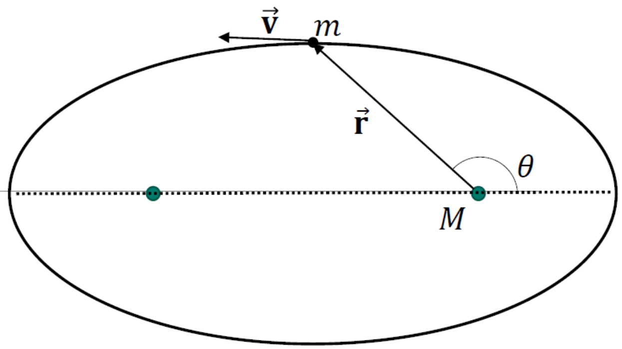
Orbital Maneuvers



- *Recall*, our orbit is fully specified with knowledge of $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$ at ay given time, t
- Orbital maneuvers are used to move a spacecraft from one orbit to another

  e.g., coplanar maneuvers, plane-change maneuvers, orbit phasing, rendezvous, gravity assists

In order to move between orbits, a thrust must be applied. There are two main maneuver types: **impulsive** or **low-thrust**

- **Impulsive maneuvers** change an orbit using one or more short duration bursts, these impulsive thrusts are treated as instantaneous changes to the velocity vector

  Examples include: single-impulse transfer, Hohmann transfer, bi-elliptic transfer

- **Low-thrust maneuvers** change an orbit by providing a small amount of thrust over long intervals, often using continual and/or constant throughout the maneuver

The most general type of orbital maneuvers rely on solving **Lambert's Problem**, which allows you to determine an orbit from two position vectors and the time taken to travel between them

