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# Vibration analysis of rotating pretwisted tapered blades made of functionally graded materials



Yutaek Oh, Hong Hee Yoo\*

School of Mechanical Engineering, Hanyang University, 222 Wangshimni-ro Seongdong-gu, Seoul 04763, South Korea

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#### ABSTRACT

This paper presents a new structural dynamic model with which vibration characteristics of rotating blades made of functionally graded materials (FGMs) can be investigated. The equations of motion of a rotating pretwisted tapered blade made of FGM are derived using the Rayleigh-Ritz assumed mode method along with the Kane's method. After converting the equations of motion into dimensionless forms, dimensionless natural frequencies are obtained by solving the modal formulation of the dimensionless equations. Various dimensionless geometric parameters like pretwist angle, taper ratios, hub radius ratio and width-to-thickness ratio as well as continuous variation of material properties along the blade thickness are considered to obtain the new structural dynamic model. The accuracy and efficiency of the proposed model are verified through a comparison study first. Then, effects of the volume fraction index, Young's modulus ratio, hub radius ratio, pretwist angle, taper ratios, width-to-thickness ratio and angular speed upon the dimensionless natural frequencies of the FG blade are investigated using the proposed model. In addition, an algebraic condition under which dimensionless natural frequencies can be minimized is obtained.

## 1. Introduction

The thermodynamic efficiency of a gas turbine system can be increased effectively by raising its operating temperature. However, the operating temperature is limited by the melting point of the materials inside the combustion chamber. That is why superalloys having high melting points are widely used for gas turbine blades. Because superalloys are very expensive, functionally graded materials (FGMs), which are made of a mixture of two or more different materials, have been investigated recently by several researchers. Because material mixing ratio varies continuously in FGMs, material properties such as density, Young's modulus and heat conductivity also vary continuously. Especially, FGMs that are made of ceramic and metal mixtures have drawn keen attention since they are less expensive and able to withstand very high temperature as well as high mechanical loads. Such FG blades are composed mostly of metal inside and ceramic outside, thereby simultaneously providing mechanical toughness and very low thermal conductivity.

To investigate the vibration characteristics of rotating blades made of FGMs, one needs to develop a structural dynamic model that simultaneously considers the continuously varying material properties along the blade thickness as well as the rotational motion. Early studies on the vibration characteristics of rotating blades began in the 1920s by

Southwell and Gough [1], who introduced a mathematical relation between the natural frequencies and angular speed of a blade. Later, Schilhansl [2] derived a governing equation for the bending vibration of a rotating blade; this equation allowed mode shapes and natural frequencies to be obtained. In the 1960s and 1970s, remarkable progress in computers and numerical methods allowed computational models to be employed to investigate the vibration characteristics of rotating blades. Putter and Manor [3] derived an eigenvalue problem for rotating blades and solved it using beam mode functions. More complicated effects could be analyzed with advanced computational models; these have included the effects of tip mass [4], pretwisted shape [5], shear deformation [6], tapered cross section [7] and multiple mixed effects [8-15] upon the vibration characteristics of rotating blades. Literature review papers [16-18] on the subject are also available. The initial concept of FGM was firstly introduced in the late 1980's by Japanese material researchers [19] as thermal barrier materials. After that, Sankar [20] derived and solved elasticity equations for the FG beam structures based on Euler-Bernoulli theory. The effects of shear deformation on the free vibration of FG beams were studied [21-23]. Kapuria et al. [24] performed the static and free vibration of layered FG beams both experimentally and theoretically. More complicated effects including the effects of varying cross-section [25] and viscoelastic support [26] upon the vibration characteristics of

E-mail addresses: ohyt1111@hanyang.ac.kr (Y. Oh), hhyoo@hanyang.ac.kr (H.H. Yoo).

<sup>\*</sup> Corresponding author.