How to Specify it!

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Autumn School

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Imagine testing **`reverse`**...

Comparison operator displaying a message if not equal

```
test_Reverse =
  reverse [1,2,3] === [3,2,1]
```

Function under test

Sample argument

Expected result

Imagine testing **`reverse`**... with QuickCheck

Tell QuickCheck to generate random ***lists of integers***

```
prop_Reverse :: [Int] -> Property
prop_Reverse xs =
  reverse xs === ???
```

A random argument

But ***what do we put here?***

Imagine testing **`reverse`**... with QuickCheck

```
prop_Reverse :: [Int] -> Property
prop_Reverse xs =
  reverse xs === predictReverse xs
```

*This is **not easier to write** than* **`reverse`***!*

*We're likely to make the **same mistakes***

Replicating the code in the tests...

Expensive! **Low value!**

What can we do instead?

```
prop_Reverse :: [Int] -> Property
prop_Reverse xs =
  reverse (reverse xs) === xs
```

Check a ***property*** *of the return value instead*

```
*Reverse> quickCheck prop_Reverse
+++ OK, passed 100 tests.
```

100 random lists!

```
reverse xs = xs
```

Passes even if we defined...

```
*Reverse> quickCheck test_Reverse
*** Failed! Falsified (after 1 test):
[1,2,3] /= [3,2,1]
```

```
prop_Wrong :: [Int] -> Property
prop_Wrong xs = reverse xs === xs
```

```
*Reverse> quickCheck prop_Wrong
*** Failed! Falsified (after 3 tests and 3 shrinks):
[0,1]
[1,0] /= [0,1]
```

Counterexample: Almost always [0,1], sometimes [1,0]

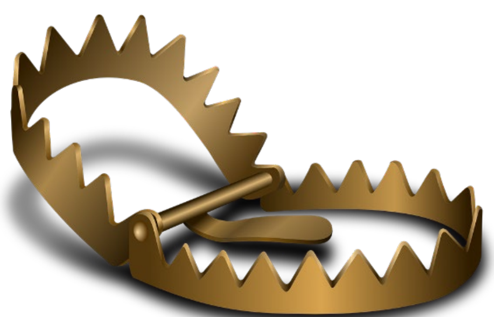
Shrinking

- Discards unnecessary list elements (we need at least two)
- Replaces integers by smaller integers (we need *distinct* integers, {0,1} are the two smallest)

