

# *t*-wise Coverage by Uniform Sampling

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## ABSTRACT

Efficiently testing large configuration spaces of *Software Product Lines* (SPLs) needs a sampling algorithm that is both scalable and provides good *t*-wise coverage. The 2019 SPLC Sampling Challenge provides large real-world feature models and asks for a *t*-wise sampling algorithm that can work for those models.

We evaluate *t*-wise coverage with one of the provided feature models using the Smarch algorithm that uniformly samples SPL configurations. While uniform sampling alone is not enough to produce 100% 1-wise and 2-wise coverage, we use standard probabilistic analysis to explain our experimental results and to conjecture how uniform sampling may enhance the scalability of existing *t*-wise sampling algorithms.

## CCS CONCEPTS

• **Theory of computation** → **Logic and verification; Automated reasoning**; • **Software and its engineering** → *Abstraction, modeling and modularity; Model-driven software engineering; Software performance.*

## KEYWORDS

software product lines, *t*-wise coverage, uniform sampling.

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## 1 INTRODUCTION

*Software Product Lines* (SPLs) are highly configurable. Building blocks of SPL products are *features* that are increments of product functionality. Each product of an SPL is defined by a unique set of features called a *configuration*. A *feature model* declares each feature and constraints among features, so that a user can identify legal configurations with desired feature combinations [4]. As the number of features increase, the size of the *configuration space*, which is the set of all possible configurations, grows exponentially.

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A large configuration space could have over a trillion ( $>10^{12}$ ) configurations and is a challenge for testing, as testing every configuration is infeasible. Instead, prior work produced a small set of configurations in order to test selected features and their interactions. The aim is to get a ‘high’ *t*-wise coverage, ideally meaning 100% of *all* combinations of *t* features are covered by at least one configuration of the set. Sometimes less than 100% is accepted. Common values for *t* include feature-wise ( $t=1$ ), pair-wise ( $t=2$ ), and three-wise coverage ( $t=3$ ).

Different approaches start with a feature model and derive samples for *t*-wise coverage [1, 2, 6, 9, 10]. However, they do not scale well for many features and complex constraints, which limited their applicability to the real-world SPLs. Thus, the proposed Challenge [16] provides large real-world feature models and asks for a sampling algorithm that can generate configuration sets with good *t*-wise coverage for those models.

In this paper, we explore *t*-wise coverage using *uniform sampling* (US). US ensures that all configurations in a configuration space have equal probability of being selected, yielding a *statistically representative sample* of the space. US can be used as a baseline against which other sampling algorithms can compare as a benchmark.

Despite its utility, US for large SPLs was considered infeasible for large SPLs until recently [11, 13]. Prior work tried different methods to make sampling as random as possible, but none achieved US for large SPLs. We use a recently developed algorithm called Smarch [8], the first to perform US of configuration spaces of size  $10^{245}$ . Smarch utilizes a #SAT solver, which counts the number of solutions to a propositional formula. [15]. We believe we are the first to explore *t*-wise coverage of US with probabilistic analyses to explain its coverage results.

Our contributions to the 2019 SPLC Sampling Challenge are:

- Demonstration of *t*-wise coverage that can be achieved by US; and
- Probabilistic analysis of configuration spaces that predicts the *t*-wise coverage by US and that may be useful for developing a practical *t*-wise sampling algorithm.

## 2 SMARCH: A US ALGORITHM

Smarch [8] is a US algorithm for SPLs based on a #SAT solver.

Let  $\phi$  be the propositional formula of a feature model [3]. A #SAT solver can count the number of configurations in  $\phi$ 's configuration space, namely  $|\phi|$ . (Each solution to  $\phi$  is a configuration, and each configuration is a solution to  $\phi$ ). A #SAT solver extends a satisfiability solver by associating the number of solutions with each truth assignment [5]. Smarch uses sharpSAT [15], a state-of-the-art #SAT solver.

Here is how Smarch achieves US: A uniform random number generator can select an integer  $r$  in the range  $[1..|\phi|]$ . Smarch creates

a one-to-one mapping that converts  $r$  into a unique configuration, so that  $\mathbb{US}$  of range  $[1..|\phi|]$  leads to  $\mathbb{US}$  of configurations.

