

Generating random tests

Beijing 2023
Autumn School

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CHALMERS

QuviQ





arbitrary

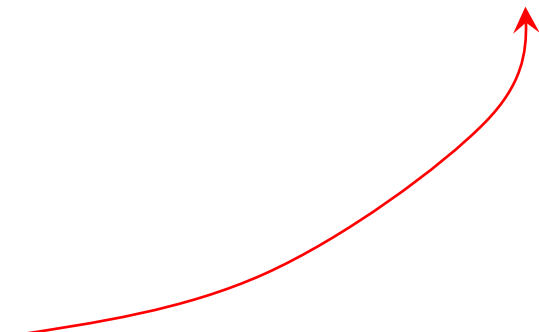


shrink



```
class Arbitrary a where
  arbitrary :: Gen a
  shrink    :: a -> [a]
```

*Given **a**, construct a list of **simpler** test cases which **a** may shrink to*



*The **generator** type; generates a random **a***



The Greedy Shrinking Search

*Placing **large** shrinking steps early makes for efficient shrinking*


- Given a failing test case **x**,
 - Replace **x** by the *first* element **y** of **shrink x** for which the test fails, and repeat
 - If no element of **shrink x** makes the test fail, report **x** as the shrunk test case

*QuickCheck runs the test for **every** element of **shrink x** before reporting the failure. Better not be too long!*

Type modifiers

- The *type* determines generation and shrinking
- We can *change* generation and shrinking by using a newtype

```
newtype NonEmptyList a = NonEmpty [a]
```

Arbitrary instance generates 
and shrinks to non-empty lists

```
newtype Positive a = Positive a
```

Arbitrary instances for numeric types 
generate and shrink to positive numbers

Testing square root

*Approximately
equal to*

```
prop_Sqrt :: Double -> _  
prop_Sqrt x =  
  x >= 0 ==> sq (sqrt x) == x
```

```
sq x = x*x
```

*The precondition discards
half the tests!*

```
prop_Sqrt' :: (NonNegative Double) -> _  
prop_Sqrt' (NonNegative x) =  
  sq (sqrt x) == x
```

*The test is
written very
similarly*

*Only non-negative
test cases are
generated*

Define an **Arbitrary** instance...

- To make your own types usable in properties
- To specify *different* generation and shrinking for an existing type

The **Gen** type: useful combinators

- **choose (m,n)**

Generate a value uniformly in the range **m** to **n**

- **elements xs**

Generate an element of **xs**, chosen uniformly

monadic
+ *operators;*
do-syntax

- **oneof [gen1, ..., genN]**

A uniform choice between generators

- **frequency [(weight1,gen1) ,..., (weightN,genN)]**

A weighted choice between generators (with integer weights)

Sampling generators

```
*Sqrt> sample (elements "hello")
```

'l'

'h'

'o'

'e'

'e'

'l'

'e'

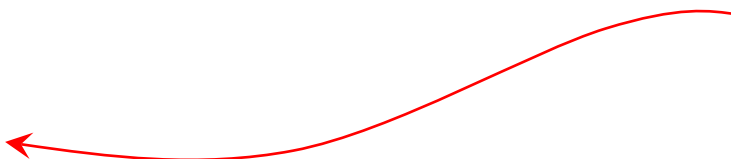
'e'

'l'


'e'

'l'

*A quick way to get an
idea of what is being
generated*



NOT a substitute for
***measuring the
distribution of
generated data***



Sampling at a type

```
Test.QuickCheck> sample (arbitrary :: Gen [Int])  
[]  
[0,0]  
[-3,-3,3,0]  
[5,5]  
[5,-5,5]  
[-2,-4,0,-6,-8,-4]  
[-1,-8]  
[-13]  
[-7]  
[10,-9,-5,6,14,0]  
[]
```

Example: ordered list insertion

```
propInsert :: Int -> [Int] -> Bool
propInsert x xs = ordered (insert x xs)
  where ordered xs = sort xs == xs
```

```
*Examples> quickCheck propInsert
*** Failed! Falsified (after 6 tests and 6 shrinks):
0
[1,0]
```

```
*Examples> insert 0 [1,0]
[0,1,0]
```

