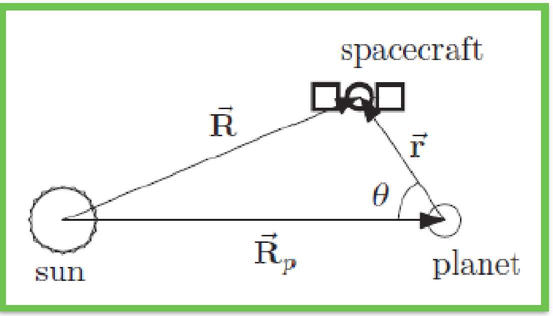
Interplanetary Transfers

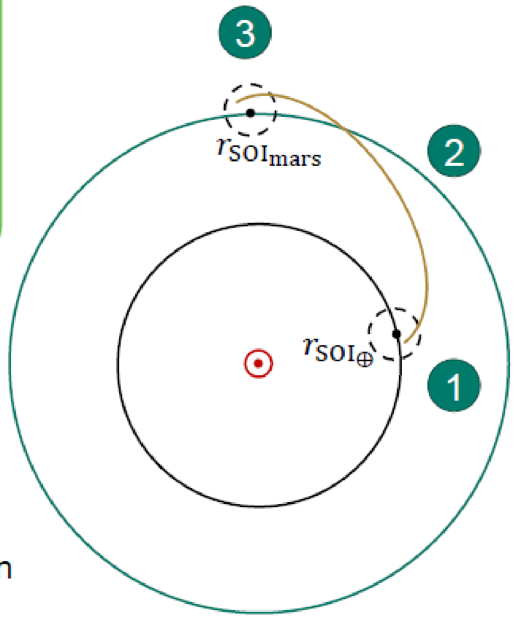


Interplanetary Hohmann Transfers (HT)

- Now that we know how to ensure planetary alignment, let's return to the main transfer problem
- But, recall, we are "patching" together three different conic sections

Example: Consider a spacecraft transferring from an orbit about the Earth to an orbit about Mars

1. In Earth's SOI → s/c is in a **hyperbolic geocentric orbit** (*while s/c escapes from Earth*)
2. Outside planetary SOIs→ s/c is in a **heliocentric elliptical orbit** (*travelling toward Mars*)
3. In Mars' SOI → s/c is in a **two-body orbit about Mars**



In general, our patch conditions at the boundary of the SOIs are:

$$\boxed{\vec{\mathbf{R}} = \vec{\mathbf{R}}_p + \vec{\mathbf{r}}}$$

$$\vec{\mathbf{V}} = \vec{\mathbf{V}}_p + \vec{\mathbf{v}}$$

$\vec{\mathbf{r}}$ = s/c position relative to the planet

$\vec{\mathbf{R}}$ = s/c position relative to the sun

$\vec{\mathbf{R}}_p$= planet position relative to the sun

$\vec{\mathbf{v}}$ = s/c velocity relative to the planet

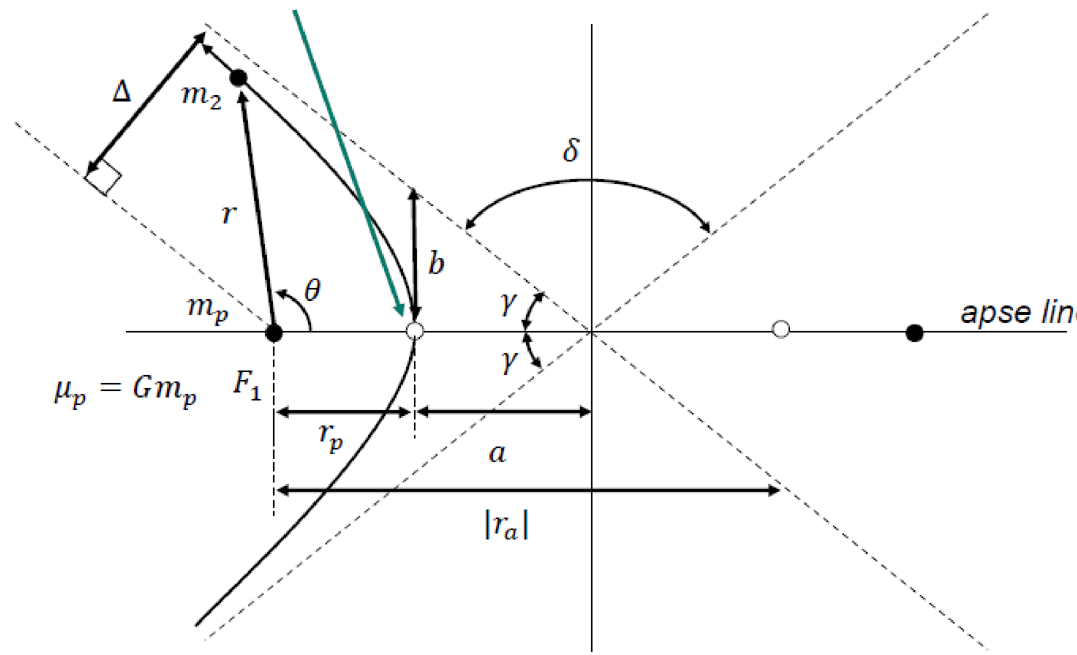
$\vec{\mathbf{V}}$ = s/c velocity relative to the sun

$\vec{\mathbf{V}}_p$= planet velocity relative to the sun

Interplanetary Transfers

Refresher on Hyperbolic Orbits

- Eccentricity, $e > 1$
- Orbital energy, $\varepsilon > 0$: $\boxed{\varepsilon = -\frac{\mu_p}{2a}}$
- Hyperbolic excess speed: $\boxed{v_\infty = \sqrt{-\frac{\mu_p}{a}}}$
- Periapsis distance ($\theta = 0$): $\boxed{r_p = a(1-e)}$
- Periapsis speed: $\boxed{v_p = \sqrt{v_\infty^2 + \frac{2\mu_p}{r_p}}}$
- Semiminor axis: $\boxed{b = ae\sin\gamma = a\sqrt{e^2-1}}$
- Turn angle: $\boxed{\delta = \pi - 2\gamma = \pi - 2\cos^{-1}\left(\frac{1}{e}\right)}$

