Matric distances:

Let X be an n-dimensional real vector space Rn ,

$$x_i, x_j \in \mathcal{X}, \quad x_i = (x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(n)})^T, \; x_j = (x_j^{(1)}, x_j^{(2)}, \ldots, x_j^{(n)})^T,$$

The LP distance of xi, xj is defined as: p >1

$$L_p(x_i, x_j) = (\sum_{l=1}^{n} |x_i^{(l)} - x_j^{(l)}|^p)^{\frac{1}{p}}$$

When p = 1, it is called Manhattan distance:

$$L_1(x_i, x_j) = \sum_{l=1}^{n} |x_i^{(l)} - x_j^{(l)}|$$

When p = 2, it is called Euclidean distance:

$$L_2(x_i, x_j) = (\sum_{l=1}^{n} |x_i^{(l)} - x_j^{(l)}|^2)^{\frac{1}{2}}$$

When p = ∞ , it is the maximum value of each coordinate distance:

$$L_\infty(x_i, x_j) = max_l \, |x_i^{(l)} - x_j^{(l)}|$$

Example

Given the three points $x_1 = (1,1)^T$, $x_2 = (5,1)^T$, $x_3 = (4,4)^T$, in two-dimensional space, we try to find the nearest neighbor of x_1 under the L_p distance when p takes different values.

Find the following Distances:

- The distance between x1 and x2:

- The Manhattan distance between x1 and x3.
- The Euclidean distance between x1 and x3.
- Minkowski distance between x1 and x3.