UNIVERSIDADE DO VALE DO ITAJAÍ

MATEMÁTICA APLICADA À ENGENHARIA

Jéssica Cristina Tironi

Prof. Walter Antonio Gontijo

1) Determine as transformadas z das seguintes funções:

a) $x(k) = -2u(k) + 0{,}7^k u(k)$

2.	$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$

Resposta:

$$x(z) = -2 * \frac{1}{1-z^{-1}} + \frac{1}{1-0.7*z^{-1}}$$

b) $x(k) = u(k-2) + \delta(k-1)$

1.	$\delta[n]$	1	All z
2.	$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$

Resposta:

$$x(z) = z^{-2}\frac{1}{1-z^{-1}} + 1 * z^{-1} =$$

c) $x(k) = (1 - 0{,}5^k)u(k)$

$x(k) = 1u(k) - 0{,}5^k u(k)$

2.	$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$

Resposta:

$$x(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-0.5*z^{-1}}$$

d) $x(k) = 2\delta(k) - 3(0{,}5^k u(k))$

1.	$\delta[n]$	1	All z
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$

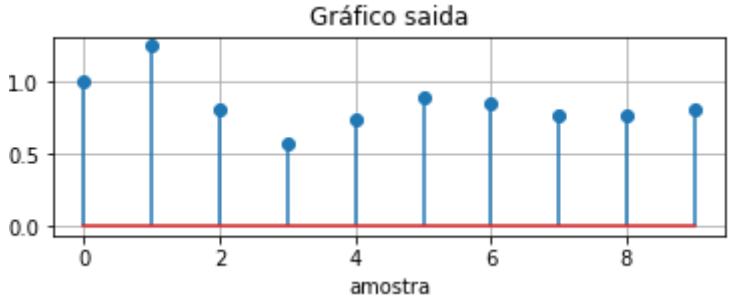
Resposta:

$$x(z) = 2 - 3 * \frac{1}{1-0.5*z^{-1}}$$

2) Considere um sistema discreto descrito pela seguinte equação diferença: $y(k) - \frac{1}{4}y(k-1) + \frac{1}{2}y(k-2) = x(k)$. Calcule a saída y(k) para uma entrada

X(k) = u(k) (degrau unitário), para −2 ≤ k ≤ 8. Apresente os gráficos de y(k) e X(k) em função de k .

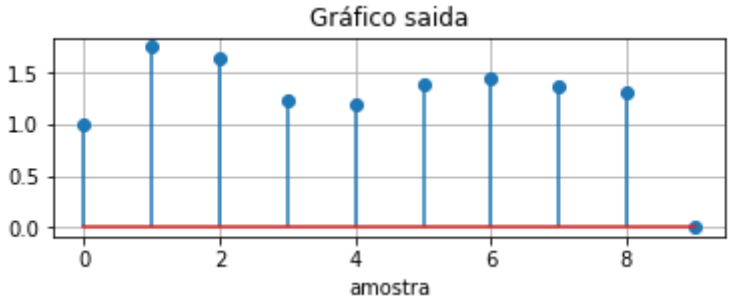
```
y[k] =  1.0
y[k] =  1.25
y[k] =  0.8125
y[k] =  0.578125
y[k] =  0.73828125
y[k] =  0.8955078125
y[k] =  0.854736328125
y[k] =  0.76593017578125
y[k] =  0.7641143798828125
y[k] =  0.8080635070800781
```



3) Repita o exercício 2 para as seguintes equações diferença:

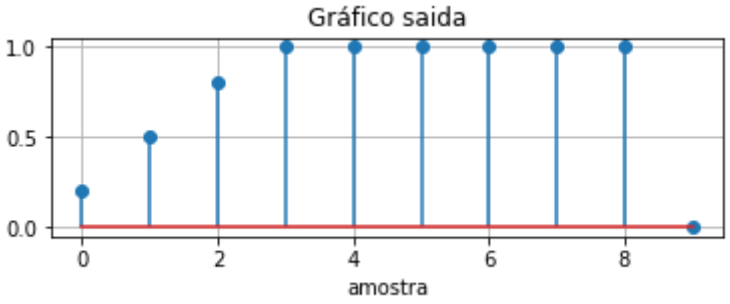
a) $y(k) - \frac{1}{4}y(k-1) + \frac{1}{2}(k-2) = x(k) + \frac{1}{2}x(k-1) + \frac{1}{5}x(k-2)$

```
y[k] =  0.0
y[k] =  0.0
y[k] =  1.0
y[k] =  1.75
y[k] =  1.6375000000000002
y[k] =  1.234375
y[k] =  1.18984375
y[k] =  1.3802734375
y[k] =  1.450146484375
y[k] =  1.3723999023437499
y[k] =  1.3180267333984377
```