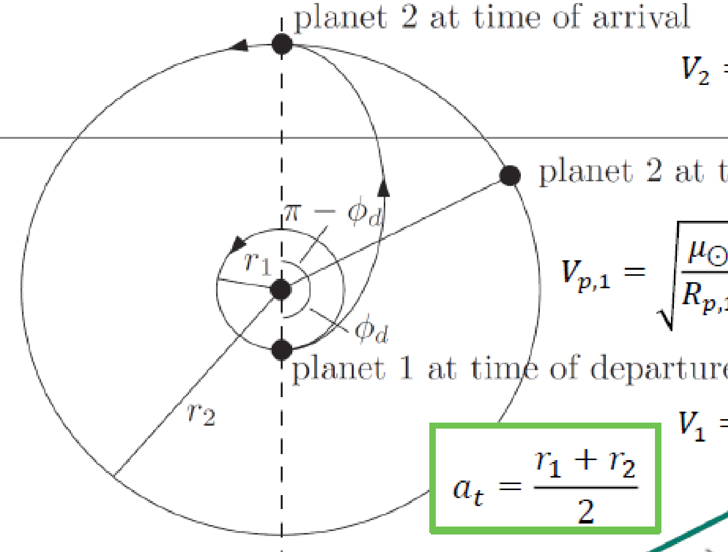


Now, let's take a look a the departure hyperbola

Interplanetary Transfers

Departure Hyperbola

- The spacecraft is in a "parking" orbit (temporary orbit) within Planet 1's SOI
- For interplanetary Hohmann transfer, the spacecraft velocity on the transfer orbit leaving planet 1 ($\vec{\mathbf{v}}_1$), must be tangent to the planet's velocity $\vec{\mathbf{V}}_{p,1}$



$$V_2 = \sqrt{\mu_\odot\left(\frac{2}{R_{p,2}} - \frac{1}{a_t}\right)}$$

$$V_{p,1} = \sqrt{\frac{\mu_\odot}{R_{p,1}}} \qquad V_{p,2} = \sqrt{\frac{\mu_\odot}{R_{p,2}}}$$

$$V_1 = \sqrt{\mu_\odot\left(\frac{2}{R_{p,1}} - \frac{1}{a_t}\right)}$$

$$\boxed{a_t = \frac{r_1 + r_2}{2}}$$

$$\boxed{\vec{\mathbf{V}} = \vec{\mathbf{V}}_p + \vec{\mathbf{v}}}$$

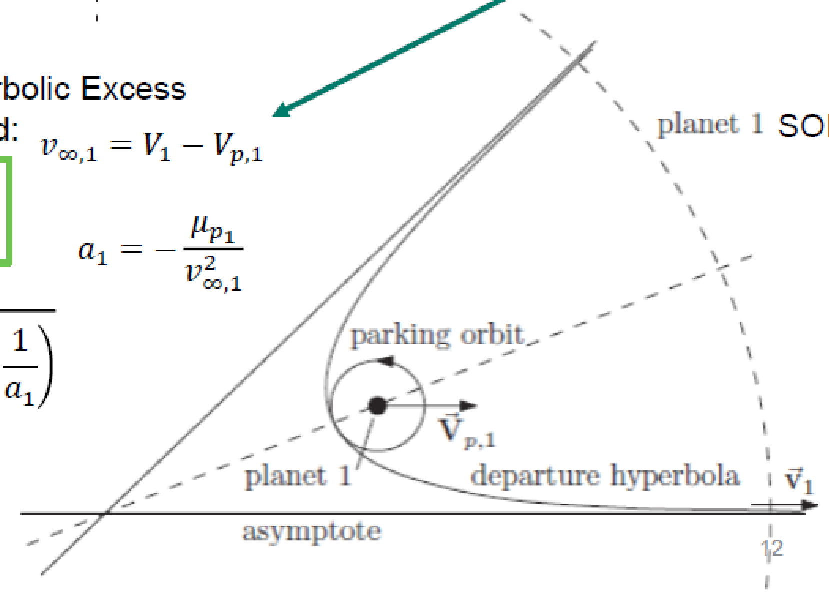
The s/c velocity relative to the sun after exiting the SOI is given by the patch condition: $\vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_{p,1} + \vec{\mathbf{v}}_1$ $\quad v_{\infty,1} = |\vec{\mathbf{v}}_1|$

Hyperbolic Excess Speed: $v_{\infty,1} = V_1 - V_{p,1}$

What is the required magnitude of the s/c heliocentric velocity (V_1)?

- It is the velocity at the start of the interplanetary HT ellipse
- This allows us to find the semimajor axis of the hyperbola, a_1

$$\boxed{v_\infty = \sqrt{-\frac{\mu_p}{a}}} \qquad a_1 = -\frac{\mu_{p_1}}{v_{\infty,1}^2}$$



Orbital Speed: (parking orbit) $\quad v_{park} = \sqrt{\frac{\mu_{p_1}}{r_{park}}}$

Orbital Speed at Periapsis of Departure Hyperbola: $\quad v_d = \sqrt{\mu_{p_1}\left(\frac{2}{r_{park}} - \frac{1}{a_1}\right)}$

- Δv to inject s/c into departure hyperbola:

$$\Delta v_{dep} = v_d - v_{park} = \sqrt{\mu_{p_1}\left(\frac{2}{r_{park}} - \frac{1}{a_1}\right)} - \sqrt{\frac{\mu_{p_1}}{r_{park}}}$$

