X	Y	f(x,y)
0	0	0.30
0	1	0.10
1	0	0.40
1	1	0.20

X \ Y	0	1	
0	0.30	0.10	0.40
1	0.40	0.20	0.60
	0.70	0.30	1

$$f_X(x) = \sum_y f_{X,Y}(x,y)$$

$$f_X(x) = 0.40\, I_{\{0\}}(x) + 0.60\, I_{\{1\}}(x)$$

$$f_Y(y) = \sum_x f_{X,Y}(x,y)$$

$$f_Y(y) = 0.70\, I_{\{0\}}(y) + 0.30\, I_{\{1\}}(y)$$

$f_{X,Y}(x,y)$: dist conjunta

$f_X(x)$: dist marginal

$F_{X,Y}(x,y)$: dist. acumulada conjunta

$F_X(x)$: dist. acumulada marginal

$$E(X+2Y) = \overset{0}{\cancel{0 \times 0.30}} + (0+2\times1)\,0.10 + (1+2\times0)\,0.40 + (1+2\times1)\,0.20 = 1.2$$

$\downarrow$

$$E(X) + 2E(Y) = (0\times0.4 + 1\times0.6) + 2(0\times0.7 + 1\times0.3) = 1.2$$

$$f_X(x) = \sum_y f_{X,Y}(x,y)$$

$$f_X(x) = 0.40\, I_{\{0\}}(x) + 0.60\, I_{\{1\}}(x) \Rightarrow$$

$$F_X(0) = 0.40, \qquad F_X(1) = 1.00$$

$$F_X(x) = 0.40\, I_{[0,1)}(x) + I_{[1,\infty)}(x)$$

$$f_Y(y) = \sum_x f_{X,Y}(x,y)$$

$$f_Y(y) = 0.70\, I_{\{0\}}(y) + 0.30\, I_{\{1\}}(y) \Rightarrow$$

$$F_Y(y) = 0.70\, I_{[0,1)}(y) + I_{[1,\infty)}(y)$$

$$F_Y(-2) = 0.70 \times 0 + 0 = 0$$

Ejemplo 2

Se define la función de probabilidad conjunta:

$$f_{X,Y}(x,y) = k(x+1)(y+2)I_{\{0,1,2,3\}}(x)I_{\{0,1,2\}}(y)$$

- Determinar la constante k para que f sea una función de probabilidad conjunta
- Obtener $f(2,2)$
- Determinar las distribuciones marginales de X e Y

a) $\sum_{x=0}^{3}\sum_{y=0}^{2} k(x+1)(y+2) = 1$

$k \sum_{x=0}^{3}(x+1)\sum_{y=0}^{2}(y+2) = 1$

$k \cdot 10 \times 9 = 1$

$\Rightarrow k = \frac{1}{90}$

b) $f_{X,Y}(x,y) = \frac{1}{90}(x+1)(y+2) I_{\{0,1,2,3\}}(x) I_{\{0,1,2\}}(y)$

$f_{X,Y}(2,2) = \frac{1}{\cancel{90}_{\cancel{30}\,15}} \times \cancel{3}^{1} \times \cancel{4}^{2} = \frac{2}{15}$

c) $f_X(x) = \sum_{y=0}^{2} \frac{1}{90}(x+1)(y+2) = \frac{(x+1)}{90}\sum_{y=0}^{2}(y+2) = \frac{9(x+1)}{90}$

✓ $f_X(x) = \frac{(x+1)}{10} I_{\{0,1,2,3\}}(x)$

$f_Y(y) = \sum_{x=0}^{3}\frac{1}{90}(x+1)(y+2) = \frac{y+2}{90}\sum_{x=0}^{3}(x+1) = \frac{10(y+2)}{90} \Rightarrow f_Y(y) = \frac{y+2}{9} I_{\{0,1,2\}}(y)$ ✓

Ejemplo 3

Un filtro automático clasifica 50 correos en:

- Spam (S): 0.10
- Promociones (P): 0.25
- Social (O): 0.15
- Principal (R): 0.50

$$(X_S, X_P, X_0, X_R) \sim Multinomial(n = 50, \pi_S = 0.10, \pi_P = 0.25, \pi_O = 0.15, \pi_R = 0.50)$$