We'll focus on **impulsive maneuvers** in this section

Clarkson

Orbital Maneuvers

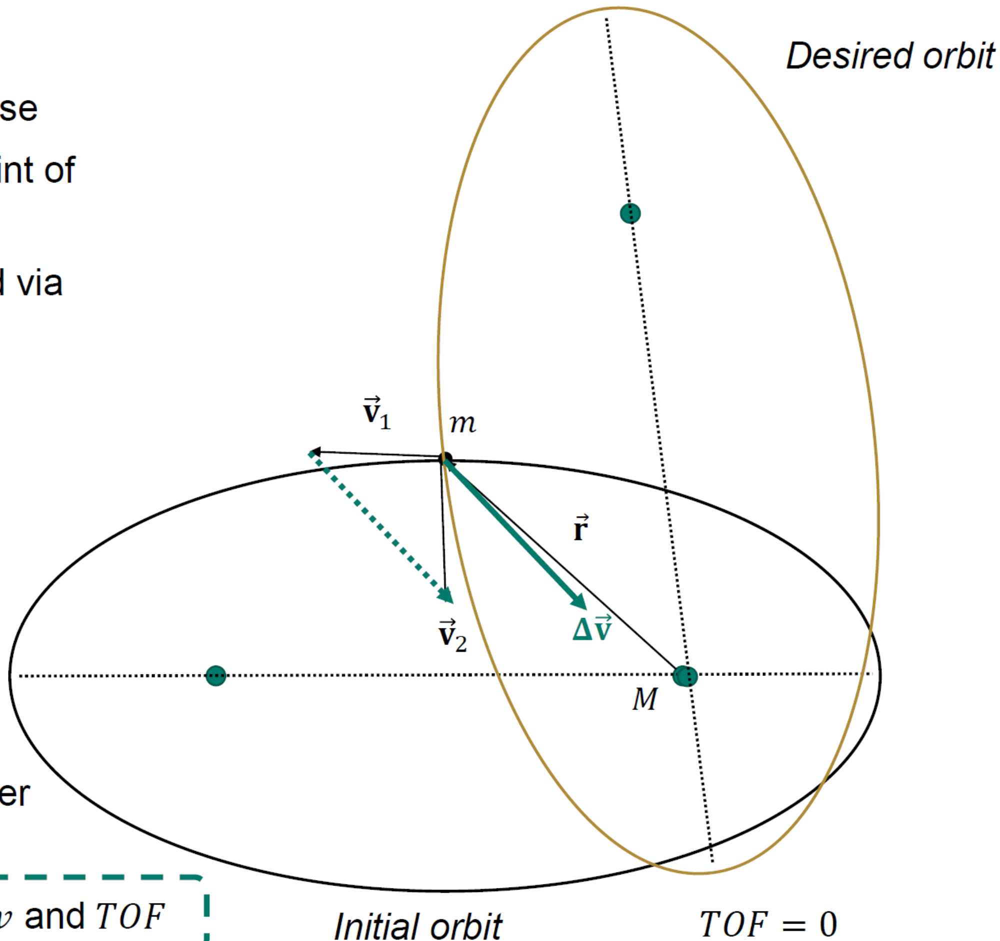
Single Impulse Maneuvers

- Simplest type of maneuver, requires only one thrust impulse
- Two orbits (initial and desired) must share at least one point of intersection, where the thrust will be applied
- Instantaneous change in the velocity vector, $\mathbf{\Delta\vec{v}}$, is applied via an impulsive thrust to change $\vec{\mathbf{v}}_1$ to $\vec{\mathbf{v}}_2$
- The change in velocity is given by: $\mathbf{\Delta\vec{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$
- The magnitude of the velocity change, "delta-v" is given by:

$$\Delta v = ||\mathbf{\Delta\vec{v}}||$$

Two important maneuver considerations:

- Delta-v (Δv): a measure of fuel consumption, minimum fuel maneuvers require minimum Δv
- *Time of Flight* (TOF): time required to complete a maneuver



Usually, a compromise is required between Δv and TOF

Clarkson

Orbital Maneuvers

Lecture 6

vis-viva equation
(“living force”)

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Circular Orbit:

$e = 0$ $\qquad v_{circ} = \sqrt{\frac{\mu}{r}}$

Coplanar Maneuvers

- In coplanar maneuvers, the only orbital elements that change are: a, e, and ω (size, shape, and orientation of orbit in plane)
- We will consider all tangential thrusts when $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$ are perpendicular (change of magnitude, not direction)

Consider: Circular to Elliptical, Elliptical to Circular, and Hohmann Transfers

Circular to Elliptical Transfer

- Transfer from orbit 1 (circular) to orbit 2 (elliptical)

Orbital speed (orbit 1)

$$v_1 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} = \sqrt{\frac{\mu}{r_1}}$$

Orbital speed (orbit 2)

$$v_2 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)}$$

If $a > r_1$, then $v_2 > v_1$

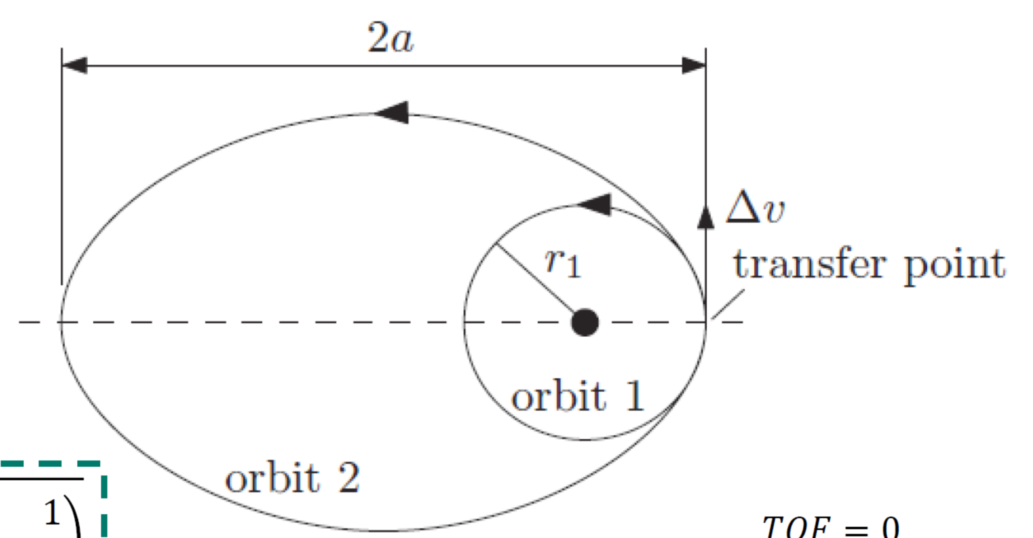
$$\Delta v = v_2 - v_1$$

$$\boxed{\Delta v = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} - \sqrt{\frac{\mu}{r_1}}}$$

