Interplanetary Transfers

Interplanetary Transfers

Interplanetary Transfers

- Up until this point, we have only considered spacecraft maneuvers in the near-vicinity of a particular body (e.g., satellite maneuvering between Earth-bound orbits)
- Now, let us consider interplanetary travel:

Example: Consider a spacecraft transferring from an orbit about the Earth to an orbit about Mars

- In general, the spacecraft is simultaneously influenced by: <u>Sun</u>, <u>Earth</u>, <u>Mars</u> (as well as <u>all other celestial bodies</u>)

However,

- Near the Earth → the main influence on the spacecraft is the Earth's gravitational field
- Near Mars → the main influence on the spacecraft is Mars' gravitational field
- For most of the transfer → the main influence is the Sun's gravitational field

For the purpose of initial mission planning, it useful to decouple these effects, <u>considering the spacecraft to be influenced by one body at a time</u>

Patched Conics

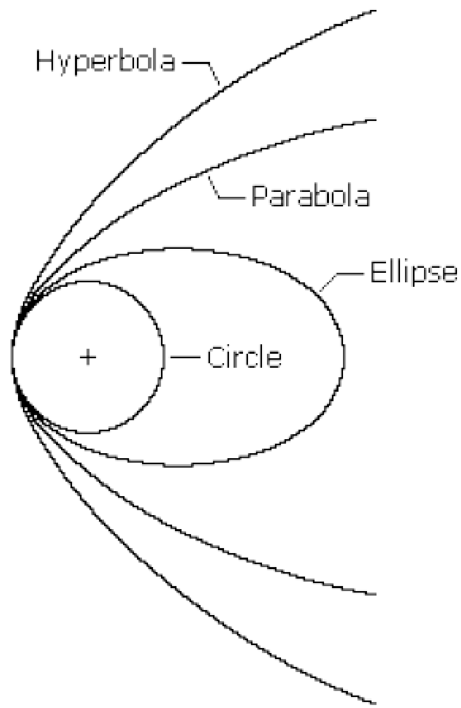
[De Ruiter, Ch. 6]

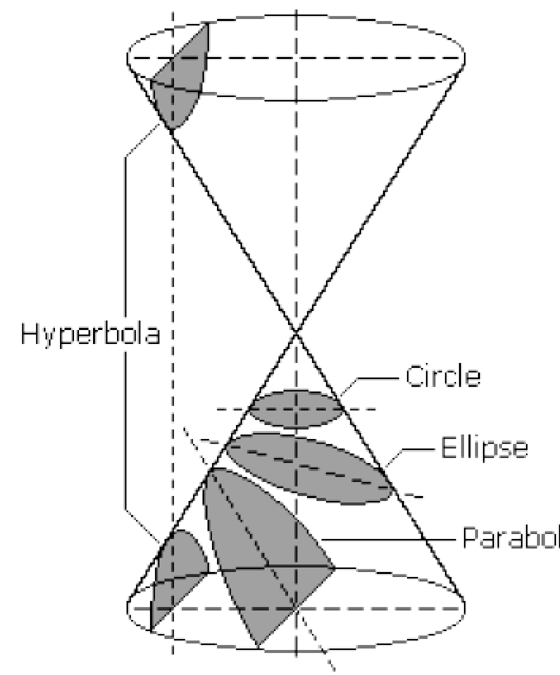
Interplanetary Transfers

Note: a typical orbit for departure or arrival at a planet is a hyperbolic orbit (i.e., $e > 1$)

Patched Conics

- "Conics" refers to the fact that two-body or Keplerian orbits are conic sections with the focus at the attracting body
- The patched conics method (or approximation) simplifies trajectory calculations by considering only one primary body acting on the spacecraft at any given time





Example: Consider a spacecraft transferring from an orbit about the Earth to an orbit about Mars

1. Near Earth → s/c is in a hyperbolic geocentric orbit (*while s/c escapes from Earth*)
2. Between Earth and Mars→ s/c is in a heliocentric elliptical orbit (*travelling toward Mars*)
3. Near Mars → s/c is in a two-body orbit about Mars

To perform patched conics, we need to know when to consider the s/c under the influence of each body

Sphere of influence

Clarkson

[Braeunig, "Orbital Mechanics," http://www.braeunig.us/space/orbmech.htm 2013]

Interplanetary Transfers

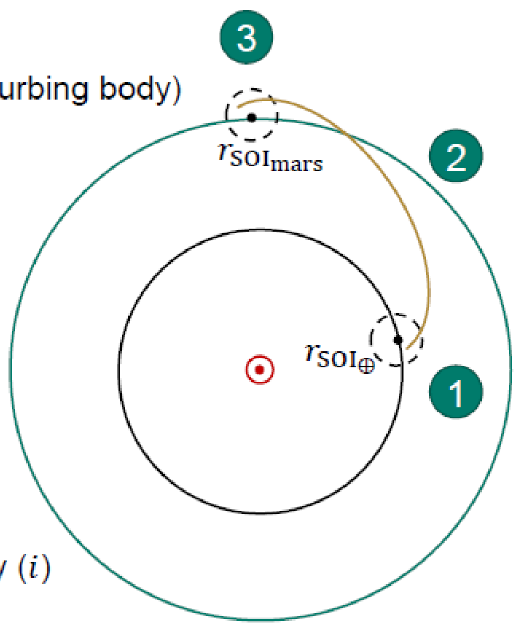
Sphere of Influence

- The <u>sphere of influence (SOI)</u> is the region in which a particular body is the dominant gravitational influence
- In the SOI, the s/c is considered to be in a two-body orbit about that body
- Outside the SOI, the s/c is considered to be in a two-body problem with the perturbing body

Example:

In the Earth's SOI, the s/c is considered to be in a two-body orbit about the Earth

Outside the Earth's SOI, the s/c is considered to be in a two-body orbit about the Sun (i.e., perturbing body)

