

# **PLATFORM DELIVERY: A GAME-THEORETIC ANALYSIS OF A NEW DELIVERY MODEL IN THE SHARING ECONOMY**

Guan-Yu Zhong  
Adviser: Ling-Chieh Kung, Ph.D.

Department of Information Management  
National Taiwan University  
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# Introduction

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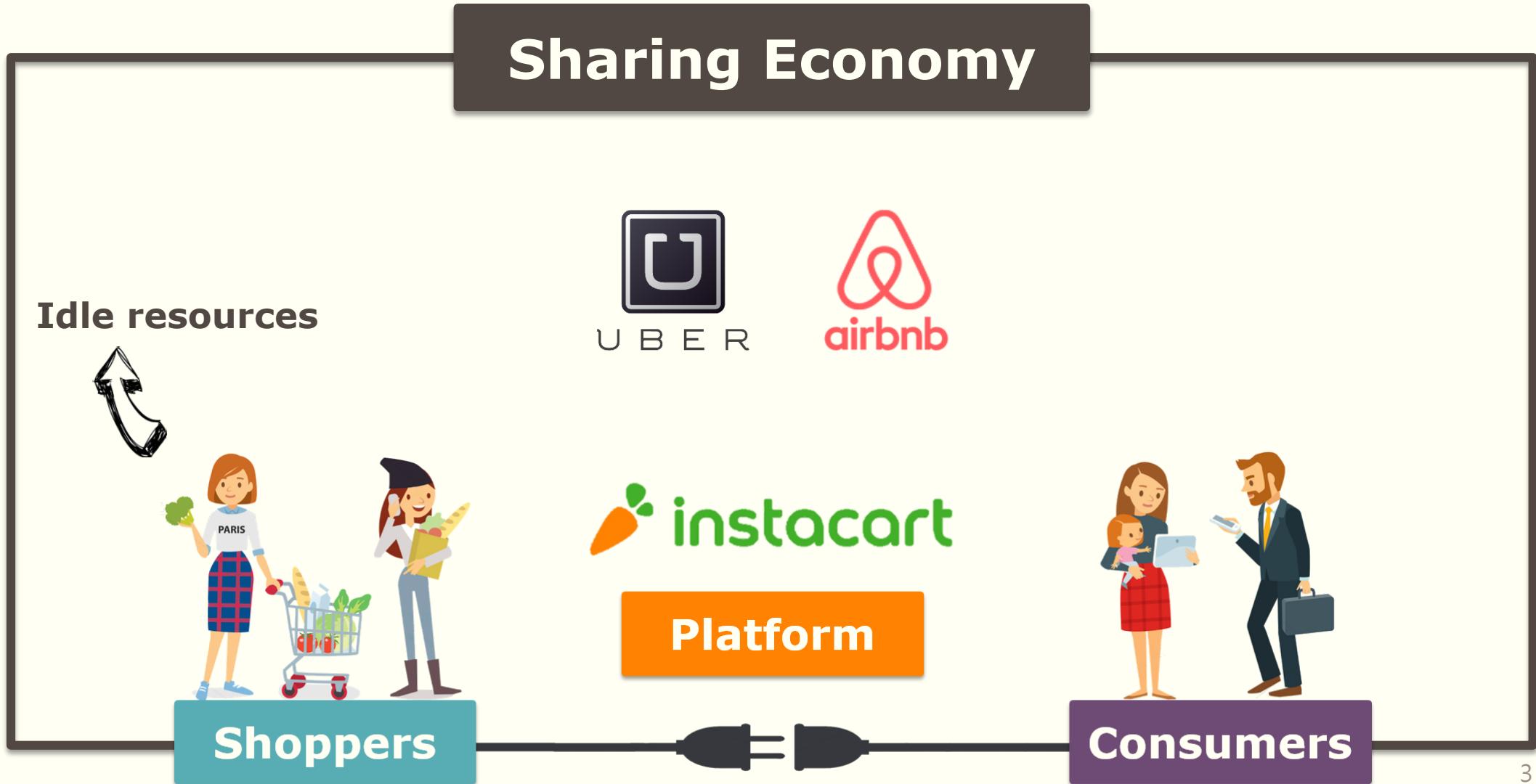
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# Introduction

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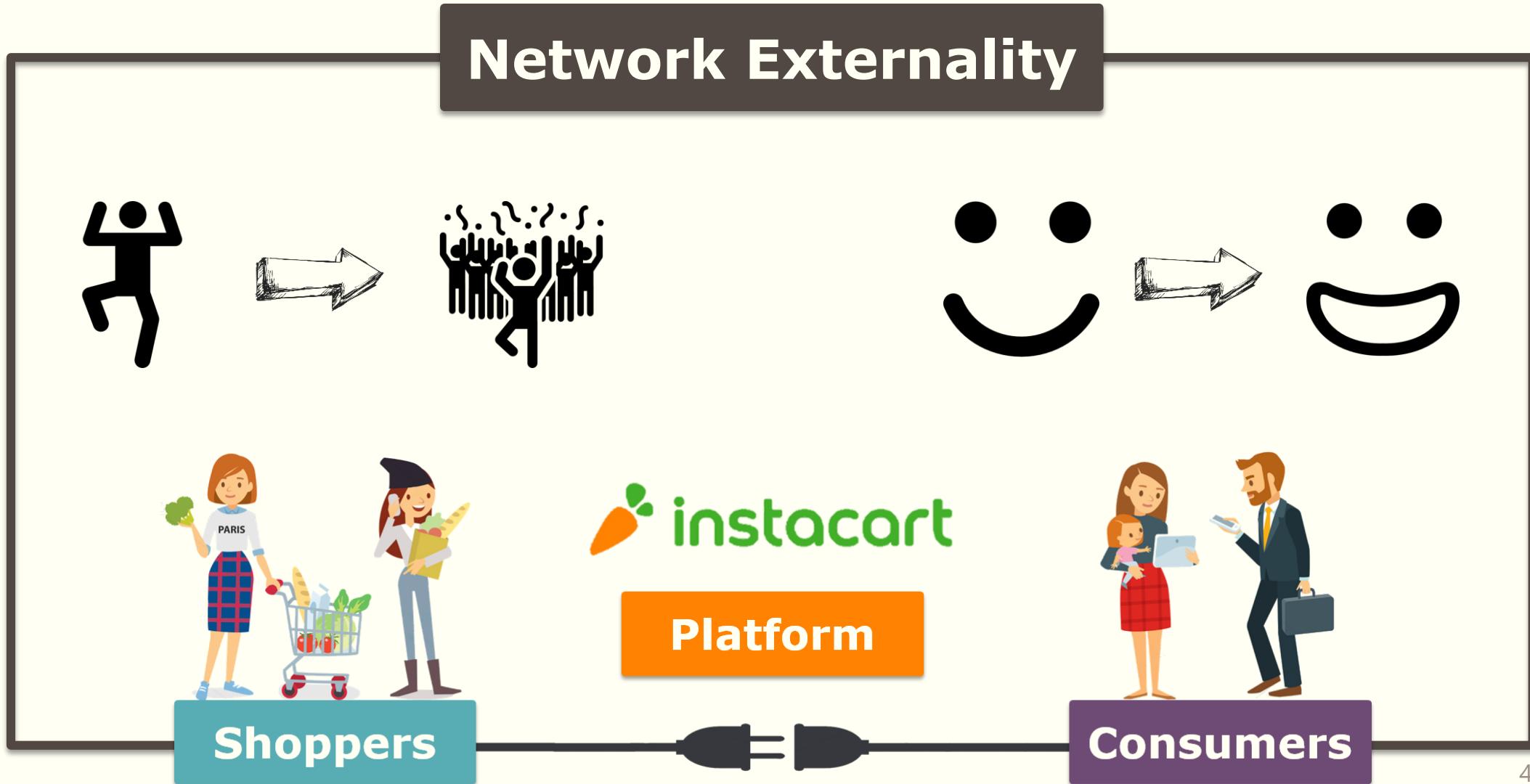
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# Introduction