Interpretar el siguiente resultado referido a simulación de muestras aleatorias multinomiales:

```
rmultinom (n = 3, size = 100 , prob = c (0.10,0.25,0.15,0.50))
```

```
     [,1] [,2] [,3]
[1,]    8    9   16
[2,]   21   23   21
[3,]   20   20   19
[4,]   51   48   44
```

En la primera muestra serían 8 correos spam, 21 promociones, 20 social y 51 principal

El 8. en el primer experimento simulado, de los 100 correos clasificados, 8 fueron etiquetados como Spam

Ejemplo 4a

Sea $\mathbf{X} = (X, Y)$ con densidad conjunta:

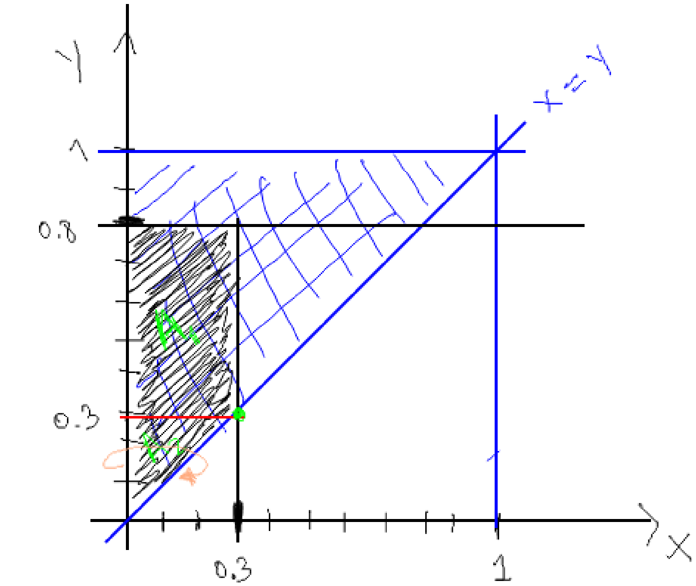
$$f(x,y) = 2\, I_{(0,y)}(x)\, I_{(0,1)}(y)$$

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 < x < y < 1, \\ 0, & \text{en otro caso.} \end{cases}$$

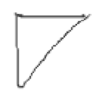
$$\int_0^1 \int_0^y 2\,dx\,dy = 1 \quad \rightarrow \int_0^1 2x\Big|_0^y \, dy = \int_0^1 2y\,dy = y^2\Big|_0^1 = 1$$

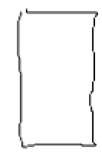
$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 < x < y < 1, \\ 0, & \text{en otro caso.} \end{cases}$$

$$P(X < 0.3,\ Y < 0.8)$$



$$\int_0^{0.3}\int_0^{y} 2\,dx\,dy + \int_{0.3}^{0.8}\int_0^{0.3} 2\,dx\,dy.$$





```r
library(pracma)
f <- function(y, x) 0*x + 0*y + 2
ymin_val <- 0
ymax_val <- 0.3
xmin_fun <- 0
xmax_fun <- function(y) y
(I1 <- integral2(f, ymin_val, ymax_val, xmin_fun, xmax_fun)$Q)
```