If $a < r_1$, then $v_1 > v_2$

$$\Delta v = v_1 - v_2$$

$$\boxed{\Delta v = \sqrt{\frac{\mu}{r_1}} - \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)}}$$



$TOF = 0$

Clarkson

Orbital Maneuvers

Lecture 6

vis-viva equation
("living force")

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Circular Orbit:

$e = 0$ $\qquad v_{circ} = \sqrt{\frac{\mu}{r}}$

Coplanar Maneuvers

- In coplanar maneuvers, the only orbital elements that change are: a, e, and ω (size, shape, and orientation of orbit in plane)
- We will consider all tangential thrusts when $\vec{\mathbf{r}}$ and $\vec{\mathbf{v}}$ are perpendicular (change of magnitude, not direction)

Consider: Circular to Elliptical, Elliptical to Circular, and Hohmann Transfers

Elliptical to Circular Transfer

- Transfer from orbit 1 (elliptical) to orbit 2 (circular)

Orbital speed (orbit 1)

$$v_1 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}$$

Orbital speed (orbit 2)

$$v_2 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} = \sqrt{\frac{\mu}{r_2}}$$

If $a > r_2$, then $v_1 > v_2$

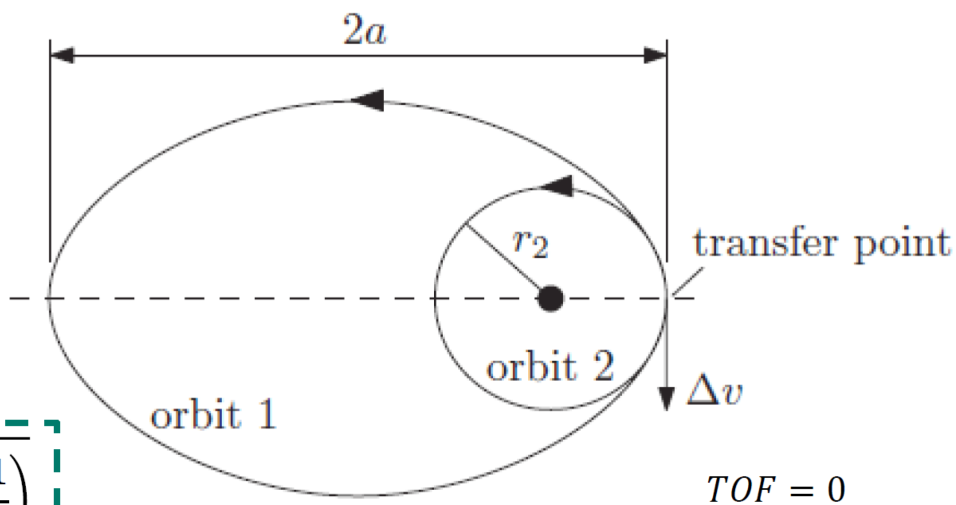
$$\Delta v = v_1 - v_2$$

$$\boxed{\Delta v = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} - \sqrt{\frac{\mu}{r_2}}}$$

If $a < r_2$, then $v_2 > v_1$

$$\Delta v = v_2 - v_1$$

$$\boxed{\Delta v = \sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}}$$



$TOF = 0$

Clarkson

Orbital Maneuvers

Quick Activity

Example

A satellite is in a circular orbit about the Earth at an altitude of $429\ \mathrm{km}$ and needs to be placed into an elliptical orbit with an apogee distance $7500\ \mathrm{km}$. Find: (a) the Δv required for the maneuver, and (b) the time of flight for transfer.

You are given: $R_{\oplus} = 6371\ \mathrm{km}$ and $\mu_{\oplus} = 398\,600\ \mathrm{km^3/s^2}$.

(a) Circular to Elliptical Transfer

- Transfer from orbit 1 (circular) to orbit 2 (elliptical)

$r_1 = R_{\oplus} + h_1 = 6371\ \mathrm{km} + 429\ \mathrm{km}$

$r_1 = 6800\ \mathrm{km}$

$r_{2a} = 7500\ \mathrm{km}$

$r_{2p} = r_1 = 6800\ \mathrm{km}$

$2a = r_{2a} + r_{2p} = 14\,300\ \mathrm{km}$

$a = 7150\ \mathrm{km}$

(a) Circular to Elliptical Transfer

- Transfer from orbit 1 (circular) to orbit 2 (elliptical)

