



PLATFORM DELIVERY: A GAME- THEORETIC ANALYSIS OF A NEW DELIVERY MODEL IN THE SHARING ECONOMY

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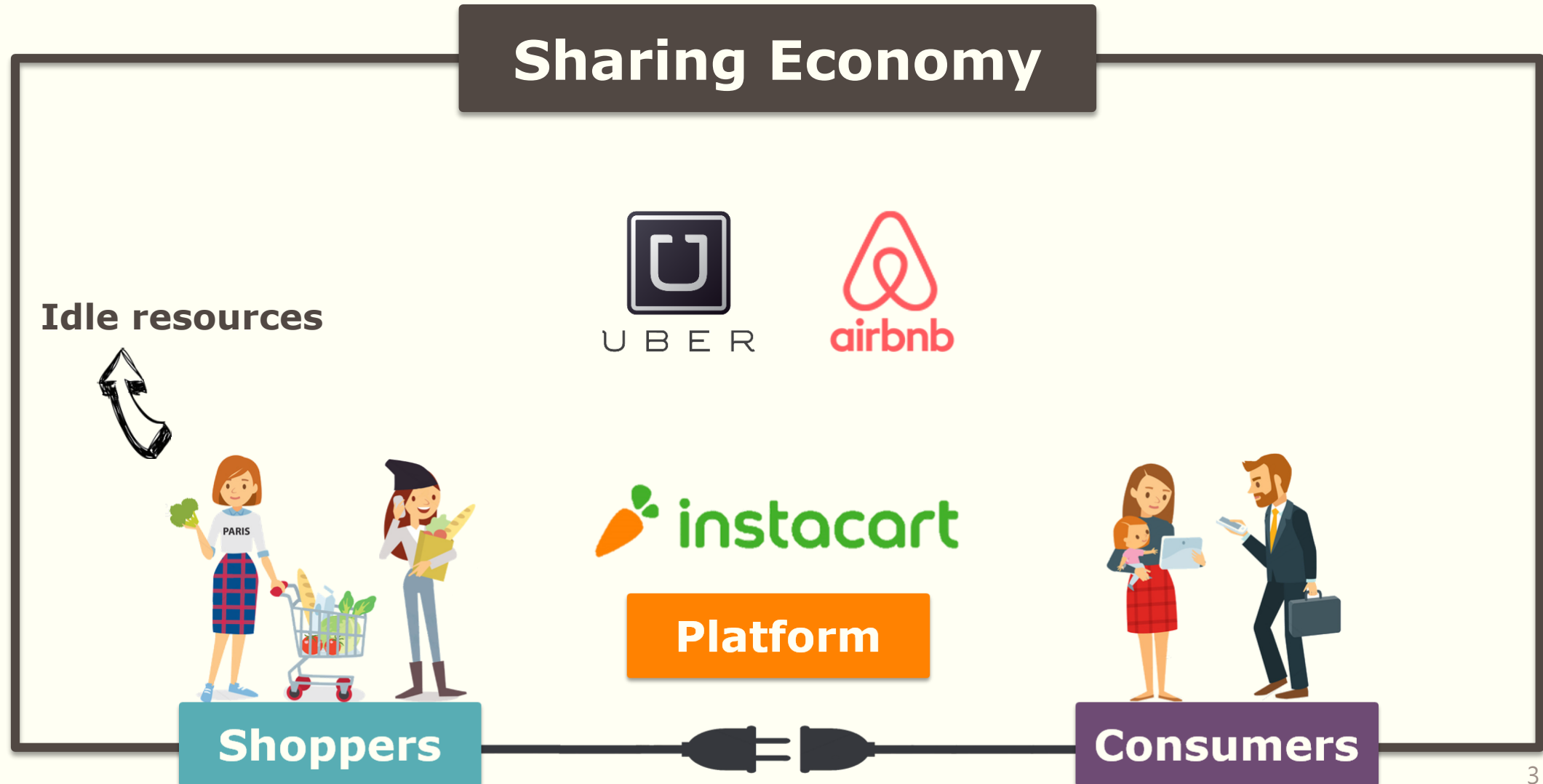
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National Taiwan University
2016/06/23



