Entropy of continuous random variables

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1 Entropy of continous random variables

1.1 Entropy of uniform distribution

Consider a uniform distributed random variable X. Determine 1. the variance (=power) and the entropy for a uniform distribution beween 0 and 1. 2. the variance and the entropy for a uniform distribution beween -1/2 and +1/2. 3. the variance and the entropy for a uniform distribution beween -2 and +2.

Note that 1. neither the variance nor the entropy change when an offset is applied (cases 1 and 2) 2. the variance scales as a^2 and the entropy is offsetted by $\log(|\det(a)|)$ when scaling the random variable with a factor a.

```
X ~ U[0.00,1.00]:
var = 0.08333; H(X) = 0.00000 bit
X ~ U[-0.50,0.50]:
var = 0.08333; H(X) = 0.00000 bit
X ~ U[-2.00,2.00]:
var = 1.33333; H(X) = 2.00000 bit
log(|det(4)|) =  2.00000
```

1.2 Entropy of Gaussian distribution

Consider a uniform distributed random variable X. Determine 1. the variance (=power) and the entropy for a Gaussian distribution with mean = 1 and variance = 1. 2. the variance and the entropy for a Gaussian distribution with zero mean and unity variance. 3. the variance and the entropy for a Gaussian distribution with zero mean and variance = 16.

Note that 1. neither the variance nor the entropy change when an offset is applied (cases 1 and 2) 2. the variance scales as a^2 and the entropy is offsetted by $\log(|\det(a)|)$ when scaling the random variable with a factor a.

```
X ~ N[0.00,1.00]:
var = 1.00000; H(X) = 2.04710 bit
X ~ N[0.00,1.00]:
var = 1.00000; H(X) = 2.04710 bit
X ~ N[0.00,16.00]:
var = 16.00000; H(X) = 4.04710 bit
log(|det(4)|) =  2.00000
```

1.3 Entropy of independent Gaussian distributions

The entropy of a set of n independent, zero-mean Gaussian distributed random variables $x_1, \dots, x_n$ with variances $\sigma_{x_1}^2, \dots, \sigma_{x_n}^2$ equals

$$\sum_{i=1}^{n} \frac{1}{2} \log(2\pi\sigma_{x_i}^2) + \frac{1}{2} \log(e).$$

Consider independent Gaussian distributions with a variance of $\sigma^2 = [1, 2, 4, 9, 3]$. Determine the total entropy of these distributions.

```
Total Entropy = 11.22753 bit
```

1.4 Gaussian distribution maximizes the entropy under fixed power constraint

Consider a uniform and a Gaussian with identical variances. Compute the entropy for equal variances and note that the entropy of the Gaussian distribution is the largest.

```
X ~ U[-1.73,1.73]:
var = 1.00000; H(X) = 1.79248 bit
X ~ N[0.00,1.00]:
var = 1.00000; H(X) = 2.04710 bit
```