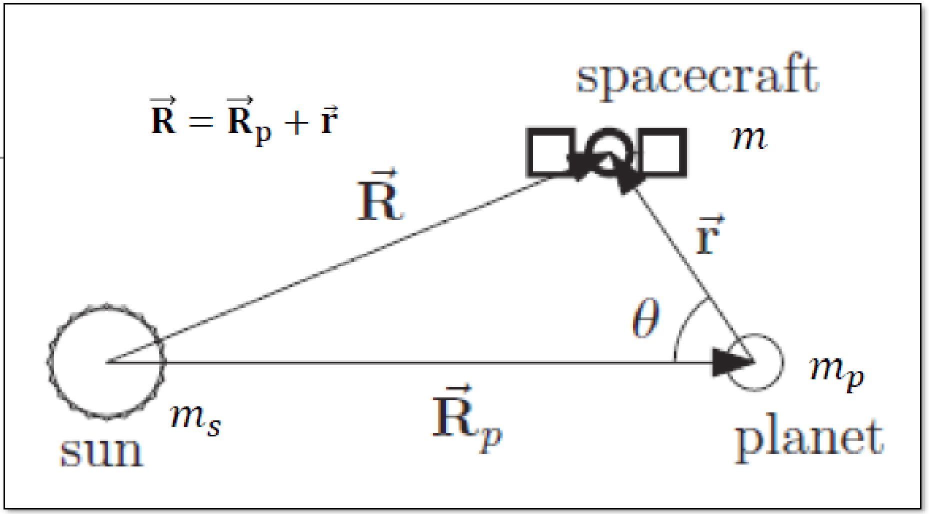


Example: Consider a spacecraft transferring from an orbit about the Earth to an orbit about Mars

1. In Earth's SOI → s/c is in a hyperbolic geocentric orbit
   (*while s/c escapes from Earth*)
2. Outside planetary SOIs→ s/c is in a heliocentric elliptical orbit
   (*travelling toward Mars*)
3. In Mars' SOI → s/c is in a two-body orbit about Mars

r_{SOI_i} ← size of SOI for a given body (i)

Interplanetary Transfers



Sphere of Influence

- We will derive the SOI based upon whether:
  (i) the sun has a larger influence over the motion about the planet, or
  (ii) the planet has a larger influence over the motion about the sun

- $\vec{\mathbf{r}}$ is the position of the s/c relative to the planet
- $\vec{\mathbf{R}}$ is the position of the s/c relative to the sun
- $\vec{\mathbf{R}}_p$ is the position of the planet relative to the sun

$$R = \left(R_p^2 + r^2 - 2R_p r\cos\theta\right)^{1/2} = R_p\left(1 + \frac{r^2}{R_p^2} - 2\frac{r}{R_p}\cos\theta\right)^{1/2}$$

- We expect that within the SOI of the planet, $r \ll R_p$, so that we can say that $R \approx R_p$

- Gravitational force of the sun and planet on the spacecraft are: $\vec{\mathbf{f}}_{\mathrm{g,s}} = -\frac{Gm_s m}{R^3}\vec{\mathbf{R}}$ $\qquad \vec{\mathbf{f}}_{\mathrm{g,p}} = -\frac{Gm_p m}{r^3}\vec{\mathbf{r}}$

- Gravitational force of the sun on the planet is: $\vec{\mathbf{f}}_{\mathrm{g,s\to p}} = -\frac{Gm_s m_p}{R_p^3}\vec{\mathbf{R}}_p$

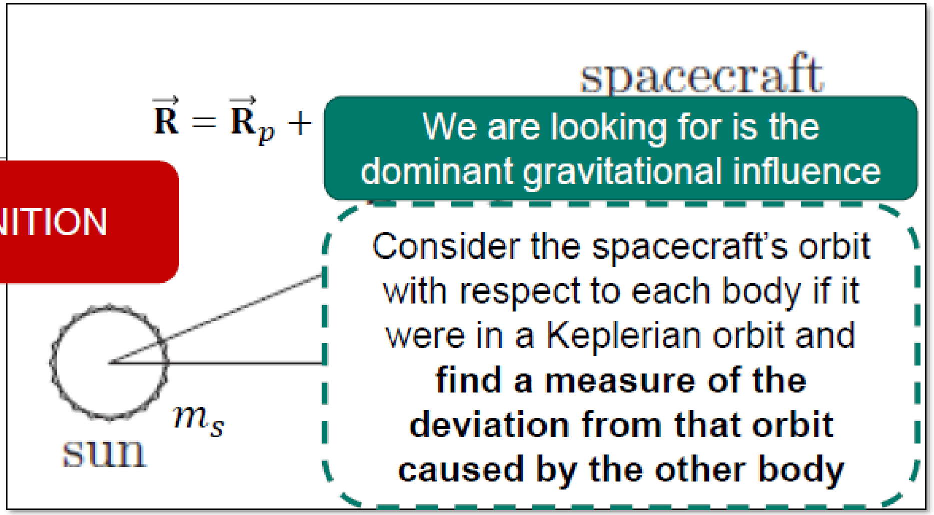
Interplanetary Transfers

WRONG DEFINITION

Sphere of Influence

Let's consider this <u>wrong</u> definition of SOI

- The bodies SOI is defined as the region wherein that body exerts the <u>largest gravitational force</u>



Pursuing this route, we could say the that the boundary of the SOI is the location where the planet's gravitational force is equal to that of the sun:

$$\boxed{\vec{\mathbf{f}}_{g,s} = -\frac{Gm_s m}{R^3}\vec{\mathbf{R}}}$$

$$\boxed{\vec{\mathbf{f}}_{g,p} = -\frac{Gm_p m}{r^3}\vec{\mathbf{r}}}$$

$$\vec{\mathbf{f}}_{g,s} = \vec{\mathbf{f}}_{g,p}$$

$$-\frac{Gm_p m}{r^3}\vec{\mathbf{r}} = -\frac{Gm_s m}{R^3}\vec{\mathbf{R}}$$

$$\frac{m_p}{r^3}\vec{\mathbf{r}} = \frac{m_s}{R^3}\vec{\mathbf{R}}$$

$$\xrightarrow{\text{Magnitude}} \quad \frac{m_p}{r^2} = \frac{m_s}{R^2}$$

$$r = R\sqrt{\frac{m_p}{m_s}}$$

$$\xrightarrow[R = R_p - r]{\text{Assume } \theta = 0}$$

$$r = (R_p - r)\sqrt{\frac{m_p}{m_s}}$$

$$r_{\mathrm{SOI}} = \frac{R_p\sqrt{\frac{m_p}{m_s}}}{\left(1 + \sqrt{\frac{m_p}{m_s}}\right)}$$