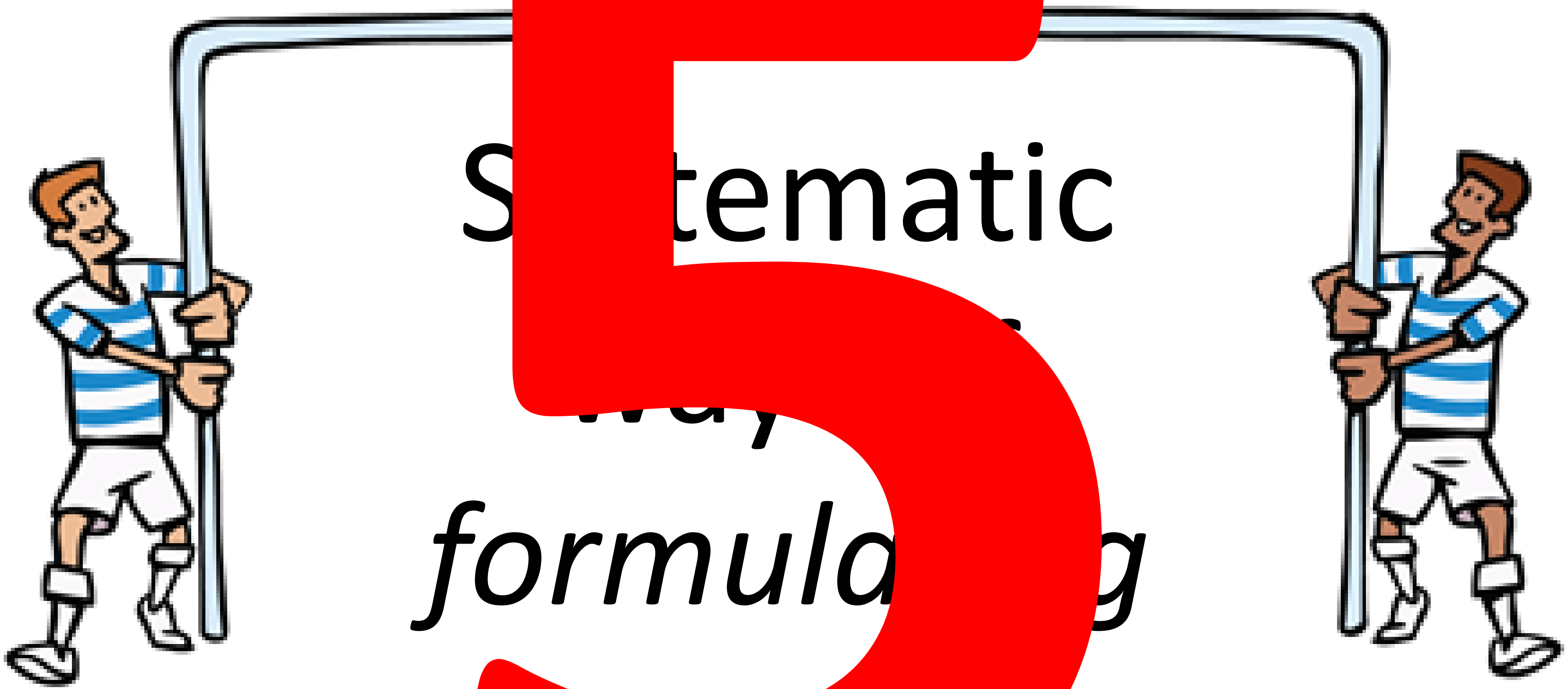
Property Based Testing

- Random generation of *lots* of test cases

- Shrinking results in *minimal* counterexamples—easy to debug



- *Replicating code in the tests* is tempting, but expensive, and low value



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Example

```
data BST k v = Leaf
             | Branch (BST k v) k v (BST k v)
  deriving (Eq, Show, Generic)

-- the operations under test
find   :: Ord k => k           -> BST k v -> Maybe v

nil    ::                                    BST k v
insert :: Ord k => k -> v      -> BST k v -> BST k v
delete :: Ord k => k           -> BST k v -> BST k v
union  :: Ord k => BST k v -> BST k v -> BST k v

-- auxiliary operations
toList :: BST k v -> [(k, v)]
keys   :: BST k v -> [k]
```

1 Is there an *invariant*?

```
valid :: Ord k => BST k v -> Bool

valid Leaf = True

valid (Branch l k v r) =
  valid l && valid r &&
  all (<k) (keys l) && all (>k) (keys r)
```

Invariant properties

```
prop_NilValid = valid (nil :: Tree)

prop_InsertValid :: Key -> Val -> Tree -> Bool
prop_InsertValid k v t = valid (insert k v t)

prop_DeleteValid :: Key -> Tree -> Bool
prop_DeleteValid k t = valid (delete k t)

prop_UnionValid :: Tree -> Tree -> Bool
prop_UnionValid t t' = valid (union t t')
```

```
type Key = Int
type Val = Int
type Tree = BST Key Val
```

2

What is the *postcondition*?

”After calling **`insert`**, we should be able to **`find`** the key inserted, and any other keys present beforehand”

```
prop_InsertPost k v t k' =
  find k' (insert k v t)
  ===
  if k==k' then Just v else find k' t
```

Rarely true?

What is the postcondition of `find`?

”After calling `find`,

—if the key is present in the tree, the result is `Just value`

—if the key is not present, the result is `Nothing`”

How can we tell this?

By construction!

