Rotational Dynamics

Jeh

2021-04-30

1 2-3M Questions

1.1 Why are curved roads banked?

A car while taking a tunr performs circulat motion.If the road is level (or horizontal road), the neccessary centripetal force is the force of static frciton between the car tyres and the road surface.

The frciton depends upon the nature of surfaces in contact and the presence of oil and water on the road surface. If the frciton is inadequate, a speeding car may skid off the road. Since the friction changes with circumstances, it cannot be relied upon to provide neccessary centripetal force. Moreover, frciton results in fast wear and tear of the tyres.

To avoid the risk of skidding as well as to reduce the wear and tear of the car tyres, the road surface at bend is tilted inward, i.e, the outer side of the road is raised above its inner side. This is called banking of road. On a banked road, the resultant of the normal reaction and the gravitational force can act as neccessary centripetal force. Thus, every car can be safely driven on such banked curve at certain optimum speed, without depending on frciton. Hence, a road should be properly banked at the bend.

The angle of banking is the angle of inclination of the surface of a banked road at bend with the horizontal.

1.2 Do we need a banked road for two wheeler?Explain

When a two wheeler takes turn along an unbanked road, the force of frciton provides the centripetal force. The two-wheeler leans inward to counteract a torque that tends to topple it outward. Firstly, frciton cannot be relied upon to provide the neccessary centripetal force on all conditions. Secondly, the frciton results in the wear and tear of the tyres. On a banked road at a turn, any vehicle can negotiate the turn without depending on friction and without straining the tyres.

1.3 On what factors does the frequency of a conical pendulum depends? Is it independent of some factors?

The frequency of a conical pendulum, of string length L and semi vertical angle θ is

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{L\cos\theta}} \tag{1}$$

where g is the acceleration due to gravity at the place.
From the above expression, we can see that

$$n \propto \sqrt{g} \tag{2}$$

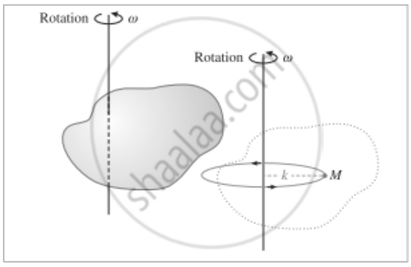
$$n \propto \frac{1}{\sqrt{L}} \tag{3}$$

$$n \propto \frac{1}{\sqrt{\cos\theta}} \tag{4}$$

(if θ increases, $\cos\theta$ decreases and n increases)
The frequency is independent of the mass of the bob.

1.4 Why is it useful to define the radius of gyration?

Defination : The radius of gyration of a body roataing about an axis is defined as the distance between the axis of roataion and point at which entire mass of the body can be supposed to be concentrated so as to give the same moment of inertia as that of the body about the given axis.



The moment of inertia (MI) of body about a given roataion axis depends upon(i) the mass of the body and (ii) the distribution of mass about axis of roataion. These two factors can be seperated by expressing the MI as the product of the mass(M) and the square of a particular distance(k) from the axis of roataion. This distance is called the radius of gyration and is defined as given above. Thus,

$$I = \sum_i m_i r_i^2 = Mk^3 \tag{5}$$

$$\therefore k = \sqrt{\frac{I}{M}} \tag{6}$$

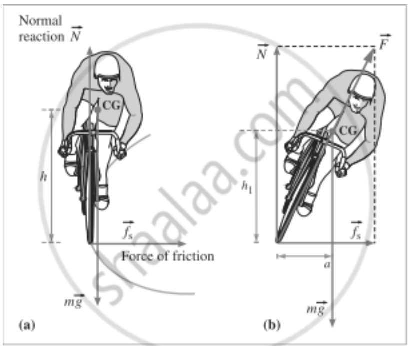
Physical significance: The radius of gyration is less if I is less, i.e., if the mass is distributed close to the axis; and it is more if I is more, i.e. if the mass is distributed away from the axis. Thus, it gives an idea about the distribution of mass about the axis of roataion.