$r_1 = R_{\oplus} + h_1 = 6371\ \mathrm{km} + 429\ \mathrm{km}$

$r_1 = 6800\ \mathrm{km}$

$r_{2a} = 7500\ \mathrm{km}$

$r_{2p} = r_1 = 6800\ \mathrm{km}$

$2a = r_{2a} + r_{2p} = 14\,300\ \mathrm{km}$

$a = 7150\ \mathrm{km}$

If $a > r_1$, then $v_2 > v_1$

$$\boxed{\Delta v = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} - \sqrt{\frac{\mu}{r_1}}}$$

$$\Delta v = \sqrt{\left(398\,600\frac{\mathrm{km^3}}{\mathrm{s^2}}\right)\left(\frac{2}{6800\ \mathrm{km}} - \frac{1}{7150\ \mathrm{km}}\right)} - \sqrt{\frac{398\,600\frac{\mathrm{km^3}}{\mathrm{s^2}}}{6800\ \mathrm{km}}}$$

$$\Delta v = 7.841\ \mathrm{km/s} - 7.656\ \mathrm{km/s}$$

$$\boxed{\Delta v = 0.185\ \mathrm{km/s}}$$

(b) Single-impulse maneuver, so: $TOF = 0$

Orbital Maneuvers

Hohmann Transfer

- Transfer between two concentric circular orbits of radius r_1 and r_2
- Both orbits have the same direction of motion
  i.e., both *prograde* or both *retrograde*
- There is no point in common, *so single impulse transfer not possible*

Consists of two tangential maneuvers in sequence:

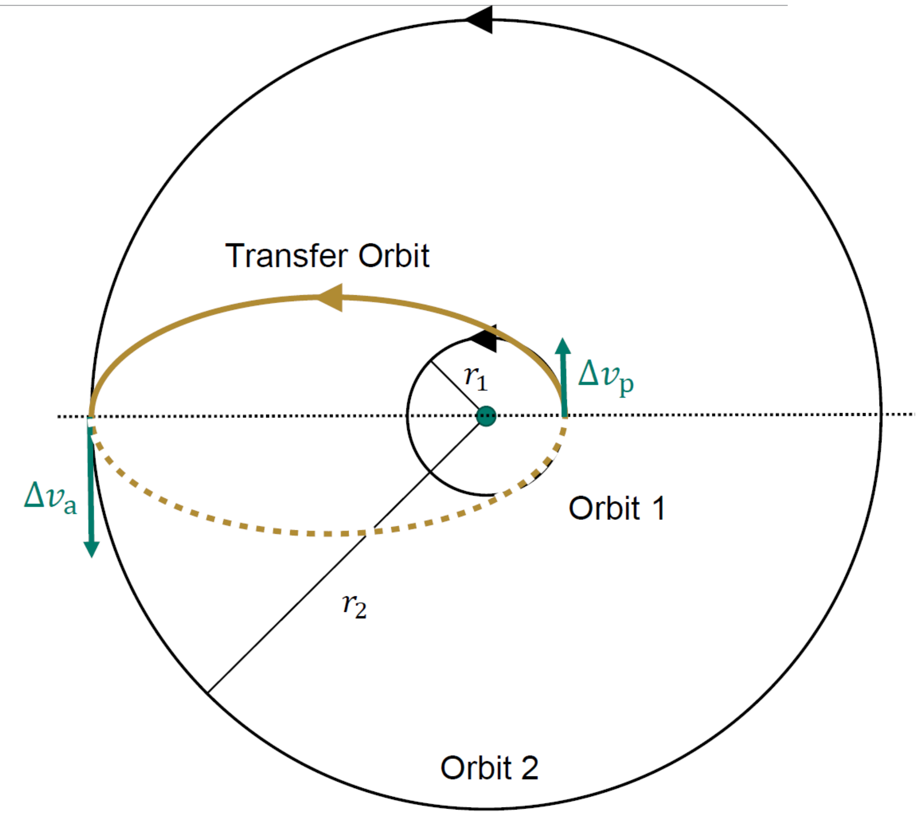
1. Circular to elliptical transfer
2. Elliptical to circular transfer

First impulse → changes orbit to an elliptic transfer orbit that is tangent to the circular orbit at its periapsis (or apoapsis, if $r_1 > r_2$)