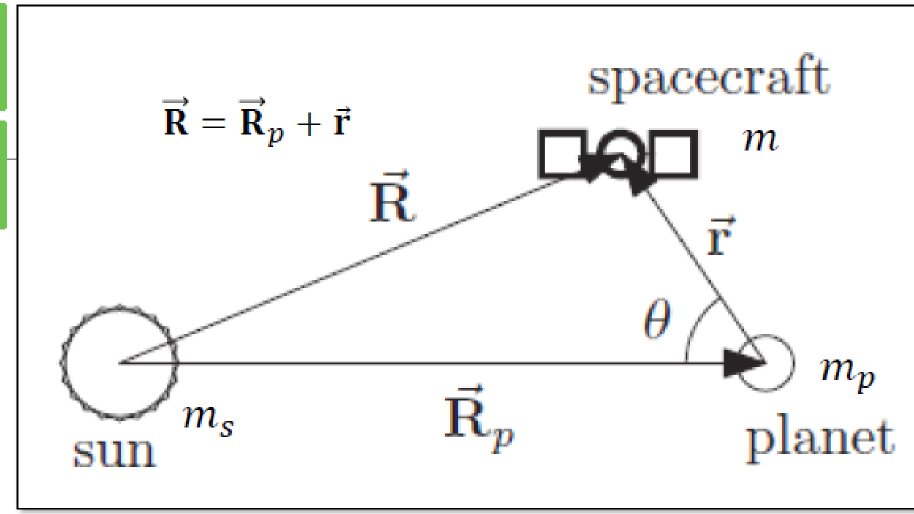
- Plugging in values for the Earth and Sun, using this equation we would get: $r_{\mathrm{SOI}} \approx 259{,}000\ \mathrm{km}$
- BUT, if that were true, then the Moon ($r_{\mathrm{moon}} = 384{,}400\ \mathrm{km}$) would not be in Earth's SOI!

WRONG DEFINITION

Interplanetary Transfers

$$\vec{\mathbf{f}}_{g,s} = -\frac{Gm_s m}{R^3}\vec{\mathbf{R}}$$

$$\mathbf{f}_{g,p} = -\frac{Gm_p m}{r^3}\mathbf{r}$$



Sphere of Influence Relative to the Sun

- Assuming the sun to be inertially fixed, the equation of motion of the spacecraft relative to the sun is:

$$m\ddot{\vec{\mathbf{R}}} = \vec{\mathbf{f}}_{g,s} + \vec{\mathbf{f}}_{g,p} = -\frac{Gm_s m}{R^3}\vec{\mathbf{R}} - \frac{Gm_p m}{r^3}\vec{\mathbf{r}}$$

- Which can be rewritten in terms of accelerations: $\ddot{\vec{\mathbf{R}}} = \vec{\mathbf{A}}_s + \vec{\mathbf{P}}_p$

$R \approx R_p$

Acceleration of the spacecraft relative to the sun due to the sun's gravitation $\vec{\mathbf{A}}_s = -\frac{Gm_s}{R^3}\vec{\mathbf{R}}$

magnitudes

$$A_s = -\frac{Gm_s}{R^2} \approx -\frac{Gm_s}{R_p^2}$$

Acceleration of the spacecraft relative to the sun due to the planet's gravitation $\vec{\mathbf{P}}_p = -\frac{Gm_p}{r^3}\vec{\mathbf{r}}$

$$P_p = -\frac{Gm_p}{r^2}$$

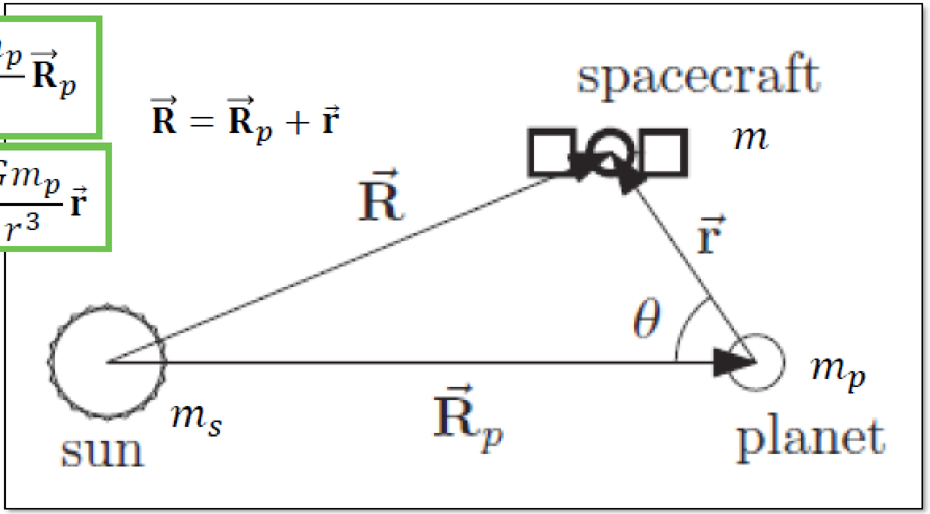
- The ratio of P_p to A_s is a measure of the influence of the planet on the spacecraft's orbit about the sun: $\frac{P_p}{A_s} = \frac{m_p}{m_s}\left(\frac{R_p}{r}\right)^2$

Now, let's obtain a similar ratio for the disturbing effect of the sun on the s/c's orbit about the planet

