



ISyE 6414

Project

Multiple Linear Regression applied to a Direct
Sales Company in different countries

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INTRODUCTION

Big companies face a lot of challenges related to the management of the supply chain in the market, ranging from offering products or services that meet consumer expectations to reducing the cost of logistics and transportation to getting a competitive edge in the market. However, one of the most important and difficult problem to deal with, is the uncertainty of the demand. This is mainly because it has a large impact on the firm's strategy, performance and finances. Although there are solutions, such as inventory management, supply- demand changes (by mixing pricing and productions decisions timely), they are either reactive or do not provide enough lead time to successfully mitigate the situation. However, forecasting or robust demand planning that can reduce uncertainty is a solution that is proactive, which can ensure a company's improved performance in this continually changing market.

Demand planning can be defined as a key component of supply chain planning to generate reliable forecasts by applying experience and/or statistical tools to estimate demand. If correctly implemented, it helps align inventory, supply and manufacturing plans according to business requirements to maximize operational performance. This project is going to focus on the development of a logical procedure to build forecasting models for Belcorp, the largest multinational cosmetic manufacturer and direct sales company in Latin America.

Belcorp has a presence in more than 13 countries in Latin America. Their products range from fragrances to make up products to body care. They have three brands, Esika, L'bel and Cyzone, which are all targeted at different age groups. The corporation has defined 18 sales campaigns (with a duration of three weeks per campaign) where every brand gets a catalog and a special magazine for the sales force. Each magazine consists of a portfolio of the 600+ products offered with the following attributes:

- Regular Price and Sale Price
- Exposition of the product in the catalog
- Type of promotion (bundle, standalone offer, conditioned promotion, etc)
- Way exposition in the catalog (product alone or promoted with a model)
- Location in the catalog (front cover, back cover, page number)
- Incentive to the sales force for selling some products
- And few others

Since we have a broad set of possibilities to work on, we will focus the project's scope to evaluate the top fragrances of the company in Colombia, Peru and Mexico, since they represent nearly 40% of total income of the company.

OBJECTIVE OF THE PROJECT

The project will focus broadly on problems faced by three departments.

1. Demand Planning department spends a lot of time and effort trying to generate forecasting models manually. This project will help increase the efficiency of the resources of the company, having a direct impact on P&L.
2. Supply Chain and Planning Department wants to increase the efficiency of their operations by adapting a better estimate of the demand, to reduce the Mean Average Percentage Error. Belcorp has internally calculated and mentioned that 1 percentage point of MAPE may be worth thousands of dollars, depending on the product.
3. Finally, the Marketing, Commercial Planning, and Branding departments always wanted to have a reliable source to take strategic decisions about their portfolio. This project will help them leverage their sales and marketing attributes to make better strategic and tactical decisions.

Considering the mentioned aspects, we are going to focus on the development of a business solution that implements Multiple Linear Regression to accomplish three objectives:

- Generate a systematic procedure to generate the Multiple Linear Regressions models per country-product.
- Provide a forecasting model to the Supply Chain and Planning department to reduce uncertainty, by accurately forecasting demand
- Build models with enough explanatory power. This will help the company identify important and influential predictors for enhanced decision making.

Additionally, another output of the project will be a set of recommendations about how to approach the problem and the potential pitfalls when working on model generation.

DATA SOURCE

Belcorp manages different transactional and information systems such as SAP APO, FP&A and PP, however they also they have their own In-House Software to operate and maintain specific data related to their Strategic and Commercial decisions. Their IT department provided us a dataset for this project. Each row in this dataset describes a sales or marketing feature of a product, that was sold in a certain campaign. The table below explains each of them;

Column Name	Explanation
Center	It corresponds to the country. The values are: CO03 (Colombia), PE03 (Peru) and MX03 (Mexico)
Brand	The brand of the product sold. The values are: Esika, Cyzone and Lbel
Year	The year of the corresponding campaign. The range goes from 2012 to 2017
Campaign	The sale campaign. The range goes from 1 to 18.
Business Unit	The category of the product. Its description can be associated with products such as Make up, Personal care, and other. In this occasion, we will focus on Fragrances.
CUC Description	Description of the Product
CUC Code	Unique identifier of the product.
Market	The subcategory of the product. There are four types of values: Cologne for Men, Cologne for Women, Perfume for Women and Unisex Cologne for Adults.

