The Global Positioning System

- GPS - Global Positioning System
  - Operated by the Department of Defense (DoD)
  - Constellation of 31 satellites in orbit around the earth (https://en.wikipedia.org/wiki/List_of_GPS_satellites)
- How it works
  -

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- How it works
  - Each satellite broadcasts a signal containing
    - Satellite's position
    - Information (code) that can be used to determine the distance (range) between the receiver and the satellite

The Global Positioning System

- How it works
  - Each satellite broadcasts a signal containing
    - Satellite's position
    - Information (**code**) that can be used to determine the distance (**range**) between the receiver and the satellite
  - Ranges from 4 or more satellites can be used to determine the position of the receiver
    - 4 unknowns: X, Y, Z (lat, lon, h) and time correction, Δt
  - This is called a code solution
    - 3 to 5 meter accuracy

The Global Positioning System

- How it works
  - **Code** solution
    - 3 to 5 meter accuracy
    - Used in recreation and mapping grade GPS receivers
    - Once an initial position is determined, only needs short time periods of signal reception to receive code
      - Generally works well under tree canopies and with intermittent reception

The Global Positioning System

- How it works
  - There is also the **carrier phase** solution
    - Uses small shifts in the wave of the satellite's signal to get higher accuracy
    - Used in survey grade GPS receivers ($$)
    - Sub-meter (cm) level accuracy
    - Needs long time periods of signal reception or signal lock
      - Interruption of the signal will cause degraded accuracy
      - Carrier phase GPS does not work well under tree canopy

The Global Positioning System

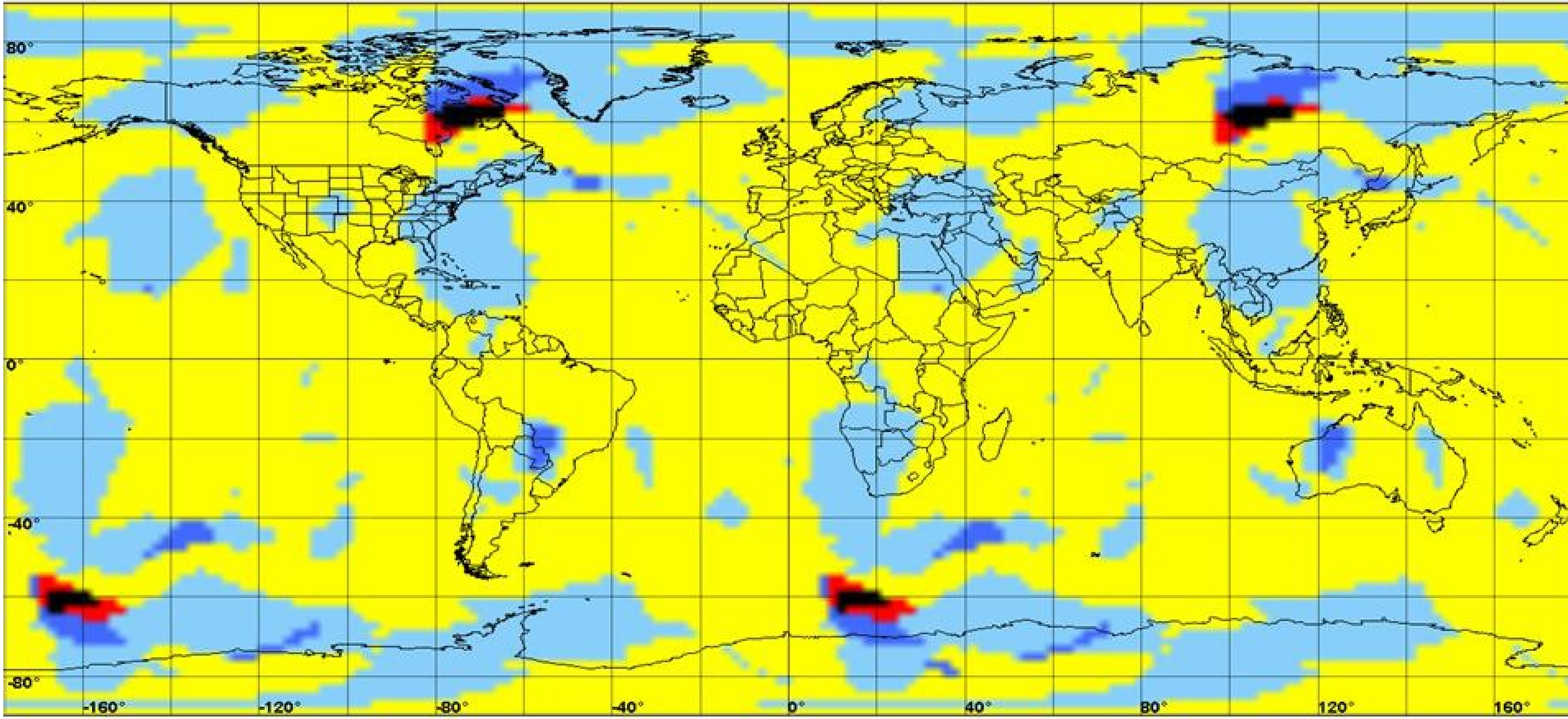
- GPS Accuracy
  - Clock Error
    - Clock on board satellite
    - Receiver clock
  - Ionospheric Delay
    - Refraction of the signal as it passes through ionosphere
    - Position errors range from 5m to 150m at the receiver
  - Tropospheric Delay
    - Refraction of signal as it passes through the denser lower atmosphere
    - Position errors range from 2m to 20m at the receiver

The Global Positioning System

- GPS Accuracy
  - Multipath
    - Caused by the signal bouncing off nearby reflective object causing the signal path (range) to be longer
    - Happens more for satellite signals closer to the horizon
    - Ground plane antennas, choke ring, elevation mask
  - Dilution of Precision (DOP)
    - Loss of accuracy due to poor geometry and too few satellites in view
    - Most mapping grade receivers will display PDOP
      - Positional Dilution of Precision
      - Want PDOP to be a lower number (below 8)

UNCLASSIFIED

World (Best 4) Max PDOP



Contour Legend

Metric: PDOP Max	Start Time: 06 Sep 2008 00:00:00Z	Number of Channels: 4	> 12.0 / 0.0 - 2.0
Production Date: 09/02/2008 20:55:29	End Time: 06 Sep 2008 23:59:00Z	Mask Angle: 5°	9.0 - 12.0
Almanac File: SEM week 471	Altitude: 0 ft HAE		6.0 - 9.0
SOF File: 2008_241_174614_v02	Latitude Increment: 02° 00'		4.0 - 6.0
PSF File: 2008_246_000000_v02	Longitude Increment: 002° 00'		2.0 - 4.0

PRN: 5 Outage: 05 Aug 2008 23:38:00 to Until Further Notice PRN: 7 Outage: 05 Sep 2008 18:00:00 to 06 Sep 2008 06:00:00
PRN: 25 Outage: 26 Aug 2008 22:19:00 to Until Further Notice

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The Global Positioning System

- GPS Accuracy
  - Signal-to-Noise Ratio (SNR)
    - Ratio of signal strength to background noise
    - Positional accuracy is degraded by weak signal (low SNR) requiring more positions to be averaged to get a solution
    - Noticed as a changing position while receiver is motionless
  - Selective Availability (SA)
    - Deliberate degradation by DoD to prevent precise positioning
    - SA was turned off on May 1, 2000

The Global Positioning System

- Differential Correction
  - Using a receiver at a known location
    - Errors can be estimated and provided to a rover
  - Rover can correct its position by removing the error
  - Examples:
    - RTK (real-time kinematic) GPS
      - Used in site surveying
    - Post processing
      - Errors are removed after data is collected – “back at the office”
    - WAAS (wide area augmentation service)
      - FAA (Federal Aviation Administration) satellite broadcasts correction
      - Available on many recreation grade GPS receivers

The Global Positioning System

- Receiver Grades
  - Recreation
    - 3 to 5 meters
    - \$100 to \$500
  - Mapping
    - 1 meter
    - \$800 to \$2000+
  - Surveying
    - Sub-meter, centimeter (cm)
    - \$10,000 to \$50,000+

The Global Positioning System

- GPS receivers record points, usually at 1 second intervals
  - GPS units will usually average subsequent points to obtain better positions
  - These points can be saved as **waypoints**
  - Points can be combined into **tracks** or **routes** and used for mapping
- GPS alone does not know direction
  - Some recreation grade receivers have electronic compasses
  - Need to be in motion and have sequential points to know direction

The Global Positioning System

- GPS web sites
  - U.S. Air Force GPS Operations
    - http://gps.afspc.af.mil/index.html
  - U.S. Coast Guard Navigation Center
    - http://www.navcen.uscg.gov/
  - U.S. Naval Observatory
    - http://tycho.usno.navy.mil/
- Other GNSS (Global Navigation Satellite System) information
  - Russian GLONASS
    - http://en.wikipedia.org/wiki/GLONASS
  - EU Galileo
    - http://en.wikipedia.org/wiki/Galileo_%28satellite_navigation%29

GPS – Overview

- History
  - 1958 Navy Navigation Satellite System (NNSS)
    - More commonly called TRANSIT
    - Used Doppler principle
    - 1967 first civilian use
  - 1978 first **NAV**igation **S**atellite **T**iming **a**nd **R**anging (NAVSTAR) satellite launched
    - Fully operational 1993

GPS – Overview

- The global positioning system can be arbitrarily divided into three parts:
  - The space segment
  - The control segment
  - The user segment

GPS – Overview

- The space segment
  - 31 total satellites
    - 24 operating satellites
    - 4 reserve satellites
  - Six orbital planes inclined at 55° to the equator
    - 24-hour coverage between latitudes 80°N and 80°S
  - Near-circular orbits
    - 20,200km mean altitude
    - 12 hour period
  - Identified by their pseudorandom noise (PRN) number or satellite vehicle number (SVN)

GPS – Overview

- The space segment
  - Each satellite carries precise atomic clocks
    - Cesium or rubidium
      - Cesium accurate to 1 sec in 300,000 years
      - Rubidium accurate to 1 second in 30,000 years

GPS – Overview

- The control segment
  - 12 monitoring stations
    - Including: Colorado Springs, Hawaii, Ascension, Diego Garcia, Kwajalein
    - Monitor signals
  - Master control
    - Colorado Springs
    - Predict orbits and clock corrections into the future
      - Uploaded to satellites and transmitted by them as part of their *broadcast message*
      - Message is used by receivers to predict satellite positions and their clock biases

GPS – Overview

- The user segment
  - Two categories of receivers, each has access to to different levels of service
    - Standard Positioning Service (SPS)
      - SPS is broadcast on L1
      - Intended to give positions accurate to 100m horizontal and 156m vertical
    - Precise Positioning Service (PPS)
      - PPS is broadcast on L1 and L2 and only available to receivers with valid cryptographic keys (ie DoD use)
      - Published accuracy 18m horizontal and 28m vertical

GPS Signal Structure

Each signal consists of:

- Carrier – *a radio wave that has one characteristic that can be changed (modulated) to carry information*
- Ranging Code – a sequence of 0s and 1s unique to each satellite that allows the receiver to determine the transit time of the signal
  - the sequences appear to be random and are called pseudo-random noise (PRN)
  - C/A (coarse/acquisition) code, or
  - P (precision) code [encrypted, P(Y)]
- Navigation Data – satellite health, ephemeris, clock bias parameters, and almanac

GPS – GPS Signal

- Each GPS satellite broadcasts unique signals on two carrier frequencies.
  - L1 1575.42 MHz
  - L2 1227.60 Mhz
- These frequencies are derived from a fundamental frequency f_0 = 10.23 Mhz
  - L1 = $154 \cdot f_0$
  - L2 = $120 \cdot f_0$

GPS – GPS Signal

- Two signals are transmitted on L1
  - C/A (coarse/acquisition) code
  - P (precise) code encrypted into the Y code
- Two signals are transmitted on L2
  - C/A (coarse/acquisition) code (L2C, 2006 Block IIRM)
  - P (precise) code encrypted into the Y code

GPS – GPS Signal

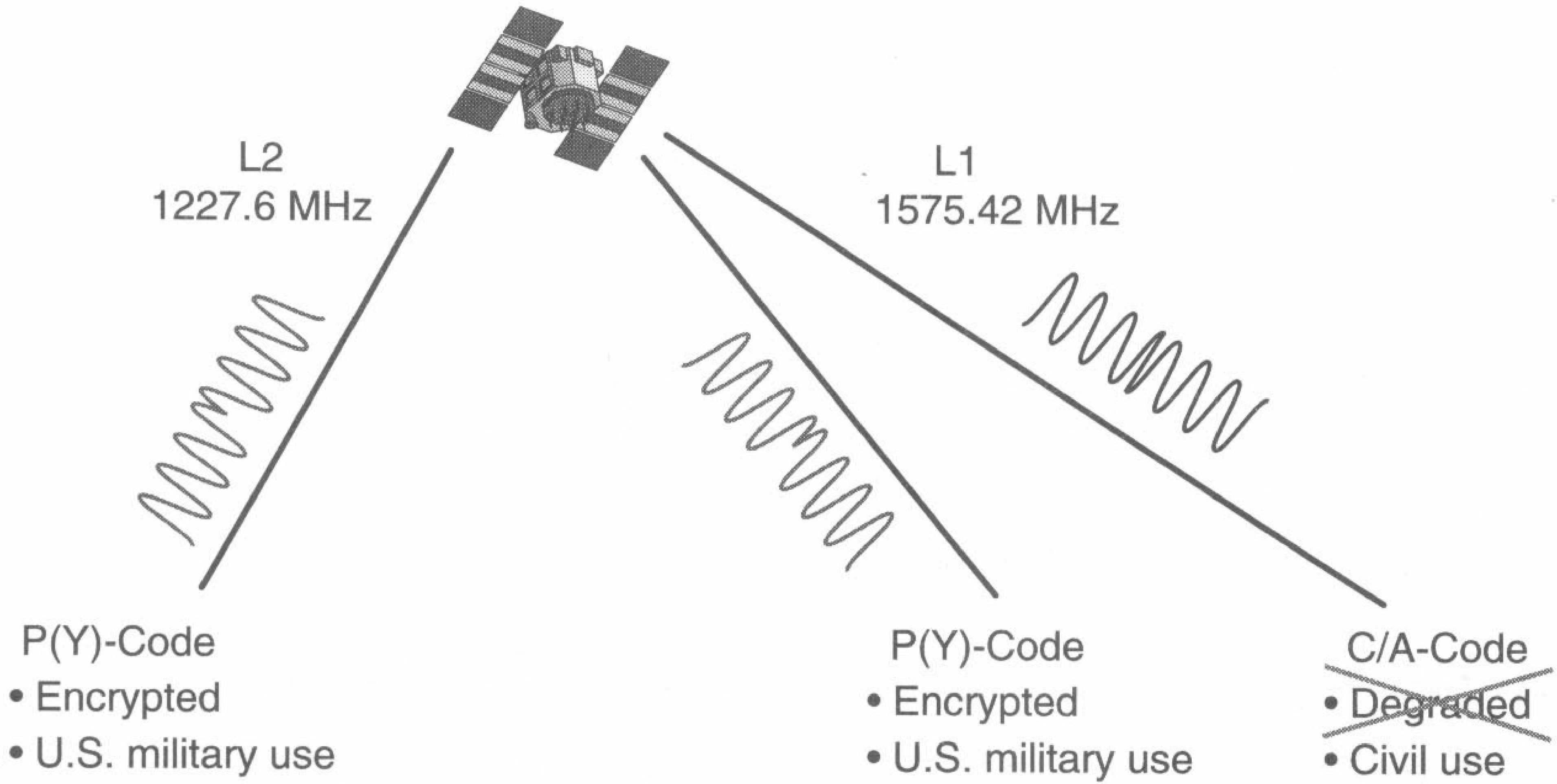


Figure 2.4 GPS signals. Currently, each GPS satellite transmits three signals, two on L1 and one on L2 frequency. The BPSK-modulated signals are shown. The signal carrying C/A-code on L1 was degraded purposely throughout the 1990s, but this practice has now ended. Access to P(Y)-code is limited to the DoD-authorized users via encryption.