Second impulse → applied at apoapsis (or periapsis, if $r_1 > r_2$), and circularizes the elliptic transfer orbit

*Hohmann transfer is the minimum Δv double-impulse transfer maneuver

(if $r_2/r_1 < 11.94$, H.T. is optimal)



Clarkson

Orbital Maneuvers

Direction Δv's are applied depends on the relative size of the two circular orbits

Hohmann Transfer

- We can see that the semimajor axis of the transfer orbit is given by:

$$a_t = \frac{r_1 + r_2}{2}$$

In the case shown, $r_2 > r_1$

(i.e., first impulse at periapsis, Δv_p, second impulse at apoapsis, Δv_a)

Orbital speeds at transfer points:

Orbit 1: $$v_1 = \sqrt{\frac{\mu}{r_1}}$$

Transfer Orbit at Periapsis: $$v_{tp} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_t}\right)}$$

Circular to Elliptical Transfer:

$a_t > r_1$, so $v_{tp} > v_1$

$$\Delta v_p = v_{tp} - v_1 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_t}\right)} - \sqrt{\frac{\mu}{r_1}}$$

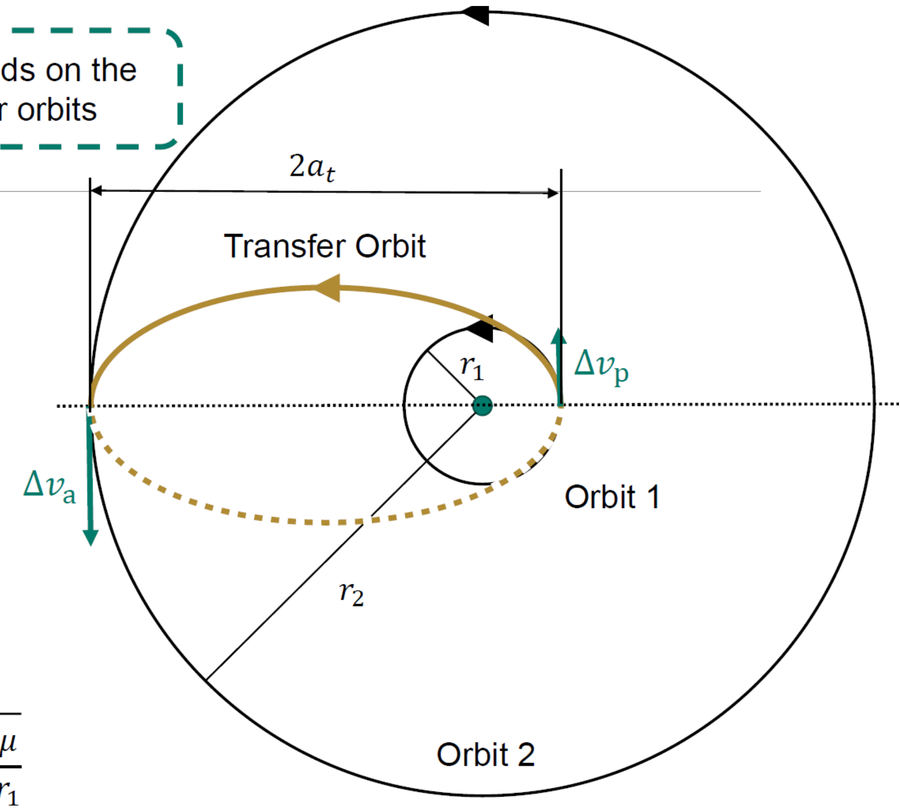
Transfer Orbit at Apoapsis: $$v_{ta} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_t}\right)}$$

Orbit 2: $$v_2 = \sqrt{\frac{\mu}{r_2}}$$

Elliptical to Circular Transfer:

$a_t < r_2$, so $v_2 > v_{ta}$

$$\Delta v_a = v_2 - v_{ta} = \sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_t}\right)}$$



