Appendix G: Explanation radial Stokes

Radial Stokes parameters Q_{phi} and U_{phi} are now a popular choice for polarimetry. However the community is using and citing different definitions of coordinate system, angles and resulting formulas. Monnier et al. (2019), a recent paper, made a statement how the conventions and formulas should be, so that the Stokes Q and U position angles fulfil the IAU standards (see Hamaker & Bregman (1996) for summary). The concept of the radial Stokes parameter is based on Q_R,U_R definitions of Schmid et al. (2006) to analysis the limb polarization of Uranus and Neptune. Q_R was constructed so that the radial polarisation was positive. For azimuthal polarized dust analysis it is more convenient to have a positive value as well, therefore one is using $Q_{phi} = -Q_R$. The stated advantage of using the radial Stokes parameter instead of showing polarized flux $p \cdot I = (Q^2 + U^2)^{1/2}$ is that the squares introduce systematic errors. The latter is especially an issue if $|Q| \approx \Delta Q$ and likewise for U.

Appendix G.1: Coordinate system and angle

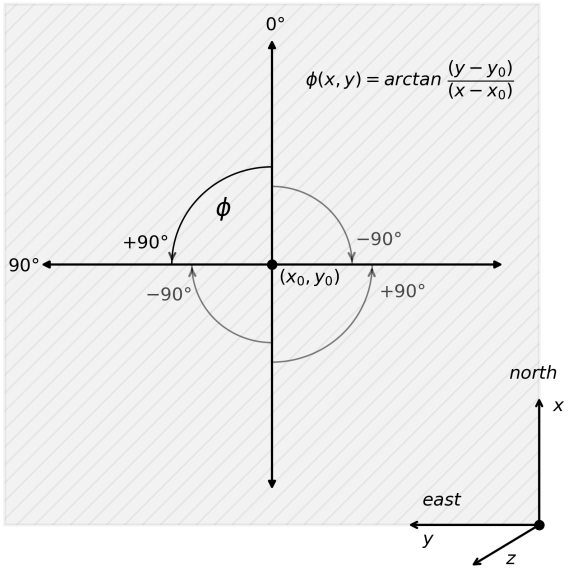
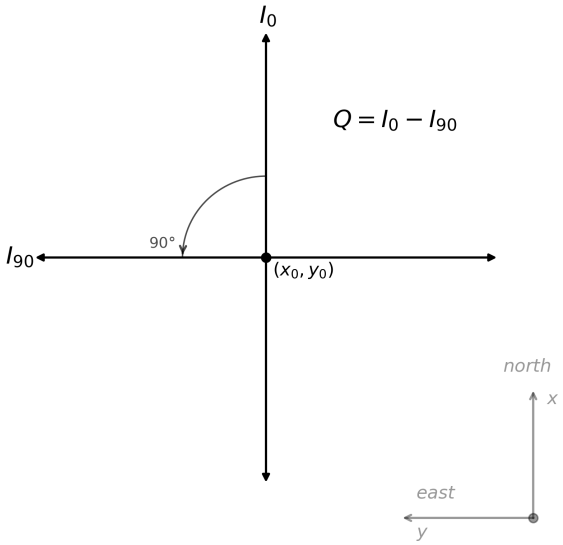


Fig. G.1: This figure shows the convention of the xyz IAU right-handed coordinate system. Radiation from the source propagates in positive z-axis towards the observer. Additionally the convention of the angle ϕ is described. x_0, y_0 denote the center of the star. The grey background represents the science frame.