b) $y(k) = 0.2x(k) + 0.3x(k-1) + 0.3x(k-2) + 0.2x(k-3)$

```
y[k] =  0.0
y[k] =  0.0
y[k] =  0.2
y[k] =  0.5
y[k] =  0.8
y[k] =  1.0
y[k] =  1.0
y[k] =  1.0
y[k] =  1.0
y[k] =  1.0
y[k] =  1.0
```



4) Determine a função de transferência e os pólos/zeros dos sistemas discretos modelados pelas seguintes equações diferença:

a) $y(k) + \frac{1}{4}y(k-1) = x(k) - \frac{1}{2}x(k-1)$

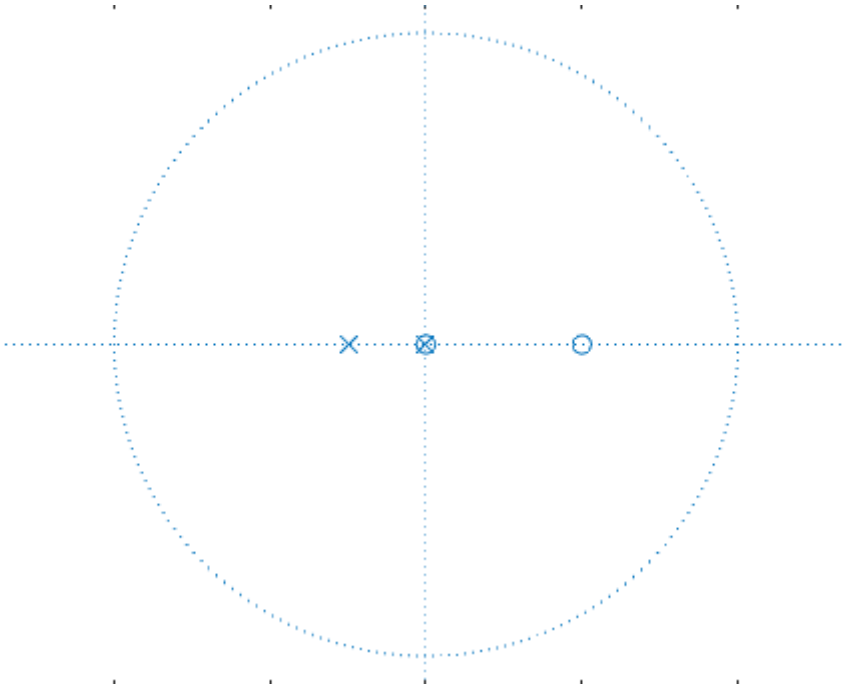
$$y(k) + \frac{1}{4}y(k-1) = x(k) - \frac{1}{2}x(k-1)$$

$$Y(z) + \frac{1}{4}z^{-1}Y(z) = X(z) - \frac{1}{2}\cdot z^{-1}\cdot X(z)$$

$$\left(1 + \frac{1}{4}\cdot z^{-1}\right)Y(z) = \left(1 - \frac{1}{2}\cdot z^{-1}\right)\cdot X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\left(1 - \frac{1}{2}\cdot z^{-1}\right)}{\left(1 + \frac{1}{4}\cdot z^{-1}\right)}$$

$$H(z) = \frac{z - \frac{1}{2}}{z + \frac{1}{4}}$$



Zero: ½
Polo: -1/4

b) $y(k)+\frac{4}{3}y(k-1)-\frac{1}{2}y(k-2)=-2x(k)$

$$Y(k)+\frac{4}{3}Y(k-1)-\frac{1}{2}Y(k-2)=-2x(k)$$

$$Y(z)+\frac{4}{3}.z^{-1}Y(z)-\frac{1}{2}.z^{-2}.Y(z)=-2X(z)$$

$$Y(z).\left(1+\frac{4}{3}.z^{-1}-\frac{1}{2}.z^{-2}\right)=-2X(z)$$

$$\frac{Y(z)}{X(z)}=\frac{-2}{1+\frac{4}{3}.z^{-1}-\frac{1}{2}.z^{-2}}\cdot\frac{z^2}{z^2}$$

$$\frac{Y(z)}{X(z)}=\frac{-2z^2}{z^2+\frac{4}{3}z-\frac{1}{2}}$$

Polos:

$\Delta=b^2-4.a.c$

$\Delta=\left(+\frac{4}{3}\right)^2-4.1.\frac{-1}{2}$

$\Delta=1,78+2$

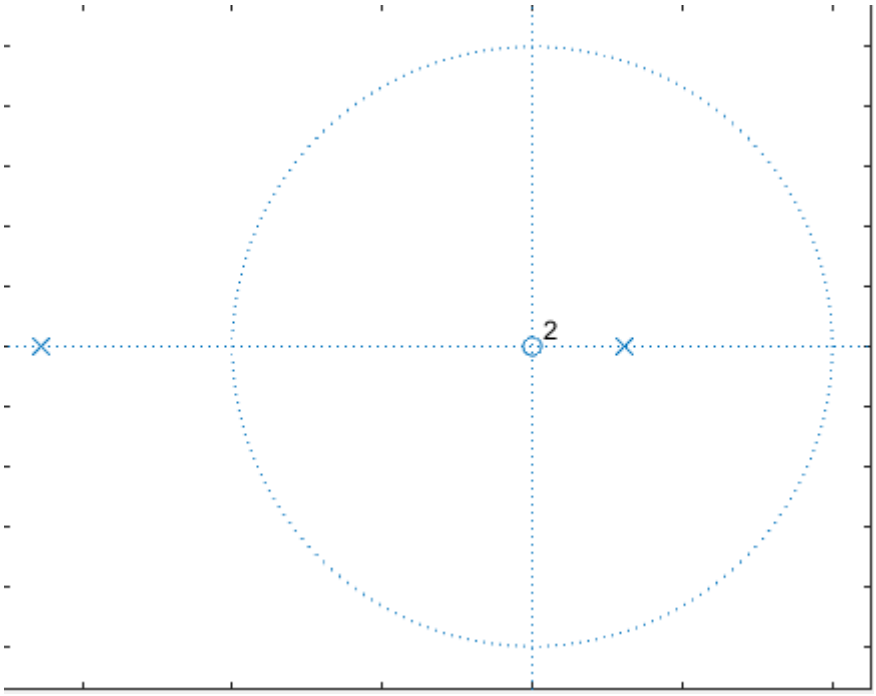
$\Delta=3,78$

$x=\frac{-b\pm\sqrt{\Delta}}{2.a}$

$x'=\frac{-1,33-1,94}{2}=-1,635$

$x''=\frac{-1,33+1,94}{2}=0,305$

2 zeros em zero.


2

5) Repita o exercício 4 para as equações diferença apresentadas no exercício 3.

a) $y(k) - \frac{1}{4}y(k-1) + \frac{1}{2}(k-2) = x(k) + \frac{1}{2}x(k-1) + \frac{1}{5}x(k-2)$

$$Y(k) - \frac{1}{4}Y(k-1) + \frac{1}{2}(k-2) = x(k) + \frac{1}{2}x(k-1) + \frac{1}{5}x(k-2)$$

$$Y(z) - \frac{1}{4}z^{-1}Y(z) + \frac{1}{2}\cdot z^{-2}Y(z) = X(z) + \frac{1}{2}\cdot z^{-1}\cdot X(z) + \frac{1}{5}\cdot z^{-2}\cdot X(z)$$

$$Y(z)\cdot\left(1 - \frac{1}{4}\cdot z^{-1} + \frac{1}{2}\cdot z^{-2}\right) = X(z)\cdot\left(1 + \frac{1}{2}\cdot z^{-1} + \frac{1}{5}\cdot z^{-2}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}\cdot z^{-1} + \frac{1}{5}\cdot z^{-2}}{1 - \frac{1}{4}\cdot z^{-1} + \frac{1}{2}\cdot z^{-2}}$$

$$H(z) = \frac{z^2 + \frac{1}{2}z + \frac{1}{5}}{z^2 - \frac{1}{4}\cdot z + \frac{1}{2}}$$