Example: using a precondition

```
propInsert :: Int -> [Int] -> Property
propInsert x xs =
    ordered xs ==> ordered (insert x xs)
```

```
*Examples> quickCheck propInsert
*** Gave up! Passed only 71 tests; 1000 discarded
tests.
```

Example: using a generator

```
propInsert :: Int -> Ordered Int -> Bool
propInsert x (Ordered xs) =
    ordered (insert x xs)
```

```
data Ordered a = Ordered [a] deriving (Eq, Show)
```

```
instance (Ord a, Arbitrary a) =>
    Arbitrary (Ordered a) where
    arbitrary = do xs <- arbitrary
                  return (Ordered (sort xs))
    shrink (Ordered xs) =
        [Ordered ys | ys <- shrink xs, ordered ys]
```

```
*Examples> quickCheck propInsert
+++ OK, passed 100 tests.
```

Recall BSTs and their invariant

```
valid :: Ord k => BST k v -> Bool
valid Leaf = True
valid (Branch l k v r) =
    valid l && valid r &&
    all (<k) (keys l) && all (>k) (keys r)
```

```
prop_NilValid          = valid (nil :: Tree)
prop_InsertValid k v t = valid (insert k v t)
prop_DeleteValid k t   = valid (delete k t)
prop_UnionValid t t'   = valid (union t t')
```

A generator and shrinker for BSTs

```
instance (Ord k, Arbitrary k, Arbitrary v) =>
  Arbitrary (BST k v) where
```

```
-- generator
```

```
arbitrary =
```

```
  do kvs <- arbitrary
```


```
  return $
```

```
    foldr (\(k,v) t -> insert k v t)
```

```
      nil
```

```
      (kvs :: [(k,v)])
```

*Generate by
inserting random
key-value pairs*



```
-- shrinker
```

```
shrink = genericShrink
```

*Shrink using a
generic QuickCheck
mechanism*





insert

=== prop_InsertValid from BSTSpec.hs:19 ===

*** Failed! Falsified (after 6 tests and 8 shrinks):

0

0

Branch Leaf 0 0 Leaf

=== prop_DeleteValid from BSTSpec.hs:22 ===

*** Failed! Falsified (after 8 tests and 7 shrinks):

0

Branch Leaf 1 0 (Branch Leaf 0 0 Leaf)

=== prop_UnionValid from BSTSpec.hs:25 ===

*** Failed! Falsified (after 7 tests and 9 shrinks):

Branch Leaf 0 0 (Branch Leaf 0 0 Leaf)

Leaf



insert

=== prop_InsertValid from BSTSpec.hs:19 ===

*** Failed! Falsified (after 6 tests and 8 shrinks):

0

0

Branch Leaf 0 0 Leaf

=== prop_DeleteValid from BSTSpec.hs:22 ===

*** Failed! Falsified (after 8 tests and 7 shrinks):

0

Branch Leaf 1 0 (Branch Leaf 0 0 Leaf)

=== prop_UnionValid from BSTSpec.hs:25 ===

*** Failed! Falsified (after 7 tests and 9 shrinks):

Branch Leaf 0 0 (Branch Leaf 0 0 Leaf)

Leaf

Testing our tests

```
prop_ArbitraryValid t = valid t
```

```
prop_ShrinkValid t =  
  all valid (shrink t)
```

```
Branch Leaf 0 0 (Branch Leaf 0 1 Leaf)  
→ Branch Leaf 0 0 (Branch Leaf 0 0 Leaf)
```

Testing our tests

```
prop_ArbitraryValid t = valid t
```

```
prop_ShrinkValid t =  
  all valid (shrink t)
```

Branch Leaf 0 0 (Branch Leaf 0 1 Leaf)

➔ Branch Leaf 0 0 (Branch Leaf 0 0 Leaf)

Testing our tests

```
prop_ArbitraryValid t = valid t
```

```
prop_ShrinkValid t =  
  all valid (shrink t)
```

Branch Leaf ~~0~~ 0 (Branch Leaf ~~0~~ 1 Leaf)

→ Branch Leaf ~~0~~ 0 (Branch Leaf ~~0~~ 0 Leaf)