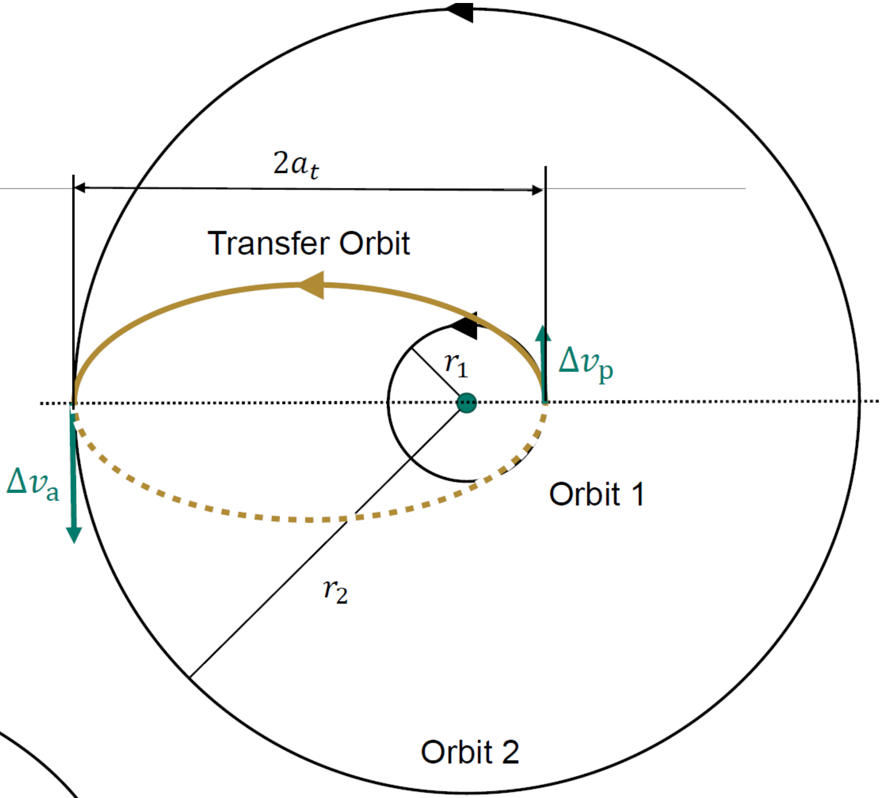
Total velocity change for the maneuver:

$$\Delta v = |\Delta v_p| + |\Delta v_a|$$

$$\boxed{\Delta v = \left|\sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_t}\right)} - \sqrt{\frac{\mu}{r_1}}\right| + \left|\sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_t}\right)}\right|}$$

Clarkson

Orbital Maneuvers



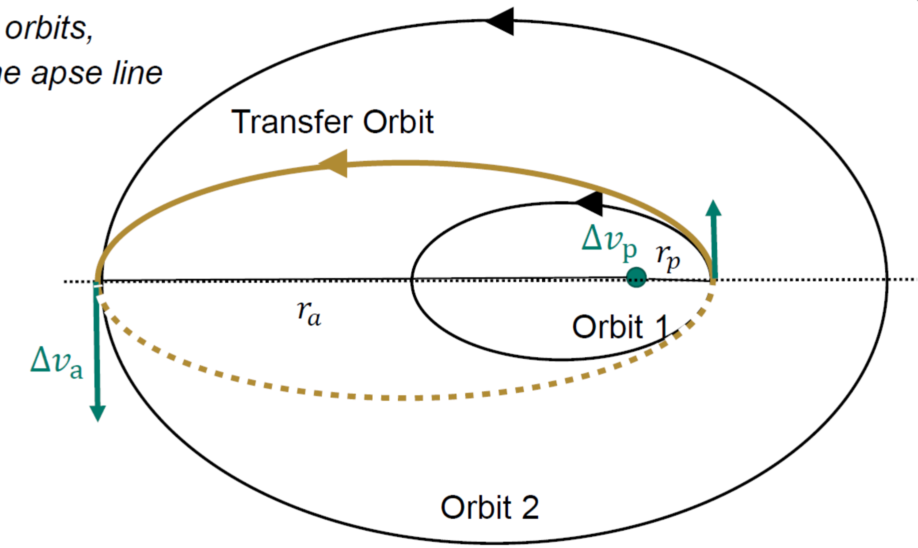
Hohmann Transfer

- With the transfer orbit, Time of Flight is no longer zero:

Recall, period for an ellipse: $\boxed{T = 2\pi\sqrt{\frac{a^3}{\mu}}}$

Time of Flight: half of the period of the transfer ellipse: $\boxed{TOF = \pi\sqrt{\frac{a_t^3}{\mu}}}$

Hohmann Transfer's are also possible between co-planar elliptical orbits, provided they have the same apse line



Clarkson