C/A Code

- Unique sequence of 1023 bits (called chips)
- Repeated each millisecond (0.001s)
- 1.023 million bits per second
- Each chip is about 1 microsecond duration

Chip width

$1\mu s * c = \sim 300m$

- Each C/A code is unique to one satellite

P Code

- 10.23 million bits per second
- Unique sequence of ~10^{14} chips
- Each satellite has a 1 week segment of the 37 week sequence
- Each satellite repeats its code every 7 days

Chip width ~30m

- Anti-Spoofing (AS)
  - P code is encrypted into the Y code
  - Meant to deny the ability to broadcast a copy-cat GPS signal

GPS – GPS Signal

- A system was devised to determine accurate travel time of the signal from the satellite to the receiver.
- The carriers are modulated with *pseudorandom noise* (PRN) codes
- Each satellite transmits its own unique PRN codes
- The receiver duplicates the code pattern

GPS – GPS Signal

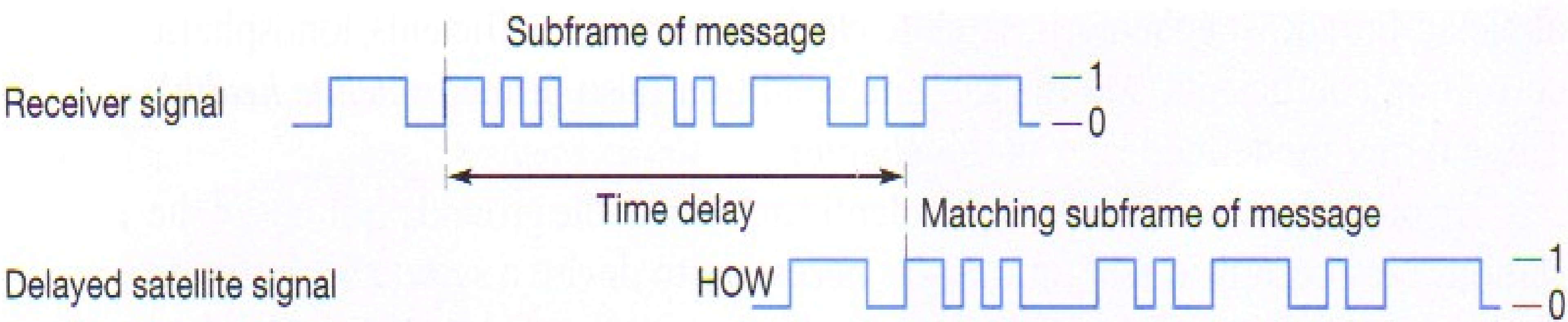


Figure 13.3 Determination of signal travel time by code matching.

GPS Signal Structure

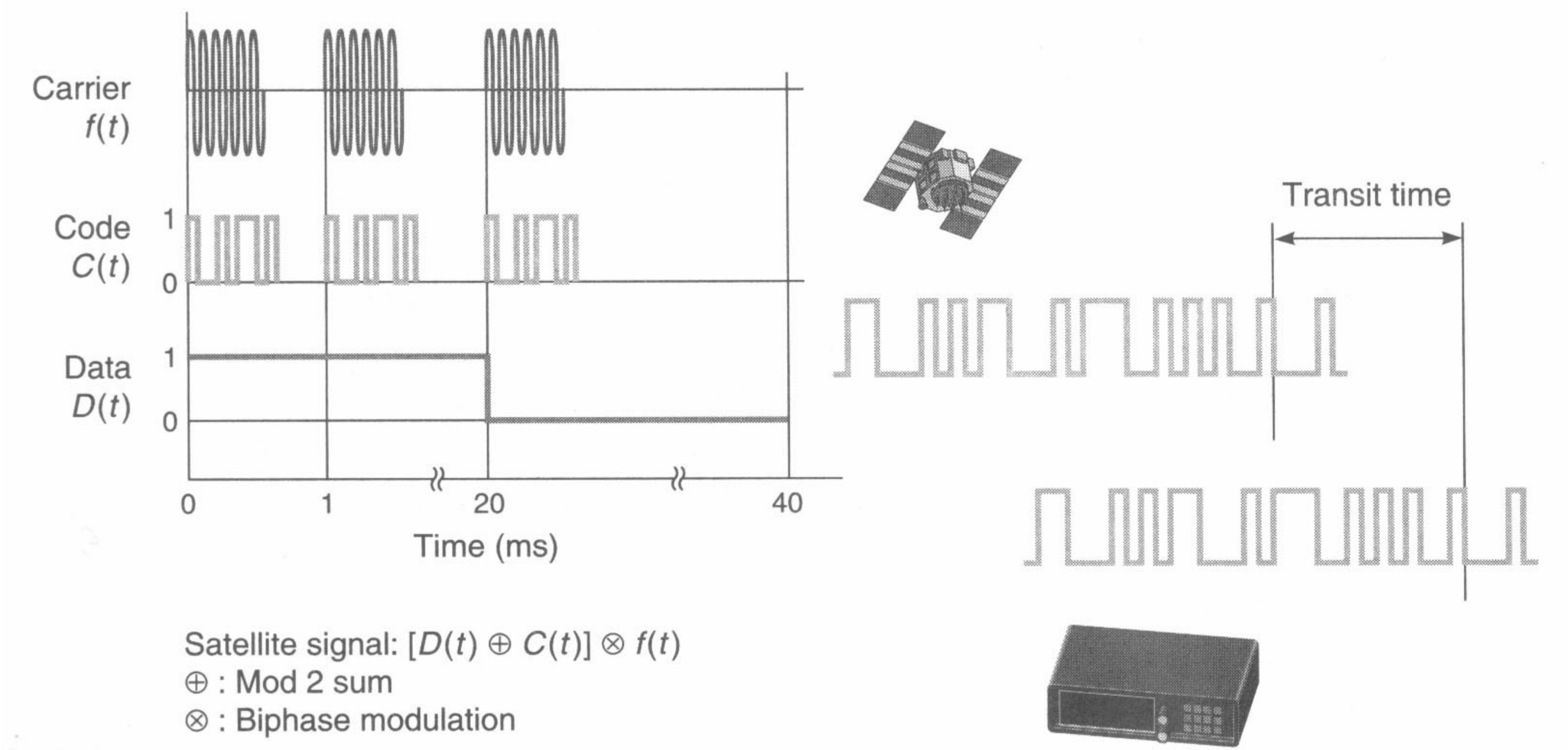


Figure 2.3 The structure of the signal available for civil use and estimation of its transit time from the satellite to a user. Each GPS signal comprises three components: an RF carrier, a unique binary pseudo-random noise (PRN) code, and a binary navigation message. The transit time is estimated by correlating the received signal with its replica generated by the receiver.

GPS – GPS Signal

- The broadcast message includes
  - Almanac
  - Broadcast ephemeris
  - Satellite clock correction coefficients
  - Ionospheric correction coefficients
  - Satellite health

Navigation Code

- 50 bits per second
- 25 frames of 1500 bits for complete message
- 30 seconds to transmit one frame
- Each frame is organized into 5 subframes
  - Subframes 1-3 repeat the same info from frame to frame

    1 sat clock corrections, health, age of data

    2 sat ephemeris

    3 sat ephemeris

    4 almanac, health, ionosphere model, UTC data

    5 almanac, health

Navigation Code

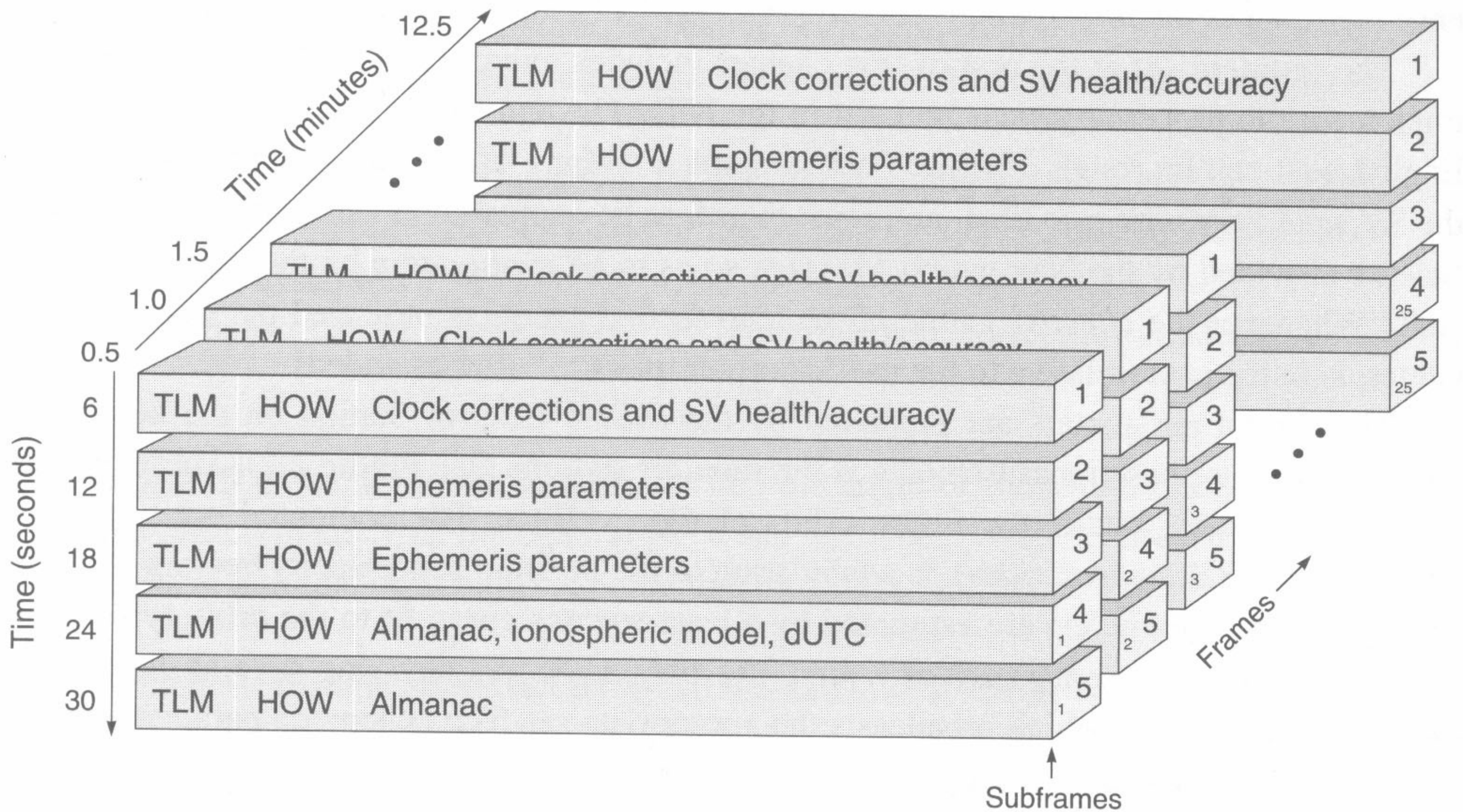


Figure 3.13 GPS navigation message organization: frames and subframes. Subframes 1, 2, and 3 repeat every 0.5 minutes; subframes 4 and 5 repeat every 12.5 minutes. Subframes 1, 2, and 3 are specific to the transmitting satellite; subframes 4 and 5 are common to all satellites. (Courtesy: Dr. Frank van Diggelen, Global Locate)

Coordinate Systems

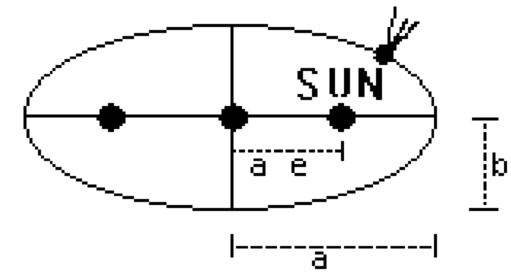
- Many reference coordinate systems
  - Satellite reference coordinate system
  - Geocentric coordinate system
    - Inertial
    - Earth Fixed
  - Geodetic coordinate system

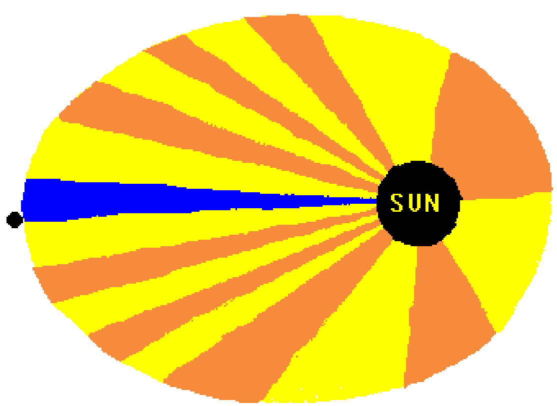
Kepler's Laws

Johannes Kepler (1571-1630)

- Orbits are ellipses with sun at one foci
- Planets revolve so that a line joining it to the sun sweeps out equal areas in equal times
- $P^2 \alpha a^3$ (square of orbital period is proportional to cube of size of orbit)

Kepler's Laws





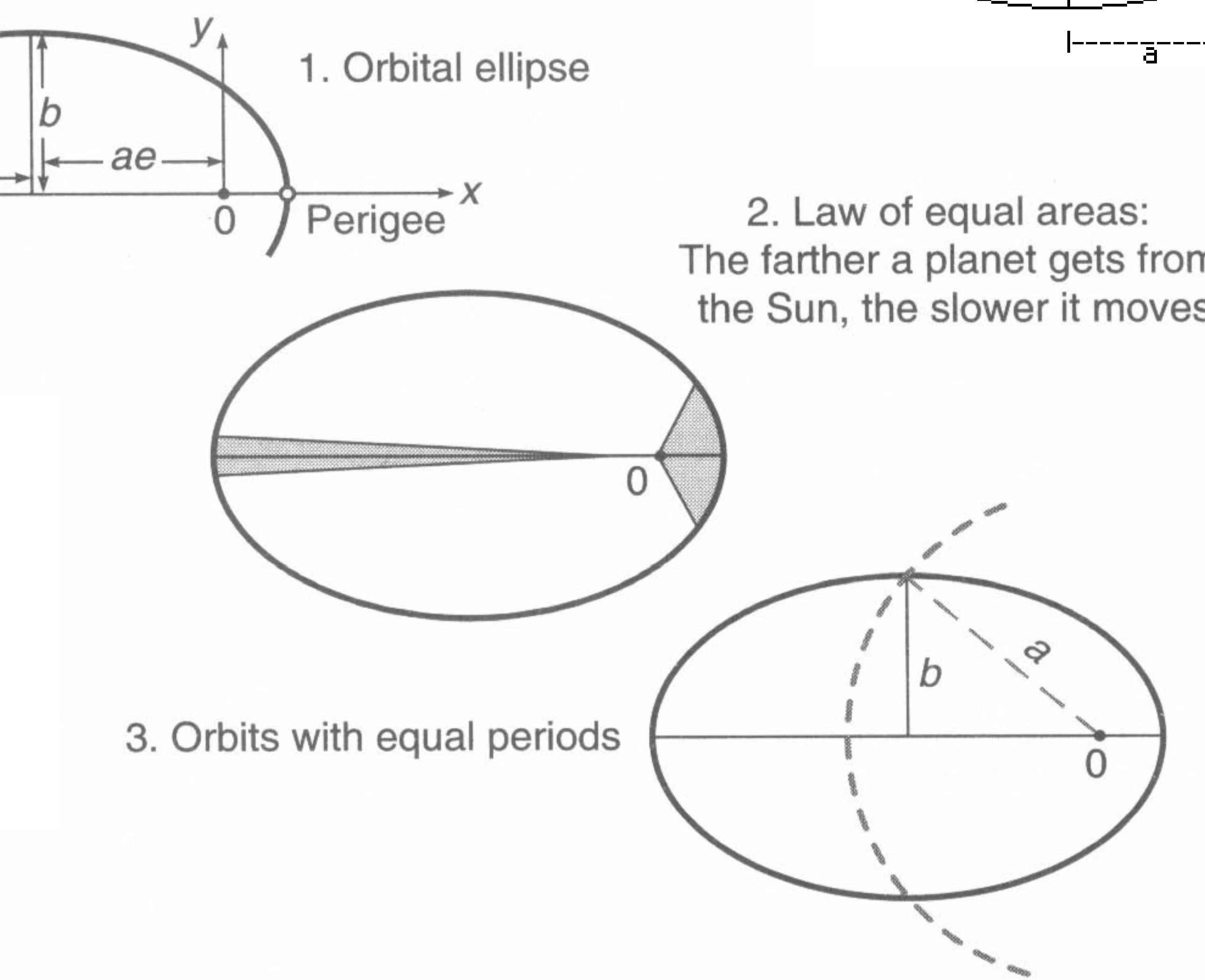


Figure 3.10 Kepler's laws of planetary motion illustrated.