To create a one-to-one mapping, Smarch recursively partitions  $\phi$  by a fixed order of variables. A variable  $v \in \phi$  partitions  $\phi$  into disjoint spaces  $(\phi \wedge \neg v)$  and  $(\phi \wedge v)$ . #SAT can compute the number of solutions for each space, i.e.,  $|\phi \wedge \neg v|$  and  $|\phi \wedge v|$  respectively.

Then, for a random number  $r \in [1..|\phi|]$ , if  $r \leq |\phi \wedge \neg v|$  the  $(\phi \wedge \neg v)$  space is selected for recursive partitioning, otherwise  $(\phi \wedge v)$  is selected and  $|\phi \wedge \neg v|$  is subtracted from  $r$  to adjust the search in  $(\phi \wedge v)$ . This process is repeated for the next variable in  $\phi$ , until all variables are considered and a unique configuration has been determined.

### 3 EVALUATION

#### 3.1 Experimental Set-Up

Among the feature models provided in the Challenge, we used 'FinancialServices01' version '2018-05-09'. This feature model was given in FeatureIDE format [14], so we used the functionality of FeatureIDE to generate its propositional formula as a dimacs file. This file has 771 variables and 7,241 clauses. Using sharpSAT [15], the size of the configuration space was determined to be  $97,451,212,554,676 \approx 9.7 \times 10^{14}$ , counted in a mere 46 milliseconds.

We evaluated  $t$ -wise coverage for  $t=1$  and  $t=2$  and did the following to find valid combinations:

- (1) We derived a list of feature selections. With 771 features, there are  $771 \times 2 = 1,542$  possible selections since we consider both a feature and its negation;
- (2) We derived all possible 1-wise and 2-wise combinations from this list. 1-wise yields  $\binom{1542}{1} = 1542$  combinations and 2-wise yields  $\binom{1542}{2} = 1,188,111$ ; and
- (3) We filtered out invalid combinations using a SAT solver. If a combination is valid, the conjunction of the combination and the feature model should be satisfiable. For example, for a feature  $f$ , a 2-wise combination  $(f, \neg f)$  is invalid as these selections conflict with each other.

For 1-wise, 1,518 valid combinations were found (some features were only mandatory). For 2-wise, 914,537 valid combinations were found.

We used Smarch to produce a  $\mathbb{US}$  set  $\mathbb{S}_n$  of  $n$  configurations.<sup>1</sup> We varied  $n$  to observe the results of increasing larger sets, using  $n = \{5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000, 1518\}$ . Then, for each set  $\mathbb{S}_n$ , we measured<sup>2</sup>:

- **Time taken to sample  $n$  configurations**, measured by the Linux 'time' tool;
- **Time taken to sample a configuration**, measured for each sample by Smarch;
- **Maximum memory used during sampling**, measured by the Linux '/usr/bin/time -v' command; and
- **$t$ -wise coverage for  $t=1$  and  $t=2$** , measured as the percentage of  $t$ -wise combinations covered in  $\mathbb{S}_n$ .

We conducted our evaluation on an Intel i7-6700@3.4Ghz Ubuntu 16.04 machine with 16GB of RAM. All the code and data for the evaluation are available at: [https://github.com/jeho-oh/Smarch\\_t\\_wise](https://github.com/jeho-oh/Smarch_t_wise).

<sup>1</sup>Smarch takes samples without replacement.

<sup>2</sup>The Challenge [16] explicitly requests sampling time and memory measurements.

#### 3.2 Experimental Results

Fig. 1a shows the total sampling time and Fig. 1b the time per sample. The X-axis is the number of samples ( $n$ ) and the Y-axis is the time in seconds. We observed:

- Total sampling time increases linearly with  $n$ ; and
- For all  $n$ , the average sampling time for a configuration was approximately 7 seconds, with standard deviation of 1 second. For all samples, the maximum sampling time was 10.1 seconds and the minimum was 3.5 seconds.
- The number of samples taken did not affect the time to sample a configuration.