Clarkson

Interplanetary Transfers

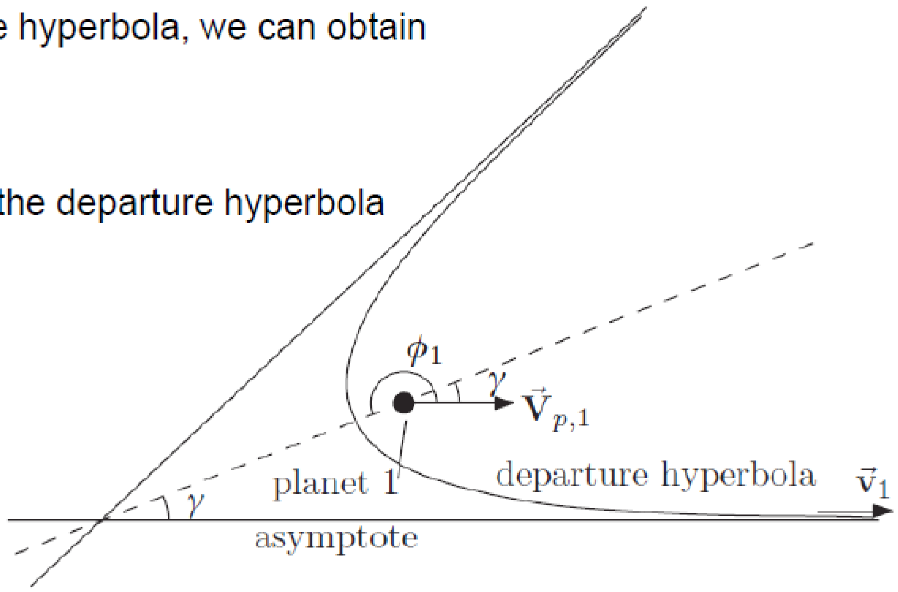
Departure Hyperbola

- We need to also determine location on the parking orbit for the transfer to the departure hyperbola (*so that the asymptote is parallel to the velocity vector of the planet*)
- Since we know r_{park} is the radius of periapsis of the departure hyperbola, we can obtain the eccentricity: $e_1 = 1 - \frac{r_{park}}{a_1}$ $\boxed{r_p = a(1-e)}$
- Now, we can find the phase, ϕ_1, of the location of transfer for the departure hyperbola relative to $\vec{\mathbf{V}}_{p,1}$:

$$\phi_1 = \pi + \gamma = \pi + \cos^{-1}\left(\frac{1}{e_1}\right)$$

N.B. The required phase above is for when $v_{\infty,1} > 0$, i.e., when the $V_1 > V_{p,1}$, which is the case when $R_{p,1} < R_{p,2}$.

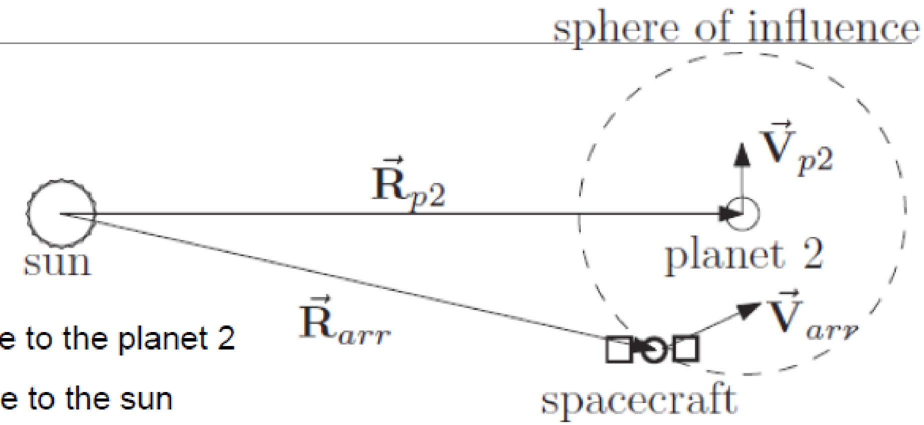
If $R_{p,1} > R_{p,2}$, then $v_{\infty,1} < 0$, and the departure asymptote must point in the opposite direction of $\mathbf{V}_{p,1}$ and $\phi_1 = \gamma$



We then move along in the heliocentric elliptic orbit until we arrive at planet 2

Let's take a look a the arrival hyperbola

Interplanetary Transfers



Arrival Hyperbola

- Let's start by setting up our patch conditions at the SOI

$$\vec{\mathbf{r}}_{arr} = \vec{\mathbf{R}}_{arr} - \vec{\mathbf{R}}_{p,2} \qquad \vec{\mathbf{v}}_{arr} = \vec{\mathbf{V}}_{arr} - \vec{\mathbf{V}}_{p,2}$$

$\vec{\mathbf{r}}_{arr}$ = s/c position relative to planet 2

$\vec{\mathbf{R}}_{arr}$ = s/c position relative to the sun

$\vec{\mathbf{R}}_{p,2}$= planet position relative to the sun

$\vec{\mathbf{v}}_{arr}$ = s/c velocity relative to the planet 2

$\vec{\mathbf{V}}_{arr}$ = s/c velocity relative to the sun

$\vec{\mathbf{V}}_{p,2}$= planet velocity relative to the sun

- We can now use this information to determine the parameters of the arrival hyperbola

