QuickSpec— Formal Specifications for Free!

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QuviQ



A simple example

Define a signature containing named operations and their types

```
*Int> quickSpec intSig
== Functions ==
  (+) :: Int -> Int -> Int
negate :: Int -> Int
```

== Laws ==

```
    x + y = y + x
    negate (negate x) = x
    x + negate x = y + negate y
    (x + y) + z = x + (y + z)
    negate x + negate y = negate (x + Distribution
    negate x + (x + y) = y
```

- All the laws are true, but some are unexpected
- No equation is printed if it follows from the previous ones

Adding 0 to the vocabulary

```
intSig0 = intSig ++ [
  con "0" (0 :: Int)
]
```

```
*Int> quickSpec intSig0
== Functions ==
   (+) :: Int -> Int -> Int
negate :: Int -> Int
     0 :: Int
== Laws ==
  1. negate 0 = 0
  2. x + y = y + x
  3. x + 0 = x
  4. negate (negate x) = x
  5. x + negate x = 0
  6. (x + y) + z = x + (y + z)
  7. negate x + negate y = negate (x + y)
  6. negate x + (x + y) = y ???
```

```
negate x + (x + y) = y
                  by 6. (x + y) + z = x + (y + z)
(negate x + x) + y = y
                  by 2. x + y = y + x
(x + negate x) + y = y
                  by 5. x + negate x = 0
0 + y = y
                  by 2. x + y = y + x
y + 0 = y
                  by 3. x + 0 = x
y = y
```

Let's try it for Float

*Float> quickSpec floatSig == Functions == (+) :: Float -> Float -> Float negate :: Float -> Float 0.0 :: Float

WARNING: The following types have no 'Arbitrary' instance declared.

You will not get any variables of the following types: Float

WARNING: The following types have no 'Ord' or 'Observe' instance declared.

You will not get any equations about the following types:

Float

== Laws == No laws!

```
*Float> quickSpec floatSig
== Functions ==
   (+) :: Float -> Float -> Float
negate :: Float -> Float
   0.0 :: Float
WARNING: The following types have no 'Arbitrary'
instance declared.
You will not get any variables of the following types:
  Float
WARNING: The following types have no 'Ord' or
'Observe' instance declared.
You will not get any equations about the following
types:
  Float
                No laws!
== Laws ==
```

Let's try it for Float, the right way

```
*Float> quickSpec floatSig
```

== Functions ==

(+) :: Float -> Float -> Float

negate :: Float -> Float

0.0 :: Float

== Laws ==

- 1. negate 0.0 = 0.0
- 2. x + y = y + x
- 3. x + 0.0 = x
- 4. negate (negate x) = x
- 5. x + negate x = 0.0
- 6. negate x + negate y = negate (x + y)
- 7. negate x + (x + x) = x
- 8. (x + y) + (x + y) = (x + x) + (y + y)
- 9. x + (x + (x + x)) = (x + x) + (x + x)

Why so many?

Comparing with Int

```
1. negate 0 = 0
                                 1. negate 0.0 = 0.0
2. x + y = y + x
                                 2. x + y = y + x
                                 3. x + 0.0 = x
3. x + 0 = x
                                 4. negate (negate x) = x
4. negate (negate x) = x
5. x + negate x = 0
                                 5. x + negate x = 0.0
6. (x + y) + z = x + (y + z)
                                 6. negate x + negate y =
7. negate x + negate y =
                                      negate (x + y)
                                 7. negate x + (x + x) = x
     negate (x + y)
                                 8. (x + y) + (x + y) =
                                     (x + x) + (y + y)
                                 9. x + (x + (x + x)) =
                                      (x + x) + (x + x)
```

Is Associativity true?

```
prop_Assoc :: Float -> Float -> Float -> Property
prop_Assoc x y z =
  (x + y) + z === x + (y + z)
```

```
*Float> quickCheck prop_Assoc
*** Failed! Falsified (after 3 tests and 6 shrinks):
1.0
0.7
0.1
1.8000001 /= 1.8
```

QuickSpec is complete!

Further investigations...