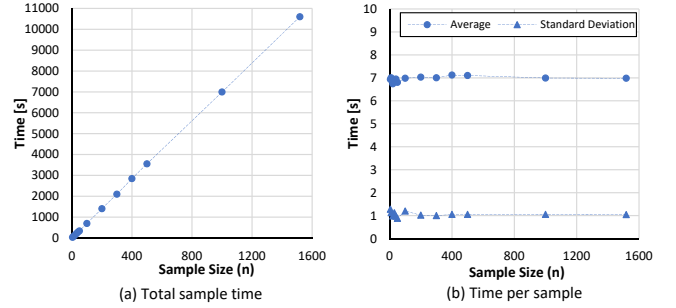


Figure 1: Sampling time.

Fig. 2 shows the maximum memory usage of Smarch, where the X-axis is the number of samples ( $n$ ) and the Y-axis is the memory size in megabytes. We observed:

- Maximum memory usage was stable, between 16.8MB and 17.1MB for all  $n$ ; and
- Sampling more configurations did not increase the maximum memory usage.

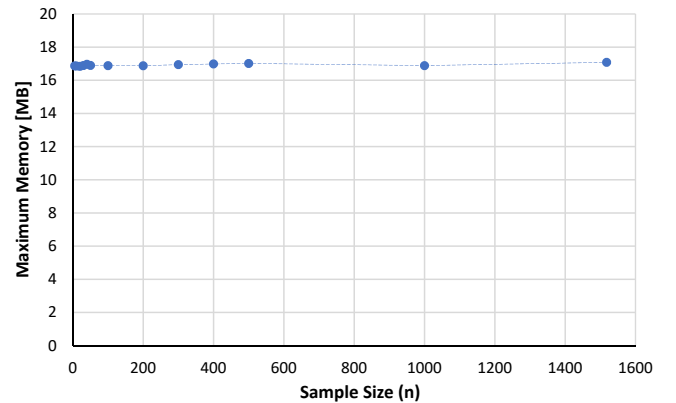


Figure 2: Maximum memory usage.

Fig. 3 shows the  $t$ -wise coverage result, where the X-axis is the number of samples ( $n$ ) and Y-axis is the percentage of the coverage. Plots with different color indicates the results for different  $t$ . We observed:

- For all values of  $n$ , coverage for  $t=1$  was higher than  $t=2$ ;
- For  $\mathbb{S}_5$ , more than half of the feature combinations were covered for  $t=1$  and over 35% for  $t=2$ ;

- For both  $t=1$  and  $t=2$ , increasing  $n$  yielded better coverage. With 1,518 samples, coverage for  $t=1$  was 61.7% and  $t=2$  was 47.6%;
- The difference in coverage between  $n=5$  and  $n=1,518$  was surprisingly small. For  $t=1$ , the difference was 6.4%. For  $t=2$ , the difference was 9.4%; and
- Although samples are expected to be statistically representative of the configuration space, their  $t$ -wise coverages seemed low. Both coverages improved imperceptibly after  $n$  exceeded 200.

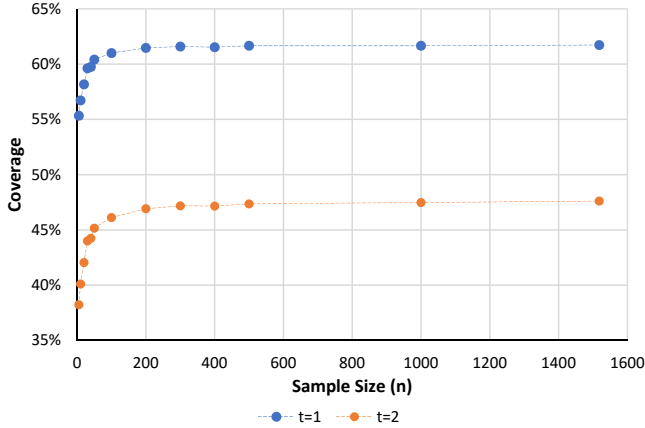


Figure 3:  $t$ -wise coverage.