Hyperbolic excess speed: $v_{\infty,arr} = |\vec{\mathbf{v}}_{arr}|$

$$\boxed{v_\infty = \sqrt{-\frac{\mu_p}{a}}}$$

Semimajor axis of the hyperbola: $a_2 = -\dfrac{\mu_{p_2}}{v_{\infty,arr}^2}$

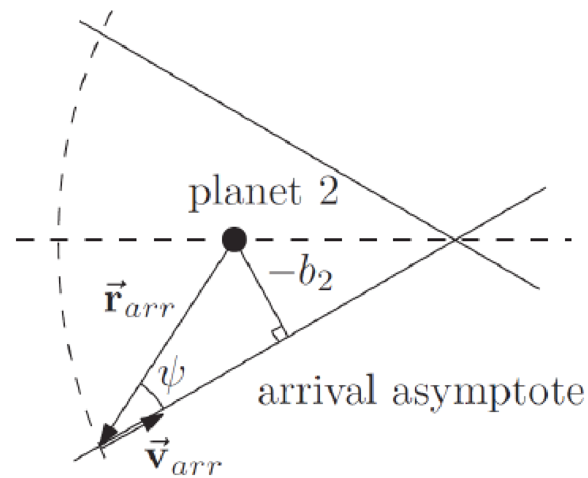
$$\boxed{b = a\sqrt{e^2 - 1}}$$

- Using a bit of geometry, we can show:

$$\psi = \cos^{-1}\left(\frac{-\vec{\mathbf{r}}_{arr} \cdot \vec{\mathbf{v}}_{arr}}{r_{arr} v_{\infty,arr}}\right) \longrightarrow b_2 = -r_{arr}\sin\psi$$

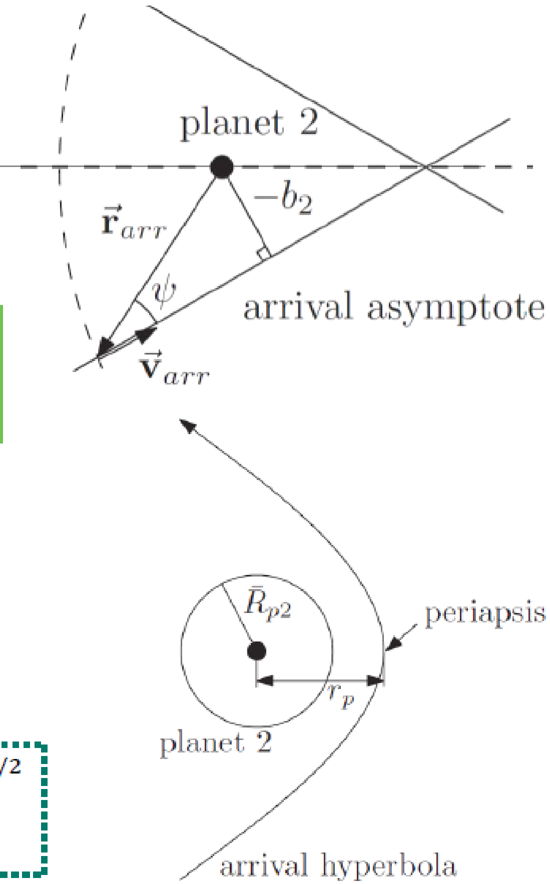
- With a_2 and b_2 we can find e_2:

$$e_2 = \sqrt{1 + \left(\frac{b_2}{a_2}\right)^2}$$



Clarkson N.B. $r_{arr} = r_{SOI,2}$

Interplanetary Transfers



Arrival Hyperbola

- Now, let's make sure we do not collide with the planet
- Determine the closest pass of the s/c w.r.t. planet 2

$$\boxed{r_p = a(1-e)} \qquad \boxed{e_2 = \sqrt{1 + \left(\frac{b_2}{a_2}\right)^2}}$$

Radius of periapsis of the arrival hyperbola: $r_{p2} = a_2\left[1 - \sqrt{1 + \left(\frac{b_2}{a_2}\right)^2}\right]$

- Remember, planet 2 has a finite size → assume spherical, with radius $\bar{R}_{p2}$

To avoid collision, $r_{p2} > \bar{R}_{p2}$

$$a_2\left[1 - \sqrt{1 + \left(\frac{b_2}{a_2}\right)^2}\right] > \bar{R}_{p2} \longrightarrow 1 - \frac{\bar{R}_{p2}}{a_2} < \sqrt{1 + \left(\frac{b_2}{a_2}\right)^2} \longrightarrow \boxed{|b_2| > |a_2|\left[\left(1 - \frac{\bar{R}_{p2}}{a_2}\right)^2 - 1\right]^{1/2}}$$

Arrival Options at Planet 2

1) Planetary Flyby → s/c enters and exits planet 2's SOI
2) Planetary Capture → s/c remains in orbit about planet 2

To avoid collision, this inequality must be satisfied.

This is useful, because b_2 can be adjusted by making small changes to the s/c velocity vector while it is still far from planet 2

Interplanetary Transfers

$$\delta = \pi - 2\gamma = \pi - 2\cos^{-1}\left(\frac{1}{e}\right)$$

Planetary Flyby

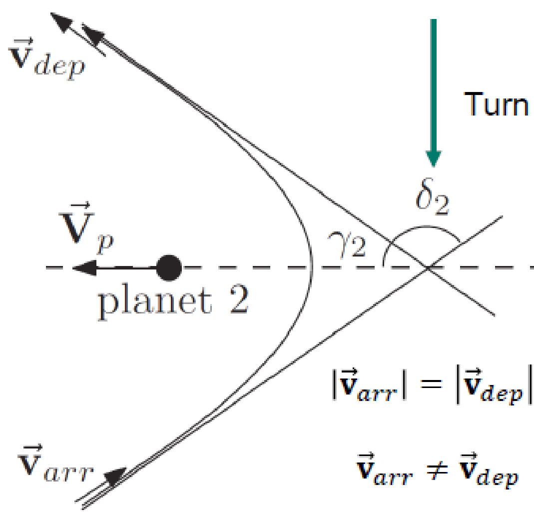
- We have already determined the parameters of the arrival hyperbola, namely, $a_2, b_2, v_{\infty,arr}$, and $\vec{\mathbf{v}}_{arr}$
- If we do not perform a maneuver, the s/c will exit the SOI of planet 2 along the other hyperbola asymptote