Haskell has exact rationals

rationalSig = [

```
monoType (Proxy :: Proxy Rational),
 con "+" ((+) :: Rational -> Rational -> Rational),
 con "negate" (negate :: Rational -> Rational),
 con "0" (0 :: Rational)
== Laws ==
  1. negate 0 = 0
  2. x + y = y + x
  3. x + 0 = x
  4. negate (negate x) = x
  5. x + negate x = 0
  6. (x + y) + z = x + (y + z)
  7. negate x + negate y = negate (x + y)
```

```
Generate terms and laws,
                      but don't print them
                                          Exact
jointSig #
  background floatSig,
  background rationalSig,
  con "toRational" (toRational :: Float -> Rational),
  con "fromRational" (fromRational :: Rational -> Float)
                                          Rounding
```

```
== Laws ==
  1. toRational 0.0 = 0
  2. from Rational 0 = 0.0
  3. from Rational (to Rational x) = x
  4. from Rational (negate x) = negate (from Rational x)
  5. negate (toRational x) = toRational (negate x)
  6. from Rational x + from Rational x = from Rational (x + x)
  7. toRational x + toRational x = toRational (x + x)
  8. from Rational (to Rational x + to Rational y) = x + y
  9. fromRational (x + toRational (fromRational x)) =
       from Rational (x + x)
prop Inverse r =
  toRational (fromRational r) === r
*Float> quickCheck prop Inverse
*** Failed! Falsified (after 2 tests and 38 shrinks):
1 % 653954092621
7572030228139149 % 4951760157141521099596496896 /= 1 % 653954092621
```

Binary Search Trees again

== Functions ==

nil :: BST Int Integer

find :: Int -> BST Int Integer -> Maybe Integer

insert :: Int -> Integer -> BST Int Integer -> BST Int Integer

== Laws ==

Doesn't matter—we always get Nothing

- 1. find x nil = find y nil
- 2. find x (insert x y z) = find x (insert x y w)

 3. find (insert x y z) = find x (insert x y w)

 Just y
- 3. find (x) (insert (x) y z) = find (x) (insert (x) y z)
- 4. find x (insert y z nil) = find y (insert x z nil)
- 5. insert x y (insert x z w) = insert x y w

Binary Search Trees again

```
*BSTSpec> quickSpec treeSig
```

== Functions ==

nil :: BST Int Integer

find :: Int -> BST Int Integer -> Maybe Integer

insert :: Int -> Integer -> BST Int Integer -> BST Int Integer

Nothing :: Maybe Integer

Just :: Integer -> Maybe Integer

== Laws ==

- 1. find x nil = Nothing
- 2. find x (insert x y z) = Just y
- 3. find x (insert y z nil) = find y (insert x z nil)
- 4. insert x y (insert x z w) = insert x y w

•
$$x == y \&\& (2) \rightarrow (3)$$

x /= y → both sides are Nothing

Adding preconditions

```
neTreeSig = [
  background treeSig,
  predicate "/=" ((/=) :: Int -> Int -> Bool)
  ]
```

== Laws ==

```
    x /= y = y /= x
    x /= x = y /= y
    x /= y => find x (insert y z w) = find x w
```

Equivalence

Why don't we get

```
4. x /= z => insert x y (insert z w t) = insert z w (insert x y t)
```

—one of the properties we wrote on Tuesday?

The trees are equivalent, but not equal

Recall t1 =~= t2 = toList t1 === toList t2

QuickSpec uses == to compare, but we can change that!

Observing a type

We need to tell QuickSpec how to observe trees

```
instance (Ord k, Ord v) =>
        Observe () [(k,v)] (BST k v) where
observe () = toList
```

We need to turn on language extensions...

```
{-# LANGUAGE MultiParamTypeClasses, FlexibleInstances #-}
```

 We need to tell QuickSpec to use the observe function for this type

```
treeEquivSig = treeSig ++ [
  monoTypeObserve (Proxy :: Proxy Tree)
]
```

== Laws ==

- 1. find x nil = Nothing
- 2. find x (insert x y z) = Just y
- 3. find x (insert y z nil) = find y (insert x z nil)
- 4. insert x y (insert x z w) = insert x y w
- 5. insert x y (insert z y w) = insert z y (insert x y w)

Swap insertions with different **keys**, but the same **value**

```
neTreeEquivSig = [
  background treeEquivSig,
  predicate "/=" ((/=) :: Int -> Int -> Bool)
  ]
```

```
== Laws ==

1. x /= y = y /= x

2. x /= x = y /= y

3. x /= y => find x (insert y z w) = find x w
```

WHAT??? No law about swapping insert?