Newton's Laws

- Issac Newton (1642-1727)

- Second Law of Motion

$$\boldsymbol{F} = m\boldsymbol{a}$$

- Law of Gravitation

$$\boldsymbol{F}_{12} = \frac{Gm_1 m_2}{r^2} \frac{\boldsymbol{r}_{12}}{r}$$

Two-Body Problem

- Newton's Laws can be arranged into this equation of motion:

$$\ddot{\boldsymbol{r}} + \frac{GM}{r^3}\boldsymbol{r} = 0$$

- Assumes small mass satellite orbiting a large mass Earth and no other forces acting on the satellite
- Second-order, nonlinear differential equation
- The solution yields 6 constants of integration that totally characterize the orbit

$$\boldsymbol{r}_0 = \left(x_0, y_0, z_0\right)$$
$$\dot{\boldsymbol{r}}_0 = \left(\dot{x}_0, \dot{y}_0, \dot{z}_0\right)$$

Keplerian Elements

- An alternate set of 6 parameters can be used to describe the orbit

size and shape of ellipse

$a = \text{semi} - \text{major axis}$

$e = \text{eccentricity}$

orientation of orbital plane relative to inertial axes

$i = \text{inclination}$

$\Omega = \text{right ascension of the ascending node}$

orientation of ellipse in orbital plane

$\omega = \text{argument of perigee}$

position of satellite in orbital plane

$\nu = \text{true anomaly (or argument of latitude, u)}$

Keplerian Elements

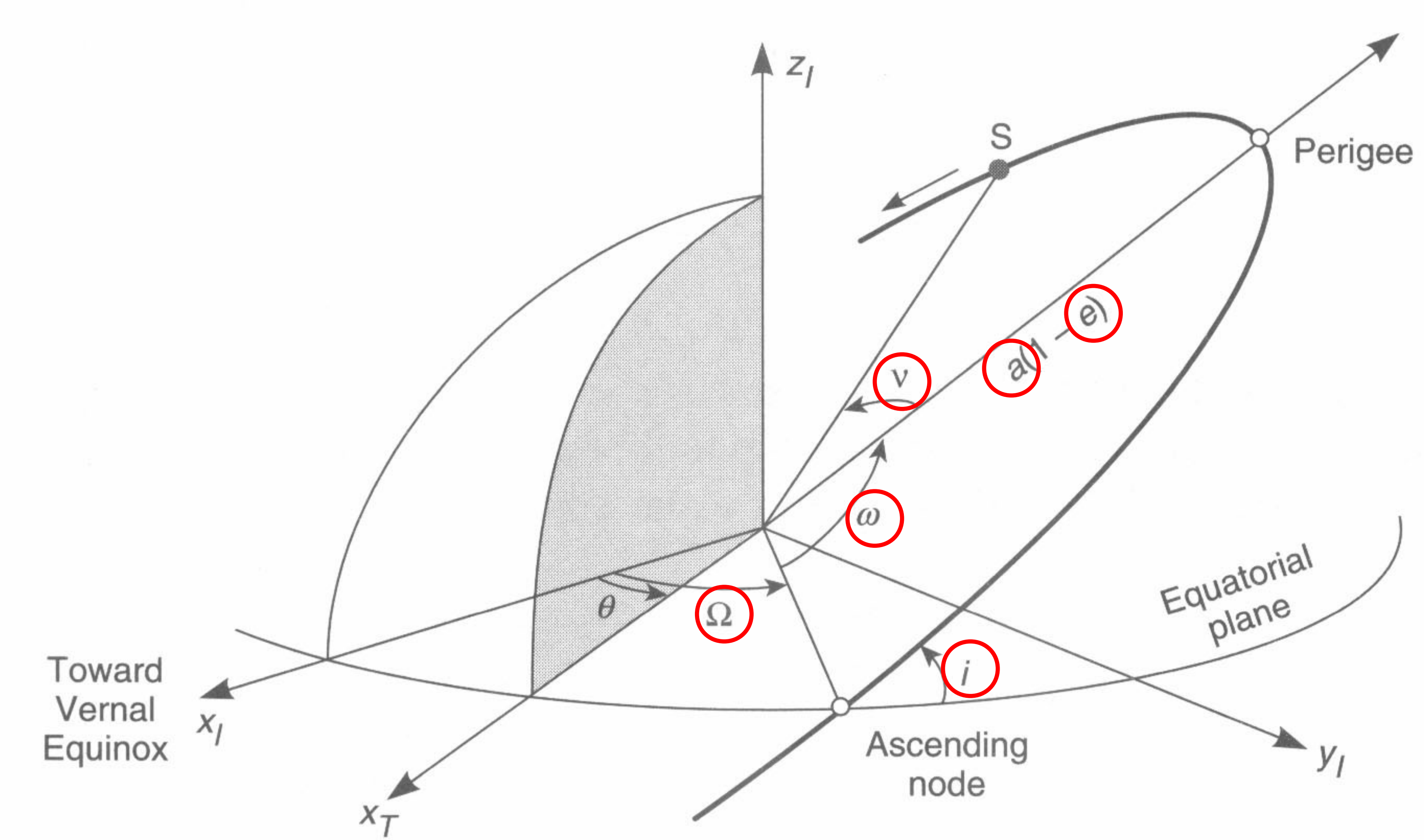


Figure 3.11 Characterization of an ideal orbit and the satellite position by Keplerian elements: $\{a, e, i, \Omega, \omega, \text{and } \nu\}$.

Orbit Plane

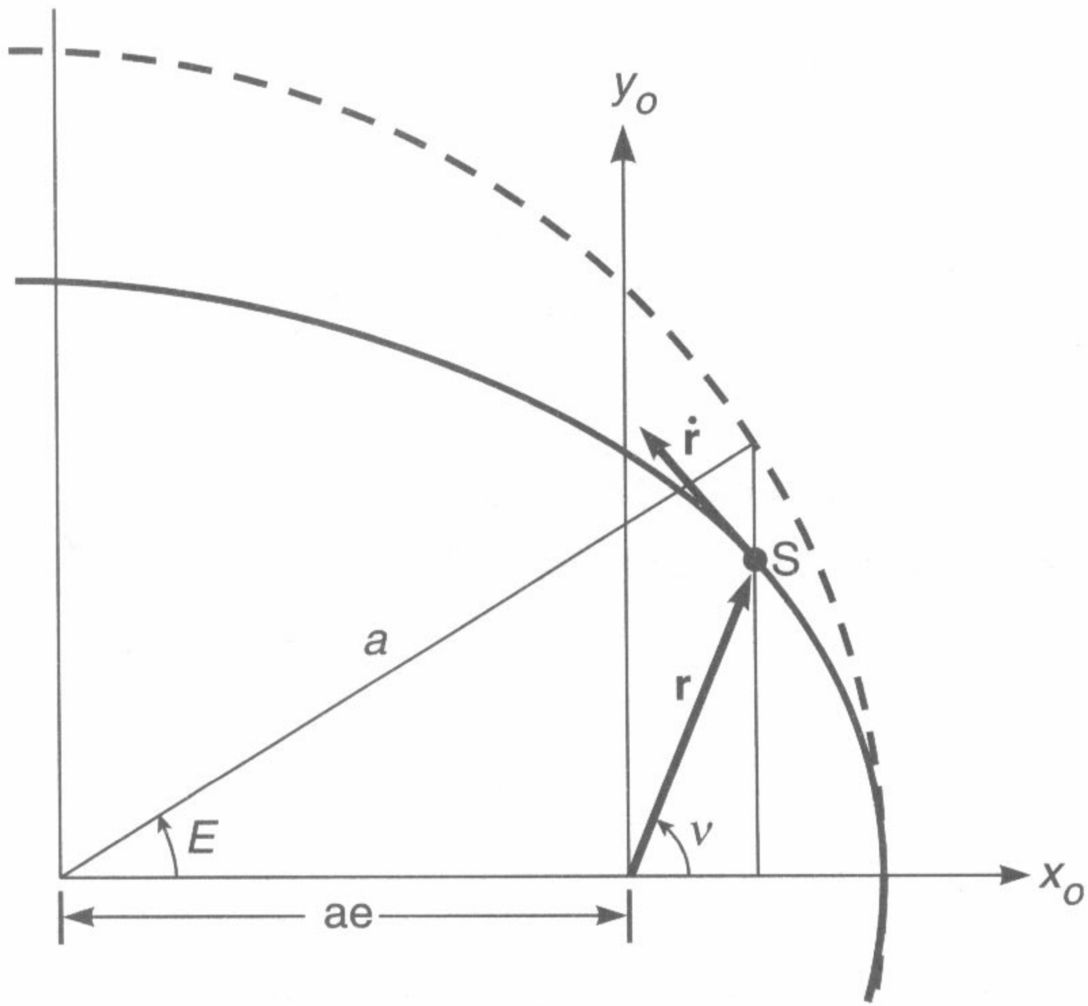


Figure 3.12 Position and velocity of a satellite in a Keplerian orbit.

Perturbations

- Non-central gravitational force
  - Earth oblateness (J_2)
  - Tidal effects
  - Lunar/Solar gravity
- Solar radiation pressure
- Air drag
- Relativistic effects

All cause the Keplerian elements to change slowly with time

Perturbations

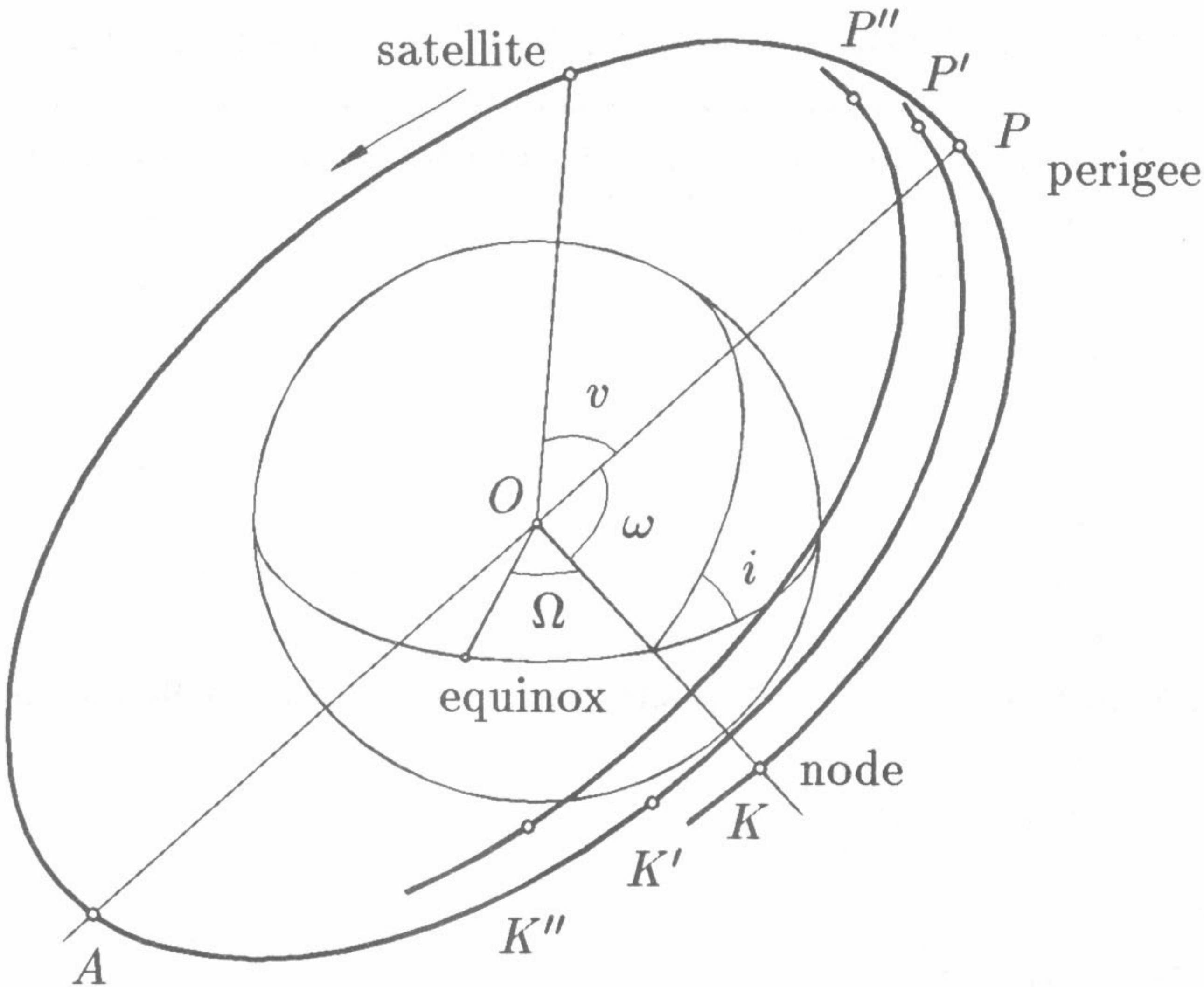


Fig. 4.3. Secular perturbations caused by the oblateness term J_2

GPS Ephemerides

- ephemeris – tabular list of a celestial body's location at regular intervals

Ephemeris	Uncertainty	Update Frequency
Almanac	kilometers	Updated at least every 6 days Broadcast by satellites for planning purposes
Broadcast	1 m	Broadcast by satellites Updated hourly Good for 4 hours
Precise	0.05 – 0.20 m	Produced by several agencies Usually available the next day