Interplanetary Transfers

$$\vec{\mathbf{f}}_{g,s\to p} = -\frac{Gm_s m_p}{R_p^3}\vec{\mathbf{R}}_p$$

$$\ddot{\vec{\mathbf{R}}} = -\frac{Gm_s}{R^3}\vec{\mathbf{R}} - \frac{Gm_p}{r^3}\vec{\mathbf{r}}$$



Sphere of Influence Relative to the Planet

- The equation of motion of the planet about the sun is:

$$m_p\ddot{\vec{\mathbf{R}}}_p = \vec{\mathbf{f}}_{g,s\to p} = -\frac{Gm_s m_p}{R_p^3}\vec{\mathbf{R}}_p \longrightarrow \ddot{\vec{\mathbf{R}}}_p = -\frac{Gm_s}{R_p^3}\vec{\mathbf{R}}_p$$

- Spacecraft's motion about the planet is described by:

$$R \approx R_p$$

$$\ddot{\vec{\mathbf{r}}} = \ddot{\vec{\mathbf{R}}} - \ddot{\vec{\mathbf{R}}}_p = -\frac{Gm_s}{R^3}\vec{\mathbf{R}} - \frac{Gm_p}{r^3}\vec{\mathbf{r}} + \frac{Gm_s}{R_p^3}\vec{\mathbf{R}}_p \approx -\frac{Gm_s}{R_p^3}\left(\vec{\mathbf{R}} - \vec{\mathbf{R}}_p\right) - \frac{Gm_p}{r^3}\vec{\mathbf{r}} = -\frac{Gm_s}{R_p^3}\vec{\mathbf{r}} - \frac{Gm_p}{r^3}\vec{\mathbf{r}} \longrightarrow \boxed{\ddot{\vec{\mathbf{r}}} = \vec{\mathbf{p}}_s + \vec{\mathbf{a}}_p}$$

Acceleration of the spacecraft relative to planet due to the sun's gravitation $\quad \vec{\mathbf{p}}_s = -\frac{Gm_s}{R_p^3}\vec{\mathbf{r}}$

magnitudes

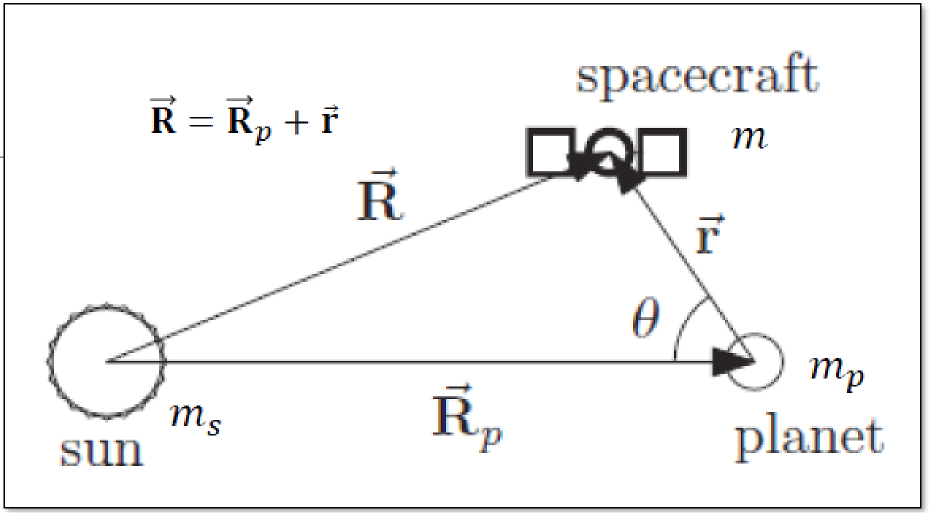
$$p_s = -\frac{Gm_s r}{R_p^3}$$

Acceleration of the spacecraft relative to the planet due to the planet's gravitation $\quad \vec{\mathbf{a}}_p = -\frac{Gm_p}{r^3}\vec{\mathbf{r}}$

$$a_p = -\frac{Gm_p}{r^2}$$

- The ratio of p_s to a_p is a measure of the influence of the sun on the spacecraft's orbit about the planet: $\boxed{\dfrac{p_s}{a_p} = \dfrac{m_s}{m_p}\left(\dfrac{r}{R_p}\right)^3}$

Interplanetary Transfers



Sphere of Influence

Measure of the influence of the planet on the s/c orbit about the sun:

$$\boxed{\frac{P_p}{A_s} = \frac{m_p}{m_s}\left(\frac{R_p}{r}\right)^2}$$

Measure of the influence of the sun on the s/c orbit about the planet:

$$\boxed{\frac{p_s}{a_p} = \frac{m_s}{m_p}\left(\frac{r}{R_p}\right)^3}$$

- If we want to consider the spacecraft to be primarily under the influence of the planet, then the influence of the sun on the s/c's orbit about the planet should be less than the influence of the planet on the s/c's orbit about the sun, i.e.,

$$\frac{p_s}{a_p} < \frac{P_p}{A_s} \quad \text{or equivalently} \quad \frac{m_s}{m_p}\left(\frac{r}{R_p}\right)^3 < \frac{m_p}{m_s}\left(\frac{R_p}{r}\right)^2$$

Magnitude of acceleration on s/c relative to the planet

Magnitude of acceleration on s/c relative to the sun

- The Sphere of Influence of a planet is defined as the boundary of this region, i.e.,

[due to sun] / [due to planet] $\boxed{\frac{p_s}{a_p} = \frac{P_p}{A_s}}$ [due to planet] / [due to sun]

