TANK HEATING LAB

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Introduction

The heating of batch storage tanks is a fundamental part of many chemical engineering applications. Because of costs related to tank heating, it is often important to pick a heating method that is efficient. In this lab, we will compare the properties and performance of a heating coil, which will be placed directly in the tank, to an external heat exchanger.

Our primary objectives in this lab consist of two things. First, we determined the overall heat transfer coefficient of each heating device. The heat transfer coefficient is a measure of how effectively heat is transferred from one median to the next in a system. Second, we determined the relative efficiency of the two systems in order to discover which method heated the tank faster. We used time to compare the efficiencies because steam is the primary cost associated with this type of tank.

The contents of this report will consist of the following:

1. Theories and equations relevant to heat exchangers
2. A description of experimental procedure
3. A comparison of experimental coil heat transfer coefficients to those resulting from correlations from literature
4. A relationship between agitator speed and the coil heat transfer coefficient for experimental and correlated values
5. A comparison of experimental heat exchanger heat transfer coefficients to those resulting from correlations from literature

6. Error propagation analysis of experimental heat transfer coefficients
7. A comparison between the two methods of heat transfer

This report does not discuss these two methods in relationship to any other methods or equipment. Additionally, the experimental values of the heat transfer coefficient cannot be easily applied to another tank due to different rates of ambient heat loss.

Theory and Working Equations

This section contains the theory and working equations used to analyze the batch tank heating experiment. An explanation of the nomenclature can be found in Appendix A.

Experimental overall heat transfer coefficient for coil heating

In order to find the experimental heat transfer coefficient (U) from data gathered while heating the tank with coils we began with a transient energy balance for the water in the tank. We assumed that the work done by the agitator was negligible so the first law became

$$\frac{dU^{CV}}{dt} = \dot{Q} . \qquad (1)$$

The internal energy of the water could then be expressed in terms of heat capacity and a temperature-time differential. Assuming that the heat loss from the tank was negligible the heat transfer into the tank could be expressed by the convection heat transfer equation. Equation (1) became [1: 462]

$$m_T C_P \frac{dT}{dt} = UA(T_{St} - T(t)) . \qquad (2)$$

Integrating (2) from $t = 0$ to $t = t$ and $T = T_0$ to $T = T_t$ gave

$$\ln\left(\frac{T_{St}-T_0}{T_{St}-T_t}\right)=\frac{UAt}{m_T C_P}. \tag{3}$$

Plotting the left-hand side vs. time resulted in a linear plot, the slope of which was a function of *U* and known values. We used the numerical value of the slope to find experimental values for *U* as a function of agitation speed.

Theoretical overall heat transfer coefficient for coil heating

In order to investigate the validity of our experimental values we used the following method to find a theoretical value for *U*. We began with a thermal resistance circuit described by

$$\frac{1}{UA}=\frac{1}{h_i A_i}+\frac{\ln\left(D_0/D_i\right)}{2\pi kL}+\frac{1}{h_o A_o}. \tag{4}$$

To find h_i we used the Chato correlation designed for steam condensing in horizontal tubes [2: 627]:

$$\bar{h}_i=0.555\left[\frac{g\rho_f\left(\rho_f-\rho_v\right)k_f^3\lambda'}{\mu_f\left(T_{sat}-T_w\right)D_i}\right]^{1/4}. \tag{5}$$

Where λ' is defined by [2: 627]

$$\lambda'\equiv\lambda+\frac{3}{8}C_{P,f}\left(T_{sat}-T_w\right). \tag{6}$$

In order to find a value for h_o we used a dimensionless correlation recommended by McCabe, Smith and Harriot [1: 460] for heating liquids in cylindrical tanks with helical coils and turbine impellers:

$$\frac{h_o D_o}{k}=0.17\left(\frac{D_I n\rho}{\mu}\right)^{0.67}\left(\frac{C_P\mu}{k}\right)^{0.37}\left(\frac{D_I}{D_T}\right)^{0.1}\left(\frac{D_o}{D_T}\right)^{0.5}\left(\frac{\mu}{\mu_w}\right)^{0.24}. \tag{7}$$

We used the hydraulic diameter of the tank to correct for its non-cylindrical shape. It is important to note that this correlation relates h_o to the speed of the agitator (n). After finding h_i and h_o we were able to find theoretical values for U as a function of agitator speed.

