IS 6489: Statistics and Predictive Analytics

Class 3

Jeff Webb

Agenda

- Announcements / Questions

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- Review: Statistical inference

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- Questions on the material or the course for this week?

Statistical inference review and practice

Statistical inference example

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- You decide to run an experiment—an A/B test—in which you randomly pick visitors to the website to see ad A or B.
- The data that is generated consists in hourly totals that are then summarized as a daily measure: average hourly clicks.
- You run the experiment for 30 days.

Statistical inference example

- The data you get back from IT (top six rows) looks like this:

Day	A	B
1	240.39	98.83
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- Your professional life flashes before your eyes. . . .

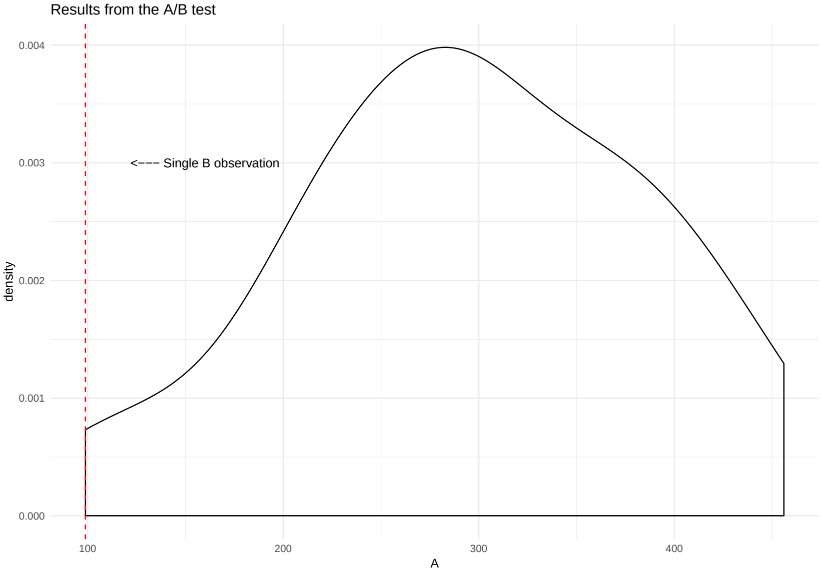
Statistical inference example

Take a closer look at the data:

```
##       Day              A               B        
##  Min.   : 1.00   Min.   :101.3   Min.   :98.83  
##  1st Qu.: 8.25   1st Qu.:237.0   1st Qu.:98.83  
##  Median :15.50   Median :292.9   Median :98.83  
##  Mean   :15.50   Mean   :295.5   Mean   :98.83  
##  3rd Qu.:22.75   3rd Qu.:362.5   3rd Qu.:98.83  
##  Max.   :30.00   Max.   :456.0   Max.   :98.83  
##                                  NA's   :29     
```

Statistical inference example

Plot the observations.



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  - Is your analysis salvageable?
- Have at it: PollEv.com/jeffwebb768

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- "is real" means: is true or actual, exists in the population.
- The problem is that samples vary and any sample statistic gives uncertain information about the population.
- Both p-values and confidence intervals (CIs) are tools for judging whether an observed difference or relationship is real.

Samples and populations

Bernoulli distribution (e.g., a single coin flip): $\mu = p$; $\sigma^2 = p(1-p)$

```r
rbinom(n = 10, size = 1, prob = .5)
```

```
##  [1] 1 1 1 0 0 0 0 0 0 0
```