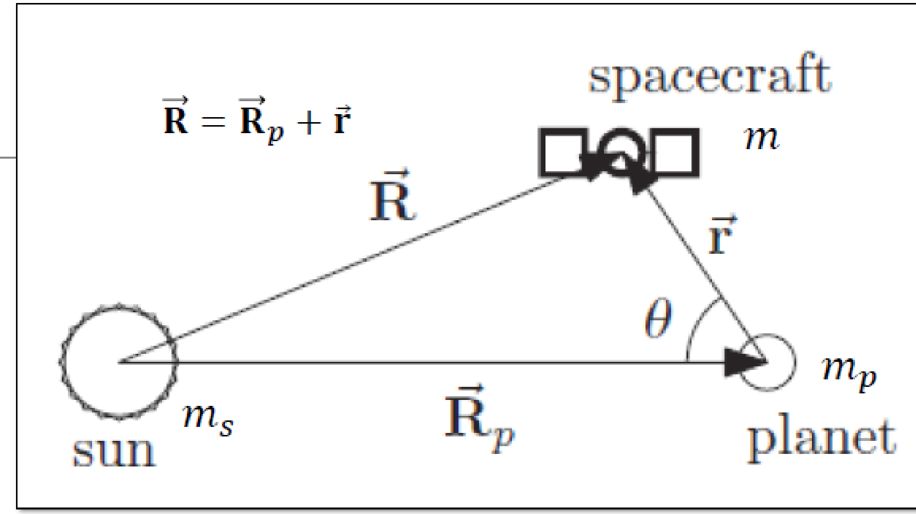
- This leads us to the following definition for the size of our sphere of influence for a planet:

$$\boxed{r_{\mathrm{SOI}} = R_p\left(\frac{m_p}{m_s}\right)^{2/5}}$$

N.B. the SOI is not an exact quantity. It is a reasonable estimate of when the planet's gravitational field dominates that of the sun

Interplanetary Transfers

Quick Activity



Sphere of Influence

Example

Compute the size of the sphere of influence for the Earth:

$m_\oplus = 5.974 \times 10^{24}$ kg

$m_\odot = 1.989 \times 10^{30}$ kg

Mean orbital radius of the Earth about the sun: $R_{\text{Earth}} = 149.6 \times 10^6$ km (i.e., 1 au)

$$\boxed{r_{\text{SOI}} = R_p \left(\frac{m_p}{m_s}\right)^{2/5}} \qquad r_{\text{SOI}\oplus} = (149.6 \times 10^6\ \text{km}) \left(\frac{5.974 \times 10^{24}\ \text{kg}}{1.989 \times 10^{30}\ \text{kg}}\right)^{2/5} = 9.25 \times 10^5\ \text{km}$$

Compared to Earth's distance to the Sun, SOI is very small

$$\frac{r_{\text{SOI}\oplus}}{R_{\text{Earth}}} = 0.0062$$

Compared to Earth's radius, SOI is very large

$$\frac{r_{\text{SOI}\oplus}}{R_\oplus} = 145.2$$

Now that we have found the SOI, we can return to our patched conics method for interplanetary transfers

Interplanetary Transfers

Interplanetary Hohmann Transfers

- Most planets in our solar system have orbits with: very low eccentricities and lie in approximately the same orbital plane
- In practice, we need to account for these differences, but to start, let us consider coplanar circular orbits about the sun
- Also, since the planetary SOIs are very small compared to their orbital radii, we will treat the start and end points of our interplanetary transfer as the location of each planet

Consider two planets:

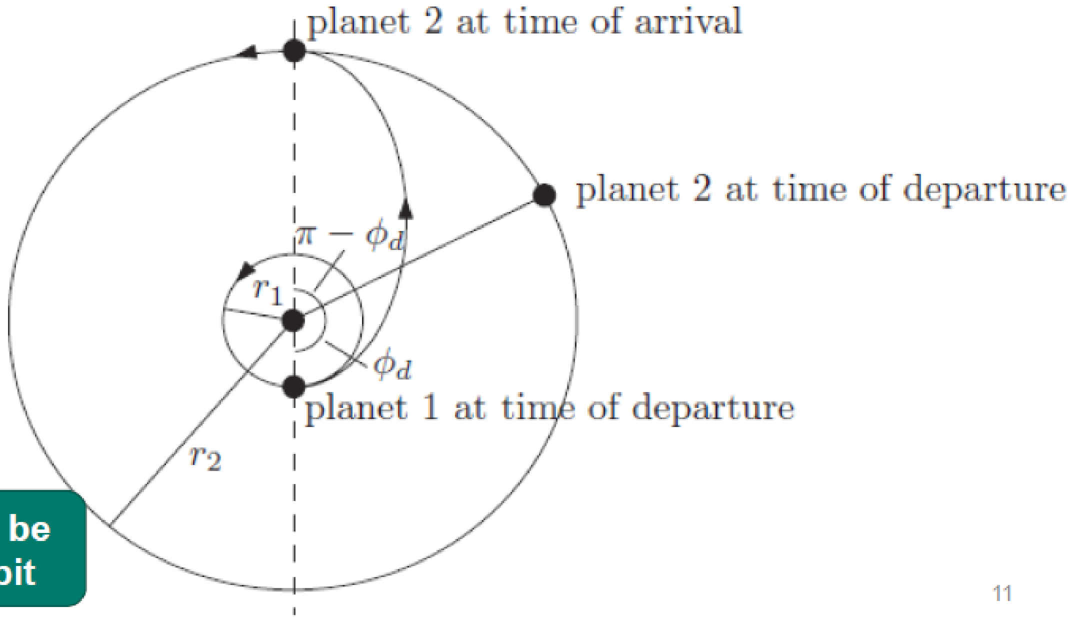
- Planet 1 with orbital radius of r_1
- Planet 2 with orbital radius of r_2

We want to perform a Hohmann transfer between the two planets

- *Recall, the semimajor axis of the transfer orbit and TOF:*

$$\boxed{a_t = \frac{r_1 + r_2}{2}} \qquad \boxed{TOF = \pi\sqrt{\frac{a_t^3}{\mu}}}$$

N.B. now, $\mu = \mu_s$ *gravitational parameter of the sun*



New constraint on the transfer! ***Planet 2* must be there when the spacecraft arrives in its orbit**

[NASA JPL, “Let’s Go to Mars! Calculating Launch Windows”, https://www.jpl.nasa.gov/edu/teach/activity/lets-go-to-mars-calculating-launch-windows/, 2016]

Interplanetary Transfers

Planetary Alignment

- In order for the s/c to intercept planet 2, planet 2 must have a certain phase (ϕ_d) with respect to planet 1 at the time of departure
- The phase angle is determined from time of flight (T_{12}), since planet 2 must advance its location by $\boxed{\pi - \phi_d}$ for intercept