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# Introduction

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Platform delivery



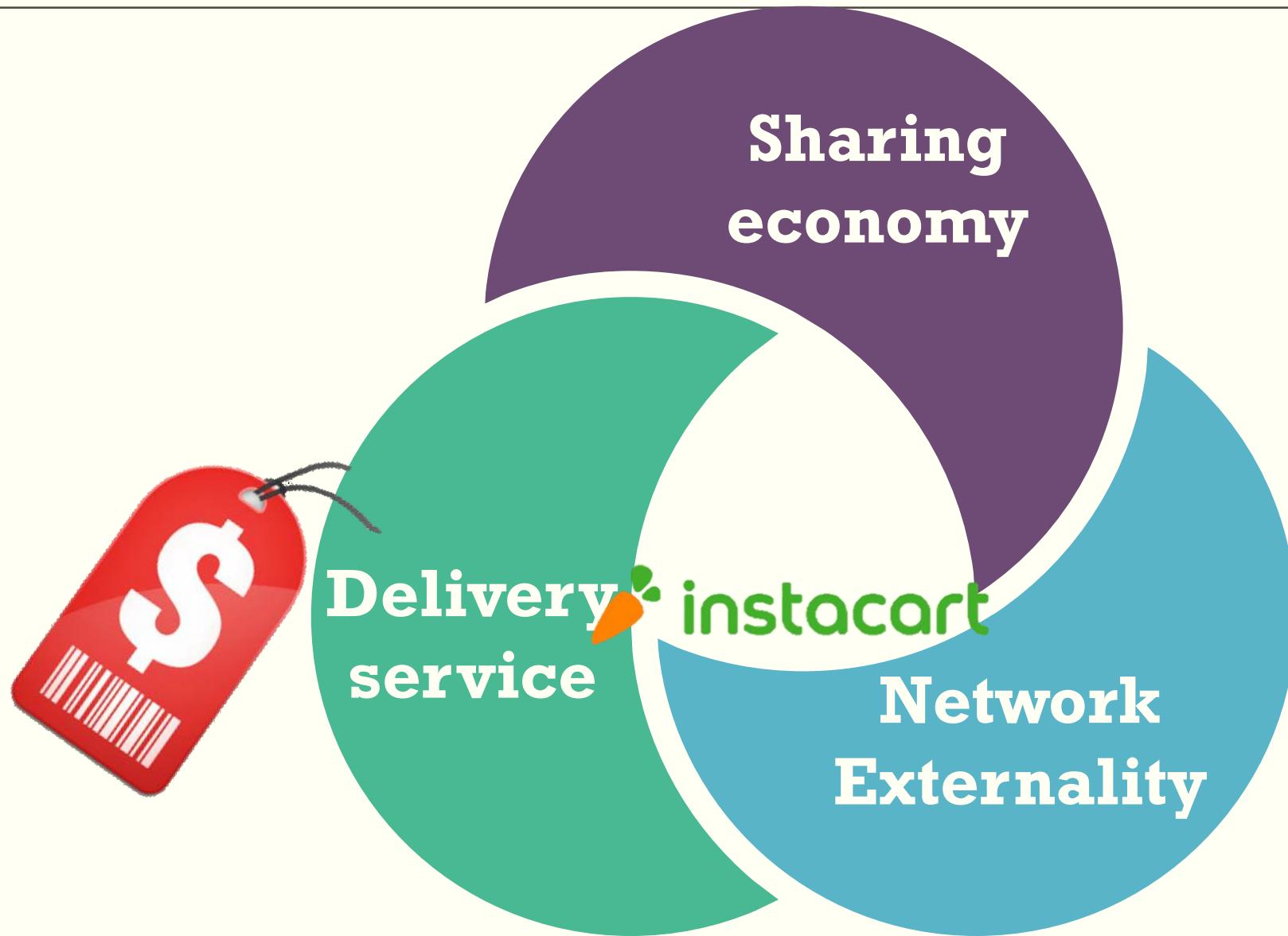
Integrated delivery



# Introduction

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# Introduction

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**Membership-based pricing**

**Transaction-based pricing**

**Cross-subsidization**

# Outline

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## 02 - MODEL



## 01 - LITERATURE REVIEW



## 03 - ANALYSIS

- Optimal profits
- Comparisons



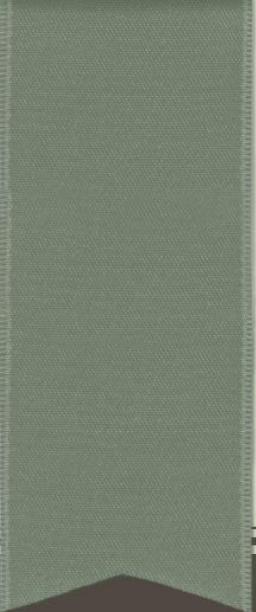
## 04 - EXTENSIONS

- Discount factor
- Marginal transaction cost
- Fixed shopper subsidization
- Price-sensitive number of orders



## 05 - CONCLUSIONS





# LITERATURE REVIEW

# Literature review

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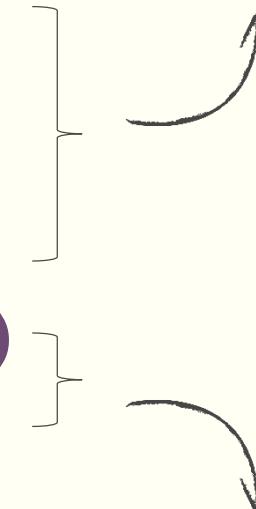
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- **Andersson et al. (2013)**
- **Santi et al. (2014)**
- **Felländer et al. (2015)**
- **Zervas et al. (2016)**
- **Rougés and Montreuil (2004)**
- **Teresa and Christy (2015)**

Reducing social cost  
Zero marginal cost

Saving inventory cost



# Literature review

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- **Katz and Shapiro (1985)**: Pioneer
- **Fudenberg and Tirole (2000)**: Competition between incumbent and entrant
- **Armstrong (2006)**: Combining demand function with network externality
- **Rochet and Tirole (2006)**: Combining two trends of literature
- **Jing (2007)**: Product line design

# Literature review

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Decentralization or centralization

- McGuire and Staelin (1983)
- Li and Lee (1994)
- So (2000)

Service speed affects service quality,  
which affects pricing strategy.

# Literature review

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## Sharing economy

Andersson et al. (2013)  
Santi et al. (2014)  
Felländer et al. (2015)  
Zervas et al. (2016)  
Rougé's and Montreuil (2004)  
Teresa and Christy (2015)



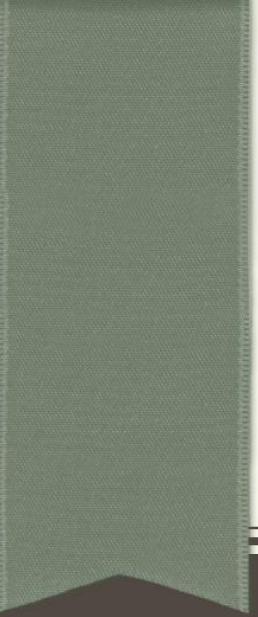
## Network externality

Katz and Shapiro (1985)  
Fudenberg and Tirole (2000)  
Armstrong (2006)  
Rochet and Tirole (2006)  
Jing (2007)



## Delivery service

McGuire and Staelin (1983)  
Li and Lee (1994)  
So (2000)



# MODEL

# Model

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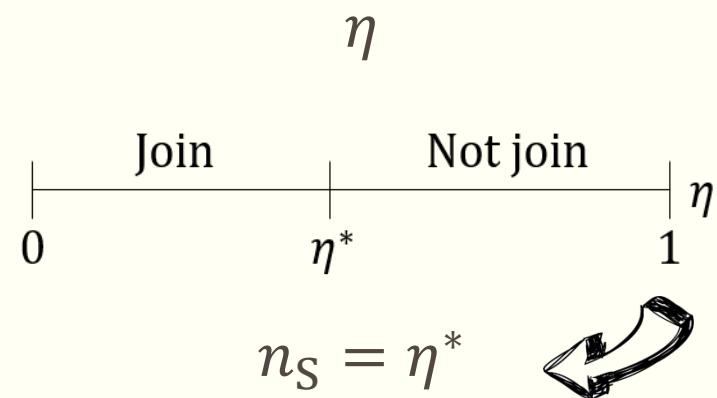
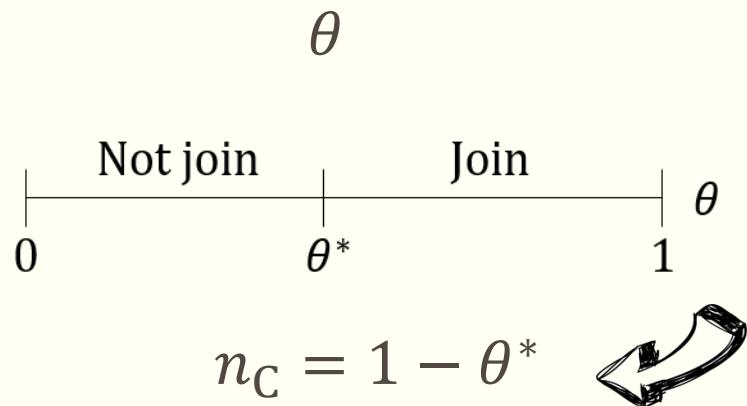