Sometimes equations are too big for QuickSpec to find

```
neTreeEquivSig = [
  background treeEquivSig,
  withMaxTermSize 8,
  predicate "/=" ((/=) :: Int -> Int -> Bool)
  ]

== Laws ==
  1. x /= y = y /= x
```

3. $x \neq y \Rightarrow find x (insert y z w) = find x w$

insert z w (insert x y x2)

4. $z \neq x \Rightarrow insert x y (insert z w x2) =$

2. x /= x = y /= y

What if the code is buggy?

```
*BSTSpec> quickSpec $ treeEquivSig ++
   [background $ predicate "/=" ((/=) :: Int -> Int -> Bool)]
...

== Laws ==
1. find x nil = Nothing
2. insert x y z = insert x y w
3. find x (insert y z w) = find y (insert x z w)
4. find x (insert x y z) = Just y
5. x /= y => find x (insert y z w) = Nothing
```

We get the wrong laws... which make the buggy behaviour pretty clear!

```
*BSTSpec> quickSpec $ treeEquivSig ++
  [background $ predicate "/=" ((/=) :: Int -> Int -> Bool)]
...

== Laws ==
  1. find x nil = Nothing
  2. find x (insert y z nil) = find y (insert x z nil)
  3. find x (insert x y nil) = Just y
  4. x /= y => find x (insert y z w) = find x w
```

All the laws are true, but...

A law to swap insertions is missing

```
*BSTSpec> quickSpec $ treeEquivSig ++
  [background $ predicate "/=" ((/=) :: Int -> Int -> Bool)]
...

== Laws ==
  1. find x nil = Nothing
  2. find x (insert y z nil) = find y (insert x z nil)
  3. find x (insert x y nil) = Just y
  4. x /= y => find x (insert y z w) = find x w
```

All the laws are true, but...

- A law to swap insertions is missing
- We expect (3) to hold in general, not just for nil

```
*BSTSpec> quickCheck $ \x y t ->
    find (x :: Int) (insert x (y :: Integer) t) === Just y

*** Failed! Falsified (after 15 tests and 9 shrinks):

12

1

Branch Leaf 12 0 Leaf

Just 0 /= Just 1

*BSTSpec> insert 12 1 (Branch Leaf 12 0 Leaf)

Branch Leaf 12 0 (Branch Leaf 12 1 Leaf)
```

```
*BSTSpec> quickSpec $ treeEquivSig ++
  [background $ predicate "/=" ((/=) :: Int -> Int -> Bool)]
...

== Laws ==

1. find x nil = Nothing
2. find x (insert y z nil) = find y (insert x z nil)
3. find x (insert x y nil) = Just y
4. x /= y => find x (insert y z w) = find x w
5. insert x y (insert x z w) = insert x z w
6. insert x y (insert z y w) = insert z y (insert x y w)
```

What use is QuickSpec?

 The laws give insight—a good design should satisfy a nice algebra

Missing laws reveal bugs or infelicities

Laws make great regression tests

A tool for design?

```
listsSig = [
  con "[]" ([] :: [Int]),
  con "++" ((++) :: [Int] -> [Int] -> [Int])
*Lists> quickSpec listsSig
== Functions ==
 [] :: [Int]
(++) :: [Int] -> [Int] -> [Int]
== Laws ==
```

3. (xs ++ ys) ++ zs = xs ++ (ys ++ zs)

1. xs ++ [] = xs

2. [] ++ xs = xs

```
consSig = [
 background listsSig,
  con ":" ((:) :: Int -> [Int] -> [Int])
*Lists> quickSpec consSig
== Functions ==
 [] :: [Int]
(++) :: [Int] -> [Int] -> [Int]
== Functions ==
```

```
== Laws ==
```

1. (x : xs) ++ ys = x : (xs ++ ys)

(:) :: Int -> [Int] -> [Int]

2. [] ++ xs = xs

Part of the **definition**

```
revSig = [
 background consSig,
  con "reverse" (reverse :: [Int] -> [Int])
*Lists> quickSpec revSig
== Laws ==
  1. reverse [] = []
  2. reverse (reverse xs) = xs
  3. reverse (x : []) = x : []
  4. reverse xs ++ reverse ys = reverse (ys ++ xs)
  5. xs ++ (x : (y : [])) = reverse (y : (x : reverse)
xs))
```

```
revSig = [
  background consSig,
  con "reverse" (reverse :: [Int] -> [Int]),
  maxPruningDepth 0
*Lists> quickSpec revSig
• • •
== Laws ==
     reverse []
  2. reverse (reverse xs) = xs
  3. reverse (x : []) = x :
  4. reverse xs ++ reverse ys = reverse (ys ++ xs)
  5. reverse (x : reverse xs) = xs ++ (x : [])
  6. reverse (xs
                                   ys ++ reverse xs
                  xs = reverse ys
  7. reverse
 8. reverse (x : xs) ++ ys = reverse xs ++ (x : ys)
                                                  [])
 9. revreverse (x): ys) (x= reverse ys
10. reverse (xs ++ (x : ys)) = reverse ys ++ (x : reverse xs)
```

