Gabarito Lista de Exercícios 3

① (a) $\omega^2 \neq \omega_0^2$:

$$u(t) = \frac{\cos(\omega t)}{\omega_0^2 - \omega^2} + c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t);$$

$\omega^2 = \omega_0^2$:

$$u(t) = \frac{t \sin(\omega_0 t)}{2\omega_0} + c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t);$$

(b) $y(t) = t^2 - 6t + 14 - 3(\sin t)/10 - 9(\cos t)/10 + c_1 e^{-t/2} + c_2 e^{-t}$;

(c) $y(t) = -(t + 2/3)e^{2t} + c_1 e^{-t} + c_2 e^{3t}$.

② (a) $y(x) = -2x^2 + c_1 x + c_2 x e^x$;

(b) $y(x) = x^2 (\ln x)^3/6 + c_1 x^2 + c_2 x^2 \ln x$;

(c) $y(x) = \sqrt{x}\,[(\ln x)\ln(\ln x) - 1] + c_1/\sqrt{x} + c_2 \sqrt{x} \ln x$.

③ (a) Singularidade regular: $x = 0$. (Todos os outros pontos são ordinários.) Índices: $\lambda_+ = \lambda_- = 0$. Soluções L.I.:

$$y_1(x) = \sum_{n=0}^{\infty} a_n x^n = 1 + x + \frac{x^2}{4} + \dots, \quad a_0 = 1, \quad a_{n+1} = \frac{a_n}{(n+1)^2},$$

$$\begin{aligned} y_2(x) &= y_1(x) \ln|x| + \sum_{n=0}^{\infty} b_n x^n \\ &= y_1(x) \ln|x| - \left(2x + \frac{3x^2}{4} + \frac{11x^3}{108} + \dots\right), \\ &b_0 = 0, \quad b_{n+1} = \frac{b_n}{(n+1)^2} - \frac{2a_{n+1}}{(n+1)}; \end{aligned}$$

(b) Singularidade regular: $x = 0$. (Todos os outros pontos são ordinários.) Índices: $\lambda_+ = 1$, $\lambda_- = 0$. Soluções L.I.:

$$y_1(x) = x\sum_{n=0}^{\infty} a_n x^n = x - \frac{x^2}{2} + \frac{x^3}{12} + ..., \;\; a_0 = 1, \;\; a_{n+1} = \frac{-a_n}{(n+2)(n+1)},$$

$$\begin{aligned} y_2(x) &= a y_1(x)\ln|x| + \sum_{n=0}^{\infty} b_n x^n \\ &= -y_1(x)\ln|x| + \left(1 - \frac{3x^2}{4} + \frac{7x^3}{36} + ...\right), \\ &a = -1, \;\; b_0 = a_0, \;\; b_1 = 0, \;\; b_{n+2} = -\frac{[b_{n+1} - (2n+3)a_{n+1}]}{(n+2)(n+1)}; \end{aligned}$$

(c) Singularidades regulars: $x = 0$ e $x = 1$. (Todos os outros pontos são ordinários.) Índices: $\lambda_+ = 1$, $\lambda_- = 0$ para ambos. Soluções L.I.:

$$y_1(x) = x\sum_{n=0}^{\infty} a_n x^n = x + \frac{x^2}{2} + \frac{x^3}{4} + ..., \;\; a_0 = 1, \;\; a_{n+1} = \frac{(n^2+n+1)a_n}{(n+2)(n+1)},$$

$$\begin{aligned} y_2(x) &= a y_1(x)\ln|x| + \sum_{n=0}^{\infty} b_n x^n \\ &= y_1(x)\ln|x| + \left(1 - \frac{x^2}{4} - \frac{x^3}{12} + ...\right), \\ &a = 1, \;\; b_0 = a_0, \;\; b_1 = 0, \;\; b_{n+2} = \frac{(n^2+n+1)b_{n+1}}{(n+2)(n+1)} + \frac{(2n^2+2n-1)a_n}{(n+2)^2(n+1)^2}. \end{aligned}$$

④ (a) $\lambda_\pm = \pm\nu$;