We conclude that although  $\mathcal{US}$  is feasible with Smarch,  $\mathcal{US}$  alone is not enough to produce a 100%  $t$ -wise coverage.

#### 4 ANALYSIS

$\mathcal{US}$  allows us to apply standard statistical analysis to explain our experimental results [7].

Let  $c$  denote a valid  $t$ -wise combination for a given  $t$ . Let  $v_c$  denote the fraction of all valid configurations that have  $c$  in the configuration space. Since every configuration has an equal probability of being selected by  $\mathcal{US}$ , the probability that a sample will have  $c$  is  $v_c$ .

$v_c$  can vary widely for different  $c$  because constraints among features may make certain combinations less frequent than others. A mandatory feature has  $v_c=1$  because it appears in all configurations. A feature with no constraints has  $v_c=0.5$ ; it can be freely enabled and disabled, making it appear in half of the valid configurations.

$v_c$  can be computed by a #SAT solver. Let  $\phi$  be the propositional formula of an SPL's feature model. Let  $\phi_c$  be the propositional formula of the conjunction of  $c$ 's features. We can use a #SAT solver to compute  $v_c$  as:

$$v_c = \frac{|\phi \wedge \phi_c|}{|\phi|} \quad (1)$$

The probability  $p(c, n)$  that at least one of  $n$  samples includes combination  $c$  is:

$$p(c, n) = 1 - (1 - v_c)^n \quad (2)$$

where the more samples taken, the higher the probability we will encounter combination  $c$ . The final probability, however, largely

depends on how often this combination appears in the configuration space, i.e.,  $v_c$ .

In our experiments of the previous section, we discovered:

- 61.5% of all 1-wise combinations have a ratio  $v_c > 0.9$ . Even with the minimum number of samples we used in the evaluation ( $n=5$ ), these combinations have more than 0.99 probability of being encountered in  $n=5$  samples; and
- 38.8% of the 1-wise combinations have a ratio of  $v_c < 0.0001$ . Even with the maximum number of samples we used in the evaluation ( $n=1815$ ), they have less than 0.15 probability of being encountered in  $n=1815$  samples.

We can use  $p(c, n)$  to predict  $t$ -wise coverage. Let  $\mathbb{C}_t$  denote the set of all valid  $t$ -wise combinations, where  $|\mathbb{C}_t|$  is the number of combinations in  $\mathbb{C}_t$ . The estimated  $t$ -wise coverage  $E(\mathbb{C}_t, n)$  for a given  $t, n$  is:

$$E(\mathbb{C}_t, n) = \frac{1}{|\mathbb{C}_t|} \sum_{c \in \mathbb{C}_t} p(c, n) = \frac{1}{|\mathbb{C}_t|} \sum_{c \in \mathbb{C}_t} (1 - (1 - v_c)^n) \quad (3)$$

Fig. 4 uses (blue) X markers to plot  $E(\mathbb{C}_1, n)$  and (brown) X markers for  $E(\mathbb{C}_2, n)$  with our experimental results (• for  $t=1$  and • for  $t=2$ ) overlaid. Eqn. (3) accurately predicts the results of our experiments and also explains why the coverage of  $t=1$  is higher than that for  $t=2$ : there are many 2-way feature combinations ( $c_{ij}$ ) that are much less likely than any 1-way combination ( $c_k$ ), meaning  $v_{c_k} \gg v_{c_{ij}}$ .

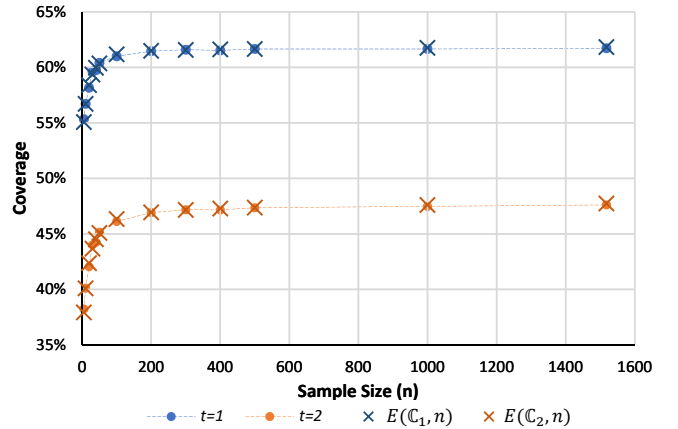


Figure 4:  $t$ -wise coverage estimation.