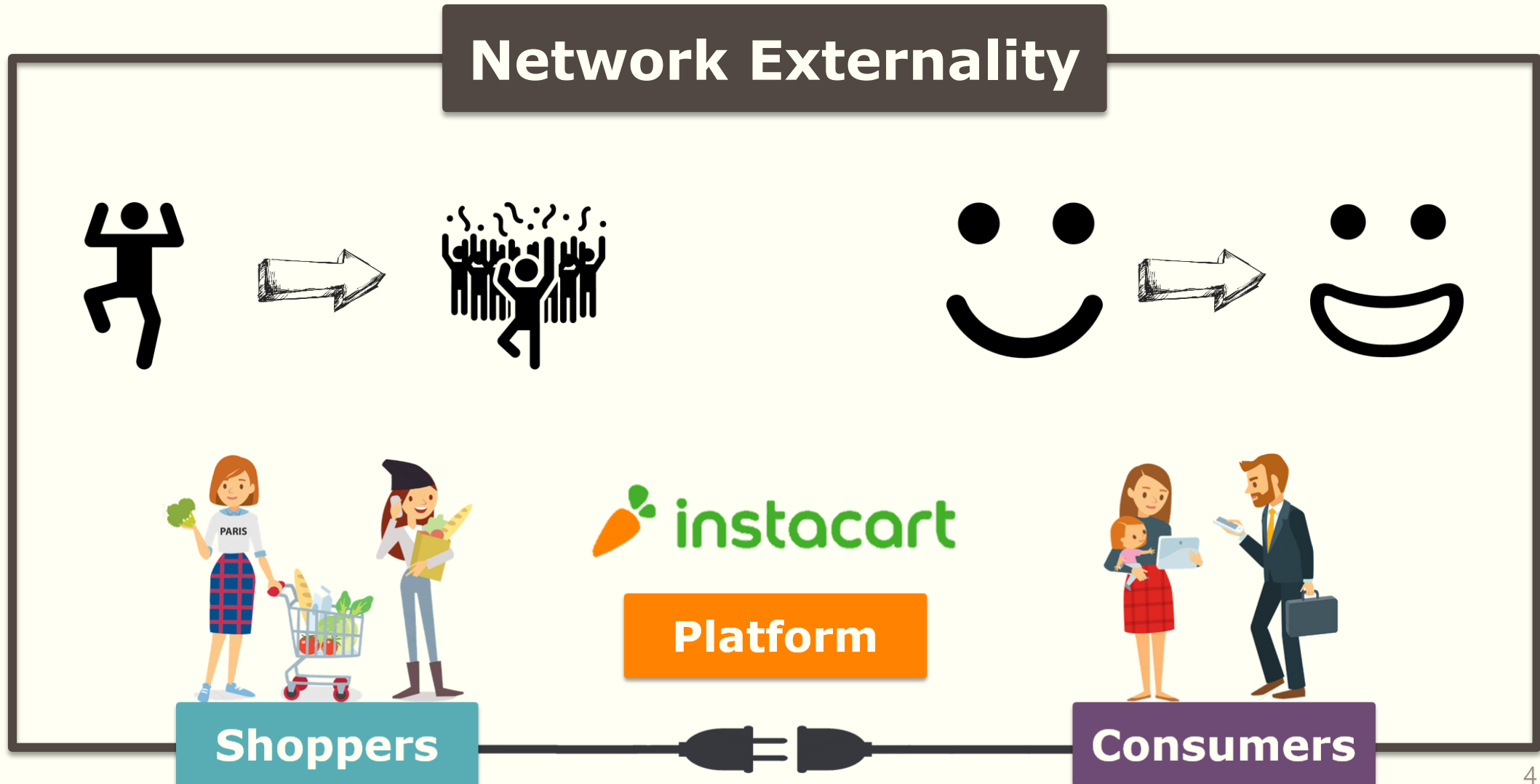
Introduction



Introduction



Introduction



Introduction

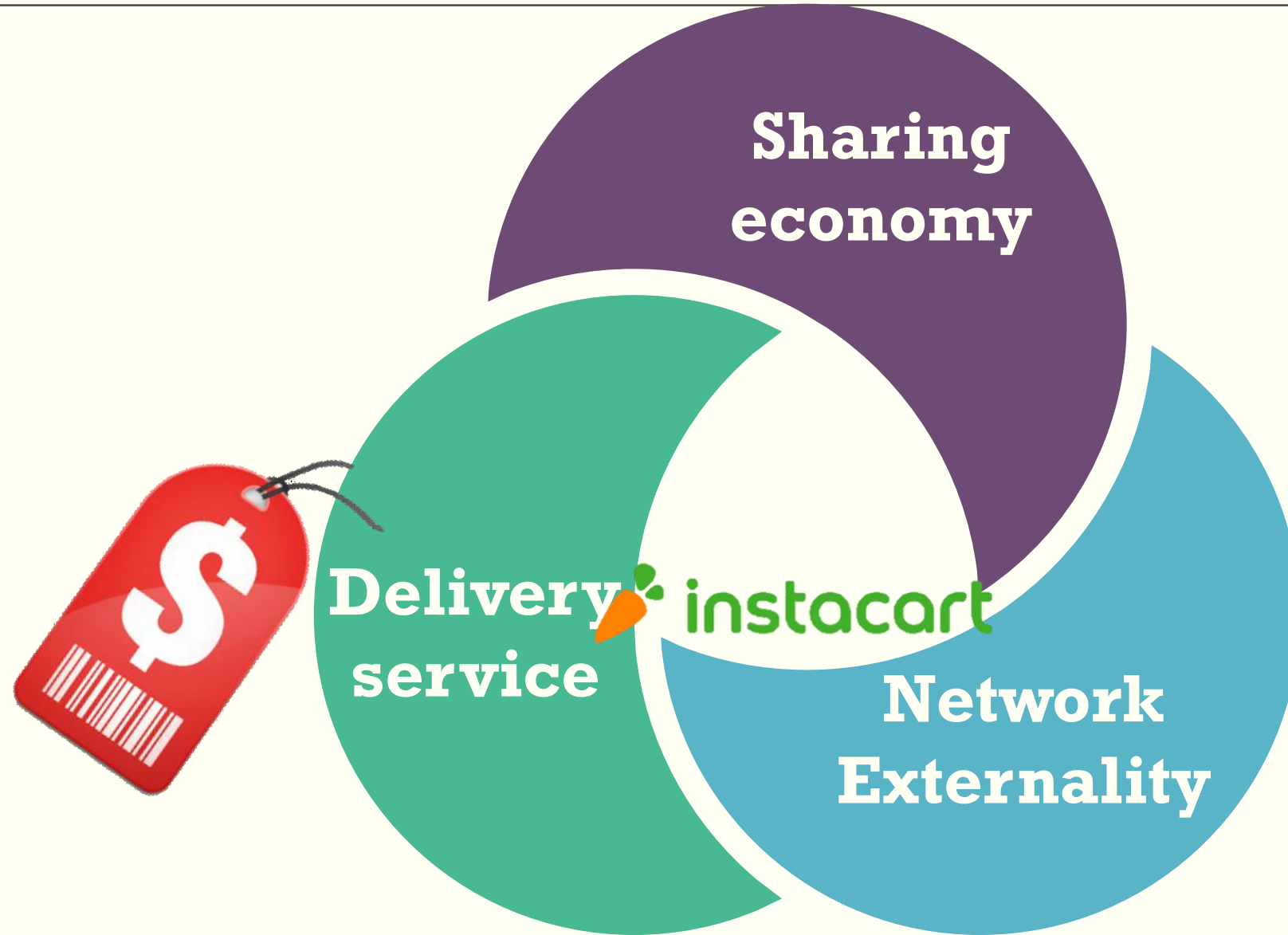


Flexibility

Efficiency

Cheap

Introduction



Introduction



**Membership-
based pricing**

**Transaction-
based pricing**

**Cross-
subsidization**

Outline



01 - LITERATURE REVIEW



02 - MODEL



03 - ANALYSIS

- Optimal profits
- Comparisons



04 - EXTENSIONS

- Discount factor
- Marginal transaction cost
- Fixed shopper subsidization
- Price-sensitive number of orders



05 - CONCLUSIONS



LITERATURE REVIEW

Literature review

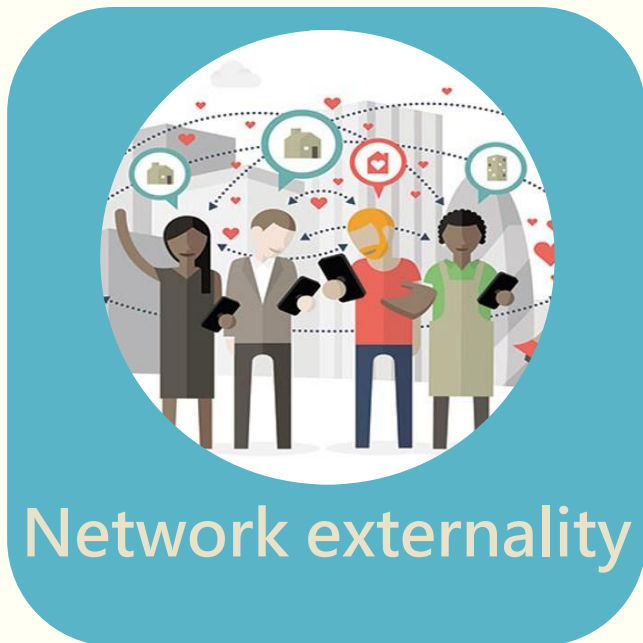


- **Andersson et al. (2013)**
- **Santi et al. (2014)**
- **Felländer et al. (2015)**
- **Zervas et al. (2016)**
- **Rougés and Montreuil (2004)**
- **Teresa and Christy (2015)**

Reducing social cost
Zero marginal cost

Saving inventory cost

Literature review



- **Katz and Shapiro (1985):** Pioneer
- **Fudenberg and Tirole (2000):** Competition between incumbent and entrant
- **Armstrong (2006):** Combining demand function with network externality
- **Rochet and Tirole (2006):** Combining two trends of literature
- **Jing (2007):** Product line design

Literature review



Decentralization or centralization

- **McGuire and Staelin (1983)**
- **Li and Lee (1994)**
- **So (2000)**

Service speed affects service quality,
which affects pricing strategy.