Consumer side	Shopper side
$n_C$ : number of consumers	$n_S$ : number of shoppers
$F_C$ : membership fee	Not consider $F_S$
$r_C$ : per-transaction fee	$r_S$ : per-transaction subsidy
$\theta$ : willingness to pay for one unit quality service	$\eta$ : cost per transaction
$N$ : consumption in one membership period	

# Model

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# Model

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$$\begin{aligned} u_C &= N(\theta Q - r_C) - F_C \\ &= N(\theta \sqrt{n_S} - r_C) - F_C \end{aligned}$$



$$\pi = Nn_C(r_C - r_S) + n_C F_C$$

$$u_S = \frac{Nn_C}{n_S}(-\eta + r_S)$$

# Model

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**M**

**Membership-based pricing**

$$r_C = 0$$



**T**

**Transaction-based pricing**

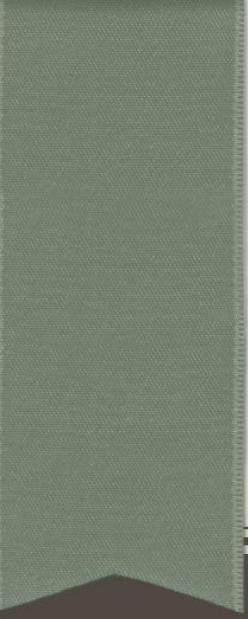
$$F_C = 0$$



**X**

**Cross-subsidization**

$$r_C = r_S$$



# ANALYSIS

# Analysis

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Consumers

$$u_C = N(\theta^* Q - r_C) - F_C = 0$$
$$n_C = 1 - \theta^*$$



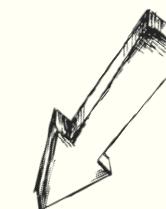
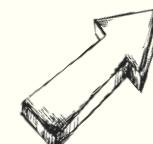
Shoppers

$$u_S = \frac{N n_C}{n_S} (-\eta^* + r_S) = 0$$
$$n_S = \eta^*$$



Platform

$$\pi = N n_C (r_C - r_S) + n_C F_C$$



$$\begin{aligned}\pi &= (1 - \theta^*) (F_C + N(r_C - r_S)) \\ &= \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right) (F_C + N(r_C - r_S))\end{aligned}$$

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Number of consumers

Earnings from each consumer

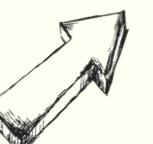
# Analysis

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$$\pi = (1 - \theta^*)(F_C + N(r_C - r_S))$$
$$= \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right)(F_C + N(r_C - r_S))$$

$$r_C = 0$$

 **M**  $\pi^M = \left(1 - \frac{\sqrt{r_S}F_C}{r_S N}\right)(F_C - Nr_S)$

$$F_C = 0$$

 **T**  $\pi^T = \left(1 - \frac{\sqrt{r_S}r_C}{r_S}\right)N(r_C - r_S)$

$$r_C = r_S$$

 **X**  $\pi^X = \left(1 - \frac{\sqrt{r_S}(r_S N + F_C)}{r_S N}\right)F_C$

# Analysis

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**M**  $\pi^M = \left(1 - \frac{\sqrt{r_S} F_C}{r_S N}\right) (F_C - N r_S)$

**Lemma 1**

$$r_S^M = \frac{1}{9}, F_C^M = \frac{2}{9}N$$

**T**  $\pi^T = \left(1 - \frac{\sqrt{r_S} r_C}{r_S}\right) N(r_C - r_S)$

**Lemma 2**

$$r_S^T = \frac{1}{9}, r_C^T = \frac{2}{9}$$

**X**  $\pi^X = \left(1 - \frac{\sqrt{r_S}(r_S N + F_C)}{r_S N}\right) F_C$

**Lemma 3**

$$r_S^X = \frac{1}{9}, r_C^X = \frac{1}{9}, F_C^X = \frac{1}{9}N$$

# Analysis - Comparisons

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## Proposition 1

$$r_S^X = r_S^M = r_S^T$$

$$r_C^T > r_C^X > 0$$

$$F_C^M > F_C^X > 0$$

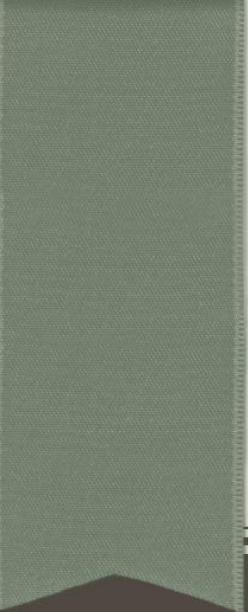
## Proposition 2

$$\pi^M = \pi^T = \pi^X$$

## Proposition 3

*A solution  $(r_C, r_S, F_C)$  is optimal to the platform's problem if and only if*

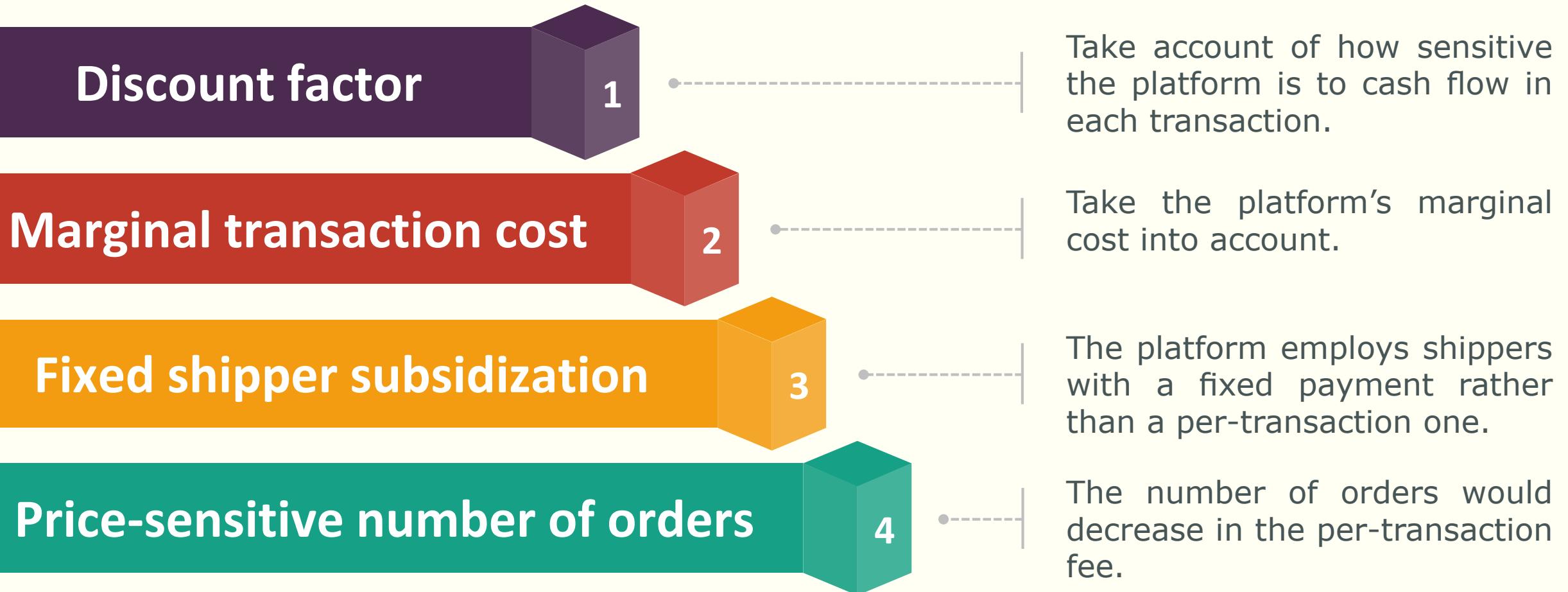
$$r_S = \frac{1}{9} \text{ and } r_C N + F_C = \frac{2}{9} N.$$



