Pushdown Automata - Simplifying the Grammar

Lecture 20
Section 2.2

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Outline

Outline

Simplifying the Grammar

Example (Simplifying a grammar)

- We will simplify the grammar

$$
\begin{aligned}
A_{pr} &\to A_{pp}A_{pr} \mid A_{pq}A_{qr} \mid A_{pr}A_{rr} \mid A_{qq} \\
A_{pp} &\to \varepsilon \\
A_{pq} &\to A_{pp}A_{pq} \mid A_{pq}A_{qq} \\
A_{qq} &\to \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon \\
A_{qr} &\to A_{qq}A_{qr} \mid A_{qr}A_{rr} \\
A_{rr} &\to \varepsilon.
\end{aligned}
$$

Outline

The Simplification Rules

- We may simplify these grammar rules considerably.
  1. Eliminate any rule involving a variable that never appears on the left.
  2. Eliminate any rule involving a variable, other than the start symbol, that never appears on the right.
  3. Eliminate any rule involving a variable that is purely recursive.
  4. Eliminate any unnecessary unit rules.
  5. Use common sense to eliminate any other rules that are useless.

Outline

The Grammar Rules

Example (Simplifying a grammar)

- Apply Rule 3 (purely recursive variables).

$$
\begin{aligned}
A_{pr} &\longrightarrow A_{pp}A_{pr} \mid A_{pq}A_{qr} \mid A_{pr}A_{rr} \mid A_{qq} \\
A_{pp} &\longrightarrow \varepsilon \\
A_{pq} &\longrightarrow A_{pp}A_{pq} \mid A_{pq}A_{qq} \\
A_{qq} &\longrightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon \\
A_{qr} &\longrightarrow A_{qq}A_{qr} \mid A_{qr}A_{rr} \\
A_{rr} &\longrightarrow \varepsilon.
\end{aligned}
$$

Simplifying the Grammar

Example (Simplifying a grammar)

- The grammar rules are reduced to

$$
\begin{aligned}
A_{pr} &\to A_{pp}A_{pr} \mid A_{pq}A_{qr} \mid A_{pr}A_{rr} \mid A_{qq} \\
A_{pp} &\to \varepsilon \\
A_{qq} &\to \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon \\
A_{rr} &\to \varepsilon.
\end{aligned}
$$

Simplifying the Grammar

Example (Simplifying a grammar)

- Apply Rule 5 (common sense).

$$A_{pr} \rightarrow A_{pp}A_{pr} \mid A_{pq}A_{qr} \mid A_{pr}A_{rr} \mid A_{qq}$$
$$A_{pp} \rightarrow \varepsilon$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon$$
$$A_{rr} \rightarrow \varepsilon.$$

- We should eliminate these rules and replace A_{pp} and A_{rr} with ε everywhere they occur.

Simplifying the Grammar

Example (Simplifying a grammar)

- Now the grammar rules are reduced to

$$A_{pr} \rightarrow A_{pr} \mid A_{pq}A_{qr} \mid A_{pr} \mid A_{qq}$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

Simplifying the Grammar

Example (Simplifying a grammar)

- Apply Rule 4 (useless unit rules).

$$A_{pr} \rightarrow A_{pr} \mid A_{pq}A_{qr} \mid A_{pr} \mid A_{qq}$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

Simplifying the Grammar

Example (Simplifying a grammar)

- We now have

$$A_{pr} \rightarrow A_{pq}A_{qr} \mid A_{qq}$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

Simplifying the Grammar

Example (Simplifying a grammar)

- Apply Rule 1 (variables never on the left).

$$A_{pr} \rightarrow A_{pq}A_{qr} \mid A_{qq}$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

Simplifying the Grammar

Example (Simplifying a grammar)

- We now have

$$A_{pr} \rightarrow A_{qq}$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

Simplifying the Grammar

Example (Simplifying a grammar)

- Apply Rule 4 (useless unit rules).

$$A_{pr} \rightarrow A_{qq}$$
$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

- Eliminate A_{pr}, making A_{qq} the new start symbol.