Literature review



Sharing economy

Andersson et al. (2013)
Santi et al. (2014)
Fell"ander et al. (2015)
Zervas et al. (2016)
Roug'es and Montreuil
(2004)
Teresa and Christy (2015)



Network externality

Katz and Shapiro (1985)
Fudenberg and Tirole (2000)
Armstrong (2006)
Rochet and Tirole (2006)
Jing (2007)



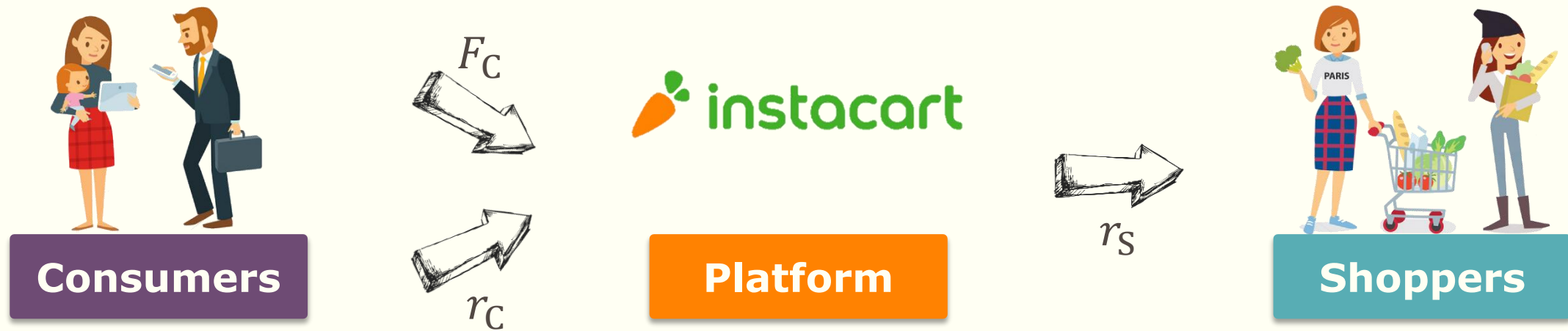
Delivery service

McGuire and Staelin (1983)
Li and Lee (1994)
So (2000)



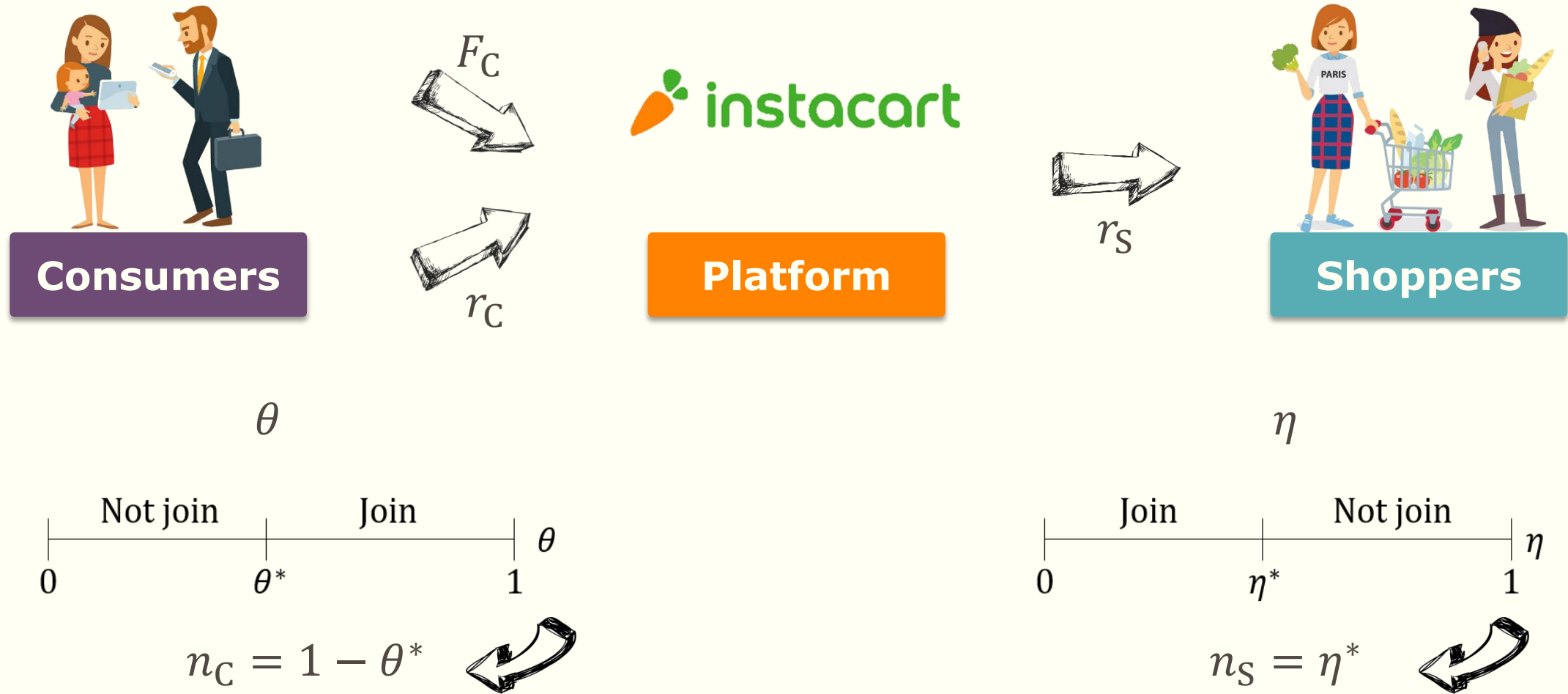
MODEL

Model

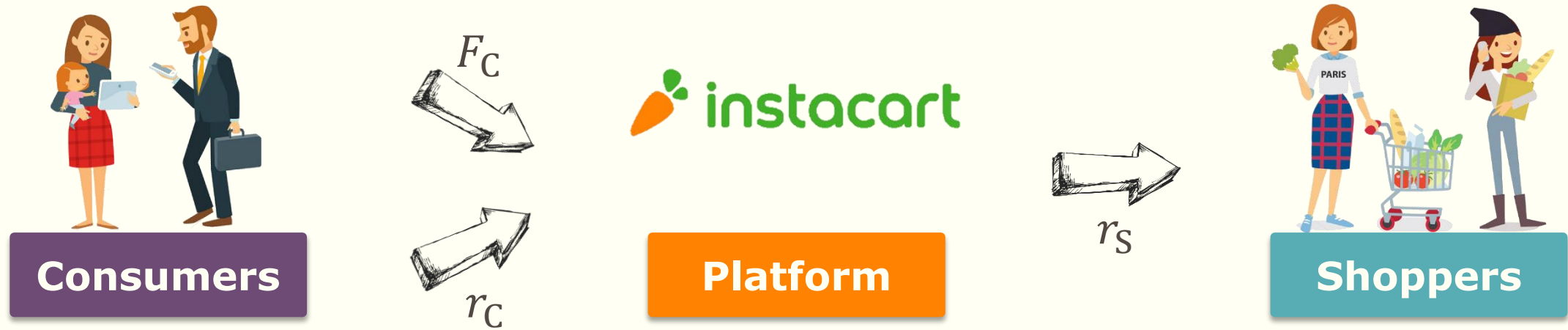


Consumer side	Shopper side
n_C : number of consumers	n_S : number of shoppers
F_C : membership fee	Not consider F_S
r_C : per-transaction fee	r_S : per-transaction subsidy
θ : willingness to pay for one unit quality service	η : cost per transaction
N : consumption in one membership period	

Model



Model



$$u_C = N(\theta Q - r_C) - F_C$$

$$= N(\theta \sqrt{n_S} - r_C) - F_C$$



$$\pi = N n_C (r_C - r_S) + n_C F_C$$

$$u_S = \frac{N n_C}{n_S} (-\eta + r_S)$$

Model

M

**Membership-
based pricing**

$$r_C = 0$$

T

**Transaction-
based pricing**

$$F_C = 0$$

X

**Cross-
subsidization**

$$r_C = r_S$$

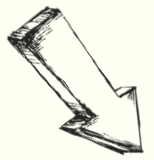


ANALYSIS

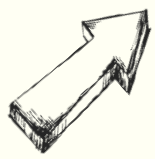
Analysis

Consumers

$$u_C = N(\theta^* Q - r_C) - F_C = 0$$

$$n_C = 1 - \theta^*$$


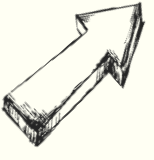
Platform

$$\pi = N n_C (r_C - r_S) + n_C F_C$$


$$\theta^* = \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}, \eta^* = r_S$$