Satellite Reference coordinate System

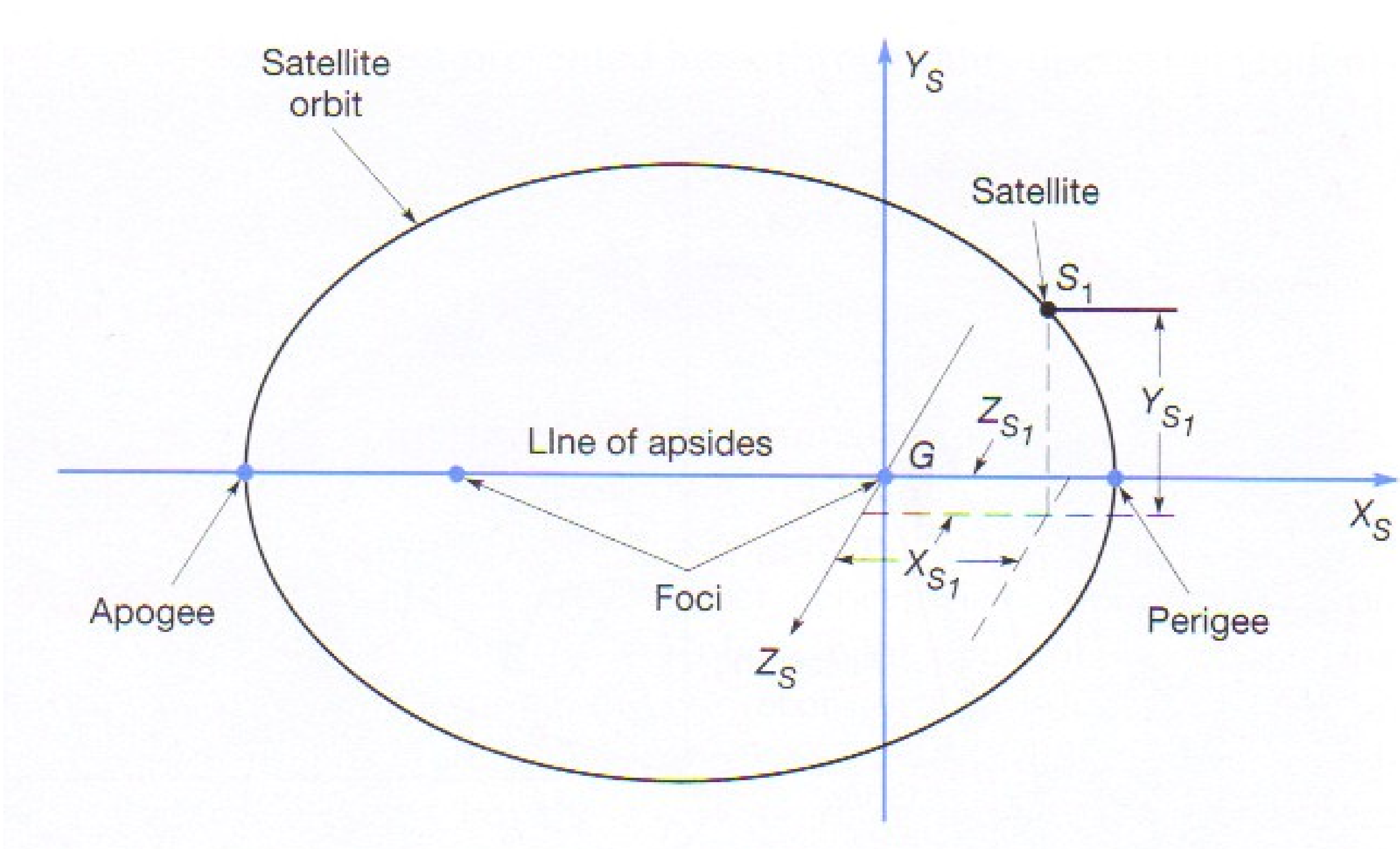


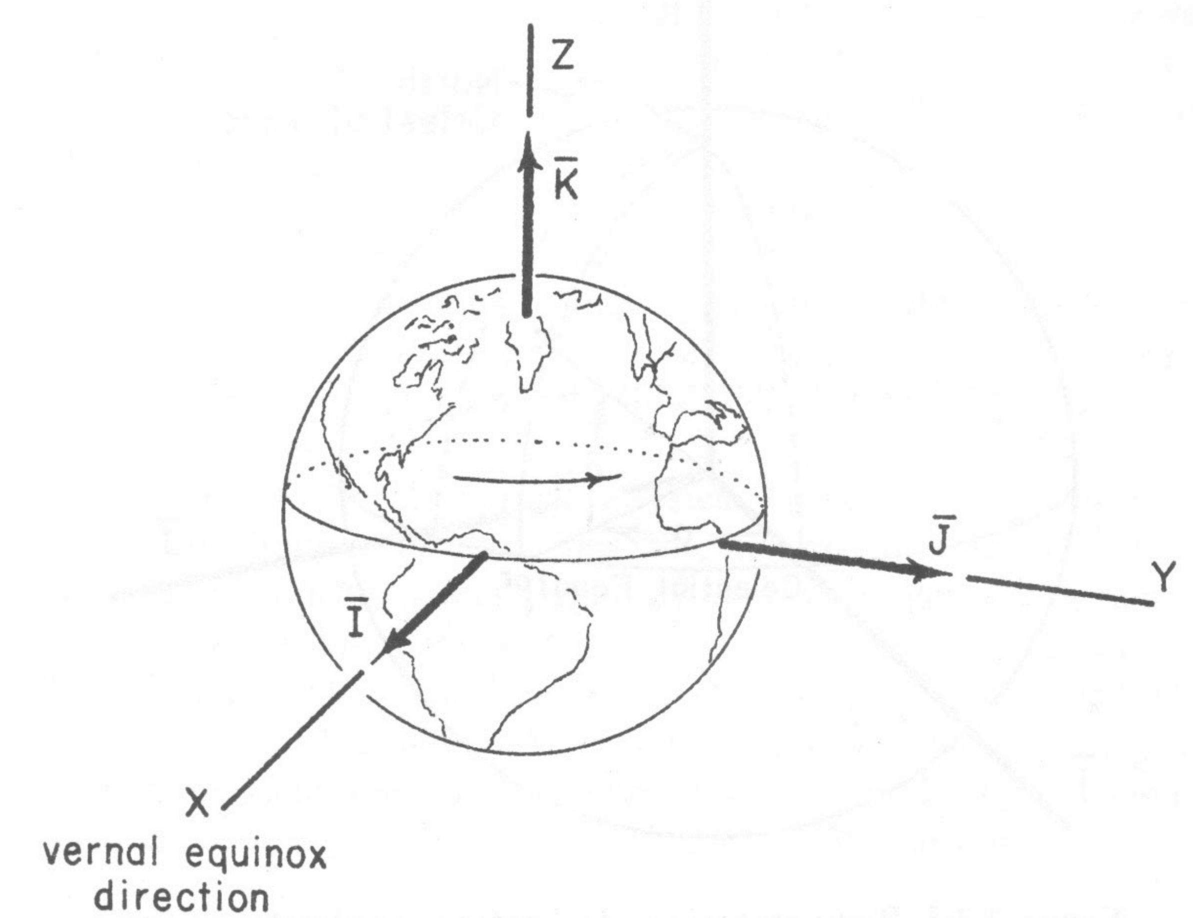
Figure 13.4
Satellite reference coordinate system.

Coordinate Systems

- Inertial
  - Fixed in space relative to “fixed stars”
  - No acceleration
  - Needed for Newton’s Laws (satellite orbits)
- Fixed to Earth
  - For defining locations on Earth
  - Moves with Earth

Inertial Reference System

- Origin: center of mass of earth
- Z-axis: along the axis of rotation
- X-axis: in equatorial plane pointing toward vernal equinox
- Y-axis: defined to complete right-handed system



Terrestrial Reference System

- Origin: center of mass of Earth
- Z-axis: along rotation axis
- X-axis: in equatorial plane through reference meridian
- Y-axis: completes right-handed system
- Aka: ECEF

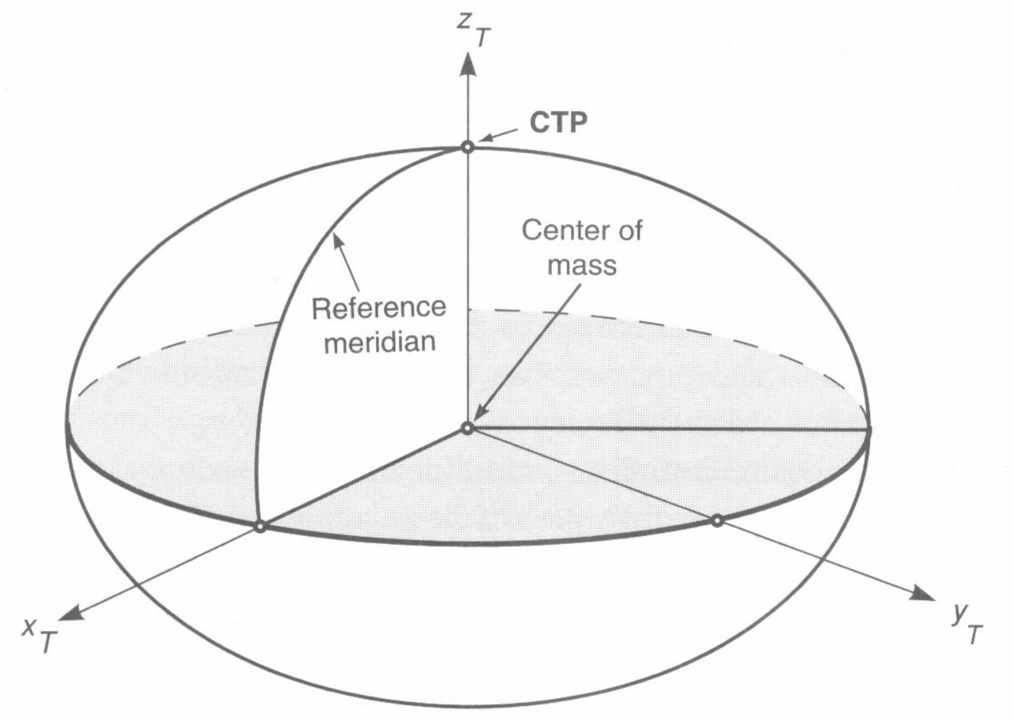


Figure 3.1 Conventional Terrestrial Reference System (CTRS).

Complications with defining ECEF

- The Earth is not a solid rigid body
- Rotation axis is not fixed with respect to the Earth
  - Polar Motion
  - <15m radius over last 100 years
- Use an average pole location to define coord system

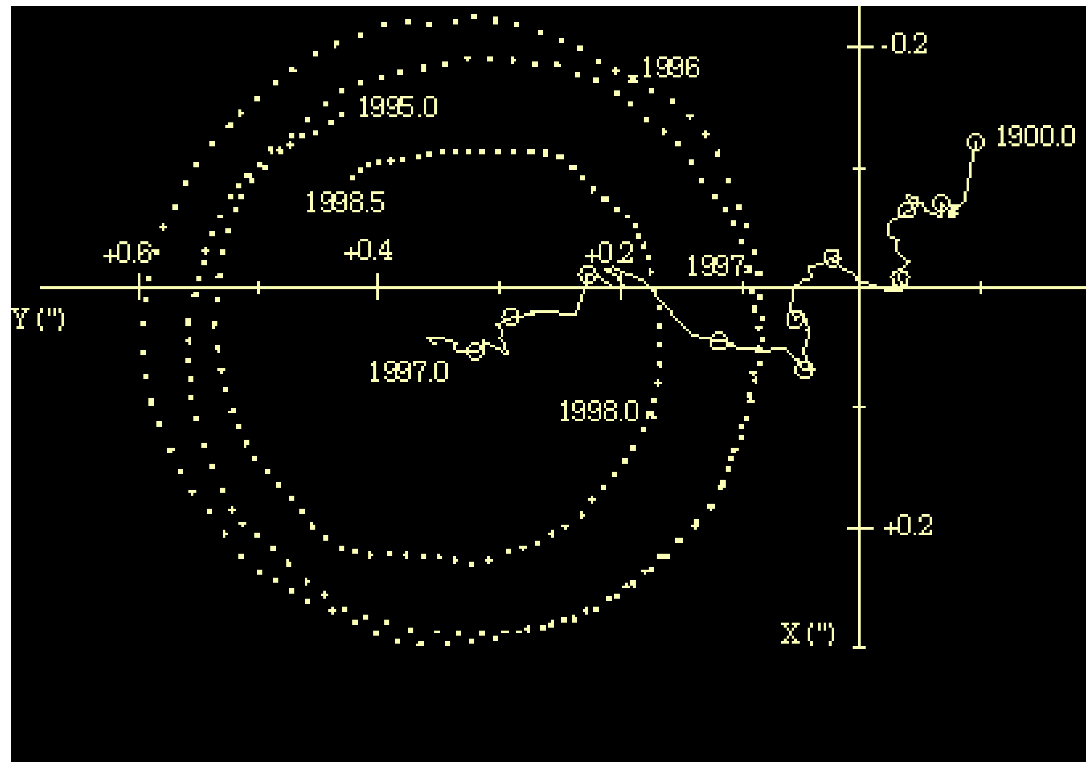


Figure O-1: Polar motion, 1995-1998. Raw value of Table O-4.
Solide line: mean pole displacement, 1900-1997

For more info: International Earth Rotation Service http://www.iers.org

Realization of ECEF is WGS 84

- World Geodetic System 1984
- A realization of a coordinate system is its *practical implementation*
- Involves known control locations around the earth whose coordinates are precisely established

Ellipsoid

- To locate positions on the surface of the earth, it is helpful to define a surface that approximates the earth's surface.
- Ellipse rotated about the z-axis (oblate)
- Center at the center of mass of the Earth
- Defined by
  - a (semi-major axis)
  - f (flattening)

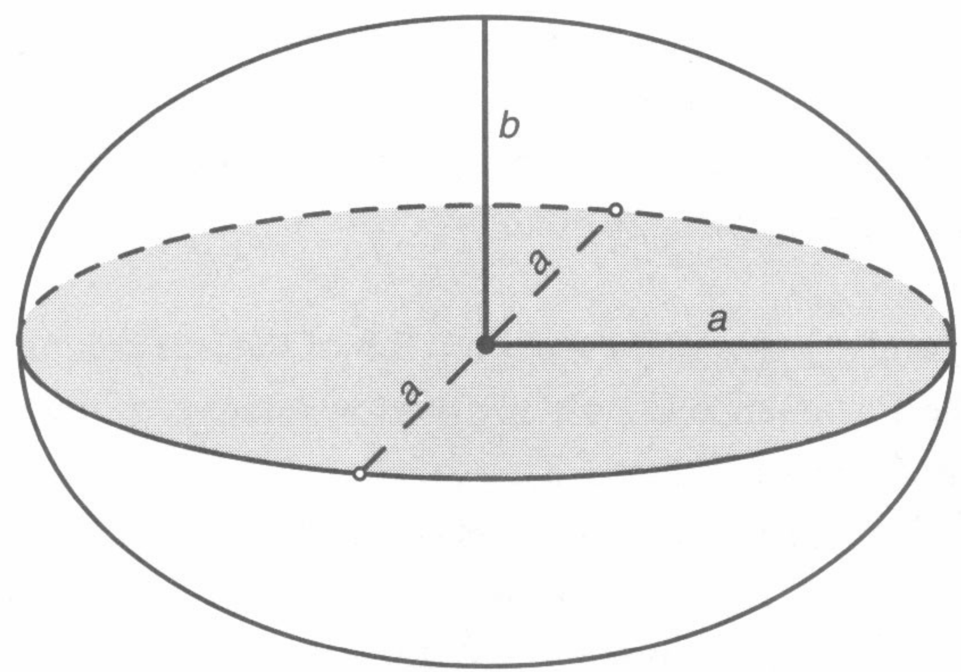


Figure 3.3 Ellipsoid of revolution (oblate ellipsoid).