Simplifying the Grammar

Example (Simplifying a grammar)

- We now have

$$A_{qq} \rightarrow \mathbf{a}A_{qq}\mathbf{b} \mid A_{qq}A_{qq} \mid \varepsilon.$$

or, more simply,

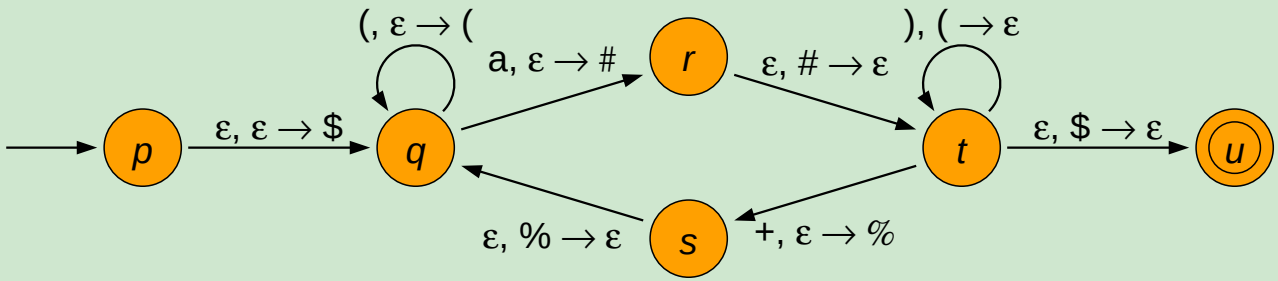
$$S \rightarrow \mathbf{a}S\mathbf{b} \mid SS \mid \epsilon$$

Outline

Example

Example (Converting a PDA to a CFG and simplifying)

- Simplify the grammar for the language of the valid algebraic expressions over the alphabet $\Sigma = \{\mathbf{a}, +, (,)\}$.



Example

Example (Converting a PDA to a CFG and simplifying)

Pushers	Poppers
$\delta(p, \varepsilon, \varepsilon) = (q, \$)$	$\delta(r, \varepsilon, \#) = (t, \varepsilon)$
$\delta(q, (, \varepsilon) = (q, ()$	$\delta(t,), () = (t, \varepsilon)$
$\delta(q, \mathbf{a}, \varepsilon) = (r, \#)$	$\delta(s, \varepsilon, \%) = (q, \varepsilon)$
$\delta(t, +, \varepsilon) = (s, \%)$	$\delta(t, \varepsilon, \$) = (u, \varepsilon)$

Example

Example (Converting a PDA to a CFG and simplifying)

- We found the grammar to be

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pp}A_{pu} \mid A_{pq}A_{qu} \mid A_{pr}A_{ru} \mid A_{pt}A_{tu} \mid A_{ps}A_{su} \mid A_{pu}A_{uu} \mid A_{qt} \\
A_{pp} &\longrightarrow \varepsilon \\
A_{pq} &\longrightarrow A_{pp}A_{pq} \mid A_{pq}A_{qq} \mid A_{pr}A_{rq} \mid A_{pt}A_{tq} \mid A_{ps}A_{sq} \\
A_{pt} &\longrightarrow A_{pp}A_{pt} \mid A_{pq}A_{qt} \mid A_{pr}A_{rt} \mid A_{pt}A_{tt} \mid A_{ps}A_{st} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qr}A_{rq} \mid A_{qt}A_{tq} \mid A_{qs}A_{sq} \mid (A_{qs} \mid \mathbf{a}A_{rs} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qr}A_{rt} \mid A_{qt}A_{tt} \mid A_{qs}A_{st} \mid (A_{qr} \mid (A_{qt}) \mid \mathbf{a}A_{rr} \mid \mathbf{a}A_{rt}) \\
A_{qu} &\longrightarrow A_{qq}A_{qu} \mid A_{qr}A_{ru} \mid A_{qt}A_{tu} \mid A_{qs}A_{su} \mid A_{qu}A_{uu} \\
A_{rr} &\longrightarrow \varepsilon \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tr}A_{rq} \mid A_{tt}A_{tq} \mid A_{ts}A_{sq} \mid \ + A_{ss} \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tr}A_{rt} \mid A_{tt}A_{tt} \mid A_{ts}A_{st} \mid \ + A_{sr} \mid \ + A_{st}) \mid \varepsilon \\
A_{tu} &\longrightarrow A_{tq}A_{qu} \mid A_{tr}A_{ru} \mid A_{tt}A_{tu} \mid A_{ts}A_{su} \mid A_{tu}A_{uu} \\
A_{ss} &\longrightarrow \varepsilon \\
A_{uu} &\longrightarrow \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Apply Rule 1.