Shoppers

$$u_S = \frac{N n_C}{n_S} (-\eta^* + r_S) = 0$$

$$n_S = \eta^*$$


$$\pi = (1 - \theta^*)(F_C + N(r_C - r_S))$$

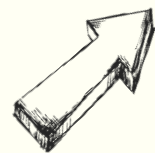
$$= \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N} \right) (F_C + N(r_C - r_S))$$

Number of consumers

Earnings from each consumer

Analysis

$$\pi = (1 - \theta^*)(F_C + N(r_C - r_S))$$
$$= \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right) (F_C + N(r_C - r_S))$$



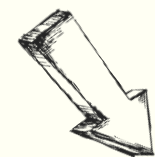
$$r_C = 0$$

$$\textcircled{\mathbf{M}} \pi^{\mathbf{M}} = \left(1 - \frac{\sqrt{r_S} F_C}{r_S N}\right) (F_C - N r_S)$$



$$F_C = 0$$

$$\textcircled{\mathbf{T}} \pi^{\mathbf{T}} = \left(1 - \frac{\sqrt{r_S} r_C}{r_S}\right) N(r_C - r_S)$$



$$r_C = r_S$$

$$\textcircled{\mathbf{X}} \pi^{\mathbf{X}} = \left(1 - \frac{\sqrt{r_S}(r_S N + F_C)}{r_S N}\right) F_C$$

Analysis

$$\textcircled{\text{M}} \quad \pi^{\text{M}} = \left(1 - \frac{\sqrt{r_S} F_C}{r_S N}\right) (F_C - N r_S)$$

$$\textcircled{\text{T}} \quad \pi^{\text{T}} = \left(1 - \frac{\sqrt{r_S} r_C}{r_S}\right) N (r_C - r_S)$$

$$\textcircled{\text{X}} \quad \pi^{\text{X}} = \left(1 - \frac{\sqrt{r_S} (r_S N + F_C)}{r_S N}\right) F_C$$

Lemma 1

$$r_S^{\text{M}} = \frac{1}{9}, F_C^{\text{M}} = \frac{2}{9} N$$

Lemma 2

$$r_S^{\text{T}} = \frac{1}{9}, r_C^{\text{T}} = \frac{2}{9}$$

Lemma 3

$$r_S^{\text{X}} = \frac{1}{9}, r_C^{\text{X}} = \frac{1}{9}, F_C^{\text{X}} = \frac{1}{9} N$$

Analysis - Comparisons

Proposition 1

$$\begin{aligned} r_S^X &= r_S^M = r_S^T \\ r_C^T &> r_C^X > 0 \\ F_C^M &> F_C^X > 0 \end{aligned}$$

Proposition 2

$$\pi^M = \pi^T = \pi^X$$

Proposition 3

A solution (r_C, r_S, F_C) is optimal to the platform's problem if and only if
$$r_S = \frac{1}{9} \text{ and } r_C N + F_C = \frac{2}{9} N.$$



EXTENSIONS

Extensions

Discount factor

1

Take account of how sensitive the platform is to cash flow in each transaction.

Marginal transaction cost

2

Take the platform's marginal cost into account.

Fixed shipper subsidization

3

The platform employs shippers with a fixed payment rather than a per-transaction one.

Price-sensitive number of orders

4

The number of orders would decrease in the per-transaction fee.

Extension 1 - Discount factor

Let $a \in [0,1]$

$$\pi_{discount} = \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right) (F_C + Na(r_C - r_S))$$

Lemma 4

$$r_S^M = \frac{a^2+1}{18}, F_C^M = \frac{(\sqrt{1+a^2}(a+a^2)+3\sqrt{2}(1+a^2))}{36\sqrt{1+a^2}} N$$

Lemma 5

$$r_S^T = \frac{1}{9}, r_C^T = \frac{2}{9}$$

Lemma 6

$$r_S^X = \frac{1}{9}, r_C^X = \frac{1}{9}, F_C^X = \frac{1}{9} N$$

Proposition 4

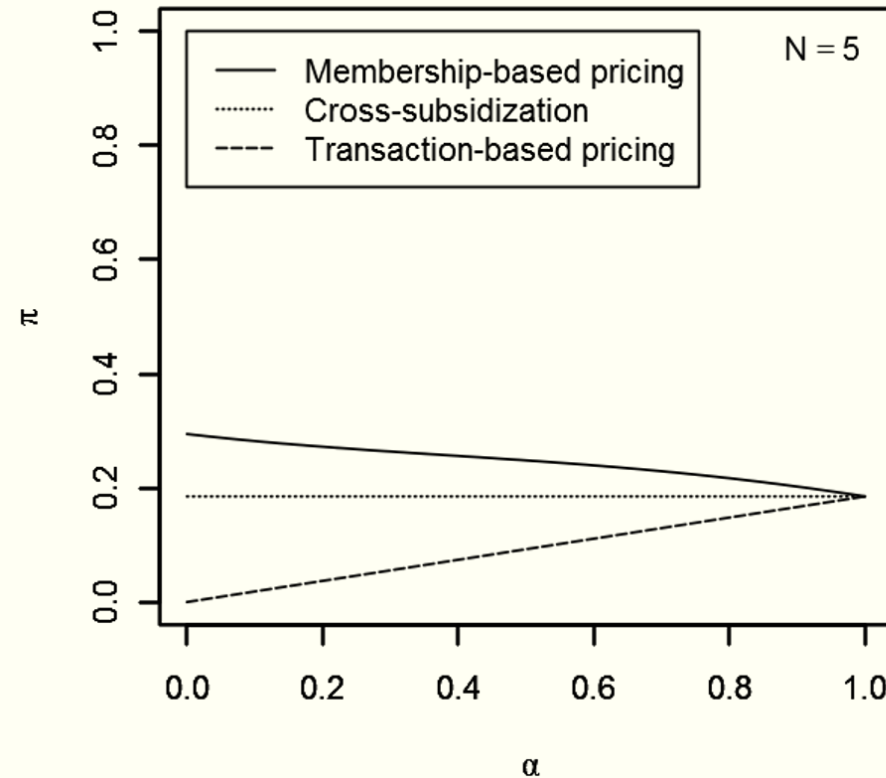
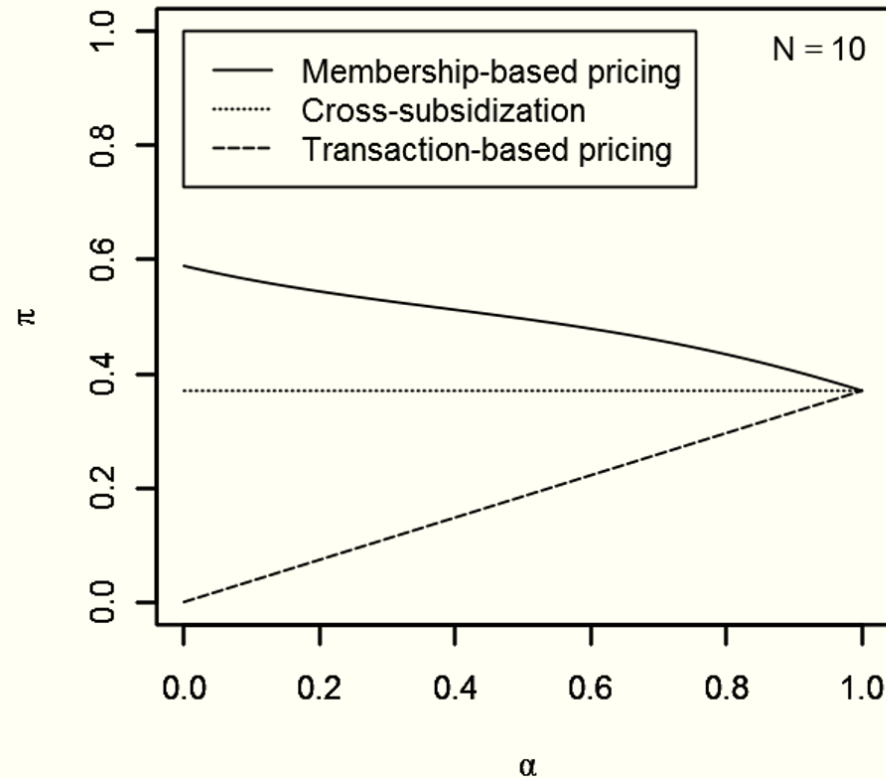
$$\begin{aligned} r_S^T &= r_S^X > r_S^M \\ r_C^T &> r_C^X > 0 \\ F_C^M &> F_C^X > 0 \end{aligned}$$