# EXTENSIONS

# Extensions

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# Extension 1 - Discount factor

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Let  $a \in [0,1]$

$$\pi_{discount} = \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right) (F_C + Na(r_C - r_S))$$

## Lemma 4

$$r_S^M = \frac{a^2+1}{18}, F_C^M = \frac{(\sqrt{1+a^2}(a+a^2)+3\sqrt{2}(1+a^2))}{36\sqrt{1+a^2}} N$$

## Proposition 4

$$\begin{aligned} r_S^T &= r_S^X > r_S^M \\ r_C^T &> r_C^X > 0 \\ F_C^M &> F_C^X > 0 \end{aligned}$$

## Lemma 5

$$r_S^T = \frac{1}{9}, r_C^T = \frac{2}{9}$$

## Lemma 6

$$r_S^X = \frac{1}{9}, r_C^X = \frac{1}{9}, F_C^X = \frac{1}{9} N$$

# Extension 1 - Discount factor

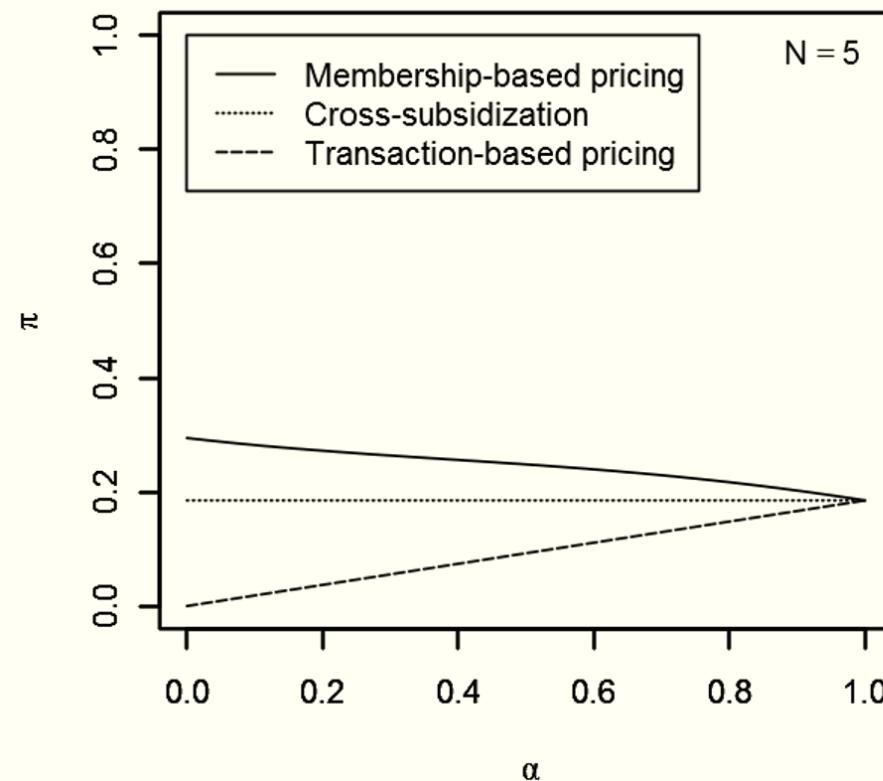
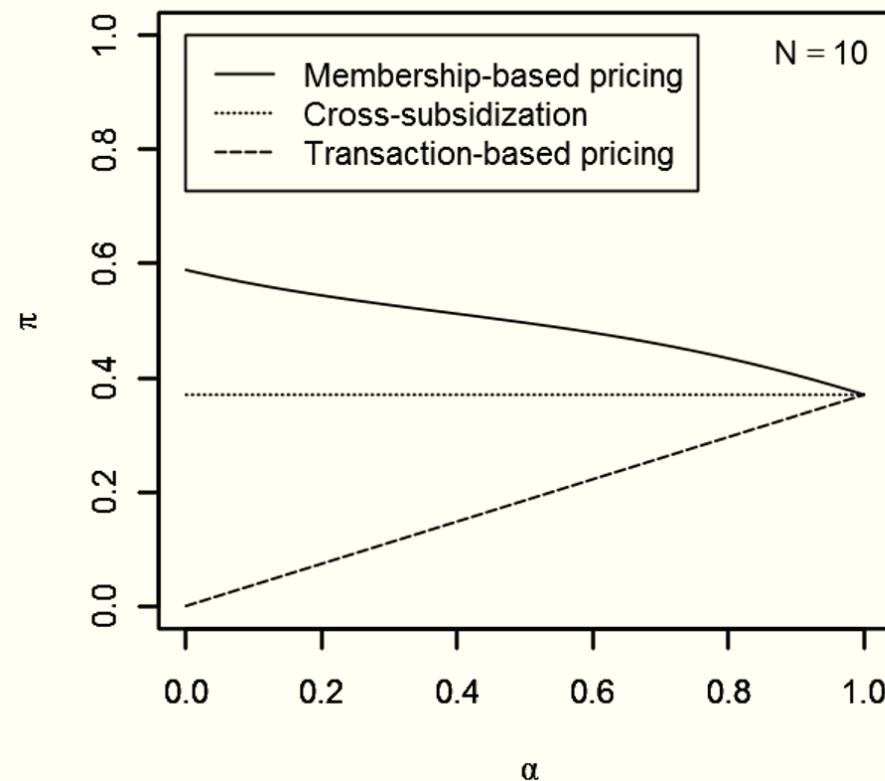
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## Proposition 5

$$\pi_{discount}^M > \pi_{discount}^X > \pi_{discount}^T$$

Why  
Instacart Express



## Extension 2 - Marginal transaction cost

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$$\pi_{cost} = \left(1 - \frac{\sqrt{r_S}(r_C N + F_C)}{r_S N}\right) (F_C + N(r_C - r_S - c))$$

**Lemma 7**

$$r_S^M = \frac{6c+1+\sqrt{12c+1}}{18}, F_C^M = \frac{(\sqrt{\sqrt{12c+1}+6c+1}(\sqrt{12c+1}+24c+1)+3\sqrt{2}(\sqrt{12c+1}+6c+1))}{36\sqrt{\sqrt{12c+1}+6c+1}} N$$

**Lemma 8**

$$r_S^T = \frac{6c+1+\sqrt{12c+1}}{18}, r_C^T = \frac{6c+1+\sqrt{12c+1}}{9}$$

**Lemma 9**

$$r_S^X = r_C^X = \frac{6c+1+\sqrt{12c+1}}{18}, F_C^X = \frac{6c+1+\sqrt{12c+1}}{18} N$$

## Extension 2 - Marginal transaction cost

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**Proposition 1**

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**Proposition 6**

$$r_S^X = r_S^M = r_S^T$$

$$r_C^T > r_C^X > 0$$

$$F_C^M > F_C^X > 0$$

**Proposition 2**

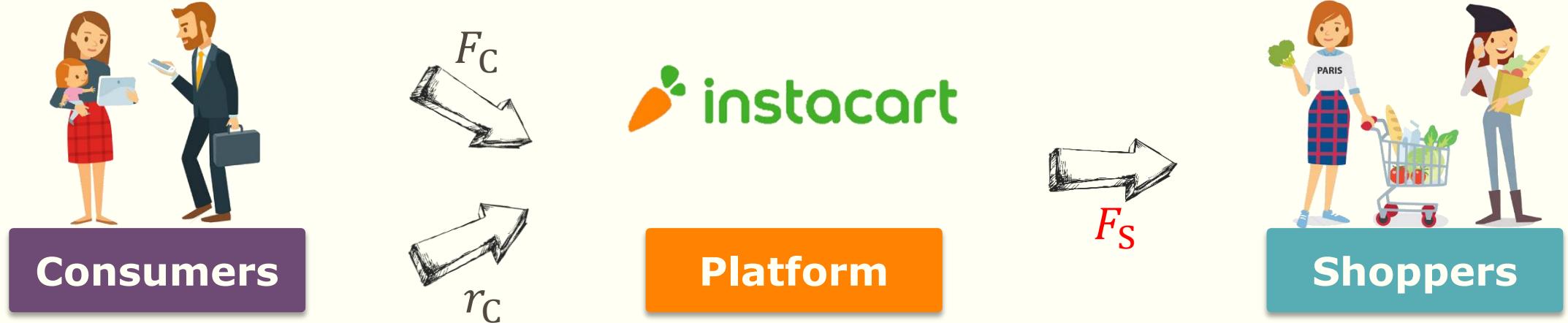
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**Proposition 7**

$$\pi_{cost}^M = \pi_{cost}^T = \pi_{cost}^X$$

## Extension 3 - Fixed shopper subsidization

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$$u_C = N(\theta Q - r_C) - F_C$$

$$\pi = Nan_C(r_C) + n_C F_C + n_S F_S$$

$$u_S = \frac{Nn_C}{n_S}(-\eta) - F_S$$

$$\pi_{fixed} = -\frac{F_S}{N} (Nar_C + F_C) + \left( \frac{r_C N + F_C}{F_S + N} \right)^2 F_S$$