Testing our tests

```
prop_ArbitraryValid t = valid t
```

```
prop_ShrinkValid t =  
  valid t ==> all valid (shrink t)
```

Branch Leaf 0 0 (Branch Leaf 1 0 Leaf)

→ Branch Leaf 0 0 (Branch Leaf 0 0 Leaf)

Fixing shrinking

```
shrink = filterValid genericShrink.
```

Completeness: can every tree be constructed just using **insert**?

```
insertions Leaf = []  
insertions (Branch l k v r) =  
    (k,v):insertions l++insertions r
```

```
prop_InsertComplete t =  
    t  
    ==  
    foldl (\t (k,v) -> insert k v t) nil (insertions t)
```

```
prop_InsertCompleteForDelete k t =  
    prop_InsertComplete (delete k t)
```

```
prop_InsertCompleteForUnion t t' =  
    prop_InsertComplete (union t t')
```

Generating trees the hard way

```
data BST k v = Leaf
              | Branch (BST k v) k v (BST k v)
  deriving (Eq, Show, Generic)
```

```
instance (Arbitrary k, Arbitrary v) =>
  Arbitrary (BST k v) where
```

```
  arbitrary = oneof [return Leaf,
                     do (l,k,v,r) <- arbitrary
                       return (Branch l k v r)]
```

```
  shrink = ...
```


*Examples> sample (arbitrary :: Gen (BST Int Int))

Leaf

Branch (Branch (Branch (Branch (Branch Leaf 1 2 Leaf) 0 1 (Branch (Branch Leaf 2 2 (Branch Leaf (-1) 2 Leaf)) (-1) 0 Leaf)) (-1) (-2) Leaf) (-2) 1 Leaf) 0 (-1) (Branch Leaf (-1) 1 (Branch Leaf 2 (-1) (Branch (Branch Leaf 2 1 Leaf) (-1) 1 (Branch Leaf (-1) 1 (Branch Leaf 0 1 (Branch Leaf 0 (-2) Leaf))))))

Branch (Branch Leaf (-2) (-1) Leaf) (-1) 4 (Branch Leaf (-2) (-4) Leaf)

Branch (Branch (Branch Leaf (-1) (-2) Leaf) (-1) (-5) (Branch (Branch (Branch Leaf 6 (-4) (Branch Leaf 6 3 Leaf)) 6 (-2) Leaf) 3 3 (Branch (Branch (Branch (Branch (Branch Leaf 4 (-2) (Branch (Branch (Branch (Branch Leaf 0 (-6) Leaf) (-1) 5 (Branch Leaf (-1) (-1) (Branch Leaf (-4) (-6) Leaf))) 1 1 (Branch Leaf 1 3 (Branch Leaf 1 (-1) Leaf))) 2 (-1) Leaf)) (-5) 3 (Branch (Branch (Branch (Branch (Branch Leaf 4 5 Leaf) 6 5 (Branch (Branch Leaf 6 (-3) Leaf) 3 (-1) Leaf)) (-5) 1 (Branch Leaf (-5) (-6) (Branch Leaf 5 1 (Branch Leaf 6 (-4) (Branch (Branch Leaf 3 5 (Branch Leaf (-3) 2 Leaf)) (-6) (-2) Leaf)))))) 0 (-3) (Branch Leaf 4 (-2) (Branch Leaf 3 6 Leaf)) (-2) 5 Leaf)) (-6) (-1) Leaf) (-4) (-5) Leaf) (-2) (-1) (Branch (Branch Leaf 6 (-4) (Branch (Branch Leaf (-2) (-2) (Branch (Branch Leaf 2 (-5) Leaf) (-4) (-1) (Branch (Branch (Branch Leaf 0 5 Leaf) (-2) (-1) Leaf) 6 3 Leaf)) (-2) (-6) (Branch Leaf 2 1 (Branch (Branch (Branch Leaf (-2) 0 (Branch Leaf (-1) (-1) Leaf)) (-3) 1 Leaf) 6 (-2) Leaf)))) (-5) 2 (Branch Leaf (-3) 3 (Branch Leaf (-1) (-6) Leaf))) 5 (-1) Leaf)) (-2) 3 Leaf

