Machine Learning 1 - Exercise 9

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December 16, 2018

1 The Dual SVM

1.a

$$\Lambda(w, \theta, \alpha) = \frac{\|w\|^2}{2} - \alpha \sum_{i=1}^{n} \left(y_i(w^T x_i + \theta) - 1 \right) \tag{1}$$

$$\Lambda(w, \theta, \alpha) = \frac{\|w\|^2}{2} - \sum_{i=1}^{n} \alpha_i \left(y_i(w^T x_i + \theta) - 1 \right) \tag{2}$$

1.b

Derivate Λ by w, θ,α:

$$\frac{\partial \Lambda(w, \theta, \alpha)}{\partial w} = 0 \tag{3}$$

$$\frac{2w}{2} - \sum_{i=1}^{n} \alpha_i y_i x_i = 0 => w = \sum_{i=1}^{n} \alpha_i y_i x_i \tag{4}$$

$$\frac{\partial \Lambda(w, \theta, \alpha)}{\partial \theta} = - \sum_{i=1}^{n} \alpha_i y_i = 0 \tag{5}$$

4) in 2)

$$\Lambda(\theta, \alpha) = \frac{1}{2} \left(\sum_{i=1}^{n} \alpha_i y_i x_i \right)^T \left(\sum_{i=1}^{n} \alpha_i y_i x_i \right) - \sum_{i=1}^{n} \alpha_i \left(y_i \left(\left(\sum_{i=1}^{n} \alpha_i y_i x_i \right)^T x_i + \theta \right) - 1 \right) \tag{6}$$

$$\Lambda(\theta,\alpha) = \frac{1}{2}\left(\sum_{i=1}^{n}\alpha_i y_i x_i\right)^T\left(\sum_{i=1}^{n}\alpha_i y_i x_i\right) - \sum_{i=1}^{n}\alpha_i y_i\left(\sum_{i=1}^{n}\alpha_i y_i x_i\right)^T x_i - \sum_{i=1}^{n}\alpha_i y_i\theta + \sum_{i=1}^{n}\alpha_i \tag{7}$$

5) in 7)

$$\Lambda(\alpha) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j x_i^T x_j - \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j x_i^T x_j + \sum_{i=1}^{n}\alpha_i \tag{8}$$

$$\Lambda(\alpha) = \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j x_i^T x_j \tag{9}$$

$$\max_{\alpha}\left(\sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j x_i^T x_j\right) \tag{10}$$

with $\alpha \geq 0$ and $\sum_{i=1}^{n}\alpha_i y_i = 0$

1.c

Kernelized versions of the primal program:

$$\min_{w,\theta}\|w\|^2 \tag{11}$$

with $y_i(w^T\Phi(x_i)+\theta) - 1 \geq 0$
Kernelized versions of the dual program:

$$\max_{\alpha}\left(\sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j k(x_i,x_j)\right) \tag{12}$$

with $\alpha \geq 0$ and $\sum_{i=1}^{n}\alpha_i y_i = 0$

2 SVMs and Quadratic Programming

2.a