WGS 84 Ellipsoid (Datum)

- a = 6378137.0 m
- $1/f$ = 298.257223563
- $f = (a-b)/a$
- eccentricity

  $e^2 = 2f - f^2$

  $e^2 = (a^2 - b^2)/a^2$
- b semi-minor axis

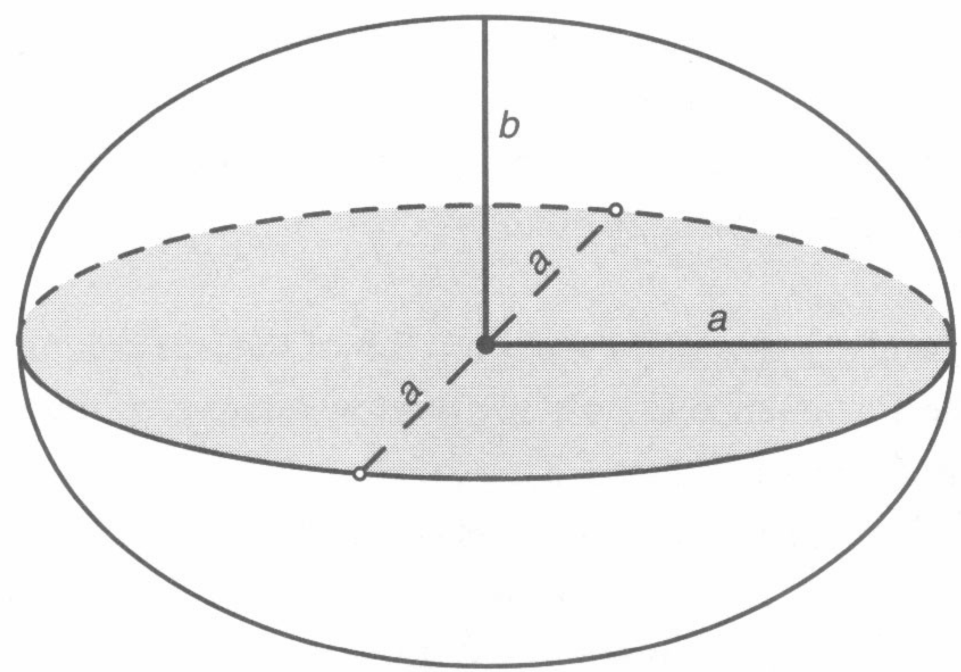


Figure 3.3 Ellipsoid of revolution (oblate ellipsoid).

∀ $\omega_E = 7292115.0\mathrm{x}10^{-11}$ rad/s

- GM = μ = $3986004.418\mathrm{x}10^{8}$ m^3/s^2

Geodetic Coordinates

- ϕ latitude
- λ longitude
- h height above ellipsoid

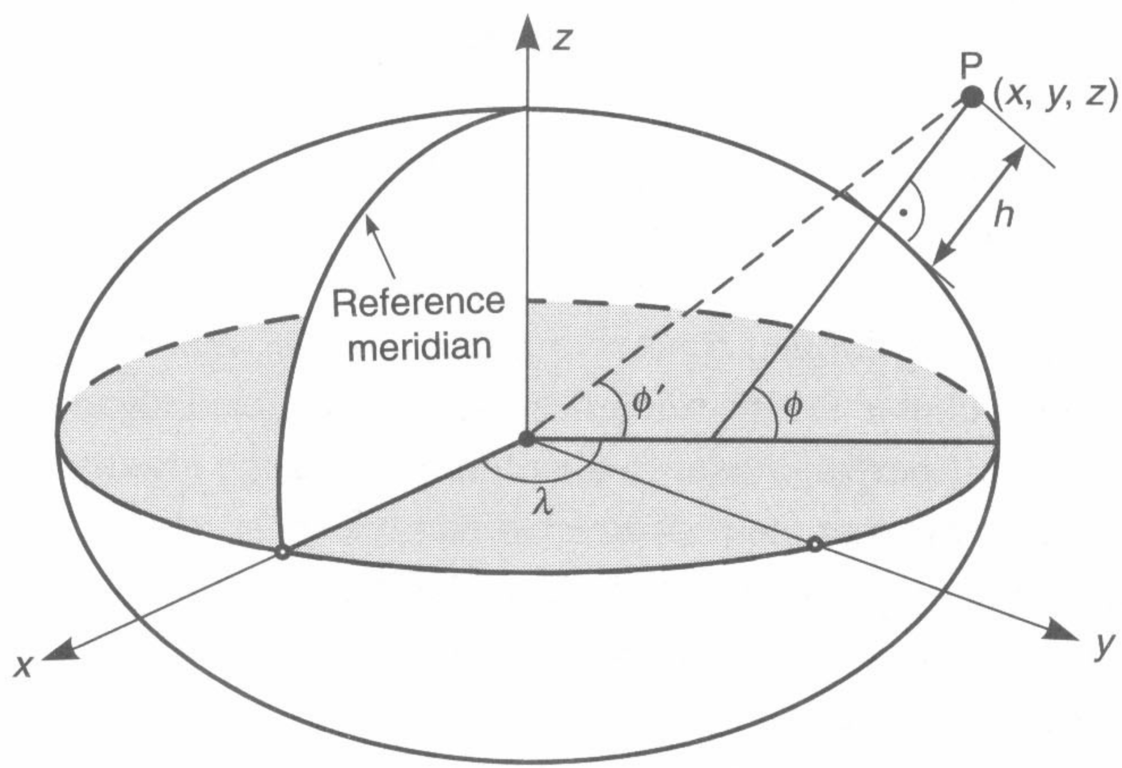


Figure 3.4 Cartesian (x, y, z) and ellipsoidal (ϕ, λ, h) coordinates. Geocentric latitude is denoted as ϕ'. We denote a right angle as ⊾.

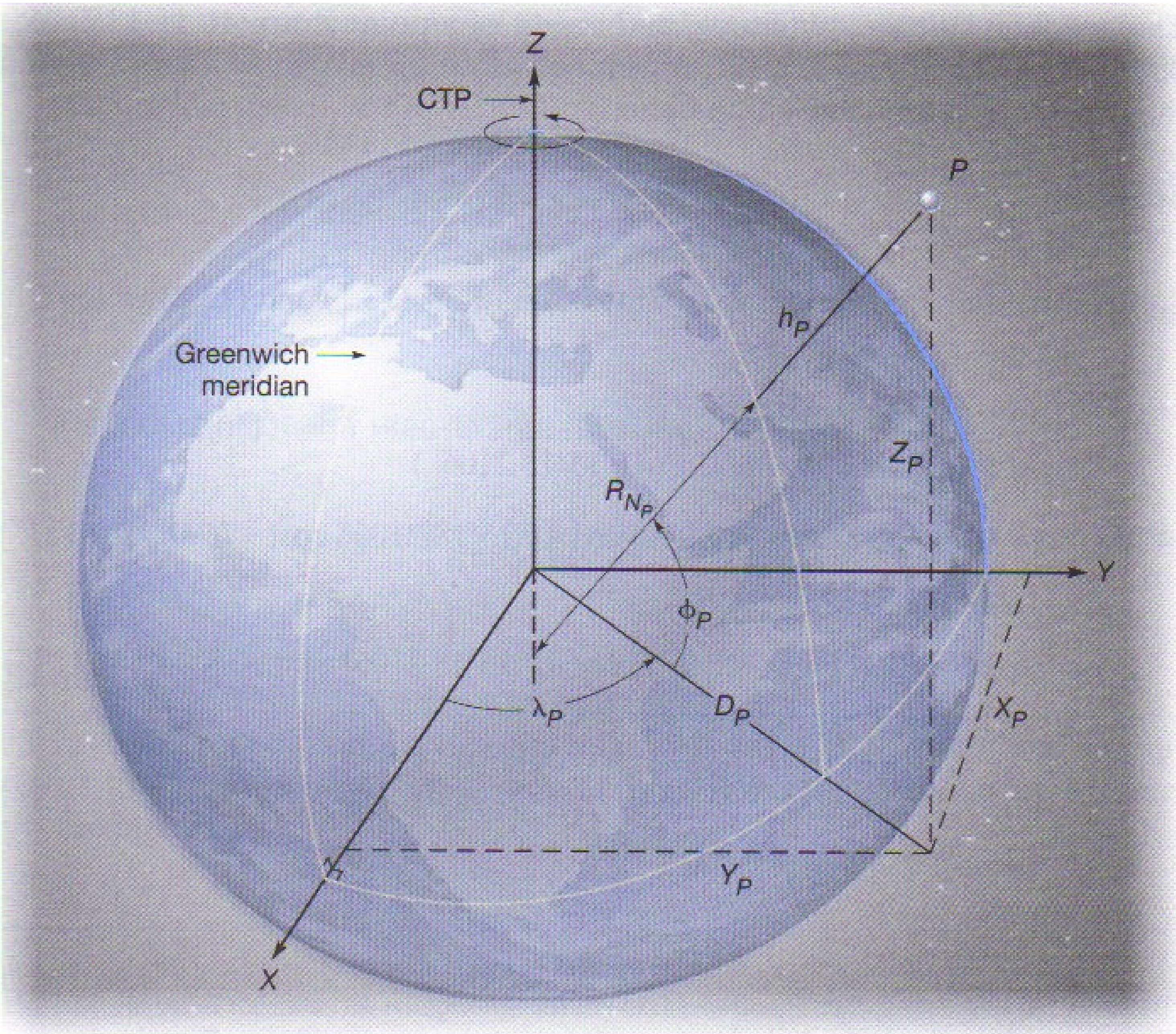


Figure 13.6 The geodetic and geocentric coordinate systems.

Datums

- A reference surface relative to which we could define the 3-D coordinates of a point

Ellipsoid

Geoid - an equipotential surface perpendicular to the local gravity vector

- direction of gravity varies slightly due to variations in mass distribution of Earth
- local gravity vector can be measured by using simple equipment (plumb line, bubble level, etc)
- MSL (mean sea level) is a geoid

Relationship between Geoid and Ellipsoid

- $H = h - N$
  - H is the *elevation* (*orthometric height*)
  - h is the *geodetic height* (from GPS)
  - N is the *geoid undulation*

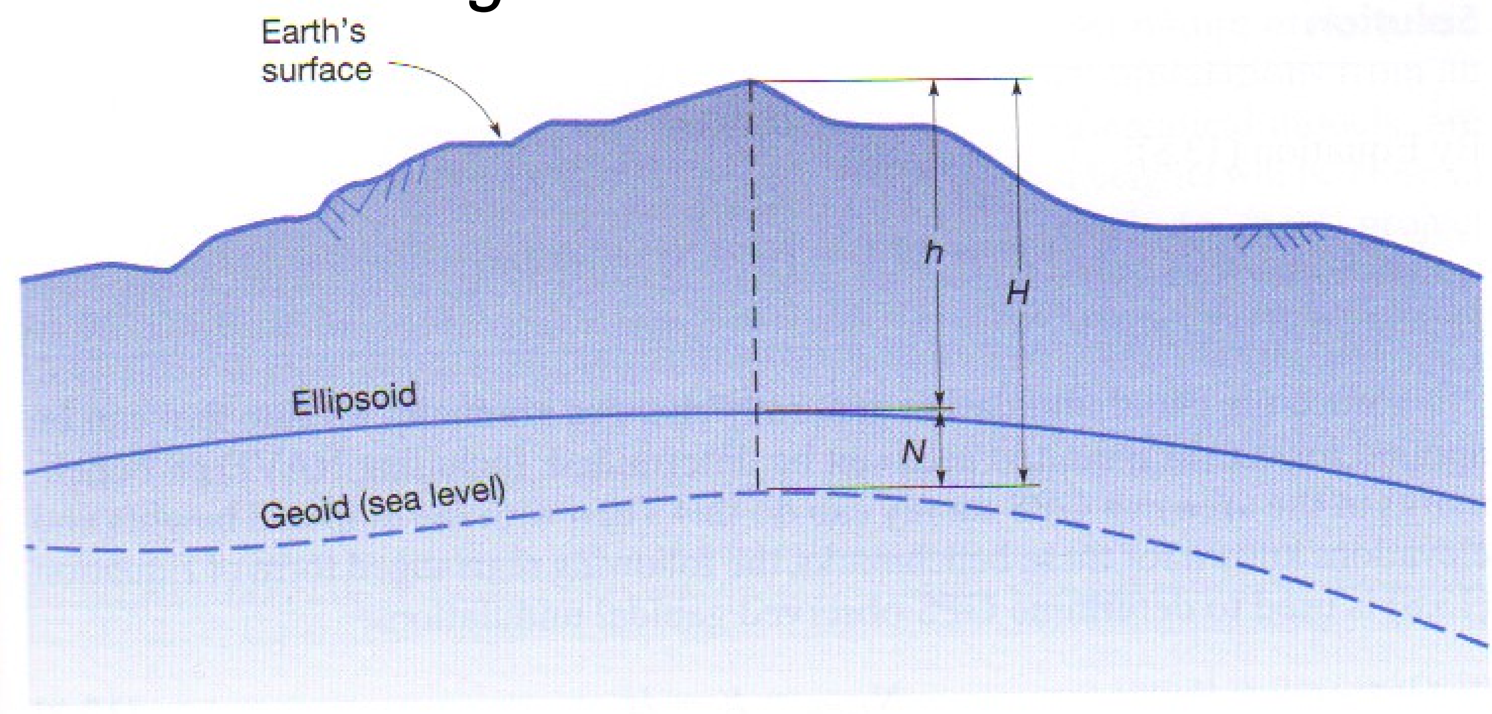


Figure 13.7 Relationships between elevation *H*, geodetic height *h*, and geoid undulation *N*.