Type of Product	A complementary description to the subcategory of the product. It is not very relevant compared to the Market Column.
Offer type	An offer type is a way to standardize the promotion of the products. For example: if the company aims at proposing an aggressive promotion of one product, they can use the offer type 11. We will provide further explanation about this later.
Cosmetic grouping OT	It is a basic description of the offer type to familiarize anyone who does not understand the meaning of the Offer type.
Way of sale	It describes the type of catalog or magazine the company uses to sell the product. It is complementary for understanding the meaning of offer type.
Real Demanded units	Number of units demanded by the customers. This will enable the company to prepare the production plan to satisfy the demand. Hence, this is the Response Variable .
Real PUP	This a ratio that calculates the proportion of demanded units per real orders.
Number of real orders	Number of sale orders that Belcorp receive per campaign. An order involves products of the three brands.
Regular Price Local Currency	Price of the product without discount in local currency.
Regular Price USD	Price of the product without discount in US Dollars
Sale Price Local Currency	Price of the product with discount in local currency.
Sale Price USD	Price of the product with discount in US Dollars.
% Discount Catalog	Percentage of discount on products sold by catalog
% Discount Demo	Percentage of discount on products sold by consultant magazine
Discover	Special zone of the catalog that can be scratched to smell the scent of the fragrance. 1 means the catalog has it, otherwise it is 0.
Exposition Percentage	The percentage of the page where a product is being shown. It has different values that go from 0 (the product does not have an image and it is shown with text), 100 (a full page), 200 (double page) or even 500 (product in a poster).
Set Flag	1 means the product is sold in a bundle with other products, otherwise 0.
Exposed Tones	A number that shows the number of tones concerning make up products.
Model Photo	A binary value. 1 means the product is shown with a model, otherwise 0.
Product Photo	A binary value. 1 means the product is shown, otherwise 0.
Bulk Photo	A binary value. 1 means the product is shown on a special paper material, otherwise 0.
Main Offer	A binary value. 1 means the product is offered with an aggressive promotion, otherwise 0.
Page number	The number of the page where the product is shown

The 'Offer Type' variable was created to standardize the different characteristics of the promotions at Belcorp. Since the list has many values, we are going to describe the most important for the project:

Offer type	Description
3	Push and pull promotion. There is an incentive offered to the sale force to sell the product. It appears in the consultant magazine. They may be sold with approximately 60% discount.
11	Aggressive offer. The promotion indicates products within a range of 45% to 60% off on the catalog. It also uses an impressive design to promote the sales.

13	A 50% off promotion on Catalog. It does not have the same impressive design as the Offer type 11, so the number of sold units would be significantly lower.
14	A promotion set for the back cover, a special poster or a set of products. It has been designed for special dates or whenever they to promote a special theme (for example: Mother's Day, Spring Beginning, and so on)
15	The product is sold with a discount of approximately 40% in the catalog. There is some visual support for this tactic.
17	A conservative promotion. The product is sold with a discount of approximately 30% in the catalog
19	Product without any promotion. No discount applied.
29	A special promotion on a set of products in the consultant magazine. The difference respect to Offer type number 3 is that this includes conditions on the number of products you can get. This is basically implemented to incentive the sale force to promote specific products.
35	Special date sale. The discounted products are sold in the consultant magazine. The range varies from 40% up to 65%.
36	Special date sale. The discounted products are sold in the catalog. The range varies from 30% up to 45% mostly
48	A special promotion on a product in the consultant magazine. The conditions are like the Offer Type 29, it just includes one product. This is basically implemented to incentive the sale force to promote specific products.
106	A 50% off promotion on Catalog. It is similar to the Offer type 13. The only difference is on the parameters they use to design the page of the product.
123	Push and pull promotion. It is similar to the offer type 3. The difference is an internal difference concerning a business rule which does not have impact in the number of units sold.

The rest of the offer types should be eliminated because they are not related to the company's sales. Most of the are ad-hoc tactics made to fix a problem or even to do some testing about a pilot.

DATA PREPARATION – Selection of predictors based on discussion with Belcorp.

Selected Products to Analyze

Since we have an enormous list of products to work, we will select the most important ones (related to sales) of each brand to build the models. We have tried to include a fractional factorial model from factors (Brand, Gender) that encompasses products contributing the highest to sales.