Experimental overall heat transfer coefficient for the heat exchanger

To find an expression for the experimental U for the heat exchanger we began with the first law for the water in the tank. Equation (8) shows the simplified first law equation with the enthalpy and internal energy expressions replaced by heat capacities:

$$m_T C_P \frac{dT_o}{d_t} = \dot{m} C_P \left(T_i - T_o(t)\right). \tag{8}$$

Next we wrote the first law for the water in the heat exchanger. Equation (9) shows the simplified equation with heat capacity substitutions. Also in (9) the subscript *o* stands for out of the tank and into the heat exchanger. The subscript *i* stands for out of the heat exchanger and into the tank. This makes the steam identification used in (8) identical to that used in (9):

$$m_T C_P \frac{d\dot{T}_m}{dt} = \dot{m} C_P (T_o(t) - T_i) + \dot{Q}. \tag{9}$$

For a heat exchanger the heat transfer can be expressed as

$$\dot{Q} = UA\Delta T_{lm}. \tag{10}$$

Substituting this into (9) and adding (8) and (9) together gives

$$m_T C_P \frac{dT_o}{dt} + \dot{m} C_P \frac{dT_m}{dt} = UA\Delta T_{lm}. \tag{11}$$

Further development of (11) into (12) can be found in Appendix B. Equation (12) is specified to work for heat exchangers in which the cooling fluid (water) is not at a constant temperature [1: 462]:

$$\ln\frac{T_o - T_i}{T_t - T_i} = \frac{\dot{m}_i}{m_T}\frac{K_1 - 1}{K_1}t\,. \tag{12}$$

K_1 is defined as

$$K_1 = \exp\left(\frac{UA}{\dot{m}_c C_P}\right). \tag{13}$$

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This allows us to plot the left-hand side of Eq. (12) vs. time and find the average value of U from the resulting slope of the linear plot.

Theoretical overall heat transfer coefficient for the heat exchanger

In order to compare our experimental U with a theoretical value we again began with the thermal circuit in Equation (4). To find a value for h_i we began by evaluating the Reynolds number of the water flowing inside the tubes of the heat exchanger. After deciding the flow was fully developed and turbulent we used the Dittus-Bolter equation [1: 347, 2: 491] for heating to find h_i:

$$Nu = \frac{h_i D_i}{k} = 0.023\mathrm{Re}^{4/5}\,\mathrm{Pr}^{0.4}\,. \tag{14}$$

A value for h_o was found by using one of the Nusselt equations for horizontal tubes [1: 379, 384, 2: 623]. Using these equations assumed that the tubes in the heat exchanger could be treated as a bank of horizontal tubes. For N number of tubes the equation is

$$h_o = 0.729\left(\frac{k_f^3 \rho_f^2 g \lambda}{N(T_{sat} - T_w) D_o \mu_f}\right)^{1/4}. \tag{15}$$

After finding the heat transfer coefficients for each side of the heat exchanger equation (4) was used to find *U*.

Comparing the two methods of heating

In order to compare the two methods of heating we compared the time it took the coil to heat the water to the time it took the heat exchanger to heat the water from 100°F to 160°F. This gave us the efficiency ratio

$$\varepsilon = \frac{t_{HX}}{t_{coil}}. \tag{16}$$

Error propagation analysis

The errors associated with coil heating stem from temperature measurements and time measurements. Applying standard error propagation analysis to Eq. (3) and assuming that the error associated with each temperature measurement is the same gives

$$\frac{\sigma_U^2}{U^2} = \frac{\dfrac{2\sigma_T^2}{(T_{St} - T_0)^2} + \dfrac{2\sigma_T^2}{(T_{St} - T_t)^2} + \dfrac{\sigma_t^2}{t^2}}{\ln\left(\dfrac{T_{St} - T_0}{T_{St} - T_t}\right)}. \tag{17}$$