S/C velocity relative to the sun upon exiting the SOI:

$$\vec{\mathbf{V}}_{dep} = \vec{\mathbf{V}}_p + \vec{\mathbf{v}}_{dep}$$

$$\delta_2 = \pi - 2\gamma_2 = \pi - 2\cos^{-1}\left(\frac{1}{e_2}\right)$$

Turn Angle, δ



If we assume $r_{SOI,2}$ is negligible compared to the orbital radius of planet 2, i.e., $|\vec{\mathbf{R}}_{arr}| \approx |\vec{\mathbf{R}}_{dep}| \approx |\vec{\mathbf{R}}_{p2}|$, then the time of SOI transit is negligible ($\vec{\mathbf{V}}_p \approx$ constant)

$$|\vec{\mathbf{v}}_{arr}| = |\vec{\mathbf{v}}_{dep}| = v_{\infty,arr}$$

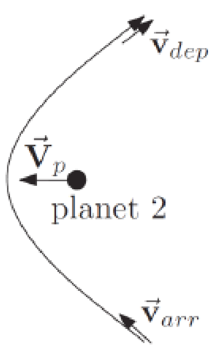
$$\vec{\mathbf{v}}_{arr} \neq \vec{\mathbf{v}}_{dep}$$

Planet 2 exerts a $\Delta\vec{\mathbf{v}}$ on the s/c

The resulting change in s/c heliocentric orbital energy is then: $\Delta\varepsilon = \frac{1}{2}\left(V_{dep}^2 - V_{arr}^2\right)$

change can be positive or negative depending on whether the s/c passes in front or behind the planet

Leading edge flyby
$\Delta\varepsilon < 0$
(in front of planet 2)

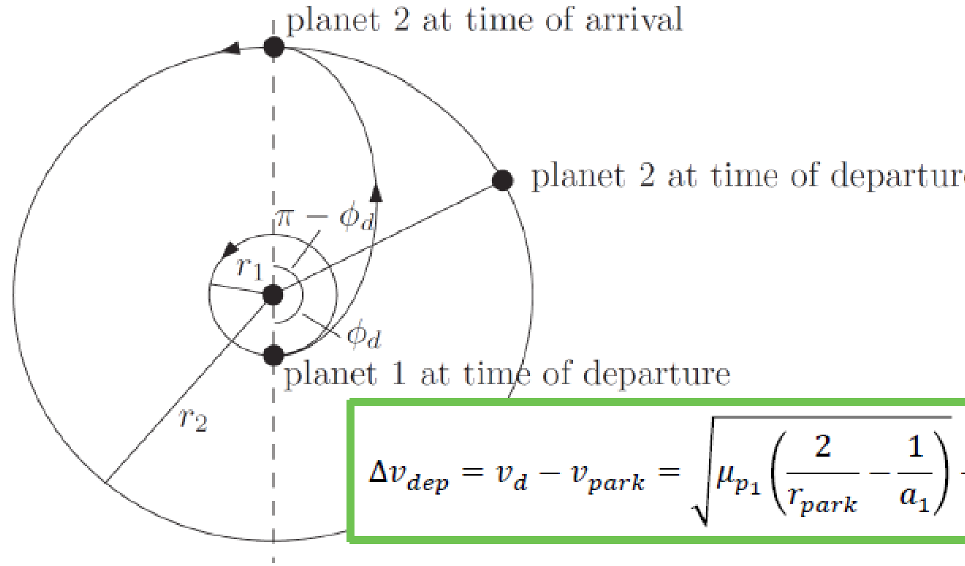


Increases s/c velocity w.r.t. the Sun →

Trailing edge flyby
$\Delta\varepsilon > 0$
(behind planet 2)

← Decreases s/c velocity w.r.t. the Sun

Interplanetary Transfers



Planetary Capture

- We have already determined the parameters of the arrival hyperbola, namely, $a_2, b_2, v_{\infty,arr}, \vec{\mathbf{v}}_{arr}$, and r_{p2}
- Now, we want to remain in the vicinity of planet 2, so we need to **perform a maneuver** to convert the arrival hyperbola to a bound orbit

$$\boxed{\Delta v_{dep} = v_d - v_{park} = \sqrt{\mu_{p_1}\left(\frac{2}{r_{park}} - \frac{1}{a_1}\right)} - \sqrt{\frac{\mu_{p_1}}{r_{park}}}}$$

Arrival Hyperbola to Circular Capture Orbit

- Determine the orbital speeds of the capture orbit and arrival hyperbola at periapsis

Orbital Speed (capture orbit)

$$v_{cap} = \sqrt{\frac{\mu_{p_2}}{r_{cap}}}$$

Orbital Speed at Periapsis of Arrival Hyperbola

$$v_{p_2} = \sqrt{v_\infty^2 + \frac{2\mu_{p_2}}{r_{p2}}}$$

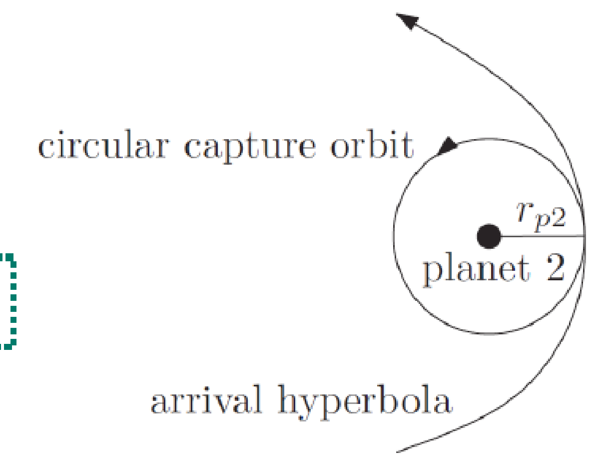
$$\boxed{v_p = \sqrt{v_\infty^2 + \frac{2\mu_p}{r_p}}}$$

- Δv required for capture is: $\Delta v_{cap} = v_{p2} - v_{cap} = \sqrt{v_\infty^2 + \frac{2\mu_{p2}}{r_{p2}}} - \sqrt{\frac{\mu_{p2}}{r_{cap}}}$