CUC Code	Code Description	Brand	Type
200012014	KALOS SPORT EAU TOILETTE 100ML	Esika	COLOGNE FOR MEN
200051232	D'ORSAY EDT 100ML	Esika	COLOGNE FOR MEN
200039855	NITRO EAU DE TOILETTE 100ML	Cyzone	COLOGNE FOR MEN
200044715	DANCING NIGHT EDT 100 ML	Cyzone	COLOGNE FOR WOMEN
200037781	LB BLUEINT EDT EDL 100 ML	L'bel	PERFUME FOR WOMEN

Eliminating the Irrelevant Columns

After evaluating the content of the columns and gathering information from the Belcorp's experts, we decided to remove the following columns from our analysis;

Column Name	Explanation
Business Unit	The category of the product. Its description can be associated with products such as Make up, Personal care, and other. In this occasion, we will focus on Fragrances.
CUC Description	We are going to keep the CUC Code to identify the product.
Market	This is a characteristic of the product that does not have impact on the sales.
Type of Product	This is a characteristic of the product that does not have impact on the sales.
Cosmetic grouping OT	Description of the offer type. It does not have any relevance to estimate the sales.
Way of sale	The description on where the product is offered does not affect the units sold.
Real PUP	The ratio is not useful since it we have the demanded units and the real orders.
Year	Since we are not focused on the relationship between response variable and time. Time stamp will not be considered as a predicting variable
Campaign	Same as “Year”, “Campaign” is also an indication of time.
Exposed Tones	This feature corresponds to make up, not fragrances.
Bulk Photo	This column just has the value 0. So, let us remove it
Product Photo	This column just has the value 1. So, let us remove it

Merging Columns

After removing the columns, we found that the discount of the product is either offered by catalog or offered by consultant magazine. Thus, we decide to merge “% Discount Catalog” and “% Discount Demo” to one column, “Discount”.

Removing Rows

After removing and merging the columns, we found that there are some observations that are very likely to be obviously wrong. Thus, before fitting any models or outlier dictations, we decide to remove data with obvious error. For instance, the observations regular price or sale price that is 0 or the observations with real demand units that is less than 10, which does not make any sense in the context of business.

DATA PREPARATION – Data cleaning based on R analysis

1. Fitting the model (1st fit)

First, we try to fit a model without modifying the dataset. Since the local currency of the three countries, Mexico, Columbia, Peru, are different in units and the difference is huge, we use “Regular Price USD” and “Sale Price USD”. However, since USD-local currency exchange rate for Mexico, Columbia, Peru varies a lot, it is not a very accurate measure across country analysis.

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Residuals:
    Min       1Q   Median       3Q      Max
-58173 -12675   -475    8054 276557

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   -6.488e+04  1.519e+04  -4.270 2.14e-05 ***
CenterMX03    -7.087e+01  6.500e+03  -0.011 0.991303
CenterPE03    -1.212e+04  3.104e+03  -3.903 0.000101 ***
as.factor(CUC.Code)200037781 -1.723e+03  2.907e+03  -0.593 0.553483
as.factor(CUC.Code)200039855 -8.094e+03  2.919e+03  -2.773 0.005661 **
as.factor(CUC.Code)200044715 -2.395e+03  3.420e+03  -0.700 0.483914
as.factor(CUC.Code)200051232  5.529e+03  2.580e+03   2.143 0.032362 *
as.factor(Offer.type)11      6.457e+03  1.046e+04   0.617 0.537247
as.factor(Offer.type)13      1.026e+04  4.168e+03   2.462 0.013985 *
as.factor(Offer.type)14      7.756e+03  8.454e+03   0.917 0.359159
as.factor(Offer.type)15      1.006e+04  5.012e+03   2.008 0.044940 *
as.factor(Offer.type)17      2.319e+04  6.271e+03   3.698 0.000229 ***
as.factor(Offer.type)29      2.386e+03  3.923e+03   0.608 0.543171
as.factor(Offer.type)35     -1.557e+03  4.537e+03  -0.343 0.731544
as.factor(Offer.type)36      9.318e+03  5.720e+03   1.629 0.103659
as.factor(Offer.type)48      2.210e+04  3.459e+03   6.390 2.55e-10 ***
as.factor(Offer.type)106    -2.082e+03  1.596e+04  -0.130 0.896236
as.factor(Offer.type)123     8.397e+03  1.572e+04   0.534 0.593420
Number.of.real.orders        2.321e-01  6.728e-02   3.450 0.000585 ***
Regular.Price.USD           -6.753e+02  2.036e+02  -3.317 0.000942 ***
Sale.Price.USD              1.668e+02  3.490e+02   0.478 0.632891
as.factor(Discover)1         1.135e+03  2.483e+03   0.457 0.647536
Exposition.Percentaje        8.329e+01  1.563e+01   5.330 1.22e-07 ***
as.factor(Set.Flag)1         -9.824e+02  3.807e+03  -0.258 0.796387
as.factor(Model.Photo)1     -5.669e+02  1.711e+03  -0.331 0.740496
as.factor(Main.Offer)1      -6.295e+03  9.142e+03  -0.689 0.491266
Page.number                  7.485e-01  2.288e+01   0.033 0.973902
Discount                   1.240e+05  1.891e+04   6.555 8.93e-11 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

