Mock Exam – Information Theory

Instructions

All answers must be clearly motivated. Since this is the practical part, it is expected from you to program your solution, preferably in Python. This is an open book exam. You can use the course material, the available solutions on Canvas, and browse the internet (classical mode and **not** AI-mode).

You are not allowed to ask for an answer from another human or AI-assistant (LLMs, agents, etc). Not sure yet about the Birds.

1 Example A: Optimal Binary Encoding

Consider a discrete information source X with the alphabet and probabilities given on the next page, together with three different binary encodings.

Answer the following questions for each of the provided encodings. You may use either a software script or explain your reasoning manually.

1. Determine the amount of information carried by each symbol.
2. Determine the information content of the source.
3. Determine the expected length of each of the three encodings.
4. Compare the encodings in terms of their performance and discuss their efficiency.
5. Identify which of the encodings can be used as prefix codes. Motivate your answer.
6. Determine the symbol lengths assigned by Shannon's binary encoding to this source.
7. Construct a binary prefix code for the source that cannot be further improved under the given constraints. Compare it to the three provided encodings.

Information source and binary encodings

Symbol	$p(x_i)$	Code 1	Code 2	Code 3
a	0.36	0	10	00
b	0.24	10	01	01
c	0.18	110	001	11
d	0.12	111	000	100
e	0.10	101	111	101

2 Example B: Channel Capacity of Independent Parallel Channels

A communication system uses independent parallel channels, each modelled as an AWGN channel. The bandwidth is identical for all channels. The signal and noise powers are listed on the additional page.

Answer the following questions. Explain carefully the assumptions behind your reasoning.

1. Determine the total information rate of the system for the given signal powers. Discuss the characteristics of the channels and the assumptions made in your calculation.
2. You are allowed to redistribute the total signal power across the channels. Determine a new distribution that increases the total information rate.
3. Compute the resulting information rate under your new power distribution.
4. Compare the two information rates and interpret the improvement.
5. Explain how the obtained results relate to the fundamental limits on communication over noisy channels.

Signal and noise parameters

Channel	Signal Power	Noise Power
1	12	5
2	7	18
3	15	9
4	5	30
5	20	4
6	11	14

Total available signal power: $P_{\text{tot}} = 70$

Bandwidth of each channel: $W = 25\,\text{kHz}$