Leaf

Leaf

Branch (Branch (Branch (Branch Leaf (-12) (-5) (Branch (Branch (Branch (Branch Leaf (-3) 12 Leaf) (-1) 3 Leaf) 6 (-1) Leaf) (-11) 1 (Branch (Branch Leaf (-9) (-8) Leaf) 9 9 Leaf))) 3 (-6) Leaf) (-11) 2 (Branch (Branch (Branch Leaf (-4) 10 Leaf) 8 0 Leaf) 6 2 (Branch (Branch Leaf 0 6 Leaf) 4 8 Leaf))) 12 12 (Branch (Branch (Branch (Branch Leaf 0 7 Leaf) (-11) (-7) Leaf) (-6) 2 Leaf) 9 12 (Branch (Branch Leaf 1 (-10) Leaf) (-11) 5 Leaf) (-11) 5 (Branch Leaf 5 (-9) (Branch Leaf (-9) 4 (Branch Leaf (-6) (-6) Leaf))))

Leaf

Leaf

Leaf

Leaf

Way too many leaves

*Way too large
test cases*

Reducing leaves

```
instance (Arbitrary k, Arbitrary v) =>
    Arbitrary (BST k v) where
    arbitrary = frequency
        [(1, return Leaf),
         (2, do (l,k,v,r) <- arbitrary
                return (Branch l k v r))]
```

```
*Examples> sample (arbitrary :: Gen (BST Int Int))
```

Leaf

Branch Leaf 1 1 Leaf

Branch (Branch (Branch (Branch (Branch (Branch Leaf 4 1 Leaf) (-2) 3 (Branch (Branch Leaf

[illegible]

Sized generators

```
instance (Arbitrary k, Arbitrary v) =>
    Arbitrary (BST k v) where
    arbitrary = sized tree
    where
```

```
    tree 0 = return Leaf
    tree n = frequency
        [(1, return Leaf),
         (2, do (k,v) <- arbitrary
                l <- child
                r <- child
                return (Branch l k v r))]
    where child = tree ((n-1) `div` 2)
```

sized **gen** generates a **gen** **n**, for some $0 \leq n \leq 100$.

```
*Examples> sample (arbitrary :: Gen (BST Int Int))
```

```
Leaf
```

```
Branch Leaf 0 (-1) Leaf
```

```
Leaf
```

```
Branch Leaf 0 2 Leaf
```

```
Branch (Branch Leaf (-5) 3 Leaf) 2 (-8) (Branch Leaf 4 2  
(Branch Leaf 1 (-4) Leaf))
```

```
Branch (Branch (Branch Leaf (-6) (-1) Leaf) 9 9 Leaf) 9 (-  
5) Leaf
```

```
Branch (Branch Leaf (-12) 2 Leaf) (-10) 10 (Branch (Branch  
Leaf 0 (-2) Leaf) 4 (-5) (Branch Leaf (-4) 1 Leaf))
```

```
Branch Leaf 4 14 Leaf
```

```
Leaf
```

```
Branch (Branch Leaf 11 (-4) (Branch (Branch Leaf 17 (-10)  
Leaf) (-3) 15 (Branch Leaf 4 (-12) Leaf))) 6 (-10) (Branch  
(Branch (Branch Leaf (-6) 11 Leaf) 3 4 (Branch Leaf (-16)  
(-7) Leaf)) (-5) (-8) Leaf)
```

```
Branch (Branch (Branch (Branch Leaf 6 4 Leaf) 8 (-7)  
(Branch Leaf (-16) (-3) Leaf)) 8 (-7) Leaf) 10 12 Leaf
```

BUT...

```
*Examples> quickCheck prop_ArbitraryValid  
*** Failed! Falsified (after 4 tests):  
Branch (Branch Leaf 3 1 Leaf) 2 0 Leaf
```

One way to fix this...

Upper and lower bounds on the keys (if present)

```
instance (Ord k, Arbitrary k, Arbitrary v) =>
  Arbitrary (BST k v) where
```

```
  arbitrary = sized (tree Nothing Nothing)
  where
```

Generate a key within the bounds (if possible)

```
    tree lb ub 0 = return Leaf
```

```
    tree lb ub n =
```

```
      frequency [(1, return Leaf),
```

```
                 (2, do maybe_k <- between lb ub
```

```
                   case maybe_k of
```

```
                     Nothing -> return Leaf
```

```
                     Just k -> do
```

```
                       v <- arbitrary
```

```
                       l <- child lb (Just k)
```

```
                       r <- child (Just k) ub
```

```
                       return (Branch l k v r))]
```

```
  where child lb ub = tree lb ub ((n-1) `div` 2)
```

...