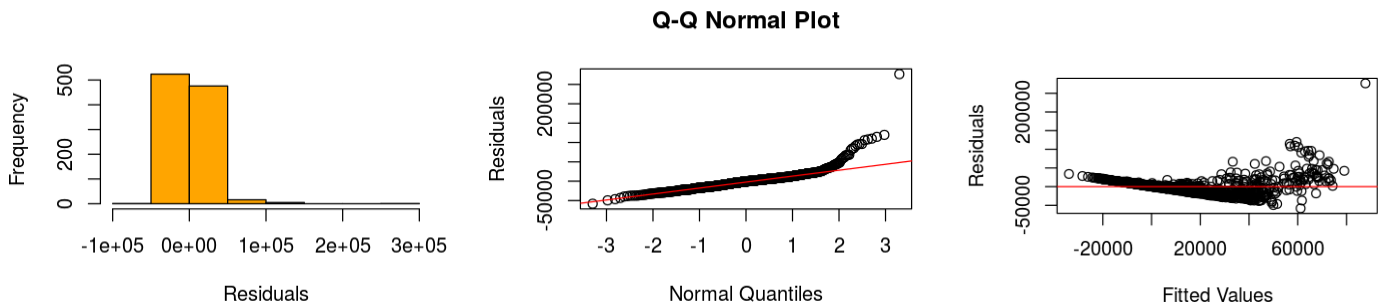
Residual standard error: 21760 on 995 degrees of freedom
Multiple R-squared:  0.5194,    Adjusted R-squared:  0.5064
F-statistic: 39.83 on 27 and 995 DF,  p-value: < 2.2e-16

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Initial Model Summary

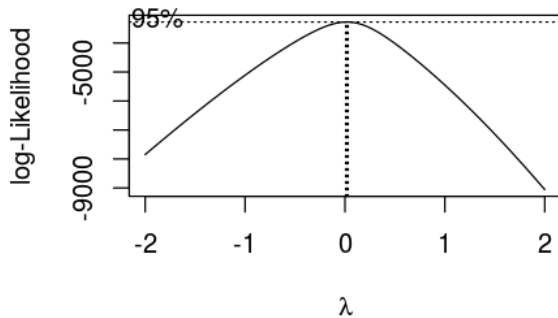
The **AIC** for the model is about 23511, **BIC** is about 23368, **R-squared** is only about 0.52, **adjusted R-squared** and a lot of predict variables are not significant at 95% significance level. Also, surprisingly, for the categorical predicting variable “CUC.Code”, levels “2000377810” and “200044715” are not statistically significant. However, we expected all the products should have a unique behavior.

1.1 Residual Analysis



According to the plots, it seems the constant variance assumption and normal assumption might **not** hold. Thus, it's reasonable to **transform** the response variable, detect and **remove** the outliers. Furthermore, we believe from the analysis that the country variable is overshadowing the significance of other predictors in the model since it is such an important factor in forecasting the demand for products. We hence have decided to split the dataset country wise and build separate models for each country.

1.1.1 Transformation of Response Variable



The, lambda returned is approximately 0.02, which means the log transformation of the response variable is necessary and helpful for modeling. In the later modeling, we will use $\log(\text{Real Demand Units})$ as response variable.

1.1.2 Outlier Detection & Model Selection

Since this is the first model before splitting the dataset, it is of no use to do variable selection and outlier removal. We will instead do those steps once the dataset is ready.

The significance of the 'country' variable is high in the model. This implies that it is statistically very significant and might be overshadowing some of the other predictors. We have hence decided to split our models based on country and have a separate model for each country (Colombia, Peru and Mexico) in order to improve the goodness of fit and predictive power. Intuitively the difference in cultures, and demographics of the 3 countries will also play a role in determining the demand of the products. Hence separate regression models seem to be a better option.

2. Fitting Separate Models for Each Country

Since the price tag of products are labeled by local currency as well, using it instead of USD is a better option.