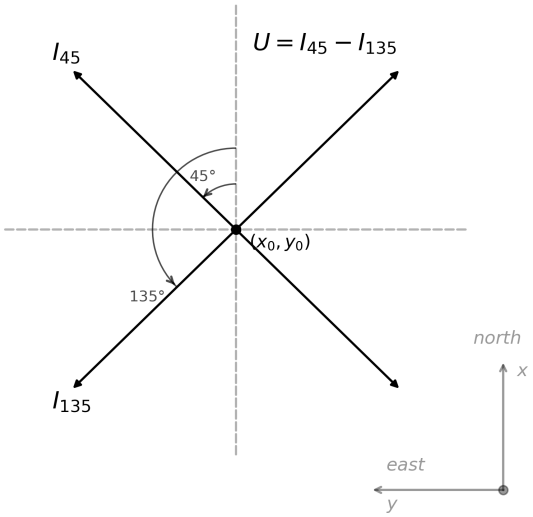
$$\phi(x, y) = \arctan\left(\frac{y - y_0}{x - x_0}\right) \tag{G.1}$$

Figure G.1 shows the xyz IAU right-handed coordinate system conventions. In other literatures such as (Canovas, H. et al. 2015), this convention and the resulting formulas are not unique. However according to the IAU/IEEE definitions of the Stokes Q and U parameter (Hamaker & Bregman 1996), positive Q value should be along positive x/north direction and -Q along +y/east direction. The conventions for Stokes U are: +U along 45 degree axis and -U along +135 degree axis. The resulting angle angle ϕ, which is used to produce the radial Stokes images is also indicate in figure G.1 and equation G.1. x_0 and y_0 denote the position of the star.

Appendix G.2: Q and U frames



(a) Construction Stokes Q frame.



(b) Construction Stokes U frame.

Fig. G.2: The positions of the polarization axis as used to measure the Stokes Q and U frames.

Figure G.2 illustrates the important polarization axis to construct the Stokes Q (G.2a) and Stokes U frames (G.2b).

$$Q = I_0 - I_{90} \tag{G.2}$$

$$U = I_{45} - I_{135} \tag{G.3}$$

Another disadvantage to analyse azimuthally polarized dust with the Stokes Q and U frames, are the cancellation effects at the boundaries between the positive and negative areas.

Appendix G.3: Q_{phi}

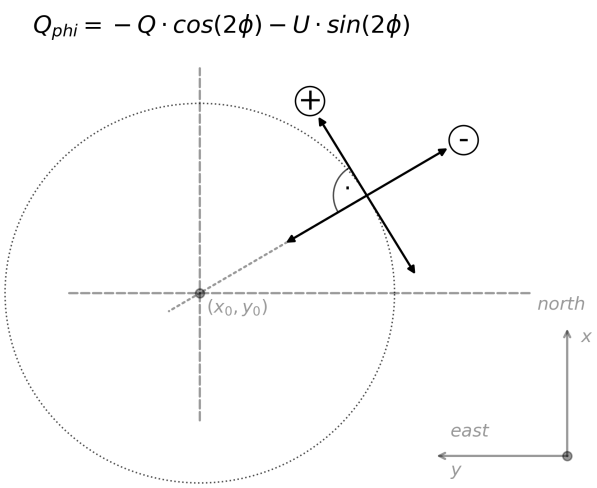
$$Q_{phi} = -Q\cos(2\phi) - U\sin(2\phi) \tag{G.4}$$

Figure G.3a shows illustrative the idea of the radial Stokes parameter Q_{phi}. The consequences for azimuthal and radial polarizations are given in figure G.4a respectively figure G.4b. Azimuthal polarization is a good approximation in our case. The dust scattering radiation produces, similar to Rayleigh scattering, polarization perpendicular to the scattering plane. This azimuthal polarization and therefore dust can be easier analysed with the Q_{phi} frames as there is no extinction along certain axis and the expected image is positive everywhere.

