Distribución geométrica

X = Número de intentos hasta obtener el primer éxito

F F F E $x = 4$

$$f(x) = \pi(1-\pi)^{x-1} I_{\{1,2,\ldots\}}(x)$$

$$\mu_X = \frac{1}{\pi}$$

Ejm: $X \sim Geom(\pi = 0.5) \Rightarrow \mu_X = 2$ intentos

$$\underbrace{P(X=4)}_{4 \text{ intentos}} = 0.5 \times 0.5^{4-1} = 0.0625$$

$$\sigma_X^2 = \frac{1-\pi}{\pi^2}$$

R

X = Número de fracasos antes de obtener el primer éxito

F F F E $x = 3$

$$f(x) = P(X = x) = \pi(1-\pi)^x I_{\{0,1,2,3,\ldots\}}(x)$$

$$\mu_X = E(X) = \frac{1-\pi}{\pi}$$

Ejm: $X \sim Geom(\pi = 0.5) \Rightarrow \mu_X = 1$ fracaso

$$\underbrace{P(X=3)}_{3 \text{ fracasos}} = 0.5 \times 0.5^3 = 0.5^4 = 0.0625$$

$$\sigma_X^2 = V(X) = \frac{1-\pi}{\pi^2}$$

Intentos: 6 7 8 9 ...

Fracasos: 5 6 7 8 ...

$P(X \geq 5) = 1 - P(X \leq 4)$

NO

```
pgeom(q = 4, prob = 0.15, lower.tail=FALSE)
```

```
[1] 0.4437053
```

```
1-pgeom(q = 4, prob = 0.15)
```

```
[1] 0.4437053
```

$$P(X > s + t | X > s) = P(X > t)$$

$S = 2, \ t = 4$

$$\left\{ \begin{array}{l} P(X > 6 | X > 2) = P(X > 4) \\ P(X > 7 | X > 3) = P(X > 4) \\ P(X > 8 | X > 4) = P(X > 4) \\ P(X > 9 | X > 5) = P(X > 4) \end{array} \right.$$

=

=

=

X

$Y = X + 1$

$\mathrm{var}(Y) = \mathrm{var}(X)$

Función de probabilidad

Se define la V.A.D. X = número de fracasos antes del r-ésimo éxito. Su función de probabilidad es:

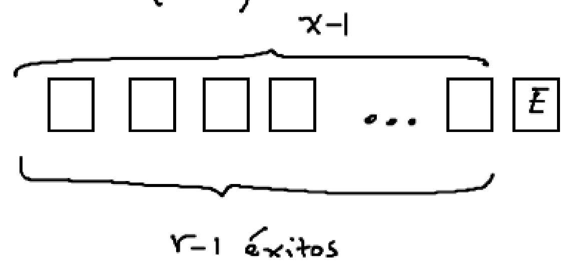
$$f(x) = P(X = x) = \binom{x+r-1}{x}(1-\pi)^x \pi^r I_{\{0,1,2,3,\ldots\}}(x)$$

$$\mu_X = E(X) = r \times \frac{1-\pi}{\pi}$$

$$\sigma^2_X = V(X) = r \times \frac{1-\pi}{\pi^2}$$

X = número de intentos hasta el r-ésimo éxito

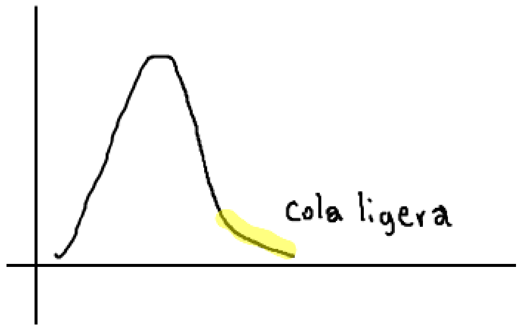
$$f(x) = \binom{x-1}{r-1}(1-\pi)^{x-r}\pi^r I_{\{r,\, r+1, \ldots\}}(x)$$

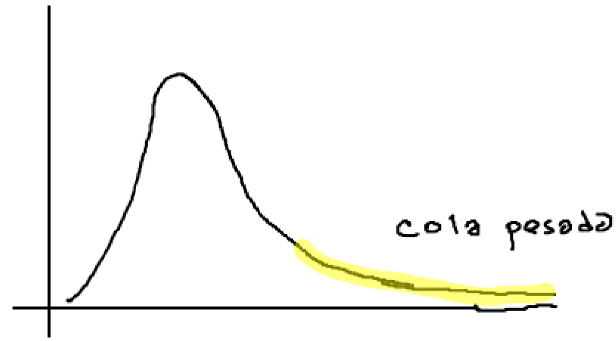


$$\mu_X = r \times \frac{1}{\pi} = \frac{r}{\pi}$$

$$\sigma^2_X = V(X) = r \times \frac{1-\pi}{\pi^2}$$

Distribución zeta





Media

$$E(X) = \frac{\zeta(s)}{\zeta(s+1)} \qquad s > 3$$

Si $0 < s \leq 1$, $E(X) = \infty$

Varianza

$$V(X) = \overset{E(X^2)}{\boxed{\frac{\zeta(s-1)}{\zeta(s+1)}}} - \overset{E(X)^2}{\left(\frac{\zeta(s)}{\zeta(s+1)}\right)^2} \qquad s > 2$$

Si $0 < s \leq 2$, $V(X) = \infty$