Eureka! rev' xs ys = reverse xs ++ ys

```
rev'Sig = [
 background revSig,
  con "rev'" (rev' :: [Int] -> [Int] -> [Int])
*Lists> quickSpec rev'Sig
== Laws ==
 1. rev' xs [] = reverse xs
 2. rev' [] xs = xs
  3. reverse (rev' xs ys) = rev' ys xs
  4. rev' xs ys = reverse xs ++ ys
            rev' xs (x:ys) = rev' (x:xs) ys
 7. rev' (rev' xs ys) 2
 8. rev' xs (x : []) = Take zs = [] ks)
 9. reverse (rev' xs y and simplify zs (rev' ys xs)
 10. rev' xs (x : ys) ++ zs = rev' (x : xs) (ys ++ zs)
 11. rev'(x : (y : xs))[] = reverse(x : (y : xs))
```

Changing representations

```
data List a = Nil | Snoc (List a) a
```

```
snoc xs x = xs ++ [x]
snocSig = [
 con "Nil" ([] :: [Int]),
 con "Snoc" (snoc :: [Int] -> Int -> [Int]),
 background [
   con "++" ((++) :: [Int] -> [Int] -> [Int]),
   con "reverse" (reverse :: [Int] -> [Int])
 withPruningDepth 0
```

```
== Laws ==
  1. reverse Nil = Nil
  2. xs ++ Nil = xs
  3. Nil ++ xs = xs
  4. reverse (Snoc Nil x) = Snoc Nil x
  5. xs ++ Snoc ys x = Snoc (xs ++ ys) x
  6. reverse (Snoc (reverse xs) x) = Snoc Nil x ++ xs
  7. xs ++ reve
                                     xs x ++ reverse ys
 8. reverse (S
                                      Snoc (Snoc Nil y) x
                                     (Snoc ys x ++ zs)
  9. Snoc (xs +
                Let xs = reverse ys
 10. xs ++ (Sno
                                     xs x ++ ys
 11. Sh
                                              (Snoc (reverse xs)
y) x)
 12. Snoc (reverse
                                  everse (Snoc (reverse ys) x ++
xs)
     reverse (Snoc ys x) = Snoc Nil x ++ reverse ys
```

Pretty-printing

while x>0 do						
x:=x-2						
end						

```
a Layout [(Int,String)]
```

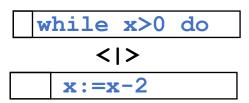
text s

S

nest k 1

	while x>0 do		
k			x:=x-2
	end		

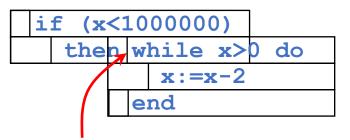
1 <|> 1'



end

<|>

1 <-> 1'



Position 1' at the end of the last line of 1, preserving layout

Semantics

```
newtype Layout = Layout [(Int,String)]
  deriving (Eq, Ord)
text s = Layout [(0,s)]
nest k (Layout nxs) =
  Layout [(n+k,x) | (n,x) < -nxs]
Layout nxs <|> Layout nxs' = Layout $ nxs ++ nxs'
Layout nxs <-> Layout ((n2,x2):nxs') = Layout $
    init nxs ++ [(n1,x1++x2)] ++
      [(n1+length x1+n'-n2,x') | (n',x') <- nxs']
    where (n1,x1) = last nxs
```