However, since we have control over b_2, we can also control r_{p2} and determine the radius for optimal capture by taking $(d\Delta v_{cap}/dr_{p2}) = 0$

$$r_{p2,opt} = \frac{2\mu_{p_2}}{v_{\infty,arr}^2}$$

$$\boxed{\Delta v_{cap,opt} = \frac{v_{\infty,arr}}{\sqrt{2}}}$$



Total Delta-v for the Interplanetary Hohmann Transfer: $\boxed{\Delta v = \Delta v_{dep} + \Delta v_{cap}}$

Interplanetary Transfers

Example Activity

As part of a preliminary study for an exploration trip to Mars, it has been decided that a Hohmann transfer will be used to travel from the Earth to Mars. You may assume that the orbits of the Earth and Mars are circular and lie in the same plane. The spacecraft is initially in a circular parking orbit around the Earth of radius $r_{park} = 100{,}000$ km. It is desired to place the spacecraft in a circular orbit around Mars of radius $r_{cap} = 50{,}000$ km. You know: $R_{earth} = 149.6 \times 10^6$ km, $R_{mars} = 227.9 \times 10^6$ km, $\mu_{sun} = 1.327 \times 10^{11}\ \mathrm{km^3/s^2}$, $\mu_{earth} = 3.986 \times 10^5\ \mathrm{km^3/s^2}$, $\mu_{mars} = 4.305 \times 10^4\ \mathrm{km^3/s^2}$.

(a) Compute the semi-major axis of the Hohmann transfer orbit.

$$\boxed{a_t = \frac{r_1 + r_2}{2}} \qquad a_t = \frac{R_{earth} + R_{mars}}{2} = 188.8 \times 10^6\ \mathrm{km}$$

(b) Compute the time-of-flight for the Hohmann transfer orbit.

$$\boxed{TOF = \pi\sqrt{\frac{a_t^3}{\mu}}} \qquad TOF = \pi\sqrt{\frac{a_t^3}{\mu_{sun}}} = 2.237 \times 10^7\,\mathrm{s} = 258.9\ \mathrm{days}$$

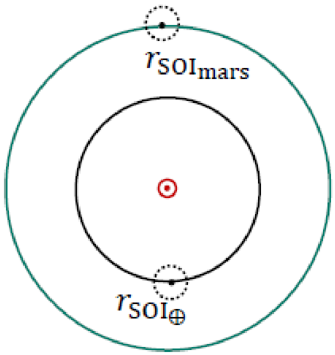
Interplanetary Transfers

$TOF = 2.237 \times 10^7 \text{s}$

Example Activity

As part of a preliminary study for an exploration trip to Mars, it has been decided that a Hohmann transfer will be used to travel from the Earth to Mars. You may assume that the orbits of the Earth and Mars are circular and lie in the same plane. The spacecraft is initially in a circular parking orbit around the Earth of radius $r_{park} = 100{,}000$ km. It is desired to place the spacecraft in a circular orbit around Mars of radius $r_{cap} = 50{,}000$ km. You know: $R_{earth} = 149.6 \times 10^6$ km, $R_{mars} = 227.9 \times 10^6$ km, $\mu_{sun} = 1.327 \times 10^{11}\ \text{km}^3/\text{s}^2$, $\mu_{earth} = 3.986 \times 10^5\ \text{km}^3/\text{s}^2$, $\mu_{mars} = 4.305 \times 10^4\ \text{km}^3/\text{s}^2$.

(c) Assuming that the Earth, Mars the Sun lie on the same line at $t = 0$, with Earth and Mars on opposite sides of the Sun, compute the time t in days of the required departure from Earth.



- Initial phase between Earth and Mars is: $\phi_0 = \pi \text{ rad} = 180°$

$$\boxed{\theta(t) - \theta(t_0) = (n_2 - n_1)t} \qquad \phi(t) = \phi_0 + (n_{mars} - n_{earth})t$$

$$\boxed{n \equiv \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}}} \qquad n_{\text{mars}} = \sqrt{\frac{\mu_{sun}}{R_{\text{mars}}^3}} = \sqrt{\frac{132.7 \times 10^9\ \text{km}^3/\text{s}^2}{(227.9 \times 10^6\ \text{km})^3}} = 1.059 \times 10^{-7}\ \text{rad/s}$$

$$n_{\text{earth}} = \sqrt{\frac{\mu_{sun}}{R_{\text{earth}}^3}} = \sqrt{\frac{132.7 \times 10^9\ \text{km}^3/\text{s}^2}{(149.6 \times 10^6\ \text{km})^3}} = 1.991 \times 10^{-7}\ \text{rad/s}$$

- Since $\phi_d < \phi_0$ and $n_{mars} - n_{earth} < 0$, then $\phi(t)$ decreasing

$$\boxed{\phi_d = \pi - n_2 T_{12}} \qquad \phi_d = \pi - n_{mars} T_{em}$$

$$= 0.7739 \text{ rad} = 44.34°$$

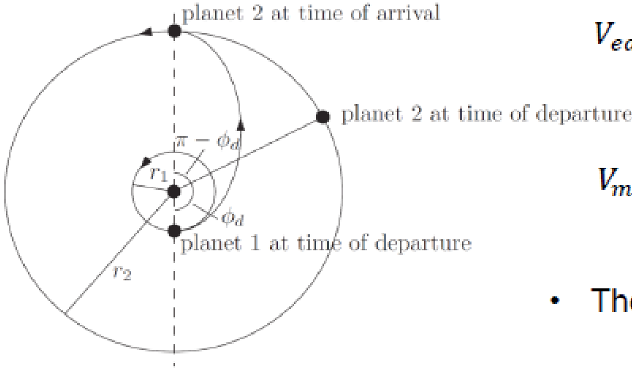
$$\boxed{\phi = \phi_d + (n_2 - n_1)t} \qquad t = \frac{\phi_0 - \phi_d}{n_2 - n_1} = 2.539 \times 10^7 \text{s} = 293.9 \text{ days}$$

