Distribución lognormal

```
> x <- rlnorm(n = 1000, meanlog = 1.5, sdlog = 0.4)
> x |> mean()
[1] 4.813539
> x |> median()
[1] 4.433556
> x |> var()
[1] 4.023704
```

$\rightarrow \bar{x}$

$\rightarrow me$

$\rightarrow s^2$

Media

$$\mu_x = E(X) = \exp\left(1.5 + \frac{0.4^2}{2}\right) = 4.855$$

Mediana

$$Me(X) = \exp(1.5) = 4.482$$

Varianza

$$Var(X) = \exp\left(3 + 0.4^2\right)[\exp\left(0.4^2\right) - 1] = 4.09$$

Propiedad de falta de memoria

$$P(X > s + t | X > s) = P(X > t)$$

Es decir, el tiempo restante no depende de cuánto ya se ha esperado.

X = Tiempo de espero en una institución pública (minutos)

$t = 3, \quad s = 2 \Rightarrow P(X > 5 | X > 2) = P(X > 3)$

$t = 3, \quad s = 3 \Rightarrow P(X > 6 | X > 3) = P(X > 3)$

$t = 3, \quad s = 4 \Rightarrow P(X > 7 | X > 4) = P(X > 3)$

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Función de distribución acumulada

$$F_X(x) = P(X \leq x) = 1 - \exp(-\lambda x) I_{(0,\infty)}(x)$$

Función de supervivencia

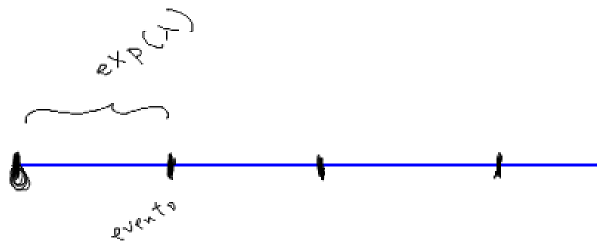
$$S_X(x) = 1 - F_X(x) = P(X > x) = \exp(-\lambda x)\, I_{(0,\infty)}(x)$$

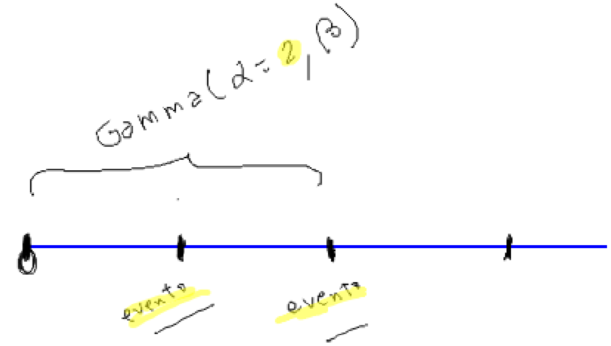
$$\big\downarrow$$

tiempo de funcionamiento

$$f_X(x) = \lambda \exp(-\lambda x) I_{(0,\infty)}(x) \qquad \text{ó} \qquad f_X(x) = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) I_{(0,\infty)}(x)$$

Distribución Gamma





Función de densidad

Se define la V.A.C. X con distribución Gamma con parámetros $\alpha > 0$ y $\beta > 0$, cuya función de densidad viene dada por:

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right) I_{[0,\infty)}(x)$$

$$\Gamma(t) = (t-1)!$$

$$\Gamma(1) = 0! = 1$$

$$\Gamma(t) = \int_0^{\infty} r^{t-1} e^{-r}\, dr$$

Si $\alpha = 1 \Rightarrow f(x) = \dfrac{1}{\underbrace{\Gamma(1)}_{1}\beta^{1}} \underbrace{x^{1-1}}_{1} \exp\left(-\dfrac{x}{\beta}\right) = \underbrace{\dfrac{1}{\beta}\exp\left(-\dfrac{x}{\beta}\right)}_{\text{exponencial}}$

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right) I_{[0,\infty)}(x)$$

en R es σ

$$\mu_X = E(X) = \alpha\beta$$

$$\sigma_X^2 = V(X) = \alpha\beta^2$$

```
> pgamma(q = 25, shape = 3, scale = 10)
[1] 0.4561869
```

$\alpha = 3$ $\beta = 10$

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-x\beta) I_{[0,\infty)}(x)$$

$$\mu_x = \alpha / \beta$$

$$\sigma_x^2 = \alpha / \beta^2$$

```
> pgamma(q = 25, shape = 3, rate = 1/10)
[1] 0.4561869
```

$\beta = 1/10$

$X \sim \text{Gamma}(\alpha = 6, \beta = 15) \longrightarrow X \sim N(\mu = 90, \sigma^2 = 1350)$

$\mu_X = 6 \times 15 = 90$

$\sigma^2_X = 6 \times 15^2 = 1350$

$\sigma_X = 36.74$

$P(X < 80)$

```
> pgamma(q = 80, shape = 6, scale = 15)
[1] 0.4423195
```

$P(X < 80)$

```
> pnorm(q = 80, mean = 90, sd = sqrt(1350))
[1] 0.3927474
```

$X \sim \text{Gamma}(\alpha = 10, \beta = 50) \longrightarrow X \sim N(\mu = 500, \sigma^2 = 25000)$