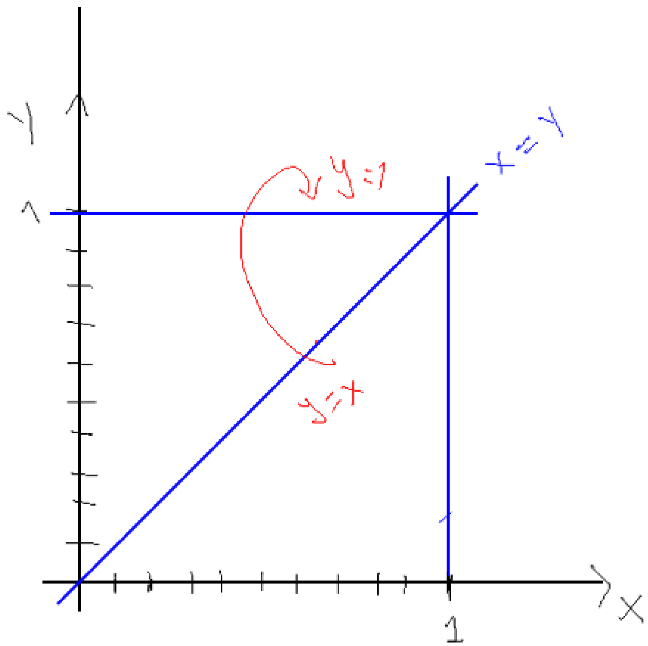
```
[1] 0.09
```

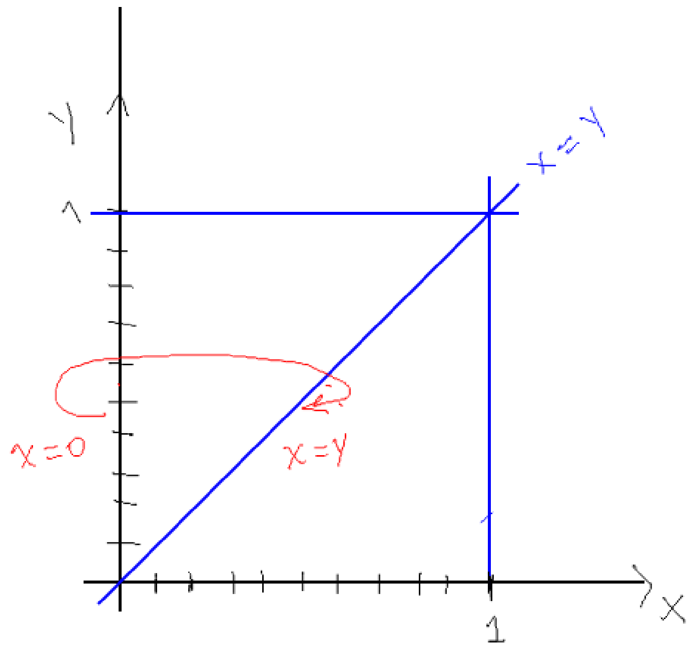
```r
ymin_val <- 0.3
ymax_val <- 0.8
xmin_fun <- 0
xmax_fun <- 0.3
(I2 <- integral2(f, ymin_val, ymax_val, xmin_fun, xmax_fun)$Q)
```

```
[1] 0.3
```

```r
I1 + I2
```

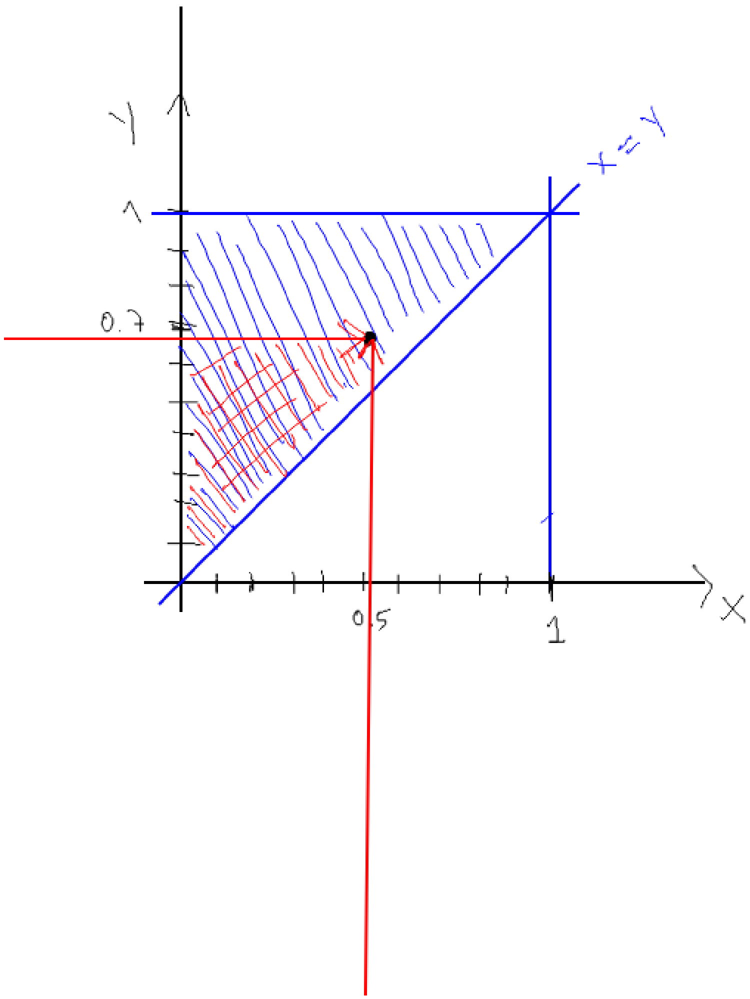
```
[1] 0.39
```





$$f_X(x) = \int_x^1 2\,dy = 2(1-x) \quad \Rightarrow \quad f_X(x) = 2(1-x)I_{(0,1)}(x)$$