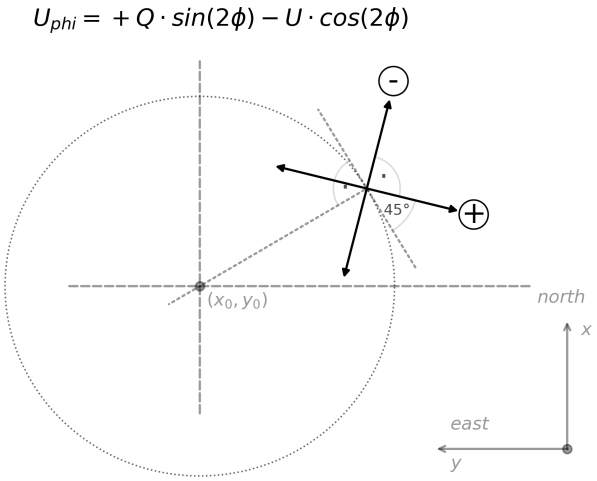
Appendix G.4: U_{phi}

$$U_{phi} = +Q\sin(2\phi) - U\cos(2\phi) \tag{G.5}$$

A summary of the situation for radial Stokes U_{phi} is given in figure G.3b. The situation for $+/-$ 45 degrees polarization compared to azimuthal direction are given in figure G.5a and figure G.5b.

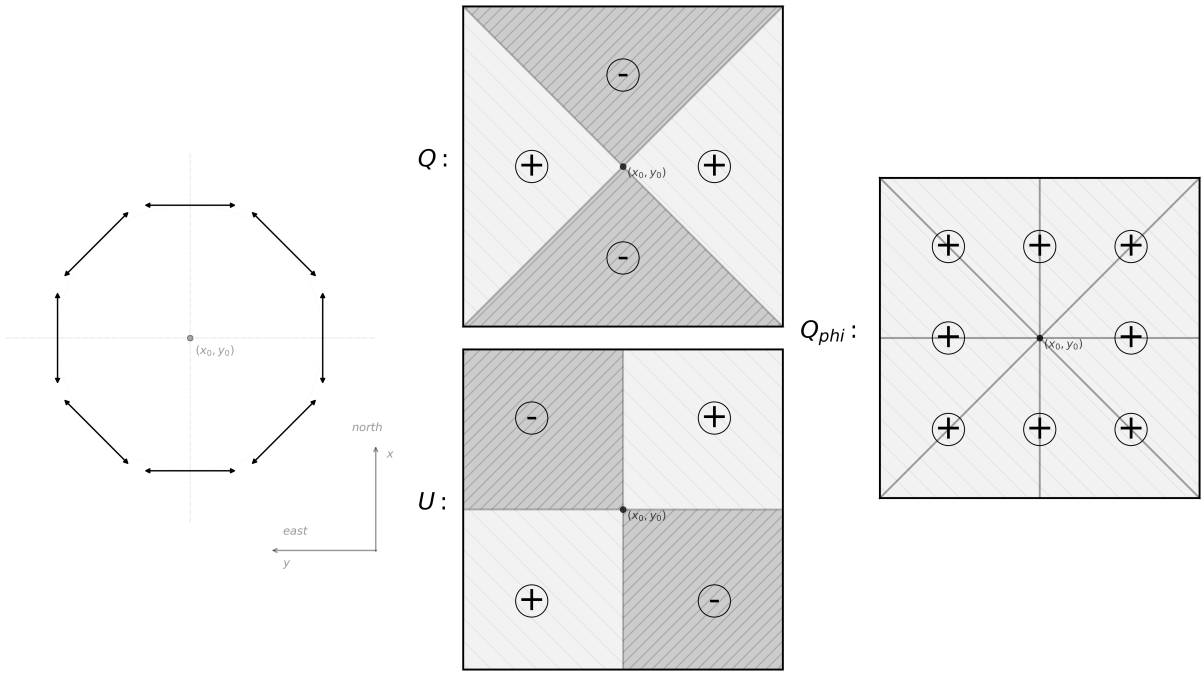


(a) Overview of the Q_{phi} idea.