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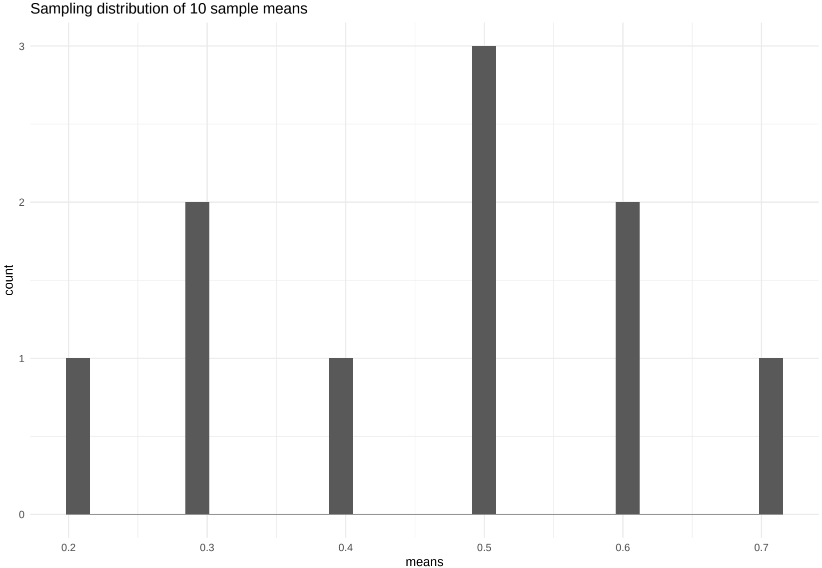
Sampling distribution

If we summarize each sample by taking the mean, those summary statistics—the means—is known as a **sampling distribution**, specifically the **sampling distribution of the sample mean**.

sample	n	mean
1	10	0.5
2	10	0.8
3	10	0.7
4	10	0.4
5	10	0.6
6	10	0.5
7	10	0.6
8	10	0.2
9	10	0.3
10	10	0.5

Sampling distribution

Why does the distribution of sample means look normal when the underlying distribution is binary?



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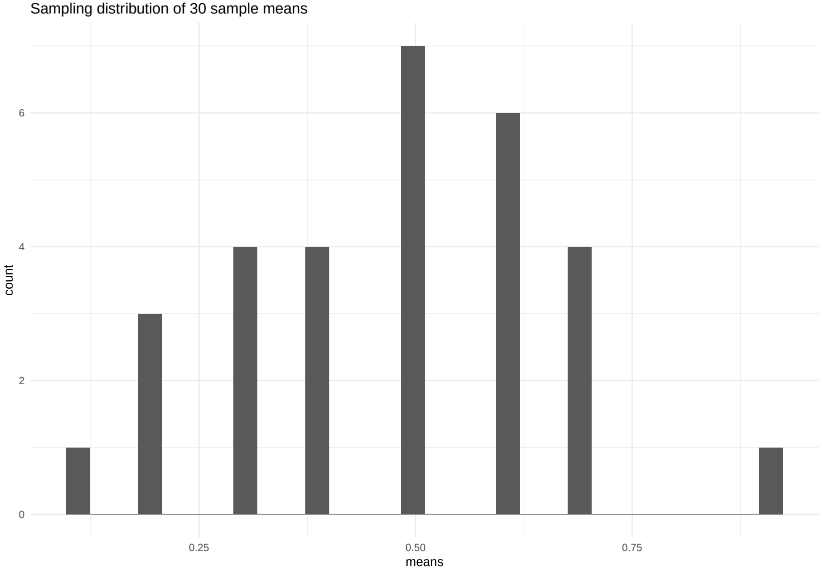