Extension 1 - Discount factor

Proposition 5

$$\pi_{discount}^M > \pi_{discount}^X > \pi_{discount}^T$$

**Why
Instacart Express**



Extension 2 - Marginal transaction cost

$$\pi_{cost} = \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right) (F_C + N(r_C - r_S - c))$$

Lemma 7

$$r_S^M = \frac{6c+1+\sqrt{12c+1}}{18}, F_C^M = \frac{(\sqrt{\sqrt{12c+1}+6c+1}(\sqrt{12c+1}+24c+1)+3\sqrt{2}(\sqrt{12c+1}+6c+1))}{36\sqrt{\sqrt{12c+1}+6c+1}} N$$

Lemma 8

$$r_S^T = \frac{6c+1+\sqrt{12c+1}}{18}, r_C^T = \frac{6c+1+\sqrt{12c+1}}{9}$$

Lemma 9

$$r_S^X = r_C^X = \frac{6c+1+\sqrt{12c+1}}{18}, F_C^X = \frac{6c+1+\sqrt{12c+1}}{18} N$$

Extension 2 - Marginal transaction cost

Proposition 1

=

Proposition 6

$$\begin{aligned} r_S^X &= r_S^M = r_S^T \\ r_C^T &> r_C^X > 0 \\ F_C^M &> F_C^X > 0 \end{aligned}$$

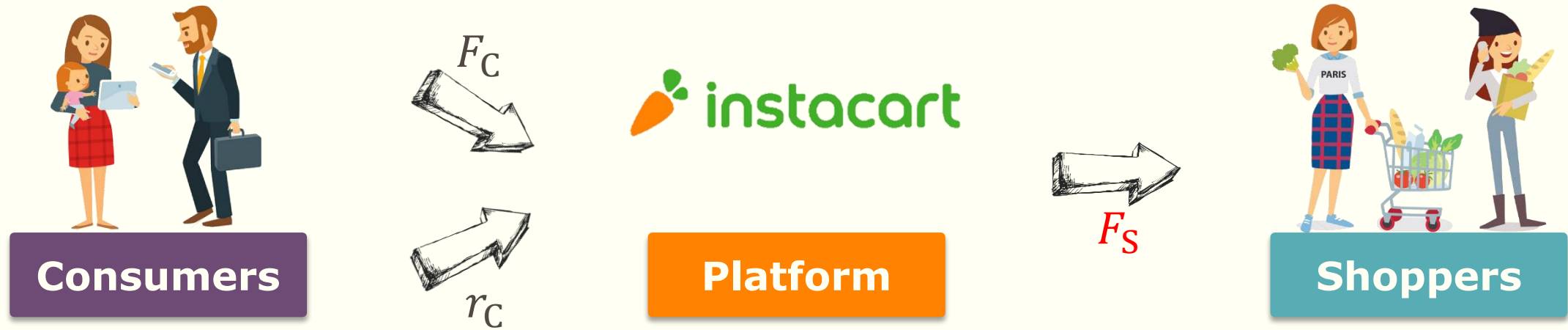
Proposition 2

=

Proposition 7

$$\pi_{cost}^M = \pi_{cost}^T = \pi_{cost}^X$$

Extension 3 - Fixed shopper subsidization



$$u_C = N(\theta Q - r_C) - F_C$$

$$\pi = N n_C(r_C) + n_C F_C + n_S F_S$$

$$u_S = \frac{N n_C}{n_S}(-\eta) - F_S$$

$$\pi_{fixed} = -\frac{F_S}{N}(N r_C + F_C) + \left(\frac{r_C N + F_C}{F_S + N}\right)^2 F_S$$

Extension 3 - Fixed shopper subsidization

Proposition 8

When we set $a = 1$, a plan (r_C, F_C, F_S) is the platform's optimal solution if and only if

$$F_S = -\frac{N}{3} \text{ and } r_C N + F_C = \frac{2}{9} N.$$

Furthermore, no matter the platform subsidizes shippers with fixed or per-transaction subsidization, the platform's profits are the same (cf. proposition 3).

**Fixed
subsidization**

=

**Per-transaction
subsidization**

Extension 3 - Fixed shopper subsidization

Proposition 9 – part 1

When we set $a < 1$, the platform's optimal solution under transaction-based and membership-based pricing strategies would be

$$F_S^T = -\frac{N}{3}, r_C^T = \frac{2}{9}a \text{ and} \\ F_S^M = -\frac{N}{3}, F_C^M = \frac{2}{9}N.$$

And the platform's optimal profits under these two strategies would have the following relation

$$\pi_{fixed}^M > \pi_{fixed}^T.$$

Extension 3 - Fixed shopper subsidization

Proposition 9 – part 2

Furthermore, no matter the platform employs which pricing strategy here, subsidizing shippers with per-transaction subsidization is better for it, i.e.,

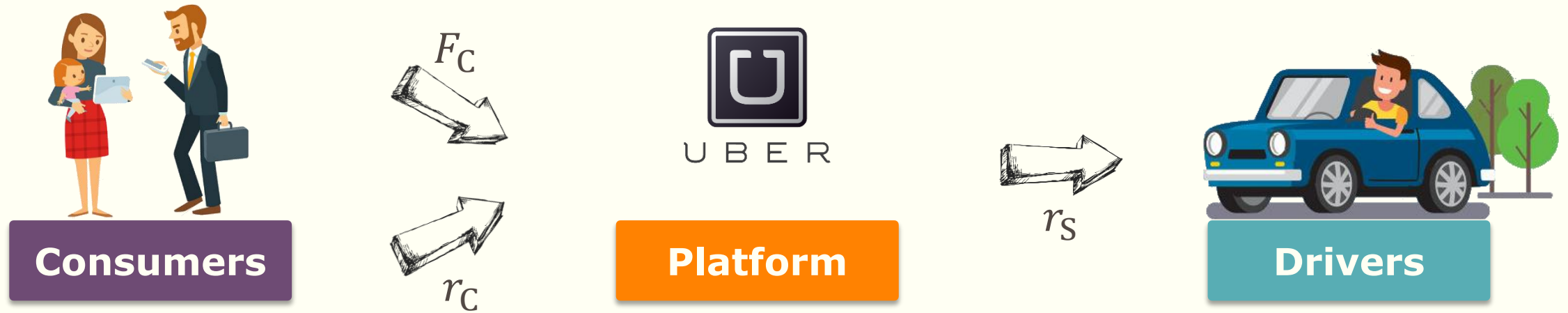
$$\pi_{discount}^M > \pi_{fixed}^M \text{ and } \pi_{discount}^T > \pi_{fixed}^T.$$

**Per-transaction
subsidization**

>

**Fixed
subsidization**

Extension 4 - Price-sensitive number of orders



$$u_C = \frac{N}{r_C} (\theta Q - r_C) - F_C$$

$$\pi = \frac{N}{r_C} n_C (r_C - r_S) + n_C F_C$$

$$u_S = \frac{N}{r_C} \frac{n_C}{n_S} (-\eta + r_S)$$

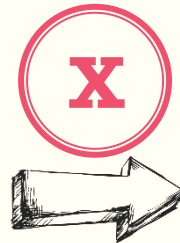
$$\text{Capacity constraint: } \frac{N}{r_C} \frac{n_C}{n_S} \leq K$$