For the heat exchanger the error stems from temperature, time, and mass measurements. Applying standard error propagation techniques to Eq. (12) and Eq. (13) gives

$$\frac{\sigma_U^2}{U^2}=\frac{\left(\frac{\sigma_{\dot{m}}^2}{\dot{m}}\right)+\left(\frac{\sigma_t^2}{t^2}\right)+\left(\frac{\sigma_m^2}{m^2}\right)+\frac{\left(\frac{2\sigma_T^2}{(T_0-T_i)^2}\right)+\left(\frac{2\sigma_T^2}{(T_t-T_i)^2}\right)}{\left[\ln\left(\frac{T_0-T_i}{T_t-T_i}\right)\right]^2}}{\left[\ln\left(\frac{(K_1-1)\dot{m}t}{m_T\ln\left(\frac{T_0-T_i}{T_t-T_i}\right)}\right)\right]^2}+\frac{\sigma_{\dot{m}}^2}{\dot{m}}. \qquad (18)$$

Experimental Methods

The purpose of this experiment was to find experimental values for the overall heat transfer coefficient for two methods of batch heating and compare them to theoretical *U* values. The first method of heating consisted of a copper coil filled with steam submerged in the tank of water. In this method *U* was a function of agitator speed. The second method of heating was a condensing steam shell and tube heat exchanger. A secondary objective was to compare the efficiencies of the two methods of heating. A schematic drawing of the apparatus used in this experiment is shown in Figure 1.

We began the experiment by turning the agitator to a setting of 30 (300 RPM) and allowing the steam to flow through the coil at a pressure of 20 psig. We set the computer to record all the temperature information and allowed the smaller steam coil (coil #2) to heat the water. During the heating process we took a measurement of steam flowrate using a bucket and stopwatch. After the tank reached approximately 170°F we turned off the agitator and the steam and allowed the tank to drain. Finally, we refilled the tank and