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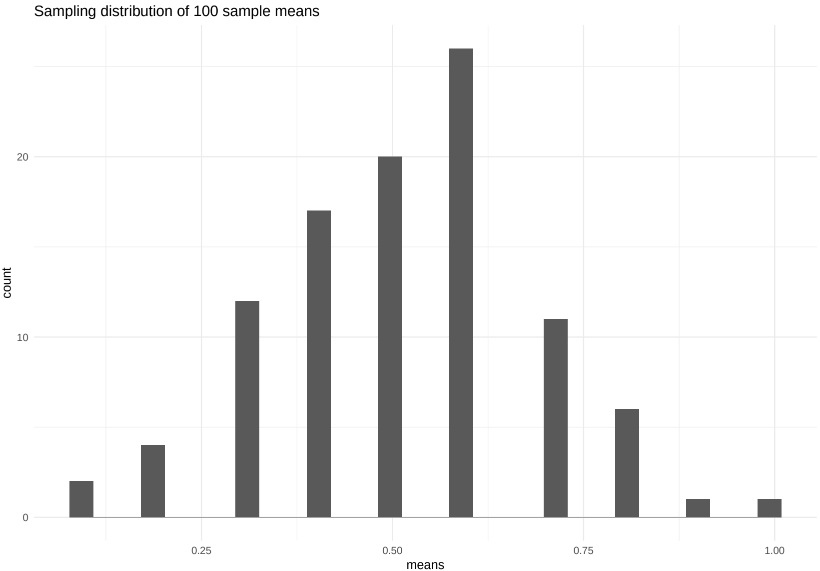

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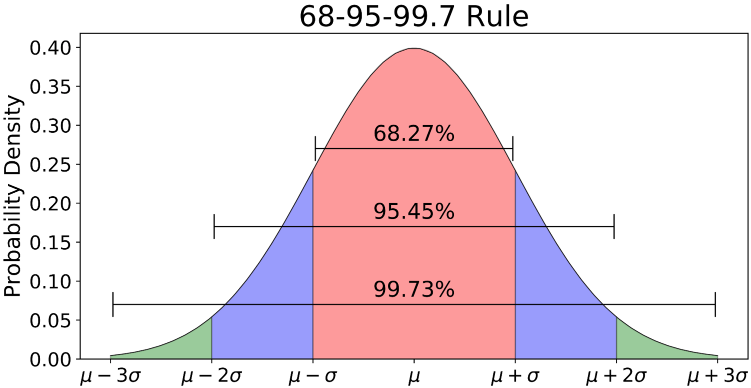
- So, if n is large enough, sampling distributions are normal.
- Why do we care?
- Well, it's a cool fact, but also...
- The normal distribution has convenient properties for doing inference, specifically: 95% of the observations are within $\pm$ 2 standard deviations of the mean.

Normal distribution



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- In the case of the standard error of the mean (SEM) the formula is: $\frac{s}{\sqrt{n}}$.
- Would the SEM be larger or smaller if the sample size was larger?

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8	286.82	NA
9	359.29	NA
10	314.02	NA

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- We can use a hypothesis test....

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- In the case of the A/B test what would our null hypothesis be?
- How would we test it?

SEM

Remember: SEM is just the standard deviation of a sampling distribution. A is already a sampling distribution, so SEM =

```
sd(df$A)
```

```
## [1] 90.39655
```

95% CI

95% CI for a mean: $\bar{x} \pm 1.96 * \text{SEM}(x)$

```
mean(df$A) - 1.96 * sd(df$A)
```

```
## [1] 118.3501
```

```
mean(df$A) + 1.96 * sd(df$A)
```

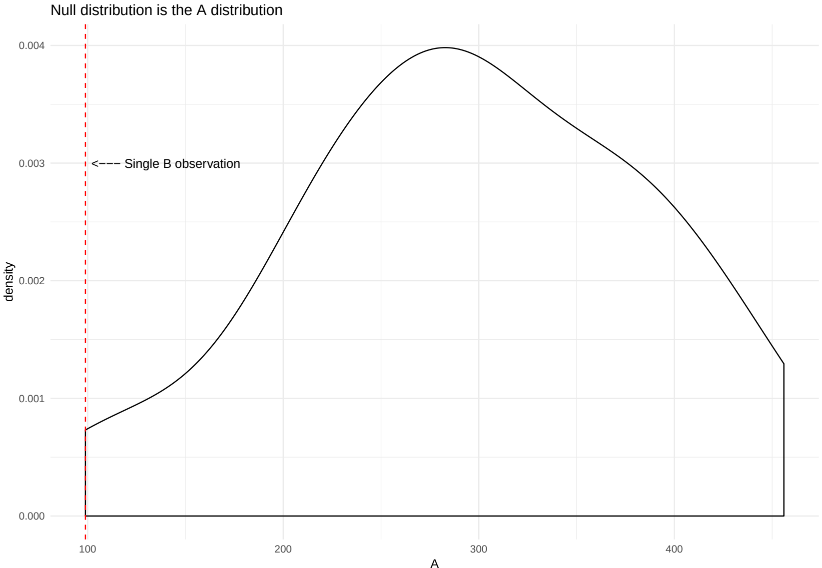
```
## [1] 472.7046
```

```
df$B[1]
```

```
## [1] 98.83
```