Recall Orbital Period and Time of Flight

$$\boxed{\dot{\theta} = n}$$

For circular orbits, orbital angular rate is simply the mean orbital motion

- Orbit 2 is circular, so mean orbital motion of planet 2 is constant and given by:

$$\boxed{\theta(t) - \theta(t_0) = n(t - t_0)}$$

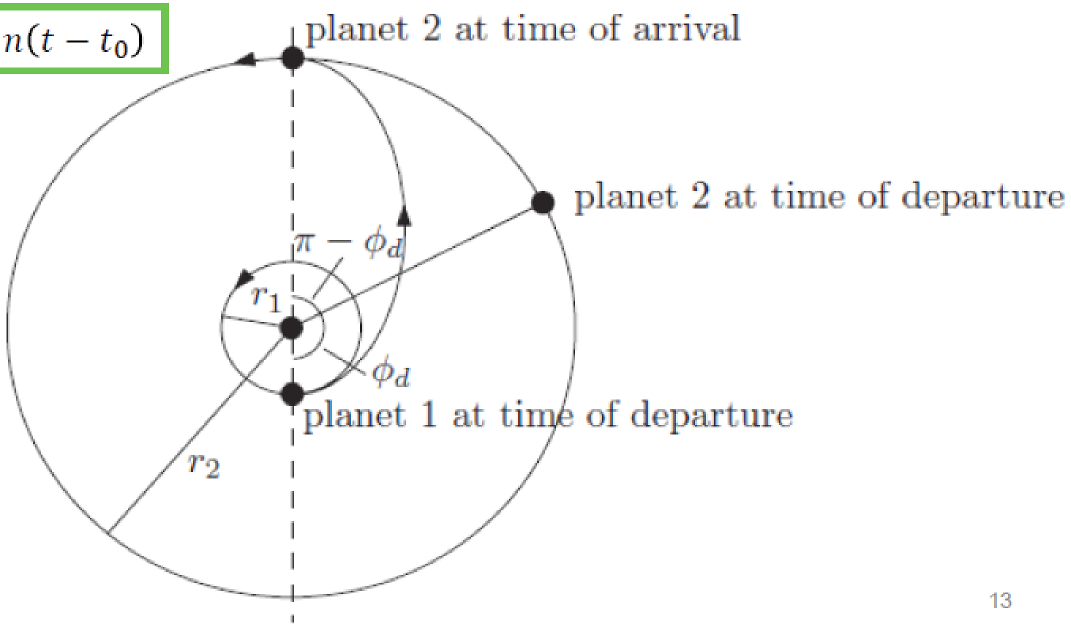
$$\boxed{n \equiv \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}}}$$

$$n_2 = \sqrt{\frac{\mu}{r_2^3}}$$

$$\pi - \phi_d = n_2 T_{12}$$

$$\boxed{\phi_d = \pi - n_2 T_{12}}$$

When planet 2 is at a phase angle ϕ_d with respect to planet 1, we say that **a launch window exists**



If we miss a launch window, how long do we need to wait until the next one?

Interplanetary Transfers

Launch Windows

- Since both planets are in circular orbits: $n_1 = \sqrt{\frac{\mu}{r_1^3}}$ $\quad n_2 = \sqrt{\frac{\mu}{r_2^3}}$

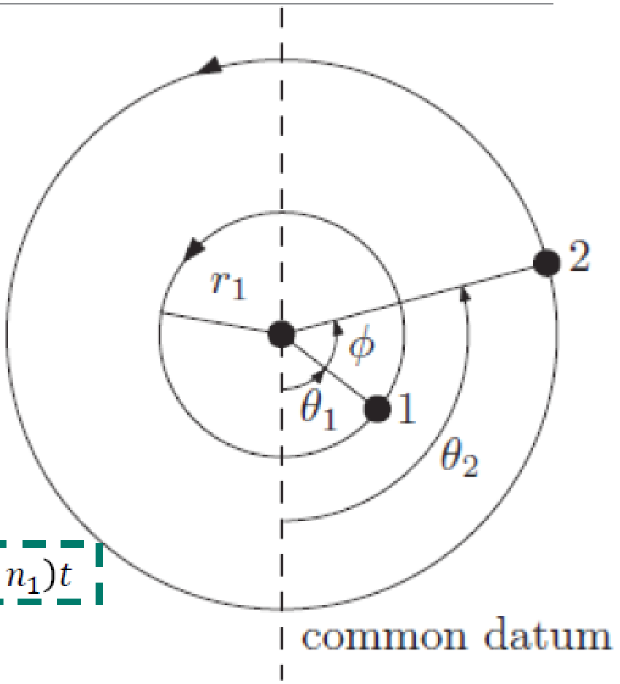
- Let's define a common datum, from which we can measure the true anomalies:

$$\boxed{\theta(t) - \theta(t_p) = n\left(t - t_p\right)} \qquad \theta_1 = \theta_{1,0} + n_1 t \qquad \theta_2 = \theta_{2,0} + n_2 t$$

$t = 0$

where time t is measured from the first launch window, such that $\phi_d = \theta_{2,0} - \theta_{1,0}$

- The phase angle for planet 2 relative to planet 1 is then: $\boxed{\phi = \theta_2 - \theta_1 = \phi_d + (n_2 - n_1)t}$



If $n_2 > n_1$, that is $r_1 > r_2$, then the next launch occurs when $\phi = \phi_d + 2\pi$

$$t = \frac{2\pi}{n_2 - n_1}$$

If $n_1 > n_2$, that is $r_2 > r_1$, then the next launch occurs when $\phi = \phi_d - 2\pi$

$$t = \frac{2\pi}{n_1 - n_2}$$

Based on this, time between launch windows is always given by:

Synodic Period $\boxed{T_{syn} = \frac{2\pi}{|n_1 - n_2|}}$

Now suppose we want to return from planet 2 to planet 1, **how long do we have to wait to get back?**

