

A binomial distribution is a discrete probability distribution that describes the probability of a specific number of "successes" in a fixed number of independent trials, where each trial has only two possible outcomes: success or failure. A binomial distribution is a discrete probability distribution that describes the probability of a specific number of "successes" in a fixed number of independent trials, where each trial has only two possible outcomes: success or failure.

PROPERTIES OF A BINOMIAL EXPERIMENT -

1. The experiment consists of a sequence of n identical trials.
2. Two outcomes are possible on each trial. We refer to one outcome as a success and the other outcome as a failure.
3. The probability of a success, denoted by p , does not change from trial to trial. Consequently, the probability of a failure, denoted by $1-p$, does not change from trial to trial.
4. The trials are independent.

The Binomial distribution is an appropriate model to use for calculating the probabilities of obtaining a certain number of successes in the given trials.

The Formula for Binomial Distribution-

The binomial distribution formula for a random variable $X=1,2,3,4,5,6,\dots,n$ is given as -

$$P(X=r) = nCr (p^r) (q)^{n-r}, r = 0, 1, 2, 3, \dots$$

where,

p = probability of success

q = probability of failure

r = number of successes

n = total number of trials

Example -

A fair coin is flipped 20 times;

Success: "Heads" ($p=0.5$).

Random variable X : Number of heads observed in 20 flips.

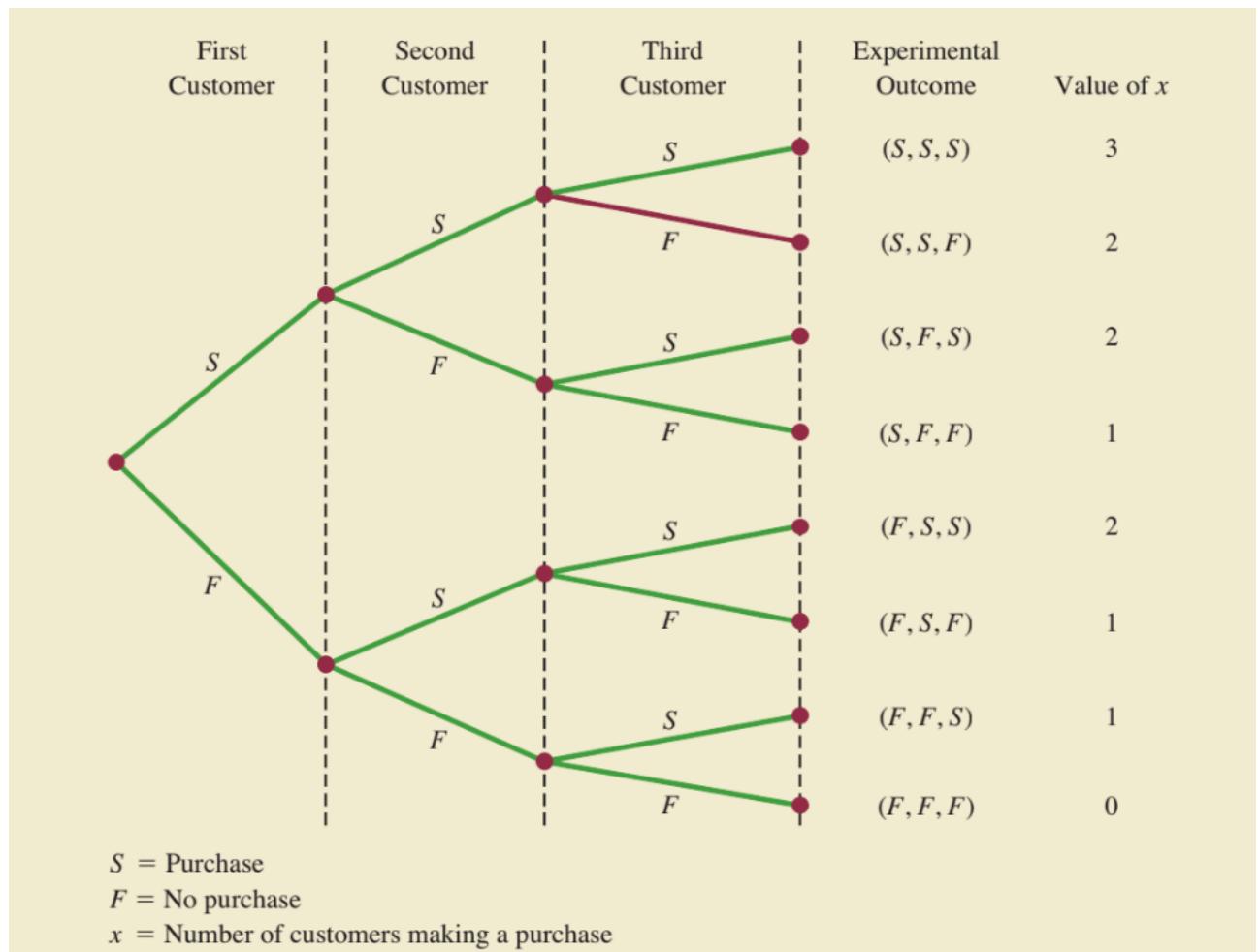
Distribution: $X \sim \text{Binomial} (n=20, p=0.5)$.

Probability of Getting exactly 10 heads is given by, ($r=10$)

$$P(X = 10) = {}^{10}C_{20} (0.5)^{10} (0.5)^{10} \approx 0.176 (17.6\%)$$

Martin Clothing Store problem -

In this scenario, Martin knows from past data that **30%** of customers make a purchase (Success), while **70%** do not (Failure). We want to find out the probability that two of the next three customers will make a purchase?



- **n (Trials):** 3 customers
- **p (Probability of Purchase):** 0.30
- **x (Desired Sales):** 2

The scenario that we are looking for is 2 out of the 3 customers purchase an item for which we have 3 possible outcomes

1. S S N (First two purchase)
2. S N S (First and third purchase)
3. N S S (Last two purchase)

Using the binomial formula, $P(X=r) = nCr (p^r) (q)^{n-r}$, we put in our values and obtain -

$$P(X=2) = (3 C 2) (0.3^2) \cdot (0.7^1)$$

$$= 1 \times 0.09 \times 0.7$$

$$= 0.063$$

This will have to be multiplied by 3 due to the number of possible outcomes being 3, therefore the final result -

$$3 \times 0.063 = 0.189$$

In percentage, 0.189 is 18.9

Therefore there is an 18.9 percent chance of 2 out of the next 3 customers making a purchase.

Expected Value and Variance in Binomial Distribution

Variance of Binomial Distribution is a measure of the dispersion of probabilities with respect to the mean value (expected value). This value tells us the typical extent to which sampled observations tend to differ from the expected value. Variance is denoted by (σ^2).

Formula for variance - $\sigma^2 = npq$, where -

N = number of trials

P = probability of success

Q = probability of failure

Expected value of any distribution is essentially its mean. To find out expected value, we simply multiply each value of the random variable by its probability and add the products.

Formula for expected value is = np , where -

N = number of trials

P = probability of success