1.5 A uniform disc and a hallow right circular cone have the same formula for their moment of inerti when rotating about their central axes. Why is it so?

A uniform disc and a hollow right circular cone have the same formula for their moment of inertia.
$MI = mr^2/2$
This is becaue when a hollow right circular cone is cut along its slanting side and the metal is stretched out, you will find that the surface of the cone will form a circle. This shape is the same as the shape of disc, which is also circle. Hence they both have same forula for MI.

1.6 While driving along an unbanked circular road, a two wheeler rider has to learn with vertical. Why is it so? With what angle the rider has to lean? Derive the relevant expression. Why such a leaning is not neccessary for a four wheeler?

1. When a bicyclist takes a turn along an unbanked road, the force of friction $\vec{f_s}$ prodvides the centripetal force; the normal reaction of the road $\vec{N}$ is vertically up. If the bicyclist does not learn inward, there will be an unbalanced outward torque about the centre of gravity, $f_s.h$, due to the frciton force that will topple the bicyclist outward. The bicyclist must lean inward to counteract this torque (and not to generate a centripetal force) such that the opposite inward torque of the couple formed by $\vec{N}$ and weight $\vec{g}$, $mg.a = f_s.h_1$



A bicyclist taking a turn to his left on a level road

2. Since the force of friction provides the centripetal force,

$$f_s = \frac{mv_2}{r} \tag{7}$$

If the cyclist leans from the vertical by an angle θ, the angle between $\vec{N}$ and $\vec{F}$

$$tan\theta = \frac{mv_2/r}{mg} = \frac{v^2}{gr} \tag{8}$$

Hence, the cyclist must lean by an angle

$$\theta = \tan^-1 = (\frac{v^2}{gr}) \tag{9}$$

3. When a car takes a turn along a level road, apart from the risk of skidding off outward, it also has a tendency to roll outward due to an outward torque, it also has a tendency to roll outward due to an outward torque about the centre of gravity due to the friction force. But a car is an extended object with four wheels. So, when the inner wheels just get lifted above the ground, it can be counterbalanced by a restoring torque of the couple formed by the normal reaction (on the outer wheels) and the weight. Consider a car of mass m taking turn of radius r along a level raod. As seen from an intertial frame of referance, the forces acting on the car are:

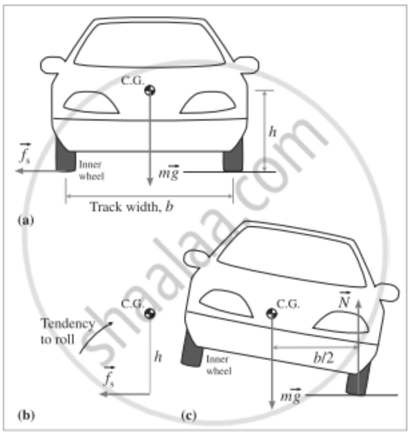
   (a) the lateral limiting force of static friction $\vec{f}$ on the wheels - acting along the axis of the wheels and towards the center of the circular path which provides the neccessary centripetal force.

(b) the weight $\vec{m}g$ acting vertically downwards at the centre of gravity (C.G.)

(c) the normal reaction $\vec{N}$ of the road on the wheels, upwards effectively at the C.G. Since the maximum centripetal force = limiting force of static friction,

$$ma_r = \frac{mv^2}{r} = f_s \tag{10}$$

In a simplified rigid-body vehicle, we consider only two parameters - the height h of the C.G. above the ground and tge average distance b between the left and right wheels called the track width.



Rolling tendency of a vehicle negotiating a bend on a level road

The friction force $\vec{f_s}$ on the wheels produces a torque τ_t that tends to overturn /rollover the car abut the outer wheel in the above figure(b). Rotation about the fron to back axis is called roll.

$$\tau_t = f_s.h = (\frac{mv^2}{r}h) \tag{11}$$

When the inner wheel just gets lifted above the ground, the normal reaction $\vec{N}$ of the road acts on the outer wheels but the weight continues to act at the C.G. Then, the couple formed by the normal reaction and the weight produces a opposite torque τ_r which tenfs to restore the car back on all four wheels in the above figure(b).

$$\tau_r = mg.\frac{b}{2} \tag{12}$$

The car does not topple as longas the restoring torque τ_r counterbalances, the toppling torque τ_r .