# Extension 3 - Fixed shopper subsidization

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## Proposition 8

*When we set  $a = 1$ , a plan  $(r_C, F_C, F_S)$  is the platform's optimal solution if and only if*

$$F_S = -\frac{N}{3} \text{ and } r_C N + F_C = \frac{2}{9} N.$$

*Furthermore, no matter the platform subsidizes shippers with fixed or per-transaction subsidization, the platform's profits are the same (cf. proposition 3).*

Fixed  
subsidization

=

Per-transaction  
subsidization

## Extension 3 - Fixed shopper subsidization

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### Proposition 9 – part 1

*When we set  $a < 1$ , the platform's optimal solution under transaction-based and membership-based pricing strategies would be*

$$F_S^T = -\frac{N}{3}, r_C^T = \frac{2}{9}a \text{ and}$$

$$F_S^M = -\frac{N}{3}, F_C^M = \frac{2}{9}N.$$

*And the platform's optimal profits under these two strategies would have the following relation*

$$\pi_{fixed}^M > \pi_{fixed}^T.$$

# Extension 3 - Fixed shopper subsidization

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## Proposition 9 – part 2

*Furthermore, no matter the platform employs which pricing strategy here, subsidizing shippers with per-transaction subsidization is better for it, i.e.,*

$$\pi_{\text{discount}}^M > \pi_{\text{fixed}}^M \text{ and } \pi_{\text{discount}}^T > \pi_{\text{fixed}}^T.$$

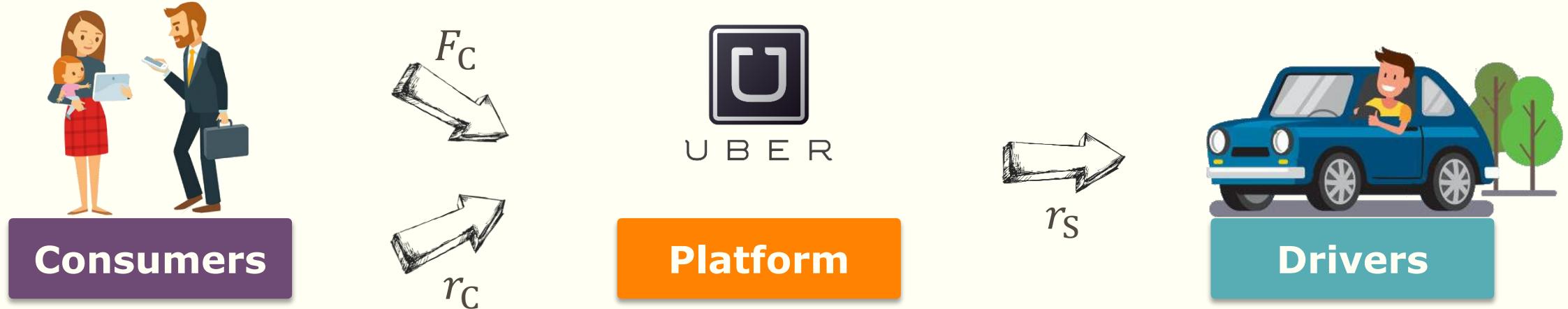
**Per-transaction  
subsidization**

>

**Fixed  
subsidization**

## Extension 4 - Price-sensitive number of orders

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$$u_C = \frac{N}{r_C} (\theta Q - r_C) - F_C$$

$$\pi = \frac{N}{r_C} n_C (r_C - r_S) + n_C F_C$$

$$u_S = \frac{N}{r_C} \frac{n_C}{n_S} (-\eta + r_S)$$

Capacity constraint:  $\frac{N}{r_C} \frac{n_C}{n_S} \leq K$

## Extension 4 - Price-sensitive number of orders

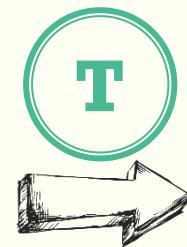
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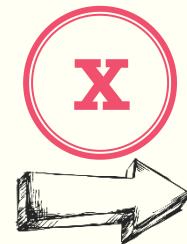
$$\max \left( 1 - \frac{\frac{F_C r_C}{N} + r_C}{\sqrt{r_S}} \right) \left( \frac{N}{r_C} (r_C - r_S) + F_C \right)$$

$$s.t. \quad r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N + F_C}$$



$$\max \left( 1 - \frac{r_C}{\sqrt{r_S}} \right) \left( \frac{N}{r_C} (r_C - r_S) \right)$$

$$s.t. \quad r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N}$$



$$\max \left( 1 - \frac{\frac{F_C r_C}{N} + r_C}{\sqrt{r_S}} \right) F_C$$

$$s.t. \quad F_C \geq \frac{N}{\sqrt{r_S}} - r_S^{\frac{3}{2}} K - N$$

## Extension 4 - Price-sensitive number of orders

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T

$$\max \quad \left(1 - \frac{r_C}{\sqrt{r_S}}\right) \left( \frac{N}{r_C} (r_C - r_S) \right)$$