Don't test for presence—ensure by construction

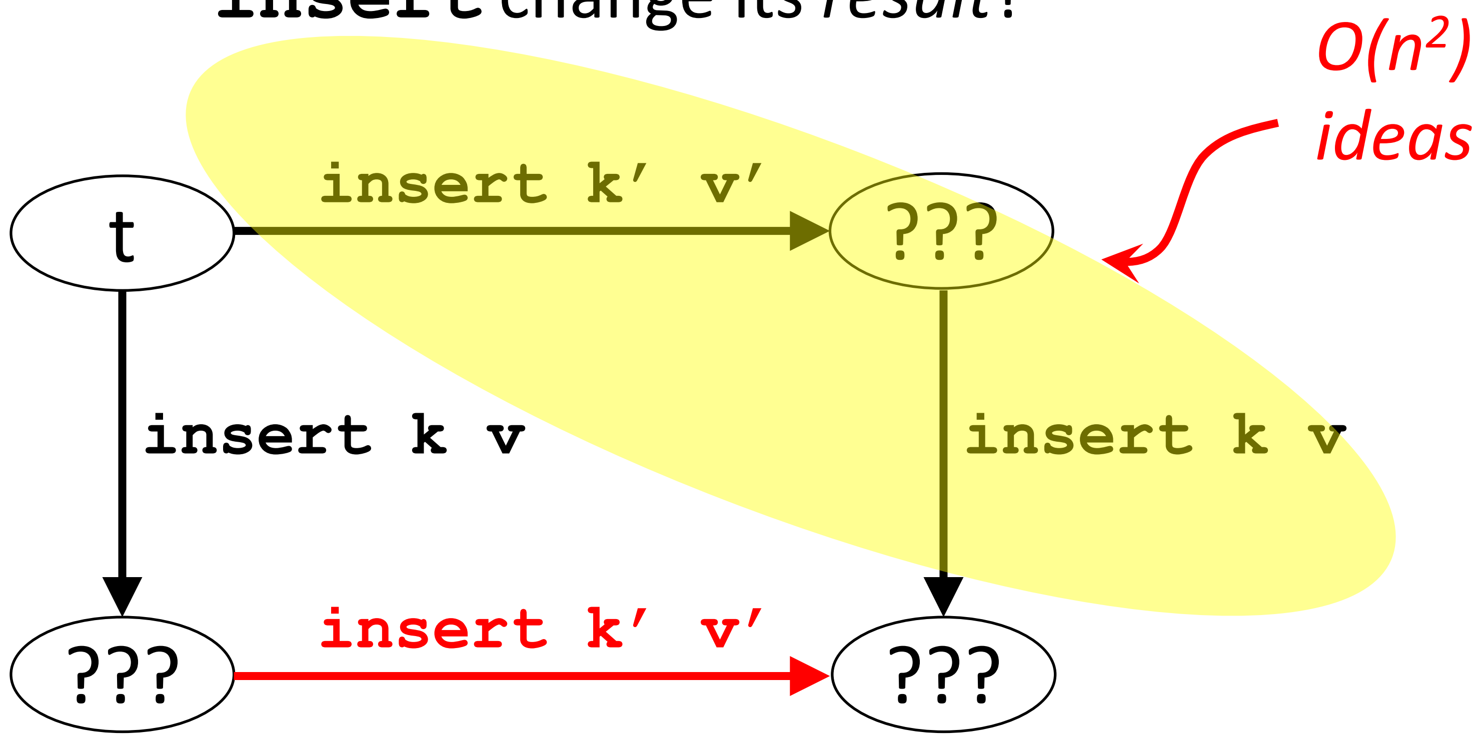
*Can **every** tree containing* **`k`** *be expressed in this form?*

```
prop_FindPostPresent k v t =
  find k (insert k v t) === Just v


prop_FindPostAbsent k t =
  find k (delete k t) === Nothing
```

3 Metamorphic properties

”How does changing the *input* of **`insert`** change its *result*?”