Zeros

$\Delta = b^2 - 4.a.c$

$\Delta = \left(\frac{1}{2}\right)^2 - 4.1.\frac{1}{5}$

$\Delta = 0,25 - 0,8$

$\Delta = -0,55$

$\frac{-b \pm \sqrt{\Delta}}{2.a}$

$x = \frac{-0,5 \pm \sqrt{-0,55}}{2}$

$x = -0,25 + 0,3708\,i$

$x = -0,25 + 0,3708\,i$

Polos

$z^2 - \frac{1}{4} \cdot z + \frac{1}{2}$

$\Delta = b^2 - 4.a.c$

$\Delta = (-0,125)^2 - 4.1.0,5$

$\Delta = 0,0625 - 2$

$\Delta = -1,9375$

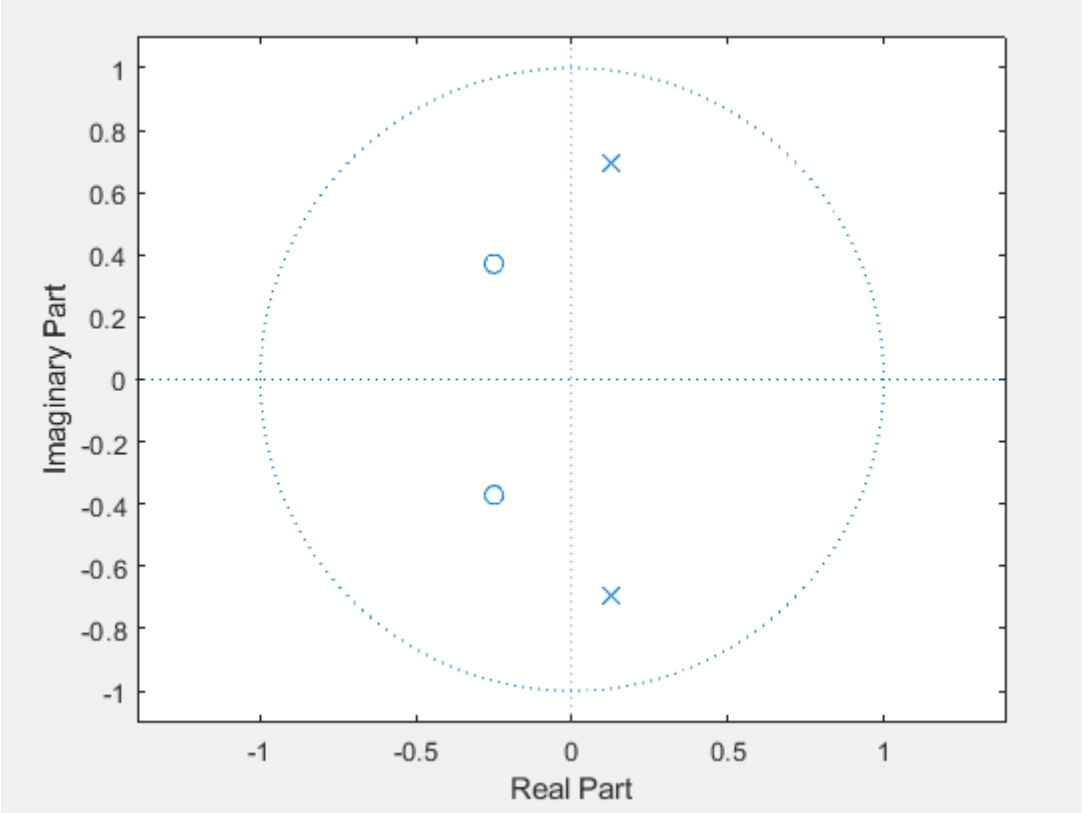
$x = \frac{-b \pm \sqrt{\Delta}}{2.a}$

$x = \frac{-0,125 \pm \sqrt{-1,9375}}{2.1}$

$x = \frac{-0,125 + 1,3919\,i}{2}$

$x = -0,125 + 0,696\,i$

$x = -0,125 - 0,696\,i$



c) $y(k) = 0.2x(k) + 0.3x(k-1) + 0.3x(k-2) + 0.2x(k-3)$