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Interplanetary Transfers

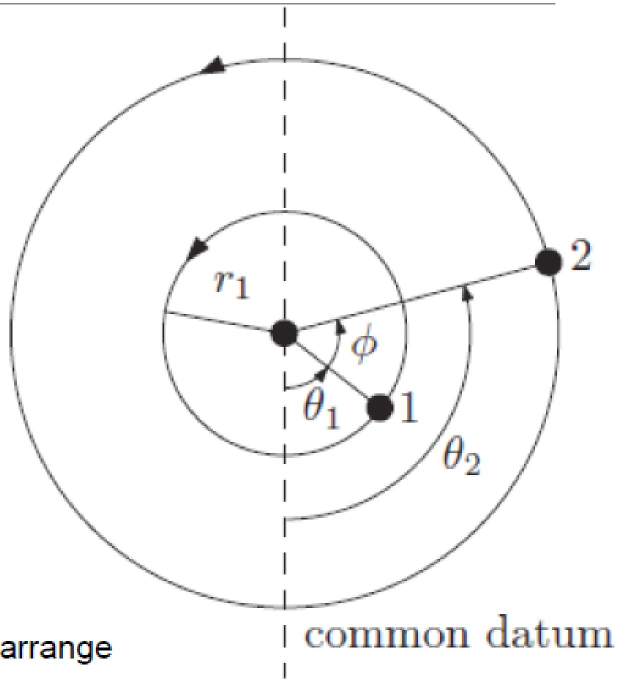
Wait Time to Return to Planet 1

- The transfer orbit to get back is the same, so the time of flight is the same as it was to get to TOF of the initial transfer, i.e., T_{12}

- Using the same approach as before, we can find the required phase angle of planet 1 with respect to planet 2

$\boxed{\phi_d = \pi - n_2 T_{12}}$ Note, the negative sign is due to the definition of ϕ as the phase of planet 2 relative to planet 1,

$-\phi_r = \pi - n_1 T_{12}$ i.e., the negative of the phase of planet 1 relative to planet 2



Starting from our phase at arrival (ϕ_{arr})

$$\boxed{\phi = \phi_d + (n_2 - n_1)t}$$

$$\phi_{arr} = \phi_d + (n_2 - n_1)T_{12}$$

Substitute in ϕ_d

$$\boxed{\phi_d = \pi - n_2 T_{12}}$$

$$\phi_{arr} = \pi - n_2 T_{12} + (n_2 - n_1)T_{12}$$

$$\phi_{arr} = \pi - n_1 T_{12}$$

$$\phi_r = -\phi_{arr}$$

Reset $t = 0$ to time of arrival at planet 2

$$\phi = \phi_{arr} + (n_2 - n_1)t$$

$$\phi_r = \phi_{arr} + (n_2 - n_1)T_{\text{wait}}$$

$$-\phi_{arr} = \phi_{arr} + (n_2 - n_1)T_{\text{wait}}$$

Rearrange

$$T_{\text{wait}} = -\frac{2\phi_{arr}}{n_2 - n_1}$$

$$T_{\text{wait}} = -\frac{2\phi_{arr} \pm N2\pi}{n_2 - n_1}$$

Problem: T_{wait} can be negative, so we add $\pm N2\pi$, s.t. N is the smallest integer that makes T_{wait} positive

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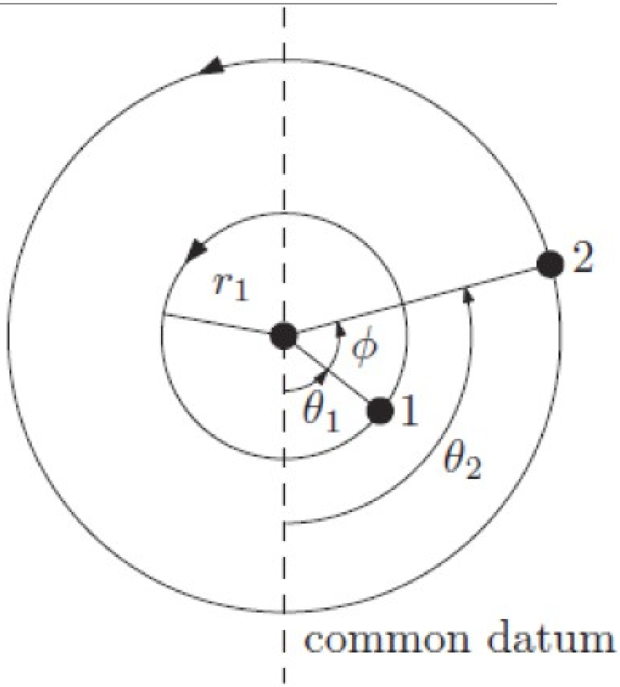
Interplanetary Transfers

Total Time for a Round Trip

- Total time for a round trip from planet 1 to planet 2, T_{trip}: $\boxed{T_{\text{trip}} = T_{\text{wait}} + 2T_{12}}$
- Times for trips from Earth to some of the planets:

Planet	T_{syn} (days)	T_{12} (days)	T_{wait} (days)	T_{trip} (days)
Mercury	115.8	105.4	66.9	277.9
Venus	583.9	146.1	467.0	759.2
Mars	779.9	258.8	454.3	972.1
Jupiter	398.8	997.5	214.6	2209.6
Saturn	378.1	2209.1	363.2	4454.5



Interplanetary Transfers

Example Example Activity

Elon Musk plans to have SpaceX's Starship travel to Mars on an uncrewed mission in 2024. In your interview with SpaceX, Musk asks you to estimate the following in days: (a) the transfer time from Earth to Mars, (b) the time required for an Earth-Mars round trip, and (c) approximately how long will they have to wait for the next launch window if they miss the 2024 launch? Assume you are using an interplanetary Hohmann transfer and that the planetary orbits are circular.