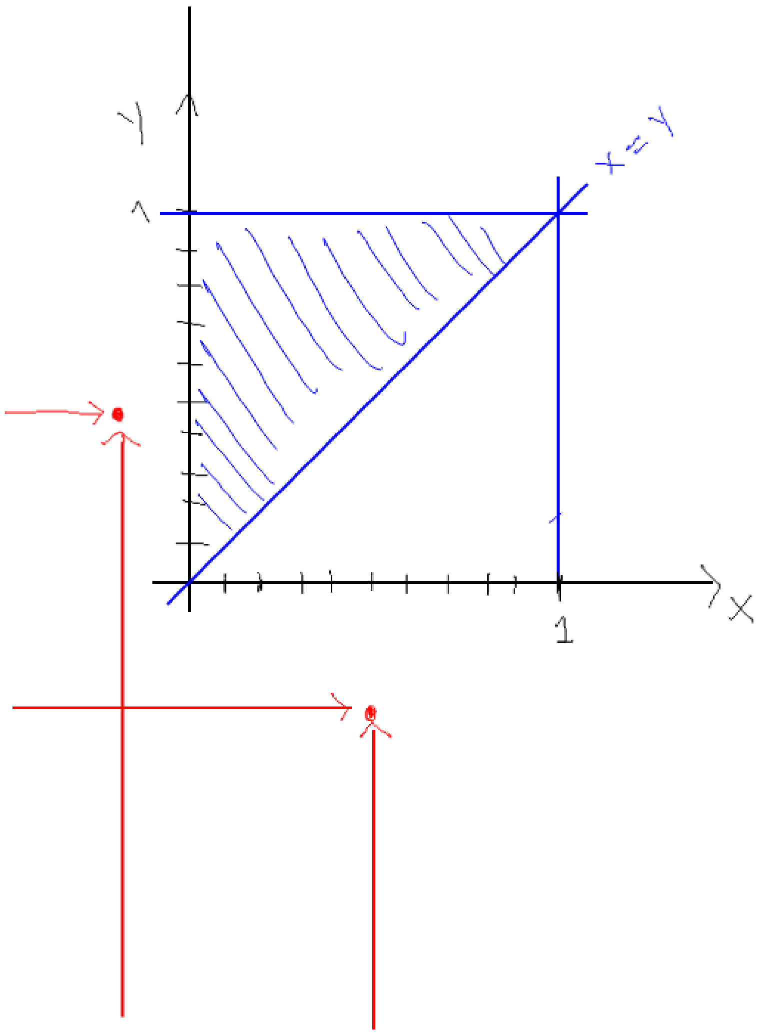
$$f_Y(y) = \int_0^y 2\,dx = 2y \quad \Rightarrow \quad f_Y(y) = 2yI_{(0,1)}(y)$$