$\mu_X = 10 \times 50 = 500$

$\sigma^2_X = 10 \times 50^2 = 25000$

$P(X < 400)$

```
> pgamma(q = 400, shape = 10, scale = 50)
[1] 0.2833757
```

$P(X < 400)$

```
> pnorm(q = 400, mean = 500, sd = sqrt(25000))
[1] 0.2635446
```

Distribución Weibull

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] I_{[0,\infty)}(x)$$

TASA DE FALLO ES CONSTANTE

Si $\alpha = 1$

$$f(x) = \frac{1}{\beta}\overset{1}{\cancel{\left(\frac{x}{\beta}\right)^{1-1}}} \exp\left(-\left(\frac{x}{\beta}\right)^{1}\right) = \frac{1}{\beta}\exp\left(-\frac{x}{\beta}\right)$$

Distribución Beta

Función de densidad

Se define la V.A.C. X con distribución Beta con parámetros $\alpha > 0$ y $\beta > 0$, cuya función de densidad es:

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} I_{(0,1)}(x)$$

donde:

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$U(0,1)$

$$\alpha = \beta = 1 \Rightarrow f(x) = \frac{1}{B(1,1)} x^{1-1} (1-x)^{1-1} I_{(0,1)}(x) = I_{(0,1)}(x)$$

$$B(1,1) = \frac{\Gamma(1)\,\Gamma(1)}{\Gamma(2)} = \frac{1 \times 1}{1} = 1$$

$$f(x \mid \theta) = \binom{n}{x} \theta^{x} (1-\theta)^{n-x}$$

$$f(\theta) = \frac{1}{B(\alpha,\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

proporción

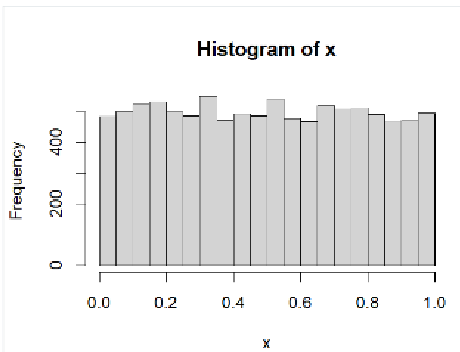
Distribuciones conjugadas

$$f(x \mid \lambda) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$f(\lambda) = \beta e^{-\beta\lambda}$$

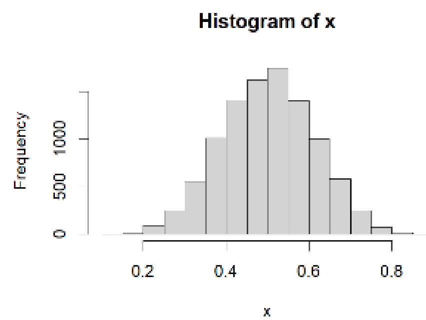
Distribuciones conjugadas

$\alpha = \beta = 1$



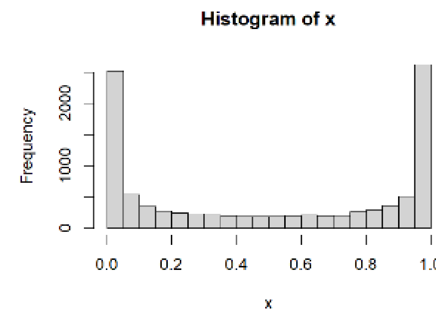
$U(0,1)$ amodal

$\alpha = \beta = 10$



$N(\mu, \sigma^2)$ unimodal

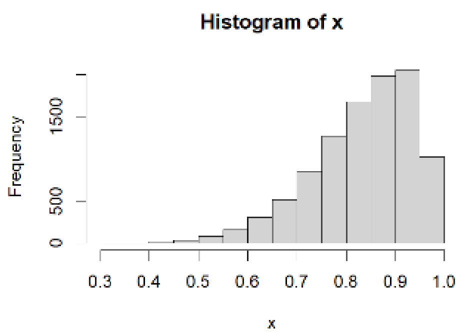
$\alpha = \beta = 0.25$



bimodal

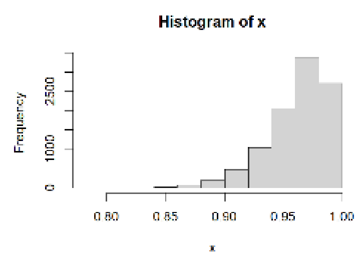
Simétrica en forma de U

$\alpha = 10, \ \beta = 2$

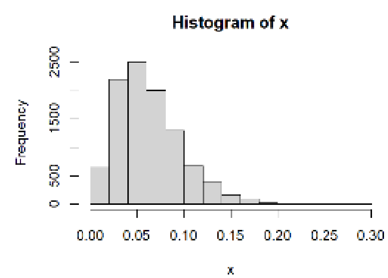


cola a la izq

$\alpha = 50, \ \beta = 2$



$\alpha = 3, \ \beta = 45$



cola a la derecha

$\alpha = 0.2, \ \beta = 0.7$