Minimizing DFAs

Lecture 11
Exercise 7.40

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Fri, Sep 21, 2012

Outline

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1. Equivalent States
2. n-Equivalence
3. Minimization Examples
4. Assignment

Equivalent States

- To minimize a DFA, we must identify states that are “equivalent.”
- When two states are equivalent, one of them may be eliminated.

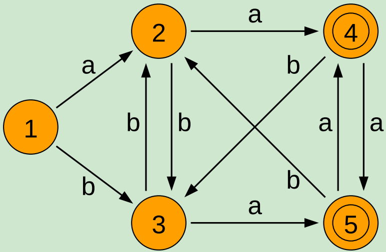
Definition (Equivalent states)

Two states in a DFA are equivalent if, for any input, the decision of whether to accept or reject it will be the same regardless of which of the two states we are currently in.

Example

Example (Equivalent states)

Clearly, states 2 and 3 are equivalent and states 4 and 5 are equivalent.



Example

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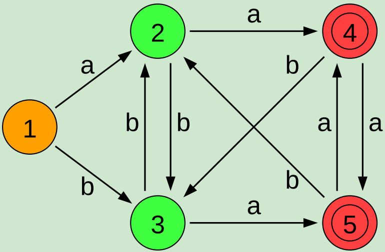

Example

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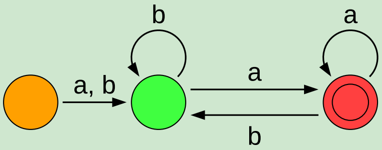


Outline

Equivalence of States

Definition (0-equivalence of states)

Two states q and q' are 0-equivalent if

- Both are accepting states, or
- Both are rejecting states.

Definition (n-equivalence of states)

Let n be a positive integer. Two states q and q' are n-equivalent if the states $\delta(q, a)$ and $\delta(q', a)$ are $(n-1)$-equivalent for all $a \in \Sigma$.

Definition (Equivalence of states)

Two states are equivalent if they are n-equivalent for all $n \geq 0$.

Determining Equivalent States

- To determine which states are equivalent,
  - Add a “dead” state, if necessary, to make the DFA fully defined.
  - Determine the 0-equivalence classes: F, $Q - F$.
  - From the 0-equivalence classes, determine the 1-equivalence classes.
  - From the 1-equivalence classes, determine the 2-equivalence classes.
  - And so on, until, for some n, the n-equivalence classes and the $(n-1)$-equivalence classes are the same.

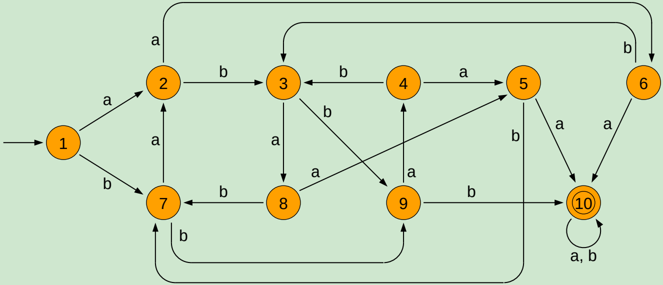
Determining Equivalent States

- At that point, the equivalence classes are the *n*-equivalence classes.
- Redraw the DFA, lumping all states of an equivalence class together as a single state.

Outline

Example (Minimizing a DFA)

Minimize the following DFA:



Example

Example (Minimizing a DFA)

- The 0-equivalence classes are

$$F = \{10\}$$

and

$$Q - F = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

Example

Example (Minimizing a DFA)

- Summarize the transitions in the following tables.

	1	2	3	4	5	6	7	8	9
a	2	6	8	5	10	10	2	5	4
b	7	3	9	3	7	3	9	7	10

	10
a	10
b	10

- Identify each entry with one of the 0-equivalence classes

	1	2	3	4	5	6	7	8	9
a	A	A	A	A	B	B	A	A	A
b	A	A	A	A	A	A	A	A	B

	10
a	B
b	B

Example

Example (Minimizing a DFA)

- There are three patterns within $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$: *AA*, *BA*, and *AB*.
- These patterns subdivide the 0-equivalence classes into the 1-equivalence classes:

$$\{1, 2, 3, 4, 7, 8\}, \{5, 6\}, \{9\}, \{10\}.$$

Example

Example (Minimizing a DFA)

- Show the transitions for the 1-equivalence classes.

	1	2	3	4	7	8
a	2	6	8	5	2	5
b	7	3	9	3	9	7

	5	6
a	10	10
b	7	3

	9
a	4
b	10

	10
a	10
b	10

- Identify each entry with a 1-equivalence class.