Two key operations

best 1

```
nestSig = [
  monoType (Proxy :: Proxy Layout),
  con "nest" (nest :: Int -> Layout -> Layout
  ]

*PP> quickSpec nestSig
== Functions ==
```

```
nest :: Int -> Layout -> Layout
== Laws ==
1. nest x (nest y z) = nest y (nest x z)
```

```
*PP> quickSpec nestSig
...

== Laws ==

1. nest 0 x = x

2. nest x (nest y z) = nest y (nest x z)

3. nest (x + y) z = nest x (nest y z)
```

```
aboveSig = [
  background nestSig,
  con "<|>" ((<|>) :: Layout -> Layout -> Layout)
  ]
```

```
*PP> quickSpec aboveSig
...
== Laws ==
1. (x <|> y) <|> z = x <|> (y <|> z)
2. nest x y <|> nest x z = nest x (y <|> z)
```

```
besideSig = [
 background aboveSig,
  con "<->" ((<->) :: Layout -> Layout -> Layout)
*PP> quickSpec besideSig
== Laws ==
  1. x \leftarrow x \rightarrow x = x \leftarrow z
  2. nest x y <-> z = nest x (y <-> z)
  3. (x <-> y) <-> z = x <-> (y <-> z)
  4. x < | > (y < -> z) = (x < | > y) < -> z
*** Failed! Falsified (after 3 tests and 5 shrinks):
Layout [(0,"a")]
Layout [(0,"")]
Layout [(0,"")]
Layout [(0,"a"),(1,"")] /= Layout [(0,"a"),(0,"")]
```

```
textSig = [
 background besideSig,
  con "text" (text :: String -> Layout),
 background [
    con "++" ((++) :: String -> String -> String),
    con "\"\"" ("" :: String)
    ],
 withMaxTermSize 10
*PP> quickSpec textSig
== Laws ==
 1. x <-> text "" = x
 3. text "" <-> (text xs <|> x) = text xs <|> x
 4. (\text{text "" } <-> x) <|> (\text{text "" } <-> x) =
     text "" <-> (x <|> x)
 5. text "" <-> ((text xs <-> x) <|> v) =
       (\text{text xs} \leftarrow x) \leftarrow y
```

```
lengthSig = [
  background textSig,
  con "length" (length :: String -> Int)
*PP> quickSpec lengthSig
== Laws ==
  1. length "" = 0
  2. length (xs ++ ys) = length (ys ++ xs)
  3. length xs + length ys = length (xs ++ ys)
  4. text xs < |> nest (length xs) x =
       text xs <-> (text "" <|> x)
  5. text (xs ++ ys) < |> nest (length xs) x =
       text xs <-> (text ys <|> x)
  6. text xs <-> (nest (length xs) x <|> x) =
        (\text{text xs } < -> x) < |> (\text{text "" } < -> x)
  7. (\text{text xs} \leftarrow x) \leftarrow |x| > \text{nest (length xs)} y =
       text xs <-> ((text "" <-> x) <|> y)
  8. text xs <-> (nest (length xs) (text ys) <|> x) =
       text (xs ++ ys) <|> x
```

From the pretty-printer paper...

```
data Doc =
    Text String
    | TextAbove String Doc
    | Nest Int Doc
    | Union Doc Doc
    | Empty
```

The implementation was *derived* by applying the algebraic laws we've seen.

Quick specifications for the busy programmer

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Abstract

QuickSpec is a theory exploration system which tests a Haskell program to find equational properties of it, automatically. The equations can be used to help understand the program, or as lemmas to help prove the program correct. QuickSpec is largely automatic: the user just supplies the functions to be tested and QuickCheck data generators. Previous theory exploration systems, including earlier versions of QuickSpec itself, scaled poorly. This paper describes a new architecture for theory exploration with which we can find vastly more complex laws than before, and much faster. We demonstrate theory exploration in QuickSpec on problems both from functional programming and mathematics.

1 Introduction

Formal specifications are a powerful tool for understanding programs. For example,

The Design of a Pretty-printing Library

John Hughes

Chalmers Tekniska Högskola, Göteborg, Sweden.

1 Introduction

On what does the power of functional programming depend? Why are functional programs so often a fraction of the size of equivalent programs in other languages? Why are they so easy to write? I claim: because functional languages support software reuse extremely well.

Programs are constructed by putting program components together. When we discuss reuse, we should ask

- What kind of components can be given a name and reused, rather than reconstructed at each use?
- How flexibly can each component be used?

Every programming language worthy of the name allows sections of a program with identical control flow to be shared, by defining and reusing a procedure. But 'programming idioms' — for example looping over an array — often cannot be defined as procedures because the repeated part (the loop construct) contains a varying part (the loop body) which is different at every instance. In a functional language there is no problem, we can define a higher order function in which the varying part is

Property-based testing

• Is an effective practical technique *and* an active research area.

 There are tools for practically every programming language.

There is much more to discover!

Above all, it's great fun!

Don't write tests!

Generate them