$$Y(k) = 0,2\,x(k) + 0,3x(k-1) + 0,3\,x(k-2) + 0,2.x(k-3)$$

$$Y(z) = 0,2\,X(z) + 0,3z^{-1}.X(z) + 0,3.z^{-2}X(z) + 0,2.z^{-3}.X(z)$$

$$Y(z) = X(z).(0,2 + 0,3.z^{-1} + 0,3.z^{-2} + 0,2.z^{-3})$$

$$\frac{Y(z)}{X(z)} = 0,2 + 0,3.z^{-1} + 0,3.z^{-2} + 0,2z^{-3}$$

$$H(z) = \frac{0,2.z^3 + 0,3.z^2 + 0,3.z + 0,2}{z^3}$$

3 polos em zero

Zeros

$$0,2.z^3 + 0,3.z^2 + 0,3.z + 0,2$$

$$(1+2).(0,2.x^2 + 0,1x + 0,2)$$

$$\Delta = b^2 - 4.a.c$$

$$\Delta = 0,1^2 - 4.0,2.0,2$$

$$\Delta = 0,01 - 0,16$$

$$\Delta = -0,15$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2.a}$$

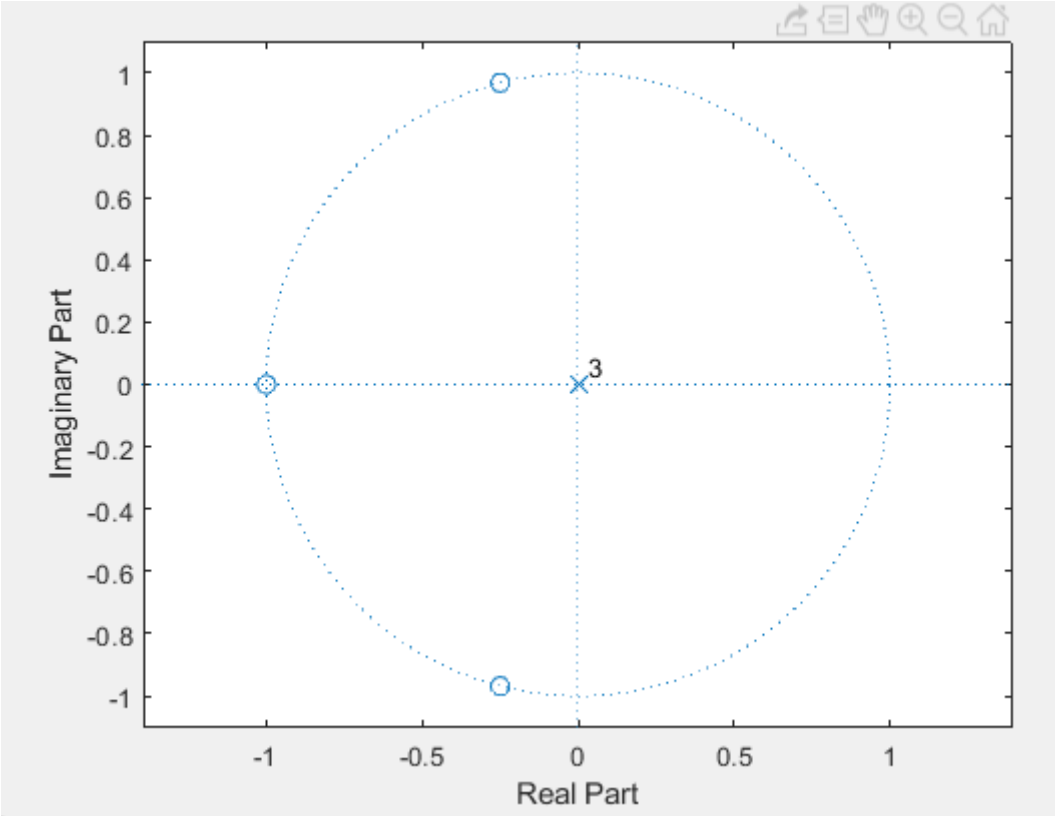
$$x = \frac{-0,1 \pm \sqrt{-0,15}}{2.0,2}$$