We are showing the below analysis only for Colombia. It is the same for Peru and Mexico.


```

Residuals:
    Min       1Q   Median       3Q      Max
-4.0135 -0.3153  0.0802  0.4057  1.6619

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    5.192e+00  1.950e+00   2.663 0.008121 **
as.factor(CUC.Code)200037781 -2.762e+00  9.737e-01  -2.837 0.004822 ***
as.factor(CUC.Code)200039855  9.135e-01  8.218e-01   1.112 0.267047
as.factor(CUC.Code)200044715  1.448e+00  9.821e-01   1.475 0.141253
as.factor(CUC.Code)200051232 -1.045e-01  6.228e-01  -0.168 0.866882
as.factor(Offer.type)11    -3.246e-01  5.423e-01  -0.599 0.549823
as.factor(Offer.type)13    -1.289e-01  2.297e-01  -0.561 0.574980
as.factor(Offer.type)14    -7.410e-01  4.294e-01  -1.725 0.085356 .
as.factor(Offer.type)15    -9.797e-01  2.651e-01  -3.696 0.000255 ***
as.factor(Offer.type)17    -1.124e+00  3.386e-01  -3.318 0.001005 **
as.factor(Offer.type)29     5.566e-02  2.533e-01   0.220 0.826188
as.factor(Offer.type)35    -1.440e-01  2.637e-01  -0.546 0.585505
as.factor(Offer.type)36    -1.575e+00  3.185e-01  -4.945 1.20e-06 ***
as.factor(Offer.type)48     4.792e-01  2.022e-01   2.370 0.018350 *
as.factor(Offer.type)106    6.651e-03  7.486e-01   0.009 0.992917
as.factor(Offer.type)123    3.554e-01  7.298e-01   0.487 0.626576
scale(Number.of.real.orders) -5.265e-03  4.474e-02  -0.118 0.906399
Regular.Price.Local.Currency  9.247e-06  1.980e-05   0.467 0.640755
Sale.Price.Local.Currency   -3.049e-06  8.101e-06  -0.376 0.706881
as.factor(Discover)1       -7.910e-02  1.200e-01  -0.659 0.510209
Exposition.Percentaje      5.001e-03  7.832e-04   6.386 5.54e-10 ***
as.factor(Set.Flag)1       -5.542e-03  2.128e-01  -0.026 0.979234
as.factor(Model.Photo)1    -2.784e-01  1.023e-01  -2.722 0.006815 **
as.factor(Main.Offer)1     -1.439e-02  4.564e-01  -0.032 0.974873
Page.number               2.871e-03  1.168e-03   2.457 0.014495 *
Discount                6.208e+00  1.017e+00   6.105 2.79e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

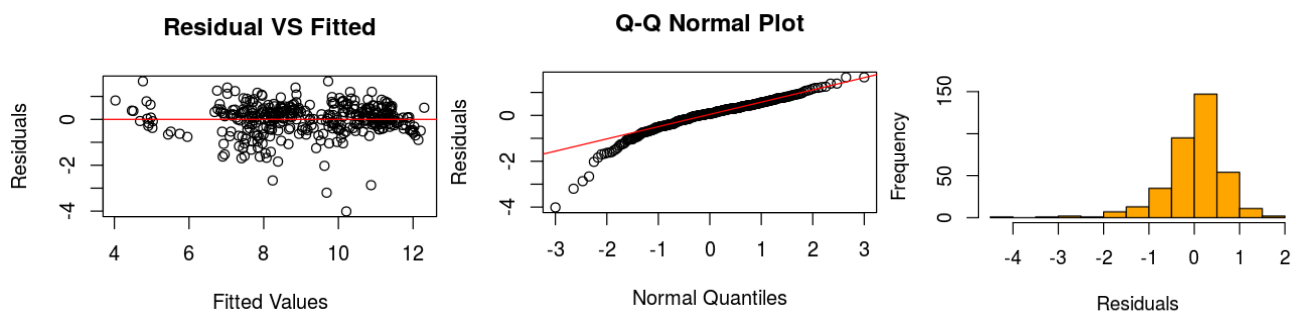
Residual standard error: 0.7024 on 343 degrees of freedom
Multiple R-squared:  0.8698,    Adjusted R-squared:  0.8603
F-statistic: 91.67 on 25 and 343 DF,  p-value: < 2.2e-16