Interplanetary Transfers $a_t = 188.8 \times 10^6$ km

Example Activity

As part of a preliminary study for an exploration trip to Mars, it has been decided that a Hohmann transfer will be used to travel from the Earth to Mars. You may assume that the orbits of the Earth and Mars are circular and lie in the same plane. The spacecraft is initially in a circular parking orbit around the Earth of radius $r_{park} = 100{,}000$ km. It is desired to place the spacecraft in a circular orbit around Mars of radius $r_{cap} = 50{,}000$ km. You know: $R_{earth} = 149.6 \times 10^6$ km, $R_{mars} = 227.9 \times 10^6$ km, $\mu_{sun} = 1.327 \times 10^{11}\ \text{km}^3/\text{s}^2$, $\mu_{earth} = 3.986 \times 10^5\ \text{km}^3/\text{s}^2$, $\mu_{mars} = 4.305 \times 10^4\ \text{km}^3/\text{s}^2$.

(d) Compute the required hyperbolic excess speed $v_{\infty,dep}$ upon exiting the Earth's SOI, and the hyperbolic excess speed $v_{\infty,arr}$ upon entering Mars' SOI.



$$V_{earth} = \sqrt{\frac{\mu_{sun}}{R_{earth}}} = 29.78\ \text{km/s} \qquad V_{dep} = \sqrt{\mu_{sun}\left(\frac{2}{R_{earth}} - \frac{1}{a_t}\right)} = 32.73\ \text{km/s}$$

$$V_{mars} = \sqrt{\frac{\mu_{sun}}{R_{mars}}} = 24.13\ \text{km/s} \qquad V_{arr} = \sqrt{\mu_{sun}\left(\frac{2}{R_{mars}} - \frac{1}{a_t}\right)} = 21.48\ \text{km/s}$$

- The hyperbolic excess speed upon departing Earth's SOI: $\boxed{v_{\infty,1} = V_1 - V_{p,1}}$

$$v_{\infty,dep} = V_{dep} - V_{earth} = 2.945\ \text{km/s}$$

- The hyperbolic excess speed upon entering Mars' SOI: $v_{\infty,arr} = V_{mars} - V_{arr} = 2.649\ \text{km/s}$

Interplanetary Transfers

$v_{\infty,dep} = 2.945 \text{ km/s}$

Example Activity

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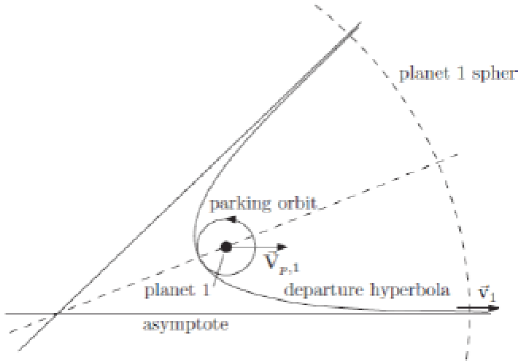
(e) Determine the location and magnitude of the Δv_{dep} required for Earth departure.

- Depart the circular parking orbit about Earth (perigee point of departure hyperbola):



Orbital speed at perigee of departure hyperbola:

$$\boxed{v_p = \sqrt{v_\infty^2 + \frac{2\mu_p}{r_p}}} \qquad v_{e,dep} = \sqrt{v_{\infty,dep}^2 + \frac{2\mu_{earth}}{r_{park}}} = 4.078 \text{ km/s}$$

Orbital speed for circular parking orbit:

$$\boxed{v_{park} = \sqrt{\frac{\mu_{p_1}}{r_{park}}}} \qquad v_{park} = \sqrt{\frac{\mu_{earth}}{r_{park}}} = 1.996 \text{ km/s}$$

Required delta-v is given by:

$$\boxed{\Delta v_{dep} = v_d - v_{park} = \sqrt{\mu_{p_1}\left(\frac{2}{r_{park}} - \frac{1}{a_1}\right)} - \sqrt{\frac{\mu_{p_1}}{r_{park}}}}$$

Now find the location…

$$\Delta v_{dep} = v_{e.dep} - v_{park} = 2.083 \text{ km/s}$$

Departing in the same direction as Earth's velocity vector, since $V_{dep} > V_{earth}$

Clarkson

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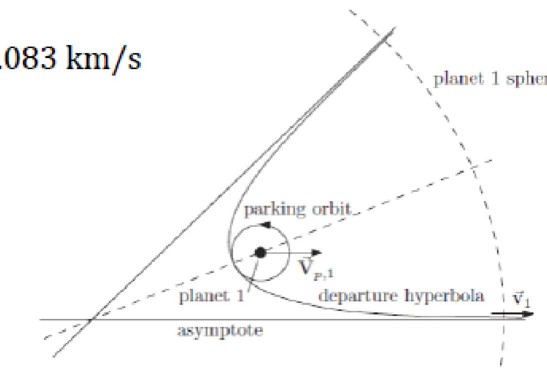
Interplanetary Transfers

$v_{\infty,dep} = 2.945 \text{ km/s}$

Example Activity

As part of a preliminary study for an exploration trip to Mars, it has been decided that a Hohmann transfer will be used to travel from the Earth to Mars. You may assume that the orbits of the Earth and Mars are circular and lie in the same plane. The spacecraft is initially in a circular parking orbit around the Earth of radius $r_{park} = 100{,}000 \text{ km}$. It is desired to place the spacecraft in a circular orbit around Mars of radius $r_{cap} = 50{,}000 \text{ km}$. You know: $R_{earth} = 149.6 \times 10^6 \text{ km}$, $R_{mars} = 227.9 \times 10^6 \text{ km}$, $\mu_{sun} = 1.327 \times 10^{11} \text{ km}^3/\text{s}^2$, $\mu_{earth} = 3.986 \times 10^5 \text{ km}^3/\text{s}^2$, $\mu_{mars} = 4.305 \times 10^4 \text{ km}^3/\text{s}^2$.