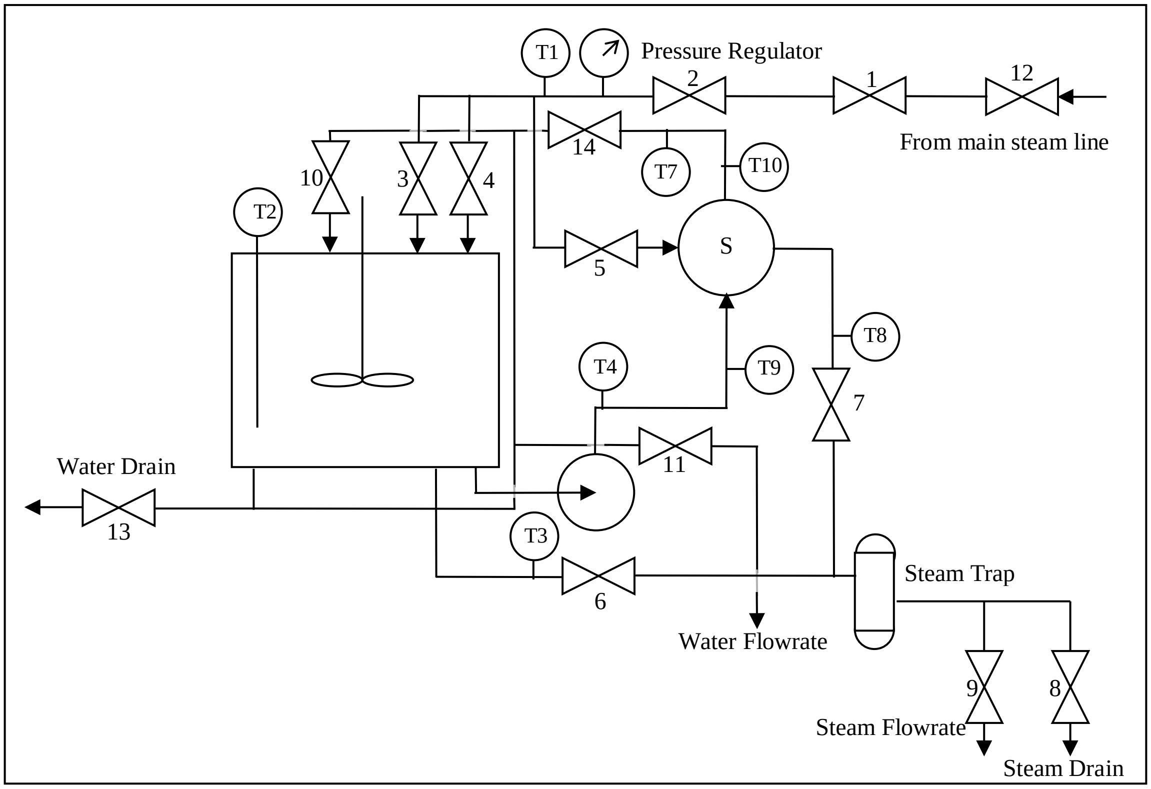

Figure1: Schematic drawing of tank heating apparatus. A complete description of the valves and thermocouples can be found in Appendix D.

repeated the above process for agitator settings of 60 (1100 RPM) and 90 (1750 RPM). This was done three times for each speed. A disk containing raw temperature data and a table describing each run are included in Appendix C.

Next we pumped water in a continuous loop through the heat exchanger and turned the agitator to 90 in order to keep the tank well mixed. We set the computer to record temperature data and allowed steam at 20 psig to flow through the heat exchanger until the water in the tank reached approximately 170°F. Next we turned off the agitator and steam and measured the water flowrate through the heat exchanger with a bucket and stopwatch. Finally we drained and refilled the tank. We repeated this portion of the experiment three times. Complete experimental data can be found in Appendix D.

Discussion of Results

This section contains the results of coil heating, external heating with a heat exchanger, a comparison between the two, and a discussion of all results.

Coil heater

Plots of the experimental data for coil heating are included in Appendix E as Figures E.1, E.2 and E.3. These figures all show excellent agreement between the three runs for each agitation speed. Plotting a trendline for each run gave us the slope of each line and allowed us to find U. The experimental U values, the error associated with each value (Eq. 16), the correlated values, and percent difference for each agitation speed are shown in Table I.

Table I: Summary of Coil Heater Results

Agitator Setting	U_{exp} [W/m2*K]	σ_U [W/m^2*K]	U_{cor} [W/m^2*K]	% difference[1]
30	2143	28	1102	94.5%
60	2572	35	2252	14.2%
90	2829	38	2819	0.3%

[1] $\%diff = \frac{|U_{corr} - U_{exp}|}{U_{exp}}$

Part of the difference between the correlated and experimental *U* values occurs because the correlation was fitted for a helical coil in a cylindrical tank. Our experiment consisted of a square coil in a square tank. The convergence of *U* values as agitation speed increases could be due to the shorter heating time allowing less heat to escape from the top of the tank. Also energy from higher agitation may compensate for heat loss from the tank.

Table I also shows that both the experimental *U* values and the correlated *U* values increase as a function of agitation speed. We expected this to occur because faster agitation causes the water in the tank to move faster. Faster water movement corresponds to more forced convection (higher h_o) and better overall heat transfer (higher *U*).

The error propagation values in the coil heating are all two orders of magnitude smaller than U_{exp} values. Considering that typical error associated with heat transfer correlations are on the same order of magnitude as the *U* values, we can be confident that error propagation does not significantly influence the accuracy of our results.

Heat Exchanger

The average *U* value for the three runs through the heat exchanger was 1608 ± 32 W/m^2K. This compares to a correlated value of 2123 W/m^2*K. The both the experimental value and the correlated value fall within the expected range for a steam condenser [2: 647]. The percent difference between the two is 24.3%.