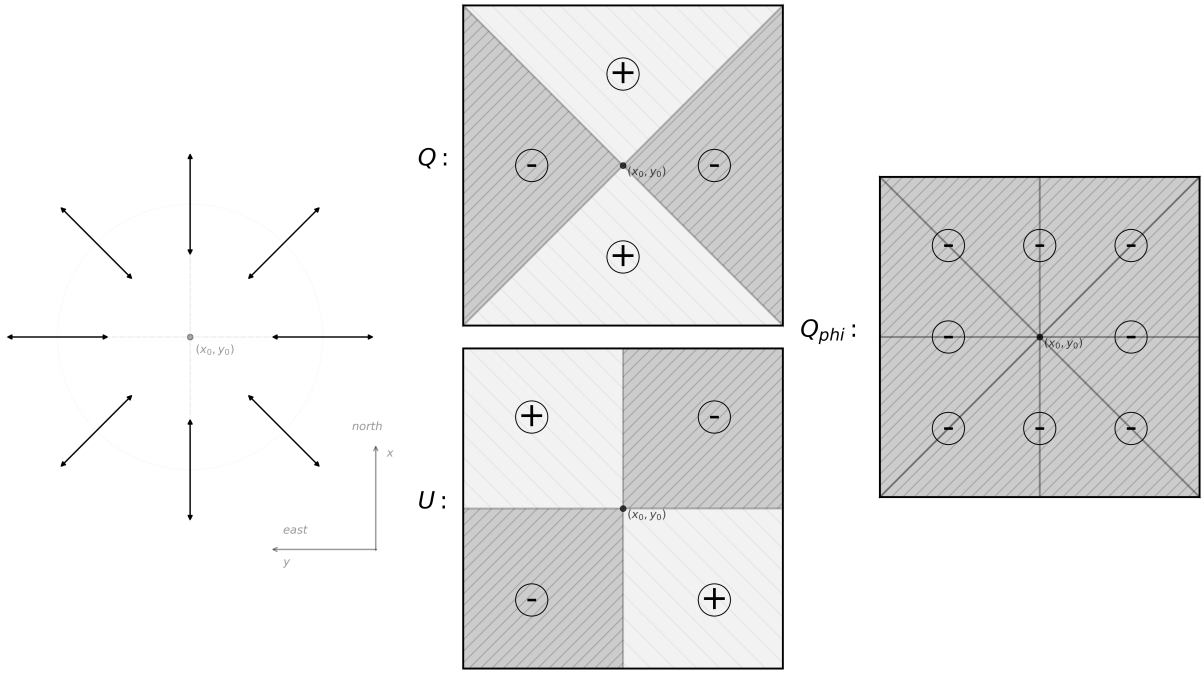


(b) Overview of the U_{phi} idea.

Fig. G.3: How to understand the values of radial Stokes parameter Q_{phi} and U_{phi}.

