

BER for AWGN Binary Symmetrical Channel

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1 Bit Error Rate (BER) for Additive White Gaussian Noise (AWGN) Binary Symmetric Channel (BSC)

The BER for for Additive White Gaussian Noise (AWGN) Binary Symmetric Channel (BSC) with signal amplitude A and additive noise variance σ^2 equals

$$BER = \frac{1}{2} \operatorname{erfc} \left(\frac{A}{\sqrt{2}\sigma} \right)$$

with the complementary error function

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

for $x > 0$.

To link the amplityde and the noise variance to the energy per bit E_b and the noise power spectral density N_0 , we need to use the relationsship that

$$E_b = A^2$$

for binary channels and that

$$N_0 = 2\sigma^2$$

Hence,

$$\frac{A^2}{2\sigma^2} = \frac{E_b}{N_0}$$

and therefore

$$BER = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right)$$

1.0.1 Note on the conversion to dB scale

A dB scale is a logarithmic scale to represent power (and other) ratio. If x is representing a power ratio P/P_0 , then the dB scale is

$$x_{dB} = 10 \log_{10}(x_{lin})$$

and

$$x_{lin} = 10^{x_{dB}/10}$$

If e.g. Voltage ratio's are used, then it is known that $P \propto V^2$. Hence, if x is presenting a voltage ratio V/V_0 , then

$$20 \log_{10}(x_{lin})$$

needs to be used.

[4]: Text(0.5, 0, 'Eb/N0 (dB)')