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pp}A_{pu} \mid A_{pq}A_{qu} \mid A_{pr}A_{ru} \mid A_{pt}A_{tu} \mid A_{ps}A_{su} \mid A_{pu}A_{uu} \mid A_{qt} \\
A_{pp} &\longrightarrow \varepsilon \\
A_{pq} &\longrightarrow A_{pp}A_{pq} \mid A_{pq}A_{qq} \mid A_{pr}A_{rq} \mid A_{pt}A_{tq} \mid A_{ps}A_{sq} \\
A_{pt} &\longrightarrow A_{pp}A_{pt} \mid A_{pq}A_{qt} \mid A_{pr}A_{rt} \mid A_{pt}A_{tt} \mid A_{ps}A_{st} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qr}A_{rq} \mid A_{qt}A_{tq} \mid A_{qs}A_{sq} \mid (A_{qs} \mid \mathbf{a}A_{rs} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qr}A_{rt} \mid A_{qt}A_{tt} \mid A_{qs}A_{st} \mid (A_{qr} \mid (A_{qt}) \mid \mathbf{a}A_{rr} \mid \mathbf{a}A_{rt}) \\
A_{qu} &\longrightarrow A_{qq}A_{qu} \mid A_{qr}A_{ru} \mid A_{qt}A_{tu} \mid A_{qs}A_{su} \mid A_{qu}A_{uu} \\
A_{rr} &\longrightarrow \varepsilon \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tr}A_{rq} \mid A_{tt}A_{tq} \mid A_{ts}A_{sq} \mid \ + A_{ss} \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tr}A_{rt} \mid A_{tt}A_{tt} \mid A_{ts}A_{st} \mid \ + A_{sr} \mid \ + A_{st}) \mid \varepsilon \\
A_{tu} &\longrightarrow A_{tq}A_{qu} \mid A_{tr}A_{ru} \mid A_{tt}A_{tu} \mid A_{ts}A_{su} \mid A_{tu}A_{uu} \\
A_{ss} &\longrightarrow \varepsilon \\
A_{uu} &\longrightarrow \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- This leaves the rules

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pp}A_{pu} \mid A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{pu}A_{uu} \mid A_{qt} \\
A_{pp} &\longrightarrow \varepsilon \\
A_{pq} &\longrightarrow A_{pp}A_{pq} \mid A_{pq}A_{qq} \mid A_{pt}A_{tq} \\
A_{pt} &\longrightarrow A_{pp}A_{pt} \mid A_{pq}A_{qt} \mid A_{pt}A_{tt} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a}A_{rr} \\
A_{qu} &\longrightarrow A_{qq}A_{qu} \mid A_{qt}A_{tu} \mid A_{qu}A_{uu} \\
A_{rr} &\longrightarrow \varepsilon \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid \ + A_{ss} \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon \\
A_{tu} &\longrightarrow A_{tq}A_{qu} \mid A_{tt}A_{tu} \mid A_{tu}A_{uu} \\
A_{ss} &\longrightarrow \varepsilon \\
A_{uu} &\longrightarrow \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Apply Rule 5.

$$A_{pu} \to A_{pp}A_{pu} \mid A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{pu}A_{uu} \mid A_{qt}$$
$$A_{pp} \to \varepsilon$$
$$A_{pq} \to A_{pp}A_{pq} \mid A_{pq}A_{qq} \mid A_{pt}A_{tq}$$
$$A_{pt} \to A_{pp}A_{pt} \mid A_{pq}A_{qt} \mid A_{pt}A_{tt}$$
$$A_{qq} \to A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon$$
$$A_{qt} \to A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a}A_{rr}$$
$$A_{qu} \to A_{qq}A_{qu} \mid A_{qt}A_{tu} \mid A_{qu}A_{uu}$$
$$A_{rr} \to \varepsilon$$
$$A_{tq} \to A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid +A_{ss}$$
$$A_{tt} \to A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon$$
$$A_{tu} \to A_{tq}A_{qu} \mid A_{tt}A_{tu} \mid A_{tu}A_{uu}$$
$$A_{ss} \to \varepsilon$$
$$A_{uu} \to \varepsilon$$

Example

Example (Converting a PDA to a CFG and simplifying)

- This leaves the rules

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pu} \mid A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{pu} \mid A_{qt} \\
A_{pq} &\longrightarrow A_{pq} \mid A_{pq}A_{qq} \mid A_{pt}A_{tq} \\
A_{pt} &\longrightarrow A_{pt} \mid A_{pq}A_{qt} \mid A_{pt}A_{tt} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{qu} &\longrightarrow A_{qq}A_{qu} \mid A_{qt}A_{tu} \mid A_{qu}A_{uu} \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon \\
A_{tu} &\longrightarrow A_{tq}A_{qu} \mid A_{tt}A_{tu} \mid A_{tu}
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Apply Rule 4.