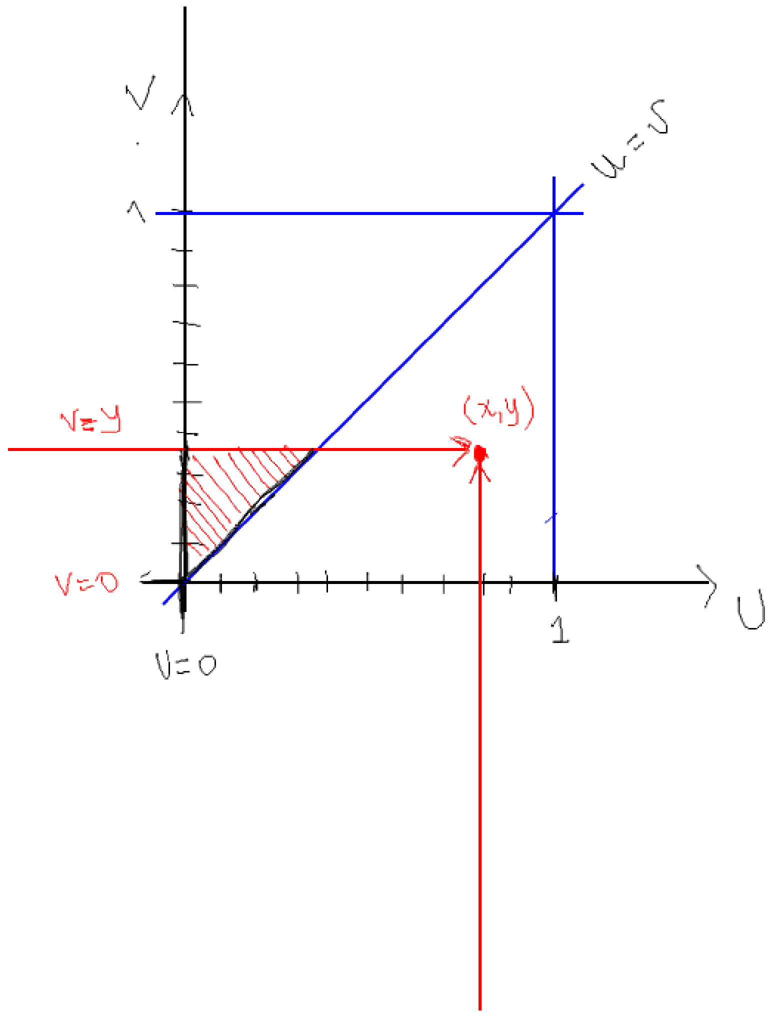
$$F(0.5, 0.7) = P(X \le 0.5,\ Y \le 0.7)$$

$$= P(-\infty < X \le 0.5,\ -\infty < Y \le 0.7)$$

$$F(x, y) = P(X \le x,\ Y \le y)$$



Si $x \le 0$ o $y \le 0$, $F_{X,Y}(x,y) = 0$



ctes

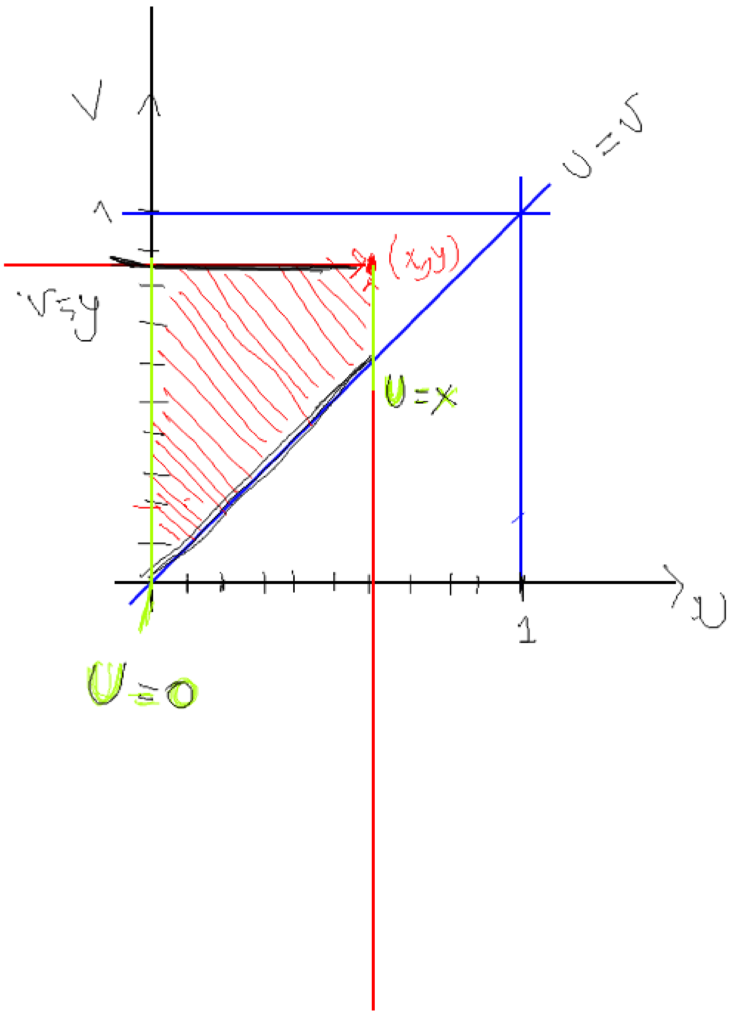
$$F(x,y) = \int_0^y \int_0^v 2du\,dv = y^2$$