Relationship between Geoid and Ellipsoid

- $h = H + N$

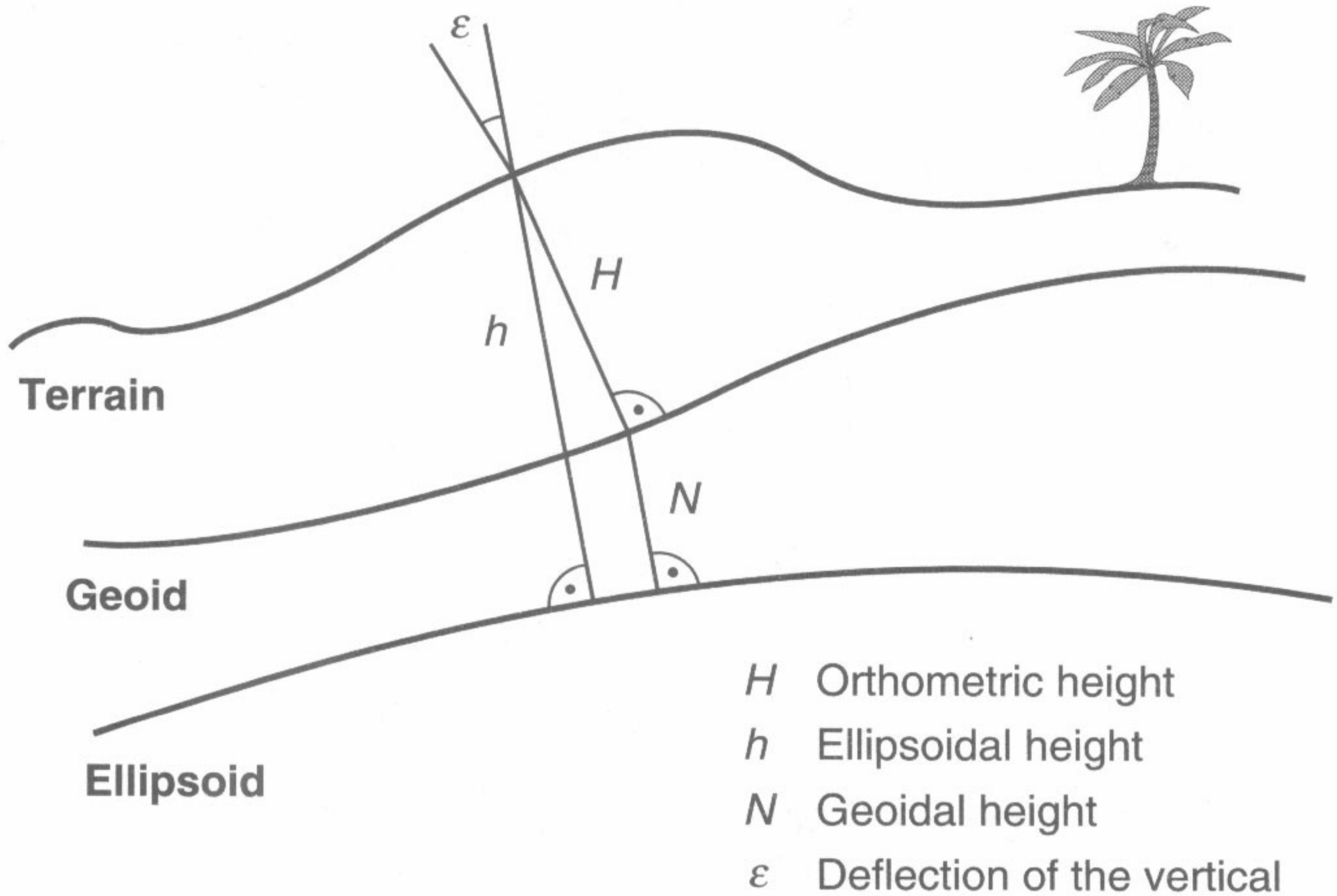
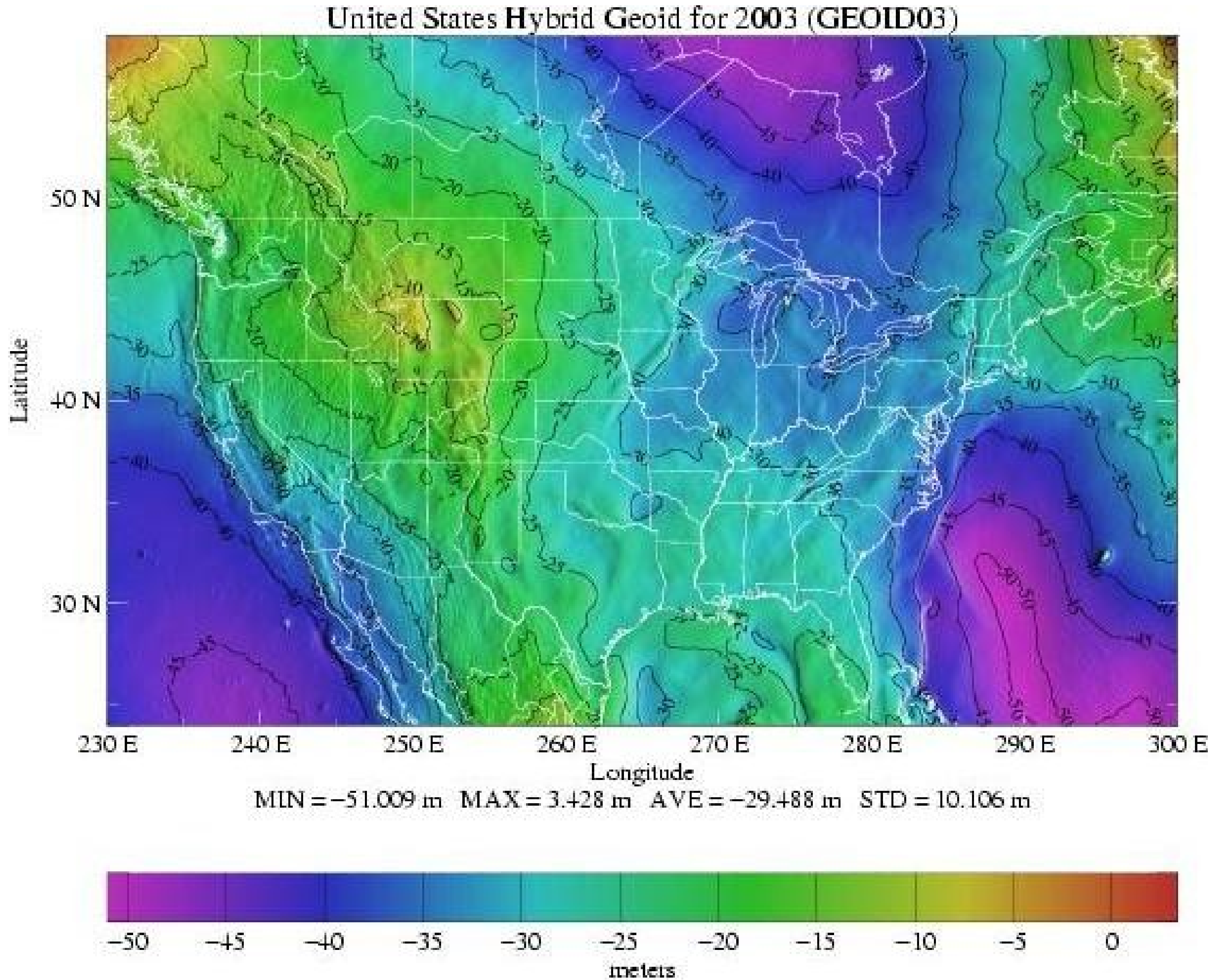


Figure 3.5 Geoid, geoidal height, and deflection of the vertical.

Geoid Height (N)



For more info: NOAA NGS http://www.ngs.noaa.gov/GEOID/

Geoid Comments

- Geoidal undulations obtained from mathematical models are not exact.
- For work requiring extremely accurate elevation differences, it is best to obtain heights by differential leveling from nearby benchmarks.
- See paper:

What does height really mean? By Tom Meyer

http://digitalcommons.uconn.edu/nrme monos/1

Regional Datums

- Fit the local geoid

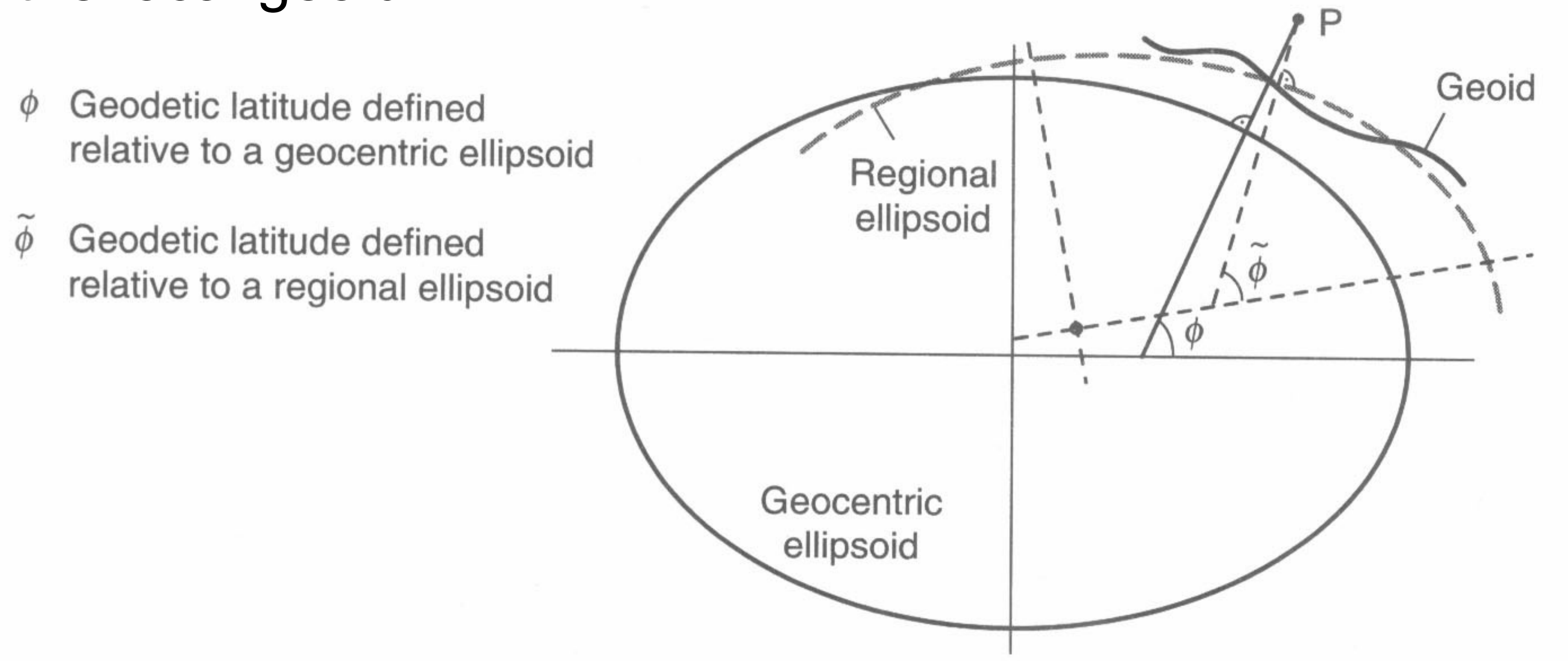
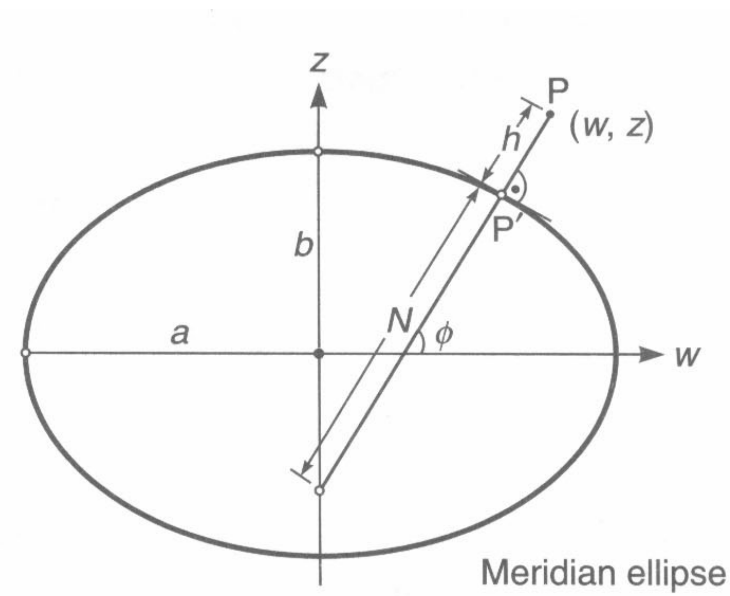


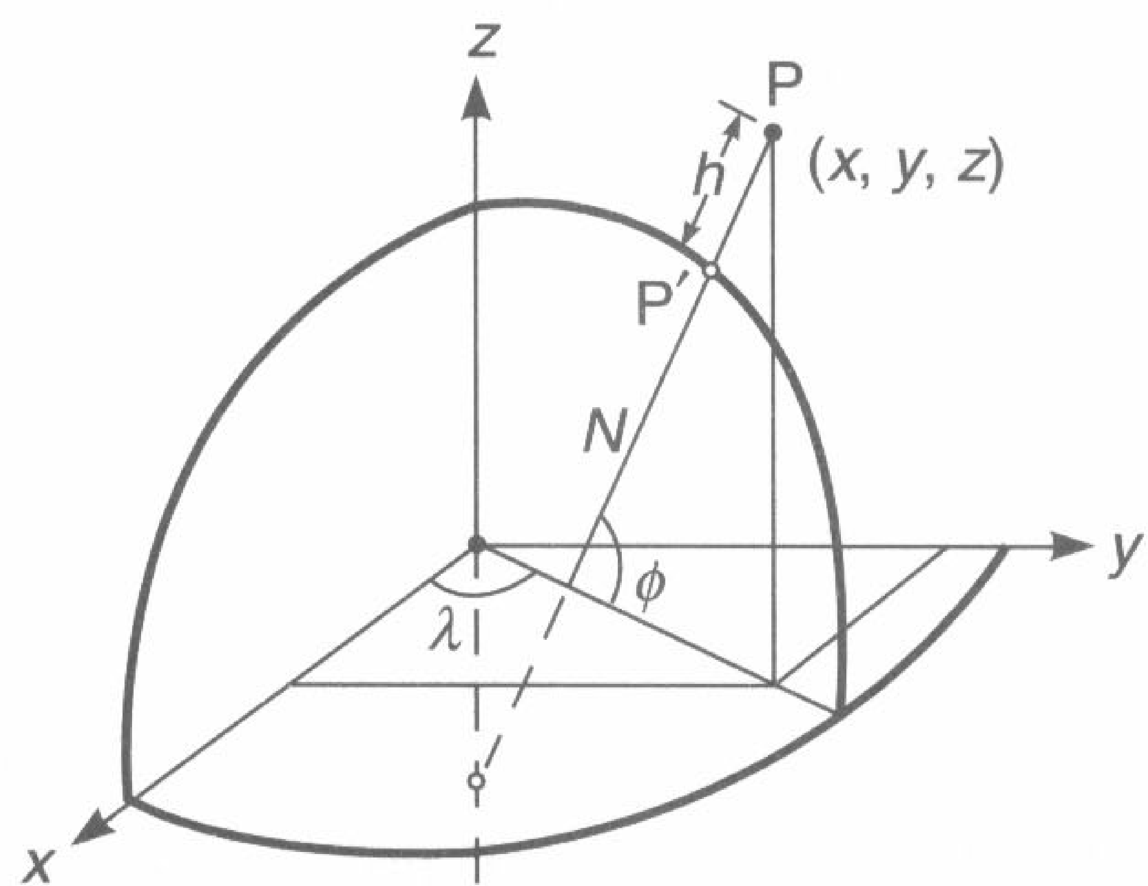
Figure 3.6 Global and regional datums.