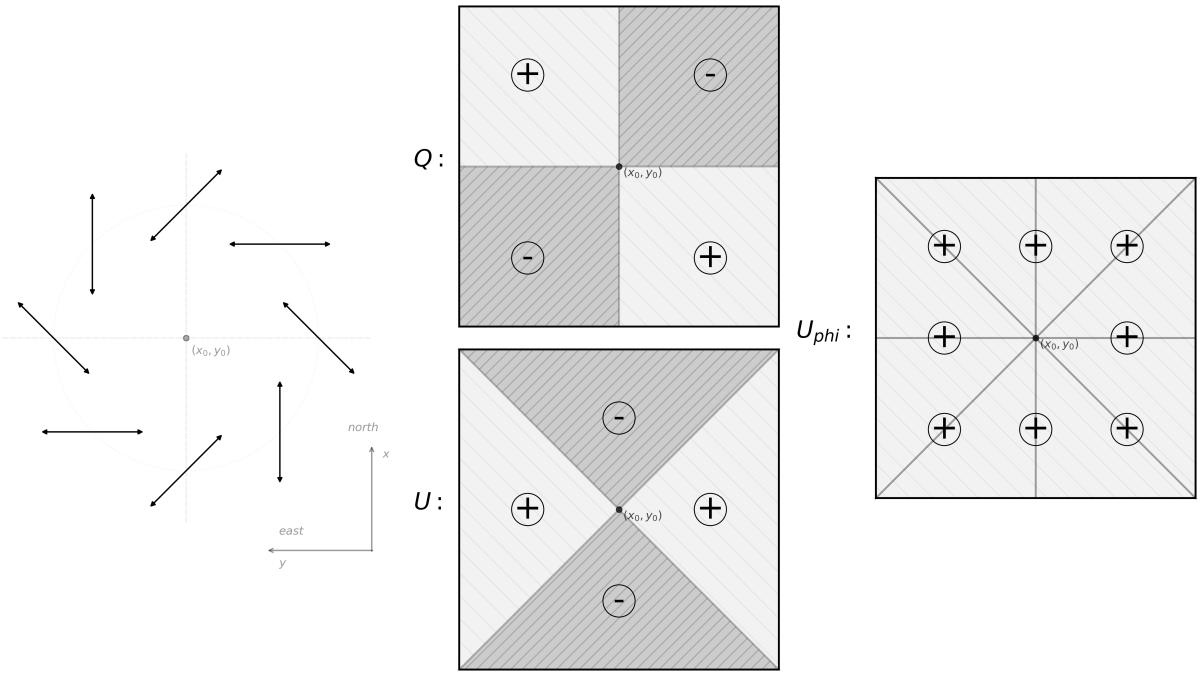


(a) Azimuthal polarization and the resulting Q, U and Q_{phi} frames.

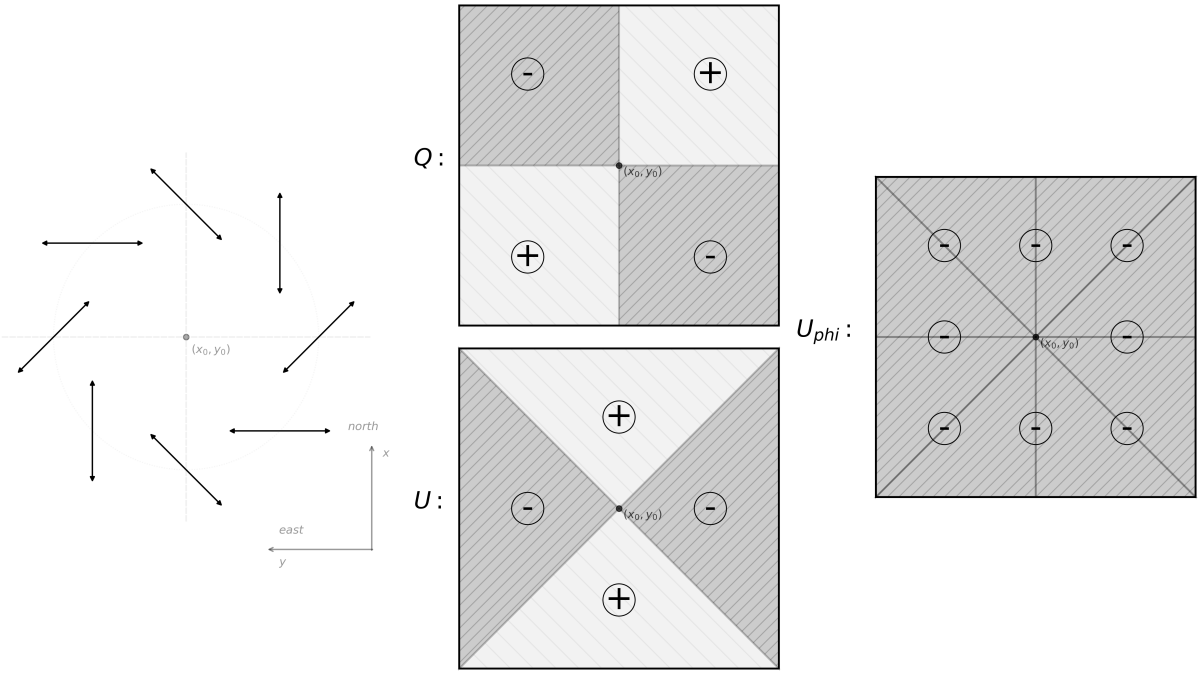


(b) Radial polarization and the resulting Q, U and Q_{phi} frames.

Fig. G.4: Radial and azimuthal polarization and the resulting Q, U and Q_{phi} frames.



(a) -45 degree polarization compared to azimuthal polarization.



(b) +45 degree polarization compared to azimuthal polarization.

Fig. G.5: $+/-45$ deg polarization compared to azimuthal and the resulting Q, U and U_{phi} frames.