(b) $\nu = 0$:

$$y_1(x) = \sum_{n=0}^{\infty} a_n x^n,$$

$$y_2(x) = y_1(x)\ln|x| + \sum_{n=0}^{\infty} b_n x^n;$$

$\nu = k/2$, com $k \in \mathbb{Z}^*$:

$$y_1(x) = |x|^{|k|/2}\sum_{n=0}^{\infty} a_n x^n,$$

$$y_2(x) = a y_1(x) \ln |x| + |x|^{-|k|/2} \sum_{n=0}^{\infty} b_n x^n;$$

$\nu \neq k/2$, para todo $k \in \mathbb{Z}$:

$$y_1(x) = |x|^{\nu} \sum_{n=0}^{\infty} a_n x^n,$$

$$y_2(x) = |x|^{-\nu} \sum_{n=0}^{\infty} b_n x^n.$$

(c) $\nu = 0$: Vide seção 5.7 do livro do Boyce&DiPrima (9^a ed.);
$\nu = 1/4$:

$$y_1(x) = |x|^{1/4} \sum_{n=0}^{\infty} a_n x^n = |x|^{1/4} \left(1 - \frac{x^2}{5} + \frac{x^4}{90} + ...\right),$$

$$a_0 = 1, \;\; a_1 = 0, \;\; a_{n+2} = \frac{-a_n}{(n+5/2)(n+2)},$$

$$y_2(x) = |x|^{-1/4} \sum_{n=0}^{\infty} b_n x^n = |x|^{-1/4} \left(1 - \frac{x^2}{3} + \frac{x^4}{42} + ...\right),$$

$$b_0 = 1, \;\; b_1 = 0, \;\; b_{n+2} = \frac{-b_n}{(n+3/2)(n+2)};$$

$\nu = 3/2$:

$$y_1(x) = |x|^{3/2} \sum_{n=0}^{\infty} a_n x^n = |x|^{3/2} \left(1 - \frac{x^2}{10} + \frac{x^4}{280} + ...\right),$$

$$a_0 = 1, \;\; a_1 = 0, \;\; a_{n+2} = \frac{-a_n}{(n+5)(n+2)},$$

$$y_2(x) = a y_1(x) \ln |x| + |x|^{-3/2} \sum_{n=0}^{\infty} b_n x^n = |x|^{-3/2} \left(1 - \frac{x^2}{3} + \frac{x^4}{42} + ...\right),$$

$$a = 0, \;\; b_0 = 1, \;\; b_1 = 0, \;\; b_{n+2} = \frac{-b_n}{(n-1)(n+2)};$$

$\nu = 2$:

$$y_1(x) = x^2 \sum_{n=0}^{\infty} a_n x^n = x^2 - \frac{x^4}{12} + \frac{x^6}{384} + ...,$$

$$a_0 = 1, \;\; a_1 = 0, \;\; a_{n+2} = \frac{-a_n}{(n+6)(n+2)},$$

$$y_2(x) = a y_1(x) \ln|x| + x^{-2} \sum_{n=0}^{\infty} b_n x^n = -\frac{1}{16} y_1(x) \ln|x| + \frac{1}{x^2} + \frac{1}{4} - \frac{x^4}{288} + ...,$$

$$a = -1/16, \;\; b_0 = a_0, \;\; b_1 = b_3 = b_4 = 0, \;\; b_2 = \frac{b_0}{4},$$

$$b_{n+4} = -\frac{[8b_{n+2} - (n+2)a_n]}{8n(n+4)}.$$

⑤ (a) $n!/(s-a)^{n+1}$, $s > a$;
(b) e^{-as}/s, $s > 0$;
(c) $n!\, e^{-as}/s^{n+1}$, $s > 0$;
(d) $(e^{-as} - e^{-bs})/s$, $s > 0$;
(e) $1/[s(1-e^{-s})]$, $s > 0$;
(f) e^{-as}.

⑥ (a) $e^{(2-t)}(t-2)u_2(t)$;
(b)$[e^{3(t-1)} + e^{(t-1)}]u_1(t)/2$;
(c) $e^{-t/2}(\cos t)/2$.