It is interesting to explore the relationship between coverage and larger sample set sizes *which are infeasible to explore experimentally*. Fig. 5 shows the estimated  $t$ -wise coverage for  $n$  up to  $10^{14}$ , which is approximately 10% of the configuration space (i.e.,  $9.7 \times 10^{14}$ ). We observed:

- With  $10^{14}$  samples, more than 99.99% of 1-wise and 2-wise combinations are expected to be covered. Of course, this is almost enumeration; and
- Many combinations will be covered with a small number of samples, over 30% of 2-way combinations are not likely to be covered even with  $10^7$  samples(!).

**We could accurately predict these results because Smarch can uniform sample from a configuration space and standard probabilistic analyses rely on  $\mathcal{US}$  [7].**

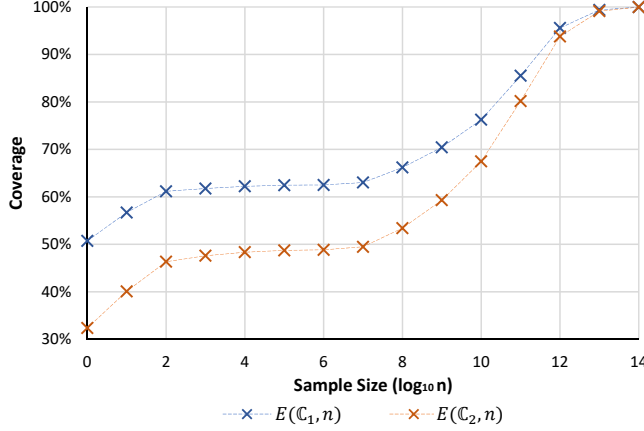


Figure 5:  $t$ -wise coverage estimation for large  $n$ .

Our analysis suggests possible enhancements to existing  $t$ -wise approaches. Once  $v_c$  values are known, we can determine which combinations can be covered by a small number of  $\mathbb{U}\mathbb{S}$ . Then, for combinations that are unlikely to be found by  $\mathbb{U}\mathbb{S}$ , we may either: 1) constrict the configuration space with constraints to (re-cursively) sample configuration sub-space of interest [12] or 2) use existing approaches that do not rely on  $\mathbb{U}\mathbb{S}$ . As sampling with many features limits the scalability of existing approaches,  $\mathbb{U}\mathbb{S}$  may improve sampling scalability by reducing the features to consider. And equally important issue is to define a reasonable  $t$ -wise coverage (percentage) for large configuration spaces (other than 100%) for practitioners to use.

## 5 CONCLUSIONS AND FUTURE WORK

As  $\mathbb{U}\mathbb{S}$  of configurations was considered infeasible, probabilistic analyses of a configuration space based on  $\mathbb{U}\mathbb{S}$  was unexplored or considered unexplorable. We used a recently developed algorithm, Smarch [8], to  $\mathbb{U}\mathbb{S}$  configurations of a configuration space. We also derived probabilistic models to explain Smarch results. We showed:

- $\mathbb{U}\mathbb{S}$  alone is **not** enough to produce 100%  $t$ -wise coverage; and
- Distribution of  $v_c$  can be used to predict the  $t$ -wise coverage of  $\mathbb{U}\mathbb{S}$ .

Our work opens new possibilities on analyzing an SPL configuration space and deriving samples for testing. As  $\mathbb{U}\mathbb{S}$  produces statistically representative samples of a configuration space, it may be possible to utilize the information from samples to improve the efficiency of existing approaches. As a future work, we plan to:

- Analyze other systems to validate and expand our insights on probabilistic analyses;
- Derive an algorithm that can utilize  $\mathbb{U}\mathbb{S}$  for  $t$ -wise coverage; and
- Enhance the performance of the Smarch algorithm.

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