eje x eje Y

$$= \int_0^y 2u \Big|_0^v dv = \int_0^y 2v\,dv = v^2 \Big|_0^y = y^2$$

$$F(0.9, 0.1) = 0.1^2 = 0.01$$

$$F(0.5, 0.1) = 0.1^2 = 0.01$$



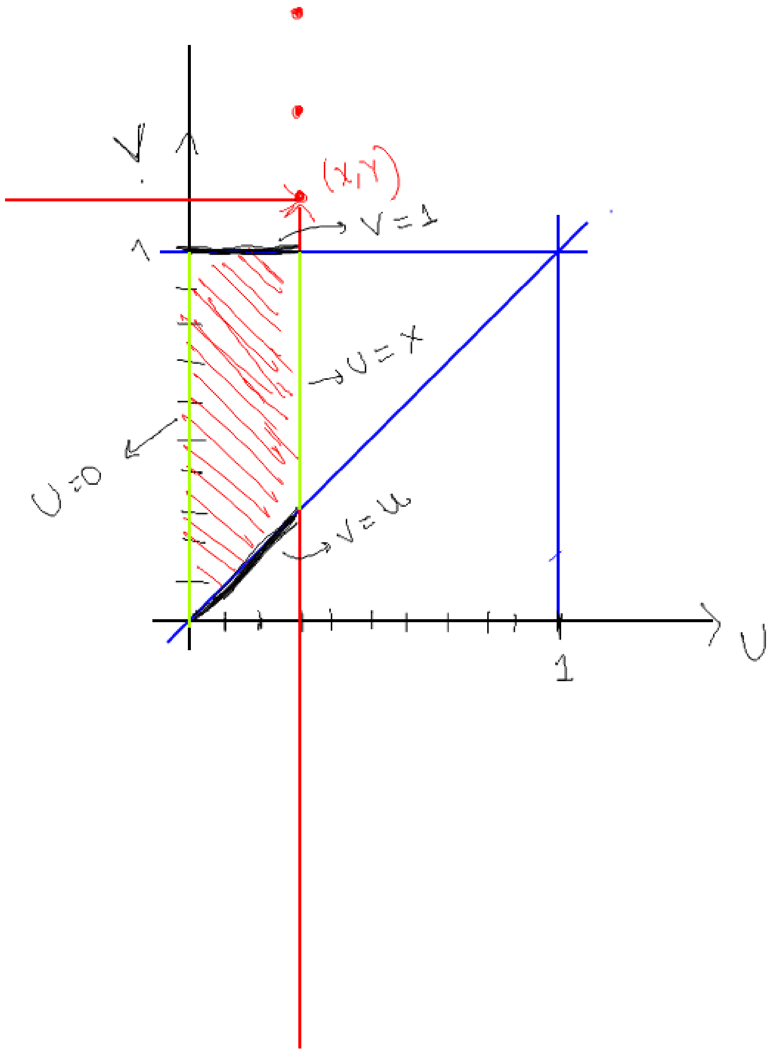
$$\overset{\text{ctes}}{} F(x,y) = \int_0^x \int_u^y 2\,dv\,du = 2yx - x^2$$