	1	2	3	4	7	8
a	A	B	A	B	A	B
b	A	A	C	A	C	A

	5	6
a	D	D
b	A	A

	9
a	A
b	D

	10
a	D
b	D

Example

Example (Minimizing a DFA)

- There are 3 different patterns within $\{1, 2, 3, 4, 7, 8\}$: *AA*, *BA*, and *AC*.
- These patterns subdivide the 1-equivalence classes into the 2-equivalence classes:

$$\{1\}, \{2, 4, 8\}, \{3, 7\}, \{5, 6\}, \{9\}, \{10\}.$$

Example

Example (Minimizing a DFA)

- Show the transitions for the 2-equivalence classes.

	1
a	2
b	7

	2	4	8
a	6	5	5
b	3	3	7

	3	7
a	8	2
b	9	9

	5	6
a	10	10
b	7	3

	9
a	4
b	10

	10
a	10
b	10

	1
a	B
b	C

	2	4	8
a	D	D	D
b	C	C	C

	3	7
a	B	B
b	E	E

	5	6
a	F	F
b	C	C

	9
a	B
b	F

	10
a	F
b	F

Example

Example (Minimizing a DFA)

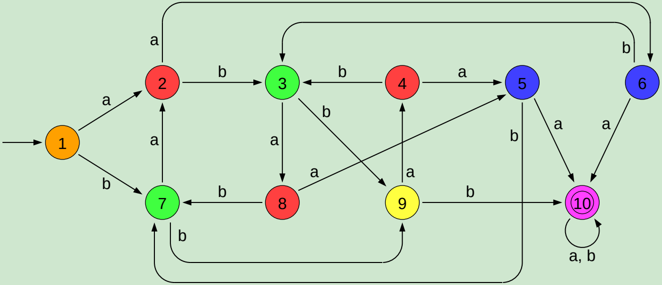
- Identify each entry with a 2-equivalence classes.
- The patterns are the same within each class.
- There is no further subdividing.
- Therefore, these are the final equivalence classes:

$$\{1\}, \{2, 4, 8\}, \{3, 7\}, \{5, 6\}, \{9\}, \{10\}.$$

Example

Example (Minimizing a DFA)

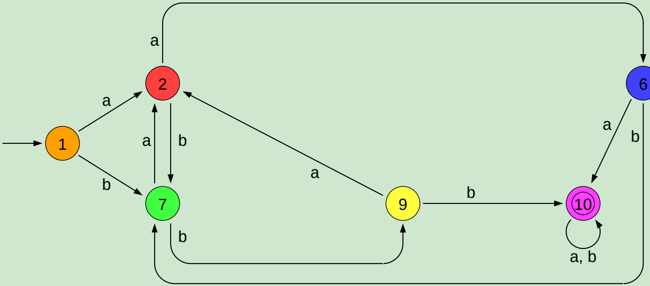
- The equivalent states:



Example

Example (Minimizing a DFA)

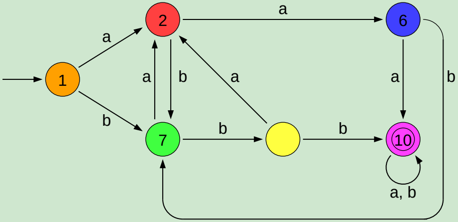
- The simplified diagram:



Example

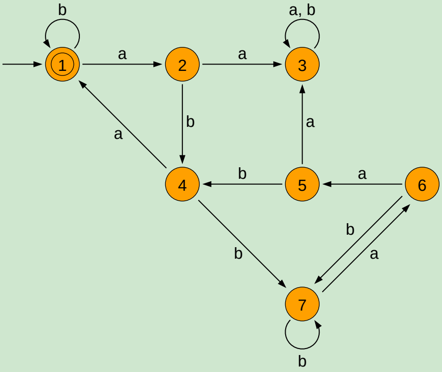
Example (Minimizing a DFA)

- The final diagram:



Example (Minimizing a DFA)

- Minimize the following NFA.



Outline

Assignment

- Read the algorithm described in Exercise 7.40, page 299. Study it until you understand how it is equivalent to the algorithm presented in class.
- Construct an NFA for the concatenation L_1L_2 of the following languages over the alphabet $\{\mathbf{a}, \mathbf{b}\}$ and then minimize it.

$$L_1 = \{w \mid \text{the length of } w \text{ is at most } 1\}$$

$$L_2 = \{w \mid \text{every odd position of } w \text{ is } \mathbf{b}\}.$$

- Construct a minimal DFA for $(L_1 \cup L_2)^*$ where

$$L_1 = \{w \mid w \text{ starts with } \mathbf{a} \text{ and has an even number of symbols}\}$$

$$L_2 = \{w \mid w \text{ starts with } \mathbf{b} \text{ and has an odd number of symbols}\}.$$