Modified call

```
prop_InsertInsert (k,v) (k',v') t =
  insert k v (insert k' v' t)
  ===
  insert k' v' (insert k v t)
```

Original call

Relationship

```
prop_InsertInsert (k,v) (k',v') t =
  insert k v (insert k' v' t)
  ===
  insert k' v' (insert k v t)
```

Is this really true?

TEST IT!!!

```
=== prop_InsertInsert from BSTSpec.hs:78 ===
*** Failed! Falsified (after 2 tests and 5 shrinks):
(0,0)
(0,1)
Leaf
Branch Leaf 0 0 Leaf /= Branch Leaf 0 1 Leaf
```

Last insertion wins

```
prop_InsertInsert (k,v) (k',v') t =
  insert k v (insert k' v' t)
  ===
  if k==k' then insert k v t else
  insert k' v' (insert k v t)
```

```
=== prop_InsertInsert from BSTSpec.hs:78 ===
*** Failed! Falsified (after 2 tests):
(1,0)
(0,0)
Leaf
Branch Leaf 0 0 (Branch Leaf 1 0 Leaf) /=
Branch (Branch Leaf 0 0 Leaf) 1 0 Leaf
```

Equivalence for trees

```
t1 =~= t2 =
  toList t1 === toList t2
```

```
prop_InsertInsert (k,v) (k',v') t =
  insert k v (insert k' v' t)
  =~=
  if k==k' then insert k v t else
  insert k' v' (insert k v t)
```

4

Inductive properties

```
prop_UnionNil t =
  union nil t === t

prop_UnionInsert t t' (k,v) =
  union (insert k v t) t'
  =~=
  insert k v (union t t')
```

`union` *must be correct, by induction on the first argument*

Completeness: can every tree be constructed just using **`insert`**?

```
insertions Leaf = []
insertions (Branch l k v r) =
  (k,v):insertions l++insertions r
```