(e) Determine the location and magnitude of the Δv_{dep} required for Earth departure. $\Delta v_{dep} = 2.083 \text{ km/s}$



- Now find the required phase of tangential impulse, starting with determining the a_{dep}

$$\boxed{a_1 = -\frac{\mu_{p_1}}{v_{\infty,1}^2}} \qquad a_{dep} = -\frac{\mu_{earth}}{v_{\infty,dep}^2} = -4.597 \times 10^4 \text{ km}$$

Find the eccentricity of the departure hyperbola: $\boxed{e_1 = 1 - \frac{r_{park}}{a_1}}$ $\quad e_{dep} = 1 - \frac{r_{park}}{a_{dep}} = 3.175$

Finally, we can find the phase of the tangential impulse: $\boxed{\phi_1 = \pi + \gamma = \pi + \cos^{-1}\left(\frac{1}{e_1}\right)}$

$$\phi_{dep} = \pi + \cos^{-1}\left(\frac{1}{e_{dep}}\right) = 4.392 \text{ rad} = 251.6°$$

Interplanetary Transfers

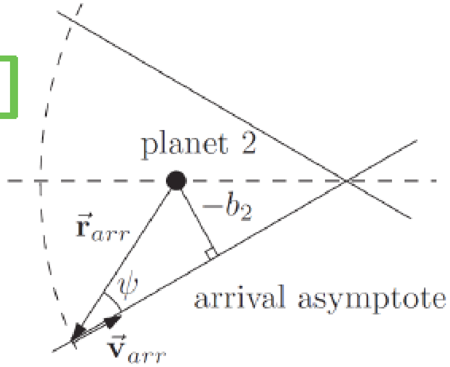
$\Delta v_{dep} = 2.083\ \mathrm{km/s}$ $\quad v_{\infty,arr} = 2.649\ \mathrm{km/s}$

Example Activity

As part of a preliminary study for an exploration trip to Mars, it has been decided that a Hohmann transfer will be used to travel from the Earth to Mars. You may assume that the orbits of the Earth and Mars are circular and lie in the same plane. The spacecraft is initially in a circular parking orbit around the Earth of radius $r_{park} = 100{,}000$ km. It is desired to place the spacecraft in a circular orbit around Mars of radius $r_{cap} = 50{,}000$ km. You know: $R_{earth} = 149.6 \times 10^6$ km, $R_{mars} = 227.9 \times 10^6$ km, $\mu_{sun} = 1.327 \times 10^{11}\ \mathrm{km^3/s^2}$, $\mu_{earth} = 3.986 \times 10^5\ \mathrm{km^3/s^2}$, $\mu_{mars} = 4.305 \times 10^4\ \mathrm{km^3/s^2}$.

(f) Determine the required arrival hyperbola asymptote offset $-b$, and compute the Δv_{arr} required for Mars capture.

- Start with finding a_{arr}: $\boxed{a_2 = -\dfrac{\mu_{p2}}{v_{\infty,arr}^2}}$ $\quad a_{\text{arr}} = -\dfrac{\mu_{mars}}{v_{\infty,arr}^2} = -6.135 \times 10^3\ \mathrm{km}$ $\quad \boxed{r_p = a(1-e)}$



- We have the desired capture orbit, r_{cap}, so we have the radius of periapsis, and can find the eccentricity: $e_{arr} = 1 - \dfrac{r_{cap}}{a_{arr}} = 9.149$
- Now find the asymptote $-b$: $\boxed{b = a\sqrt{e^2 - 1}}$ $\quad b = a_{arr}\sqrt{e_{arr}^2 - 1} = -55{,}800\ \mathrm{km}$

$$-b = 55{,}800\ \mathrm{km}$$

- Find the orbital speed at periapsis of the arrival hyperbola and of the circular capture orbit: $\boxed{\Delta v_{cap} = v_{p2} - v_{cap}}$

$$\boxed{v_{p_2} = \sqrt{v_\infty^2 + \frac{2\mu_{p_2}}{r_{p2}}}} \quad v_{p,arr} = \sqrt{v_{\infty,arr}^2 + \frac{2\mu_{mars}}{r_{cap}}} = 2.956\ \mathrm{km/s} \quad \boxed{v_{cap} = \sqrt{\frac{\mu_{p2}}{r_{cap}}}} \quad v_{cap} = \sqrt{\frac{\mu_{mars}}{r_{cap}}} = 0.9279\ \mathrm{km/s}$$

$$\Delta v_{cap} = v_{p,arr} - v_{cap}$$
$$\Delta v_{cap} = 2.028\ \mathrm{km/s}$$

Clarkson

Interplanetary Transfers

$\Delta v_{dep} = 2.083 \text{ km/s}$ $\Delta v_{cap} = 2.028 \text{ km/s}$

Example Activity

As part of a preliminary study for an exploration trip to Mars, it has been decided that a Hohmann transfer will be used to travel from the Earth to Mars. You may assume that the orbits of the Earth and Mars are circular and lie in the same plane. The spacecraft is initially in a circular parking orbit around the Earth of radius $r_{park} = 100{,}000$ km. It is desired to place the spacecraft in a circular orbit around Mars of radius $r_{cap} = 50{,}000$ km. You know: $R_{earth} = 149.6 \times 10^6$ km, $R_{mars} = 227.9 \times 10^6$ km, $\mu_{sun} = 1.327 \times 10^{11} \text{ km}^3/\text{s}^2$, $\mu_{earth} = 3.986 \times 10^5 \text{ km}^3/\text{s}^2$, $\mu_{mars} = 4.305 \times 10^4 \text{ km}^3/\text{s}^2$.

(f) Compute the total Δv for the trip.

$$\boxed{\Delta v = \Delta v_{dep} + \Delta v_{cap}}$$

$$\Delta v = \Delta v_{dep} + \Delta v_{cap} = 4.111 \text{ km/s}$$