```

1 Model Summary for the Initial Model after splitting by Country

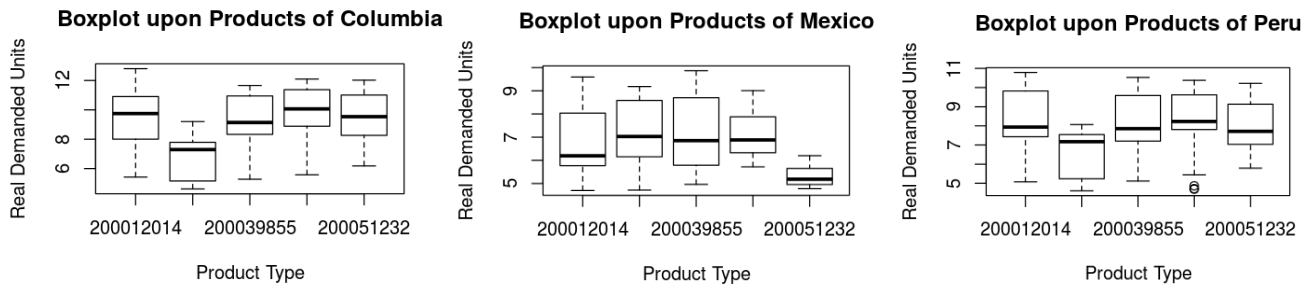
The **AIC** for the model is about **814**, **BIC** is about **919**, **R-squared** is about **0.87**, and **adjusted R-squared** is about **0.86**. As we can see that after splitting the dataset by country the AIC, BIC, R-squared, and adjusted R-squared improved a lot. As for significance level, the pattern of sales of product types truly varied in some situations. For instance, for different products, the prices will vary a lot.

2.1 Residual Analysis for the Colombia Model



Also, as shown in the graphs, the constant variance assumption and normality assumption seems to hold.

Boxplots for Product Types (country wise)



3. *Splitting the Dataset further*

After splitting the dataset by countries, we split the dataset by both country and product type. We had an intuition that the predictors would affect different products differently in each country and hence a separate model for each country-product combination would give us a model with the best predictive power. This approach was confirmed by Belcorp Inc to be logical and hence we continued our further analysis on this split dataset.

Changes in R^2 values before and after splitting the dataset

	Before Splitting	After Splitting
R^2	0.52	0.87

MODEL SELECTION AND ANALYSIS

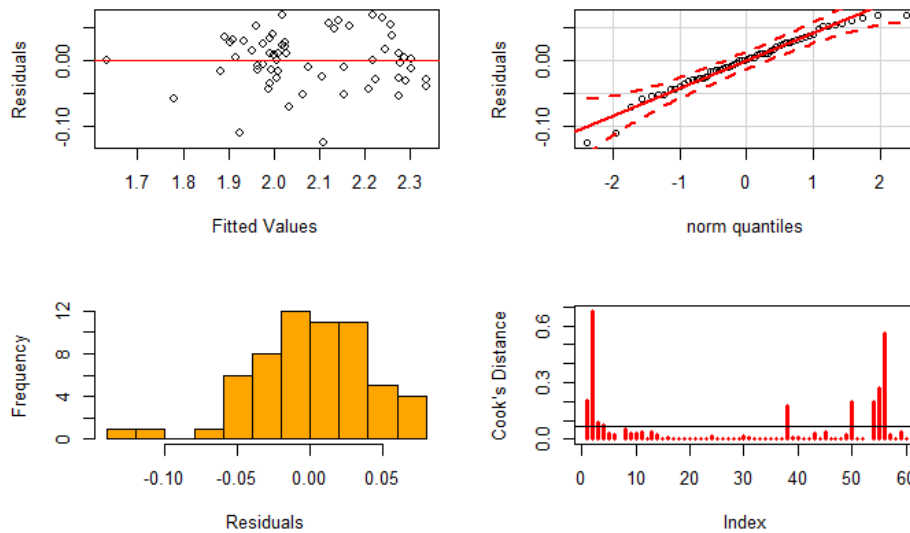
After splitting the dataset by country (Colombia, Mexico and Peru) and products (5 products selected from the entire dataset), we explore this data subset. The below steps are for Country: Peru and Product: Nitro Eau De Toilette

1. *Data Preparation*

We are transforming the response variable with a log transformation to account for the large range of data, after looking at the residual plots. This transformation reduces the spread of response variable. Since we need to test the prediction power of the final model, we have split the data into training and test sets. The testing accuracy will help correct for the downward bias in the training accuracy.

2. *Initial model*

The initial model was plotted with all the predictors after transforming the response variable. Certainly not all the predictors were not statistically significant in this model. However, plots for normality and variance of residuals show some skewness due to the presence of outliers.