$$
\begin{aligned}
A_{pu} &\to A_{pu} \mid A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{pu} \mid A_{qt} \\
A_{pq} &\to A_{pq} \mid A_{pq}A_{qq} \mid A_{pt}A_{tq} \\
A_{pt} &\to A_{pt} \mid A_{pq}A_{qt} \mid A_{pt}A_{tt} \\
A_{qq} &\to A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\to A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{qu} &\to A_{qq}A_{qu} \mid A_{qt}A_{tu} \mid A_{qu}A_{uu} \\
A_{tq} &\to A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\to A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon \\
A_{tu} &\to A_{tq}A_{qu} \mid A_{tt}A_{tu} \mid A_{tu}
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- This leaves the rules

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{qt} \\
A_{pq} &\longrightarrow A_{pq}A_{qq} \mid A_{pt}A_{tq} \\
A_{pt} &\longrightarrow A_{pq}A_{qt} \mid A_{pt}A_{tt} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{qu} &\longrightarrow A_{qq}A_{qu} \mid A_{qt}A_{tu} \mid A_{qu}A_{uu} \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon \\
A_{tu} &\longrightarrow A_{tq}A_{qu} \mid A_{tt}A_{tu}
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Apply Rule 3.

$$
\begin{aligned}
A_{pu} &\to A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{qt} \\
A_{pq} &\to A_{pq}A_{qq} \mid A_{pt}A_{tq} \\
A_{pt} &\to A_{pq}A_{qt} \mid A_{pt}A_{tt} \\
A_{qq} &\to A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\to A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{qu} &\to A_{qq}A_{qu} \mid A_{qt}A_{tu} \mid A_{qu}A_{uu} \\
A_{tq} &\to A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\to A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon \\
A_{tu} &\to A_{tq}A_{qu} \mid A_{tt}A_{tu}
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- This leaves the rules

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{qt} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Apply Rule 1.

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{pq}A_{qu} \mid A_{pt}A_{tu} \mid A_{qt} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- This leaves the rules

$$
\begin{aligned}
A_{pu} &\longrightarrow A_{qt} \\
A_{qq} &\longrightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\longrightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{tq} &\longrightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\longrightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Apply Rule 4.

$$
\begin{aligned}
A_{pu} &\rightarrow A_{qt} \\
A_{qq} &\rightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{qt} &\rightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{tq} &\rightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\rightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon
\end{aligned}
$$

and make A_{qt} the new start symbol.

Example

Example (Converting a PDA to a CFG and simplifying)

- This leaves the rules

$$
\begin{aligned}
A_{qt} &\rightarrow A_{qq}A_{qt} \mid A_{qt}A_{tt} \mid (A_{qt}) \mid \mathbf{a} \\
A_{qq} &\rightarrow A_{qq}A_{qq} \mid A_{qt}A_{tq} \mid \varepsilon \\
A_{tq} &\rightarrow A_{tq}A_{qq} \mid A_{tt}A_{tq} \mid + \\
A_{tt} &\rightarrow A_{tq}A_{qt} \mid A_{tt}A_{tt} \mid \varepsilon
\end{aligned}
$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Now give them simpler names: $S = A_{qt}$, $A = A_{qq}$, $B = A_{tt}$, and $C = A_{tq}$.

$$S \to AS \mid SB \mid (S) \mid \mathbf{a}$$
$$A \to AA \mid SC \mid \varepsilon$$
$$C \to CA \mid BC \mid +$$
$$B \to CS \mid BB \mid \varepsilon$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Use the grammar to derive the string $(\mathbf{a} + \mathbf{a}) + \mathbf{a}$.

$$S \rightarrow AS \mid SB \mid (S) \mid \mathbf{a}$$
$$A \rightarrow AA \mid SC \mid \varepsilon$$
$$B \rightarrow BB \mid CS \mid \varepsilon$$
$$C \rightarrow CA \mid BC \mid +$$

Example

Example (Converting a PDA to a CFG and simplifying)

- Use the grammar to derive the string $(\mathbf{a} + \mathbf{a}) + \mathbf{a}$.

$$
\begin{aligned}
S &\Rightarrow SB \\
&\Rightarrow (S)B \\
&\Rightarrow (AS)B \\
&\Rightarrow (SCS)B \\
&\Rightarrow (SCS)CS \\
&\Rightarrow (\mathbf{a} + \mathbf{a}) + \mathbf{a}
\end{aligned}
$$

Outline

Assignment

Assignment

- Read Section 2.2, pages 115 - 123.
- Simplify the grammar created in the previous homework problem, with PDA