Extension 4 - Price-sensitive number of orders

$$\begin{aligned} \max \quad & \left(1 - \frac{\frac{F_C r_C}{N} + r_C}{\sqrt{r_S}} \right) \left(\frac{N}{r_C} (r_C - r_S) + F_C \right) \\ \text{s.t.} \quad & r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N + F_C} \end{aligned}$$



$$\begin{aligned} \max \quad & \left(1 - \frac{r_C}{\sqrt{r_S}} \right) \left(\frac{N}{r_C} (r_C - r_S) \right) \\ \text{s.t.} \quad & r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N} \end{aligned}$$



$$\begin{aligned} \max \quad & \left(1 - \frac{\frac{F_C r_C}{N} + r_C}{\sqrt{r_S}} \right) F_C \\ \text{s.t.} \quad & F_C \geq \frac{N}{\sqrt{r_S}} - r_S^{\frac{3}{2}} K - N \end{aligned}$$

Extension 4 - Price-sensitive number of orders

T

$$\begin{aligned} \max \quad & \left(1 - \frac{r_C}{\sqrt{r_S}}\right) \left(\frac{N}{r_C} (r_C - r_S)\right) \\ \text{s.t.} \quad & r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N} \end{aligned}$$

n_C : **Number of consumers**

m : **Earnings from each consumer**

X

$$\begin{aligned} \max \quad & \left(1 - \frac{\frac{F_C r_C + r_C}{N}}{\sqrt{r_S}}\right) F_C \\ \text{s.t.} \quad & F_C \geq \frac{N}{\sqrt{r_S}} - r_S^{\frac{3}{2}} K - N \end{aligned}$$

$$TC = \frac{N}{r_C} n_C$$

Extension 4 - Price-sensitive number of orders



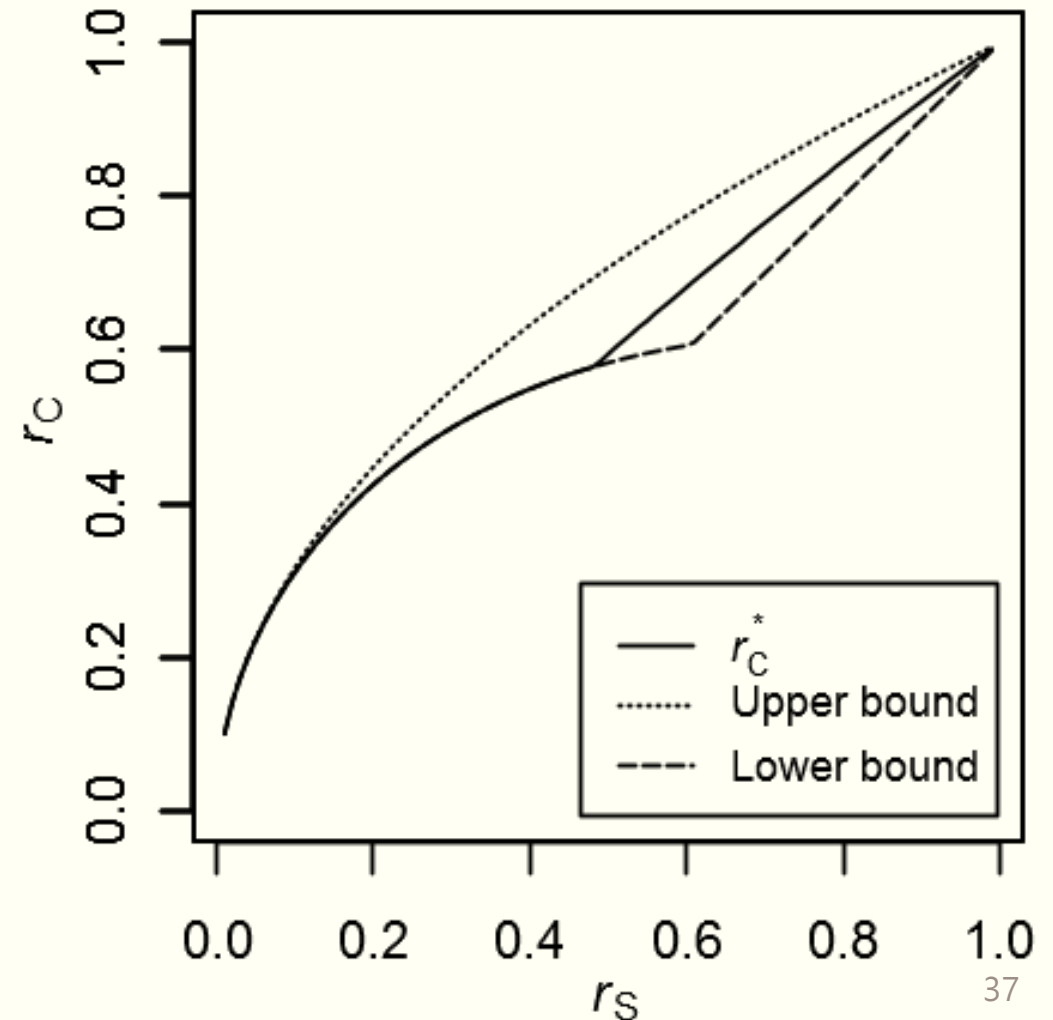
$$\max \left(1 - \frac{r_C}{\sqrt{r_S}} \right) \left(\frac{N}{r_C} (r_C - r_S) \right)$$

