



$$y = \alpha_0 + \alpha_1 P_1(x) + \alpha_2 P_2(x) + \dots + \alpha_k P_k(x) + \epsilon$$

$$\mathbf{y} = X\alpha + \epsilon$$

$$X = \begin{pmatrix} P_0(x_1) & P_1(x_1) & \dots & P_k(x_1) \\ P_0(x_2) & P_1(x_2) & \dots & P_k(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ P_0(x_n) & P_1(x_n) & \dots & P_k(x_n) \end{pmatrix}$$

$$X^T X = \begin{pmatrix} \sum_{i=1}^n P_0^2(x_i) & 0 & \dots & 0 \\ 0 & \sum_{i=1}^n P_1^2(x_i) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sum_{i=1}^n P_k^2(x_i) \end{pmatrix} \rightarrow (X^T X)^{-1} = \begin{pmatrix} 1/\sum P_0^2(x_i) & 0 & \dots & 0 \\ 0 & 1/\sum P_1^2(x_i) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1/\sum P_k^2(x_i) \end{pmatrix}$$

$$P_1(x_i) = \lambda_1 \left(\frac{x_i - \bar{x}}{d} \right)$$

$$P_2(x_i) = \lambda_2 \left[\left(\frac{x_i - \bar{x}}{d} \right)^2 - \left(\frac{n^2 - 1}{12} \right) \right]$$

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(P1 = 2*(x-mean(x))/25)
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```
[1] -11 -9 -7 -5 -3 -1 1 3 5 7 9 11
```

```
(P2 = 3*(((x-mean(x))/25)**2 - 143/12))
```

```
[1] 55 25 1 -17 -29 -35 -35 -29 -17 1 25 55
```

	$n = 12$				
	P_1	P_2	P_3	P_4	P_5
1	1	-35	-7	28	20
3	3	-29	-19	12	44
5	5	-17	-25	-13	29
7	7	1	-21	-33	-21
9	9	25	-3	-27	-57
11	11	55	33	33	33
A_{ii}	572	12012	5148	8008	15912
λ_i	2	3	$\frac{2}{3}$	$\frac{7}{24}$	$\frac{3}{20}$

```
poly(x,3)[1:nrow(datos),]
```

	P1	P2	P3
[1,]	-0.4599331	0.501828160	-0.45993311
[2,]	-0.3763089	0.228103709	0.04181210
[3,]	-0.2926847	0.009124148	0.29268470
[4,]	-0.2090605	-0.155110522	0.34843417
[5,]	-0.1254363	-0.264600302	0.26480997
[6,]	-0.0418121	-0.319345193	0.09756157
[7,]	0.0418121	-0.319345193	-0.09756157
[8,]	0.1254363	-0.264600302	-0.26480997
[9,]	0.2090605	-0.155110522	-0.34843417
[10,]	0.2926847	0.009124148	-0.29268470
[11,]	0.3763089	0.228103709	-0.04181210
[12,]	0.4599331	0.501828160	0.45993311

polinomio lineal

```
sum(poly(x,3)[,1]*poly(x,3)[,2])
```

```
[1] -7.806256e-18 ≈ 0
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```
sum(poly(x,3)[,1]*poly(x,3)[,3])
```

```
[1] -2.428613e-17 ≈ 0
```

```
sum(poly(x,3)[,2]*poly(x,3)[,3])
```

```
[1] -1.374768e-16 ≈ 0
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ORTOGONALIDAD

				sqrt(CME)					
	Modelo	pvalor	R ² aj	sigma	pvalor Shapiro	pvalor Breusch Pagan	pvalor DW	AIC	VIF
1	y ~ x	0,404	-0,022	15,96	0,2778	0,6505	1,298*10 ⁻⁷	104,3542	
2	y ~ x + l(x ²)	1,18*10 ⁻⁸	0,9789	2,296	0,4560	0,5543	0,0033	58,55	19,107
3	y ~ P1 + P2	1,18*10 ⁻⁸	0,9789	2,296	0,4560	0,5543	0,0033	58,55	1
4	y ~ poly(x,3)	1,82*10 ⁻⁹	0,9928	1,337	0,0870	0,7188	0,2355	46,17	1
5	y ~ poly(x,4)	1,55*10 ⁻⁸	0,9940	1,224	0,6834	0,6491	0,7474	44,43	1

* X no necesariamente tiene el mismo espaciamento

* pueden haber más variables predictoras:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 p_1(x) + \hat{\beta}_2 p_2(x) + \hat{\beta}_3 p_3(x) + \underbrace{\hat{\beta}_4 x_2} + \underbrace{\hat{\beta}_5 x_3}$$

* Todo lo realizado debe ejecutarse en el TRAINING.

TESTING \rightarrow evaluar el modelo.