...and between

*Can't use **choose** because
that only works for numbers*

```
between lb ub =  
  arbitrary  
    `suchThatMaybe` (\x ->  
      (lb==Nothing || lb < Just x) &&  
      (ub==Nothing || Just x < ub))
```

suchThatMaybe tries repeatedly to generate a value satisfying the predicate, but may fail and generate **Nothing**

Maybe values are compared by comparing their components

```
*Examples> quickCheck prop_ArbitraryValid
+++ OK, passed 100 tests.
```

```
*Examples> sample (arbitrary :: Gen (BST Int Int))
```

```
Leaf
```

```
Branch Leaf (-1) (-1) Leaf
```

```
Leaf
```

```
Branch (Branch Leaf (-2) 3 Leaf) (-1) 4 (Branch
Leaf 0 (-1) Leaf)
```

```
Branch (Branch Leaf (-3) 4 Leaf) (-2) (-6) Leaf
```

```
Branch Leaf (-10) (-3) Leaf
```

```
→ Branch Leaf 10 (-3) (Branch Leaf 14 0 (Branch Leaf
19 12 Leaf))
```

```
Branch Leaf (-10) (-8) (Branch (Branch Leaf (-6) 6
Leaf) 14 5 Leaf)
```

```
Leaf
```

```
Branch (Branch (Branch (Branch Leaf (-10) (-10)
Leaf) (-1) (-18) Leaf) 2 (-10) Leaf) 12 (-18)
(Branch Leaf 15 8 (Branch Leaf 17 (-17) (Branch
Leaf 20 (-8) Leaf)))
```

```
Leaf
```


Are our tests effective?

- What size are the trees we generate?

```
size Leaf = 0
```

```
size (Branch l _ _ r) = size l + 1 + size r
```

```
prop_MeasureSize :: BST Int Int -> Property
```

```
prop_MeasureSize t = collect (size t) True
```

Data to collect

*The rest of the
property*

```
*Examples> quickCheck prop_MeasureSize
```

```
+++ OK, passed 100 tests:
```

```
9% 21
```

```
6% 6
```

```
5% 11
```

```
5% 42
```

```
4% 23
```

```
4% 3
```

```
4% 47
```

```
3% 10
```

```
3% 17
```

```
3% 22
```

```
3% 24
```

```
3% 38
```

```
2% 1
```

```
2% 14
```

```
2% 16
```

```
2% 20
```