$$s.t. \quad r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N}$$

$n_C$ : Number of consumers

$m$ : Earnings from each consumer

X

$$\max \quad \left(1 - \frac{\frac{F_C r_C}{N} + r_C}{\sqrt{r_S}}\right) F_C$$

$$s.t. \quad F_C \geq \frac{N}{\sqrt{r_S}} - r_S^{\frac{3}{2}} K - N$$

$$TC = \frac{N}{r_C} n_C$$

## Extension 4 - Price-sensitive number of orders

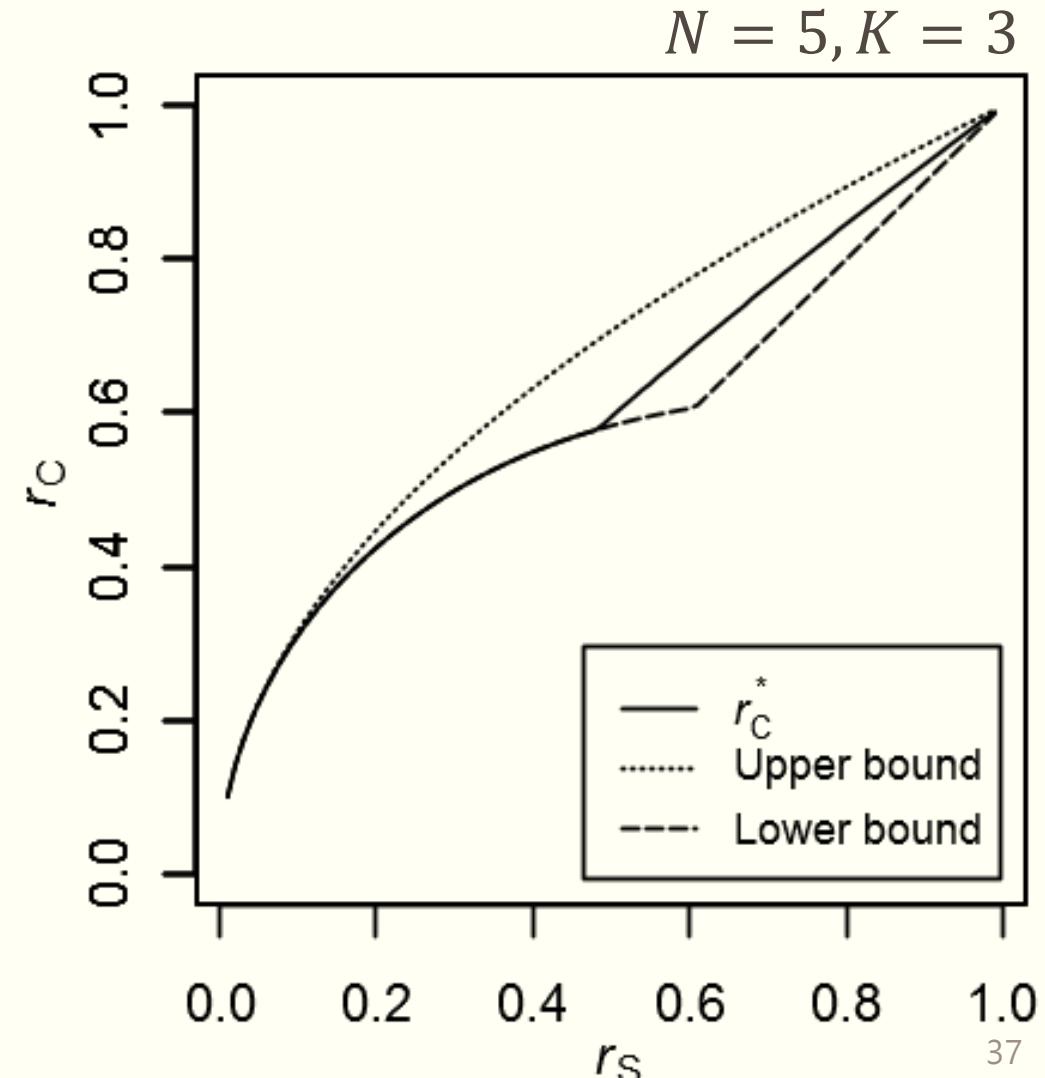
T

$$\max \quad \left(1 - \frac{r_C}{\sqrt{r_S}}\right) \left(\frac{N}{r_C} (r_C - r_S)\right)$$

$$s.t. \quad r_C \geq \frac{\sqrt{r_S} N}{r_S^{3/2} K + N}$$

$$1 - \frac{r_C}{\sqrt{r_S}} > 0 \text{ and } r_C - r_S > 0$$

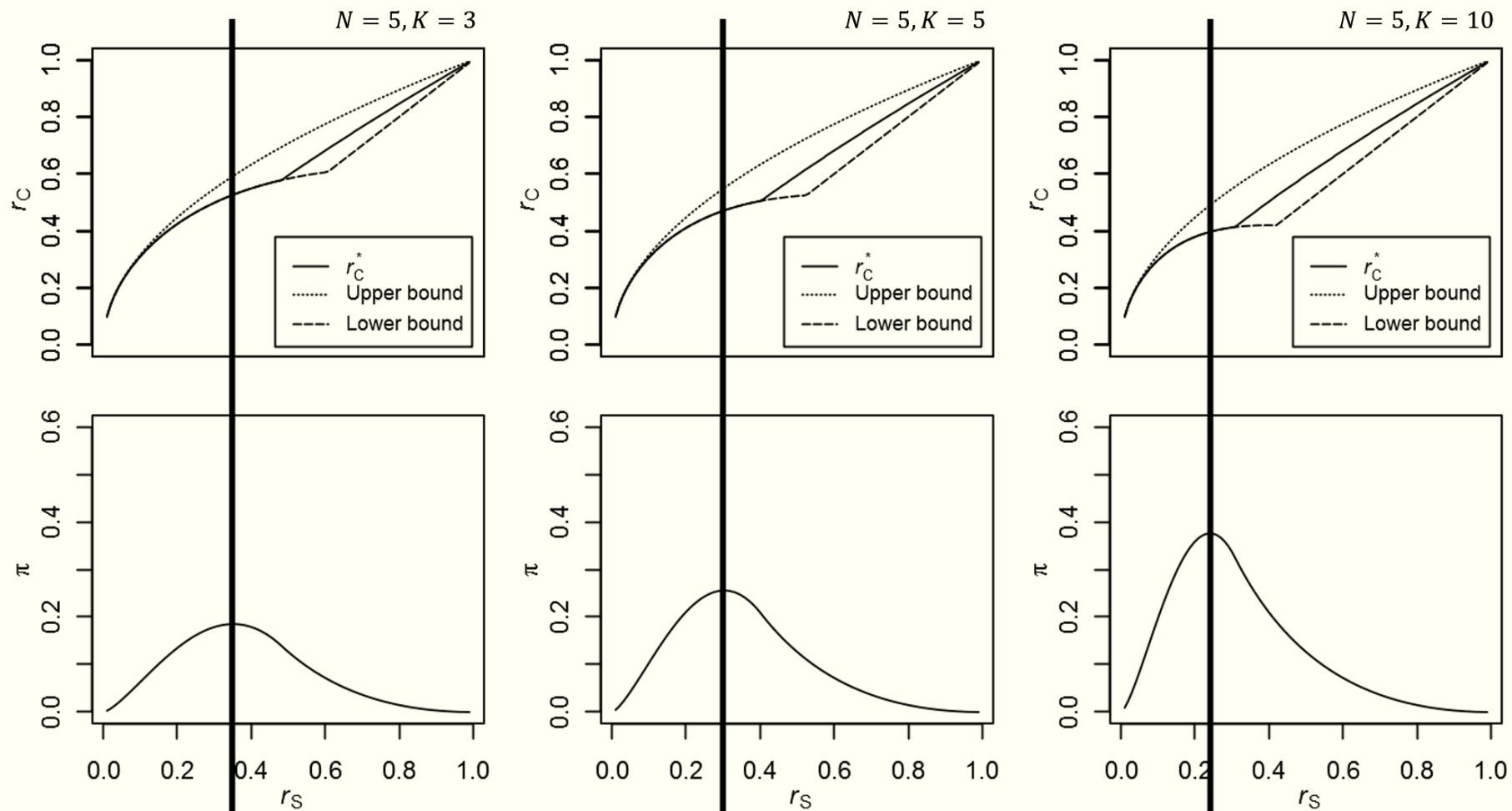
→  $r_S < r_C < \sqrt{r_S}$



# Extension 4 - Price-sensitive number of orders

## Observation 1

*Given an arbitrary set of  $N$  and  $K$ , the optimal solution would bind to the capacity constraint under pure-transaction pricing and cross-subsidization strategies.*