$$x = \frac{-0,1 + 0,38729i}{0,4}$$

$$x^{I} = -0,25 + 0,9682\,i$$

$$x^{II} = -0,25 - 0,9682\,i$$

$$x^{III} = -1 + 0,000\,i$$


Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1
3

6) Determine e esboce no plano complexo z (desenhe também o círculo de raio unitário!) os pólos e zeros das seguintes funções de transferência:

a) $H(z) = \frac{z+0.6}{(z^2+0.6z+0.2)*(z-1)}$

Zeros = -0,6

Polos:

$(z^2+0,6z+0,2).(z-1)$

$\Delta = b^2-4.a.c$

$\Delta = 0,6^2-4.1.0,2$

$\Delta = 0,36-0,8$

$\Delta = -0,44$

$X = \frac{-b \pm \sqrt{\Delta}}{2.a}$

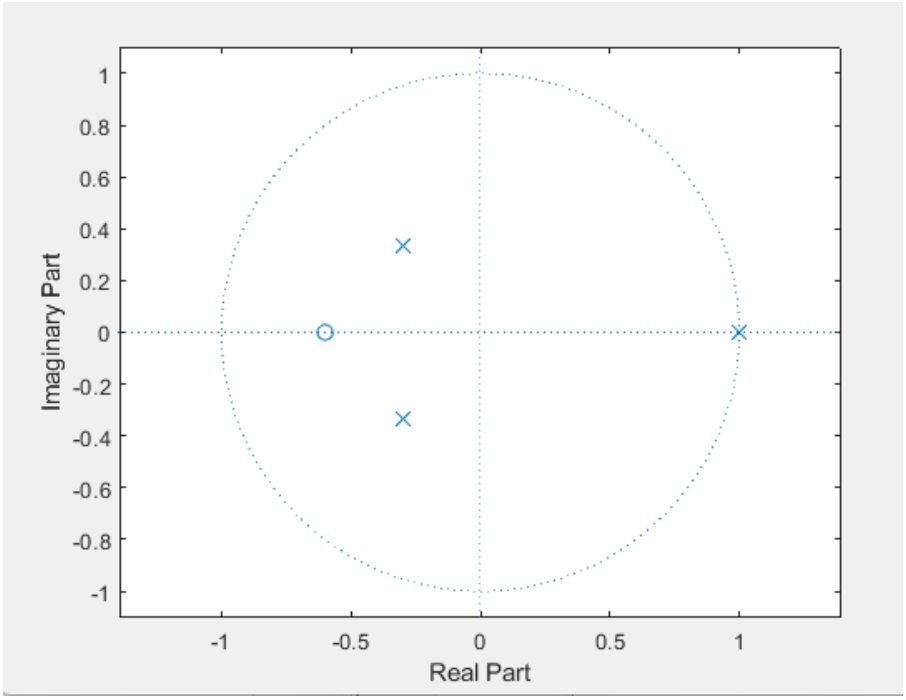
$X = \frac{-0,6 \pm \sqrt{-0,44}}{2.1}$

$X = \frac{-0,6 + 0,663\,i}{2}$

$X' = -0,3 + 0,3317\,i$

$X'' = -0,3 - 0,3317\,i$

$X'' = 1$



b) $H(z) = \frac{z^{-1}+0.8z^{-2}}{1+z^{-1}+0.41z^{-2}}$

$$H(z) = \frac{z+0,8}{z^2+z+0,41}$$

$$2\omega_0 = 0,8$$

Polos:

$$\Delta = b^2 - 4.a.c$$

$$\Delta = 1^2 - 4.1.0,41$$

$$\Delta = 1 - 1,64$$

$$\Delta = -0,64$$

$$\frac{-b \pm \sqrt{\Delta}}{2.1}$$

$$\frac{-1 \pm \sqrt{-0,64}}{2.1}$$

$$x' = \frac{-1 + 0,8i}{2}$$

$$x' = -0,5 + 0,4i$$

$$x'' = -0,5 - 0,4i$$