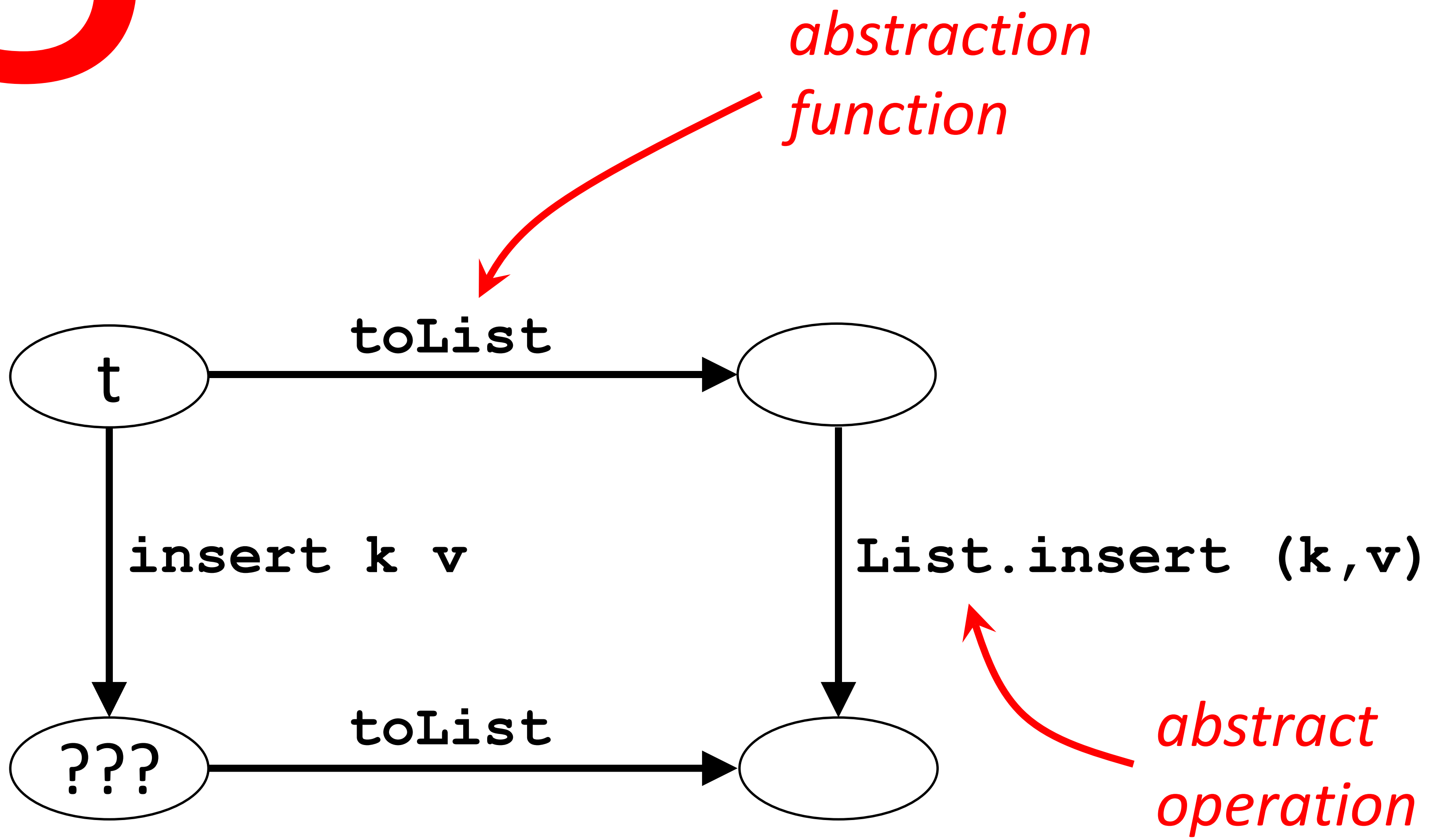
```
prop_InsertComplete t =
  t
  ===
  foldl (\t (k,v)-> insert k v t) nil (insertions t)
```

Structurally equal to!

```
prop_InsertCompleteForDelete k t =
  prop_InsertComplete (delete k t)
```

```
prop_InsertCompleteForUnion t t' =
  prop_InsertComplete (union t t')
```

5 Model-based properties



```
prop_InsertModel (k,v) t =
  toList (insert k v t)
  ===
  L.insert (k,v) (toList t)
```

```
*BSTSpec> quickCheck prop_InsertModel
*** Failed! Falsified (after 13 tests and 7 shrinks):
(1,0)
Branch Leaf 1 0 Leaf
[(1,0)] /= [(1,0),(1,0)]
```

duplicated key

```
prop_InsertModel (k,v) t =
  toList (insert k v t)
  ===
  L.insert (k,v) (deleteKey k $ toList t)
```

Acta Informatica 1, 271—281 (1972)