$$s.t. \quad r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N}$$

$$1 - \frac{r_C}{\sqrt{r_S}} > 0 \text{ and } r_C - r_S > 0$$

$$\Rightarrow r_S < r_C < \sqrt{r_S}$$

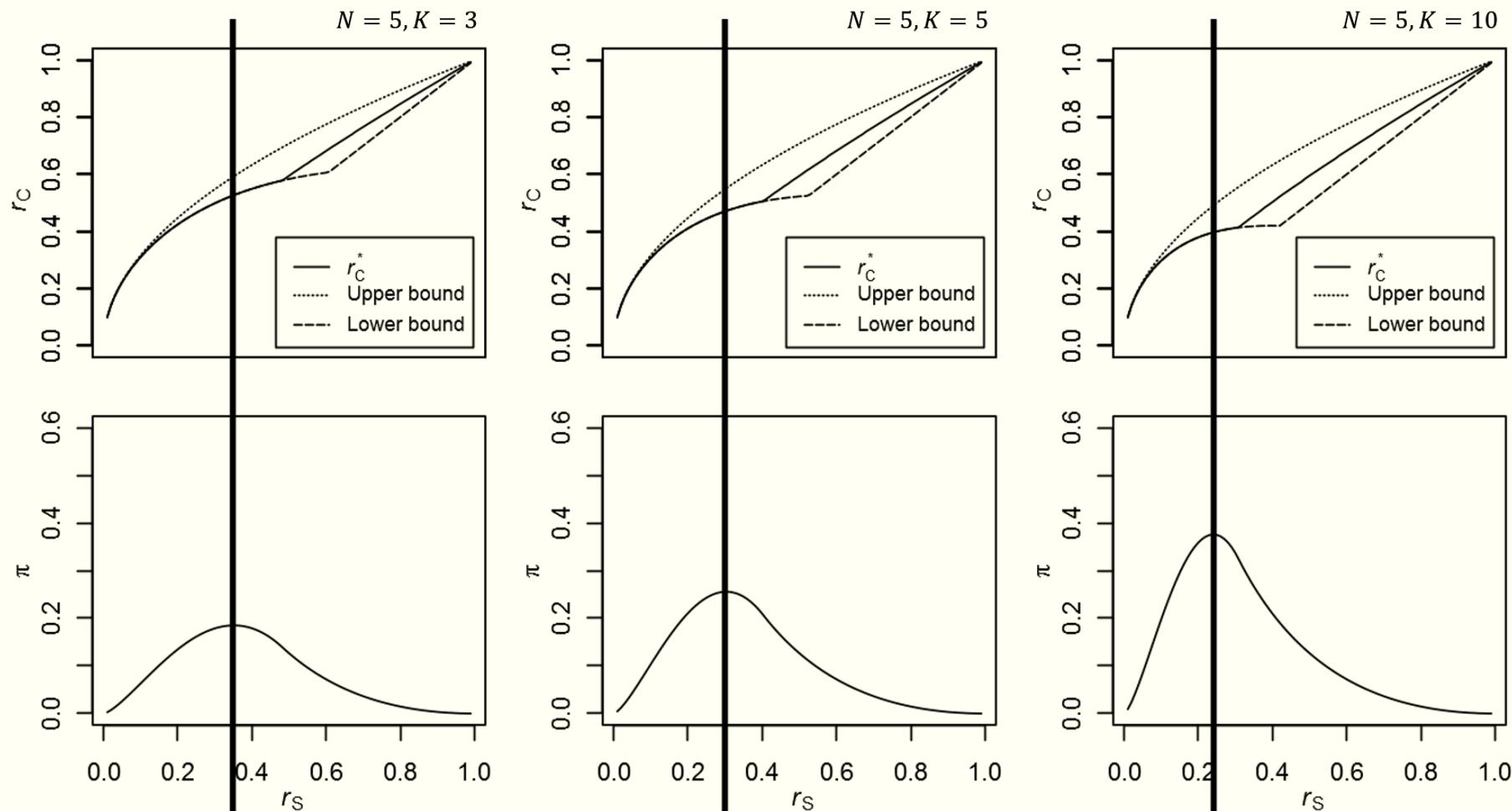
$$N = 5, K = 3$$



Extension 4 - Price-sensitive number of orders

Observation 1

Given an arbitrary set of N and K , the optimal solution would bind to the capacity constraint under pure-transaction pricing and cross-subsidization strategies.

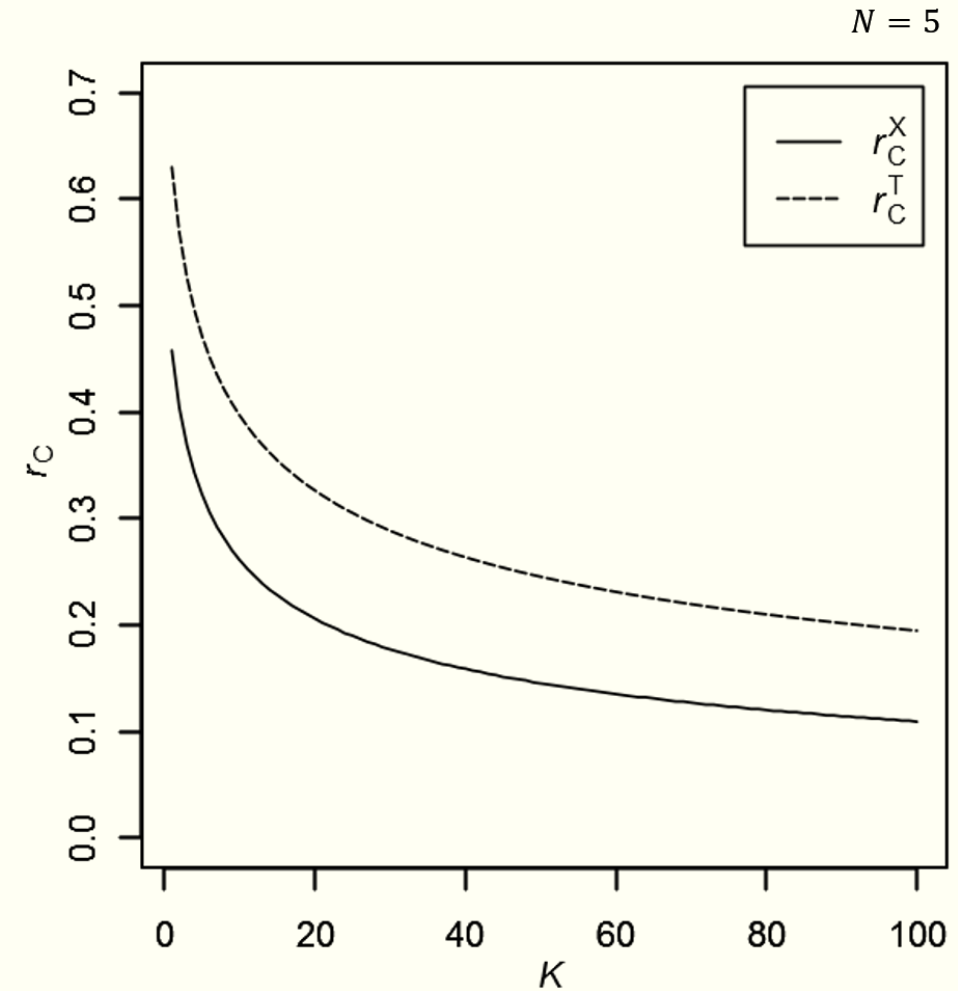
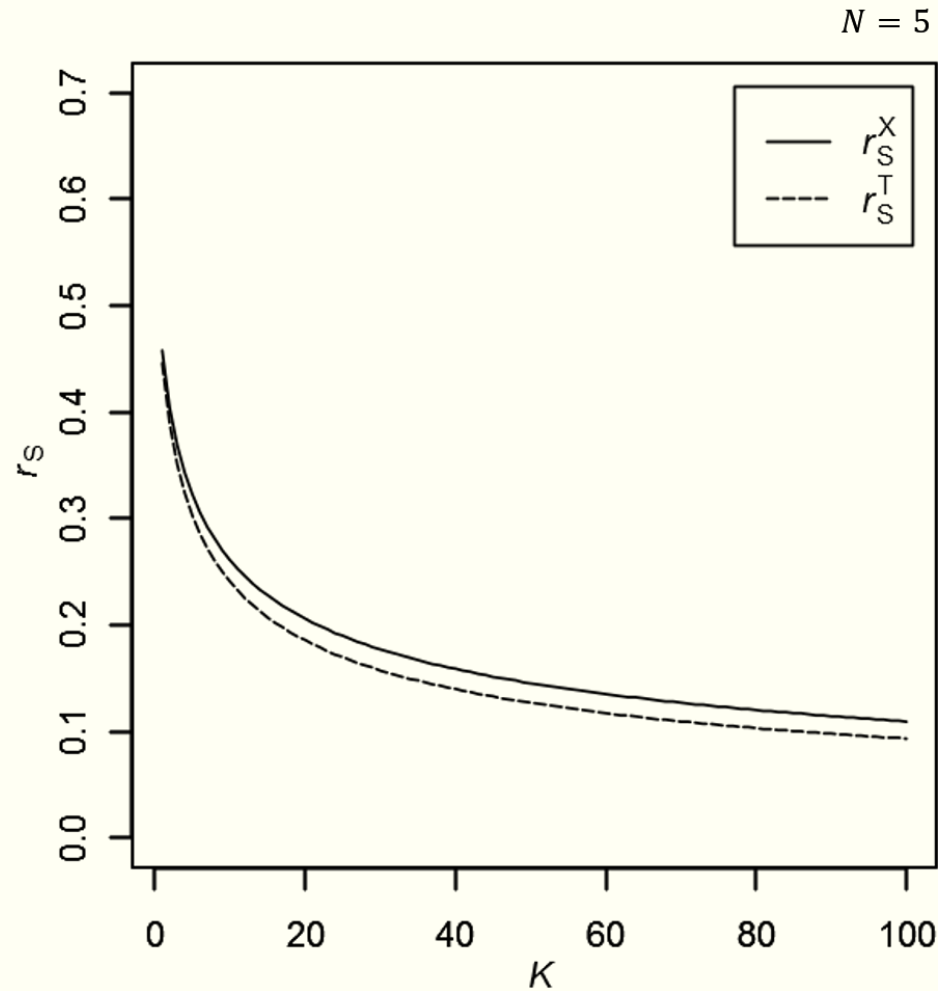


