C is used directly for the payoff.

1.0.2 At Maturity

At maturity, depending on the clause, the convertible can either be fully redeemed at par or other fixed amount, optimally redeemed, or (soft) mandatory redeemed

This will be expressed by as the terminal value, either as

$\$Par$, $max(\$Par, \alpha_r(T)S_T)$, $\alpha_r(T)S_T$, $\$Par + \alpha_r(T)S_T$

Here we need to make a distinction between the monetary value \$ and the stock value. The difference is that the stock delivery is credit-risk free, while the monetary delivery should include credit risk. The monetary value is thus calculated by the weighted average of the non-defaulted and defaulted value of the Par weighted by the probability of default at the last grid. While the converted stock value would be between pre-default value and the defaulted value (which is zero according to our jump-to-ruin assumption)

1.1 ASCOT calculation

ASCOT (recall value + option) 's recall swap is calculated by the recall spread(RS), This is the credit spread on top of the floating (credit risk-free rate), from the perspective of the Converitble bond holder, the recall swap is a payer swap.

$$\text{Recall Swap} = \sum_i NPV(Libor_i + RS_i - C_{CB_i}, DF_i)$$

Here $NPV(CF, DF)$ is the NPV of the CF at time i, discounted back to time 0 with discount factor DF_i We can see for the special case, that (i) Assuming $CS_i = r_{risky,i}$, the risky-bond spread that gives us the equivalent risky bond floor of the same issuer (ii) the discounting $DF_i = DF_{Libor_i}$, that we are discounting all the cashflow with the same risk-free rate (single-curve setting), then under these 2 assumptions:

$$\begin{aligned}
\text{Recall Swap} &= \sum_i^N NPV(Libor_i + r_{risky,i} - C_{CB_i}, DF_{Libor,i}) \\
&= \sum_i^N NPV(Libor_i, DF_{Libor_i}) + NPV(Par, DF_{Libor_N}) \\
&- \left(\sum_i^N NPV(C_{CB_i} - r_{risky,i}, DF_{Libor,i}) + NPV(Par, DF_{Libor_N}) \right) \\
&= Par - \left(\sum_i^N NPV(C_{CB_i} - r_{risky,i}, DF_{Libor,i}) + NPV(Par, DF_{Libor_N}) \right) \\
&\approx Par - NPV(CF_{CB}, DF_{Libor+r_{risky}}) \\
&= Par - \text{Risky Bond Floor}
\end{aligned}$$

Here CF_{CB} is the collection of all Convertible Bond coupon cash flow, The approximation is good enough if $r_{risky} << 1$

Recall value is defined as:

$$\text{Recall Value} = Par - \text{Recall Swap}$$

And the ASCOT value is calculated as (intrinsic value only):

$$\text{ASCOT} = \max(\text{CB} - \text{Recall Value}, 0)$$

The idea is to separate the convertible into the credit (bond floor) part and the equity component. And to underwrite an option on just the equity component (often the credit seller will exchange the convertible with recall value and ASCOT, thus only exposed to the equity risk and mitigated the credit exposure).

If we look at the special case with recall spread equal to the real credit spread, and under single-curve setting. Then Recall Value = Risky Bond Floor. Such that the ASCOT value will simply be the intrinsic value of a call with strike equals to the risky bond floor of the convertible bond.

In reality, the Recall Spread chosen will not be exactly the risky bond spread, is subject to liquidity and decided by the market participants; nor will the discount factor be the floating rate in general sense. So the sum of Risky Bond Floor and Recall Swap will not sum up to Par.

2 Reset

If $S_{\text{Calc}} \leq (\text{Reset.Trig} \cdot \text{Initial Conv Price})$:

$$\text{New Conv Price} = \max(\min(S_{\text{Calc}} \cdot \text{Mul.}, \text{Eff.Cap}), \text{Eff.Floor})$$

Here S_{Calc} is the calculated stock average level, $Mul.$ is the multiplier, Reset.Trig is the reset trigger level, and Eff.Cap, Eff.Floor is effective cap and floor respectively.