Solution (a)

$$\boxed{TOF = \pi\sqrt{\frac{a_t^3}{\mu}}}$$

N.B. now, $\mu = \mu_s$ *gravitational parameter of the sun*

$$\boxed{a_t = \frac{r_1 + r_2}{2}}$$

$$a_t = \frac{(149.6 \times 10^6 \text{ km}) + (227.9 \times 10^6 \text{ km})}{2} = 188.75 \times 10^6 \text{ km}$$

$$T_{12} = \pi\sqrt{\frac{a_t^3}{\mu}} = \pi\sqrt{\frac{(188.75 \times 10^6 \text{ km})^3}{(132.7 \times 10^9 \text{ km}^3/\text{s}^2)}} = 2.236 \times 10^7 \text{ s}$$

$$\boxed{T_{12} = 258.8 \text{ days}}$$

Consider the interplanetary variables:

Earth's average orbital radius of $r_E = 149.6 \times 10^6$ km (1 au)

Mars' average orbital radius of $r_M = 227.9 \times 10^6$ km (1.53 au)

Gravitational parameter for the Sun: $\mu_s = 132.7 \times 10^9 \text{ km}^3/\text{s}^2$

Interplanetary Transfers

$$r_E = 149.6 \times 10^6 \text{ km} \qquad a_t = 188.75 \times 10^6 \text{ km}$$
$$r_M = 227.9 \times 10^6 \text{ km}$$
$$\mu_s = 132.7 \times 10^9 \text{ km}^3/\text{s}^2 \qquad T_{12} = 2.236 \times 10^7 \text{ s}$$

$$\boxed{T_{12} = 258.8 \text{ days}}$$

Example Example Activity

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Solution (b)

$$\boxed{T_{\text{trip}} = T_{\text{wait}} + 2T_{12}}$$

$$\boxed{T_{\text{wait}} = -\frac{2\phi_{arr} \pm N2\pi}{n_2 - n_1}}$$

Mean orbital motion:

$$\boxed{n \equiv \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}}}$$

$$n_{\text{M}} = \sqrt{\frac{\mu_s}{r_{\text{M}}^3}} = \sqrt{\frac{132.7 \times 10^9 \text{ km}^3/\text{s}^2}{(227.9 \times 10^6 \text{ km})^3}} = 1.059 \times 10^{-7} \text{ rad/s}$$

$$n_{\text{E}} = \sqrt{\frac{\mu_s}{r_{\text{E}}^3}} = \sqrt{\frac{132.7 \times 10^9 \text{ km}^3/\text{s}^2}{(149.6 \times 10^6 \text{ km})^3}} = 1.991 \times 10^{-7} \text{rad/s}$$

$$T_{\text{wait}} = -\frac{2\phi_{arr} \pm N2\pi}{n_{\text{M}} - n_{\text{E}}}$$

Phase at arrival:

$$\boxed{\phi_{arr} = \pi - n_1 T_{12}}$$
$$\phi_{arr} = \pi - n_{\text{E}} T_{12}$$
$$\phi_{arr} = \pi - (1.991 \times 10^{-7} \text{ rad/s})(2.236 \times 10^7 \text{ s})$$
$$\phi_{arr} = -1.311 \text{ rad}$$

Problem: T_{wait} can be negative, so we add $\pm N2\pi$, s.t. N is the smallest integer that makes T_{wait} positive

$$n_{\text{M}} - n_{\text{E}} = -9.32 \times 10^{-8} \text{ rad/s}$$

$$T_{\text{wait}} = -\frac{-2.622 \pm N2\pi}{-9.32 \times 10^{-8} \text{ rad/s}}$$

$$T_{\text{wait}} = \frac{-2.622 \pm N2\pi}{9.32 \times 10^{-8} \text{ rad/s}}$$

Looking at the numerator, $N = +1$ would yield a positive value for time (but $N = 0$ would be negative)

$$T_{\text{wait}} = \frac{-2.62208 \pm (1)2\pi}{9.32 \times 10^{-8} \text{ rad/s}} = 3.928 \times 10^7 \text{ s}$$

$$T_{\text{wait}} = 454.6 \text{ days}$$

$$T_{\text{trip}} = T_{\text{wait}} + 2T_{12} = 454.6 \text{ days} + 2(258.8 \text{ days})$$

$$\boxed{T_{\text{trip}} = 972.3 \text{ days}}$$

Clarkson

Interplanetary Transfers

$\phi_{arr} = -1.311$ rad $\quad r_E = 149.6 \times 10^6$ km $\quad a_t = 188.75 \times 10^6$ km

$r_M = 227.9 \times 10^6$ km

$T_{\text{wait}} = 454.6$ days $\quad \mu_s = 132.7 \times 10^9$ km³/s² $\quad T_{12} = 2.236 \times 10^7$ s

$\boxed{T_{\text{trip}} = 972.3 \text{ days}}$ $\quad n_M - n_E = -9.32 \times 10^{-8}$ rad/s $\quad \boxed{T_{12} = 258.8 \text{ days}}$

Example Example Activity

Elon Musk plans to have SpaceX's Starship travel to Mars on an uncrewed mission in 2024. In your interview with SpaceX, Musk asks you to estimate the following in days: (a) the transfer time from Earth to Mars, (b) the time required for an Earth-Mars round trip, and (c) approximately how long will they have to wait for the next launch window if they miss the 2024 launch? Assume you are using an interplanetary Hohmann transfer and that the planetary orbits are circular.

Solution (c)

Synodic Period
(*time between launch windows*):

$$\boxed{T_{\text{syn}} = \frac{2\pi}{|n_1 - n_2|}}$$

$$T_{\text{syn}} = \frac{2\pi}{|n_E - n_M|} = \frac{2\pi}{9.32 \times 10^{-8} \text{ rad/s}} = 6.742 \times 10^7 \text{ s}$$

Mean orbital motion:

$$\boxed{n_M - n_E = -9.32 \times 10^{-8} \text{ rad/s}}$$

$$|n_E - n_M| = |n_M - n_E|$$

$$|n_E - n_M| = 9.32 \times 10^{-8} \text{ rad/s}$$

$$\boxed{T_{\text{syn}} = 780.3 \text{ days}}$$

$$T_{\text{syn}} \approx 26 \text{ months}$$