We began analyzing the heat exchanger data with equations (12) and (13). A plot of this data is included in Appendix E as Figure E.4. When we plotted trendlines for the data, the resulting slopes gave negative values for K_1. This made it impossible to solve (13) for U because the natural logarithm of a negative number is not defined. After checking the spreadsheet several times for errors we were unable to resolve this problem.

An alternative to equations (12) and (13) is equation (3) which was used to analyze the coil heater. As per a conversation with Professor Ely [3], equation (3) can be used to analyze the heat exchanger if $(K_1-1)/K_1$ is small. Our correlated value of U gives this expression equal to 0.36. We decided this was small enough to justify using equation (3) but remain aware that our experimental U values will be changed by this approximation. The amount of error propagation was calculated using (17) rather than (18). Figure E.5 shows the plot resulting from this simplification.

The experimental U value is lower than the actual U value for several reasons. The first reason is the equation simplification discussed above. Another problem in comparing our experimental and theoretical values occurs because of an error in our experimental method. Studying the steam temperature data collected during lab (Appendix D) shows that the steam was completely condensed when leaving the heat exchanger. This means that some of the heat transfer was caused by convection rather than a phase change. Because amount of heat transfer from convection is less than that of condensation, the experimental U value (condensation and convection) is lower than the correlated U value (condensation only). This experimental error may also explain why equations (12) and (13) were unsuccessful in finding U.

Comparison of coil heater and heat exchanger

We found that the heat exchanger was able to heat the water 1.49 times faster than the coil heater at the best agitation rate. It took the coil heater approximately 485 seconds to heat the water from 100°F to 160°F at an agitation speed of 90. The heat exchanger took only 325 seconds to effect the same amount of heating.

Conclusions and Recommendations

The experimental and correlated values for the overall heat transfer coefficient for batch heating with a coil heater were found to increase with agitation speed. An agitation rate of 30 gave an experimental U value of 2143 ± 28 W/m^2*K. This compares to a correlated value of 1102 W/m^2*K. An agitation rate of 60 gave U_{exp} as 2573 ± 35 W/m^2*K and U_{cor} as 2252 W/m^2*K. The experimental U for an agitation rate of 90 was 2829 ± 38 W/m^2*K. The correlated value was 2819 W/m^2*K. The difference between experimental values and correlated values decreases dramatically as the agitation speed increases. One reason this may occur is that our experimental approximation of a perfectly insulated tank was better at higher agitation rates. If we could repeat the experiment again we would use a larger range of agitation rates in order to find a more precise mathematical relationship between agitation speed and heat transfer coefficient.

The experimental overall heat transfer coefficient for the heat exchanger was found to be 1608 ± 32 W/m^{2}K. This compares to a correlated value of 2123 W/m^2*K. One reason the correlated value is larger than the experimental value is that the steam in the experiment was allowed to completely condense. This means that the experimental mechanisms of heat transfer were condensation and convection. The correlation used accounts only for condensation. This error could be corrected in further experiments by lowering the mass flow rate of water through the heat exchanger. Another reason the experimental value may be lower than the correlated value is that we were unable to analyze our data with equation (12). So we used equation (3). This adds additional error to our analysis that in not accounted for in the experimental range of U values.

We found that the heat exchanger was 1.49 times faster than the coil heater. This ratio compare the average time it took to heat the water with the heat exchanger to the average time it took to heat the water with the coil heater with an agitation rate of 90. Additional experiments in which that water flowrate through the heat exchanger was reduced may increase this ratio by increasing the efficiency f the heat exchanger.

List of References

[1] W. L. McCabe, J. C. Smith, and P. Harriot, *Unit Operations of Chemical Engineering*, New York: McGraw-Hill Higher Education, 2001.

[2] F. P. Incropera and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer* (Fifth Edition), New York: John Wiley and Sons, 2002.

[3] Discussion between James Esteban and Jim F. Ely Ph.D., Department Head of the Colorado School of Mines Department of Chemical Engineering, Golden, CO. July 12, 2002.