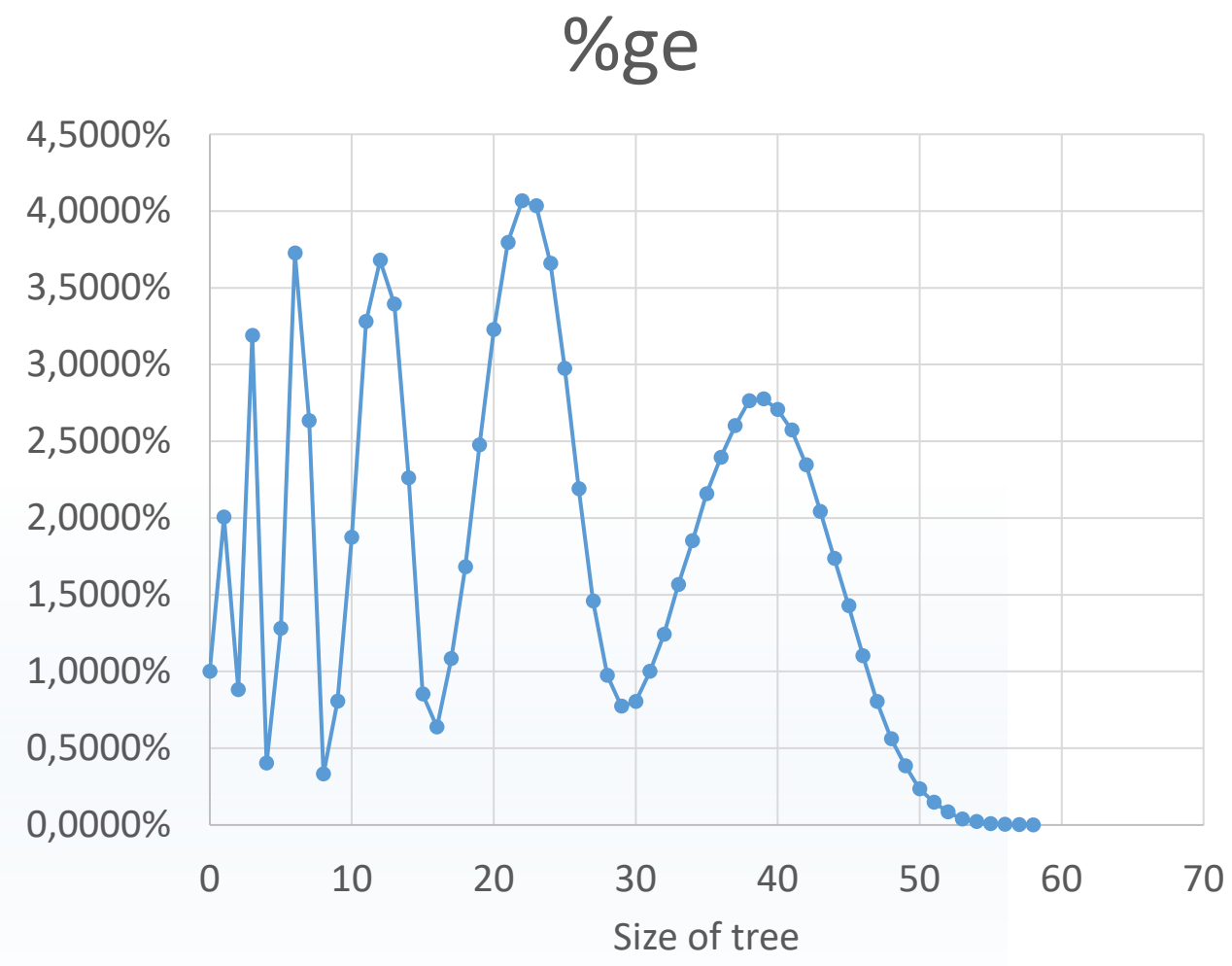
```
2% 29
```

*Way too small for
reliable statistics*

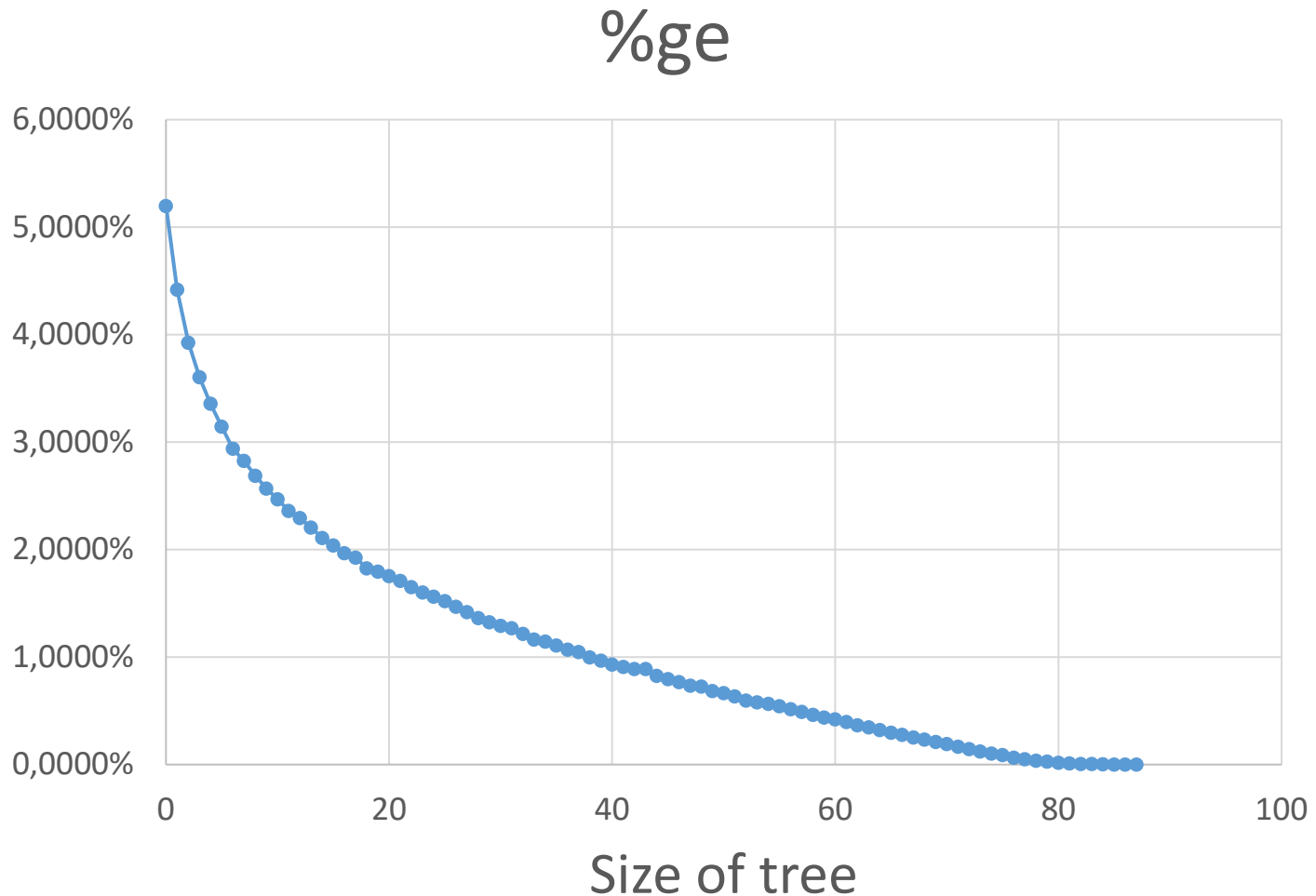
*Decent looking
variety of sizes, not
too small, not too big*

```
*Examples> quickCheck . withMaxSuccess 1000000 $ prop_MeasureSize
+++ OK, passed 1000000 tests:
```

4.0675%	22
4.0348%	23
3.7954%	21
3.7257%	6
3.6792%	12
3.6602%	24
3.3938%	13
3.2810%	11
3.2282%	20
3.1886%	3
2.9750%	25
2.7764%	39
2.7644%	38
2.7062%	40
2.6339%	7
2.6007%	37
2.5724%	41
2.4761%	19
2.3949%	36
2.3465%	42



Generator based on insertion



Are our tests effective?

```
prop_NilValid          = valid (nil :: Tree)
prop_InsertValid k v t = valid (insert k v t)
prop_DeleteValid k t   = valid (delete k t)
prop_UnionValid t t'   = valid (union t t')
```

- What *unit tests* would you write?
- How often is **k** present in **t**?
- How often *should* **k** be present in **t**?

 *About half the time?*

Let's find out...

```
prop_MeasureMembership
  :: Int -> BST Int Int -> Property
prop_MeasureMembership k t = label l True
  where l | k `elem` keys t = "present"
          | otherwise      = "absent"
```

```
*Examples> quickCheck . withMaxSuccess 100000 $
              prop_MeasureMembership
+++ OK, passed 100000 tests:
79.258% absent
20.742% present
```

It depends on the type of keys...

```
prop_MeasureMembership                                not (null k) ==>
  :: String -> BST String Int -> Property
prop_MeasureMembership k t = label 1 True
  where l | k `elem` keys t = "present"
          | otherwise       = "absent"
```

```
*Examples> quickCheck . withMaxSuccess 100000 $
prop_MeasureMembership
+++ OK, passed 100000 tests:
98.558% absent
1.442% present
```

```
*Examples> quickCheck . withMaxSuccess 100000 $
prop_MeasureMembership
+++ OK, passed 100000 tests; 16228 discarded:
99.987% absent
0.013% present
```