## Extension 4 - Price-sensitive number of orders

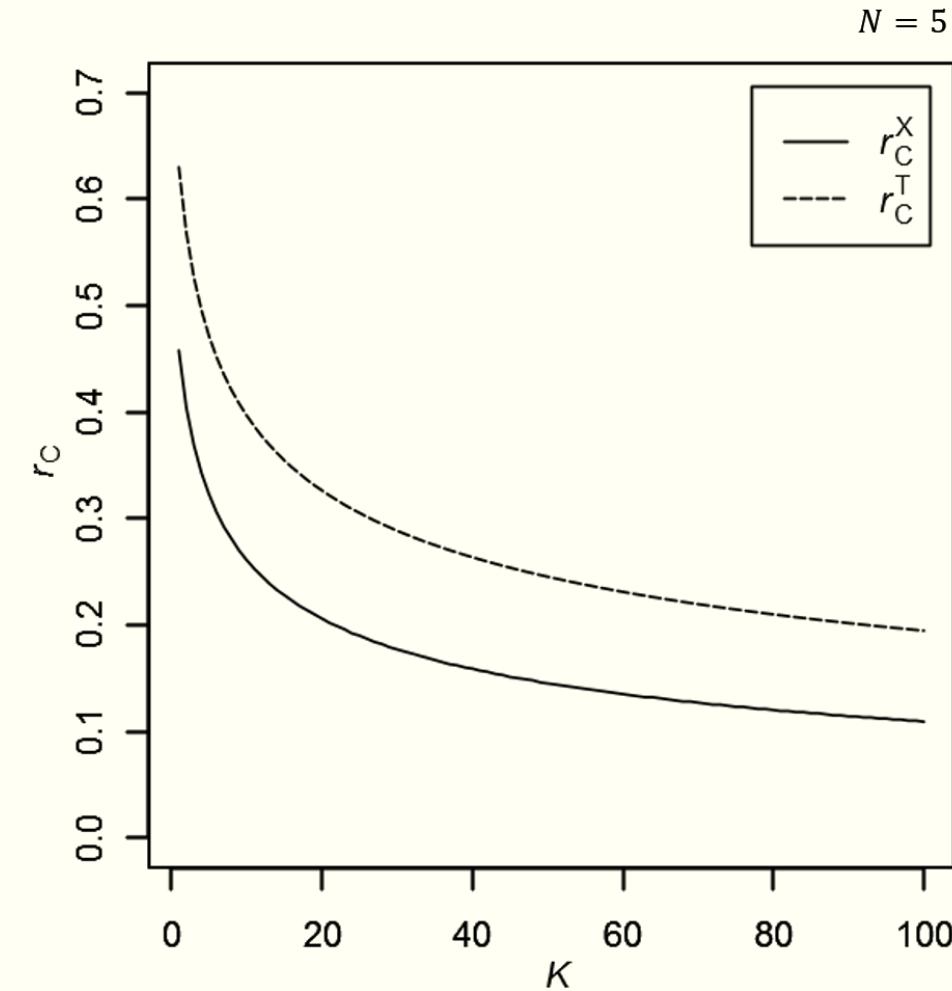
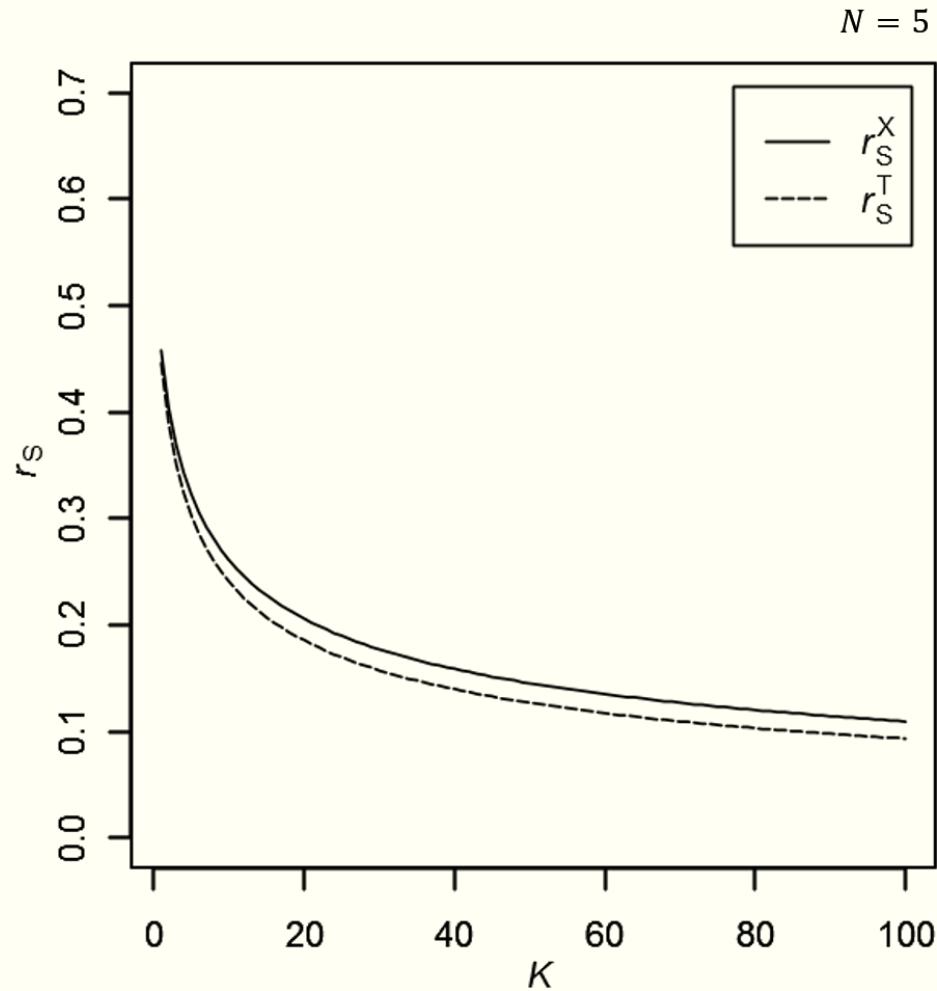
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### Observation 2

$$r_C^T > r_C^X \\ = r_S^X > r_S^T$$

$$n_S^X > n_S^T$$

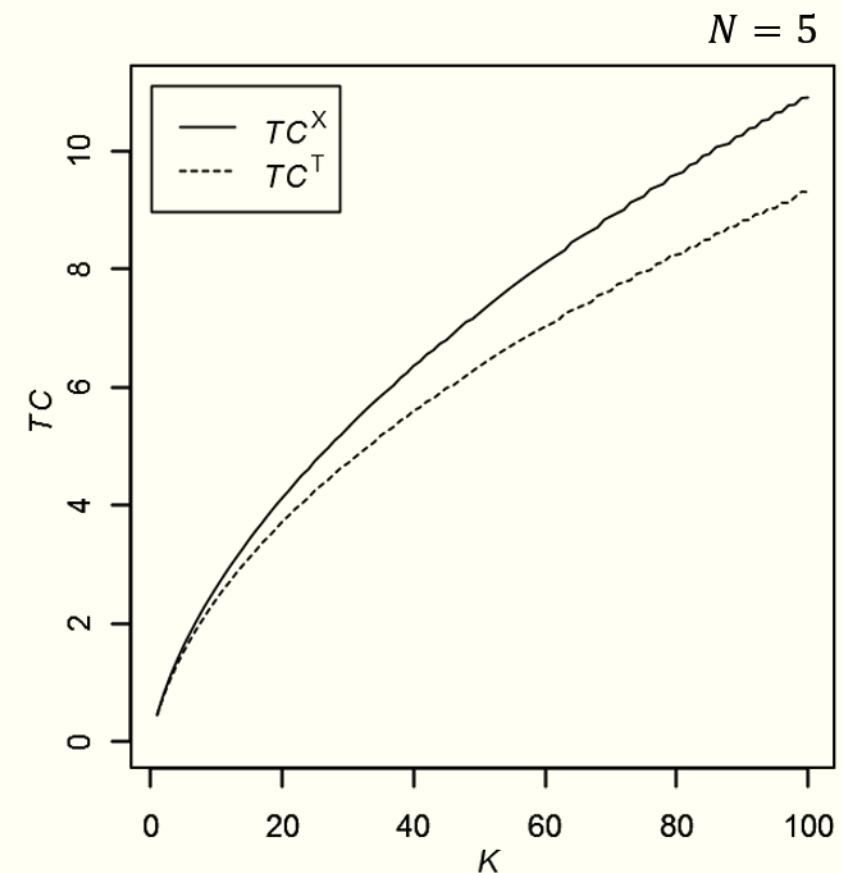
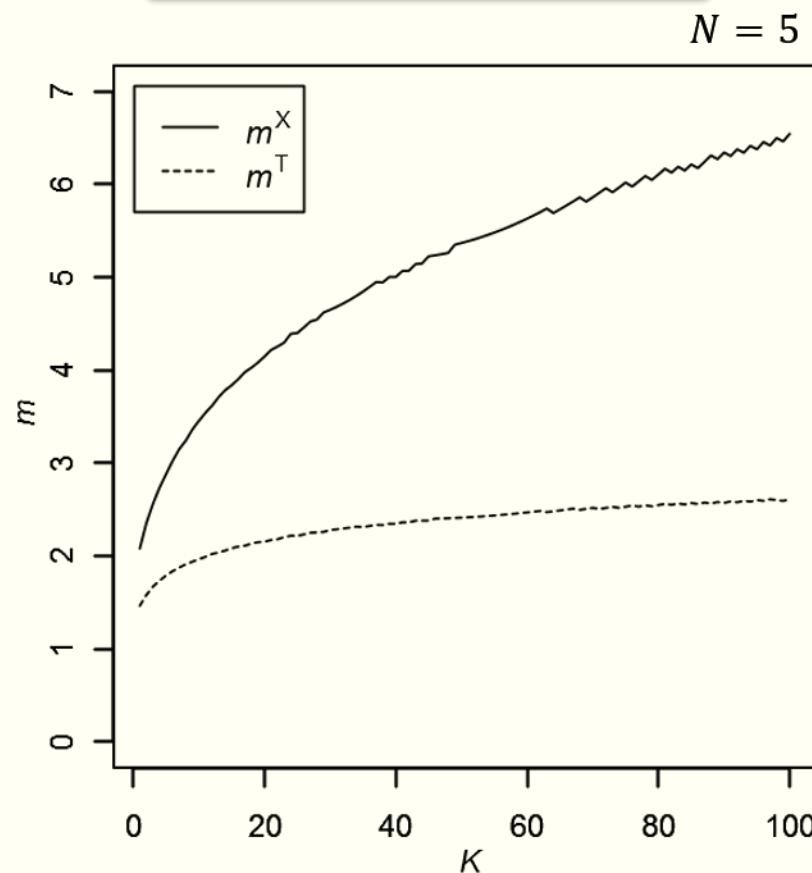
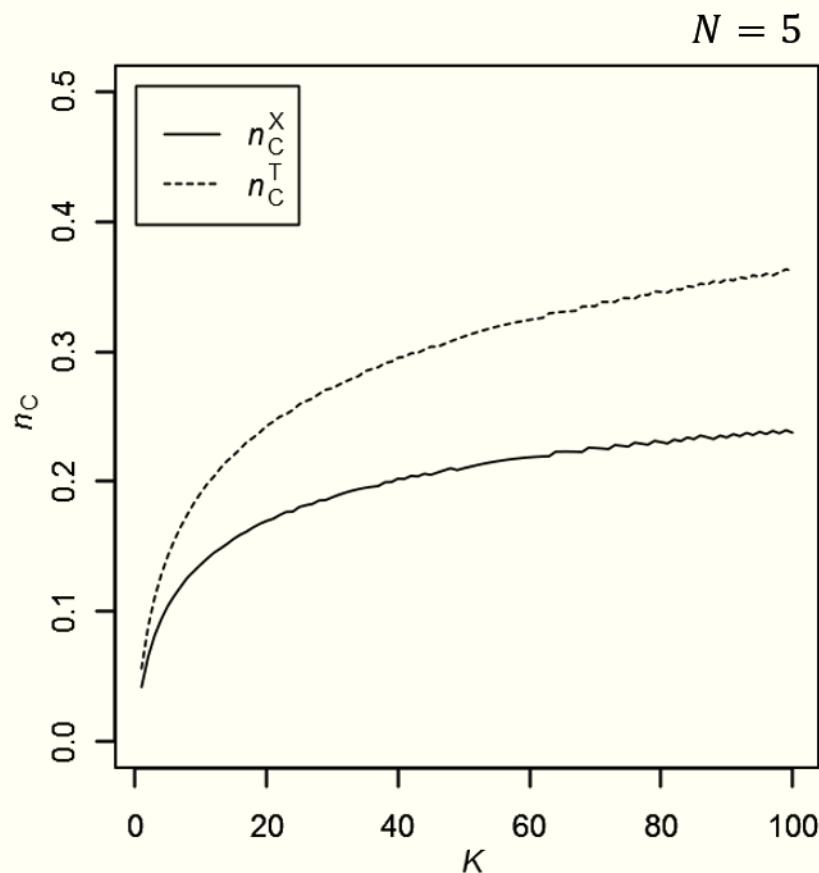