Residual and Outlier plots for the initial model

3. Outlier detection

The outliers from the data were removed based on Cook's Distance $> 4/n$ (shown in the figure above)

4. Correlation



Correlation Matrix

The variables for **Sales Price** and **Discount** are **highly negatively correlated** which is logical that higher the discount, lower would be the sales price. This also confirms our choice of splitting the dataset by products, since the sales price for a product (say product A) after a discount of 50% might still be higher than the price of product B (without discount).

Furthermore, **Regular Price** is somewhat **negatively correlated** with **Number of Real Orders** which might imply that the cost of making the product varies by the number of orders. Higher the number of orders, lower is the regular price of the product.

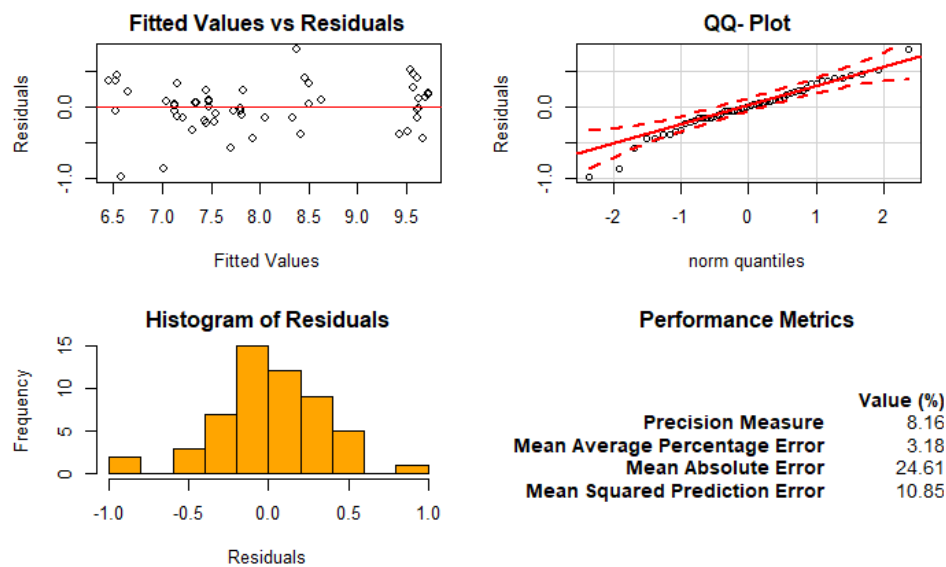
The marketing and sales data hence has a lot of correlation different predictors somewhat affect the values of the other predictors. We hence need to account for this collinearity and select only those limited number of predictors that can fit the model without adding too much bias.

5. Elastic Net Model

We have implemented elastic net by running the GLMNET function for different values of alpha (0, 0.5, 1) and cross-validated those for different values of lambda. The final models for elastic net are generated using values of lambda that minimize the mean squared error.

6. Residual Analysis

Residual Analysis for each of the elastic net models confirm that the assumptions of linear regression is followed. Find below the graphs for LASSO (alpha = 1).



Residual Analysis for the LASSO model

7. Prediction Accuracy

Prediction Accuracy has been calculated for both the training set (downward biased) and the test set to give a wholesome picture of the predictive power of the model.

From the training set measures, we can see that the **mean average percentage error** is **3.18%**. However, since this is a downward biased estimate, we check the same on our test set, which gives us a value of **4.07%**. We are using MAPE to analyze our results, since it a popular metric in supply chain analysis.

Precision Measure also helps us understand how precise our predictions are over the dataset. The value of PM over the training dataset is 8.16 while the value on the test set is 8.13. We expect the test data to perform poorly as compared to the training set, and this might be attributed to the fitting of random effects in the test data by the model.

8. Conclusions

- The sales of the product Nitro Eau De Toilette in Peru, is a lot dependent on the **Discount**, **Discover** and **Set Flag** variables. This can be analyzed looking at the coefficients of the model.