Draw keys from a smaller set

```
prop_MeasureMembership
  :: Key -> BST Key Int -> Property
prop_MeasureMembership k t = label 1 True
  where 1 | k `elem` keys t = "present"
         | otherwise       = "absent"
```

Is this code correct?

```
newtype Key = Key Int deriving (Eq, Ord, Show)
```

```
instance Arbitrary Key where
  arbitrary = Key <$> scale (`div` 3) arbitrary
  shrink (Key n) = map Key (shrink n)
```

52.114% absent

47.886% present

labelledExamples

```
*Examples> labelledExamples prop_MeasureMembership
```

```
*** Found example of absent
```

```
Key 0
```

```
Leaf
```



Maybe not such a good example?

```
*** Found example of present
```

```
Key 0
```

```
Branch Leaf (Key 0) 0 Leaf
```

```
+++ OK, passed 100 tests:
```

```
50% absent
```

```
50% present
```

```
prop_MeasureMembership
  :: Key -> BST Key Int -> Property
prop_MeasureMembership k t = label 1 True
  where 1 | t == nil          = "nil"
         | k `atRoot` t      = "at root"
         | k `elem` keys t   = "present"
         | otherwise         = "absent"
```

```
*Examples> labelledExamples . withMaxSuccess 100000 $
              prop_MeasureMembership
*** Found example of nil
Key 0
Leaf

*** Found example of at root
Key 0
Branch Leaf (Key 0) 0 Leaf

*** Found example of absent
Key 0
Branch Leaf (Key 1) 0 Leaf

*** Found example of present
Key 0
Branch Leaf (Key (-1)) 0 (Branch Leaf (Key 0) 0 Leaf)

+++ OK, passed 100000 tests:
47.128% absent
41.766% present
 5.907% at root
 5.199% nil
```

} *These are important test cases too*

What if someone changes the generator?

- State *coverage requirements* as a property!

```
prop_CoverMembership
  :: Key -> BST Key Int -> Property
prop_CoverMembership k t =
  cover 5  (t == nil)                "nil"      $
  cover 5  (k `atRoot` t)            "at root"  $
  cover 40 (k `elem` keys t &&
    not (k `atRoot` t))              "present"  $
  cover 40 (k `notElem` keys t &&
    t /= nil)                        "absent"   $
  True
```

Required percentage of test cases



```
*Examples> quickCheck . checkCoverage $  
                prop_CoverMembership  
+++ OK, passed 51200 tests:  
46.666% absent  
42.246% present  
 5.844% at root  
 5.244% nil
```

SEQUENTIAL TESTS OF STATISTICAL HYPOTHESES

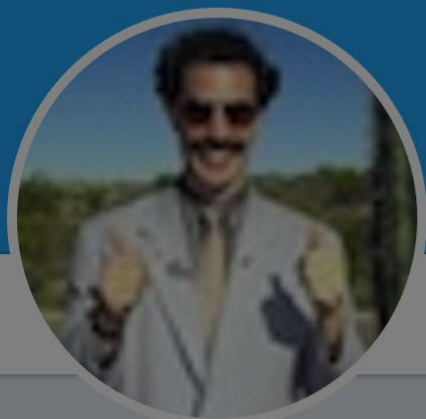
By A. WALD

Columbia University

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How often is it OK
for a test to fail
*when there is no
bug?*



Agile Borat

@AgileBorat

Hello, I am Borat! Am Agile Coach, Scrum Master and Product Owner too also.

📅 Joined April 2011



Agile Borat

@AgileBorat

Follow



My friend Azamat is very good developer, he is always have all unit test green. If unit test is fail, it is remove. Is best practice.

8:35 AM - 5 May 2011

256 Retweets 25 Likes



256



25



2



1

How often is it ok for a test to fail
when there is no bug?

**Never in the
lifetime of the
project!**

10^{-6} ?

10^{-9} ?

Summary

- *Generators* and *shrinkers* must be written, for new types or custom distributions
- *Reuse* existing code as far as possible
- Include *tests* for generators and shrinkers
- *Measure* the distribution of tests
- *Label* test cases with unit test ideas
- *Debug* labelling with **labelledExamples**
- *Tune* generators to make test effective
- *State* and *test* coverage requirements with **cover/checkCoverage**.