# Extension 4 - Price-sensitive number of orders

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## Observation 3



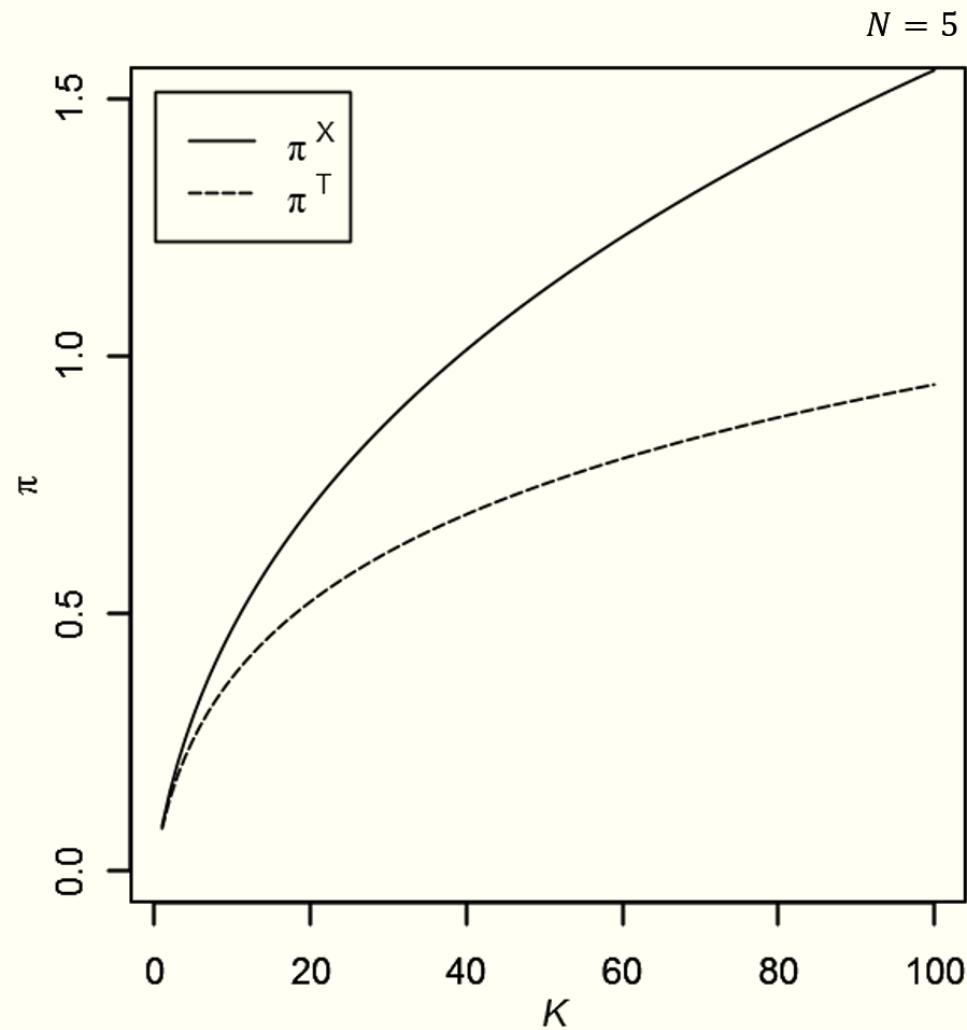
## Extension 4 - Price-sensitive number of orders

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### Observation 4

$$\pi^X > \pi^T$$

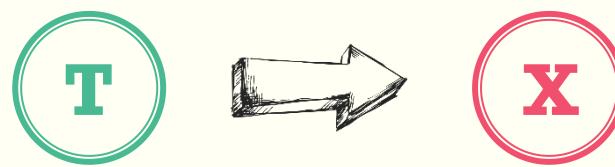


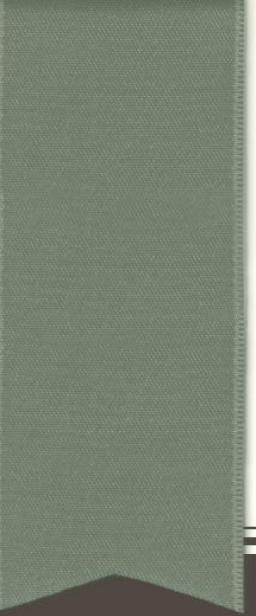
## Extension 4 - Price-sensitive number of orders

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	$r_S$	$r_C$	$n_S$	$n_C$	$m$	$TC$	$\pi$
X	higher		more		more	more	higher
T		higher		more			





# CONCLUSIONS

# Conclusions

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## 01 Basic model

All the three strategies are equivalent.

## 02 Extension 1

Discount factor: Membership-based pricing is the best.

## 03 Extension 2

Marginal cost: It does not matter.

## 04 Extension 3

Fixed subsidy rather than the per-transaction one: Per-transaction subsidy is better for platform to adopt.

## 05 Extension 4

Price-sensitive number of orders: Cross-subsidization strategy is better.

# Conclusions

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## 06 Future work

- Competition between integrated delivery and platform one.
- Take Uber's *surge pricing* strategy into account.



Q & A



& Thank You!