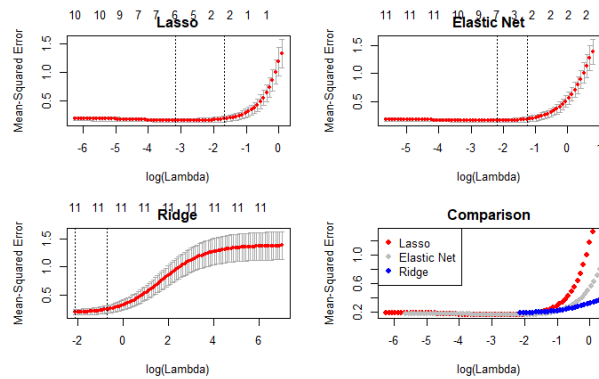
- We can however not comment on the explanatory power of each of these variables, since they are selected using LASSO and LASSO in general removes **any one** of the highly correlated predictors.
- We can however comment that a higher discount (correlated with lower sales price) affects the demand quite a lot, holding all other predictors constant.
- Furthermore, demand for this product specifically also depends on if it is sold along with some other product (set flag) and if the sales catalog has a scratch out for this perfume (discover). These factors will certainly vary across products and even across countries for the same product (as confirmed by our EDA on the same).
- We hence affirm the decision to split the dataset country wise and product wise. It also seconds our intuition that, certain products might be perceived differently in different countries and hence analyzed differently.
- The project can generate models in an automated way. So, this is useful for a big company like Belcorp which has more than 600 products per country.
- In most of the companies, the most popular accuracy metric used by the department of supply chain is Mean Average Performance Error, however it is important to complement that analysis by including the Precision Measure.
- Currently, Belcorp' s MAPE for fragrances can reach level of 15%. The project provides a better accuracy thanks to the implementation of Regularized Regression Methods to select the most important variables, and the data preparation process.
- A company should have two models: one that explains the influence of the predicting variables and the response variable, and another that predicts accurately the response variable. Our solution may provide both models.
- We can state the importance of transforming the sales variable to build regression models. From our experience, this situation happens frequently in almost every industry.
- By performing a detailed exploratory analysis at the beginning, we could set the appropriate granularity level analysis to build the models. This matched with the statements of the employees of Belcorp.
- The project builds an automated way to generate models, however it is important to monitor the output frequently to detect any new anomaly that might present over time.
- We can observe that one product may have different relevant sale behaviors per country. The coefficients of the regression models reflect those characteristics.

SCALING UP AND SCALING OUT THE MODEL

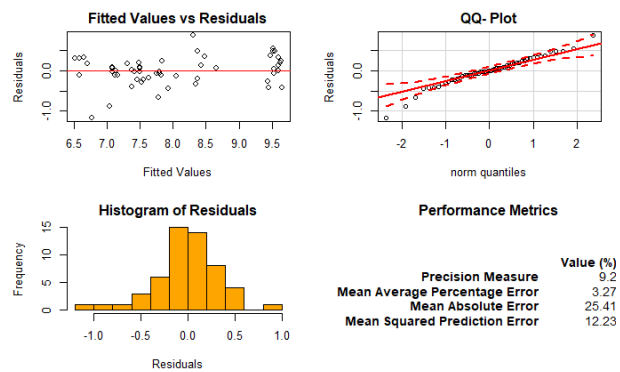
The models built for product-country combinations had good predictive power and certainly could be scaled up to larger datasets. However, such models are used with really large datasets. With larger datasets, the variability explained by these models would reduce and we would have to include more predictors in the model. The R code developed is robust enough to scale up to larger datasets without much manual change. Furthermore, the developed code is also easy enough to tweak to handle the different company-product combinations and hence scaling out should not be an issue either.

APPENDIX

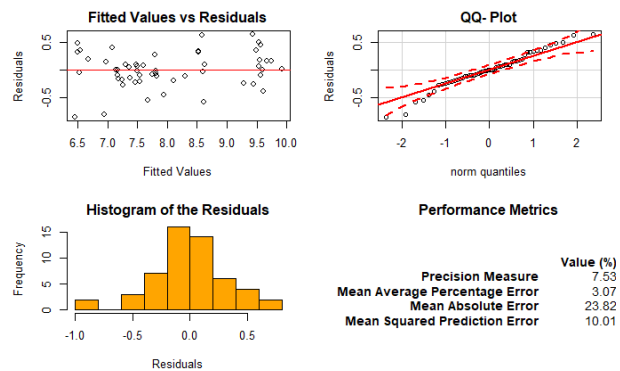
Comparison between the 3 elastic net models – MSE for different values of lambda



Residual Analysis for the Elastic Net model ($\alpha = 0.5$)



Residual Analysis for the Elastic Net model ($\alpha = 0$)



Test-set prediction measures for the 3 different models

	Lasso	Elastic Net	Ridge
<i>Precision Measure</i>	8.13	8.63	9.41
<i>Mean Average Percentage Error</i>	4.07	4.27	4.3
<i>Mean Absolute Error</i>	31.27	33.06	33.38
<i>Mean Squared Prediction Error</i>	15.68	16.65	18.16