Extension 4 - Price-sensitive number of orders

Observation 2

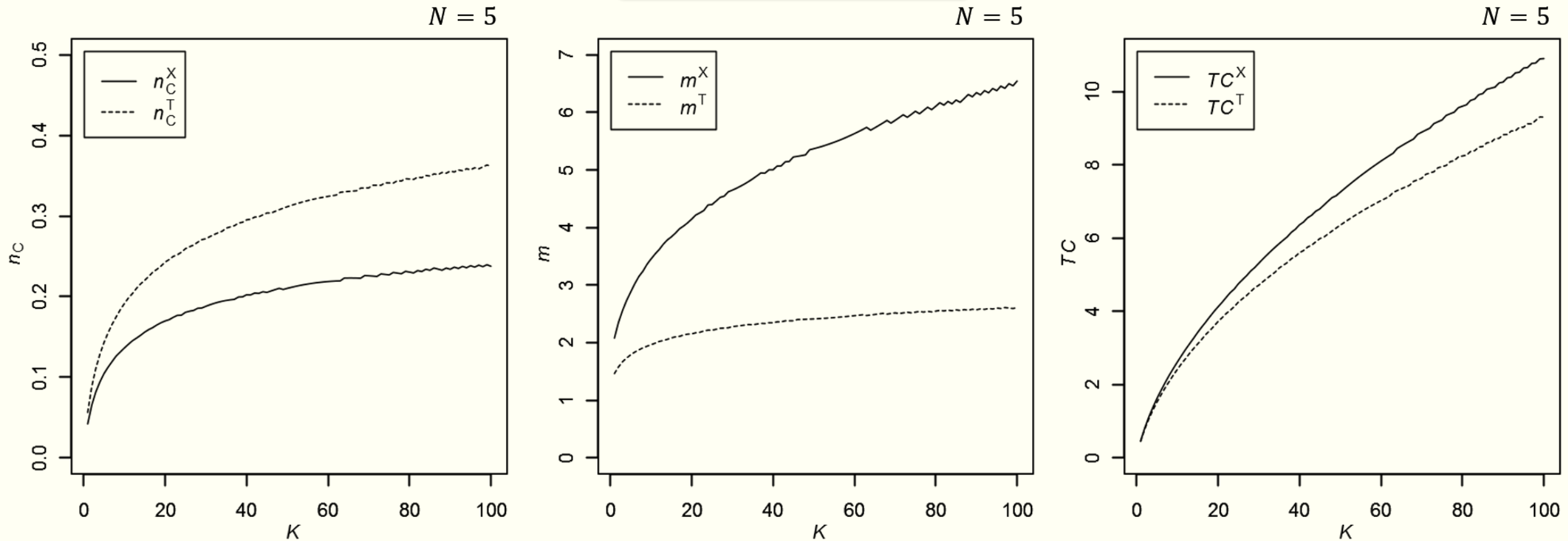
$$r_C^T > r_C^X \\ = r_S^X > r_S^T$$

$$n_S^X > n_S^T$$



Extension 4 - Price-sensitive number of orders

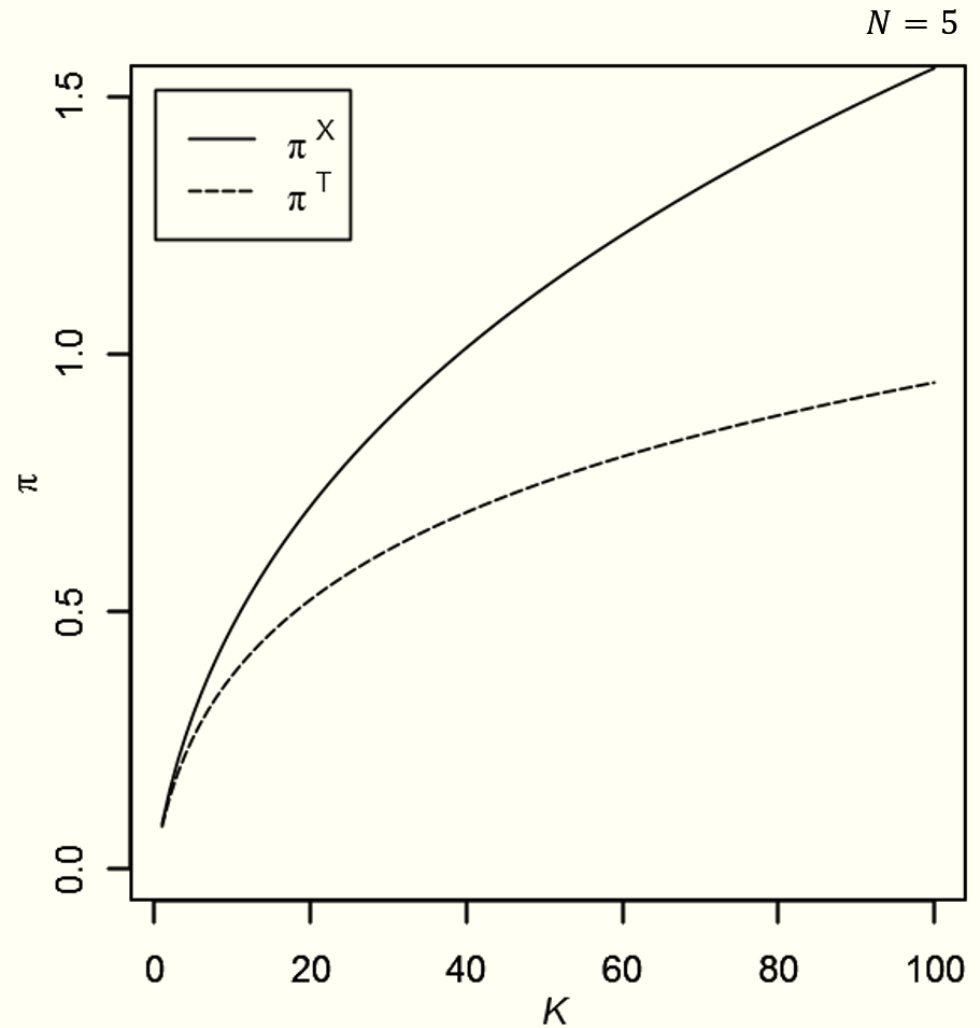
Observation 3



Extension 4 - Price-sensitive number of orders

Observation 4

$$\pi^X > \pi^T$$



Extension 4 - Price-sensitive number of orders

	r_S	r_C	n_S	n_C	m	TC	π
X	higher		more		more	more	higher
T		higher		more			





CONCLUSIONS

Conclusions

01 Basic model

All the three strategies are equivalent.

02 Extension 1

Discount factor: Membership-based pricing is the best.

03 Extension 2

Marginal cost: It does not matter.

04 Extension 3

Fixed subsidy rather than the per-transaction one: Per-transaction subsidy is better for platform to adopt.

05 Extension 4

Price-sensitive number of orders: Cross-subsidization strategy is better.

Conclusions

06 Future work

- Competition between integrated delivery and platform one.
- Take Uber's *surge pricing* strategy into account.