- May offset the origin from the CM of the Earth
- NAD 83 (a regional datum) is ‘virtually identical’ to WGS 84

Cartesian/Geodetic Coordinates

- Cartesian (x,y,z)
- Geodetic (ϕ,λ,h)





Geodetic to Cartesian

- Compute N (radius of curvature in prime vertical)

$$N = \frac{a^2}{(a^2 \cos^2 \phi + b^2 \sin^2 \phi)^{1/2}} = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

- Compute X, Y, Z

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (N + h) \quad \cos\phi \cos\lambda \\ (N + h) \quad \cos\phi \sin\lambda \\ (N(1 - e^2) + h) \quad \sin\phi \end{bmatrix}$$

Cartesian to Geodetic – basic eqn's

- Given x, y, z compute ϕ, λ, h

$$\tan \lambda = \frac{y}{x}$$

$$p = \sqrt{x^2 + y^2}$$

$$N = \frac{a^2}{(a^2 \cos^2 \phi + b^2 \sin^2 \phi)^{1/2}} = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

$$h = \frac{p}{\cos \phi} - N$$

$$\tan \phi = \frac{z}{p}\left(1 - e^2 \frac{N}{N + h}\right)^{-1}$$

Cartesian to Geodetic – approx sol'n

- Given x, y, z compute ϕ, λ, h

TABLE 2.2 Approximate Closed-Form Solution for the ECEF Rectangular-to-Geodetic Coordinate Transformation

$$p = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{z * a}{p * b}\right)$$

$$(e')^2 = \frac{a^2 - b^2}{b^2}$$

$$\lambda = \arctan\left(\frac{z + (e')^2 b \sin^3(\theta)}{p - e^2 a \cos^3(\theta)}\right)$$

$$\phi = \arctan2(y, x)$$

$$h = \frac{p}{\cos(\lambda)} - N(\lambda)$$

Note: λ = latitude
ϕ = longitude

Cartesian to Geodetic – closed form

TABLE 2.1 Closed-Form Solution [65, 81] for the ECEF Rectangular-to-Geodetic Coordinate Transformation

$$p = \sqrt{x^2 + y^2}$$

$$E^2 = a^2 - b^2$$

$$F = 54b^2z^2$$

$$G = p^2 + (1 - e^2)z^2 - e^2E^2$$

$$c = \frac{e^4Fp^2}{G^3}$$

$$s = \left(1 + c + \sqrt{c^2 + 2c}\right)^{1/3}$$

$$P = \frac{F}{3(s + \frac{1}{s} + 1)^2 G^2}$$

$$Q = \sqrt{1 + 2e^4P}$$

$$r_0 = -\frac{Pe^2p}{1 + Q} + \sqrt{0.5a^2\left(1 + \frac{1}{Q}\right) - \frac{P(1 - e^2)z^2}{Q(1 + Q)} - 0.5Pp^2}$$

$$U = \sqrt{(p - e^2r_0)^2 + z^2}$$

$$V = \sqrt{(p - e^2r_0)^2 + (1 - e^2)z^2}$$

$$z_0 = \frac{b^2z}{aV}$$

$$e' = \frac{a}{b}e (= 0.00820944379496 \text{ for WGS-84})$$

$$h = U\left(1 - \frac{b^2}{aV}\right)$$

$$\lambda = \arctan\left(\frac{z + (e')^2z_0}{p}\right)$$

$$\phi = \arctan2(y, x)^*$$

* $\arctan2(y, x)$ is a four quadrant inverse tangent function.

- Given x, y, z compute ϕ, λ, h

Note: λ = latitude
ϕ = longitude

Cartesian to Geodetic – iterative

- Given x, y, z compute ϕ, λ, h

1. Compute $p = \sqrt{X^2 + Y^2}$.

2. Compute an approximate value $\varphi_{(0)}$ from
$$\tan \varphi_{(0)} = \frac{Z}{p}(1 - e^2)^{-1}.$$

3. Compute an approximate value $N_{(0)}$ from
$$N_{(0)} = \frac{a^2}{\sqrt{a^2 \cos^2 \varphi_{(0)} + b^2 \sin^2 \varphi_{(0)}}}.$$

4. Compute the ellipsoidal height by
$$h = \frac{p}{\cos \varphi_{(0)}} - N_{(0)}.$$

5. Compute an improved value for the latitude by
$$\tan \varphi = \frac{Z}{p}\left(1 - e^2 \frac{N_{(0)}}{N_{(0)} + h}\right)^{-1}.$$

6. Check for another iteration step: if $\varphi = \varphi_{(0)}$ then the iteration is completed otherwise set $\varphi_{(0)} = \varphi$ and continue with step 3.

GPS Positioning

- *Range*:
  - Distance to the satellite
  - Determined by the travel time of the signal

$$r = C \cdot t$$

  - Two methods used to determine range:
    - Code pseudoranges
    - Carrier phase-shifts

GPS Observables

Observables

- Code Pseudoranges
  - C/A code ranging precision ~3 meters
  - P code ranging precision ~30 cm

- Carrier Phase
  - Few millimeter precision

Accuracy vs. Precision

- Accuracy: distance from true position
- Precision: resolution of measurement, fineness
- Receiver noise ‘smudges’ the signal, reducing the precision
- Multipath introduces interfering signals that can cause a loss of accuracy

Code Pseudoranges

The receiver generates its own copy of the code and correlates it to the code from the satellite.

The amount of time needed to shift the receiver's code to match the satellite code is the transit time.

The transit time is multiplied by the speed of light to get the range to the satellite.

The problem is that both the satellite clock and the receiver's clock have biases (offsets) so this calculated range is not the true range. It is called a pseudorange.

Code Pseudoranges

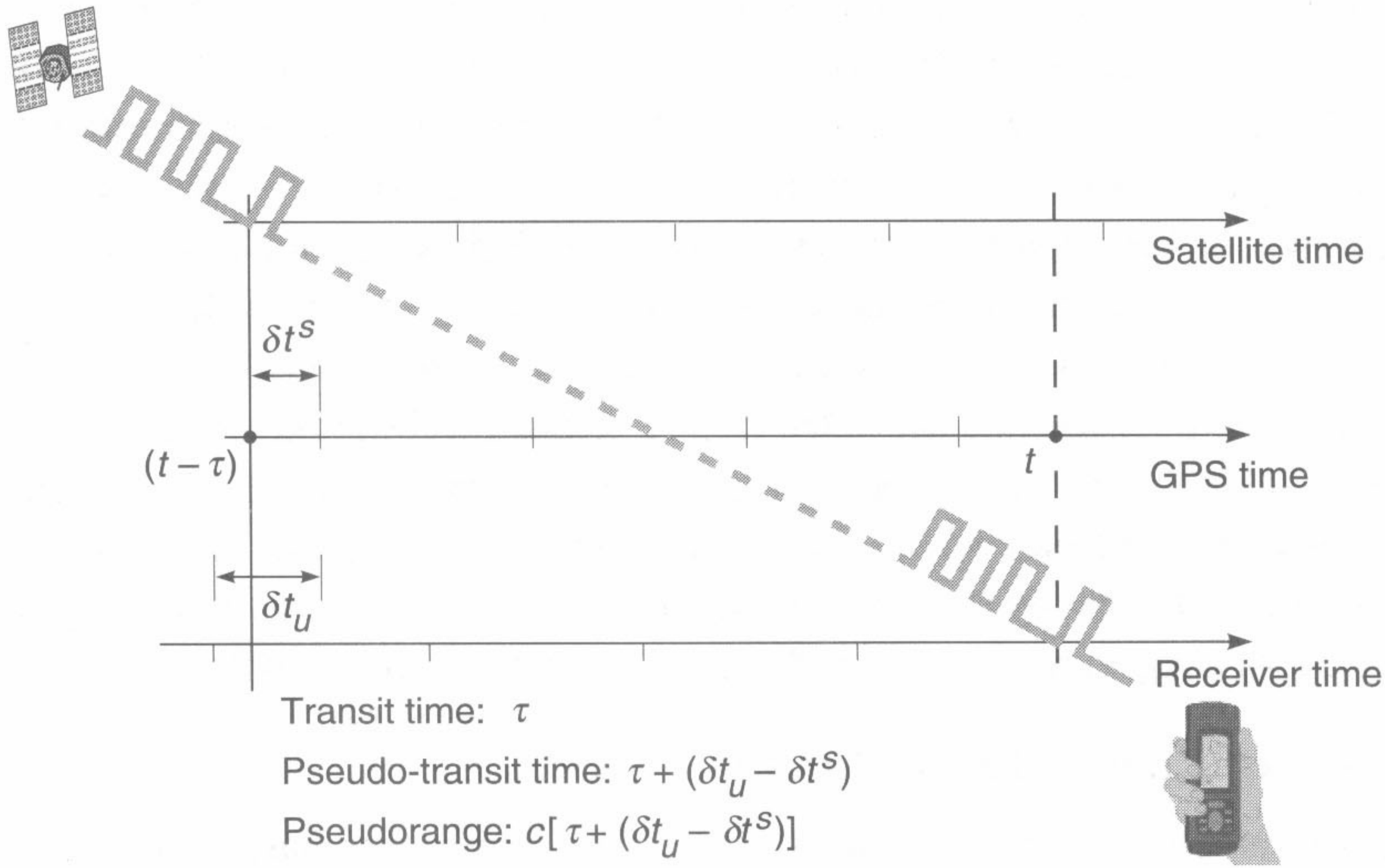


Figure 4.1 A conceptual view of the pseudorange measurements. The receiver and satellite clocks are unsynchronized. The apparent transit time (pseudo-transit time) is the difference between the signal reception time according to the receiver clock and the transmission time imprinted upon the signal in accordance with the satellite clock.

GPS Range

- Signal from <u>one satellite</u> determines your location on a sphere
- Two satellites: a circle (the intersection of two spheres)
- Three Satellites: two points (the intersection of two circles)

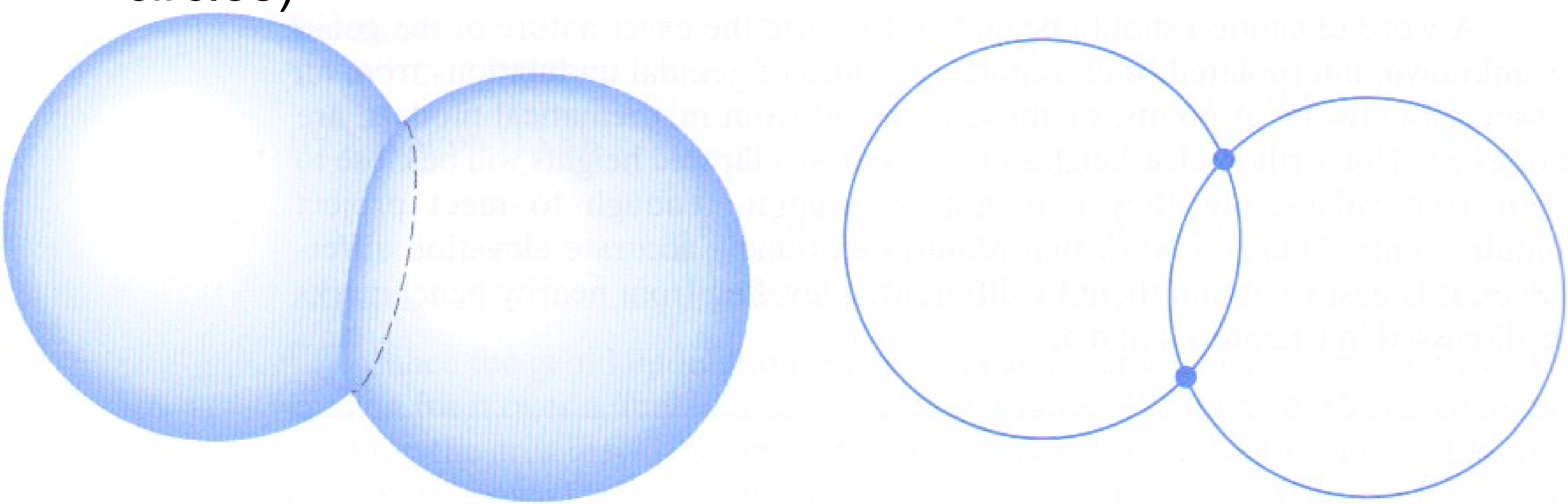


Figure 13.8 (a) The intersection of two spheres and (b) the intersection of two circles.

GPS Range

- Geometric range
  - Satellite positions (X^n, Y^n, Z^n) **known**
  - Receiver position (X_A, Y_A, Z_A) **unknown**

$$\rho_A^1 = \sqrt{(X^1 - X_A)^2 + (Y^1 - Y_A)^2 + (Z^1 - Z_A)^2}$$
$$\rho_A^2 = \sqrt{(X^2 - X_A)^2 + (Y^2 - Y_A)^2 + (Z^2 - Z_A)^2}$$
$$\rho_A^3 = \sqrt{(X^3 - X_A)^2 + (Y^3 - Y_A)^2 + (Z^3 - Z_A)^2}$$

GPS Pseudorange

- The quantity actually measured by the GPS receiver is *pseudorange*
  - Includes clock biases and delays caused by travel through the atmosphere
  - Adding a fourth satellite allows mathematical solution

$$R_A^1(t) = \rho_A^1(t) + c\left(\delta^1(t) + \delta_A(t)\right)$$
$$R_A^2(t) = \rho_A^2(t) + c\left(\delta^2(t) + \delta_A(t)\right)$$
$$R_A^3(t) = \rho_A^3(t) + c\left(\delta^3(t) + \delta_A(t)\right)$$
$$R_A^4(t) = \rho_A^4(t) + c\left(\delta^4(t) + \delta_A(t)\right)$$

Pseudorange Equation

$$R_A^n(t) = \rho_A^n(t) + c(\delta^n(t) + \delta_A(t)) + I_R + T_R + \epsilon_R$$

R_A^n = pseudorange between sat n rec A

ρ_A^n = geometric (true) range

c = speed of light

δ^n = satellite clock bias

δ_A = receiver clock bias

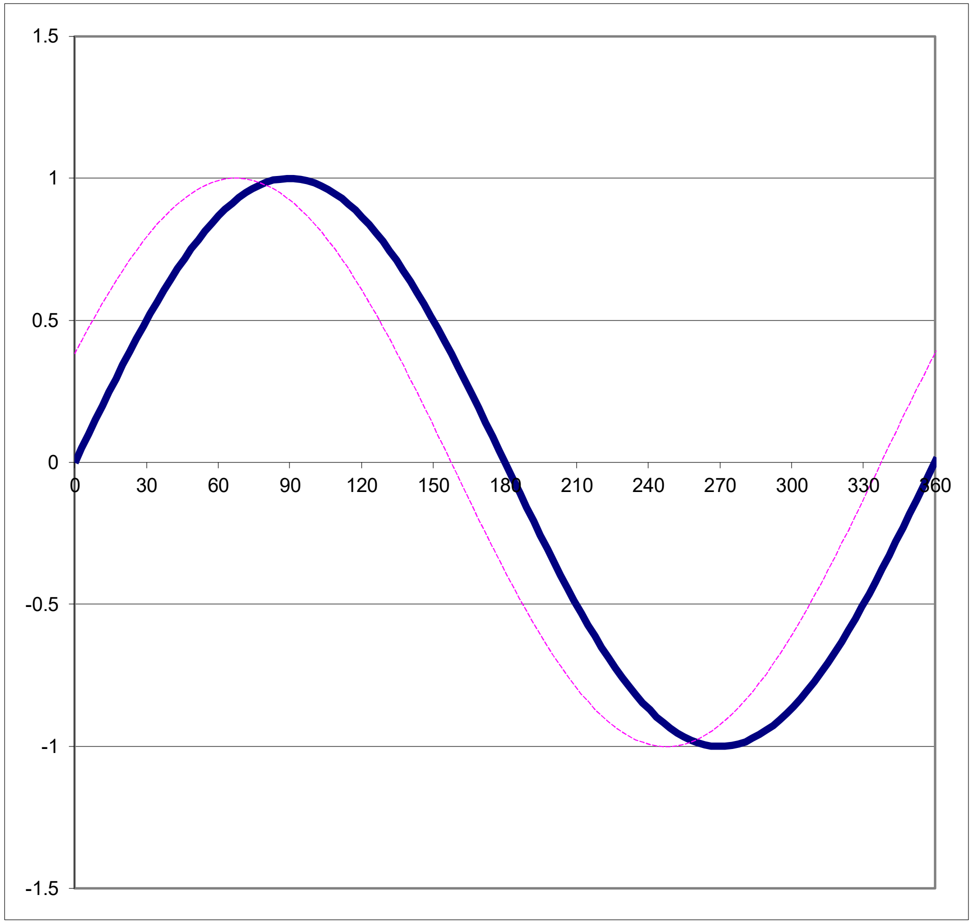
I_R = ionospheric delay range error

T_R = tropospheric delay range error

ϵ_R = other measurement and modeling errors

Carrier Phase

Carrier Phase – the amount the carrier wave generated by the receiver is shifted with respect to the wave from the satellite



