Shannon Hartley theorem for independent variables

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1 Shannon Hartley theorem for independent variables

1.1 Channel capacity of a Gaussian channel

Assume a Gaussian channel with a signal variance $\sigma_x^2 = 2$ and the noise variance for $\sigma_n^2 = [1, 2, 4, 9, 3]$. Determine the individual channel capacities and the total channel capacity for independent Gaussian channels.

```
Individual capacities= [0.79248125 0.5        0.29248125 0.14475331 0.3684828 ]
bit/sample
Total capacity= 2.09820 bit/sample
```

1.2 Waterfilling algorithm

Consider that the noise on the channels are independent Gaussian distributions with variance/power of $\sigma_n^2 = [1, 2, 4, 9, 3]$. Assume furthermore that a total signal power of all the independent sigals $P_i = \sigma_{xi}^2$ equals

$$P_{tot} = \sum_i P_i = 10.$$

Allocate the transmit powers P_i to maximize the channel capacity using the waterfilling algorithm.

Compute the total channel capacity and remark that this capacity is larger than when a uniform power level is used (previous excercise).

```
Intermediate result:
B = 5.8
Px = [ 4.8  3.8  1.8 -3.2  2.8]
Intermediate result:
B = 5.0
Px = [4. 3. 1. 0. 2.]
Power Allocation: [4. 3. 1. 0. 2.]
Total capacity= 2.35137 bit/sample
```

1.3 Power distribution for independent parallel channels

Consider a wireless communication with the following properties

- the systems uses 8 different frequency carriers and all carriers can be considered as indepen-dent channels

- the channels have a frequency dependent behavior due to multipath introduced by relections described by the transfer functions

- $|H_i| = [100, 50, 10, 3, 1, 2, 15, 25] * 10^{-6}$

- the bandwidth is identical for each channel and equals 1kHz
  the noise power spectral density equals $N_0 = k_B T$ for all channels with Boltzmann's constant $k_B = 1.380\text{x}10^{-23} JK^{-1}$ and a room temperature of $T = 300K$.

Determine

1. The noise power for each independent noise carrier
2. Determine the channel capacity (in bits/s) if a signal power of $P_{tot} = 1\mu W$ is uniformly distributed over all carriers

```
Individual capacities= [8242.85192252 6257.06893955 2006.95276064  346.80276353
42.91497382
  164.49403763 2962.26735264 4312.5760875 ] bit/s
Total capacity= 24335.92884 bit/s
```

Determine the optimal power distribution amongst the carriers and compute the resulting channel capacity.

```
Intermediate result:
B = 8.3793675e-07
Px = [ 8.37522750e-07  8.36280750e-07  7.96536750e-07  3.77936750e-07
 -3.30206325e-06 -1.97063250e-07  8.19536750e-07  8.31312750e-07]
Intermediate result:
B = 2.54749e-07
Px = [ 2.54335e-07  2.53093e-07  2.13349e-07 -2.05251e-07  0.00000e+00
  0.00000e+00  2.36349e-07  2.48125e-07]
Intermediate result:
B = 2.136988e-07
Px = [2.132848e-07 2.120428e-07 1.722988e-07 0.000000e+00 0.000000e+00
 0.000000e+00 1.952988e-07 2.070748e-07]
Power Allocation: [2.132848e-07 2.120428e-07 1.722988e-07 0.000000e+00
0.000000e+00
 0.000000e+00 1.952988e-07 2.070748e-07]
Total capacity= 26940.87424 bit/s
```