Statistical inference in a picture

Null hypothesis: B does not differ from A (is sampled from the same population)



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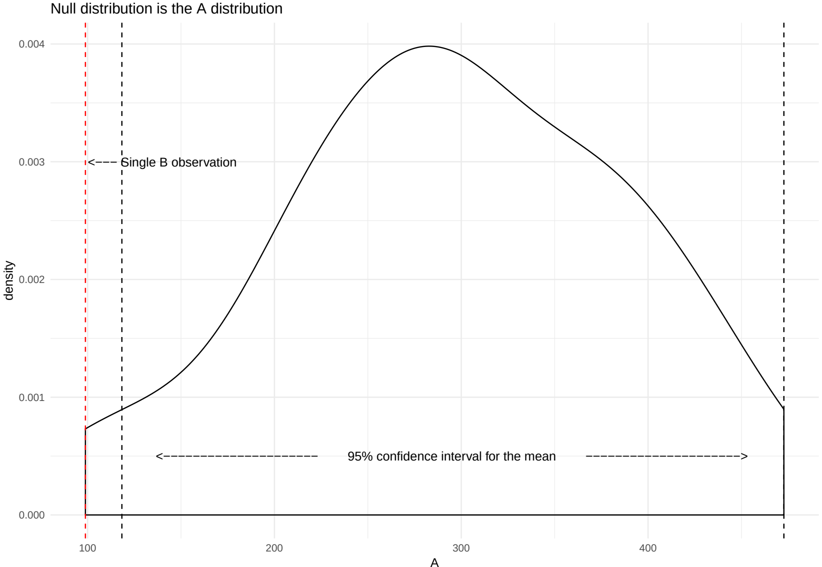
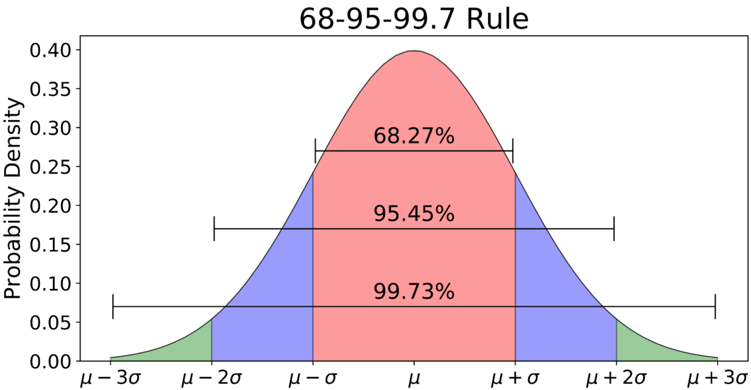


Use the normal distribution for inference



P-value from a 1-sample t-test

t statistic $= \frac{\bar{x}-\mu}{SE}$

```
(t <- (98.83 - mean(df$A)) / sd(df$A)) # compute t
```

```
## [1] -2.175939
```

```
2*pt(q = t, df = 29) # compute p-value
```

```
## [1] 0.0378509
```

Conclusion: The probability of observing average hourly clicks as low as 98.83 is very small. Thus, the assumption there is no difference between B and A is not supported by the data, which allows us to "reject the null." one sample t-test

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- Do this analysis using Rstudio Cloud. Find the .Rmd file at Canvas.