Analogous to EDM measurements

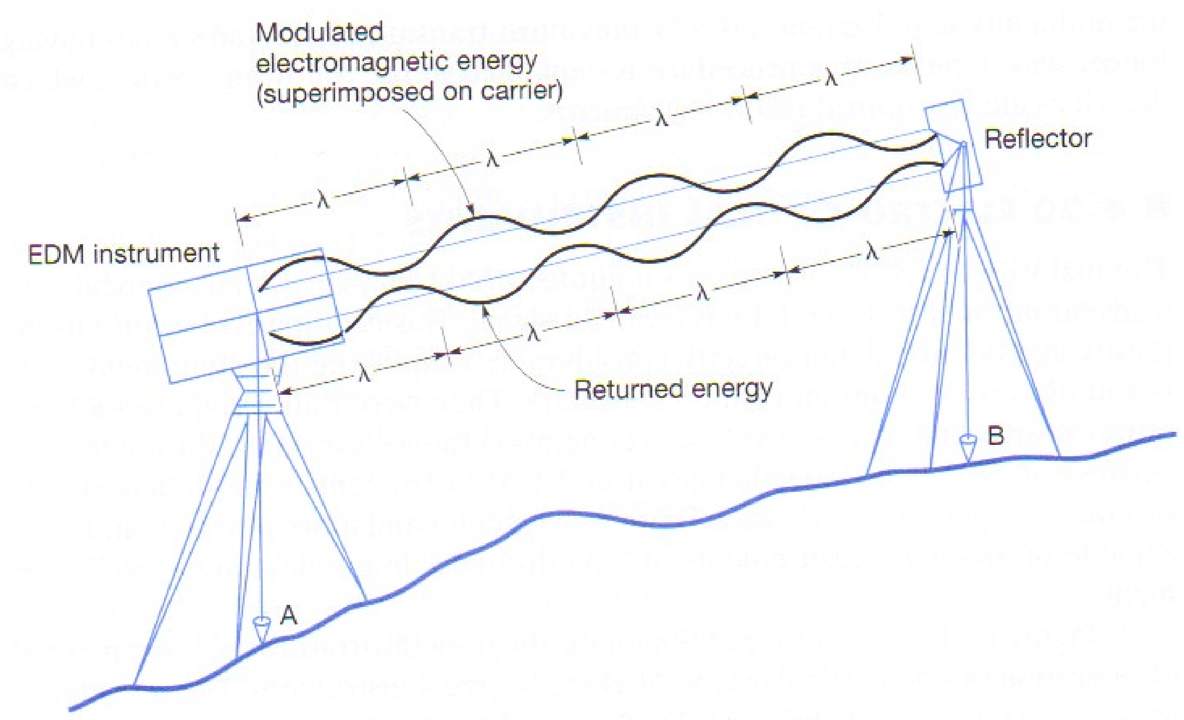


Figure 6.9
Generalized EDM procedure.

$$L = \frac{n\lambda + p}{2}$$

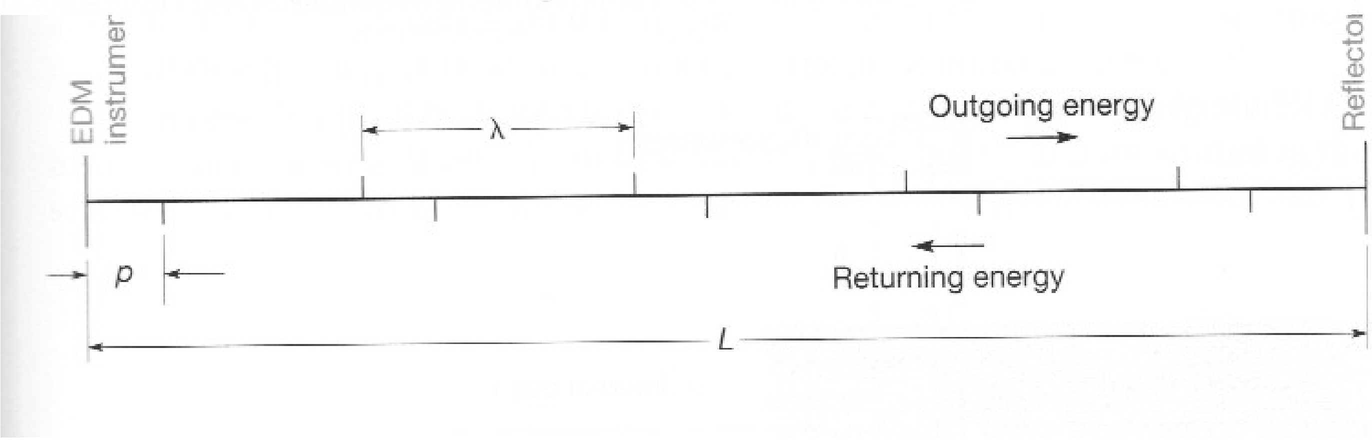


Figure 6.10
Phase difference measurement principle.

Carrier Phase

Idealized case – synchronized clocks, no relative motion

The distance between the satellite and the receiver is an integer number of whole cycles plus a fraction of the last cycle of the carrier wave

We can measure the fraction of a cycle from the carrier phases but the number of whole cycles is unknown.

This is referred to as the <u>integer ambiguity</u>.

Integer Ambiguity (N)

To compensate for integer ambiguity the receiver counts the number of whole cycles it sees as the distance changes.

To do this the receiver acquires ‘phase lock’, counts the number of whole cycles, and keeps track of the fractional cycle at each epoch.

If the receiver ‘loses lock’, possibly from loss of reception of the satellite’s signal due to an obstruction, it must begin the counting of cycles over.

A collection of measurements over a ‘short’ time can be used to estimate N.

Carrier Phase Measurement Equation

$$\phi_i^j(t) = \frac{\rho_i^j(t)}{\lambda} + N_i^j + \frac{c}{\lambda}\left(\delta^j(t) + \delta_i(t) + I_\phi + T_\phi + \epsilon_\phi\right)$$

$\phi_i^j(t) =$ carrier phaseshift measurement between rec i sat j

$\rho_i^j =$ geometric (true) range

$N_i^j =$ integer ambiguity

$c =$ speed of light

$\lambda =$ wavelength of the signal

$\delta^j =$ satellite clock bias

$\delta_i =$ receiver clock bias

$I_\phi =$ ionospheric delay range error

$T_\phi =$ tropospheric delay range error

$\epsilon_\phi =$ other measurement and modeling errors

Sources of Error

- Errors:
  - Noise: quickly varying error that averages out to zero over a ‘short’ time interval
  - Bias: tends to persist over a period of time

- Control Segment errors
- Propagation errors
- Receiver noise

URE (or UERE)

- User Range Error (User Equivalent Range Error)

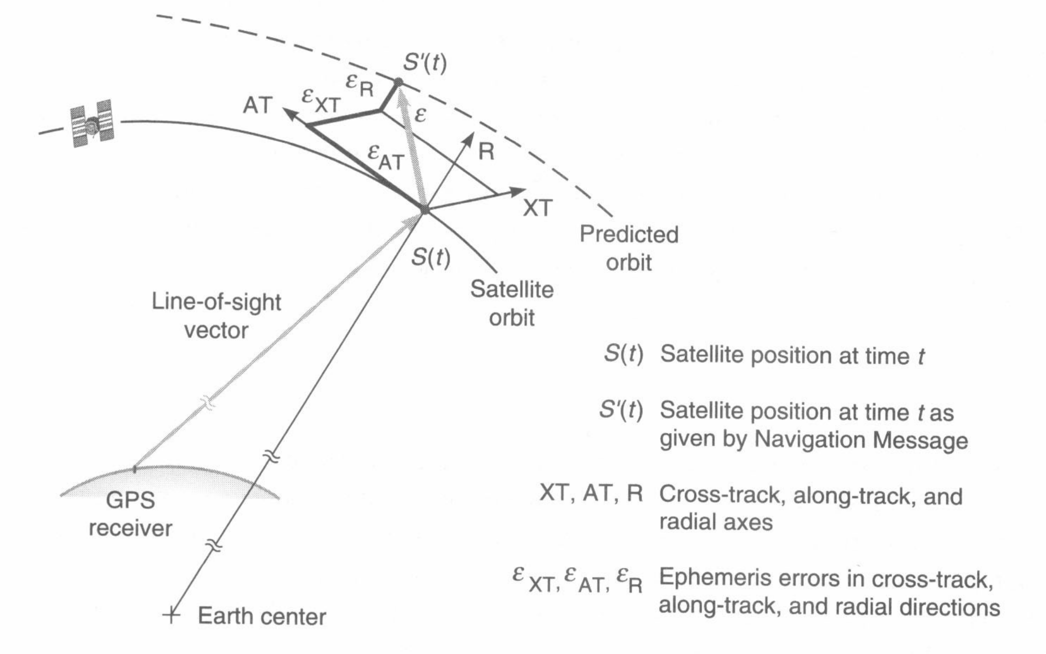
- Effect of errors on pseudorange measurements
- Given in units of meters

Control Segment Errors

- Ephemeris error
  - Orbit bias
  - Satellite clock bias, drift (up to 300km)

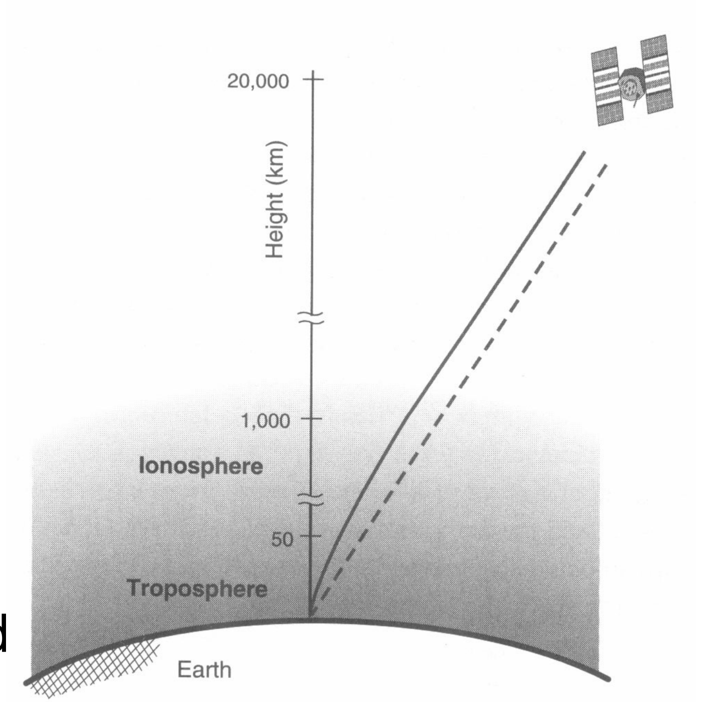
- Now under 3m

- During SA
  25m



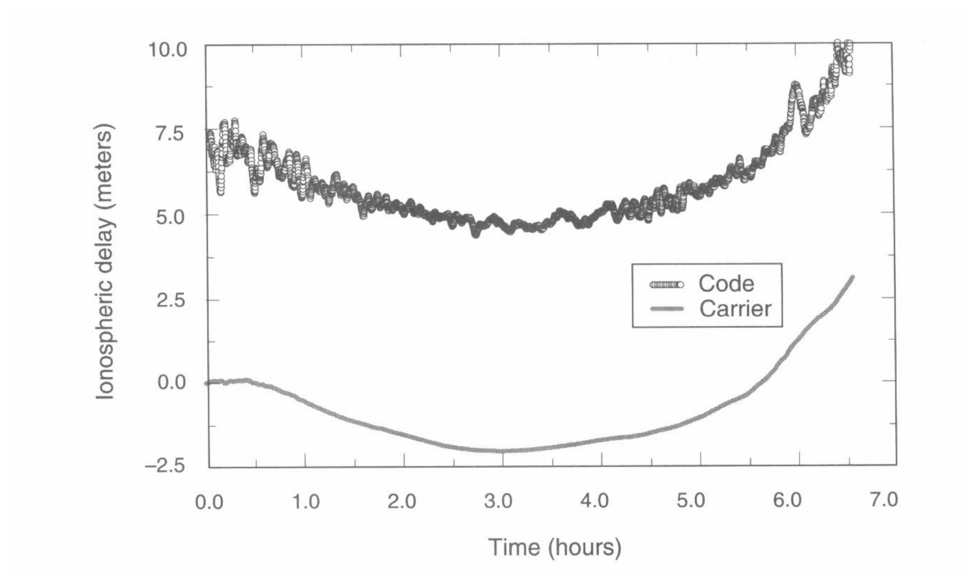
Propagation Errors

- Signal path: 20,000km to 26,000km
- Ionosphere height = 1000km
  - Charged particles
- Troposphere height = 40km
  - Denser neutral gases
- Refraction
  - Changes in
    - Direction
    - Speed
  - Refractive Index
    - Depends on frequency in ionosphere (‘d



Ionospheric Effect

- Due to Refraction
- Depends on:
  - layer thickness
  - TEC (total electron count)
- Result
  - Code delay
  - Phase advance



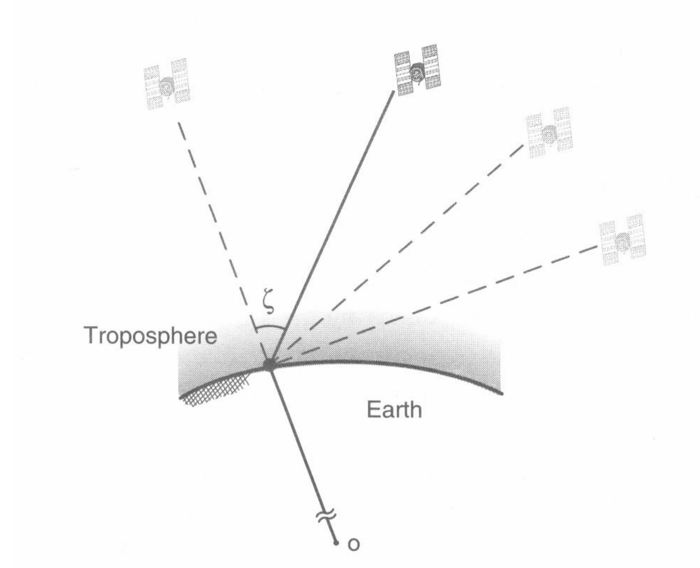
Ionospheric Effect

- 1-3m during night
- 5-15m mid-afternoon
- Up to 36m during peak solar cycles

- Can be estimated using:
  - Dual Frequency Receiver (L1 and L2)
    - ~1m during day
  - Broadcast Model (Klobuchar model)
    - 1m to 10m during day

Tropospheric Effect

- Refraction
  - Not dependent on frequency
  - Signal speed is lower
  - Ranges are longer
  - Depends on
    - Density of atmosphere
    - Water vapor content
    - Path Length (satellite elevation)



- Use models based on average meteorological conditions to estimate bias
- URE 2-10m without model, ~1m with model

Receiver Noise

- Def’n: noise caused by other RF signals or introduced by equipment in processing
- Varies with signal strength (which varies with satellite elevation)
- Depends on quality of receiver

Multipath Errors

- Range errors caused by reflected signals
- Up to several meters in magnitude
- Effects both code and carrier phase measurements
  - 1m to more than 5m for code
  - 1-5cm for carrier phase
- Can be reduced by:
  - Better antenna design
    - Elevation mask (choke ring)
    - Ground plane
  - Improved signal processing
  - Avoid poor sites