Thus, to avoid the risk of rollover, the maximum speeed that the care can have is given by

$$(\frac{mv^2}{r})h = mg.\frac{b}{2} \tag{13}$$

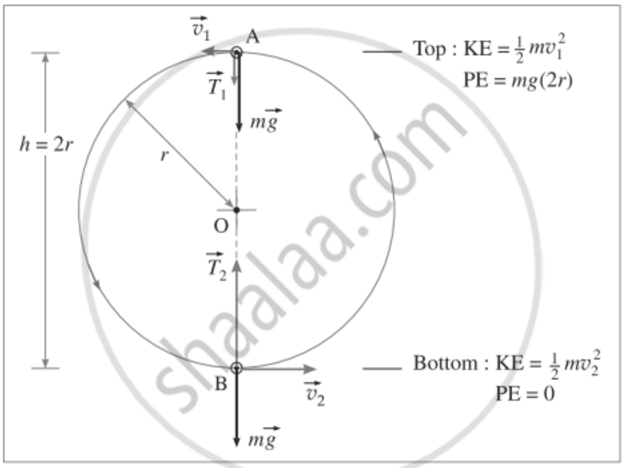
$$\therefore v_m ax = \sqrt{\frac{rbg}{2h}} \tag{14}$$

Thus, the vehicle to roll when the radial acceleration reaches a point where inner wheels of the four-wheeler are lifted off of the ground and the vehicle is rotated outward. A rollover occurs when the gravitational force $\vec{mg}$ passes through the pivot point of the outer wheels, i.e the C.G. is above the line of contact of the outer wheels. Equation(3) shows that this maximum speed is high for a car with larger track widtg and lowwer center of gravity.

1.7 Using energy conservation, derive the expression for the minimum speeds at different loctions along a vertical circular motion controlled by gravity.

Consider a particle of mass m attached to a string and revolved in a vertical circle of radius r. At every instant of its motion, the particle is acted upon by its weight $\vec{mg}$ and the tesnion $\vec{T}$ in the string. Let v_2 be the speed of the body and T_2 be the tesnion in the string at the lowest point B. We take the referance level for the zero potential energy to be bottom of the circle. Then, the body has only kinetic energy $\frac{1}{2}mv_2^2$ at the lowest point.

$$T_2 = \frac{mv_2^2}{r} + mg \tag{15}$$



Vertical circular motion (schematic)

and the total energy at the bottom - KE + PE

$$\frac{1}{2}mv_2^2 + 0 \tag{16}$$

Let v_1 be the speed and T_1 the tension in the string at the highest point A. As the body goes from B to A, it rises through a height h = 2r.

$$\therefore T_1 = \frac{mv_1^2}{r} - mg \tag{17}$$

and the total energy at A = KE + PE

$$= \frac{1}{2}mv_1^2 + mg(2r) \tag{18}$$

Then, from Eqs(15) and (17),

$$T_2 - T_1 = \frac{mv_2^2}{r} + mg - (\frac{mv_1^2}{r} - mg) \tag{19}$$

$$= \frac{m}{r}(v_2^2 - v_1^2) + 2mg \tag{20}$$

Assuming that the total energy of the body is conserved, the total energy at the bottom = total energy at the top
Then, from Eqs(16) and (18)

$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + mg(2r) \tag{21}$$

$$\therefore v_2^2 - v_1^2 = 4gr \tag{22}$$

Subsittuting this in eq(20),

$$T_2 - T_1 = \frac{m}{r}(4gr) + 2mg \tag{23}$$

$$= 4mg + 2mg = 6mg \tag{24}$$

Therefore, the difference in the tensions in the string at the highest and lowest points is 6 times the weight of the body.