Proof of Correctness of Data Representations

C. A. R. Hoare

Received February 16, 1972

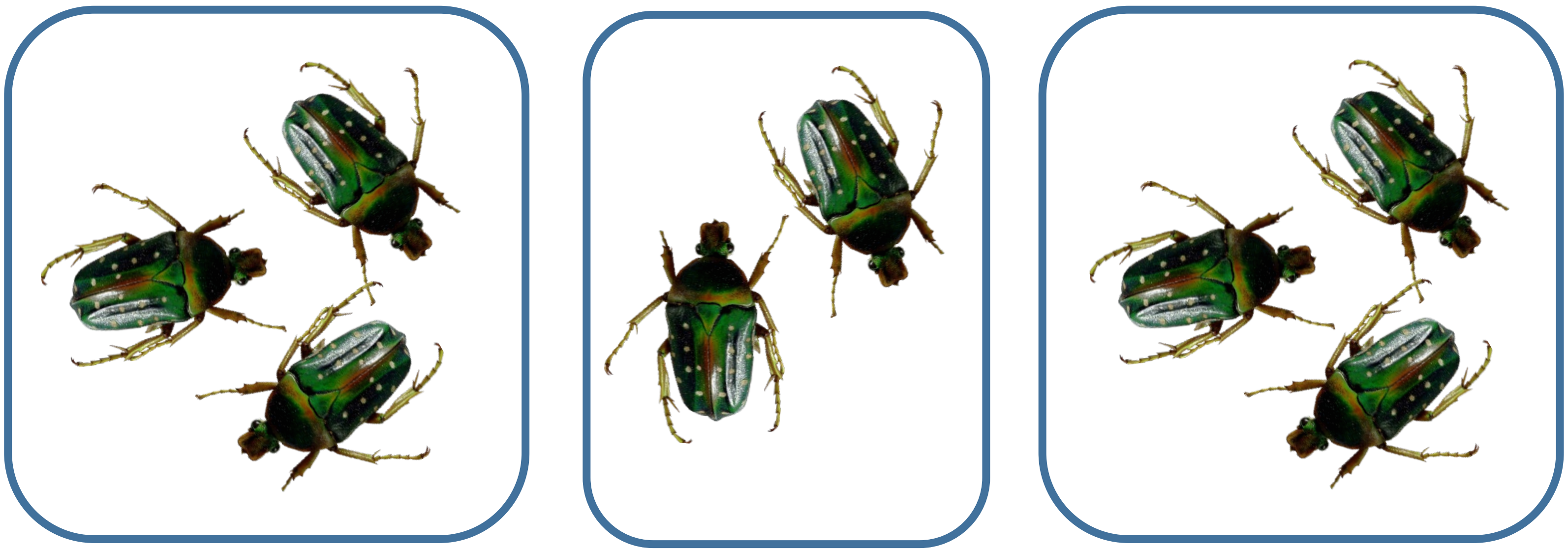
Summary. A powerful method of simplifying the proofs of program correctness is suggested; and some new light is shed on the problem of functions with side-effects.

1. Introduction

In the development of programs by stepwise refinement [1–4], the programmer is encouraged to postpone the decision on the representation of his data until after he has designed his algorithm, and has expressed it as an "abstract" program operating on "abstract" data. He then chooses for the abstract data some convenient and efficient concrete representation in the store of a computer; and finally programs the primitive operations required by his abstract program in terms of this concrete representation. This paper suggests an automatic method of accomplishing the transition between an abstract and a concrete program, and also a method of proving its correctness; that is, of proving that the concrete representation exhibits all the properties expected of it by the "abstract" pro-

Type of property	Number of properties
Invariant	4
Postcondition	5
Metamorphic	16
Model-based	5

insert delete union



Type of property	Number of properties	Bugs missed
Invariant	4	**5**
Postcondition	5	**0**
Metamorphic	16	**0**
Model-based	5	**0**

Effectiveness

```
prop_FindPostPresent k v t =
  find k (insert k v t) === Just v
```

May find bug in **`find`** *or* **`insert`**

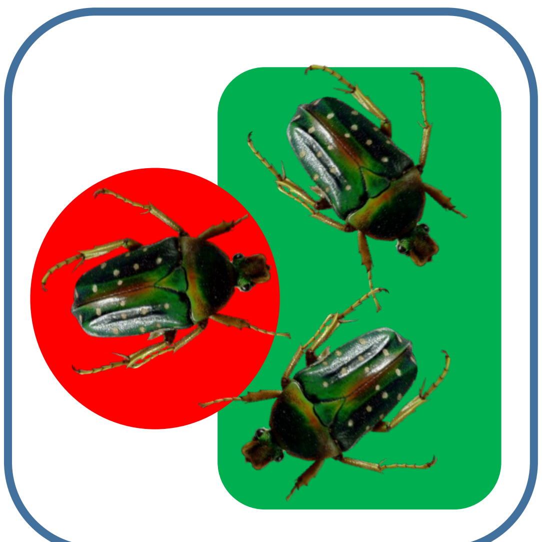
*Will **not** find bugs in* **`delete`** *or* **`union`**