$dv \downarrow$ eje y, $du \downarrow$ eje x

$$\int_0^x 2v\Big|_u^y du = \int_0^x 2y - 2u \, du$$

$$= \left(2yu - u^2\right)\Big|_0^x$$

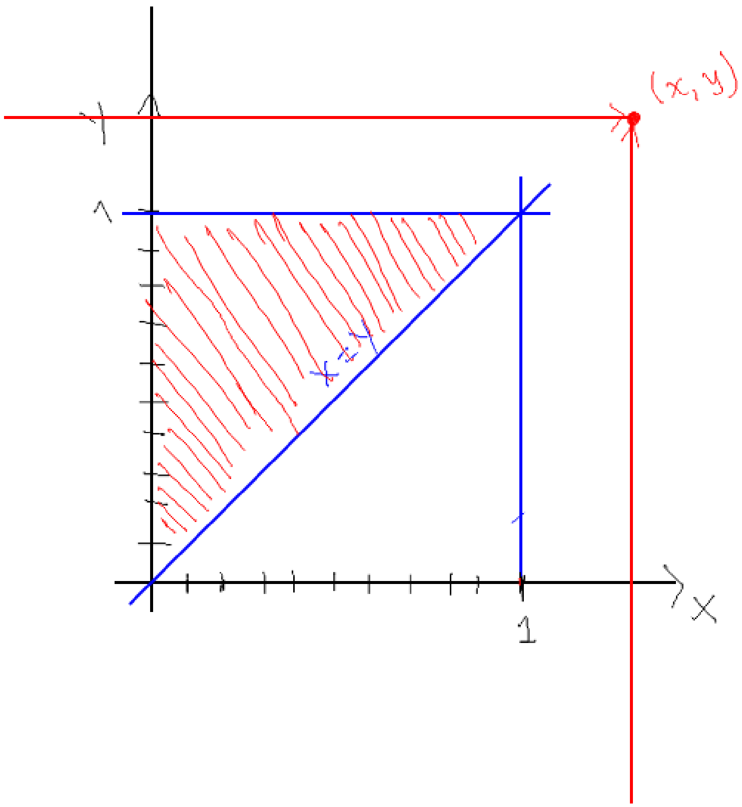
$$= 2xy - x^2$$



$$F(x,y) = \int_0^x \int_u^1 2dvdu = 2x - x^2$$

$F(0.2, 1.1) = 2 \times 0.2 - 0.2^2 = 0.40 - 0.04 = 0.36$

$F(0.2, 1.8) = 2 \times 0.2 - 0.2^2 = 0.36$



Entonces, finalmente:

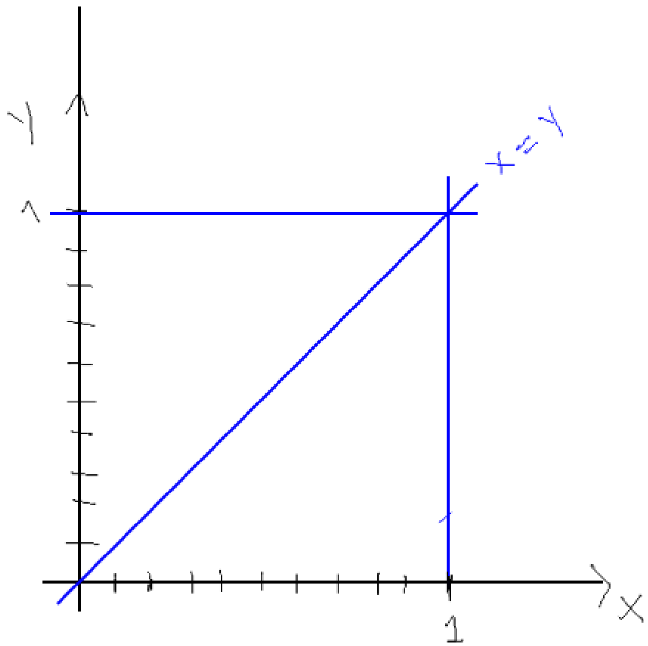
$$F_{X,Y}(x,y) = \begin{cases} 0 & x \leq 0 \text{ o } y \leq 0 \\ 2xy - x^2 & 0 < x < y < 1 \\ y^2 & 0 < y < 1,\ x \geq y \\ 2x - x^2 & 0 < x < 1,\ y \geq 1 \\ 1 & x \geq 1,\ y \geq 1 \end{cases}$$

$$F_X(x) = \lim_{y\to\infty} F_{X,Y}(x,y) \qquad F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$$

$$F_X(x) = \lim_{y\to\infty} F_{X,Y}(x,y) = 2x - x^2$$

$$F_Y(y) = \lim_{x\to\infty} F_{X,Y}(x,y) \qquad F_Y(y) = \int_{-\infty}^{y} f_Y(v)\,dv$$

$$F_Y(y) = \lim_{x\to\infty} F_{X,Y}(x,y) = y^2$$



Y
1
x = y
1
X