Effectiveness

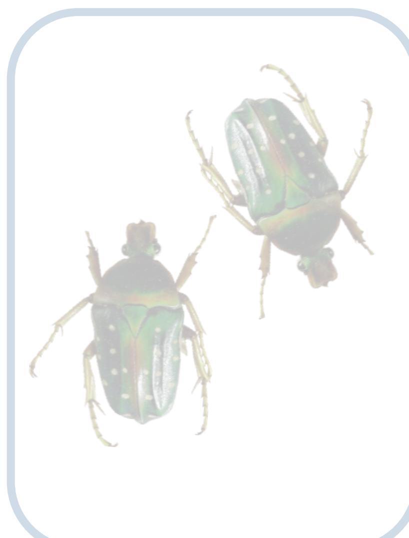
2/3

```
prop_FindPostPresent k v t =
  find k (insert k v t) === Just v
```

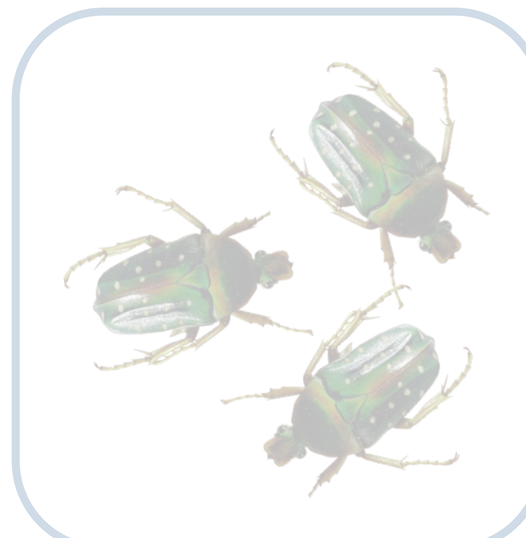
insert



delete



union



Type of property	Number of properties	Bugs missed	Effectiveness
Invariant	4	5	**38%**
Postcondition	5	0	**79%**
Metamorphic	16	0	**90%**
Model-based	5	0	**100%**

```
=== prop_UnionPost from BSTSpec.hs:75 ===
Mean time to failure: 50.04595404595405
```

```
=== prop_InsertUnion fr[illegible]ec.hs:117 ===
Mean time to failure[illegible]5374626

=== prop_DeleteUnion fr[illegible]pec.hs:145 ===
Mean time to failure: 4[illegible]696303694

=== prop_UnionDeleteIns[illegible]n BSTSpec.hs:167 ===
Mean time to failure: 7[illegible]32767233

=== prop_UnionUnionAsso[illegible]BSTSpec.hs:185 ===
Mean time to failure: 8[illegible]95104895

=== prop_FindUnion from[illegible].hs:206 ===
Mean time to failure[illegible]72827
```

```
=== prop_UnionModel from[illegible]pec.hs:290 ===
Mean time to failure: 8.36[illegible]31368631368
```

Logically equivalent!

```
prop_UnionPost t t' k =
  find k (union t t')
  ===
   (find k t <|> find k t')
```

```
prop_UnionModel t t' =
  toList (union t t′)
  ===
  List.sort
    (List.unionBy
      ((==) `on` fst)
      (toList t)
      (toList t'))
```

Mean time to failure

Property type	Min	Max	Mean
Postcondition	9.7	160	68
Metamorphic	1	401	61.6
Model-based	5	6.5	5.8

Averaged over seven bugs, and all properties of each type that detect the bugs

Model-based

- Easier to think of than postconditions
- Require fewer properties than metamorphic approach
- Are the most effective properties
- Find bugs fastest
- *Complete* specification

Metamorphic

- *Do not require a model*
- Easiest to write
- Good effectiveness